Are Neutrinos different ?
Are neutrinos different?

- Masses are very small (one could even vanish); we only know the differences of their squares.
- "Cabibbo" mixing is important, might even be more complicated (extra phases if Majorana, mixing with steriles)
- We don’t even know the number of degrees of freedom (Majorana vs Dirac)
- They violate the separate conservation of electron, muon and tau numbers

Facts

Conjectures

- They might violate the global lepton number (neutrinoless double beta)
- they could explain the Defeat of Antimatter (leptogenesis)
- They suggest (via See-Saw or other) the presence of new particles, new scales, and could even accommodate extra dimensions
They pester us with re-learning about Dirac, Majorana, degrees of freedom, oscillations, ... 

while the rest of the fermions seem so simple by comparison!
Can we tell 4 from 2?

Outline

Part 1: masses
- Basic notations, spinors, vectors, Dirac equation
- Who thought neutrinos should be massless?
- Neutrino masses: Majorana, Dirac ...
- Can we tell 4 from 2 components?
  - Neutrino-Antineutrino oscillations ??? (no!)
  - Cosmology?
  - Magnetic moments? - a new inequality
Sorry if this seems too basic, but ... we will very soon be in some tricky concepts, so better to establish the basis!

In classical physics (3D for now), we know of the scalars, pseudoscalars, vectors, tensors, (and axial vectors)

Just to fix the ideas:

- A scalar is something invariant under the rotations; (e.g. : the temperature in this room)
- A vector is characterized by a direction : for instance, the speed \( \mathbf{v} \) of a particle. (3 components) and transforms as such under rotations
- A pseudovector is similar to a vector, but the pseudovector does not change sign under a mirror reflexion (e.g. an angular momentum : \( \mathbf{J} = \mathbf{r} \times \mathbf{v} \)) while both \( \mathbf{r} \) and \( \mathbf{v} \) flip sign. (technically, it is an antisymmetric tensor, but with components reorganized in a pseudo-vector)
- A pseudoscalar is similar to a scalar, but it does flip under a mirror reflexion. (e.g. the “scalar” product \( \mathbf{v} \cdot \mathbf{J} \) is in fact a pseudoscalar while \( \mathbf{v} \cdot \mathbf{v} \) is a true scalar.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>Rotations</th>
<th>Mirror reflexion (Parity P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar</td>
<td>Invariant</td>
<td>+</td>
</tr>
<tr>
<td>PseudoScalar</td>
<td>Invariant</td>
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<tr>
<td>Vector</td>
<td>Transforms as vector</td>
<td>-</td>
</tr>
<tr>
<td>Pseudo Vector</td>
<td>Transforms as vector</td>
<td>+</td>
</tr>
</tbody>
</table>
Relativistic mechanics

A Massive scalar: its Energy and Momentum

\[ \vec{p} \text{ also noted } \mathbf{p} \]

\[ p^\mu = (p^0, \vec{p}) \]

\[ p^0 = E/c \]

Einstein’s famous equation \( E = m \ c^2 \) is actually written:

\[ p^\mu p_\mu = p^2 = (p^0)^2 - \vec{p}^2 = m^2 \ c^2 \]

From now on, set \( c = 1 \) (choice of unit speed), and get

\[ p^0 = \pm \sqrt{\vec{p}^2 + m^2} \equiv \pm \omega \]
Massive particles

A scalar, massive particle kinematics is just described by its 4-momentum (Energy + 3- momentum)
Idem for a pseudoscalar: these have “no spin”

A Vector massive particle is more complicated. If we go to the rest frame, (p=0) we must still specify in which
direction points the “spin” (i.e. : even “at rest” the particle carries angular momentum). The “spin” is a 3-(pseudo)vector
which can point in any of the 3 directions. We say that the particle has 3 degrees of freedom. (same would be true for
a pseudo-vector). It carries one unit of quantum spin If we project the spin in any direction, it can have eigenvalues
(+1, 0, -1)

A spinor (think of the electron as usually described in chemistry) also has spin, but only a half-unit.
When described naïvely in chemistry, the electron spin is either “up” or “down”, which means only 2 degrees
of freedom. This is a strange situation (spin projection on an arbitrary axis has eigenvalues (+1/2 , -1/2)
Massless particles

For a massless scalar, nothing changes: just the 4-momentum, but with
\[ p^0 = \pm \sqrt{\vec{p}^2} \]
since \( \sqrt{\frac{\vec{p}}{c} = \frac{\vec{p}}{E}} \) it must however move at the speed of light \( m = 0 \Rightarrow |v| = c \)

A massless Vector is more complicated to describe. Remember that for a massive one, we went to the rest frame, \((p=0)\) but this is impossible here! We cannot define the spin direction that way.
Instead, we introduce “helicity”: the spin in the direction of motion. But such helicity is only “forward” or "backward": only 2 states (degrees of freedom), with \( h = (-1, +1) \). Often, we will call this left-handed or right-handed polarization. Remember that the photon indeed has only 2 degrees of polarization.
Remarkd: they are directly related, but don’t confuse the spin (here forward or backward) with the direction of polarization of the electric field (here purely transverse)

A massless spinor is also moving at the speed of light: can’t go into the rest frame. We describe the helicity as the spin in the direction of motion, just like for photons, but since we have only \( \frac{1}{2} \) units of spin, it has eigenvalues \( (+1/2, -1/2) \) (2 degrees of freedom, unchanged).
This is the “minimal block”, called a Weyl spinor, and it describes a massless fermion (in fact, a “left-handed” one)

\[(p^0 + \vec{p} \cdot \vec{\sigma}) \psi_L = 0\]

\[h = \frac{\vec{p}}{|\vec{p}|} \cdot \vec{s}\]

\[\left( \frac{\vec{p}}{|\vec{p}|} \cdot \vec{\sigma} \right) \psi_L = -\frac{p^0}{|\vec{p}|} \psi_L\]

\[(\vec{s} \cdot \vec{p}) \psi_L = -1/2 \text{ sign}(p^0) \psi_L\]

\[\psi = \left( \begin{array}{c} \psi_1 \\ \psi_2 \end{array} \right)\]

\[p_0 < 0\]

\[p_0 > 0\]
For a spinor moving in “z” directions, we have 2 solutions:

spin (in z direction) = -1/2 , and energy >0
Spin (in z direction) = +1/2 and energy <0 .... I want to re-interpret this:

Instead of “creating a particle of -E, s, and p”
I speak of “destroying an antiparticle of E, -s, -p
For a spinor moving in “z” directions, we have 2 solutions:

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\[ \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]

- \[ p_0 < 0 \] and energy >0
- \[ p_0 > 0 \] and energy <0

I want to re-interpret this:

Instead of “creating a particle of -E, s, and p”
I speak of “destroying an antiparticle of E, -s, -p”
In summary, this 2-component spinor (Left-Handed) represents 2 possible propagations.

A particle, with helicity $-1/2$ (left-handed)
An antiparticle, with helicity $+1/2$ (right-handed)

Neutrinos are not very intuitive, so let us turn shortly to electrons to illustrate the process:
Describes 2 things: the destruction of a L-handed electron and the creation of a R-handed positron.

We can choose to use the electron or the positron for our description. These 2 are CP conjugates (not C!).

\[
\begin{align*}
\left(\overline{e^-}_L\right) &= \left(\overline{e^-}\right)_R = e^+_R
\end{align*}
\]

But... this is not sufficient to represent the full electron: we miss the R-handed electron and the L handed positron!
We must then introduce a 2\textsuperscript{nd} 2-component spinor, and are lead to the 4-component Dirac spinor

\[
\begin{pmatrix}
e_L \\
e_R
\end{pmatrix}
= \begin{pmatrix}
e_{L1} \\
e_{L2} \\
e_{R1} \\
e_{R2}
\end{pmatrix}
\]

The Dirac spinor breaks down into 2 « Weyl » spinors of opposite « chirality » (L/R)

Gauge interactions talk separately to the L (left-handed) and R (right-handed)
In the Standard Model, think of the Z, photon, but also the W boson.
Describes 2 things: the destruction of a L-handed electron and the creation of a R-handed positron.

We can choose to use the electron or the positron for our description. These 2 are CP conjugates (not C!).

But \( e_L \) does not describe the other 2 states..

\[
\begin{align*}
\left( e_L^- \right) & \equiv \left( e^- \right)_R \equiv e_R^+ \\
\left( e_R^+ \right) & \equiv \left( e^+ \right)_L
\end{align*}
\]
Should neutrinos have been massless for the Standard Model?

**NO!**

*Once upon a time (has it completely ended? ) people used to blame P violation on the absence of right-handed neutrinos ...*

P violation was clearly demonstrated in the Wu experiment..

It is easy to explain if only left-handed electrons are produced in a charged vector current.

*Killing the right-handed neutrino allows for parity violation in charged currents, even if the coupling is pure vector*
Killing the right-handed neutrino allows for parity violation in charged currents, even if the coupling is pure vector.

This was NEVER a solution ... Assuming the whole world to be symmetrical under P, and taking the right-handed neutrino as the BAD GUY was NO SOLUTION.

- Not a solution today: we know the Standard Model has neutral currents which violate P (parity violation in atoms, asymmetrical couplings of Z to quarks).

- Even at the time of Wu’s experiment, it was not a solution ... this experiment was only a confirmation, a demonstration of P violation, known from the $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ (the $\Theta \tau$ puzzle) where neutrinos don’t play!

Still, in a way the doublet $(\nu_L, e_L)$ was at the basis of the Standard Model, but the actual symmetry was experimentally found to be $SU(2)_L$, acting on left-handed spinors applied to all known fermions, including quarks.
The simplest building block of gauge interactions only introduces the left-handed Weyl spinor, C and P are violated, but CP is conserved: \textit{this is THE symmetry of gauge interactions},

The natural symmetry of Gauge interactions is CP, not P or C.
Spinors, Examples of masses

To represent a 2-component spinor, we use a 1 column matrix, each element a complex number (or rather, quantum field):

\[ \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]

To write a mass term in a Lagrangian:

The Lagrangian is scalar, the mass itself is scalar, we must thus arrange the spinors to form spin 0.

The solution is easy \( \psi \psi \) with a configuration \( \psi_1 \psi_2 \) or \( \psi_2 \psi_1 \). To get a more symmetrical result, we might be tempted to write \( \psi_1 \psi_2 + \psi_2 \psi_1 \) but this does not work!

Indeed, remember that the Pauli principle requests that 2 fermions (neutrinos are fermions) cannot be in the same state: formulated differently, this means that fermions anticommute (hence, we cannot have pure “numbers”):

\[ \psi_1 \psi_2 = - \psi_2 \psi_1 \]

and thus \( \psi_1 \psi_2 + \psi_2 \psi_1 \) vanishes!

The answer is then \( \psi_1 \psi_2 - \psi_2 \psi_1 \) the antisymmetric combination, and indeed \( m(\psi_1 \psi_2 - \psi_2 \psi_1) \) is precisely the mass term we are looking for. **Later we will call this a “Majorana mass term”:**

**it is in fact just the “obvious” mass term for 2-component fermions.**

\[ m \left( \begin{pmatrix} \uparrow & \downarrow \\ \downarrow & \uparrow \end{pmatrix} - \begin{pmatrix} \downarrow & \uparrow \\ \uparrow & \downarrow \end{pmatrix} \right) \]

In a more elegant form, for the experts:

\[ \epsilon_{ij} \psi_i \psi_j \]
How can we write a mass term – (continued)?

A « mass » term must be invariant under proper Lorentz transformations (but we don’t impose P or C, which are broken in the SM.

