



# Neutrino interactions

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**VSON9, ICISE, July 2025**

# Who I am?



- 2017 - 2023: PhD student at IFIRSE, ICISE
- 2023-now: JSPS postdoctoral fellow at Kyoto U.
- Student of VSON2
- Lecturer of VSON8 & VSON9
- T2K collaborator since 2019
- Now working mostly on WAGASCI-BabyMIND analysis

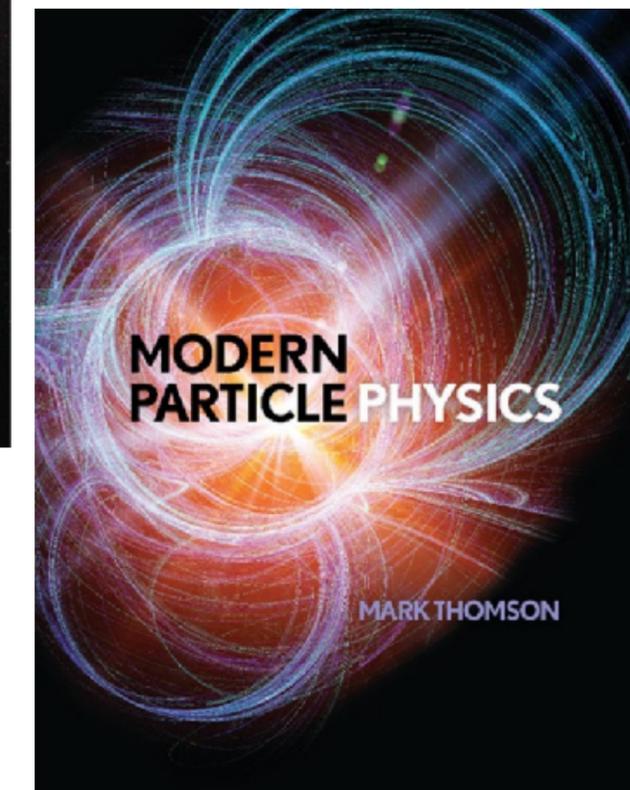
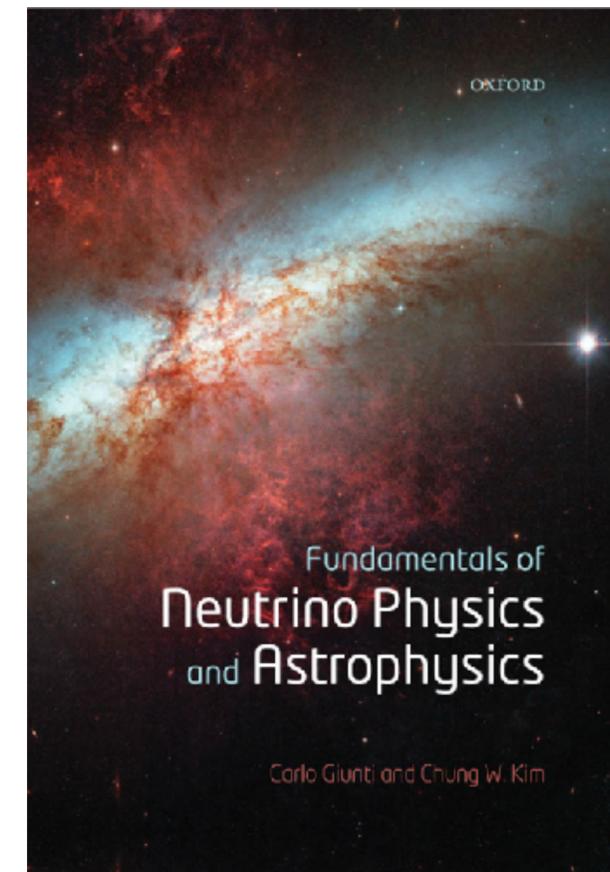
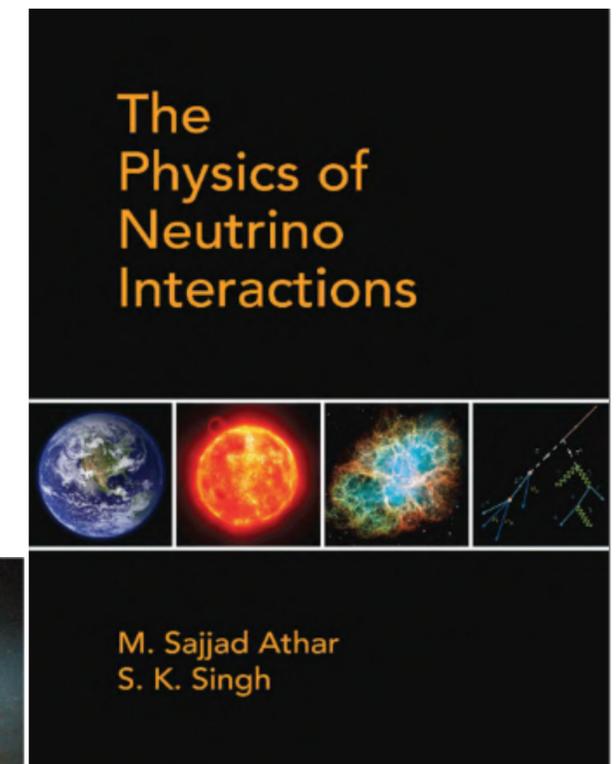
# Disclaimers

- To prepare for this lecture, I have borrowed from many sources: books, papers, lectures, internet, ... I can not cite them all here.
- Time is limited (my knowledge too :)) ), content is unlimited! I can not cover all things and go into details within 90 mins.
- If your questions can not be answered in the lecture, discuss later in break or refer to references for more information.
- If you see some complicated formulae and don't even know anything about them. Don't worry, me too !!!



# References

- M. Sajjad Athar & S. K. Singh, **Physics of neutrino interactions** (2020): very modern and cover almost aspects of neutrino interactions.
- Carlo Giunti & Chung W. Kim, **Fundamentals of Neutrino Physics and Astrophysics** (2007): detailed calculation
- Mark Thomson, **Modern particle physics** (2013): very good in physical interpretation
- Rev. Mod. Phys. 84, 1307 (2012)  
(arXiv:1305.7513v1) **From eV to EeV: Neutrino Cross-Sections Across Energy Scales**



# Why do we need to study neutrino interactions?

**Neutrino interactions:** means to **experimentally** and **theoretically** investigate how neutrinos scatter off matter, with the goal of understanding **cross sections**, nuclear effects, final-state particles, and potential signs of new physics.

1. Crucial for measuring fundamental properties of neutrinos (mass, spin, ...)
  2. Essential for detector performance and data analysis:
    - Improve detection efficiency
    - Optimize event reconstruction
    - Reduce systematic uncertainties
  3. Windows for looking into the universe and stellar interiors:
    - Neutrinos are key of astrophysical and cosmological processes
    - Low energy cross sections are vital for supernova neutrino detections and geoneutrino studies
  4. Search for new physics beyond Standard Model
    - Sterile neutrinos
    - Non-standard interactions
- **Theoretical calculation:** based on QFT, Feynman rules, ...
  - **Experimental measurement:** based on data analysis

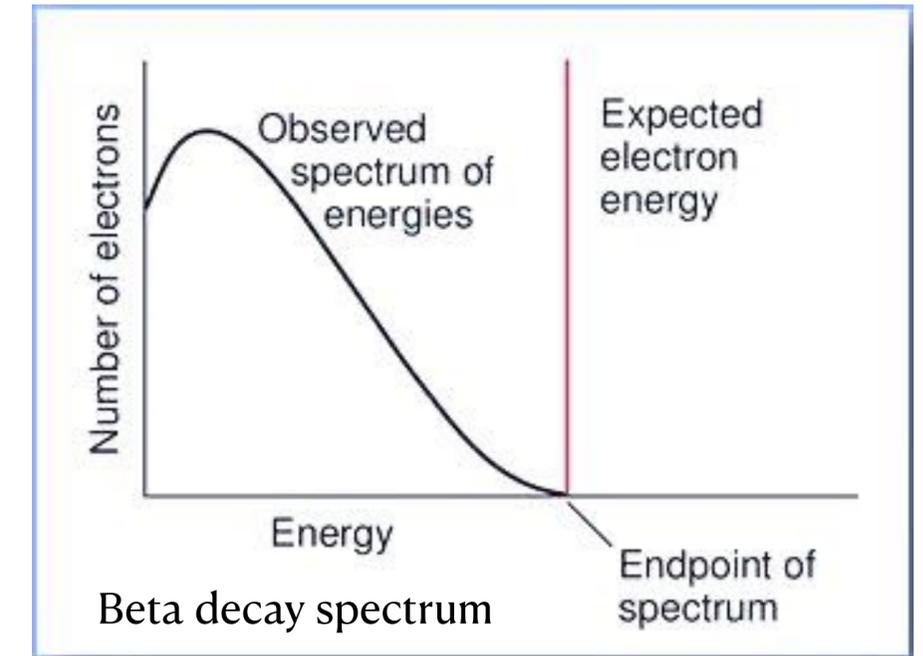
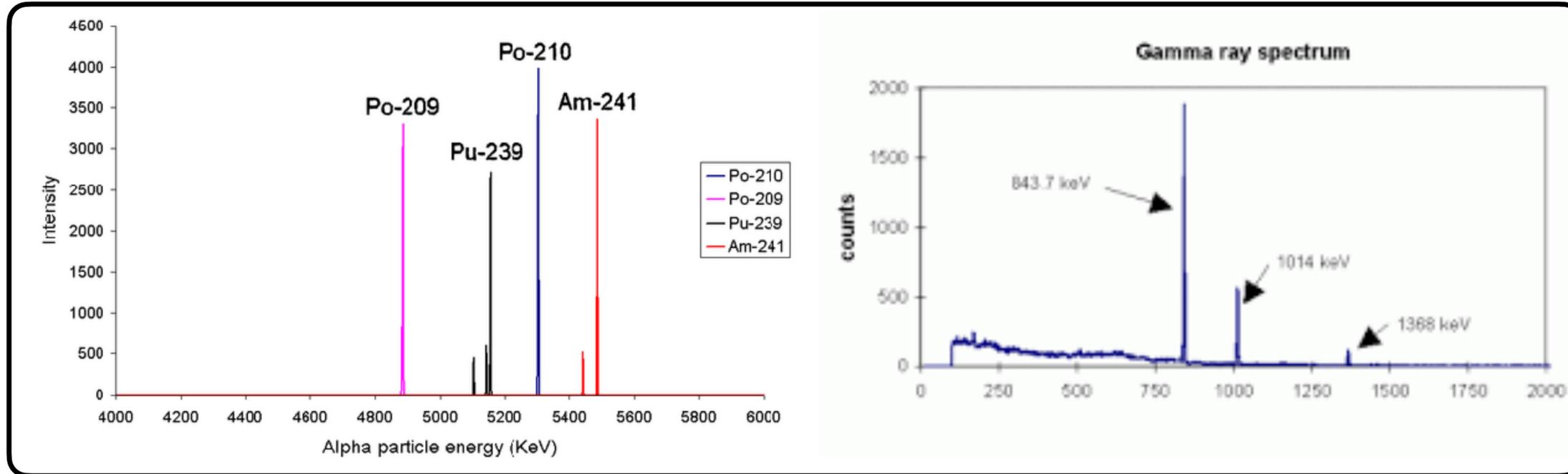
# Contents

1. A brief history and properties of neutrinos
2. Cross sections
3. Neutrino interactions
4. Introduction to NEUT - an event generator
  - 4.1: Neutrino event generator
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  - 4.3: Practice with NEUT
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# 1. A brief history and properties of neutrinos



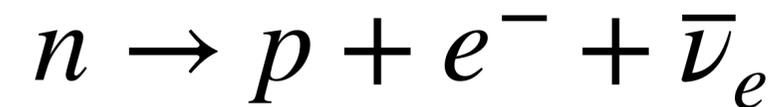
- Alpha and gamma decay spectra are discrete
- Two particles detected in the beta decay



$$K_e = (m_n - m_p - m_e); \quad (c = 1)$$

- Beta spectrum was expected to be discrete as alpha and gamma spectra
- **But it was not**

- Beta continuous spectrum: energy is missing => energy is not conserved or what else?
- In 1930, Pauli proposed an extremely light, neutral particle: neutrino ( $\nu$ )



$$K_e = (m_n - m_p - m_e) - K_\nu$$

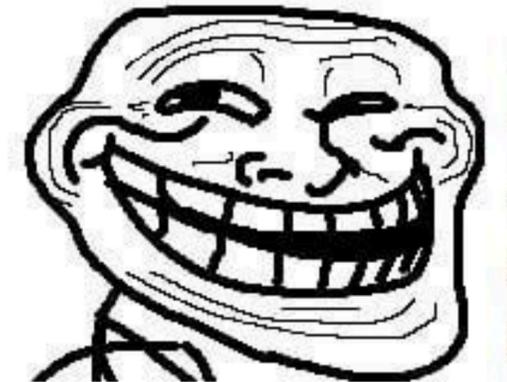
- Beta energy now can have any value between 0 and maximum

# Tips for doing physics and math



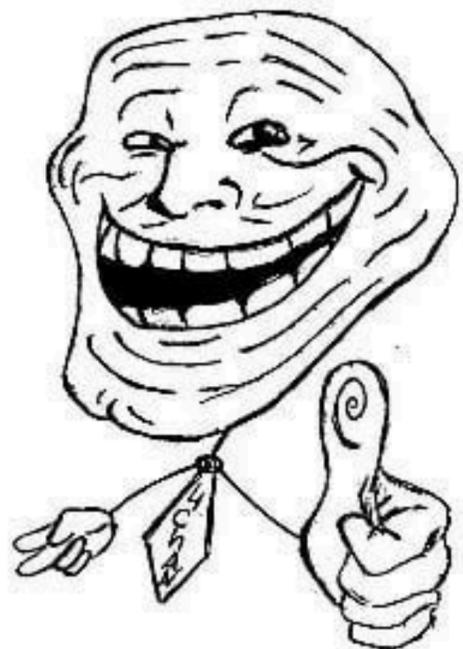
$$1+1=3$$

Calculations don't add up?



$$1+1+1=3$$

Add a hypothetical "dark number" to account for inaccuracy.



Now you're doing math like a physicist!

- This philosophy is so true for the cases of neutrino, dark matter, dark energy, and many other discoveries
  - Don't know what is the reason of missing energy in beta decay: propose "undetectable particle" (neutrinos)
  - Don't know what cause the unusual gravitational effects in galaxies: propose dark matter
  - Don't know what cause the acceleration of the universe: propose dark energy
- However, these hypotheses are **not arbitrary** but based on **rigorously mathematical calculations** or **carefully experimental data analyses**

# 1. A brief history and properties of neutrinos



**W. Pauli**

“I have done a terrible thing:  
I have postulated a **particle that cannot be detected**”

- Neutrinos:
  - Mass  $\sim 0$ : almost **no gravitational interaction**
  - Neutral: **no electromagnetic interaction**
  - **No strong interaction**
  - **Only participate in weak interaction**



**The ghost particle**

- Neutrinos are **detectable** but they interact **extremely weak** with matter

# 1. A brief history and properties of neutrinos

- An example to see how weak neutrino-matter interaction is

- Neutrino mean free path:  $L = \frac{m}{\rho\sigma}$ 
  - $m$  is mass of nucleon
  - $\rho$  is material density
  - $\sigma$  is neutrino cross section
- $m_p \approx m_n \approx 1.67 \times 10^{-27} \text{kg}$
- $\sigma \approx 10^{-47} \text{m}^2$  at 1 MeV
- $\rho_{H_2O} = 1000 \text{kg/m}^3$ ,  
 $\rho_{Pb} = 11400 \text{kg/m}^3$

- $L_{H_2O} = \frac{1.67 \times 10^{-27} \text{kg}}{1000 \text{kg/m}^3 \times 10^{-47} \text{m}^2} = 1.7 \times 10^{17} \text{m}$
- $L_{Pb} = \frac{1.67 \times 10^{-27} \text{kg}}{11400 \text{kg/m}^3 \times 10^{-47} \text{m}^2} = 1.5 \times 10^{16} \text{m}$

1 light year  $\approx 9.5 \times 10^{15} \text{m}$

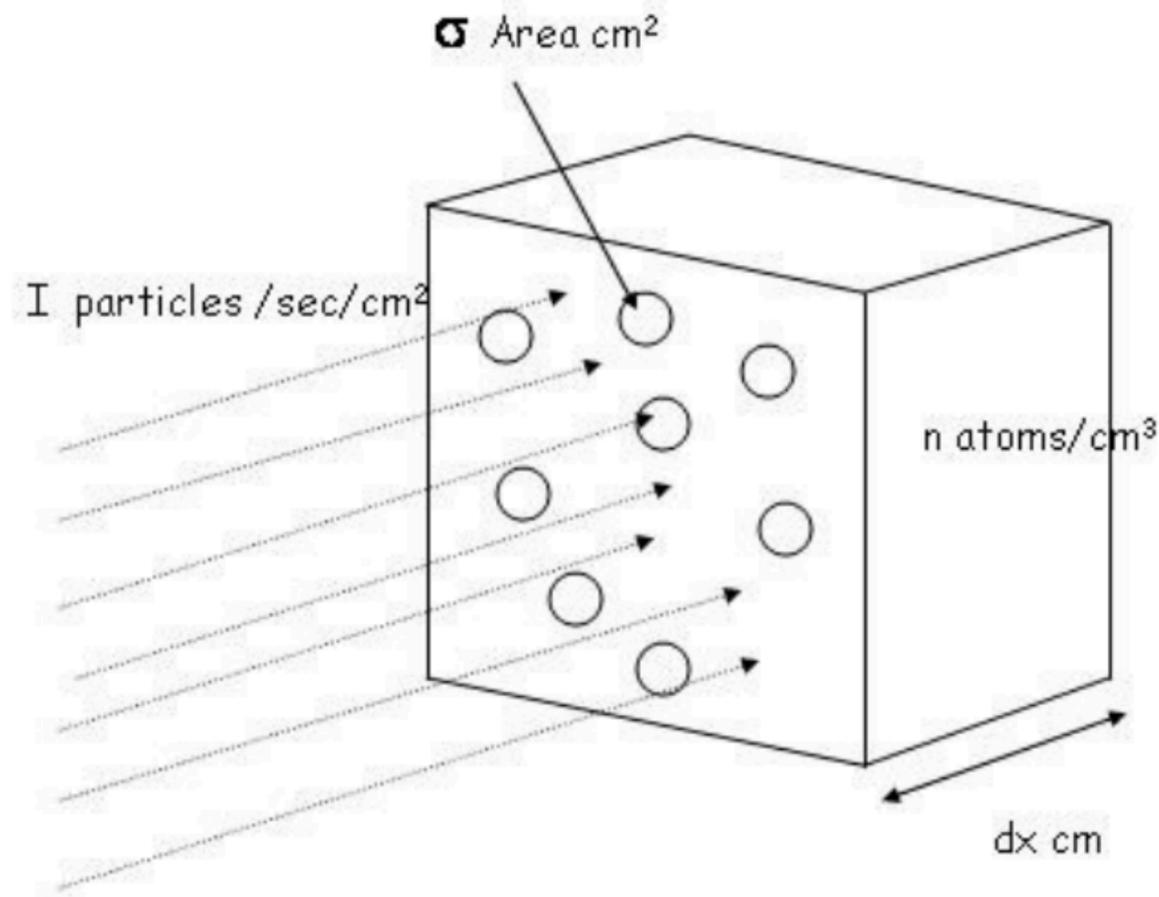
- **Need a block of lead with 1-light-year length to stop neutrinos!!!!**
- **It is a real challenge to study neutrino interactions (to measure cross sections)**

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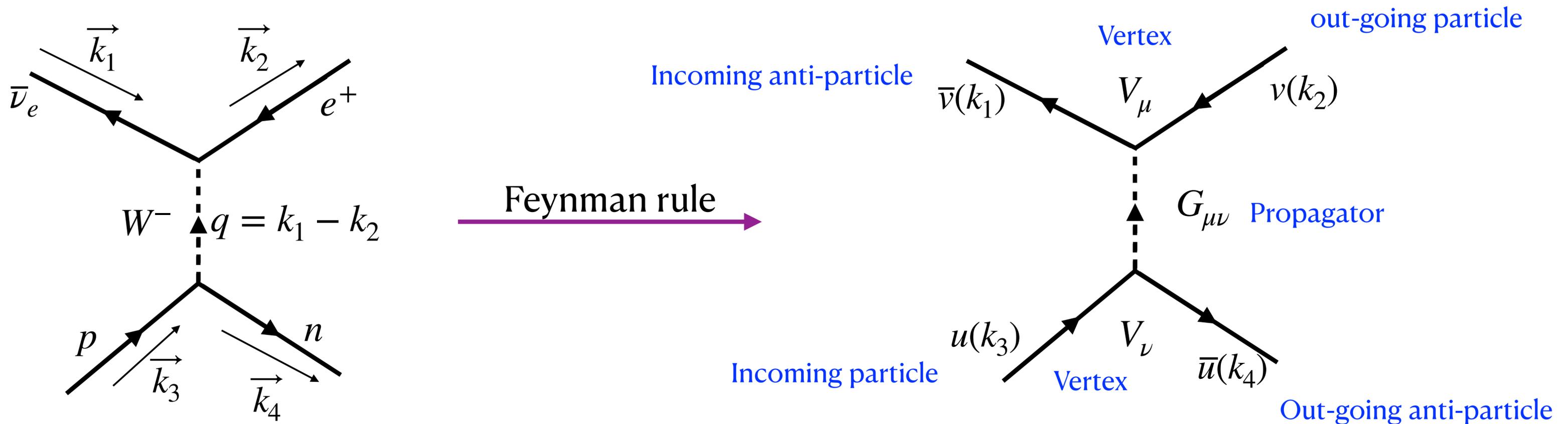
## 2. Cross section (1): definition

- **Cross section (Xsec):** quantum mechanical probability that an interaction will occur.
  - “If I shoot a neutrino beam at a target, how likely is it that a neutrino will interact?”
- **Units:** the Xsec has unit of *area*<sup>2</sup> (barn: b),  $1b = 10^{-28} m^2$
- Neutrino Xsec are **extremely small:**  $\sigma_\nu \approx 10^{-62} m^2 - 10^{-35} m^2$



## 2. Cross section (2): theoretical calculations

- Using QFT:  $\sigma \propto |M_{fi}|^2$  (1) ; i: initial states, f: final states
- $M_{fi}$ : Scattering amplitude, calculated using Feynman rules
- **Example:** Inverse beta decay (**IBD**) => important for reactor neutrino detection:



## 2. Cross section (2): theoretical calculations

- Calculate scattering amplitude:

$$iM_{fi} = \bar{\nu}(k_1) V_\mu \nu(k_2) G_{\mu\nu} \bar{u}(k_4) V_\nu u(k_3)$$

- Taking square of  $|M_{fi}|^2$  and inserting into eq. (1)

$$\sigma \approx 10^{-43} \left( \frac{E_\nu}{MeV} \right) cm^2$$

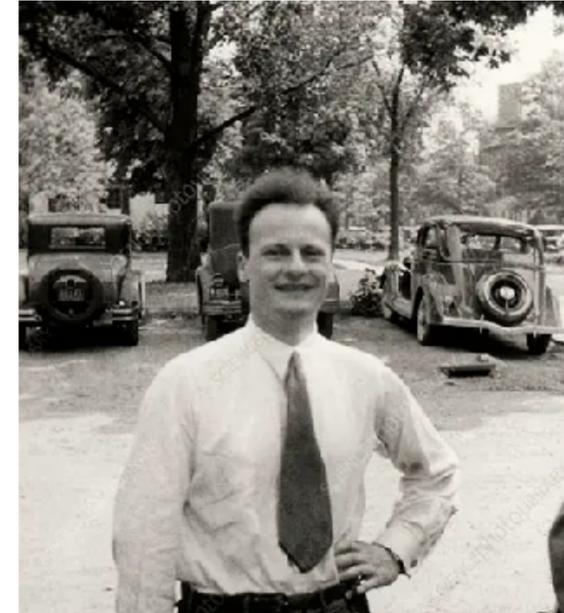
- Reactor neutrino energy  $\sim 0$  MeV - 10 MeV

$$\sigma_{\bar{\nu}_e}^{reactor} \approx 10^{-44} - 10^{-42} cm^2 \longrightarrow$$

- How to improve the number of neutrino interactions?

$$N = \Phi \left[ \frac{1}{time * area^2} \right] \times \sigma [area^2] \times T$$

flux # of target particles



Hans Bethe



Rudolf Peierls

- 1934: Hans Bethe & Rudolf Peierls were the **first people** who theoretically **calculated neutrino cross section**
- They claimed: “**no practically possible way of observing the neutrino.**”
- 1956: F. Reines and C. Cowan et. al. confirmed “**neutrinos are detectable**”
- H. Bethe humorously replied Reines: “**Well, you shouldn't believe everything you read in the papers.**”

# 2. Cross section (2): theoretical calculations

- Calculate scattering amplitude:

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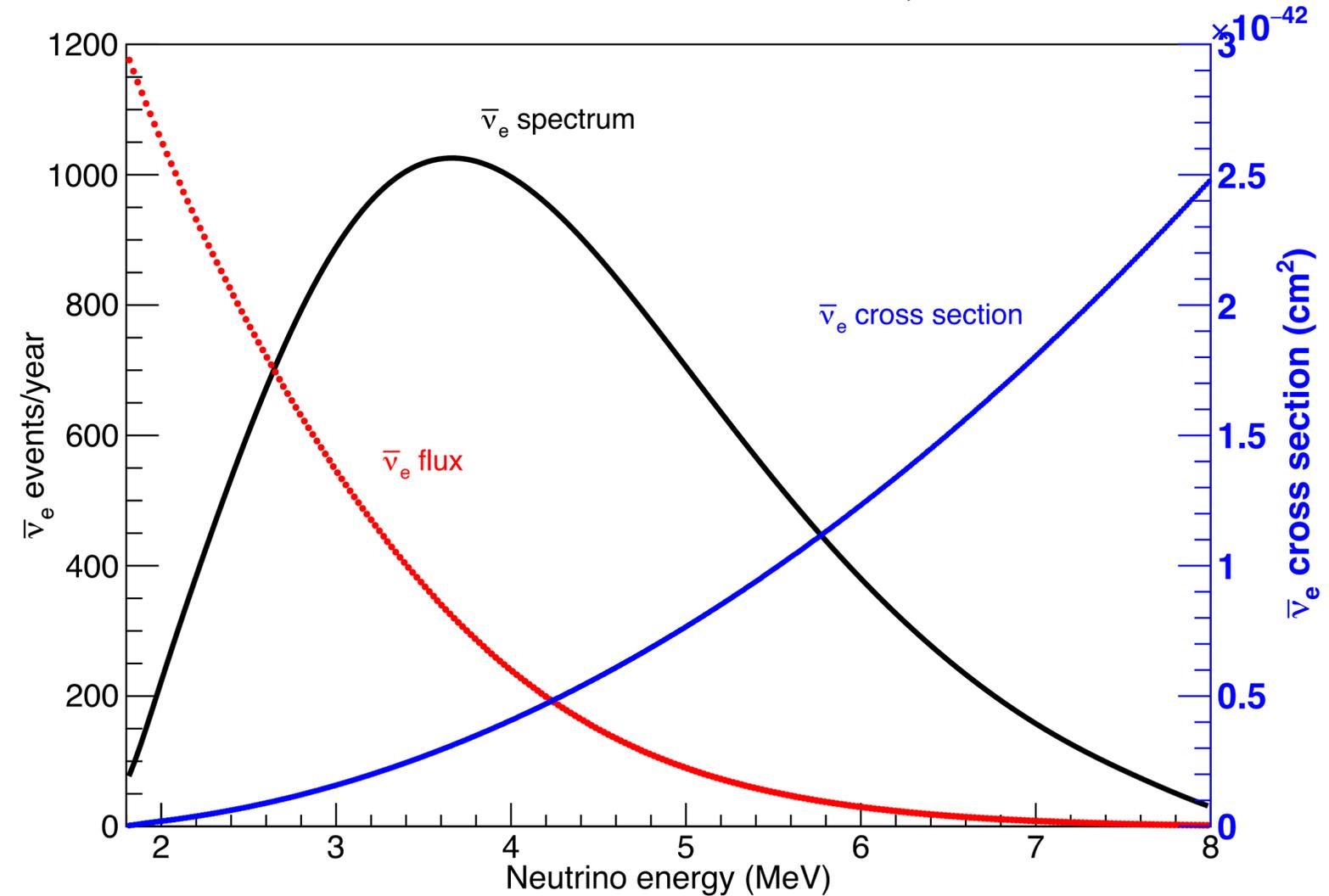
- How to improve the number of neutrino interactions?

$$N = \phi \left[ \frac{1}{time * area^2} \right] \times \sigma [area^2] \times T$$

flux

# of target particles

GLOBES simulation @1000MWth, L = 20m



## 2. Cross section (2): theoretical calculations

### ○ Threshold for IBD to be occurred:

- Energy-momentum conservation =>

$$E_{\nu}^{thr} = \frac{(m_e + m_n)^2 - m_{\nu}^2 - m_p^2}{2m_p}$$

- With:

$$m_{\nu} \approx 0; m_e = 0.511 \text{ MeV}$$

$$m_p = 938.272 \text{ MeV}; m_n = 939.565 \text{ MeV}$$

- Threshold energy:  $E_{\nu}^{thr} \approx 1.8 \text{ MeV}$
- $E_{\nu}^{reactor} \approx 0 - 10 \text{ MeV}$ , IBD occurs
- $E_{\nu}^{solar}(pp) \approx 0.1 - 1 \text{ MeV}$ , IBD does not occurs

## 2. Cross section (3): types of Xsec measured in experiments

**1. Total Xsec:** The **integrated probability** of *any interaction* between the incoming particle and the target, over all possible final states:  $\sigma = \frac{N}{\phi \cdot T}$ , N: number of interactions,  $\phi$ : neutrino flux,  $T$ : number of target particles

### 2. Differential Xsec:

- **Differential in scattering angle:**  $\frac{d\sigma}{d(\cos \theta)}$ 
  - Tells how the **outgoing lepton or hadron** is distributed in angle relative to the incoming neutrino.
  - Important for understanding **interaction dynamics**
- **Differential in energy of final-state lepton:**  $\frac{d\sigma}{dE_l}$ 
  - Useful for neutrino energy reconstruction.
  - Helps in estimating how much energy is transferred to the lepton.
- **Differential in momentum transfer:**  $\frac{d\sigma}{dQ^2}$ 
  - $Q^2 = -q^2 = -(k_1 - k_2)^2$ : negative four-momentum transfer squared.
  - High  $Q^2$ : hard scattering, go to Deep Inelastic Scattering (DIS)
  - Low  $Q^2$ : elastic or quasi-elastic scattering (QE)

## 2. Cross section (3): types of Xsec measured in experiments

3. **Double Differential Cross Section:**  $\frac{d^2\sigma}{dE_l d(\cos\theta)}$

- Gives **simultaneous dependence** on both energy and angle of outgoing lepton.
- Important in precise energy reconstruction and detector simulation.

4. **Inclusive Xsec:** All final states with at least one specified particle, sum over unobserved particles

5. **Exclusive Xsec:** Final state is completely specified

6. **Semi-inclusive Xsec:** Some particles specified, others summed over

7. **Charge current (CC) Xsec:** involves charged lepton flavor changing, important for flavor tagging

8. **Neutral current (NC) Xsec:** involves neutral current interactions, used for defining neutral backgrounds.

### Example

Measurement of the charged-current electron (anti-)neutrino inclusive cross-sections at the T2K off-axis near detector ND280

T2K Collaboration • K. Abe (Kamioka Observ.) et al. (Feb 27, 2020)

Published in: *JHEP* 10 (2020) 114 • e-Print: [2002.11986](https://arxiv.org/abs/2002.11986) [hep-ex]

## 2. Cross section (4): measurement in experiments

- The **true** number of interactions at the detector:

$$N_{true} = \underbrace{\phi}_{\text{flux}} \left[ \frac{1}{\text{time} * \text{area}^2} \right] \times \sigma[\text{area}^2] \times T \quad \leftarrow \text{\# of target particles}$$

- Ex: calculate number of expected events observed at detector of Reines & Cowan experiment:  $\bar{\nu}_e + p \rightarrow e^+ + n$

- Reactor neutrino cross section @1MeV:  $\sigma_{\bar{\nu}_e}^{reactor} \approx 10^{-43} \text{ cm}^2$

- Neutrino flux:  $\phi_{reactor} = 3.7 \times 10^{13} \left[ \frac{1}{s * \text{cm}^2} \right]$

- Target:  $200l \text{ H}_2\text{O} + 40 \text{ kg CdCl}_2 \Rightarrow T_p \approx 2.68 \times 10^{28}$  protons

- Expected events:  $N_{expected} \approx 357$  events/h

- Measured events:  $N_{measured} \approx 3$  events/h (~1% efficiency)

How to deal with this discrepancy?

C. Cowan

F. Reines



Project Poltergeist (1956)

## 2. Cross section (4): measurement in experiments

- The **true** number of interactions at the detector:

$$N_{true} = \phi \left[ \frac{1}{time * area^2} \right] \times \sigma[area^2] \times T$$

- There is no perfect detector, what you **measured (reconstructed)** is **smear**ed from the **true** information  $N_{recon} \neq N_{true}$ . You need to account for this difference and make your measurement closer to the true information as much as possible:

- Take into account the **detector efficiency**  $\epsilon$
- Construct a **smearing matrix** (migration matrix)  $U_{smearing}$  to migrate from **true quantity** to **reconstructed quantity**

$$N_{recon} = U_{smearing} \cdot \epsilon \cdot \phi(E_\nu) \cdot \sigma_{true}(E_\nu) \cdot T$$

- Cross section:

$$\sigma_{true}(E_\nu) = \frac{U_{smearing}^{-1} N_{recon}}{\epsilon \cdot \phi(E_\nu) \cdot T} = \frac{U_{unfolding} N_{recon}}{\epsilon \cdot \phi(E_\nu) \cdot T}$$

## 2. Cross section (4): measurement in experiments

- Differential cross section with respect to an observable  $x$ :

$$\frac{d\sigma(E_\nu, x_{true})}{dx_{true}} = \frac{U_{unfolding} N(x_{recon})}{\phi(E_\nu) \cdot T \cdot \epsilon \cdot dx_{true}}$$

- Where  $U_{unfolding} = U_{smearing}^{-1}$

- In general

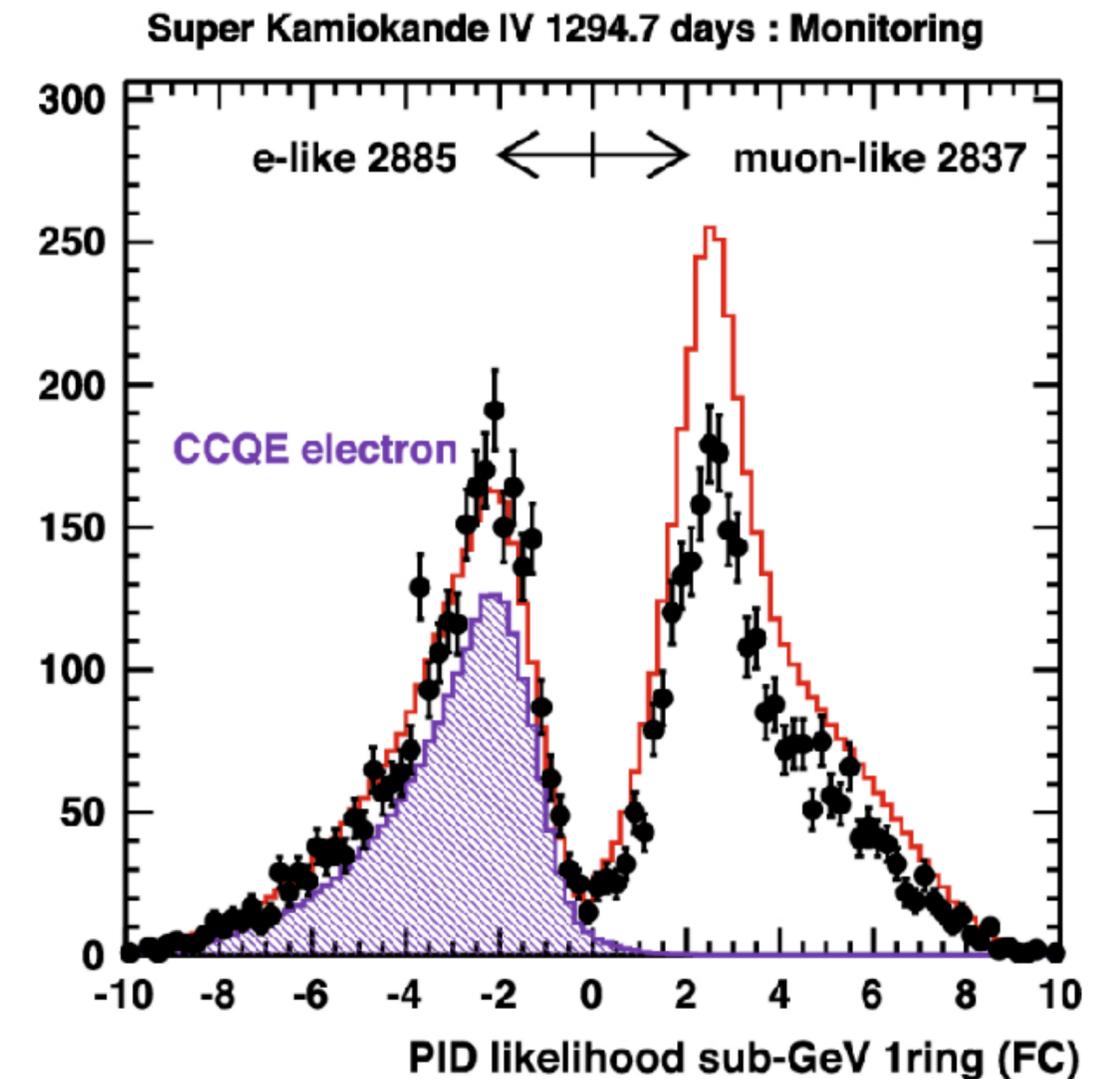
$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

- $U_{i\alpha}$ : unfolding matrix (unsmearing matrix)
- $N_{data}^{selected}$ : total number of selected events after applying all cuts
- $N_{data}^{bckg}$ : number of backgrounds in selected events
- $\phi$ : neutrino integrated flux
- $T$ : number of target particles
- $\epsilon$ : detector efficiency
- $\Delta x$ : bin width

## 2. Cross section (4): measurement in experiments

- **An event:** a single interaction recorded by the detector
  - Reconstructed tracks
  - Energy deposits
  - Timing info
  - Particle ID likelihoods, etc
- **Selected events** are events chosen from total dataset that match the physics signature you're interested in (e.g.  $\nu_\mu \rightarrow \nu_e$  appearance) and pass quality cuts:
  - Correct topology (e.g., one track, one shower),
  - Energy within a certain range
  - Vertex within fiducial volume,
  - Timing consistent with beam spill,
  - Particle ID matching the expected signal.

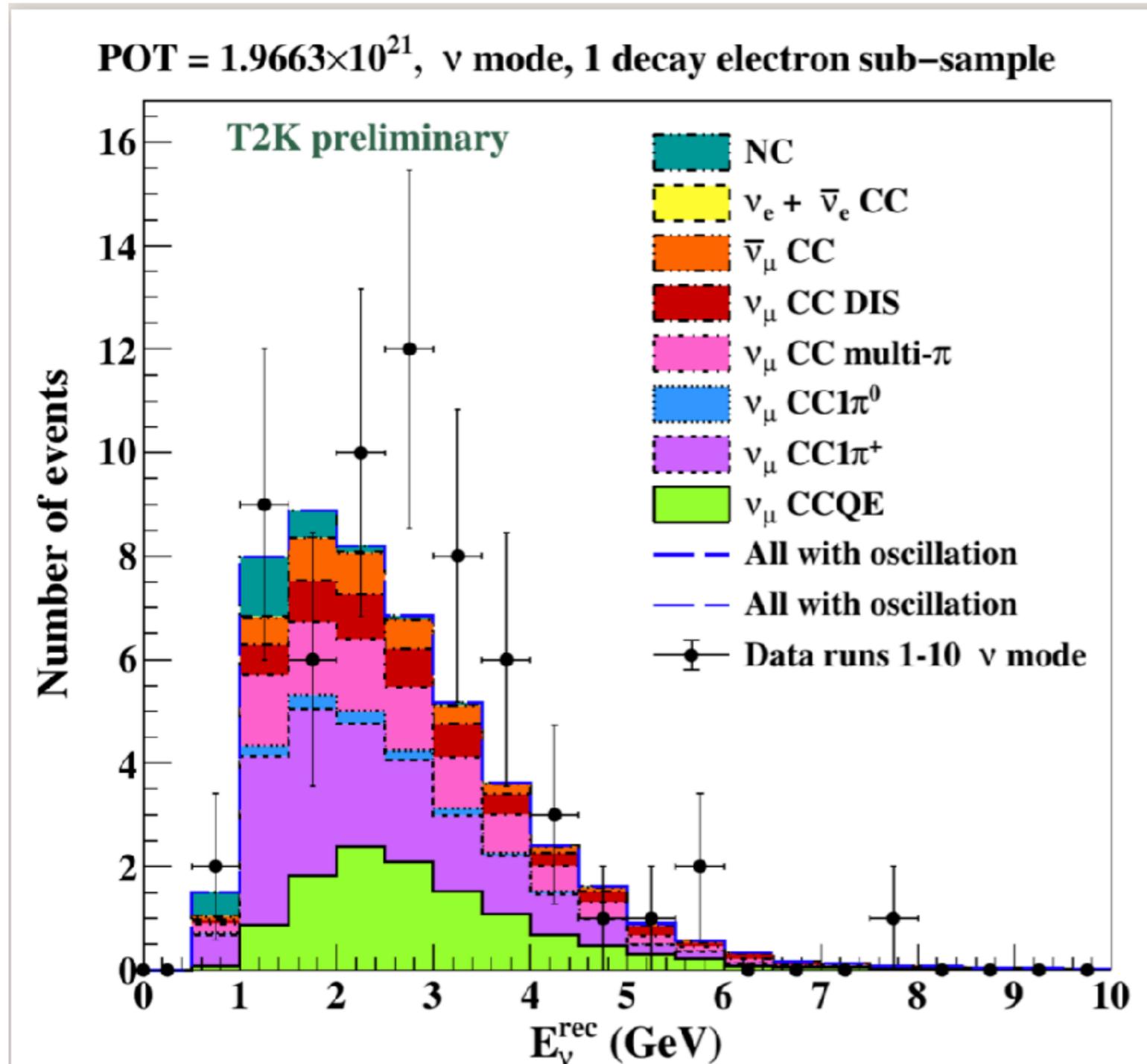
$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$



## 2. Cross section (4): measurement in experiments

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

- **Signal** refers to the type of event you are trying to study or measure. It matches the physics process of interest.
- **Background** refers to **unwanted events** that **mimic or obscure the signal**, often due to other physical processes or noise that **pass our cuts**
- **Selected events = Signals + Backgrounds**

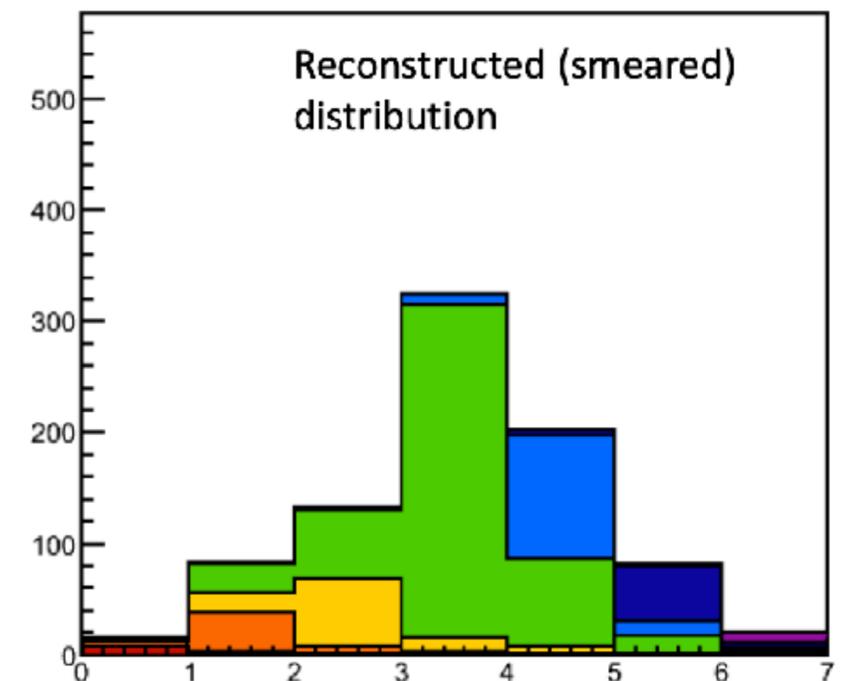
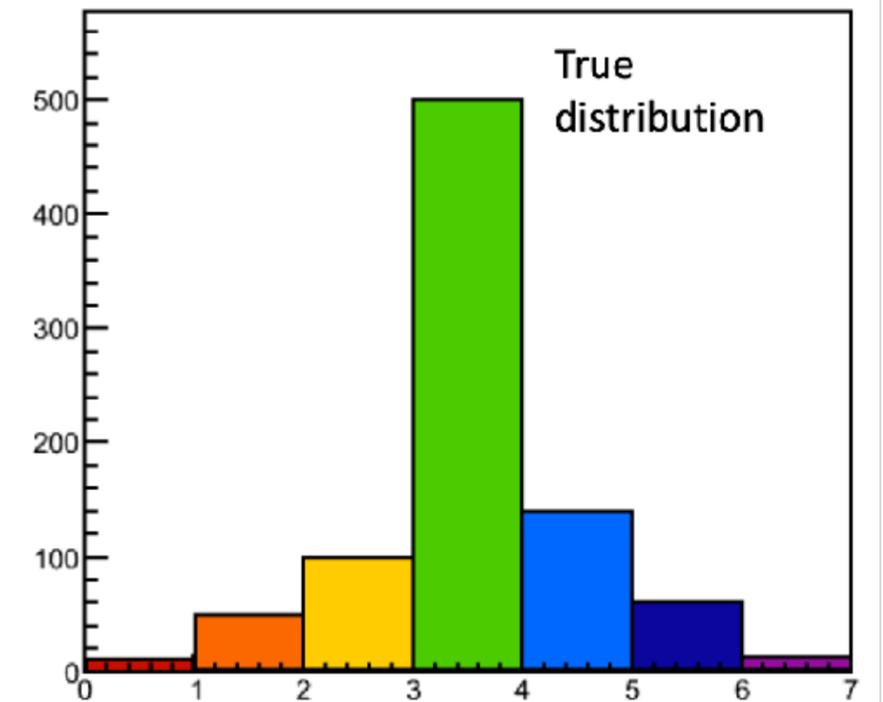


## 2. Cross section (4): measurement in experiments

### ○ Unfolding:

- Our reconstruction is not perfect, an event can be reconstructed into a wrong bin
- $U_{i\alpha}$  presents the probability that an event observed in bin  $i$  actually happened in bin  $\alpha$
- **We can use our Monte Carlo to construct a smearing matrix (migration matrix) which indicates the fraction of events generated in bin  $\alpha$  be mis-reconstructed into bin  $i$**
- If the detector has good resolution, the matrix should be close to diagonal
- Bin width is also important. If the bins are too small compared to our resolution, the matrix may not be diagonal

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

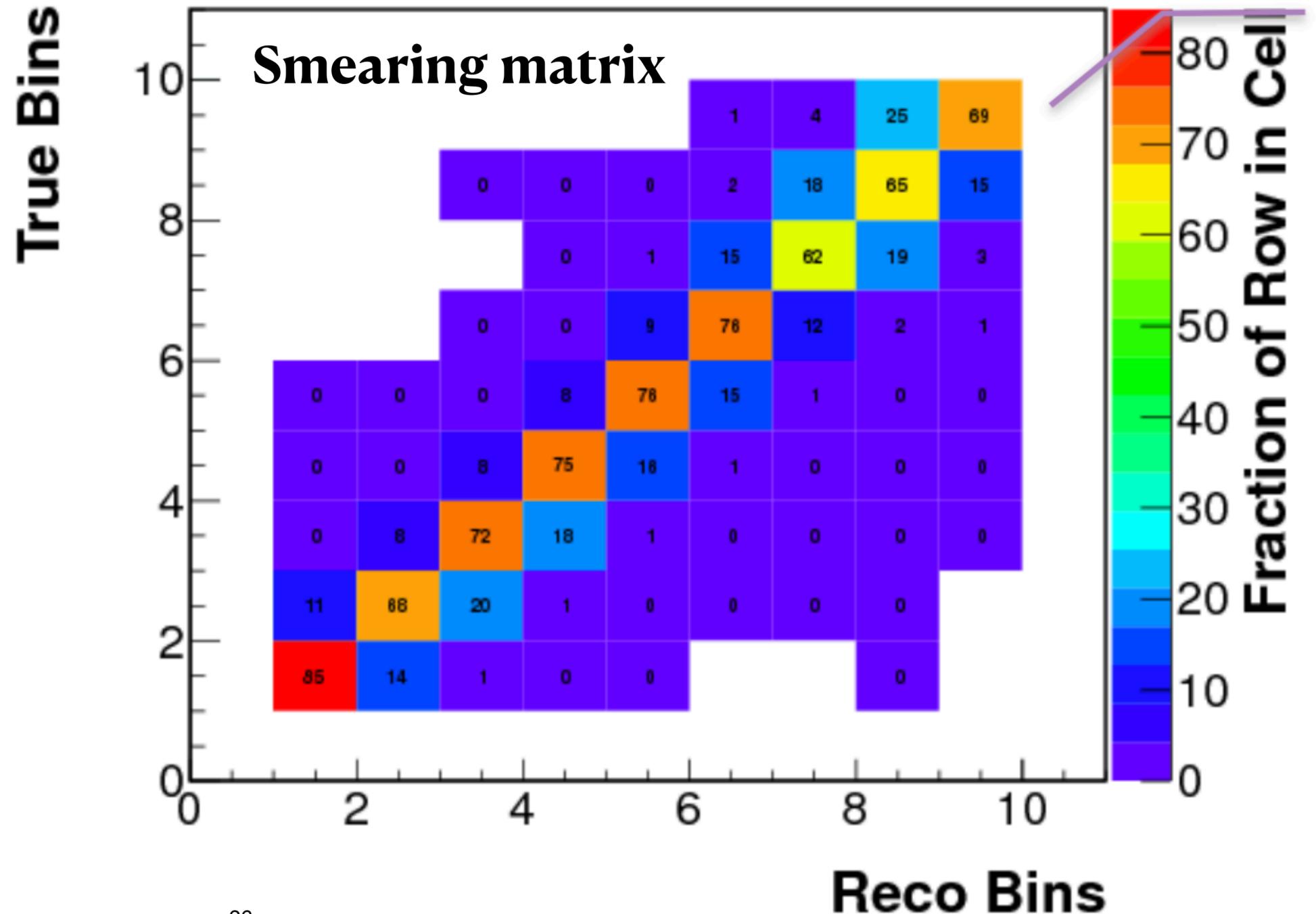


## 2. Cross section (4): measurement in experiments

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

- **Unfolding matrix** (**unsmearing matrix**) can be obtained by inverting the **smearing matrix** (**migration matrix**)

$$U_{unfolding} = U_{smearing}^{-1}$$



## 2. Cross section (4): measurement in experiments

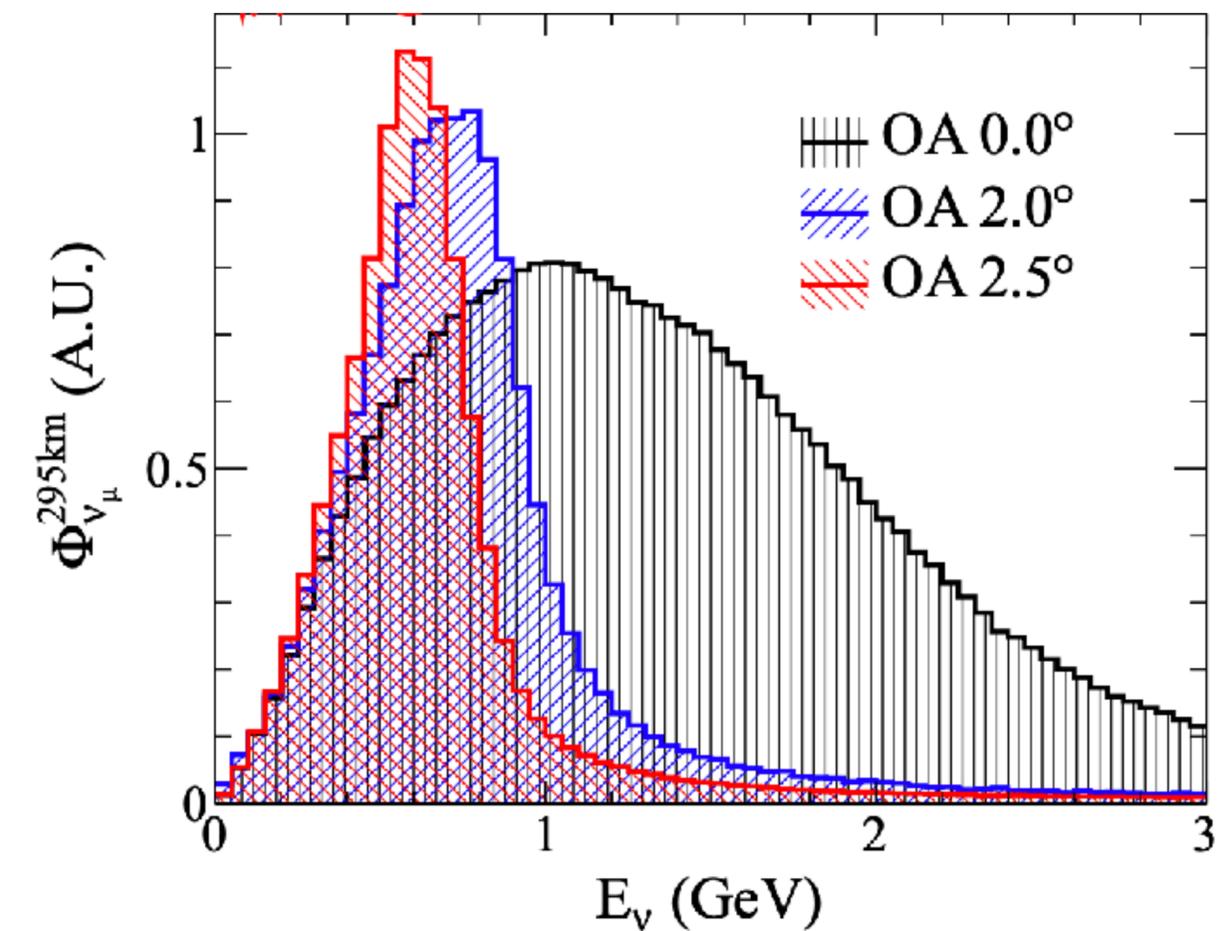
$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

### ○ Flux:

- Integrated flux over all neutrino energies

$$\phi = \frac{\int dE_\nu \phi(E_\nu)}{dE_\nu}$$

- It is hard to calculate
- It is an important source of systematic uncertainty
- In T2K, flux uncertainty is about 10%



## 2. Cross section (4): measurement in experiments

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

### ○ **T: number of nucleons in target**

- $T = \frac{M}{m} \cdot N_A * (n_p + n_n)$

- M: target mass in grams

- m: atomic mass

- $n_p, n_n$ : number of protons and neutrons per atom

- $N_A = 6.02214 \times 10^{23} \text{ mol}^{-1}$ :  
Avogadro's number

### ○ **How many nucleons are there in Super-K detector?**

- $M = 50\text{kt} = 5 \times 10^{10} \text{ g}$

- $m_{H_2O} = 18 \text{ g}$

- $(n_p + n_n)_{H_2O} = (10 + 8) = 18$

$$T = \frac{5 \times 10^{10}}{18} * 6.0214 \times 10^{23} * (10 + 8) = 3 \times 10^{34}$$

## 2. Cross section (4): measurement in experiments

- $\epsilon$ : efficiency

Number of selected signal events

$$\epsilon = \frac{\text{Number of selected signal events}}{\text{Number of true signal events generated in the fiducial volume}}$$

- Why do we need **efficiency correction**?