Equations of motion must lead to

\[ p^2 = |m|^2 \]

We now introduce here 2 spinors, We assume both to be L, (if not, perform a CP transformation)

\[
\begin{pmatrix}
\psi_L \\
O
\end{pmatrix}
= 
\begin{pmatrix}
\psi_{L1} \\
\psi_{L2} \\
O
\end{pmatrix}
\]

; 

\[ \xi_L \]

\[
\begin{pmatrix}
\psi_{L1} \xi_{L2} - \psi_{L2} \xi_{L1} = \epsilon_{ij} \psi_{Li} \xi_{Lj}
\end{pmatrix}
\]

This expression covers ALL cases! (Majorana+Dirac)
\[ \psi_{L1} \xi_{L2} - \psi_{L2} \xi_{L1} = \epsilon_{ij} \psi_{Li} \xi_{Lj} \]

2 special cases:

- Creates (or destroys) 2 units of fermionic number: « Majorana mass term »

- « Dirac mass term »

If we can assign the same fermionic number to \( \eta \) and \( \xi \), Fermion number is now conserved.
For the electron, only the « Dirac » mass term is allowed – the « Majorana » one does not even conserve electric charge!

\[\begin{align*}
&\begin{array}{c}
\hat{e}_L \\
\end{array} \quad \begin{array}{c}
e^- \\
\end{array} \\
\end{align*}\]

\[\begin{align*}
&\begin{array}{c}
e^- \\
\end{array} \quad \begin{array}{c}
\hat{e}_L \\
\end{array} \\
\end{align*}\]

On the other hand, for the neutrino, charge is not a problem, and we can use the « Majorana » mass. It violates leptonic number, but if the mass is small enough, this escapes detection.

It is thus possible to have Neutrino masses without introducing the right-handed neutrino (but NOTHING forbids to have a right-handed neutrino and a standard Dirac mass)
A number of confusions
From the «absence of $\nu_R$» to «massless neutrinos»

In some frequently repeated folklore, the «absence of $\nu_R$» was supposed to account for P violation (WRONG) supposed meant that «ordinary» (Dirac) masses were excluded ...

This fitted well the fact that very small neutrino masses (at least for the electron neutrino) were requested from $\beta$ decay kinematics, and for a long time, experiments remained compatible with zero neutrino mass

...and this lead to the WRONG legend that neutrinos had to be massless in the Standard Model

In fact, masses were simply omitted in the first version for simplicity (and this first version also lacked quarks, families, CP violation..)

But .. Evidence for neutrino masses!
The sign (or phase) of the mass.

The parameter $m$ in the Lagrangian is in general a complex number.

In the case of one family, in the Dirac case, we can always re-define $m$ to be real,

just by changing the sign of $\eta_R$, which does not couple to anyone.
Neutrinoless double beta decay

Is currently the main experimental signature explored for Majorana masses.

It hinges on the non-conservation of the lepton number

Indeed, a nuclear decay (L=0) produces 2 electrons (L=2)
The sign of the fermion mass – Majorana case

Here, we cannot re-define the sign of the mass without affecting the interactions ... *we can bring $m$ to be real* by re-defining $\xi \rightarrow i \xi$

But in any case, the sign of the amplitude remains

Neutrinoless Double Beta decay is sensitive to the weighted sum of masses, including Majorana phases.
For experts

**Advanced remark**: Special case: for one flavor, Dirac can be seen as 2 semi-spinors with equal but opposite masses and equal couplings

For later use: if the cancellation occurs not in one family, but across families « Pseudo-Dirac »

When several types of neutrinos contribute the signs of the masses count, and can lead to cancellations *(of course, M’s are complex numbers in general)*

\[
\sum \left( \frac{M_i}{g^2 - m_i^2} \cdot g^2 \cdot V_{ei}^2 \right)
\]
This far we spoke of Weyl neutrino, Majorana mass terms, but not of Majorana spinors...

In fact, they are not needed in 3+1 dim ... just another (confusing but convenient) notation

Let us see 2 ways in which we can write 2 components spinors in a 4-component mechanism

\[ \psi = \begin{pmatrix} \lambda \\ \rho \end{pmatrix} \]

Since all other fermions are represented in Dirac spinors, on which act Dirac matrices, it is not convenient to keep 2-component Weyl spinors

If we want to write a 2-components spinor as a 4-components one, there is one obvious way: projection. L is a projector, we have \( L \cdot L = L, L \cdot R = 0, R \cdot R = R, L + R = 1 \)

\[
\begin{pmatrix} e_L \\ e_R \end{pmatrix} \quad \Rightarrow \quad L \begin{pmatrix} e_L \\ e_R \end{pmatrix} = \begin{pmatrix} e_L \\ 0 \end{pmatrix}
\]

This projection is the simplest way, and adapted when we don’t have Majorana masses
Majorana spinors...

Another way to write 2-component spinors as 4-component: this time, we use a **REDUNDANT NOTATION**

\[ \psi = \begin{pmatrix} \lambda \\ \rho \end{pmatrix} \]

Majorana or Weyl spinors?

In 4-D: logically equivalent, **Majorana is just a REDUNDANT way to write Weyl spinors**

We keep the bottom term, but express it as a “repetition” of the top one

\[ \psi^c = \psi \]

Expect factors of $\frac{1}{2}$ in the equations!

\[ \lambda_i = \epsilon_{ij} \rho^t \]

**NOTE**: Majorana – Weyl equivalence does not hold in $> 3+1$ dimensions in general
Exercise (for the experts): A Dirac spinor can indeed be seen as the sum of 2 Majorana spinors of equal and opposite masses.

\[ m \bar{\psi} \psi \]

\[ \chi = \frac{1}{\sqrt{2}} (\psi + \psi^c) \]

\[ \lambda = \frac{1}{\sqrt{2}} (\psi - \psi^c) \]

\[ \frac{m}{2} \bar{\chi}^c \chi - \frac{m}{2} \bar{\lambda}^c \lambda \]
Back to some “experimental” considerations.

One big question now is to know if we have Dirac or Majorana masses

(in other terms: **total** lepton number conserved or not)
Beyond the Neutrinoless Double beta decay, Can we probe the Majorana nature of neutrino masses?

Could we have neutrino-antineutrino oscillations?

In principle, Yes, but in practice, NO
Even though the lepton number is not conserved, **angular momentum suppresses this reaction**

The $\nu_L$ stays linked to $e^-_L$, and not to $e^+_R$ by the W’s in the SM (remember: P violation is in the gauge interactions, NOT in the presence or absence of right-handed neutrino)

As long as the detector and emitter don’t have large relative speeds (in comparison to the neutrino), helicity is conserved up to factor of m/E in amplitude. Even for 1MeV neutrinos, this gives a suppression of $10^{-12}$ in probability
Could the cosmological counting of neutrinos help us?

Could cosmological neutrino counting help?

Planck: $N_{\text{eff}} = 3.15 \pm 0.23$


... 2 or 4-components? ... not sensitive!
Magnetic moments?

For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics..

Is it a way to exclude Majorana masses?

↑↑
Magnetic moments?

For ONE Weyl neutrino, a magnetic moment is forbidden by Fermi statistics ..

Is it a way to exclude Majorana masses?

BUT, TRANSITION magnetic moments are still allowed ...

and undistinguishable!

(since we do NOT detect the out-going neutrino!)
\[ H_{\text{eff}} = \frac{\mu_{IJ}}{2} \bar{\nu}_c^i \sigma_{\alpha \beta} P_L \nu_J F^{\alpha \beta} + \text{h.c.}, \]

\[ \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau \mu}|^2} \left( \bar{\nu}_X^c \sigma_{\alpha \beta} \nu_\mu F^{\alpha \beta} \right), \]

\[ \bar{\nu}_X^c \equiv \frac{(\mu_{e\mu} \bar{\nu}_e^c + \mu_{\tau \mu} \bar{\nu}_\tau^c)}{\sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau \mu}|^2}}. \]
Effective electromagnetic moment for the muon neutrino
In WEYL (Majorana) case: it is simulated by the combination
of the transition moments.

\[ |\mu_{\nu\mu}| \equiv \sqrt{ |\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2 } \]
Effective electromagnetic moment for the muon neutrino:

\[ |\mu_{\nu_\mu}| \equiv \sqrt{|\mu_{e\mu}|^2 + |\mu_{\tau\mu}|^2} \]

Figure 1: \( |\mu_{\nu_J}| \) forms a right triangle with \( |\mu_{I,J}| \) and \( |\mu_{K,I}| \) (for \( I \neq J \neq K \)). \( |\mu_{\nu_{1,J,K}}| \) thus also form a triangle (shown in thick blue), in general not with right angles.

JMF, J Heeck, S Mollet  arXiv:1506.02964  
It is then easy to work out the inequalities ..

\[
|\mu_{\nu_\tau}|^2 \leq |\mu_{\nu_e}|^2 + |\mu_{\nu_\mu}|^2,
|\mu_{\nu_\mu}|^2 \leq |\mu_{\nu_\tau}|^2 + |\mu_{\nu_e}|^2,
|\mu_{\nu_e}|^2 \leq |\mu_{\nu_\mu}|^2 + |\mu_{\nu_\tau}|^2,
\]

These are stronger than the more obvious « triangle inequalities »:
(none of the angles can be \(> 90^\circ\))
\[
||\mu_{\nu_\tau}|-|\mu_{\nu_K}|| \leq |\mu_{\nu_\tau}| \leq |\mu_{\nu_\tau}| + |\mu_{\nu_K}|
\]

Current limits (terrestrial)
\[
|\mu_{\nu_e}| < 2.9 \times 10^{-11} \mu_B, \quad |\mu_{\nu_\mu}| < 6.8 \times 10^{-10} \mu_B, \quad |\mu_{\nu_\tau}| < 3.9 \times 10^{-7} \mu_B.
\]

Perspectives : SHiP (CERN SPS) could improve considerably the \(\tau\) neutrino limit ..
Current limits (terrestrial)

\[ |\mu_{\nu_e}| < 2.9 \times 10^{-11} \mu_B, \quad |\mu_{\nu_\mu}| < 6.8 \times 10^{-10} \mu_B, \quad |\mu_{\nu_\tau}| < 3.9 \times 10^{-7} \mu_B. \]

Current limits (astrophysics – in fact sum over all neutrinos)

\[ 4.5 \times 10^{-12} \mu_B \]

Hopeless for terrestrial measurements?

NO ...

if there is a 4th light (sterile) neutrino, with mass \( > \) keV,

astro limits don’t apply

and a large electromagnetic moment could be observed ... SHiP is in business!

(by the way, light extra neutrinos are considered as components of Dark Matter...)

JM Frère Neutrino School Quy Nhon, 2019
Update ....

Borexino brings interesting new bounds (from « oscillated » Solar neutrinos)

Using these numbers, we have (if we saturate the bounds) 31.36 > 16 + 9.61 .... is there hope to improve and get an actual check at the 10^{-11} level?
Outline (2/3)

Part 2 : oscillations – the basics -- R neutrinos

• 2 families oscillations in vacuum
• Oscillations - the polarized light analogy (demonstration)
• Neutrinos in matter
• Mass mechanisms :
  • The simplest (and boring?) SM Dirac masses
  • The immensely popular See-Saw
    • Connection with “ The Defeat of Antimatter”
    • What about scalar triplets and pure Majorana masses?
• R neutrinos put to use : leptogenesis – falsifiable by light $W_R$
• Right-handed neutrinos: which scale ?
• R – neutrinos for your accelerator builder friend?
• Observing heavy R neutrinos (and their decay) would make Dirac-Majorana distinction easy
To discuss neutrino oscillations: we will assume that “neutrinos have mass”, and not worry for now on the mechanism of mass generation, or the Majorana/Dirac nature (we will return to this later).

For 3 neutrino families, we can consider 2 basis, describing the same particles. This is just like the situation for quarks!