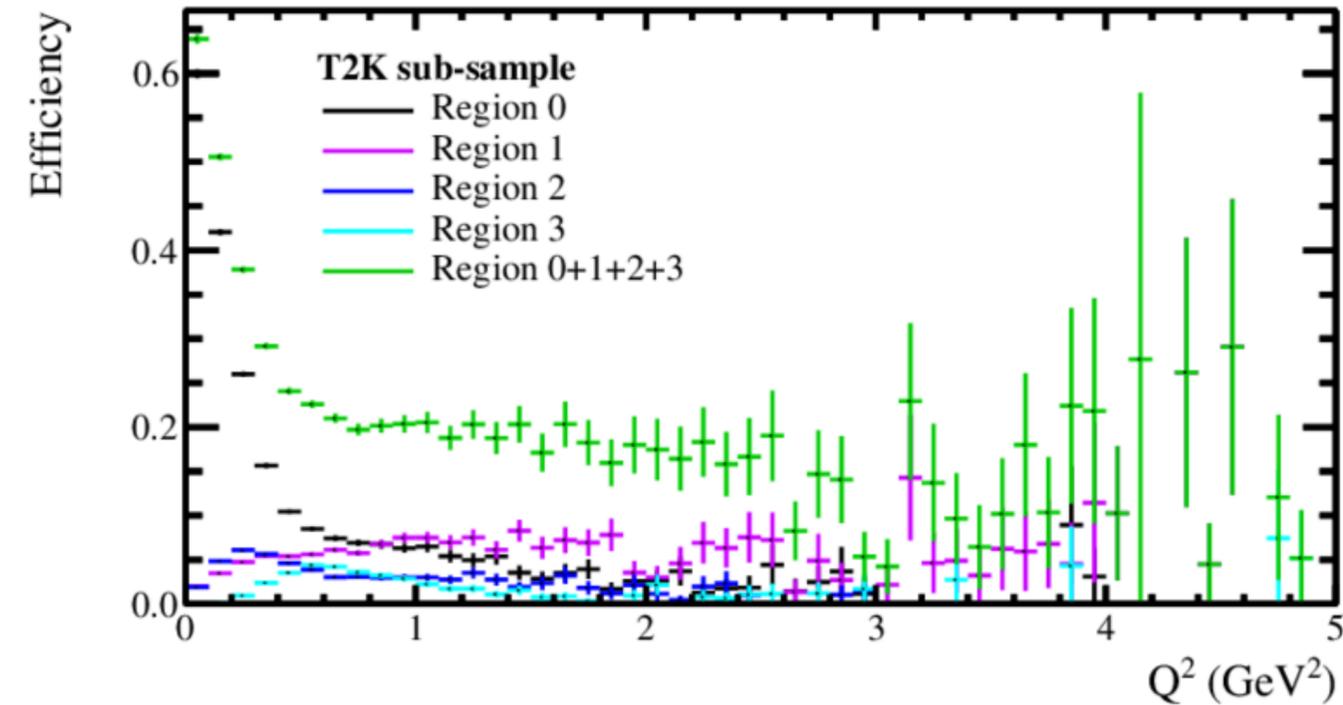
Inefficiency comes from misreconstruction and detector geometry

- Some events aren't detected,
- Some are misidentified or fail selection cuts.

- How to calculate efficiency? - use **Monte Carlo simulation**

- Generate many simulated neutrino interactions.
- Count how many fall within your detector's fiducial volume.
- Run them through detector simulation + reconstruction + event selection.
- Take the ratio of **selected/total** to get  $\epsilon$

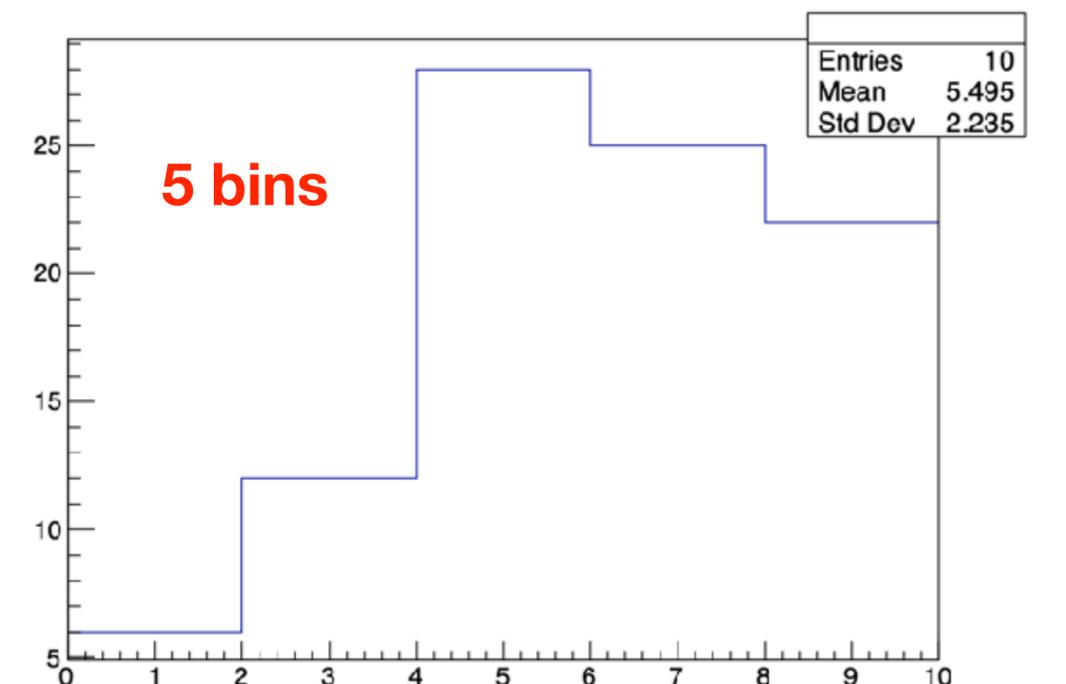
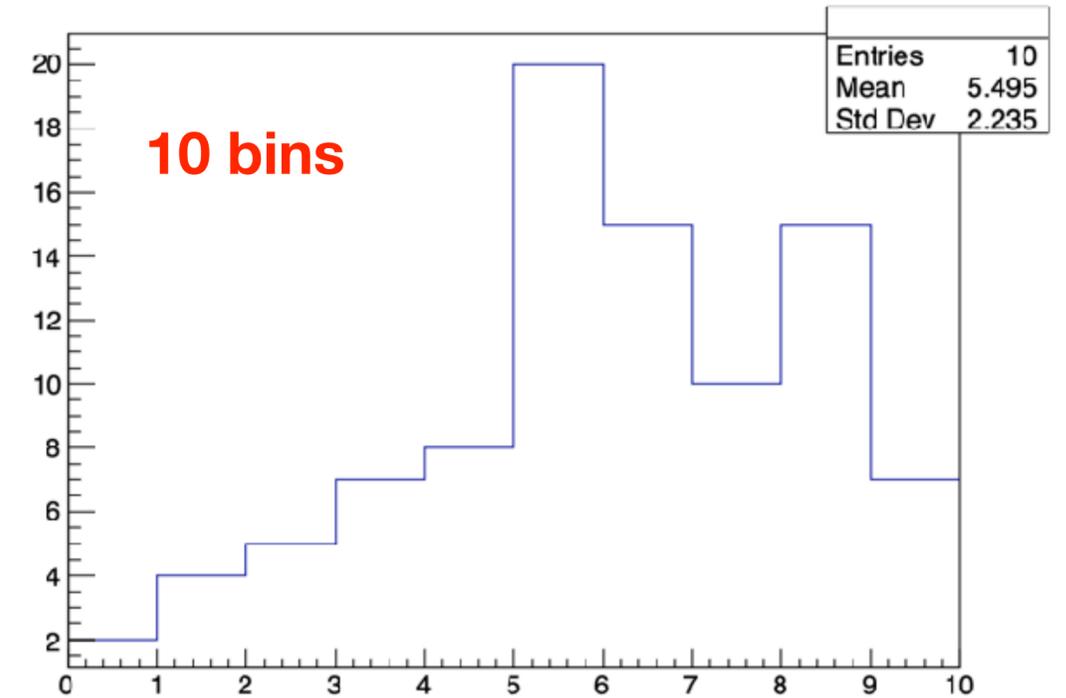
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## 2. Cross section (4): measurement in experiments

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha} (N_{data,i}^{selected} - N_{data,i}^{bckg})}{\phi \cdot T \cdot \epsilon \cdot \Delta x}$$

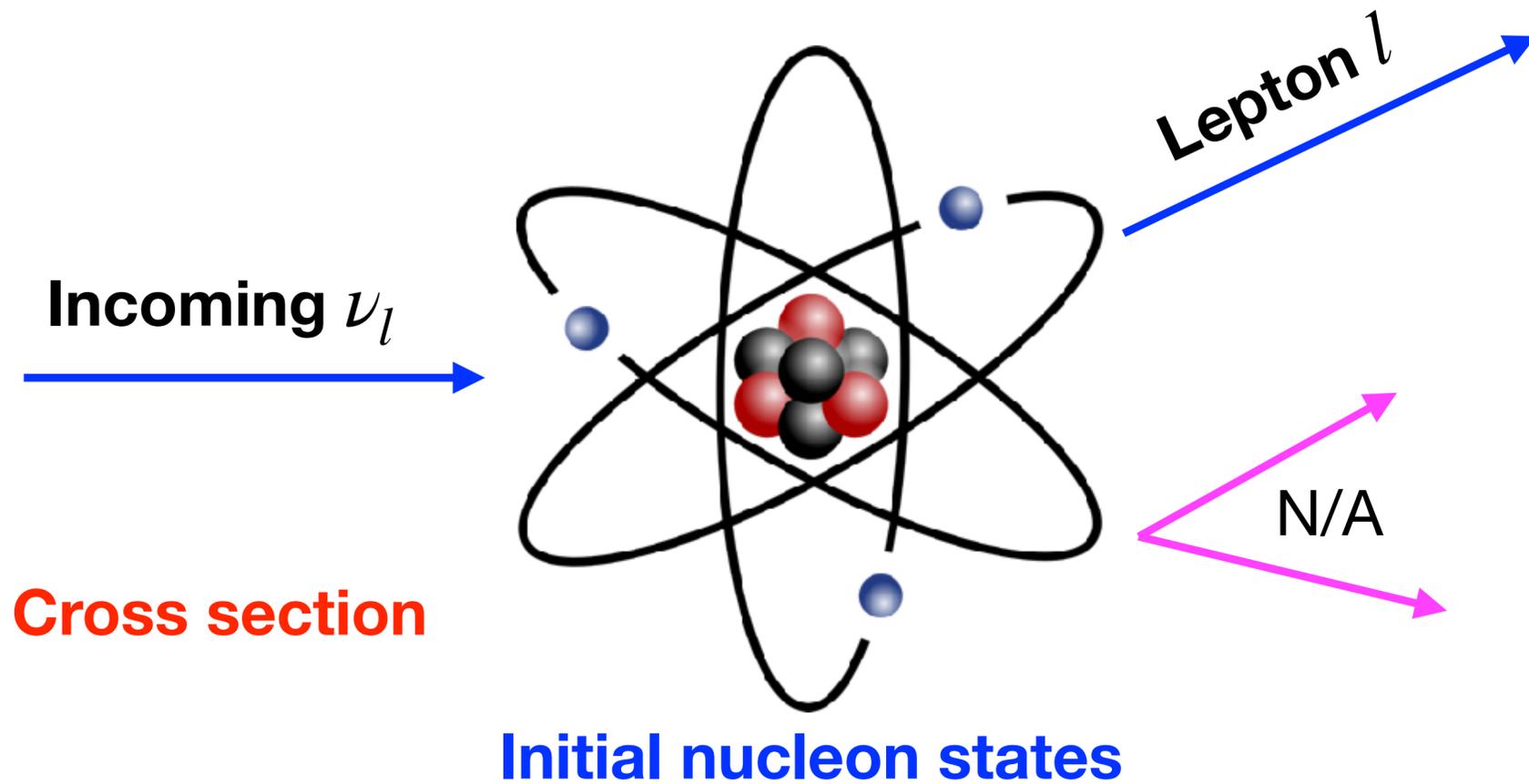
- **Bin width  $\Delta x$** 
  - Bin width is defined based on the number of expected events in each bin
  - More bins (smaller bin width) you have, better you can distinguish the features of the distribution
  - However, more bins you have, less events in each bin you get (worse statistics)



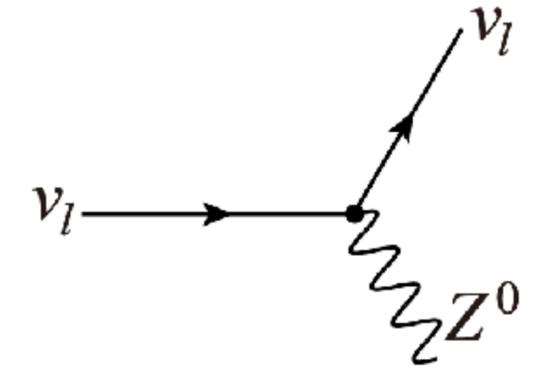
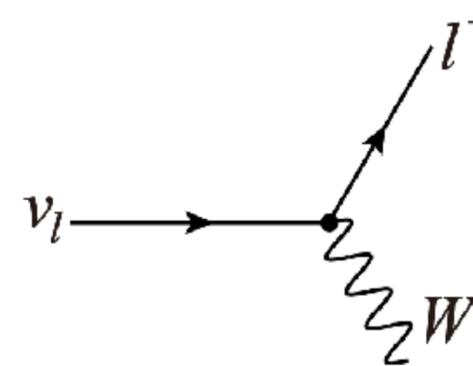
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# 3. Neutrino interactions



- Basically there are 2 types of interactions:
  - Charge current (CC) by exchange  $W^\pm$  bosons
  - Neutral current (NC) by exchange  $Z^0$  boson



Neutrino can interact with electron (e), the whole nucleus (A), nucleon (N), or quark (q) depending on its energy

# 3. Neutrino interactions

- **Threshold at which neutrino interactions happen**  $\nu + A \rightarrow \sum X$

$$E_{\nu}^{th} = \frac{(\sum m_X)^2 - m_A^2}{2m_A}$$

- **Example: CCQE at detectors**  $\nu_l + n \rightarrow l^- + p$

$$\begin{aligned}
 m_p &= 938.272 \text{ MeV} & m_e &= 0.511 \text{ MeV} \\
 m_n &= 939.565 \text{ MeV} & m_{\mu} &= 105.66 \text{ MeV} \\
 & & m_{\tau} &= 1776.9 \text{ MeV}
 \end{aligned}$$

$$\begin{aligned}
 E_{\nu_e}^{th} &= 0.0 \text{ MeV} \\
 E_{\nu_{\mu}}^{th} &\approx 0.1 \text{ GeV} \\
 E_{\nu_{\tau}}^{th} &= 3.5 \text{ GeV}
 \end{aligned}$$

Detector	Target	Typical E	nu_e	nu_mu	nu_tau
SK	H2O	~ MeV - few GeV	✓	✓	✗
SNO	Heavy water	5 MeV - 20 MeV	✓	✗	✗
OPERA	Emulsion cloud chamber	~ 17 GeV	✓	✓	✓
IceCube	Ice	~ GeV - PeV	✓	✓	✓

# 3. Neutrino interactions

Interaction type	Reaction	Threshold energy	Main experiments	Dominant for
Elastic scattering with electrons	$\nu_\alpha + e^- \rightarrow \nu_\alpha + e^-$	$E_\nu > 0 \text{ MeV}$	Super-K, SNO, Borexino	Solar neutrinos
Quasi-elastic scattering with electron	$\nu_\mu + e^- \rightarrow \nu_e + \mu^-$ $\nu_\tau + e^- \rightarrow \nu_e + \tau^-$	$E_\nu > 10.92 \text{ GeV}$ $E_\nu > 3089 \text{ GeV}$	IceCube, ANTARES, ARIANNA	High and ultra-high energy neutrinos
Inverse beta decay	$\bar{\nu}_e + p \rightarrow e^+ + n$	$E_\nu > 1.8 \text{ MeV}$	KamLAND, DayaBay, JUNO, Double Chooz	Reactor, supernova neutrinos
Coherent pion production	$\nu_l + A \rightarrow l^- + A + \pi^+$	$E_\nu > \sim \text{keV}$	COHERENT	Low-E neutrinos
Resonant meson production	$\nu_l + N \rightarrow \Delta(1232) \rightarrow l^- + N' + m$	$E_\nu > 200 \text{ MeV}$	T2K, MINERvA, NOvA, MicroBooNE	Medium-E neutrino beams
Quasi-elastic scattering	$\nu_l + n \rightarrow l^- + p$	$E_{\nu_e}^{th} = 0.0 \text{ MeV}$ $E_{\nu_\mu}^{th} \approx 0.1 \text{ GeV}$ $E_{\nu_\tau}^{th} = 3.5 \text{ GeV}$	T2K, NOvA, MiniBooNE	Oscillation at accelerator neutrinos
Inelastic scattering	$\nu_l + A \rightarrow l^-/\nu + X$	$E_\nu > 10s \text{ MeV}$	SNO, Super-K	Detector backgrounds, SN modeling
Deep inelastic scattering	$\nu_l + N \rightarrow l^- + X$	$E_\nu > 1 \text{ GeV}$	IceCube, ANTARES, ARIANNA, NOvA	High and ultra-high energy neutrinos

# 3. Neutrino interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

• **Quasi-elastic scattering (QE):**

- CCQE:  $\nu_l + n \rightarrow l^- + p$
- NC:  $\nu_l + N \rightarrow \nu_l + N'$

• **Resonant meson production (RES):**

- CC RES:  $\nu_l + N \rightarrow l^- + N' + m$
- NC RES:  $\nu_l + N \rightarrow \nu_l + N' + m$

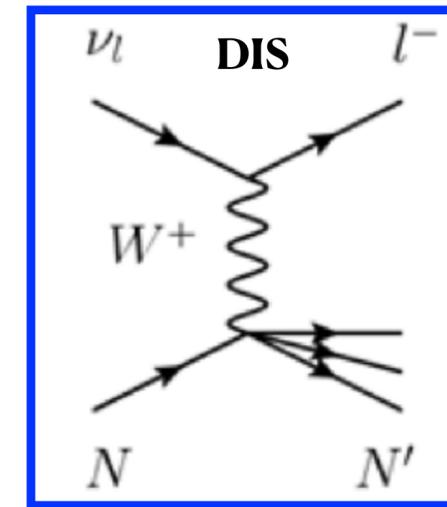
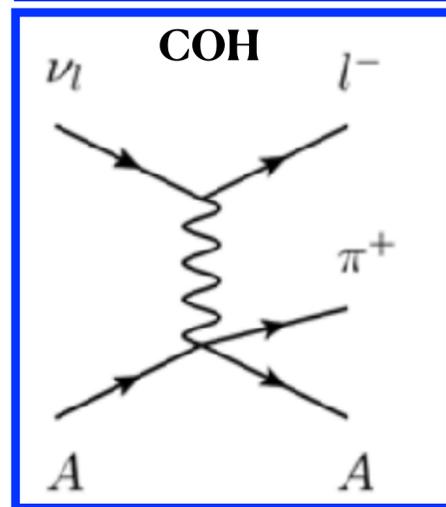
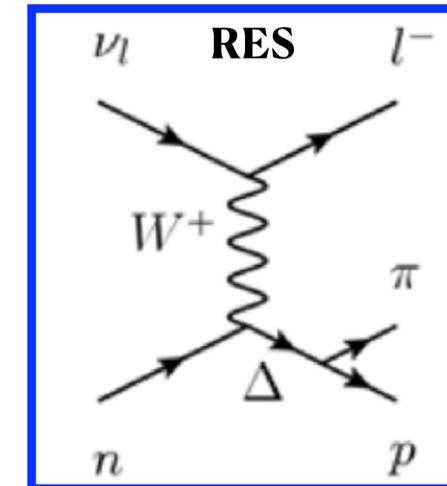
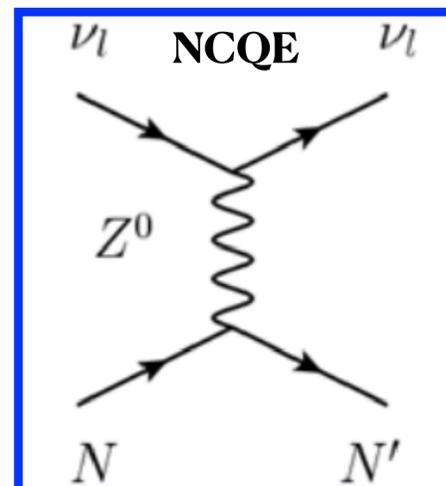
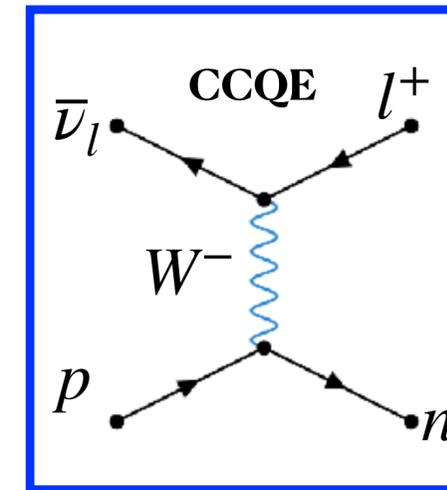
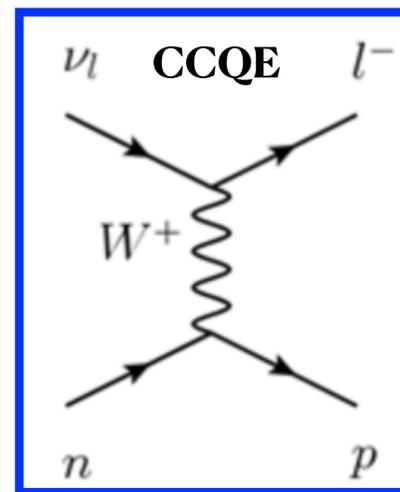
• **Coherent pion production (COH):**

- CC COH:  $\nu_l + A \rightarrow l^- + A + \pi^+$
- NC COH:  $\nu_l + A \rightarrow \nu_l + A + \pi^0$

• **Deep inelastic scattering (DIS):**

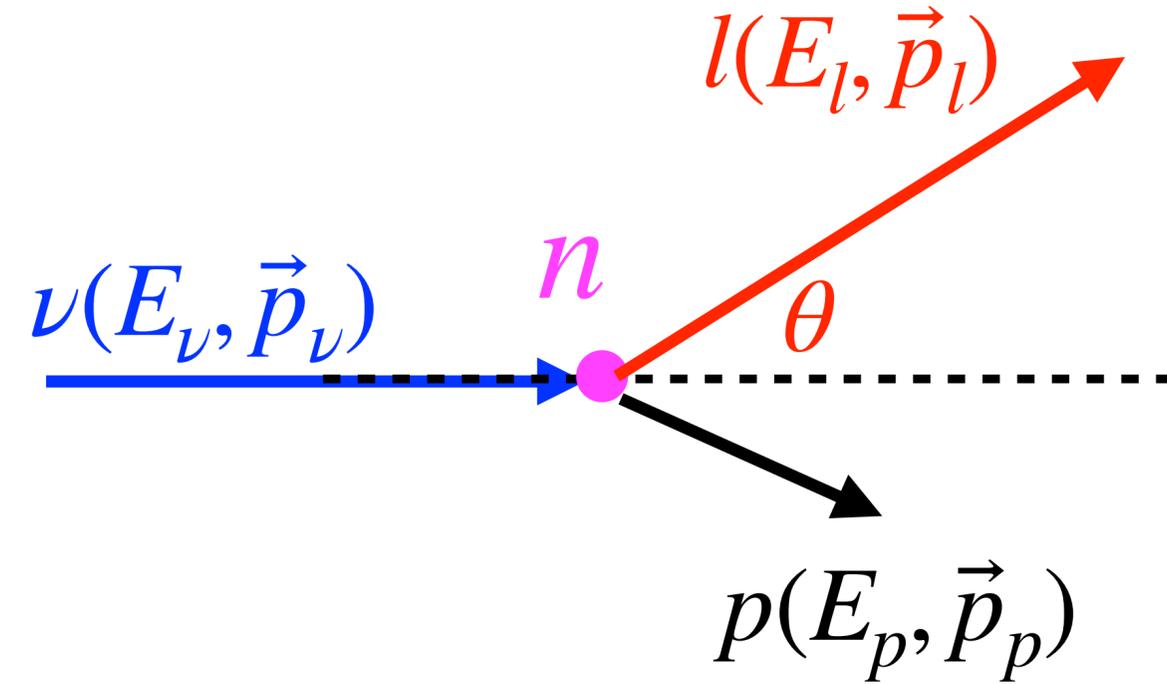
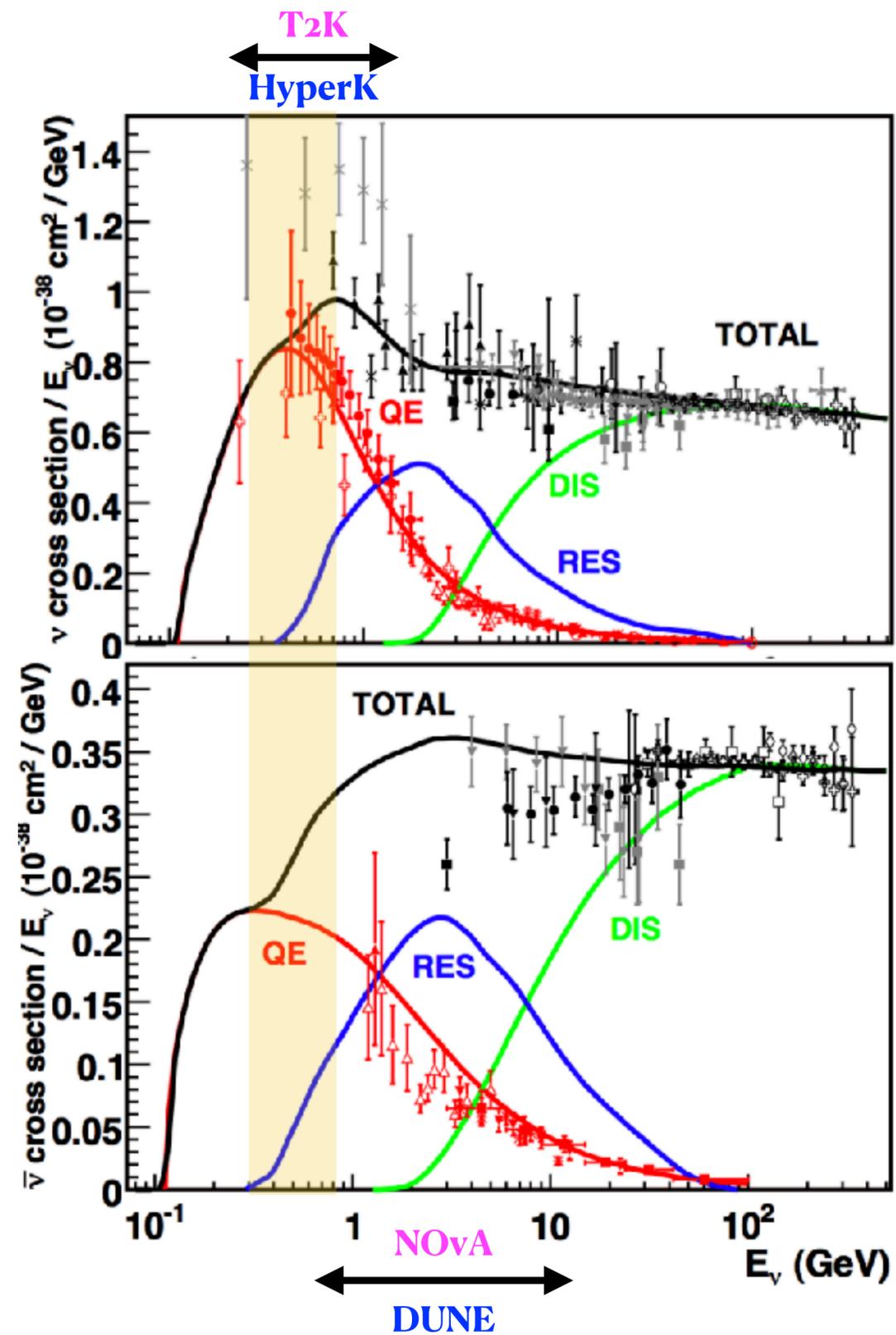
- CC DIS:  $\nu_l + N \rightarrow l^- + N' + \text{hadrons}$
- NC DIS:  $\nu_l + N \rightarrow \nu_l + N' + \text{hadrons}$