- The mass basis (which we would use for particles at rest, or, equivalently for particles in free propagation)
  \[ (\nu_1 \nu_2 \nu_3) \]
- The “current” basis, where the gauge interaction is diagonal
  \[ (\nu_e \nu_\mu \nu_\tau) \]

Those bases are related by a unitary matrix:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= V
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]
For instance, in pion decay

\[ \pi^- \rightarrow \mu^- + \nu + \bar{\nu} \]

"current" eigenstate

\[ V_\mu = V_2 \cdot j \cdot j \]

mass eigenstate
Now, when they propagate, the most convenient basis is the **mass basis**

Each massive neutrino moves at its own speed, determined by its mass.

\[ E = \sqrt{\vec{p}^2 + m^2} \quad \frac{\vec{v}}{c} = \frac{\vec{p}}{E} \]

\[ \nu_1(t) = \nu_1(t = 0) \quad e^{-i\left( E_1 t - \vec{p}_1 \cdot \vec{x} \right)} \]

For more details (and wave-packet formulation) see:

Neutrino physics

P. Hernandez : C09-03-15 Proceedings

e-Print: arXiv:1010.4131 PDF
If we only look at 2 neutrinos, the mixing matrix elements are simply sin and cos

\[ |\nu_2\rangle = \cos \theta |\nu_1\rangle \rightarrow e + \sin \theta |\nu_2\rangle \rightarrow e \]

if \( \nu_2 \neq \nu_1 \), emerging state is no longer \( \nu_\mu \).
\[ \nu(t) = e^{i \theta \nu_1} \left( \cos \theta \nu_1 + e^{i \theta} \sin \theta \nu_2 \right) \]

If \( \nu_2 \neq \nu_1 \), emerging state is no longer \( \nu_\mu \)
\[ \nu_\mu = \cos \theta \nu_\nu + \nu_e \]

If \( \nu_2 \neq \nu_1 \), emerging state is no longer \( \nu_\mu \)

\[ \text{here, } \Delta \text{ phase } = \pi \\
\text{"half wave"} \]

\[ i E(\nu, -\nu_2) L = (\cdot) \]

and \( \theta = \frac{\pi}{4} \)
Suggested do-it-yourself demo:
Use anisotropic medium between crossed polarizers ...

\[ \nu_e \]

\[ \nu_\mu \]

\[ i E_1 (\nu_e - \nu_\mu) L = (-1) \]

\[ \text{(here, } \Delta \text{ plane } = \pi \]

"half wave"

C2

C1
Suggested do-it-yourself demo:
Use anisotropic medium between crossed polarizers ...
Why would be the propagation speed of neutrinos 1 and 2 differ?

It could be MASS,

\[ E^2 = \overrightarrow{p}^2 + m^2 \]
\[ v = \frac{\overrightarrow{p}}{E} \]
\[ v = \sqrt{1 - (m/E)^2} \]
\[ (v_1 - v_2) \ L = \frac{(m_2^2 - m_1^2) \ L}{2E} \]

The effect is the same for neutrinos and antineutrinos, does not depend on the type of mass (Majorana or Dirac)

But also any kind of interaction affecting differently 1 and 2
Well-known example: MSW effect
In the case of mass, we get simply (for 2 neutrino oscillations)

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right), \quad \alpha \neq \beta.
\]

These formulas are standard ... see for instance P. Hernandez reference
Why would be the propagation speed of neutrinos 1 and 2 differ?

It could be MASS,

\[ E^2 = \overrightarrow{p}^2 + m^2 \]
\[ v = \frac{\overrightarrow{p}}{E} \]
\[ v = \sqrt{1 - \left(\frac{m}{E}\right)^2} \]
\[ (v_1 - v_2) L = \frac{(m_2^2 - m_1^2) L}{2E} \]

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Well-known example: MSW effect

\[
\text{Lagrangian} \supset \overline{\psi_{\nu_e}} \left( p^0 \gamma^0 - \overrightarrow{p} \gamma \right) \psi_{\nu_e} - V
\]

\[
V \supset \kappa G_F \overline{\psi_{\nu_e}} \gamma^\mu \psi_e \gamma_\mu \psi_{\nu_e}
\]

After Fierzing,

\[
\kappa G_F \overline{\psi_{\nu_e}} \gamma^\mu \psi_e \gamma_\mu \psi_{\nu_e} = \kappa' G_F \overline{\psi_{\nu_e}} \gamma^\mu L \psi_{\nu_e} \psi_e \gamma_\mu L \psi_e = \kappa'' \overline{\psi_{\nu_e}} \gamma^0 L \psi_{\nu_e} G_F \rho_e
\]

This means that we simply get an effective mass \( \ldots \)
which differs for neutrino and antineutrino (CPT violation \ldots
expected: we interact with MATTER and not anitmatter \()
Mass models
Neutrinos masses in the Standard Model .. And a bit beyond...

The simplest...
Just treat them like other fermions,

**DIRAC mass for neutrinos**
Introduce $\nu_R$ and a Yukawa coupling $\lambda$.

$$\lambda \, \bar{\nu}_R \, \phi^+ \left( \nu_L \right) \, e_L \, \, h.c.$$  

$$\lambda \, < \, m_\nu \, / \, m_W \, < \, 10^{-11}$$

A bit inelegant, but there are other large/small Yukawa ratios in the SM (top/ electron = 3 $10^5$)

In this context, the $\nu_R$ is all but unobservable, as its sole role is in giving mass.
We can also try to do without the $\nu_R$, and use a Majorana mass for the sole $\nu_L$

$$
\begin{array}{c}
\text{SU(2)} \\
m & \nu \\
\vdots & \vdots \\
\end{array}
\Rightarrow
\begin{array}{c}
m & \nu & \nu \\
\vdots & \vdots & \vdots \\
\end{array}

-- But such a term breaks SU(2) invariance, and

we would need a scalar triplet, with a vev through spontaneous symmetry breaking.
we would need a scalar triplet, with a vev through spontaneous symmetry breaking.