Where  $l^- = \{e^-, \mu^-, \tau^-\}$ ;  $N = \{p, n\}$ ;  
 $m = \{\pi, \eta, K\}$ ;  $A$  : nucleus



- There are similar channels for anti-neutrino
- For RES at low E,  $CC1\pi$  &  $NC1\pi$  dominates
- We will focus on neutrino interactions at this energy range

# 3. Neutrino interactions: Quasi-elastic scattering (QE):



CCQE:  $\nu_l + n \rightarrow l^- + p$

- Dominated channel at accelerator-based experiments such as T2K, Hyper-K

### 3. Neutrino interactions: Quasi-elastic scattering (QE):

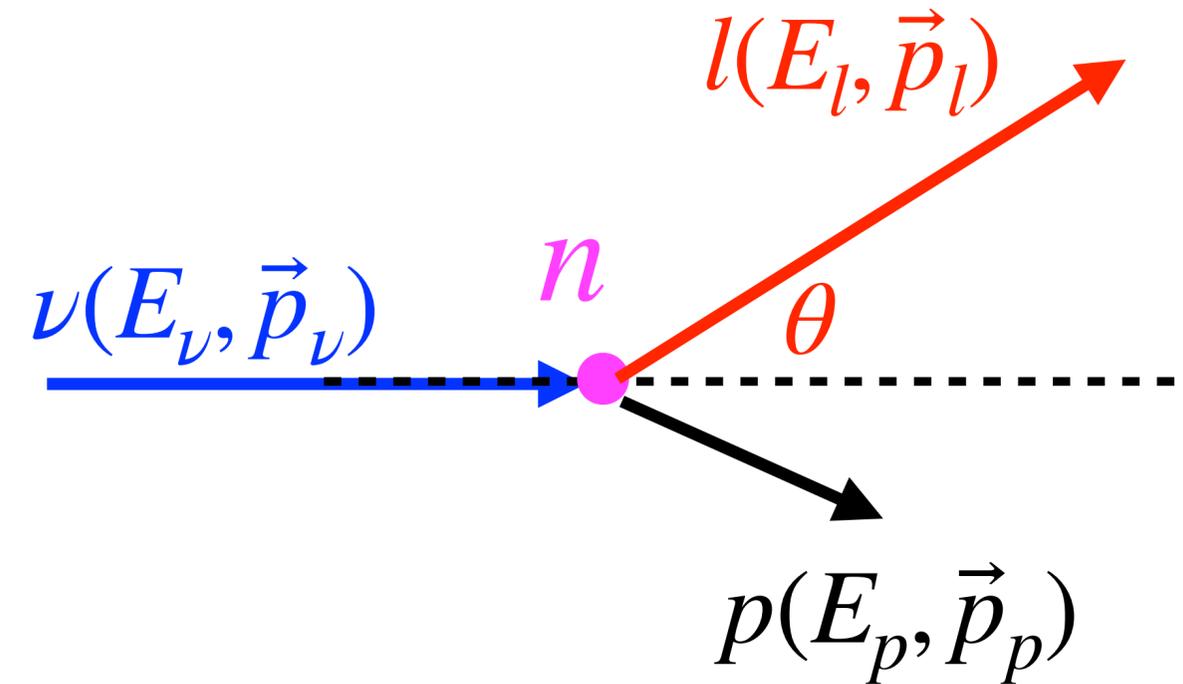
- **Reconstructed energy**

- CCQE:  $\nu_l + n \rightarrow l^- + p$
- Neutrino reconstructed energy (HW: derive it)

$$E_\nu = \frac{2m_n E_l - m_l^2}{2(m_n - E_l + p_l \cos \theta)}$$

$p_l, E_l, \theta$  : out-going lepton momentum, energy, scattered angle

$m_p, m_n, m_l$  : proton, neutron, lepton masses



# Summary

- Three active neutrinos ( $\nu_e, \nu_\mu, \nu_\tau$ ) and their anti-particles interact with matter by exchange  $W^\pm$  via CC or  $Z^0$  via NC
- Active neutrinos only participate in weak interaction with extremely small  $\lambda_{\text{sec}}$
- A neutrino can have elastic, quasi-elastic, or inelastic scattering with electron, nucleus, or nucleon depending on its energy
- Theoretical calculation of  $\lambda_{\text{sec}}$  is difficult when dealing with nuclear effects
- Experimental measurement of  $\lambda_{\text{sec}}$  is difficult due to tiny  $\lambda_{\text{sec}}$  and technical barriers
- Interactions in medium energy region ( $\sim \text{GeV}$ ) is important because they cover mostly accessible and abundant neutrino sources (solar, reactor, atmospheric, and accelerator)

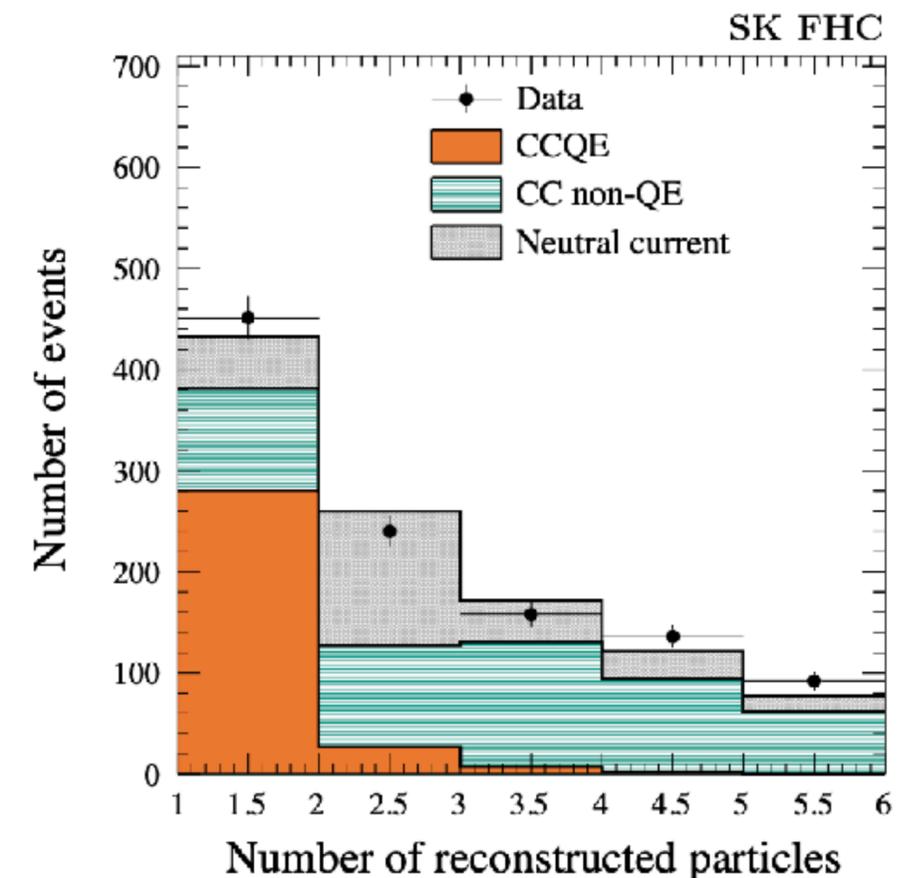
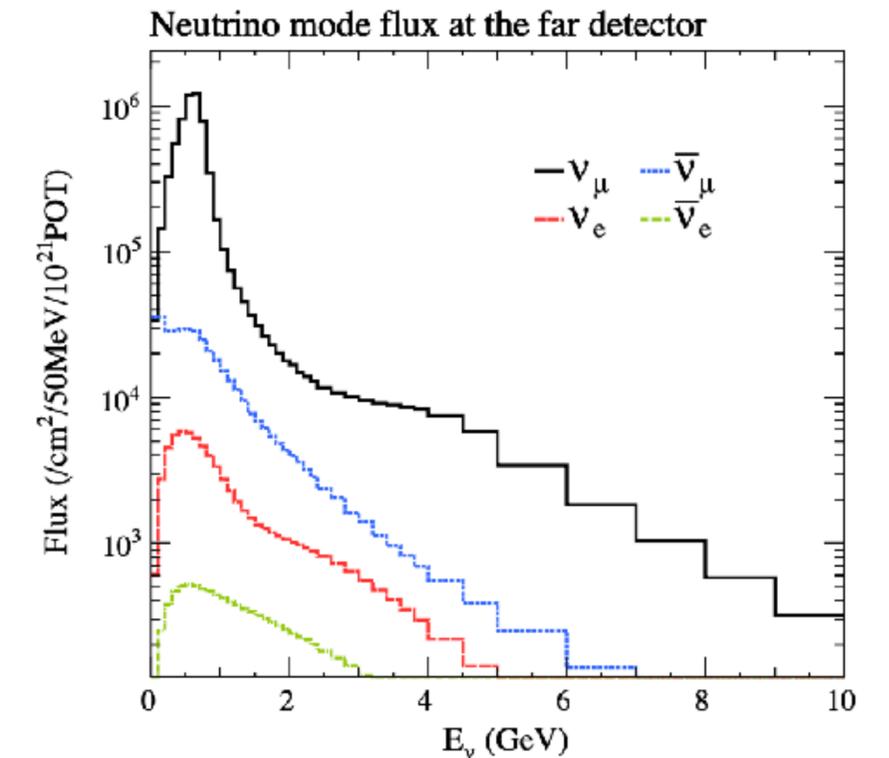
# Contents

1. A brief history and properties of neutrinos
2. Cross sections
3. Neutrino interactions
4. **Introduction to NEUT - an event generator**
  - 4.1: Neutrino event generator
  - 4.2: Introduction to NEUT
  - 4.3: Practice with NEUT
  - 4.4: Exercises

# 4. Introduction to NEUT - an event generator

## 4.1: Neutrino event generators

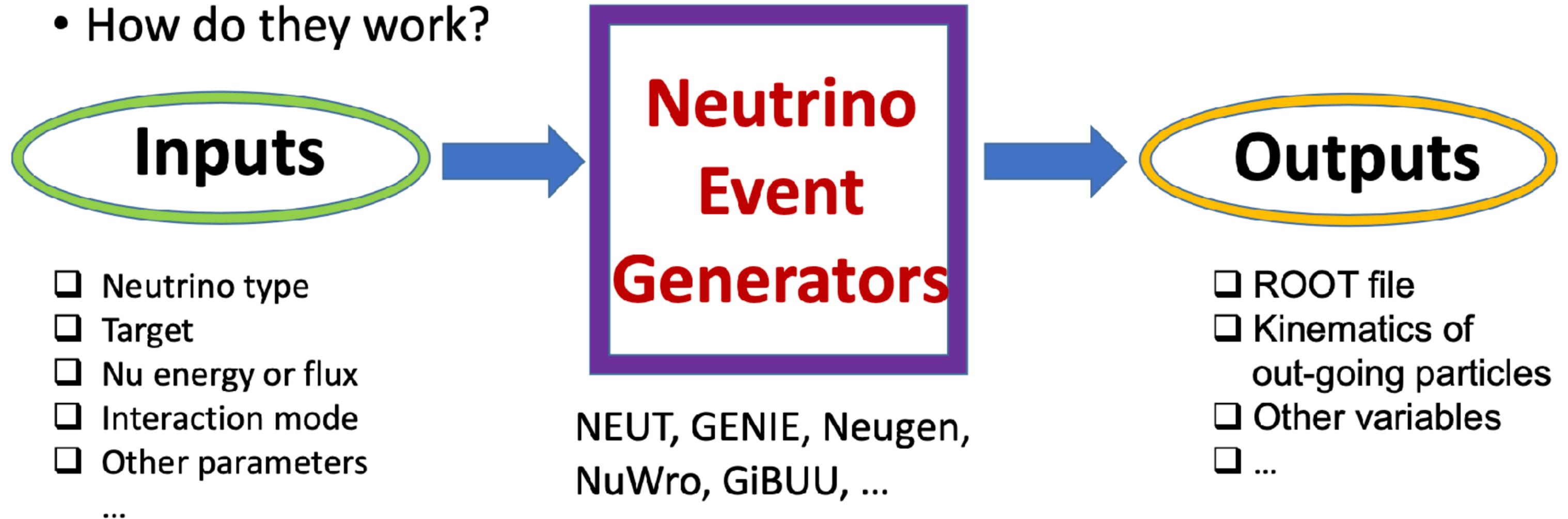
- Neutrino event generators are MC softwares used for simulating neutrino interactions
- They are bridges connecting between theory and experiment
- They play an important role in oscillation analysis:
  - Predicting neutrino flux
  - Simulate interaction channels
  - Simulate signals and backgrounds
  - Calculate efficiencies and systematics uncertainties



# 4. Introduction to NEUT - an event generator

## 4.1: Neutrino event generators

- How do they work?



From H. Van

# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT

- Originally developed for Kamiokande experiment to predict neutrino-induced backgrounds for nucleon decay search
- Has been officially used for Super-K and T2K (currently NEUT 5.4.0)
- Mainly written in Fortran, not open source yet
- Primarily simulate neutrino-nucleus/nucleon interactions in wide range of energy from 100 MeV to few TeV ( $10^{12}$  eV)
- Incorporates nuclear effects
- Recently implemented CCQE and multi-nucleon models, single pion production models, and electron - nucleus scattering

# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT: interaction models

Process	Model in NEUT
CCQE	Llewellyn-Smith + RFG / SF / LFG
2p2h / MEC	Nieves et al. (empirical + theoretical)
Resonance (RES)	Rein-Sehgal
Coherent Pion (COH)	Rein-Sehgal / Berger-Sehgal
Deep Inelastic (DIS)	PDFs (GRV98), hadronization from PYTHIA
Final State Interactions	Semi-classical cascade (INTRANUKE-style)
Elastic / NC scattering	Standard QFT processes

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

- **Working space is IOP cluster**
- CERNLIB and ROOT are needed for ROOT installation (**already available on IOP cluster**)
- **Source code:** [/home/vson/neut\\_5.4.0](#)
- **Car file:** [/home/vson/neut\\_5.4.0/src/neutsmpl/Cards/neut\\_5.4.0\\_nd5\\_C\\_ccqe.card](#)

```
[vson@tcp ~]$ pwd
/home/vson
[vson@tcp ~]$ ls
neut_5.4.0
[vson@tcp ~]$ ls neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card
neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card
```

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### 1. Log in IOP cluster (2024)

```
$ ssh -XY vson@202.151.162.171  
pass: son...
```

### 2. Create a new folder with your\_name (ex: tvngoc)

```
$ mkdir your_name
```

### 3. Copy source folder to your working directory you have just created

```
$ cp -r -p neut_5.4.0/ your_name
```

### 4. Go to your working directory

```
$ cd your_name/neut_5.4.0/
```

```
$ ls
```

```
include lib setup_env_neut540.sh src
```

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### 5. Check where you are

```
$ pwd
```

```
/home/vson/your_name/neut_5.4.0
```

**6. Before working with neut, we need to setup environment.** Use vim or nano or other text editors to open and modify the file *setup\_env\_neut540.sh*

Change the default directory to your directory

```
$ vi setup_env_neut540.sh
```

### 7. Press “i” to modify:

```
#export NEUT_ROOT=/home/vson/neut_5.4.0
```

```
export NEUT_ROOT=/home/vson/your_name/neut_5.4.0 (same as output when  
you pwd)
```

Press “**esc**”, then “**:wq**”, press “**enter**” to save and close the file

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### 8. Source the environment file

```
$ source setup_env_neut540.sh
```

### 9. Go to working directory

```
$ cd /src/neutsmpl
```

### 10. Clean and make to create binary files

```
$ /bin/csh Cleanneutsmpl.csh (or ./Cleanneutsmpl)
```

```
$ /bin/csh Makeneutsmpl.csh (or ./Makeneutsmpl)
```

11. Press “ls” then “enter” to see if there are “**neutroot2**” binary file created

### 12. Open card file to see what inside

```
$ vi Cards/neut_5.4.0_nd5_C_ccqe.card
```

Press “i” to enter modified mode, press “esc” and then “:wq” to save and exit.

The letter “C” at the beginning of the line means “comment out” that line.

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

- NEUT input: Setup card file in `/home/vson/your_name/neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card`
  - Used to specify models, parameters, interaction modes, ...
  - When setting your card file, check carefully there is no letter **C** at the beginning of the line (**C**: **comment out that line**)

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

Define **number of events** and **neutrino flavor**

```
C-----  
C Number of events to generate  
C EVCT-NEVT 1000000  
EVCT-NEVT 1000  
C-----  
  
C-----  
C Neutrino flavor ( PDG code )  
C 12 : nue ( -12 : anti-nue )  
C 14 : numu ( -14 : anti-numu )  
C 16 : nutau ( -16 : anti-nutau )  
C  
C neutrino flavor setting  
EVCT-IDPT 14  
C-----
```

Define **flux file** and **histogram**

```
C off-axis numu => EVCT-IDPT 14  
EVCT-FILENM  
'nd5_tuned13av1.1_13anom_run1-7c_fine.root'  
EVCT-HISTNM 'enu_nd5_tuned13a_numu'  
  
C off-axis numubar => EVCT-IDPT -14  
C EVCT-FILENM  
'nd5_tuned13av1.1_13anom_run5c-7b_antinumode_fine.root'  
C EVCT-HISTNM 'enu_nd5_tuned13a_numub'  
  
C on-axis numu => EVCT-IDPT 14  
C EVCT-FILENM 'nd34_tuned_11bv3.1_250ka.root'  
C EVCT-HISTNM 'ing3_tune_numu'  
  
C on-axis numubar => EVCT-IDPT -14  
C EVCT-FILENM 'run5c_tune_INGRID_13a_1_1.root'  
C EVCT-HISTNM 'ing3_tune_numub'
```

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### Define target: CH

```
C-----  
C **** TARGET INFORMATION ****  
C NUMBNDN : total number of neutron  
C      (e.g. CH => 6, H2O => 8, Ar => 22, Fe => 30)  
C NUMBNDP : total number of bound proton  
C      (e.g. CH => 6, H2O => 8, Ar => 18, Fe => 26)  
C NUMFREP : total number of free proton  
C      (e.g. CH => 1, H2O => 2, Ar => 0, Fe => 0)  
C NUMATOM : atomic number of atom heavier than  
hydrogen  
C      (e.g. CH => 12, H2O =>16, Ar => 40, Fe => 56)  
  
NEUT-NUMBNDN 6  
NEUT-NUMBNDP 6  
NEUT-NUMFREP 1  
NEUT-NUMATOM 12
```

### Define interaction mode: CCQE

```
C-----  
C MODE : Interaction mode  
C      0 : normal ( default )  
C      -1 : input cross section by CRSNEUT  
C      n : select one mode ( n > 0 ) See nemodsel.F  
C      n = 1 : charged current Q.E.  
C      n = 11,12,13  
C      : charged current Single pi production  
C      n = 16 : coherent Single pi production  
C      n = 21 : charged current Multi pi production  
C      n = 31,32,33,34  
C      : neutral current Single pi production  
C      n = 36 : coherent Single pi production  
C      n = 41 : neutral current Multi pi production  
C      n = 51,52 : neutral current elastic  
C      n = 22,42,43 : single eta production  
C      n = 23,44,45 : single K production  
C  
NEUT-MODE 1
```

# 4. Introduction to NEUT - an event generator

## 13. Run neut with card file you have just edited.

```
$ ./neutroot2 Cards/neut_5.4.0_nd5_C_ccqe.card output_numu_offaxis_ your_name.root
```

o Executable **neutroot2** will:

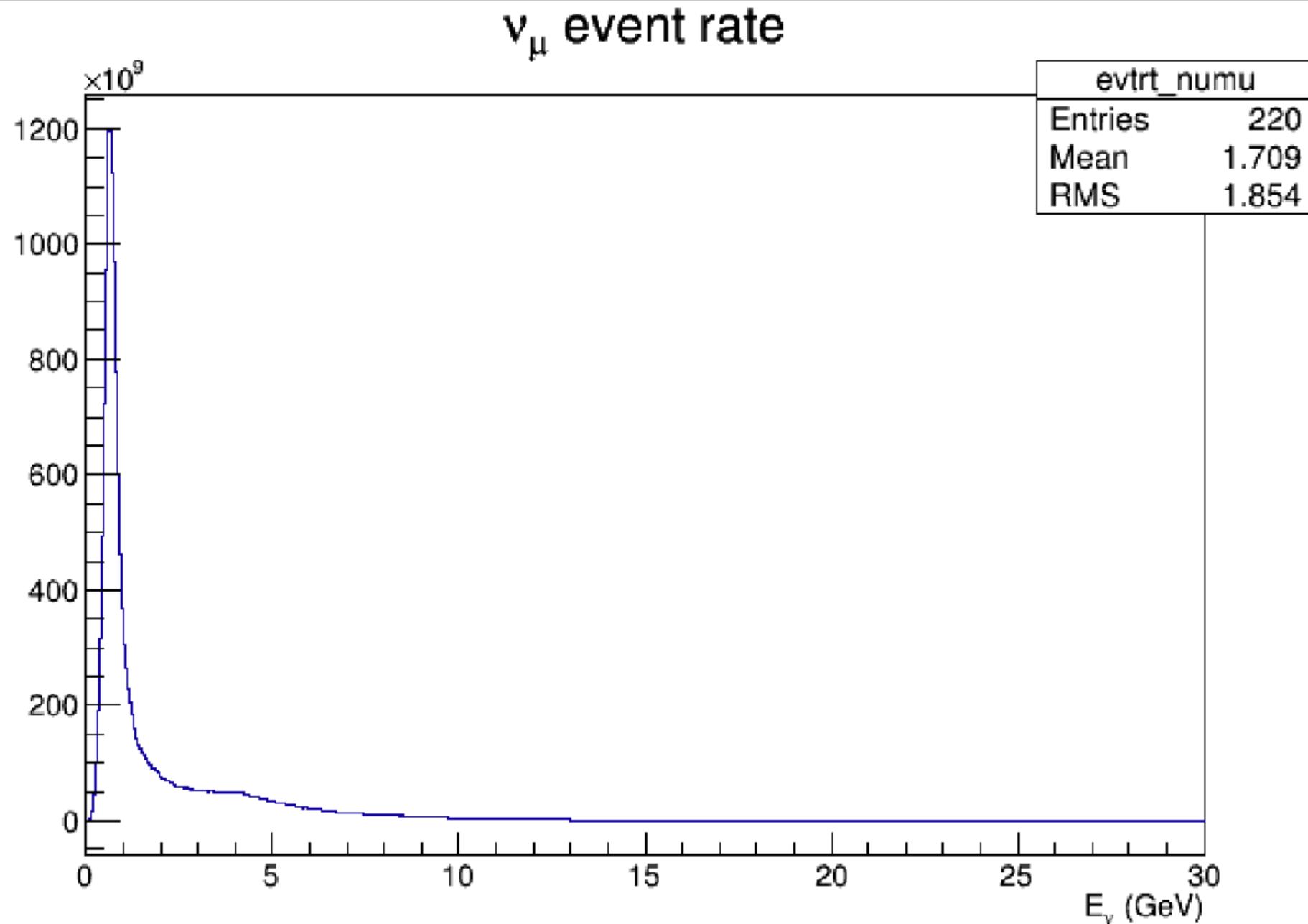
- Reads the card file with `neutcore/necard.F` (neut\_5.4.0\_nd5\_C\_ccqe.card)
- Fills the interaction model with `neutcore/nefillmodel.F`
- Creates output root file (**output\_numu\_offaxis\_ **your\_name**.root**)
- Read input flux histogram (**enu\_nd5\_tuned13a\_numu** in **nd5\_tuned13av1.1\_13anom\_run1-7c\_fine.root** flux file)
- Start the loop over the generated events (**1000 events**)
  - Set the vertex position; the neutrino direction and energy.
  - Draw the event rate (if there is an input flux histogram)
  - Generate events with `neutsmpl/nevecgen.F`
  - Call `neutcore/nevent.F` to compute the kinematics for each event
  - Consider other effects (radiative corrections, nucleon re-scattering)
- Saves all the information in the ROOT output file (**output\_numu\_offaxis\_ **your\_name**.root**)

# 4. Introduction to NEUT - an event generator

```
[vson@tcp neutsmpl]$ root -l output_numu_offaxis_tvngoc.root
root [0]
root [1] .ls
TFile** output_numu_offaxis_tvngoc.root
TFile*  output_numu_offaxis_tvngoc.root
KEY: TH1Dflux_numu;1#nu_{#mu} flux
KEY: TH1Devtrt_numu;1#nu_{#mu} event rate
KEY: TH1Dfluxhisto;1#nu_{#mu} flux
KEY: TH1Dratehisto;1#nu_{#mu} event rate
KEY: TTree neuttree;1Neut Tree
root [2] neuttree->Print()
*****
*Tree      :neuttree  : Neut Tree *
*Entries   :    1000  : Total =      4090013 bytes File Size =    1335697 *
*          :          : Tree compression factor =    3.05 *
*****
*Branch    :vectorbranch *
*Entries   :    1000  : BranchElement (see below) *
*.....*
*Br       0 :fUniqueID : UInt_t *
*Entries   :    1000  : Total Size=    4814 bytes File Size =    432 *
*Baskets   :         4 : Basket Size=    1100 bytes Compression= 10.00 *
*.....*
*Br       1 :fBits     : UInt_t *
*Entries   :    1000  : Total Size=    9546 bytes File Size =    3283 *
*Baskets   :         12 : Basket Size=    1100 bytes Compression= 2.74 *
```