Possible, if the vev of D is << than the vev of F... (otherwise, W/Z mass ratio affected)
Such a breaking $V_L$ would upset the mass ratio $W/Z$.

But is acceptable if small enough, for instance ..

\[
<\Delta^0> = v_L < \sqrt{100} \quad \frac{g\nu}{2} = m_W
\]

This solution is not more costly in terms of « degrees of freedom » than the introduction of right – handed neutrinos, ... it deserves study at the LHC.

Searches at LHC ?
(does not couple to quarks, but couples to leptons, and could be very heavy)
If we really want Majorana Masses, and don’t want a triplet ...

We still introduce a right-handed neutrino (like in the Dirac case), but remark that, since the right-handed “neutrino” is a SINGLET for $\text{SU}(2) \times \text{U}(1)$, it can have a (large) Majorana mass.

This particle is simply NOT coupled to the Standard model!
We can build an « effective triplet » from the Standard Model doublet, and, right-handed neutrinos ..

After diagonalization, 2 Weyl spinors

\[ \text{SU(2) imposes} \]

\[ M_1 = 0 \]

For \( m = \lambda \nu \ll M_2 = M \) we get

\[ |m_1| \approx \frac{m^2}{M} \]

\[ |m_2| \approx M \]

\[ \lambda_1 \approx \xi_L - \frac{m}{M} \epsilon \cdot \eta_R^+ \]

\[ \lambda_2 \approx \eta_R + \frac{m}{M} \epsilon \cdot \xi_L^+ \]
\[ \lambda_1 \approx \xi_L - m/M \epsilon \cdot \eta_R^+ \]
\[ \lambda_2 \approx \eta_R + m/M \epsilon \cdot \xi_L^+ \]

We end up with something close to a low Majorana mass left-handed neutrino, In principle, such schemes could be differentiated from the triplet by the small admixture of the R mode, which leads to a departure from unitarity in the mixing matrix. However such effects are of order m/M and thus unobservable.

Some models may make this presence detectable, they tend however to be quite artificial ... for instance:
What are Right-handed neutrinos good for?

Heavy $\nu_R$ (= N) are found in grand unified theories like SO(10) and above, but are specially useful for inducing the defeat of antimatter.

CP violating decay creates L<0, converted into B>0 by an anomaly-related mechanism (instantons).
Assume that we have some population of heavy $N$ particles... (either initial thermal population, or re-created after inflation); due to their heavy mass and relatively small coupling, $N$ become easily relic particles.

**Generation of lepton number**

$N_1 \rightarrow L \phi$

$N_1 \rightarrow \bar{L} \phi$

$N_1 \rightarrow L \phi$

$N_1 \rightarrow \bar{L} \phi$

CP violation + Interference term leads to excess of $L$ or anti-$L$
Constraints:

Heavy neutrinos must decay out of equilibrium

\[ \tau(X) \gg H^{-1} \]

\[ H = \dot{a}/a \] is the Hubble constant,

\[ \tau^{-1} = \Gamma \approx g^2 M \]

\[ H = \sqrt{g^*} \frac{T^2}{10^{19} \text{GeV}} \]

\( g^* \) is the number of degrees of freedom at the time

at decay: \( T \approx M \),

Need enough CP violation;
for large splitting between neutrino masses, get

\[ \mathcal{E}_i^\vartheta = -\frac{3}{16\pi} \frac{1}{[\bar{\lambda}_\nu \lambda_\nu^\dagger]_{ii}} \sum_{j \neq i} \text{Im} \left( [\lambda_\nu \lambda_\nu^\dagger]_{ij} \right)^2 \frac{M_i}{M_j}. \]
Some rough estimations…

…What are the suitable values of $\lambda$ and $M$?
Assume there is only one generic value of $\lambda$ (in reality, a matrix)

\[ \epsilon < \chi^4/\lambda^2 \approx \lambda^2 > 10^{-8} \]

\[ m_\nu = m^2/M \approx \lambda^2/M \approx 0.01\text{eV} \]

rough estimate of $M$ scale (in GeV) needed…

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>light neutrino $\approx 0.01\text{eV}$</th>
<th>decay out of equil. $M &gt;$</th>
<th>enough CP viol</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^7$</td>
<td>$10^8$</td>
<td>need tuning</td>
</tr>
<tr>
<td>0.0001</td>
<td>$10^9$</td>
<td>$10^{10}$</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>$10^{11}$</td>
<td>$10^{12}$</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>$10^{13}$</td>
<td>$10^{14}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$10^{17}$</td>
<td>$10^{18}$</td>
<td>large</td>
</tr>
</tbody>
</table>

At the difference of baryogenesis, the Yukawa matrix $\lambda$ leaves a lot of freedom

similar to $\tau$ lepton
Can leptogenesis be falsified?

In general, no, since most mass ranges are unaccessible. But .. Presence of $\nu_R$ suggest a larger symmetry, like SO(10) or SU(2)$_L \times$ SU(2)$_R$

with the gauge inclusion

\[
\epsilon_1 = \frac{\epsilon_1^0}{1 + X}
\]

\[
\begin{align*}
M_{W_R} &< M_{N_1} \\
M_{W_R} &> M_{N_1}
\end{align*}
\]

(competing effect: the presence of $W_R$ allows a faster build-up of the N population after inflation)

S Carlier, JMF, FS Ling *Phys.Rev. D60* (1999) 096003
JMF, T Hambye, G Vertongen *JHEP 0901* (2009) 051
Bounds on $m(W_R)$ & $m(N_R)$

For $\varepsilon_{CP} = 1$

For $\varepsilon_{CP} = \varepsilon_{DI}$

$m(W_R) \geq 18 \text{ TeV}$

$m(W_R) \geq 10^{11} \text{ TeV}$

See T Hambye’s talk
Leptogenesis is by far the most attractive way to generate the current baryon asymmetry. It is extraordinarily sturdy and resilient, and almost hopeless to confirm.

BUT

finding a $W_R$ at a collider near you would kill at least the « type 1 » leptogenesis (= through asymmetrical $N$ decay)

probably the only realistic way to EXCLUDE simple leptogenesis!
Outline (3/3)

Part 3 : CP
CP and Complex Conjugation
Gauge interactions

CP symmetry and Complex conjugation

\[ \overline{\tau^L_{1L}} \overline{\mu^L_{2L}} W^+ \]

\[ W^- e^- \]

\[ g \]

\[ \gamma^\mu \gamma^L \bar{\nu}_L \]

\[ \gamma^\mu \gamma^L \bar{\nu}_L \]

\[ e^+ e^- \]

\[ W^+ \gamma^\mu \gamma^L \bar{\nu}_L \]

\[ g^+ e^- \]

\[ \text{herm. conj.} \]
CP symmetry and Complex conjugation

Gauge interactions
CP symmetry and Complex conjugation

Gauge interactions
CP violation is the natural symmetry of gauge interactions

and is automatic, even for the minimal fermion content (one L mode)!

Other symmetries (C, P- would need to introduce more particles (for instance, both L and R fermions, to ensure parity or charge conjugation symmetry .. But these are not needed or realized in the Standard Model.

Since we observe CP violation (at a small level) in the Standard Model,
It has to come from other interactions!
Scalar interactions

CP symmetry and Complex conjugation

\[ \mathcal{L}_Y = \lambda_{ij}^d \tilde{d}_{iR} \Phi_1^+ Q_{jL} + \lambda_{ij}^u \tilde{u}_{iR} \Phi_2^+ Q_{jL} + h.c. \]

Where \( \Phi_1 \) and \( \Phi_2 \) are SCALAR fields \((\text{or Brout-Englert-Higgs scalars})\)

Need to open 2 parenthesis
- on could use just one field, by replacing \( \Phi_2 \) by \( \Phi_1 \equiv i\sigma_2 \Phi_1^* \)
- The scalar field was indeed introduced first by Brout and Englert (and slightly later by Higgs)
Temporary conclusions

P and C violation present in the Standard Model, expected from the use of chiral fermions (L and R spinors have different interactions

CP is a symmetry of the « PURE GAUGE » part

A small CP violation is observed, and can be attributed to complex coefficients in the scalar sector

→ CP violation probes the Scalar Sector.
→ CP is linked to PHASES between processes ... how can these phases be made observable?
CP violation is linked to the Scalar Sector

Standard Model (Brout-Englert-Higgs)
Scalar Boson
Some like to claim that Brout-Englert → mechanism, while Higgs → Boson. Some even claim that the Scalar boson is hard to find in Brout-Englert paper …
Let us look closer …
… we need to go all the way to Equation 1

This is the Abelian case, and $\phi_1$ is « The » Scalar, $\phi_2$ being absorbed…

**FIG. 1.** Broken-symmetry diagram leading to a mass for the gauge field. Short-dashed line, $\langle \phi_1 \rangle$; long-dashed line, $\phi_2$ propagator; wavy line, $A_\mu$ propagator. (a) $\rightarrow (2\pi)^4 i e^2 g_{\mu \nu} \langle \phi_1 \rangle^2$, (b) $\rightarrow (2\pi)^4 i e^2 (q_\mu q_\nu / q^2) \times \langle \phi_1 \rangle^2$. 

**Looks familiar? From you SM course?**
Exercise (Standard Model)

Due to SU(2) symmetry (and gauge invariance), one is free to choose any direction for the « symmetry breaking » (vev).

Show how
\[
\langle \Phi_1 \rangle \equiv \begin{pmatrix} v / \sqrt{2} \\ 0 \end{pmatrix}
\]
can be used instead of
\[
\langle \Phi_1 \rangle \equiv \begin{pmatrix} 0 \\ v / \sqrt{2} \end{pmatrix}
\]

(Answer: ok, but \( (\nu_e) \) in doubt, \( Q = -T_3 + \frac{y}{2} \))

(end of parenthesis on BEH scalars in the SM)
Scalar interactions

CP symmetry and Complex conjugation

\[ \mathcal{L}_Y = \lambda_{ij}^d \bar{d}_i R \Phi_1^+ Q_j L + \lambda_{ij}^u \bar{u}_i R \Phi_2^+ Q_j L + h.c. \]
Scalar interactions

CP symmetry and Complex conjugation

\[ \mathcal{L}_Y = \lambda_{ij}^d \bar{d}_{iR} \Phi_1 Q_{jL} + \lambda_{ij}^u \bar{u}_{iR} \Phi_2 Q_{jL} + h.c. \]
Scalar interactions

CP symmetry and Complex conjugation

\[ \mathcal{L}_Y = \lambda_{i \bar{j}}^d \bar{d}_i R \Phi_1 Q_j \bar{L} + \lambda_{i \bar{j}}^u \bar{u}_i R \Phi_2 Q_j \bar{L} + h.c. \]
For your accelerator building friends
(relatively) light Right-handed neutrinos for your accelerator – lobbying friend ...
« Double see-saw »

\[ M_\nu = \begin{pmatrix} 0 & m & 0 \\ m^T & 0 & M^T \\ 0 & M & m_\sigma \end{pmatrix} \]

\[ m = \lambda \nu \]

\[ \lambda \] can then be large, and lead to observable effects, since the light neutrino mass is proportional to \( m_\sigma \).

\[ m_{\nu_1} \approx \left( \frac{m}{M} \right)^2 m_\sigma, \quad m_{\nu_{2,3}} \approx M \pm m_\sigma/2, \]

(remark: this is an example of « pseudo-Dirac »,

since \( V_R + V_S \) act as a Dirac pair, whose contributions to the light neutrino compensate.

(an old idea, .. Langacker, Mohapatra, Antoniadis, 1986-88, jmf+Liu, recently revived...)
Notice that the decay of a heavy ("right") Majorana neutrino could give an easy proof of "Majorana"
Counting the phases ...

or How Kobayashi and Maskawa earned a Nobel
CP violation requires OBSERVABLE phases; obviously, those which can be removed from the Lagrangian cannot have a physical effect...