# 4. Introduction to NEUT - an event generator

```
root [3] evtrt_numu->Draw()
```



# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

14. Root macro file for analyzing the output. Edit this file if you want to do the exercises.

\$ vi make\_histos\_standalone\_neut540\_ccqe\_simple.cc

```
#include <iostream>
using namespace std;
//pre-define function
void fill_histos(char *, char *, Double_t );
//in_fname is the input file name you get after run neutroot2
//out_fname is the output file name wher eyou want to put the interesting information
void make_histos_standalone_neut540_ccqe_simple(char *in_fname, char *out_fname){
    //loading needed libraries
    gSystem->Load("../neutclass/neutnucfsistep.so");
    gSystem->Load("../neutclass/neutnucfsivert.so");
    gSystem->Load("../neutclass/neutvtx.so");
    gSystem->Load("../neutclass/neutpart.so");
    gSystem->Load("../neutclass/neutfsipart.so");
    gSystem->Load("../neutclass/neutfsivert.so");
    gSystem->Load("../neutclass/neutvect.so");
    fill_histos(in_fname,out_fname);
}

//this is the main function to get information for each event and make plot
void fill_histos(char *in_fname, char *out_fname){
    // To check if you really have input file (file created by neutroot2)
    TFile *f = new TFile(in_fname,"READ");
    if ( f == NULL ){
        cout << "Failed to open " << in_fname << endl;
        return;
    }
    // Link the branches of the file.
    TTree *tn = (TTree*)(f->Get("neuttree"));
    NeutVtx *nvtx = new NeutVtx();
    tn->SetBranchAddress("vertexbranch",&nvtx);
    NeutVect *nvect = new NeutVect();
    tn->SetBranchAddress("vectorbranch",&nvect);
}
```

Load NEUT libraries

Open root file created by neutroot2

Get branches of the ttree

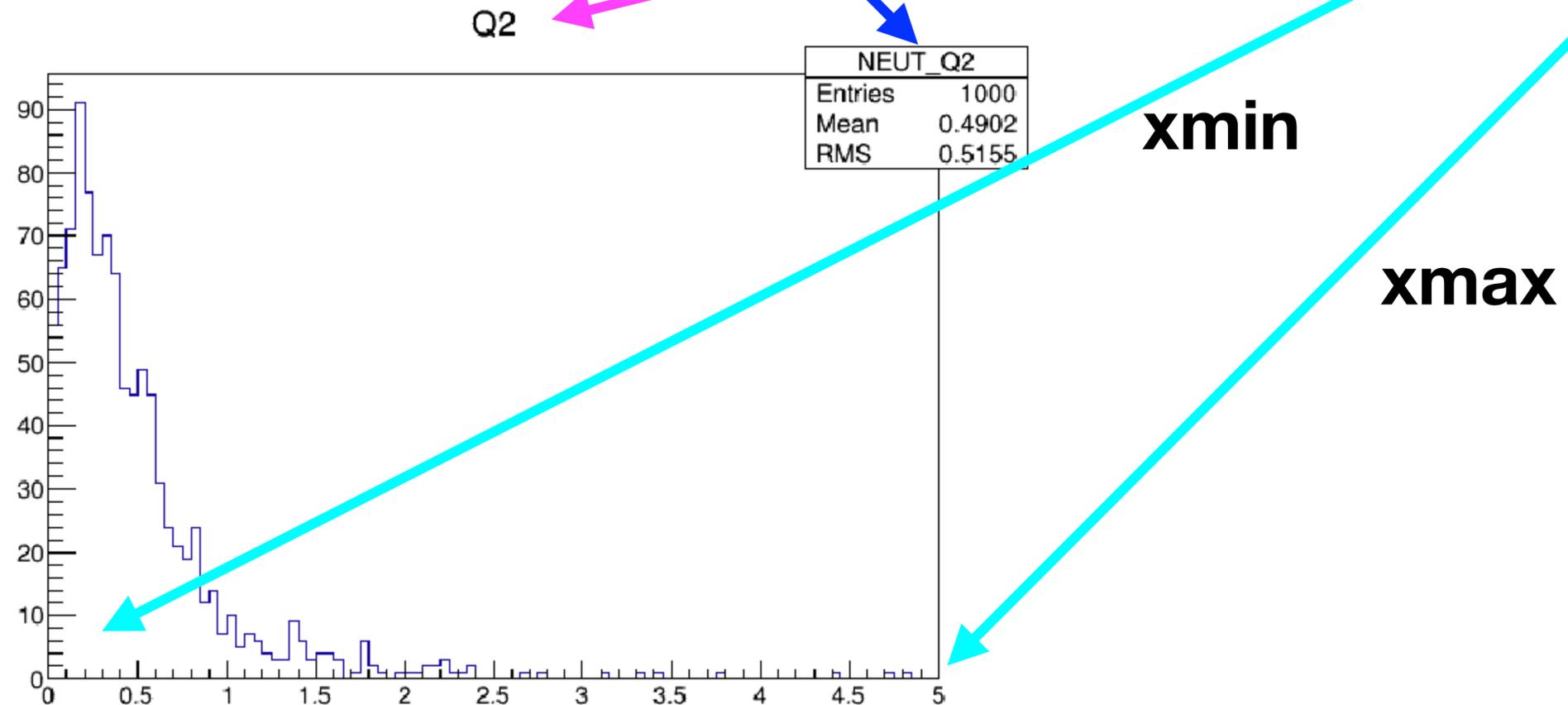
# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

14. Root macro file for analyzing the output. Edit this file if you want to do the exercises.

Define histograms

```
TH1D *NEUT_Q2 = new TH1D("NEUT_Q2", "Q2", 100, 0., 5.);
```



# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

```
//make a loop over all event you generated
for ( Int_t j = 0 ; j < nevents ; j++ ){
  tn->GetEntry(j);
  if (j%iprintProcess == 0) cout<<"Processing "<<int(j*100./nevents)<<"% of events"<<endl;
  //we consider CCQE only
  //it Mode is equal to 1 (check card file for interaction mode)
  //for CCQE, particle index is follow: 0 is incoming neutrino, 1 is nucleon, 2 is muon, 3 is proton
  Int_t nu_index, neutron_index, muon_index, proton_index;
  for(Int_t i=0 ;i<nvect->Npart(); i++){
    if(nvect->PartInfo(i)->fPID == 13) {muon_index=i; cout<<"This is a muon with the index: "<<muon_index<<endl;}
    else if(nvect->PartInfo(i)->fPID == 14){nu_index=i; cout<<"This is a Muon Neutrino with the index: "<<nu_index<<endl;}
    else if(nvect->PartInfo(i)->fPID == 2212){proton_index=i; cout<<"This is a proton with the index: "<<proton_index<<endl;}
  }

  if (abs(nvect->Mode) == 1){ // select only CCQE
    ++nevent_ccqe;
    //to check if it's really muon
    if((nvect->PartInfo(muon_index))->fPID != 13){ // Oh Look ! There is a muon
      cout<<"Oops, for event "<<j<<" particle w/ index 2 is not muon"<<endl;
    }
    // - Fill basic info. -
    // -----
    //fill interaction mode
    NEUT_mode->Fill(nvect->Mode);
    //fill neutrino energy
    double e_nu =(nvect->PartInfo(nu_index))->fP.E()/1000.; //true energy from MC in GeV
    NEUT_enu->Fill(e_nu);
  }
}
```

Loop all over events

Declare particle index

Loop over all particles in an event

Define true neutrino energy

Fill histogram

# 4. Introduction to NEUT - an event generator

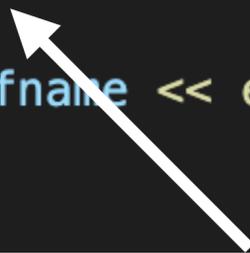
## 4.3: Practice with NEUT

```
// Create the output root file analysed file that will contain all our info
TFile *f2 = new TFile(out_fname,"RECREATE");
if ( f2 == NULL ){
    cout << "Failed to (re)create " << out_fname << endl;
    return;
}

gStyle->SetOptStat(0);
gStyle->SetOptTitle(0);
gStyle->SetLabelSize(0.05, "x");
gStyle->SetLabelSize(0.05, "y");

// write histograms into root files:
NEUT_mode->Write();
NEUT_enu->Write();

NEUT_pmu->Write();
NEUT_anglemu->Write();
NEUT_cosanglemu->Write();
NEUT_mu_pvscosangle->Write();
```



Open root file to save histogram



Save histograms into root file

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### 15. Run the file to create basic histograms.

```
$ root -b -q 'make_histos_standalone_neut540_ccqe_simple.cc("output_numu_offaxis_your_name.root",  
"output_numu_offaxis_your_name_basichisto.root")'
```

Your defined output, which stores the histograms

Output of neutroot2

#### o The following histograms have already created

KEY: TH1D NEUT\_mode;1 NEUT mode  
KEY: TH1D NEUT\_enu;1 Neutrino energy  
KEY: TH1D NEUT\_pmu;1 muon momentum  
KEY: TH1D NEUT\_anglemu;1 muon angle  
KEY: TH1D NEUT\_cosanglemu;1 cos muon angle  
KEY: TH2D NEUT\_mu\_pvscosangle;1 muon momentum vs. cos(#theta)  
KEY: TH1D NEUT\_enurec;1 (equation (**Enurec2**) in slide 48)  
KEY: TH1D flux\_numu;1 #nu\_{#mu} flux  
KEY: TH1D evtrt\_numu;1 #nu\_{#mu} event rate

# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

### 16. Open created root file to check the histograms

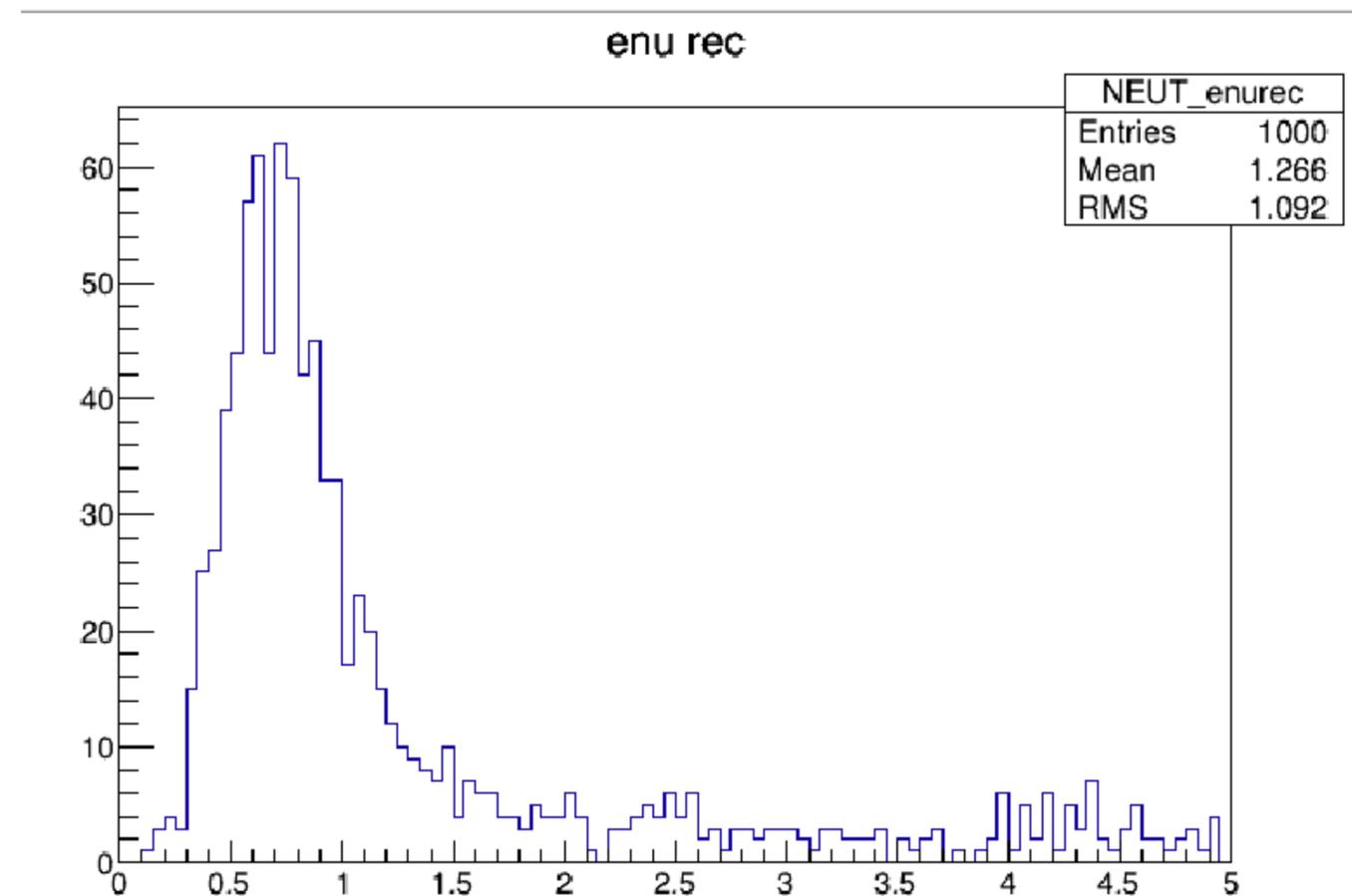
```
$ root -l output_numu_offaxis_ your_name_basichisto.root
```

```
root [0]
```

```
Attaching file output_numu_offaxis_ your_name_basichisto.root as _file0...
```

```
root [1] .ls
```

```
root [2] NEUT_enurec ->Draw();
```



# 4. Introduction to NEUT - an event generator

## 4.3: Practice with NEUT

- Working space is IOP cluster

- Summary

- 1. Source environment:

- ```
$ source setup_env_neut540.sh
```

- 2. Run NEUT to generate events

- ```
$ ./neutroot2 Cards/neut_5.4.0_nd5_C_ccqe.card output_numu_offaxis_your_name.root
```

- 3. Analyze the output

- ```
$ root -b -q
```

- ```
'make_histos_standalone_neut540_ccqe_simple.cc("output_numu_offaxis_your_name.root", "output_numu_offaxis_your_name_basichisto.root")'
```

- The macro is at `neut_5.4.0/src/neutsmpl/`

Name it yourself



# 4. Introduction to NEUT - an event generator

## 4.4: Exercises

- Edit `make_histos_standalone_neut540_ccqe_simple.cc` file to produce the following histograms

- Proton momentum
- Proton angle
- Cosine of proton angle

- 4 momentum transfer (equation (**Q2rec**) in slide Kinematic variables)
- Neutrino reconstructed energy (equation (**Erec1**) in slide Nu energy)
- Neutrino true energy (get from neut output), plot in the same canvas to compare with **Erec1** and **Erec2**

# 4. Introduction to NEUT - an event generator

## 4.4: Exercises

- **Hints:** edit [make\\_histos\\_standalone\\_neut540\\_ccqe\\_simple.cc](#)

### 1. Define proton ID: 2212

Where to fill

Particle	Code	Particle	Code
Proton	2212	Xi-zero	3322
Antiproton	-2212	Antixi-zero	-3322
Electron	11	Negative Xi	3312
Positron	-11	Positive Xi	-3312
Electron Neutrino	12	Omega-minus	3334
Electron Antineutrino	-12	Antiomega	-3334
Photon	22	Positive Tau	-15
Neutron	2112	Negative Tau	15
Antineutron	-2112	Tau neutrino	16
Positive Muon	-13	Tau antineutrino	-16
Negative Muon	13	D-plus	411
Kaon-zero long	130	D-minus	-411
Positive Pion	211	D-zero	421
Negative Pion	-211	AntiD-zero	-421
Positive Kaon	321	D_s-plus	431
Negative Kaon	-321	D s-minus	-431

```
95  Int_t nu_index, neutron_index, muon_index, proton_index;
96  for(Int_t i=0 ;i<nvect->Npart(); i++){
97      if(nvect->PartInfo(i)->fPID == 13) {muon_index=i; cout<<"This is a muon with the index: "<<muon_index<<endl;}
98      else if(nvect->PartInfo(i)->fPID == 14){nu_index=i; cout<<"This is a Muon Neutrino with the index: "<<nu_index<<endl;}
99      else if(nvect->PartInfo(i)->fPID == 2212){proton_index=i; cout<<"This is a proton with the index: "<<proton_index<<endl;}
100 }
```

Check if the particle is a proton

Assign proton index

Kaon-zero	311	AntiXi_c-minus	-4232
Antikaon-zero	-311	AntiXi_c-zero	-4132
Muon neutrino	14	AntiXi'_c-minus	-4322
Muon antineutrino	-14	AntiXi'_c-zero	-4312
Antisigma-minus	-3222	AntiOmega_c-zero	-4332
Antisigma-zero	-3212		
Antisigma-plus	-3112		

# 4. Introduction to NEUT - an event generator

## 4.4: Exercises

- **Hints:**

### 2. Define new histogram

```
TH1D * NEUT_epro = new TH1D("NEUT_epro","Ep", 100, 0., 5.);
```

### 3. Define what to fill in your histogram (Ex: Energy (GeV))

```
double epro = (nvect->PartInfo(proton_index))->fP.E()/1000.;
```

### 4. Fill histogram:

```
NEUT_epro -> Fill(epro);
```

### 5. Write histogram into root file:

```
NEUT_epro -> Write();
```

### 6. Rerun:

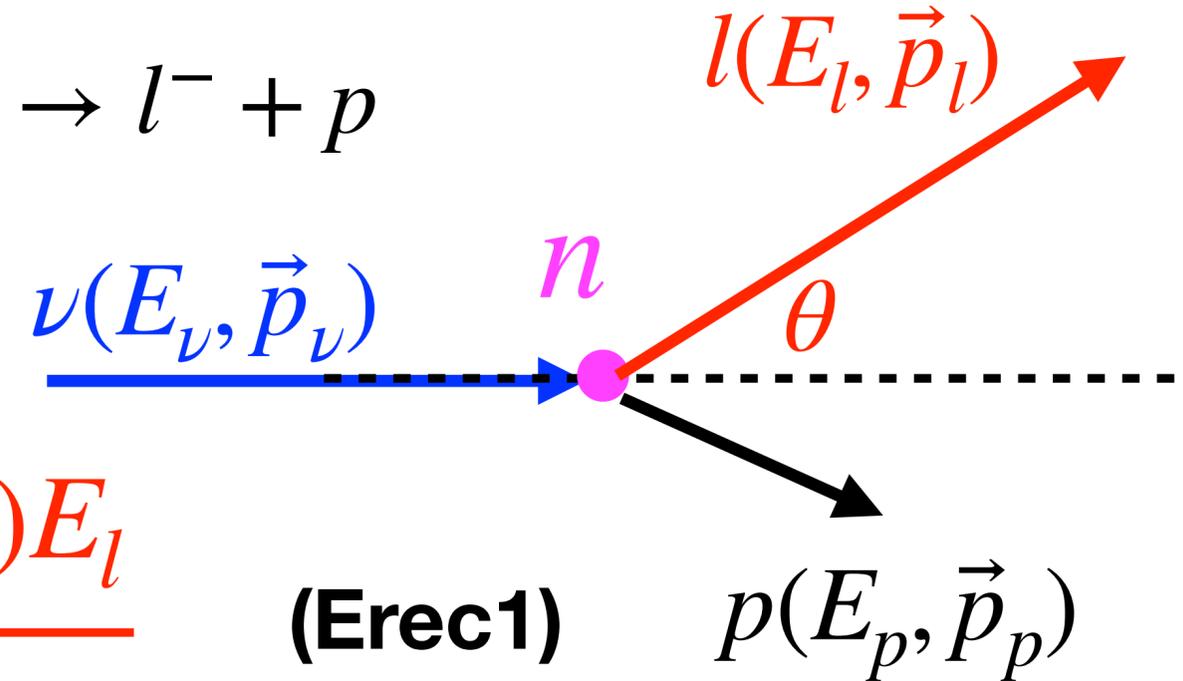
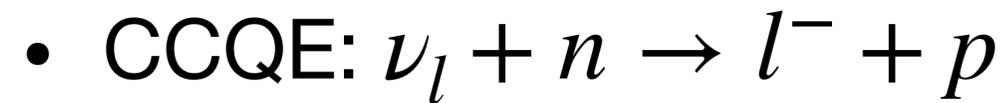
```
$ root -b -q 'make_histos_standalone_neut540_ccqe_simple.cc("output_numu_offaxis_  
your_name.root", "output_numu_offaxis_ your_name_basichisto.root")'
```

# 4. Introduction to NEUT - an event generator

## 4.4: Exercises

- **Useful information**

- **Neutrino reconstructed energy**



$$E_\nu = \frac{m_p^2 - m_l^2 - (m_n - E_b)^2 + 2(m_n - E_b)E_l}{2[(m_n - E_b) - E_l + p_l \cos \theta]} \quad \text{(Erec1)}$$

$p_l, E_l, \theta$  : out-going lepton momentum, energy, scattered angle

$E_b = 27 \text{ MeV}$  : neutron binding energy in Oxygen

$m_p, m_n, m_l$  : proton, neutron, lepton masses

- Remove  $E_b$  and consider  $m_p \approx m_n$ : 
$$E_\nu = \frac{2m_n E_l - m_l^2}{2(m_n - E_l + p_l \cos \theta)} \quad \text{(Erec2)}$$

# 4. Introduction to NEUT - an event generator

## 4.4: Exercises

- **Useful information**
  - **Kinematic variables**

- Inelasticity:
$$y = \frac{p_l \cdot q}{p_l \cdot p_\nu}$$

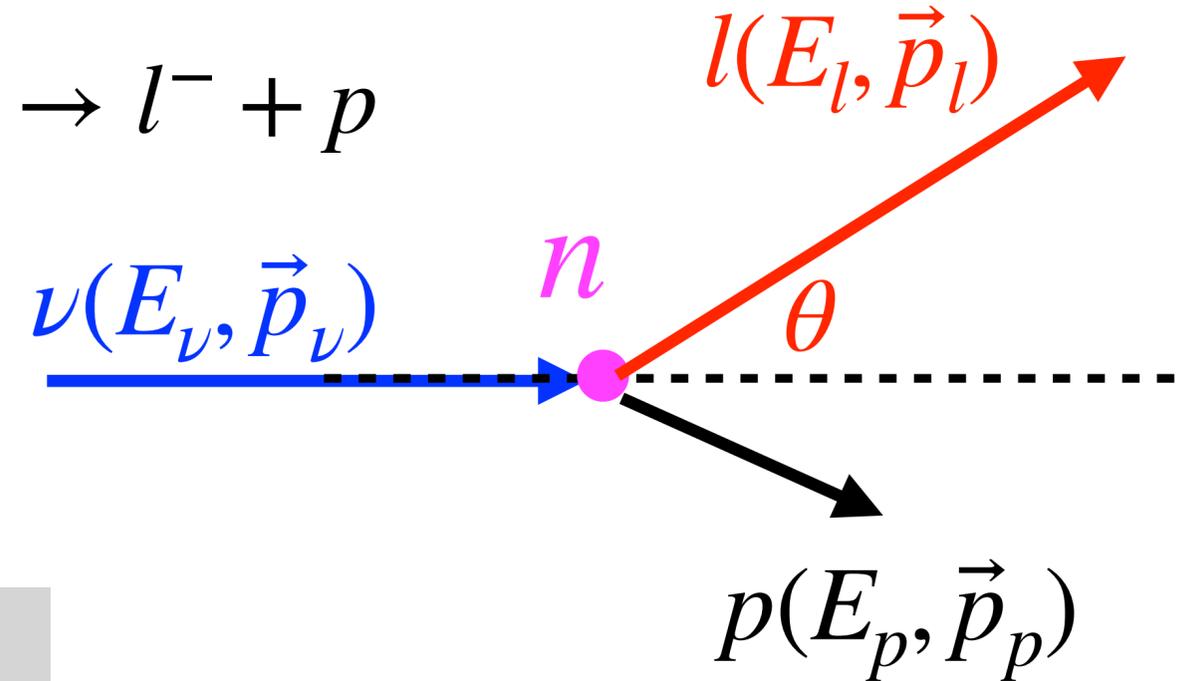
- Hadronic invariant mass:

$$W = \sqrt{E_X^2 - p_X^2}$$

- 4-momentum transferred:

$$Q^2 = -q^2 = 2E_\nu(E_l - p_l \cos \theta) - m_l^2 \quad (\mathbf{Q2rec})$$

- CCQE:  $\nu_l + n \rightarrow l^- + p$



# Backup

# Why do we need to study neutrino interactions?

## 1. Improve Detection Efficiency

- **Interaction cross sections:** Knowing how likely a neutrino is to interact with a given target helps design detectors with appropriate **target materials** (e.g., water, liquid argon).
- **Final-state particles:** Understanding what particles are produced (e.g., electrons, muons, pions) enables detector optimization for **signal signatures**.
- **Energy-dependent behavior:** Different energies lead to different dominant interaction modes (QE, RES, DIS), influencing which **energy ranges** the detector is most sensitive to.

## 2. Optimize Event Reconstruction

Neutrino interactions are needed to **reconstruct the incident neutrino's energy, flavor, and direction:**

- **Interaction models:** Precise modeling (via generators like NEUT, GENIE) helps simulate how a neutrino deposits energy in the detector.
- **Topology identification:** Knowing interaction signatures helps classify events (e.g., single-ring vs. multi-ring in Super-K).
- **Kinematic calculations:** For CCQE, the neutrino energy can be reconstructed from the outgoing lepton's momentum using interaction kinematics.

**Without accurate interaction models, event reconstruction suffers from energy bias, misidentification, and poor resolution.**

## 3. Reduce Systematic Uncertainties

Interaction uncertainties are among the **dominant systematics** in neutrino experiments:

- **Cross-section modeling:** Errors in  $\nu$ -nucleus interaction models propagate to uncertainties in oscillation parameters.
- **Nuclear effects:** Fermi motion, binding energy, and final state interactions (FSI) must be accounted for to avoid misestimating energy.
- **Detector response:** Understanding how particles from interactions behave inside the detector helps reduce modeling errors.