We can for instance change the phase of $u_L$ without affecting its mass $m_\mu$ if we change in the same time the phase of $u_R$ ... In the Standard Model, the phase then disappears, since the $R$ matrix does not contribute (it IS different in Left-Right models, for instance ..)

\[
\begin{align*}
\begin{pmatrix} u_L' \\
\end{pmatrix} &= e^{i\alpha} \begin{pmatrix} e^{i\alpha_1} & 0 \\
0 & e^{i\alpha_2} \end{pmatrix} \begin{pmatrix} u_L'' \\
\end{pmatrix}
\end{align*}
\]

(approximately 3 phases
\[
\begin{pmatrix} \alpha_1 \\
\alpha_2 \\
\end{pmatrix}
\]
for up, and same for d

(n phases for n families)
In fact, only the combination $\alpha - \beta$ appears (not $\alpha + \beta$). So we can only remove 2n-1 phases for n families.

Initially, the unitary matrix $K$ has 2 $n^2$ real parameters (angles and phases) (general $n \times n$ matrix)
- $n^2$ « real conditions for unitarity
  = $n^2$ parameters

Remains $n^2 - (2n-1)$ parameters after phase redefinition (std model only)
Of these, we can count the angles by comparing to a real orthogonal matrix
Thus $n(n-1)/2$ parameters are angles

Remains $(n^2 - 2n + 1) - (n(n-1)/2) = (n^2 - 3n + 2)/2 = (n-1)(n-2)/2$ phases

CP violation requires thus AT least 3 families for quarks (predicted by KM when the 2nd family was not yet established)
At least 3 families (with mixing) necessary for CP violation

Implies:
• all families must be distinguishable (\( \rightarrow \) masses non-degenerate)
• all mixings must be present (otherwise 2+1 families ..

Many different parametrisations possible, but one invariant characterisation due to Cecilia Jarlskog

\[ X = \det \left[ S_u, S_d \right] = 2i J_v(S_u) v(S_d) \]

\[ S_u \equiv M_u M_u^\dagger, \quad S_d \equiv M_d M_d^\dagger. \]

\[ v(S_u) = (m_u^2 - m_c^2)(m_c^2 - m_t^2)(m_t^2 - m_w^2) \]

\[ v(S_d) = (m_d^2 - m_s^2)(m_s^2 - m_b^2)(m_b^2 - m_w^2) \]

In a familiar parametrisation, \( J \) is

\[ J = \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) \sin(\delta) \]

To realize that the CP violation is VERY SMALL, we must consider a dimensionless parameter, \( X/(100 \text{ GeV})^{12} \sim 10^{-17} \ldots \) we will see this is too small for generating baryon asymmetry in the Universe.

But by looking at specific (rare) decay channels, it can be seen in well-prepared experiments ...
What about phase counting with Majorana spinors? (for the neutrinos)

In principle, the calculation is quite different (in particular to diagonalize the mass matrix), but ... the result is in practice the same, with one exception: since we cannot reabsorb phases in the right-handed neutrino spinors, 2 extra phases remain. They are observable in neutrinoless double beta decay, for instance.
How to observe CP violation
– setting up the scene
CP vs TCP
Therefore, $X$ and $\bar{X}$ have the same mass and same lifetime!
No hope that one decays before the other!

But having the same mass lifetime does not mean that they die the same way ....
But having the same mass lifetime does not mean that they die the same way.

\[ X \to a \quad (hagg) \quad \Gamma_a \]

\[ \bar{X} \to \bar{a} \quad \Gamma_{\bar{a}} \]

\[ \Gamma_X = \Gamma_a + \Gamma_{\bar{a}} \]

\[ \bar{X} \to b \quad (bnn) \quad \Gamma_b \]

\[ \Gamma_X = \Gamma_b + \Gamma_{\bar{b}} \]

\[ \Gamma_a + \Gamma_b = \Gamma_{\bar{a}} + \Gamma_{\bar{b}} \]

\( \Gamma_a = \Gamma_{\bar{a}} \)
How does this apply to neutrinos? In particular to neutrino oscillations?

TCP

\[
< \nu_\alpha | S | \nu_\beta > = < \overline{\nu}_\beta | S | \overline{\nu}_\alpha >
\]

This implies that the “survival” probabilities are equal!

\[
P_{\nu_\alpha \rightarrow \nu_\alpha} = P_{\nu_\alpha \rightarrow \overline{\nu}_\alpha}
\]
So, a CP effect is possible, provided the sum of the e to mu and e to tau contributions are the same for neutrino and antineutrino ... the 2 channels mus “know of each other”.
\[
P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{k > j} \text{Re}[W_{\alpha\beta}^{jk}] \sin^2 \left( \frac{\Delta m_{jk}^2 L}{4E_\nu} \right) \pm 2 \sum_{k > j} \text{Im}[W_{\alpha\beta}^{jk}] \sin \left( \frac{\Delta m_{jk}^2 L}{2E_\nu} \right)
\]

(see for instance eq 37, 38, 91 of P. Hernandez arXiv:1010.4131 PDF)

\[
A^{CP(T)-odd}_{\nu_\alpha \nu_\beta} = \frac{\sin \delta c_{13} \sin 2\theta_{13} \sin 2\theta_{12} \frac{\Delta m_{12}^2 L}{4E_\nu} \sin 2\theta_{23} \sin^2 \frac{\Delta m_{13}^2 L}{4E_\nu}}{P^{CP-even}_{\nu_\alpha \nu_\beta}}.
\]

Compare to the Jarlskog determinant!

+ for neutrino
- for antineutrino
Much ado has been made about neutrinos...

Future experiments will tell us if they are

• Boring Dirac particle with very small coupling to scalar bosons (simplest, and I am afraid most likely)
• The signal of new physics (see-saw, heavy $M$, defeat of antimatter ...)

In any case they force us to review our concepts in some depth!