## 2. Cross section (2): theoretical calculations

- Using QFT (momentum space):  $1 + 2 \rightarrow 3 + 4$

$$\sigma = \frac{(2\pi)^4}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \int \frac{d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{d^3\vec{p}_4}{(2\pi)^3 2E_4} \delta^4(p_f - p_i) |M_{fi}|^2$$

i: initial states  
f: final states

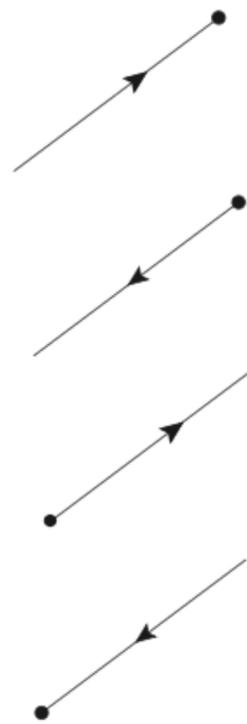
- $M_{fi}$ : Scattering amplitude, calculated using Feynman rules

- External lines:

- Incoming particle:  $u(p, s)$
- Incoming anti-particle:  $\bar{v}(p, s)$
- Outgoing particle:  $\bar{u}(p, s)$
- Outgoing anti-particle:  $v(p, s)$

Momentum

Spin



Incoming anti-particle:  $\bar{v}(p, s)$

Outgoing particle:  $\bar{u}(p, s)$

Incoming particle:  $u(p, s)$

Outgoing anti-particle:  $v(p, s)$



Time

## 2. Cross section (2): theoretical calculations

- $M_{fi}$ : Scattering amplitude, calculated using Feynman rules

o W and Z propagators

- For  $W^\pm$ :  $\frac{-ig^{\mu\nu}}{q^2 - m_W^2}$
- For  $Z^0$ :  $\frac{-ig^{\mu\nu}}{q^2 - m_Z^2}$

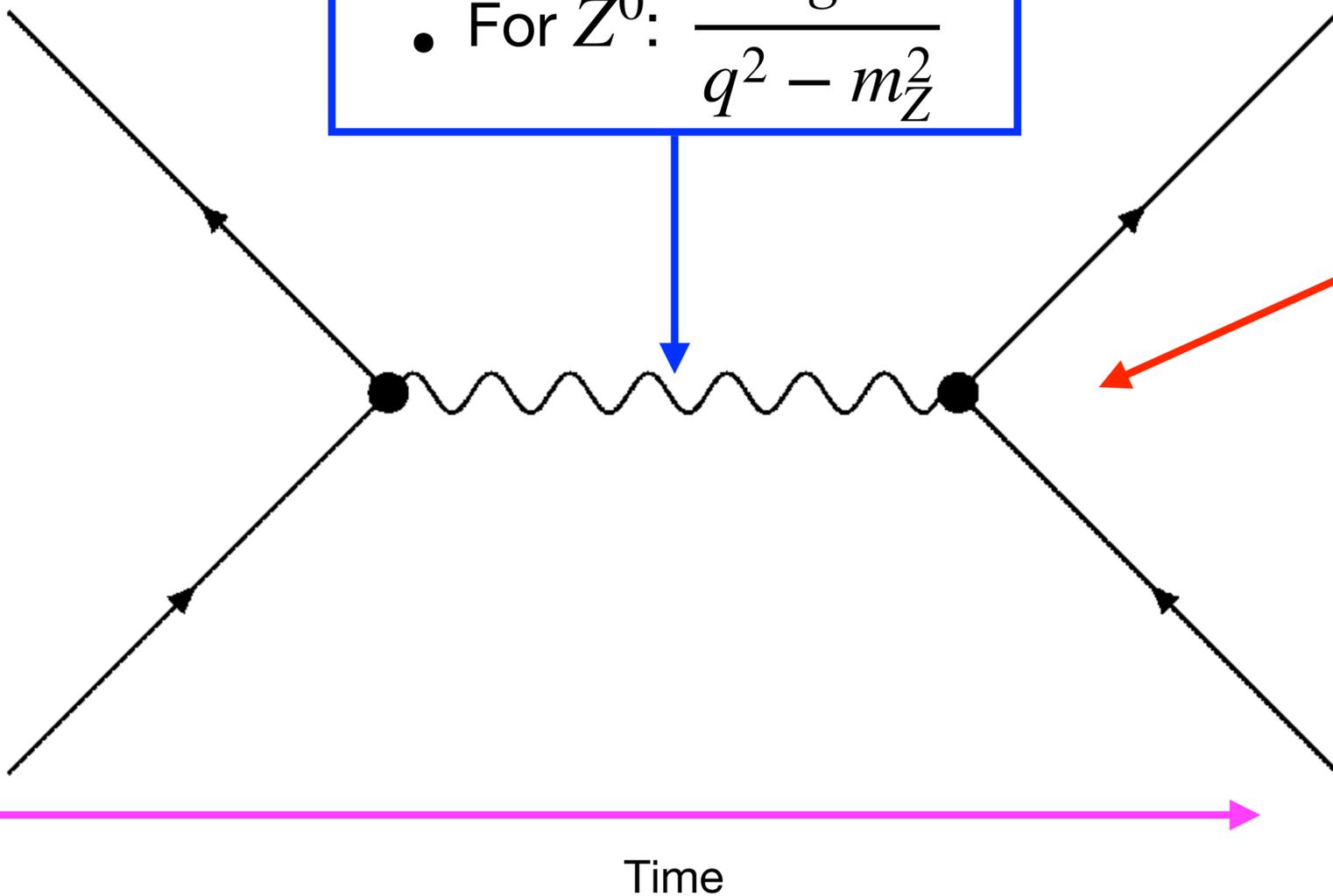
o Vertexes:

- For  $\nu l W$ :  $\frac{-ig}{2\sqrt{2}}\gamma^\mu(1 - \gamma^5)$
- For  $\nu\nu Z$ :  $\frac{-ig}{4\cos\theta_W}\gamma^\mu(1 - \gamma^5)$
- For  $ffZ$ :  $\frac{-ig}{\cos\theta_W}\gamma^\mu(g_V - g_A\gamma^5)$

For charged leptons:

$$g_V = -\frac{1}{2} + 2\sin^2\theta_W, \quad g_A = -\frac{1}{2}$$

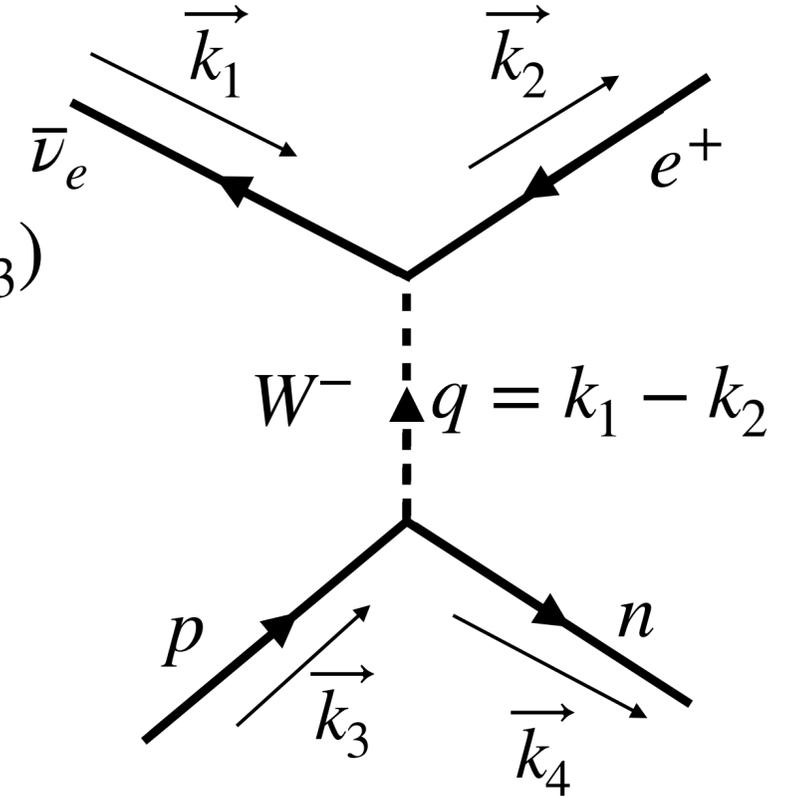
- For  $qqW$ :  $\frac{-ig}{2\sqrt{2}}\gamma^\mu(g_V - g_A\gamma^5)$



- $q$ : four-momentum of W/Z
- $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ : metric tensor
- $g$ : weak coupling constant

# 2. Cross section (2): theoretical calculations

- o **Example:** Inverse beta decay (important for reactor neutrino detection):



- $$iM_{fi} = \bar{v}(k_1) \frac{-ig}{2\sqrt{2}} \gamma^\mu (1 - \gamma^5) v(k_2) \left( \frac{-ig_{\mu\nu}}{q^2 - m_W^2} \right) \bar{u}(k_4) \frac{-ig}{2\sqrt{2}} \gamma^\nu (g_V - g_A \gamma^5) u(k_3)$$

- In low energy  $q^2 \ll m_W^2$ , recover to Fermi theory of point interactions

$$M_{fi} = \frac{G_F}{\sqrt{2}} \left[ \bar{v}(k_1) \gamma^\mu (1 - \gamma^5) v(k_2) \right] \left[ \bar{u}(k_4) \gamma_\mu (g_V - g_A \gamma^5) u(k_3) \right] = \frac{G_F}{\sqrt{2}} J_L^\mu J_{H\mu}$$

- \* Where  $G_F = \frac{g^2}{4\sqrt{2}m_W^2} \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$ : Fermi constant

- \*  $g_V = 1$  (vector coupling);  $g_A \approx 1.27$  (axial-vector coupling)

- Taking square and average over initial spins and sum over final spins

$$\langle |M_{fi}|^2 \rangle = \frac{G_F^2}{2} \cdot \frac{1}{2} \sum_{spins} |J_L^\mu J_H^\mu|^2$$

$$= 16G_F^2 \left[ (g_V^2 + 3g_A^2)(p_\nu \cdot p_p)(p_e \cdot p_n) + (g_V^2 - g_A^2)(p_\nu \cdot p_e) \cdot m_p m_n \right]$$

$$\sigma \approx \frac{G_F^2}{\pi} (g_V^2 + 3g_A^2) \left( E_\nu - (m_n - m_p) \right)^2$$

For reactor neutrinos,  $E_\nu \sim \text{MeV}$

$$\sigma \approx 9.5 \times 10^{-44} \text{ cm}^2 \left( \frac{E_\nu}{\text{MeV}} \right)^2$$

## 2. Cross section (2): theoretical calculations

- **Threshold for IBD to be occurred:** energy-momentum conservation

$$(p_\nu + p_p)^2 = (p_e + p_n)^2 \equiv (E_\nu + E_p)^2 - (p_\nu + p_p)^2 = (E_e + E_n)^2 - (p_e + p_n)^2$$

- $E_\nu = E_\nu^{thr}$  when  $p_e = p_n = 0 \Rightarrow E_e = m_e; \quad E_n = m_n$

- also p at rest  $\Rightarrow p_p = 0$  and  $E_p = m_p$

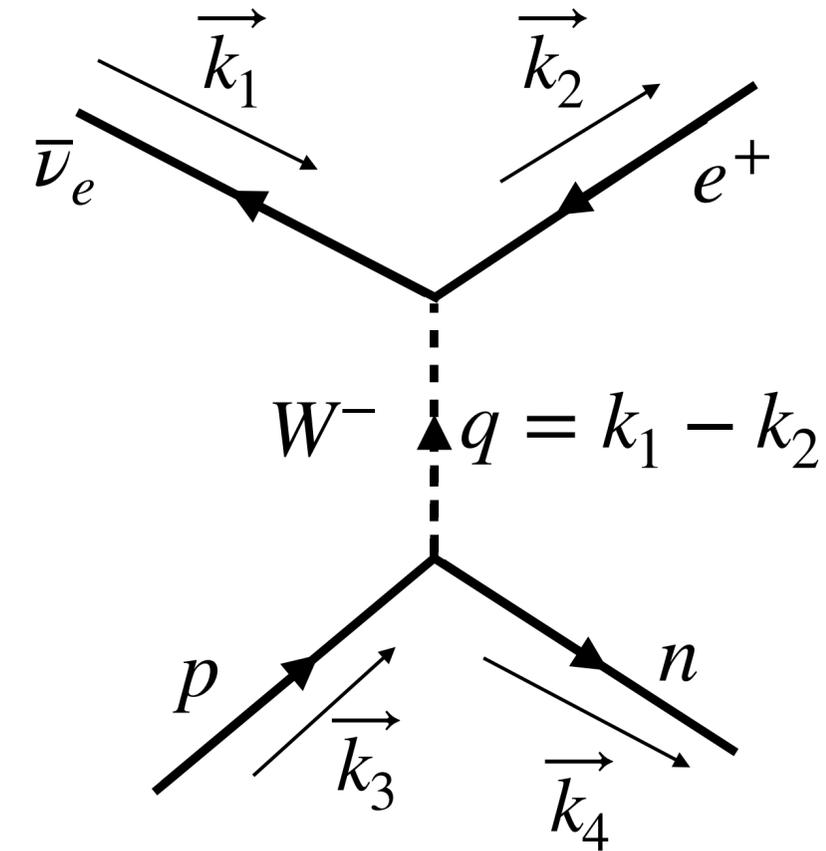
- Then  $E_\nu^{thr} = \frac{(m_e + m_n)^2 - m_\nu^2 - m_p^2}{2m_p}$

- With:  $m_\nu \approx 0; m_p = 938.272 \text{ MeV}; m_n = 939.565 \text{ MeV}; m_e = 0.511 \text{ MeV}$

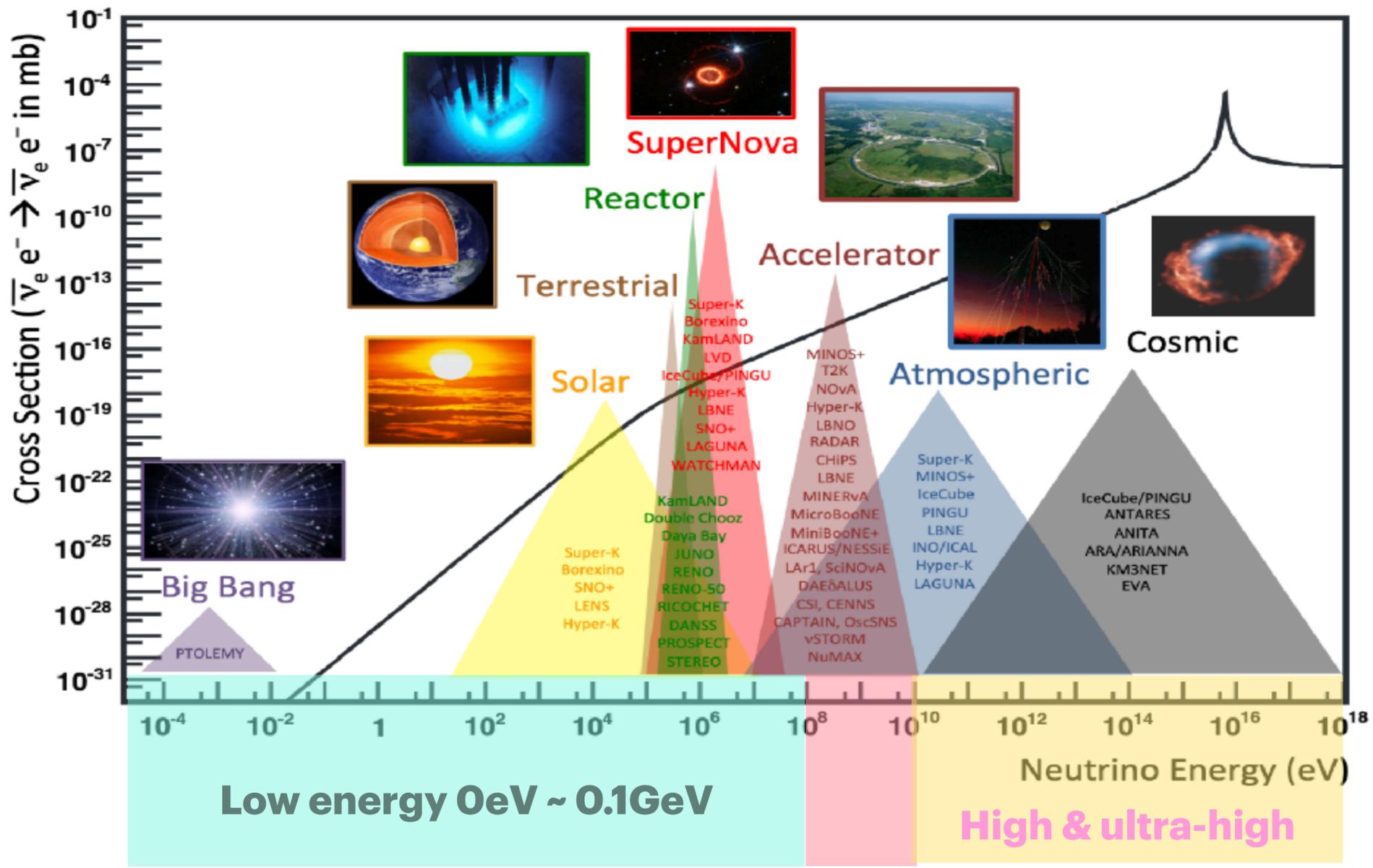
$$E_\nu^{thr} = 1.8 \text{ MeV}$$

- $E_\nu^{reactor} \approx 2 - 8 \text{ MeV}$ , IBD occurs

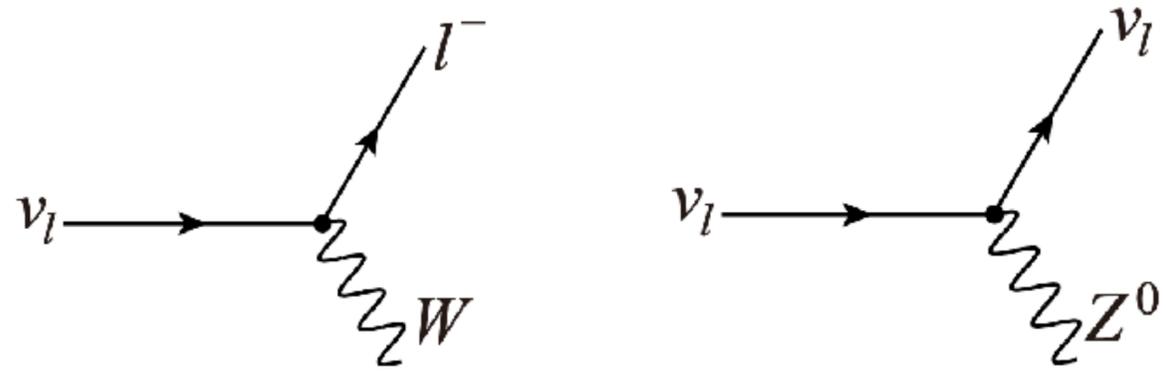
- $E_\nu^{solar}(pp) \approx 0.1 - 1 \text{ MeV}$ , IBD does not occurs



# 3. Neutrino interactions



- Basically there are 2 types of interactions:
  - Charge current (CC) by exchange  $W^\pm$  bosons
  - Neutral current (NC) by exchange  $Z^0$  boson



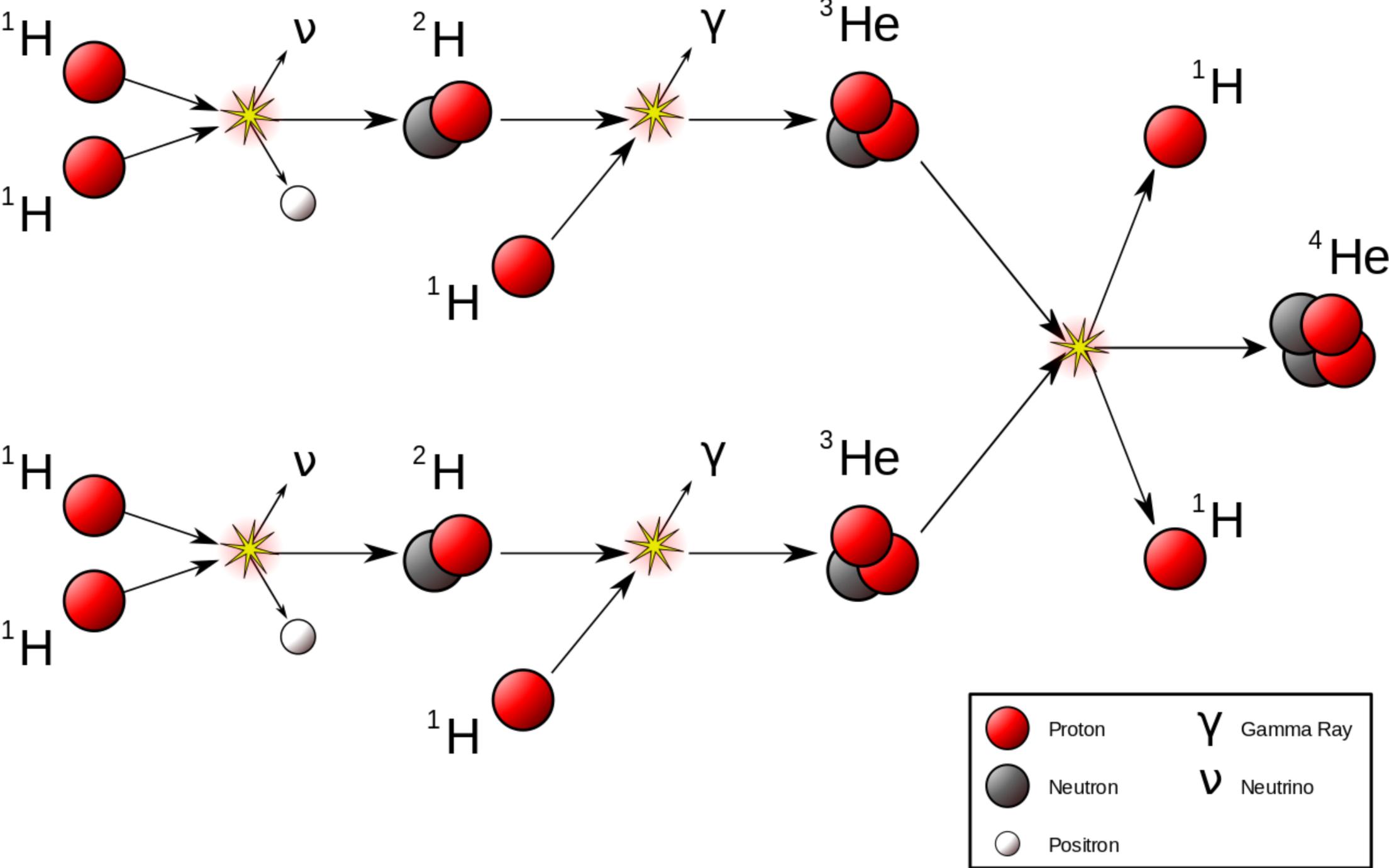
$$j_{CC}^\mu = \frac{g_W}{\sqrt{2}} \bar{\psi} \gamma^\mu \frac{1}{2} (1 - \gamma^5) \phi,$$

$$j_{NC}^\mu = g_Z \bar{\psi} \gamma^\mu \frac{1}{2} (g_V - g_A \gamma^5) \psi.$$

Interact with e    Interact with nucleus    Interact with nucleon

When neutrino energy is increased, it can interact with deeper stuff inside the atoms

# pp chain in the Sun



# Beta decay

- 2 out-going particle ( $c = 1$ ):

- $E_n = m_n$

- Proton almost at rest:  $E_p \approx m_p$

- Case  $m_e \ll p_e$ :  $E_e = \sqrt{m_e^2 + p_e^2} \approx m_e(1 + p_e/m_e)$

- Energy conservation:  $E_n = E_p + E_e \rightarrow p_e = m_n - m_p - m_e$

- 3 out-going particle ( $c = 1$ ):

$$E_n = E_p + E_e + e_\nu \rightarrow p_e = (m_n - m_p - m_e) - p_\nu$$

# 3.1 Neutrino - electron interactions

- Neutrino - electron elastic scattering:

$$\nu_{\alpha} + e^{-} \rightarrow \nu_{\alpha} + e^{-}$$

$$\bar{\nu}_{\alpha} + e^{-} \rightarrow \bar{\nu}_{\alpha} + e^{-}$$

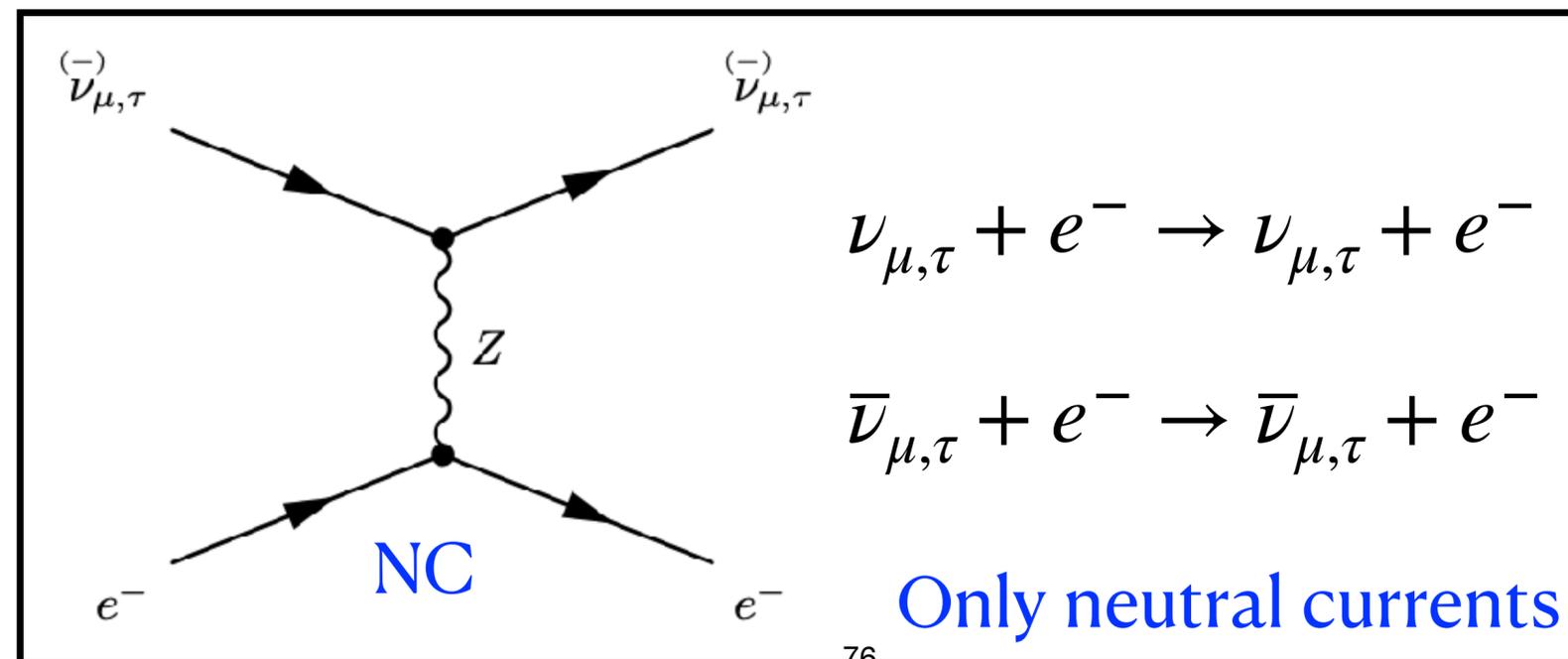
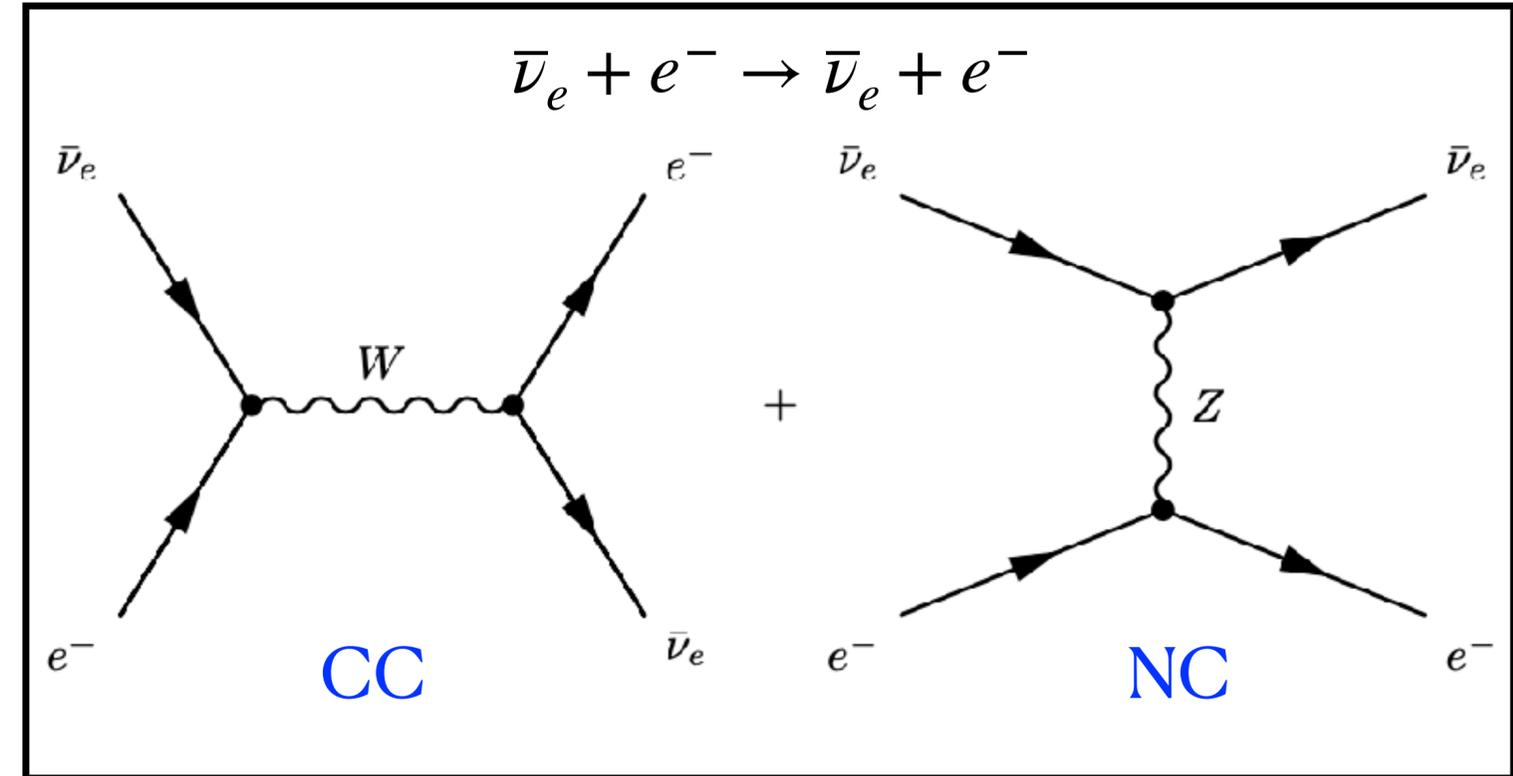
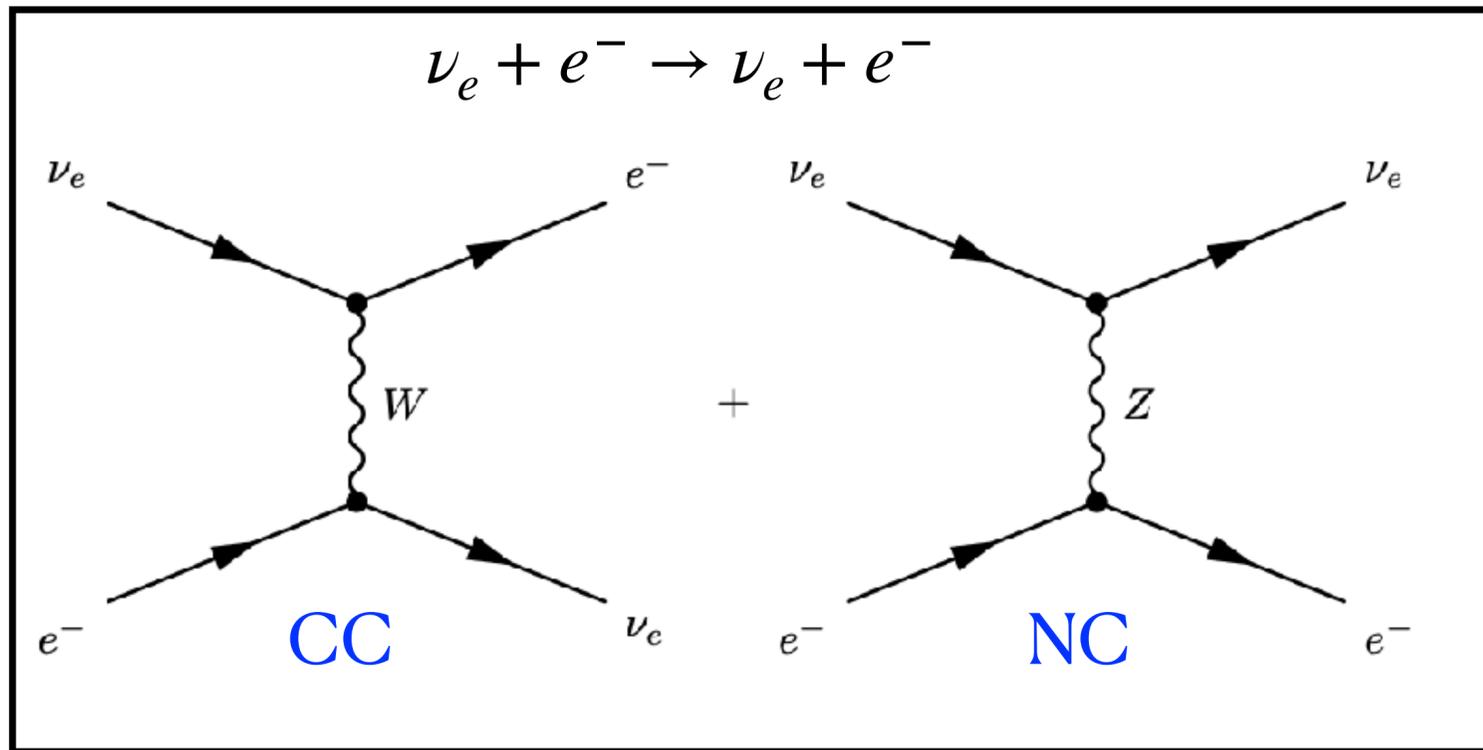
- Neutrino - electron quasi-elastic scattering:

$$\nu_{\mu} + e^{-} \rightarrow \nu_e + \mu^{-} \quad (E_{\nu} > 10.92 \text{ GeV})$$

$$\nu_{\tau} + e^{-} \rightarrow \nu_e + \tau^{-} \quad (E_{\nu} > 3089 \text{ GeV})$$

# 3.1 Neutrino - electron interactions

## ○ Neutrino - electron elastic scattering



# 3.1 Neutrino - electron interactions

## ○ Neutrino - electron elastic scattering

TABLE 5.1. Total neutrino–electron elastic scattering cross-sections for  $\sqrt{s} \gg m_e$ . The numerical values are in units of  $10^{-46} \text{ cm}^2$ .

Process	Total cross-section
$\nu_e + e^-$	$(G_F^2 s/4\pi) \left[ (1 + 2 \sin^2 \vartheta_W)^2 + \frac{4}{3} \sin^4 \vartheta_W \right] \simeq 93 s/\text{MeV}^2$
$\bar{\nu}_e + e^-$	$(G_F^2 s/4\pi) \left[ \frac{1}{3} (1 + 2 \sin^2 \vartheta_W)^2 + 4 \sin^4 \vartheta_W \right] \simeq 39 s/\text{MeV}^2$
$\nu_{\mu,\tau} + e^-$	$(G_F^2 s/4\pi) \left[ (1 - 2 \sin^2 \vartheta_W)^2 + \frac{4}{3} \sin^4 \vartheta_W \right] \simeq 15 s/\text{MeV}^2$
$\bar{\nu}_{\mu,\tau} + e^-$	$(G_F^2 s/4\pi) \left[ \frac{1}{3} (1 - 2 \sin^2 \vartheta_W)^2 + 4 \sin^4 \vartheta_W \right] \simeq 13 s/\text{MeV}^2$

- $G_F = 1.17 \times 10^{-5} \text{ GeV}^2$  : Fermi constant
- $s = (E_\nu + E_e)_i^2 = m_e^2 + 2m_e E_\nu$  : Mandelstam variable
- $\sin^2 \theta_W = 0.2324 \pm 0.0058 \pm 0.0059$  (1994)

# 3.1 Neutrino - electron interactions

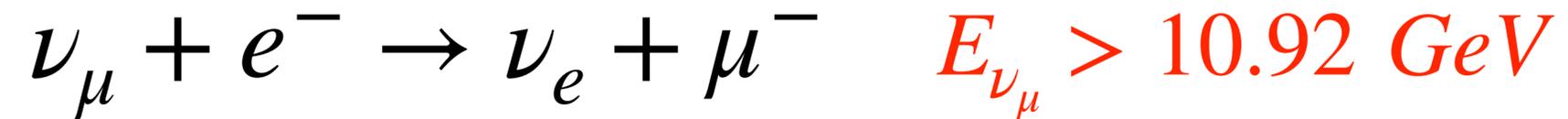
- Neutrino - electron elastic scattering
- Differential cross section

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{\pi} \left[ g_1^2 + g_2^2 \left( 1 - \frac{Q^2}{2p_\nu p_e} \right)^2 - g_1 g_2 m_e^2 \frac{Q^2}{2(p_\nu p_e)^2} \right]$$

- $g_1(\nu_e) = g_2(\bar{\nu}_e) = \frac{1}{2} + \sin^2 \theta_W \approx 0.73$
- $g_2(\nu_e) = g_1(\bar{\nu}_e) = \sin^2 \theta_W \approx 0.23$
- $g_1(\nu_{\mu,\tau}) = g_2(\bar{\nu}_{\mu,\tau}) = -\frac{1}{2} + \sin^2 \theta_W \approx -0.27$
- $g_2(\nu_{\mu,\tau}) = g_1(\bar{\nu}_{\mu,\tau}) = \sin^2 \theta_W \approx 0.23$

# 3.1 Neutrino - electron interactions

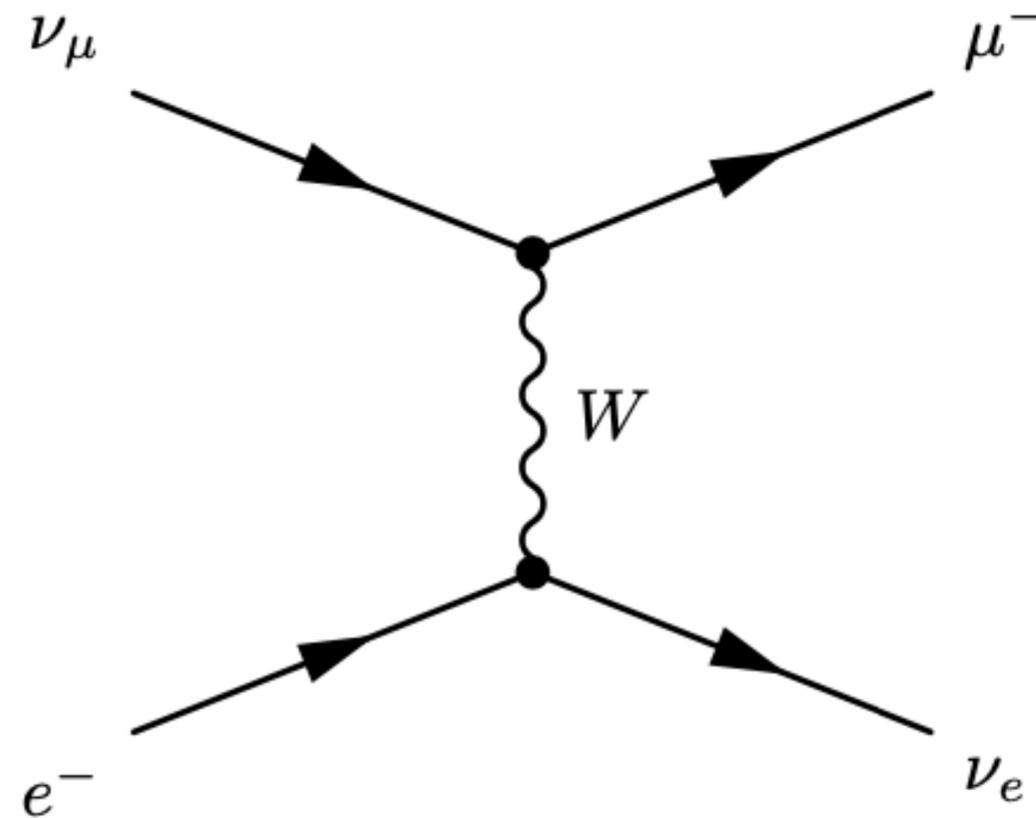
- Neutrino - electron quasi-elastic scattering



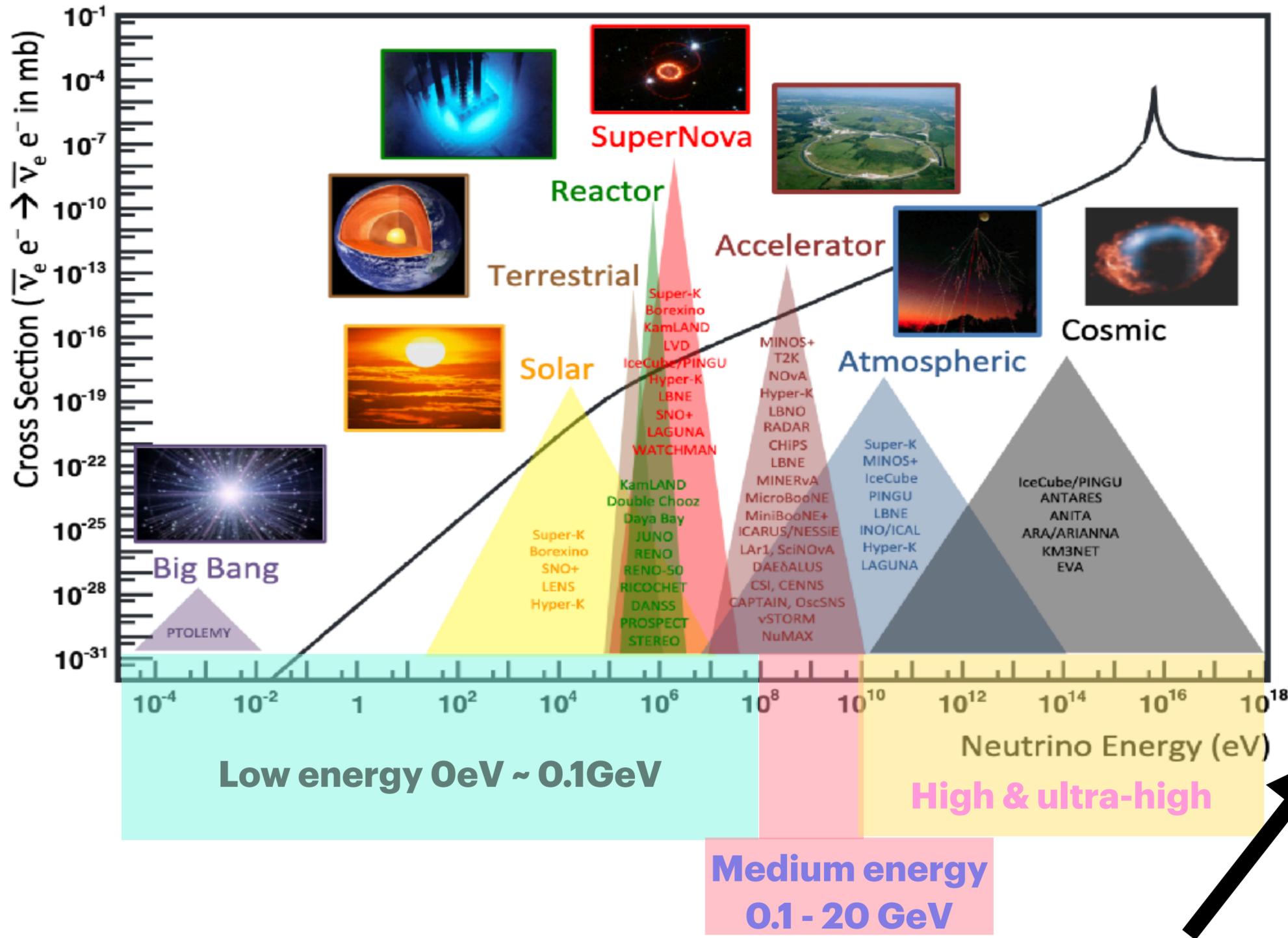
- Total cross section

$$\sigma = \frac{G_F^2 s}{\pi} \left( 1 - \frac{m_{\mu}^2}{s} \right)$$

$$s = (p_{\nu_{\mu}} + p_e)^2$$



# 3.2 Neutrino - nucleon interactions



- Neutrino interactions at 0-1 MeV energy:
  - Coherent scattering (COH):
  - Neutrino capture on radioactive nuclei:

- Neutrino interactions at 1-100 MeV energy:
  - Inverse beta decay
- Neutrino interactions at 0.1-20 GeV energy:
  - Quasi-elastic scattering (QE):
  - Resonant meson production (RES):
  - Coherent pion production (COH):
  - Deep inelastic scattering (DIS):

- Neutrino interactions at high and ultra-high energy (> 20 GeV):
  - Deep inelastic scattering (DIS):

Important for reactor & accelerator neutrino oscillation

## 3.2 Neutrino - nucleon interactions

### ○ Neutrino interactions at 0-1 MeV energy:

#### • Coherent scattering (COH):



$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi} Q_W^2 M_A \left(1 - \frac{M_A T}{2E_\nu^2}\right) F(Q^2)^2$$

#### • Neutrino capture on radioactive nuclei:



$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 |V_{ud}|^2 F(Z_f, E_e)}{2\pi\beta_\nu} E_e p_e f_V^2(0) \left( (1 + \beta_e \beta_\nu \cos\theta) + 3\lambda^2 \left(1 - \frac{1}{3} \beta_e \beta_\nu \cos\theta\right) \right)$$

- $\beta_e, \beta_\nu$ : electron, neutrino velocities
- $E_e, p_e, \cos\theta$ : electron energy, momentum, scattering angle
- $\lambda^2$ : axial-to-vector coupling ratio,  $f_V^2(0)$ : coupling strength
- $|V_{ud}|^2$ : Cabibbo angle
- $F(Z_f, E_e)$ : Fermi function

**Not experimentally measured yet**

- $G_F = 1.17 \times 10^{-5} \text{ GeV}^2$
- Q: momentum transfer
- R: nuclear radius
- T: recoil kinetic energy
- $M_A$ : target mass
- $E_\nu$ : neutrino energy
- $F(Q^2)$ : nucleon form factor
- $Q_W$ : weak current term

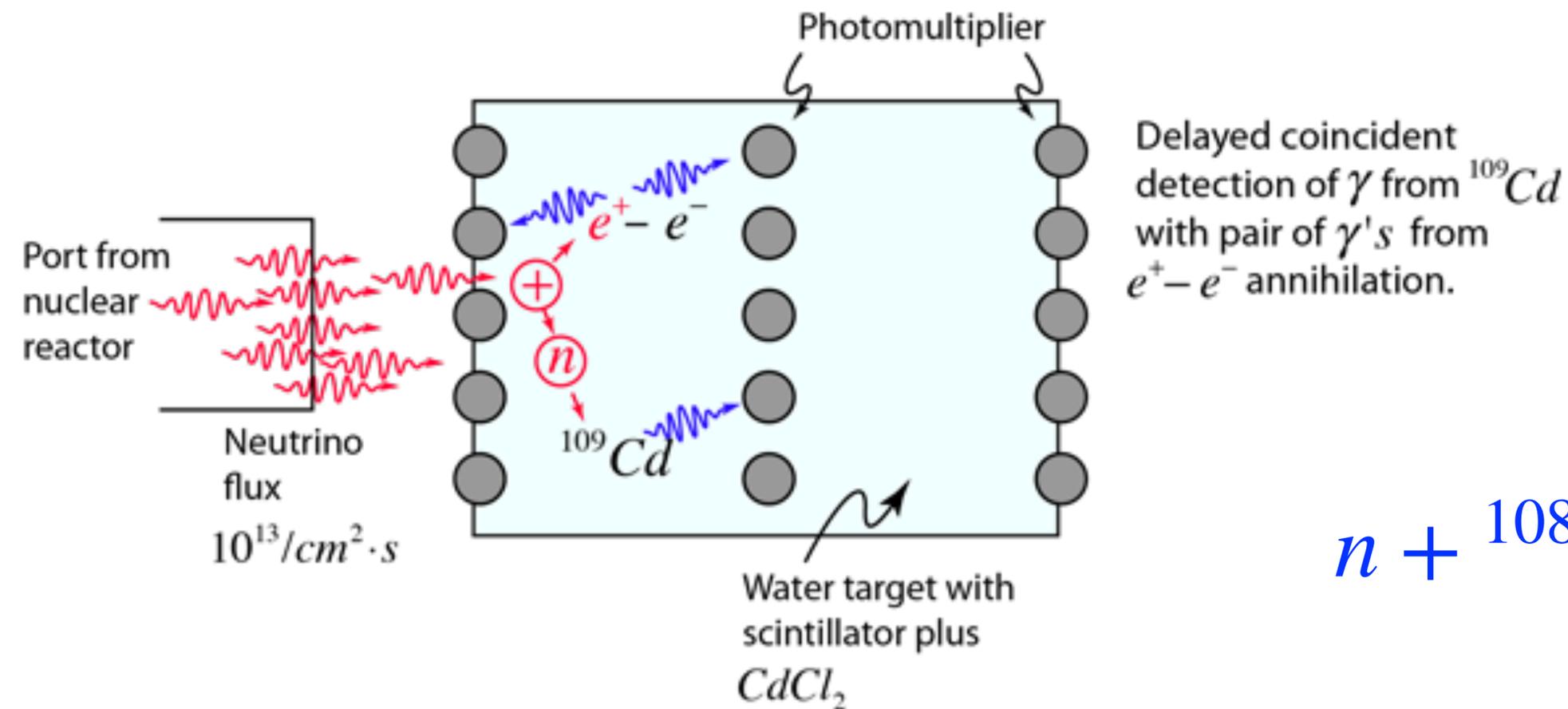
# 3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 1-100MeV energy:

- Inverse beta decay:



- Scattering process used in Reines & Cowan experiment to confirm the existence of neutrino



0.5 MeV prompt signals of gamma rays



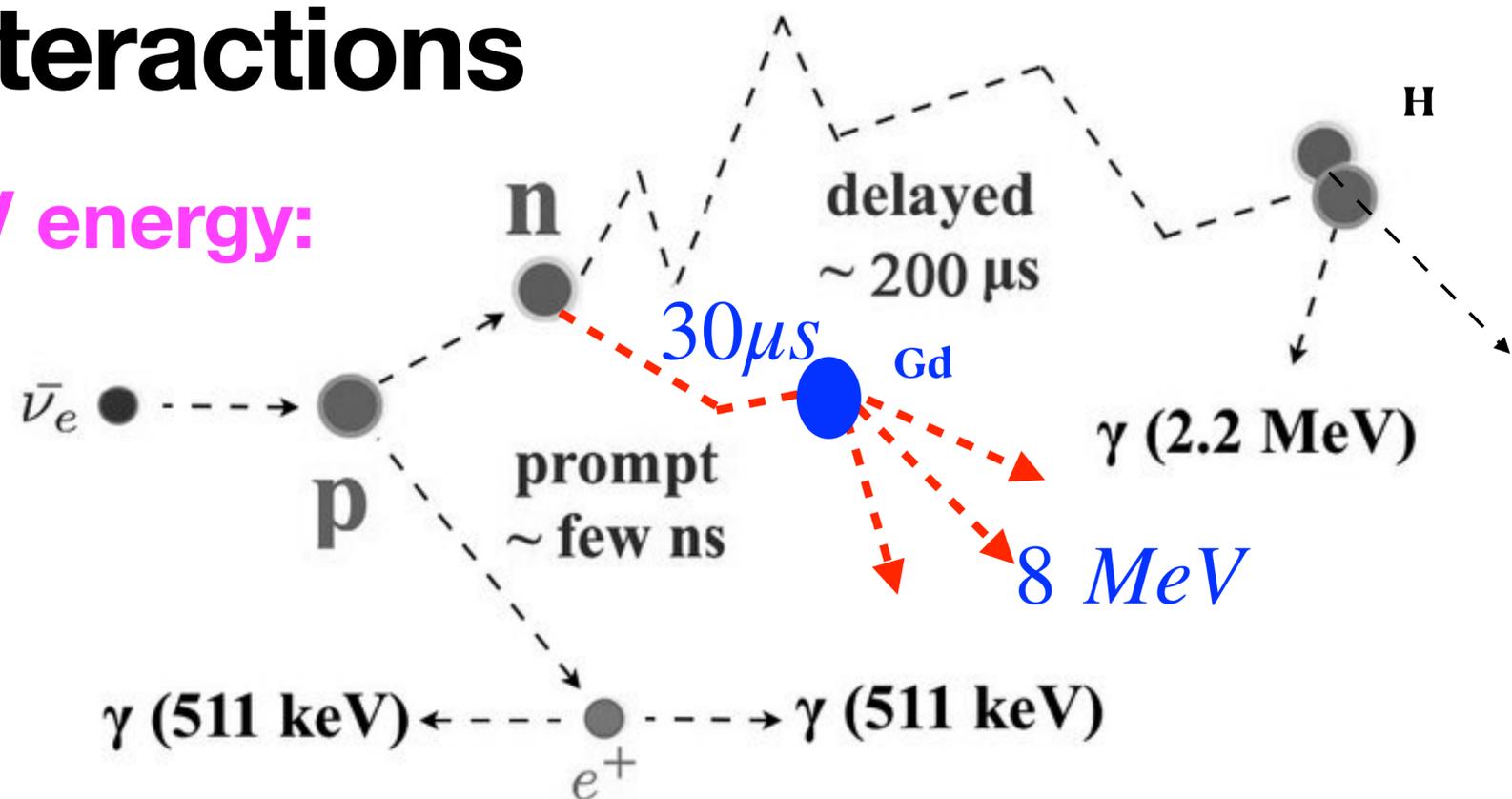
Delayed gamma signal after  $5\mu\text{s}$



# 3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 1-100MeV energy:**

- **Inverse beta decay:**



$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 |V_{ud}|^2 E_e p_e}{2\pi} \left( f_V^2(0)(1 + \beta_e \cos\theta) + 3f_A^2(0)\left(1 - \frac{1}{3}\beta_e \cos\theta\right) \right)$$

- $f_V$ : nuclear **vector (Fermi transition)**,  $f_A$ : **axial-vector (Gamov-Teller transition)**
- Important for detecting reactor neutrino experiments ( $1.806 < E_\nu \leq 10 \text{ MeV}$ )
- Important for understanding supernova explosion mechanism ( $10 < E_\nu \leq 20 \text{ MeV}$ )

## 3.2 Neutrino - nucleon interactions

### ○ Neutrino interactions at 0.1-20GeV energy:

Charge current:  $W^+$  exchange

- **Quasi-elastic scattering**  
(**target change but not break up**):
  - CCQE:  $\nu_l + n \rightarrow l^- + p$
- **Resonant meson production**  
(**target is excited**):
  - CCRES:  
 $\nu_l + N \rightarrow l^- + \Delta(1232) \rightarrow l^- + N' + m$
- **Coherent pion production**  
(**target unchange**):
  - CCCOH:  $\nu_l + A \rightarrow l^- + A + \pi^+$
- **Deep inelastic scattering**  
(**nucleon broken**):
  - CCDIS:  $\nu_l + N \rightarrow l^- + X$

Neutral current:  $Z^0$  exchange

- **Elastic scattering**  
(**target change but not break up**):
  - NC:  $\nu_l + N \rightarrow \nu_l + N'$
- **Resonant meson production**  
(**target is excited**):
  - NCRES:  
 $\nu_l + N \rightarrow \nu_l + \Delta(1232) \rightarrow \nu_l + N' + m$
- **Coherent pion production**  
(**target unchange**):
  - NCCOH:  $\nu_l + A \rightarrow \nu_l + A + \pi^0$
- **Deep inelastic scattering**  
(**nucleon broken**):
  - NCDIS:  $\nu_l + N \rightarrow \nu_l + X$

Where  $l^- = \{e^-, \mu^-, \tau^-\}$ ;  $N = \{p, n\}$ ;  $m = \{\pi, \eta, K\}$ ;  $A$  : nucleus;  $X$  : hadrons

## 3.2 Neutrino - nucleon interactions

### ○ Anti-neutrino interactions at 0.1-20GeV energy:

Charge current:  $W^-$  exchange

- **Quasi-elastic scattering**  
(**target change but not break up**):
  - CCQE:  $\bar{\nu}_l + p \rightarrow l^+ + n$
- **Resonant meson production**  
(**target is excited**):
  - CCRES:  
$$\bar{\nu}_l + N \rightarrow l^+ + \Delta(1232) \rightarrow l^+ + N' + m$$
- **Coherent pion production**  
(**target unchanged**):
  - CCCOH:  $\bar{\nu}_l + A \rightarrow l^+ + A + \pi^-$
- **Deep inelastic scattering**  
(**nucleon broken**):
  - CCDIS:  $\bar{\nu}_l + N \rightarrow l^+ + X$

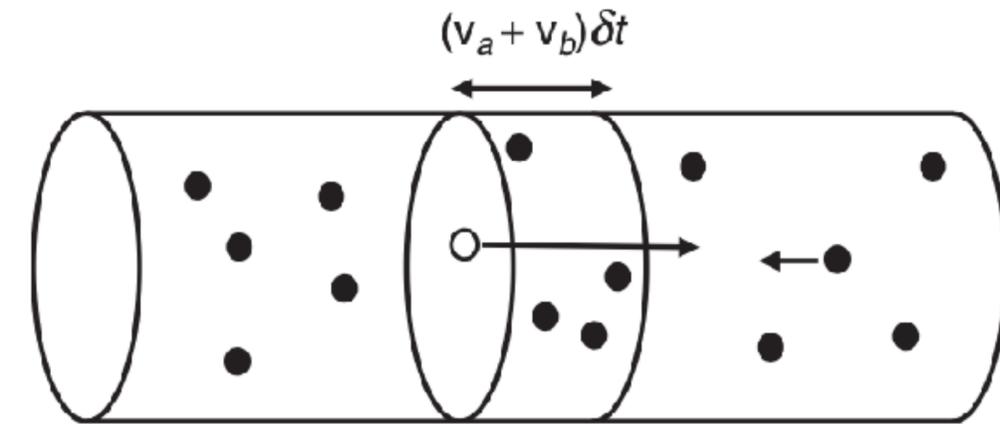
Neutral current:  $Z^0$  exchange

- **Elastic scattering**  
(**target change but not break up**):
  - NC:  $\bar{\nu}_l + N \rightarrow \bar{\nu}_l + N'$
- **Resonant meson production**  
(**target is excited**):
  - NCRES:  
$$\bar{\nu}_l + N \rightarrow \bar{\nu}_l + \Delta(1232) \rightarrow \bar{\nu}_l + N' + m$$
- **Coherent pion production**  
(**target unchanged**):
  - NCCOH:  $\bar{\nu}_l + A \rightarrow \bar{\nu}_l + A + \pi^0$
- **Deep inelastic scattering**  
(**nucleon broken**):
  - NCDIS:  $\bar{\nu}_l + N \rightarrow \bar{\nu}_l + X$

Where  $l^- = \{e^-, \mu^-, \tau^-\}$ ;  $N = \{p, n\}$ ;  $m = \{\pi, \eta, K\}$ ;  $A$  : nucleus;  $X$  : hadrons

## 2. Cross section

- Consider a beam of particles **a** with flux  $\phi_a$ , pass through a volume  $V = Av\Delta t$  of particle **b**:
- $v = v_a + v_b$  ( $\vec{v}_a, \vec{v}_b$ : velocities of a and b)
- $N_a = n_a V$ : total number of incident particles in volume V
- $N_b = n_b V$ : total number of target particles in volume V
- $\phi_a = \frac{N_a}{A\Delta t} = n_a v$ : flux of particle **a**  $\left[ \frac{1}{[time] \cdot [area^2]} \right]$
- $N = \phi_a N_b \sigma$ : #interactions = flux x number of target particles x cross section
- **Cross section is a measure of the probability of an interaction to be occurred**



$$\sigma = \frac{N}{\phi_a \cdot N_b} \quad [area^2]$$

## 2. Cross section

- Consider a scattering:  $a + b \rightarrow 1 + 2$
- Cross section is related to the transition rate:

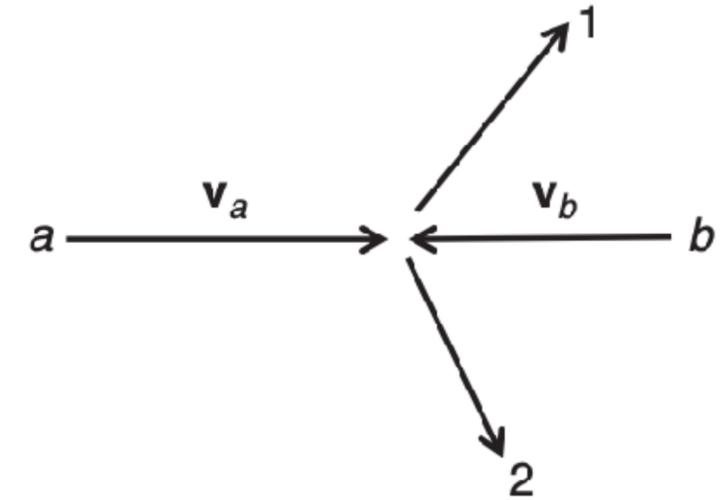
$$\sigma = \frac{\Gamma_{fi}}{\phi_a N_b} = \frac{\Gamma_{fi}}{n_a n_b (v_a + v_b) V}$$

- Normalising the wavefunctions to one particle per unit volume  $n_a = n_b = 1/V$

$$\sigma = \frac{\Gamma_{fi} V}{v_a + v_b}$$

- Because the factors of  $V$  in the expression for the flux will ultimately be cancelled by the corresponding factors from the wavefunction normalisation and phase space ( $\sim 1/V$ ), the volume  $V$  will not appear in the final result

$$\sigma = \frac{\Gamma_{fi}}{v_a + v_b}$$



## 2. Cross section

- In terms of matrix element  $M_{fi}$

$$\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}} \int |M_{fi}|^2 \sigma(E_i - E_f) \sigma^3(\vec{p}_i - \vec{p}_f) \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}$$

- $M_{fi} = \sqrt{2E_a \cdot 2E_b \cdot 2E_1 \cdot 2E_2} T_{fi}$ ,  $T_{fi}$ : transition matrix element
- $E_i = E_a + E_b$ ;  $E_f = E_1 + E_2$ ;  $\vec{p}_i = \vec{p}_a + \vec{p}_b$ ;  $\vec{p}_f = \vec{p}_1 + \vec{p}_2$
- Lorentz-invariant flux factor

$$F = 4(v_a + v_b)E_a E_b = 4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}$$

- In center-of-mass (COM) frame:

$$\sigma_{COM} = \frac{p_f}{64\pi^2 s p_i} \int |M_{fi}|^2 d\Omega$$

- $\vec{p}_i = \vec{p}_a = -\vec{p}_b$  and  $\vec{p}_f = \vec{p}_1 = -\vec{p}_2$  (in COM frame)
- $s = (E_a + E_b)^2$  (in COM frame)
- $d\Omega = d(\cos \theta) d\phi$  ( $\theta$ : polar or zenith angle;  $\phi$ : azimuthal angle)

## 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:

- Quasi-elastic scattering (QE): (Llewellyn-Smith, 1972)

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 m_N^2 |V_{ud}|^2}{8\pi (P_\nu \cdot p_{N_i})_\nu^2} \left[ A \pm \frac{(s-u)}{m_N^2} B + \frac{(s-u)^2}{m_N^4} C \right]$$

$$A = \frac{(m_l^2 + Q^2)}{m_N^2} \left[ (1 + \eta)F_A^2 - (1 - \eta)F_1^2 + \eta(1 - \eta)F_2^2 + 4\eta F_1 F_2 - \frac{m_l^2}{4m_N^2} \left( (F_1 + F_2)^2 + (F_A + 2F_P)^2 - \left( \frac{Q^2}{m_N^2} + 4 \right) F_P^2 \right) \right]$$

$$B = \frac{Q^2}{m_N^2} F_A (F_1 + F_2) \quad C = \frac{1}{4} (F_A^2 + F_1^2 + \eta F_2^2)$$

- (-) +: (anti-) neutrino scattering

- $G_F$ : Fermi constant

- $Q^2$ : squared 4-momentum transferred ( $Q^2 = -q^2 > 0$ )

- $m_N$ : nucleon mass

- $m_l$ : lepton mass

- $E_\nu$ : neutrino energy

- $(s-u) = 4m_N E_\nu - Q^2 - m_l^2$

- $F_1, F_2, F_A, F_P$ : Dirac, Pauli, axial-vector, and pseudoscalar form factors

- $\eta = Q^2/4m_N^2$

## 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:

- Quasi-elastic scattering (QE): (Llewellyn-Smith, 1972)

- Axial-vector form factor  $F_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2}$

- Axial mass  $M_A = 1.026 \pm 0.021 \text{ GeV}$   $g_A = F_A(0) = 1.2694 \pm 0.0028$

- Dirac form factor  $F_1(Q^2) = F_1^p(Q^2) + F_1^n(Q^2)$

$$F_1^p(0) = 1, \quad F_1^n(0) = 0,$$

- Pauli form factor  $F_2(Q^2) = F_2^p(Q^2) + F_2^n(Q^2)$

$$F_2^p(0) = \frac{\mu_p}{\mu_N} - 1, \quad F_2^n(0) = \frac{\mu_n}{\mu_N},$$

- For  $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$  scattering, the term related to A can be neglected. In this case, cross sections do not depend on the pseudoscalar form factor  $F_P$

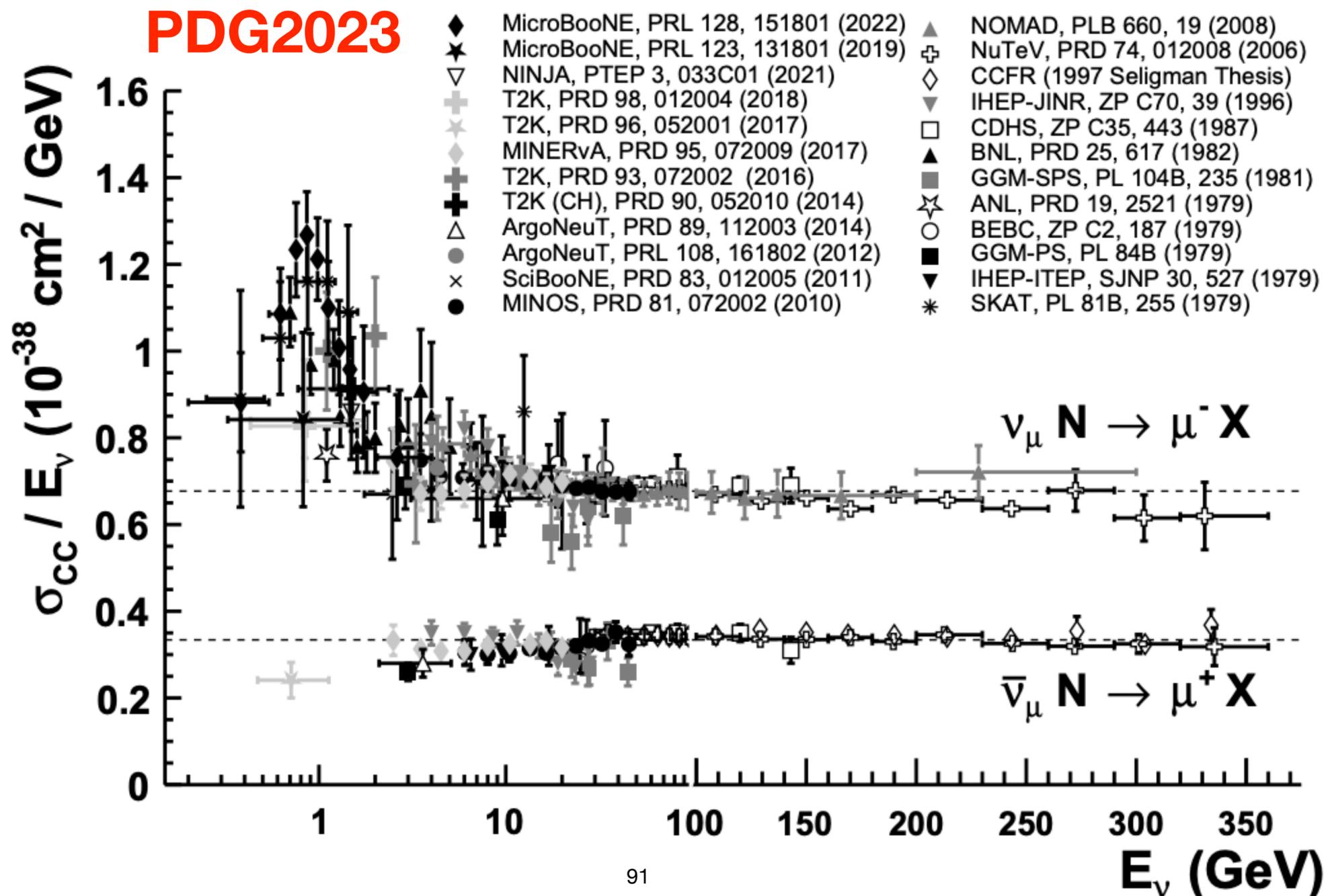
- Nuclear magneton  $\mu_N = e\hbar/2m_p$

- Proton magnetic moment  $\mu_p$

- Neutron magnetic moment  $\mu_n$

# 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:
- Quasi-elastic scattering (QE): measurement



# 3.2 Neutrino - nucleon interactions

- **Neutrino interactions at 0.1-20GeV energy:**

- **NC elastic scattering: Similar to formulae of CCQE but without  $|V_{ud}|^2$ , and different form factors**

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 m_N^2}{8\pi (P_\nu \cdot p_{N_i})^2} \left[ A \pm \frac{(s-u)}{m_N^2} B + \frac{(s-u)^2}{m_N^4} C \right] \quad (-)+ \text{ for (anti-)neutrino scattering}$$

$$F_1(Q^2) = \left( \frac{1}{2} - \sin^2 \theta_W \right) \left[ \frac{\tau_3(1 + \eta(1 + \mu_p - \mu_n))}{(1 + \eta)(1 + Q^2/M_V^2)} \right] - \sin^2 \theta_W \left[ \frac{1 + \eta(1 + \mu_p + \mu_n)}{(1 + \eta)(1 + Q^2/M_V^2)^2} \right] - \frac{F_1^s(Q^2)}{2}$$

$$F_2(Q^2) = \left( \frac{1}{2} - \sin^2 \theta_W \right) \frac{\tau_3(\mu_p - \mu_n)}{(1 + \eta) \left( 1 + \frac{Q^2}{M_V^2} \right)^2} - \sin^2 \theta_W \frac{\mu_p + \mu_n}{(1 + \eta) \left( 1 + \frac{Q^2}{M_V^2} \right)^2} - \frac{F_2^s(Q^2)}{2}$$

$$F_A(Q^2) = \frac{g_A \tau_3}{2 \left( 1 + \frac{Q^2}{M_A^2} \right)^2} - \frac{F_A^s(Q^2)}{2}$$

- $\tau_3 = +1$  ( $-1$ ) for proton (neutron) scattering
- $F_{1,2}^s(Q^2)$ : strange vector form factors
- Strange axial-vector form factor ( $\Delta s$ : strange quark contribution to nucleon spin)

$$F_A^s(Q^2) = \frac{\Delta s}{(1 + Q^2/M_A^2)^2}$$

## 3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

• Resonant meson production (CC1 $\pi$  dominates at low energy)

$$\begin{aligned} \nu_\mu p &\rightarrow \mu^- p \pi^+, & \bar{\nu}_\mu p &\rightarrow \mu^+ p \pi^- \\ \nu_\mu n &\rightarrow \mu^- p \pi^0, & \bar{\nu}_\mu p &\rightarrow \mu^+ n \pi^0 \\ \nu_\mu n &\rightarrow \mu^- n \pi^+, & \bar{\nu}_\mu n &\rightarrow \mu^+ n \pi^- \end{aligned}$$

$$\begin{aligned} \nu_\mu p &\rightarrow \nu_\mu p \pi^0, & \bar{\nu}_\mu p &\rightarrow \bar{\nu}_\mu p \pi^0 \\ \nu_\mu p &\rightarrow \nu_\mu n \pi^+, & \bar{\nu}_\mu n &\rightarrow \bar{\nu}_\mu n \pi^0 \\ \nu_\mu n &\rightarrow \nu_\mu n \pi^0, & \bar{\nu}_\mu n &\rightarrow \bar{\nu}_\mu n \pi^0 \\ \nu_\mu n &\rightarrow \nu_\mu p \pi^-, & \bar{\nu}_\mu n &\rightarrow \bar{\nu}_\mu p \pi^- \end{aligned}$$

$$\frac{d\sigma(\nu N \rightarrow l N \pi)}{dk^2 dW d\Omega_\pi} = \frac{G_F^2}{2} \frac{1}{(2\pi)^4} \frac{|\mathbf{q}|}{4} \frac{-k^2}{(kL)^2} \sum_{\lambda_2, \lambda_1} \left\{ \begin{aligned} & \left| C_{L-} (\tilde{F}_{\lambda_2 \lambda_1}^{eL}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{eL}(\theta, \phi)) + C_{R-} (\tilde{F}_{\lambda_2 \lambda_1}^{eR}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{eR}(\theta, \phi)) + C_- (\tilde{F}_{\lambda_2 \lambda_1}^{e-}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{e-}(\theta, \phi)) \right|^2 \\ & + \left| C_{L+} (\tilde{F}_{\lambda_2 \lambda_1}^{eL}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{eL}(\theta, \phi)) + C_{R+} (\tilde{F}_{\lambda_2 \lambda_1}^{eR}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{eR}(\theta, \phi)) + C_+ (\tilde{F}_{\lambda_2 \lambda_1}^{e+}(\theta, \phi) - \tilde{G}_{\lambda_2 \lambda_1}^{e+}(\theta, \phi)) \right|^2 \end{aligned} \right\}$$

**arXiv:1711.02403v3**

• For anti-neutrino, swap  $C_{L_\pm}$  and  $C_{R_\pm}$

## 3.2 Neutrino - nucleon interactions

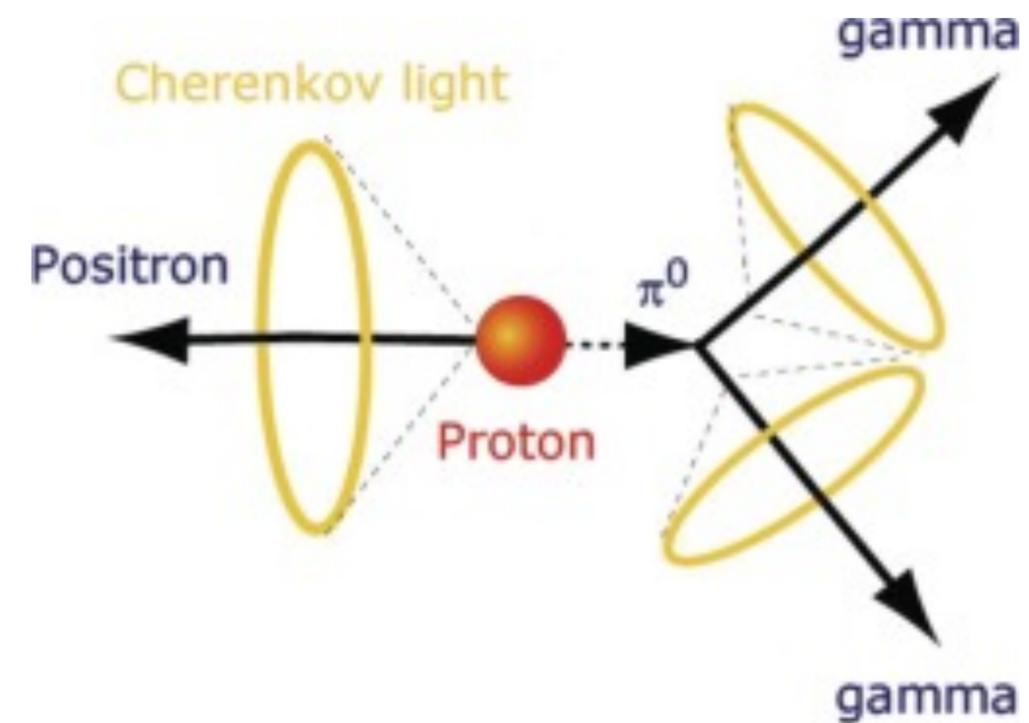
- Neutrino interactions at 0.1-20GeV energy:

- Resonant meson production (CC1 $\pi$  dominates at low energy)

$$\begin{array}{ll} \nu_{\mu} p \rightarrow \mu^{-} p \pi^{+}, & \bar{\nu}_{\mu} p \rightarrow \mu^{+} p \pi^{-} \\ \nu_{\mu} n \rightarrow \mu^{-} p \pi^{0}, & \bar{\nu}_{\mu} p \rightarrow \mu^{+} n \pi^{0} \\ \nu_{\mu} n \rightarrow \mu^{-} n \pi^{+}, & \bar{\nu}_{\mu} n \rightarrow \mu^{+} n \pi^{-} \end{array}$$

$$\begin{array}{ll} \nu_{\mu} p \rightarrow \nu_{\mu} p \pi^{0}, & \bar{\nu}_{\mu} p \rightarrow \bar{\nu}_{\mu} p \pi^{0} \\ \nu_{\mu} p \rightarrow \nu_{\mu} n \pi^{+}, & \bar{\nu}_{\mu} n \rightarrow \bar{\nu}_{\mu} n \pi^{0} \\ \nu_{\mu} n \rightarrow \nu_{\mu} n \pi^{0}, & \bar{\nu}_{\mu} n \rightarrow \bar{\nu}_{\mu} n \pi^{0} \\ \nu_{\mu} n \rightarrow \nu_{\mu} p \pi^{-}, & \bar{\nu}_{\mu} n \rightarrow \bar{\nu}_{\mu} p \pi^{-} \end{array}$$

- Main backgrounds of nucleon decay search: same produced particles

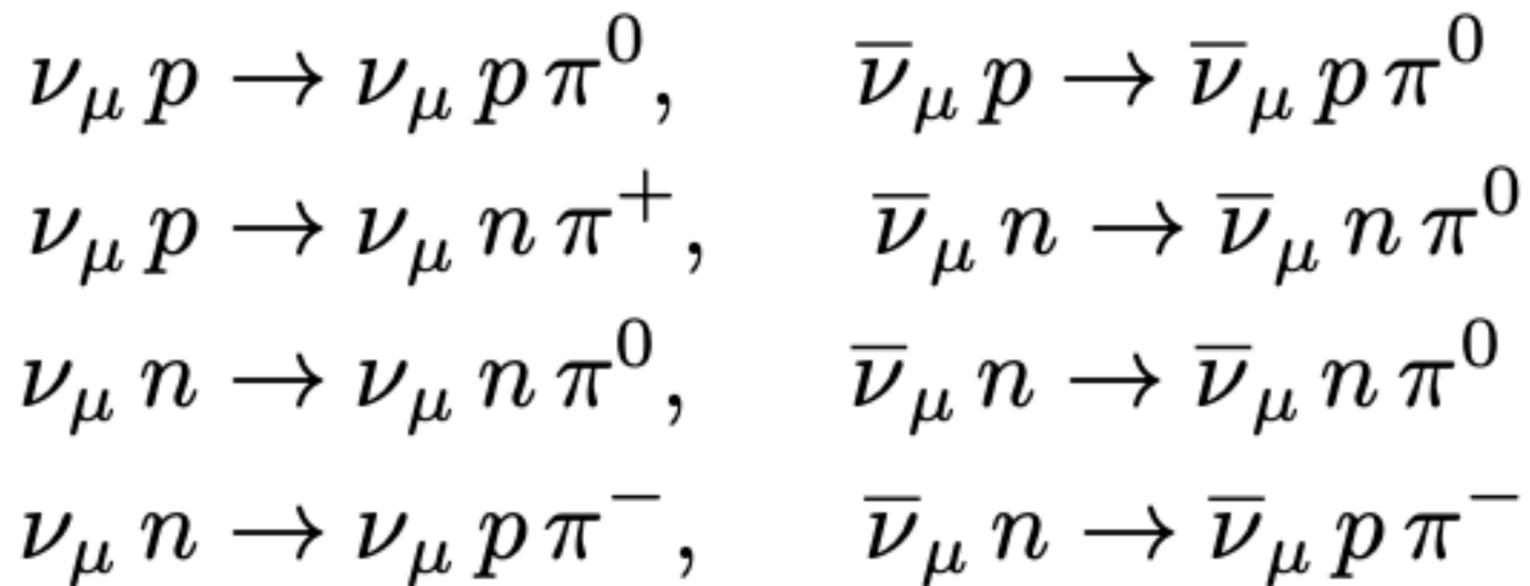
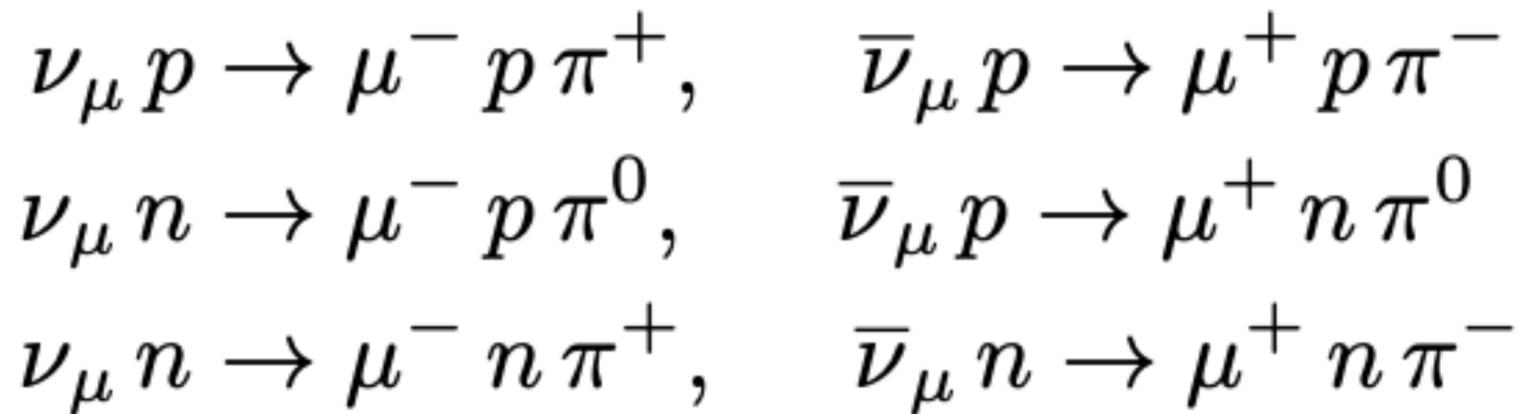


$$p \rightarrow e^{+} + \pi^{0}$$

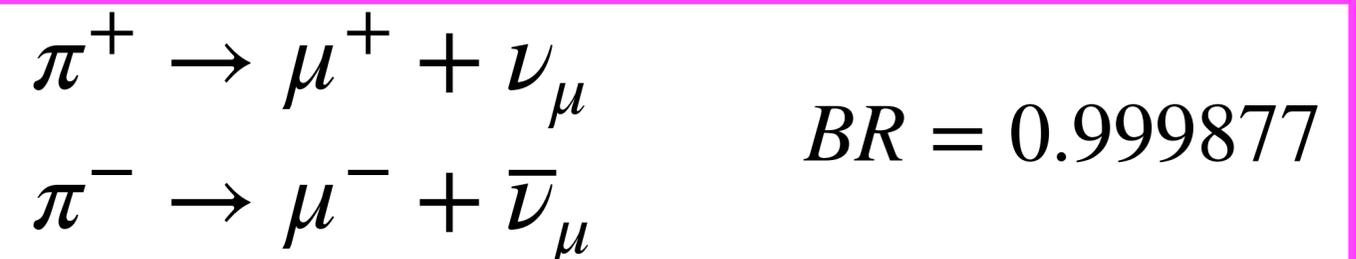
## 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:

- Resonant meson production (CC1 $\pi$  dominates at low energy)



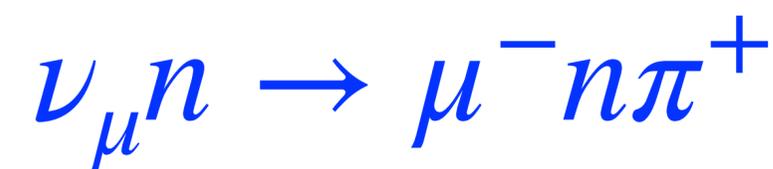
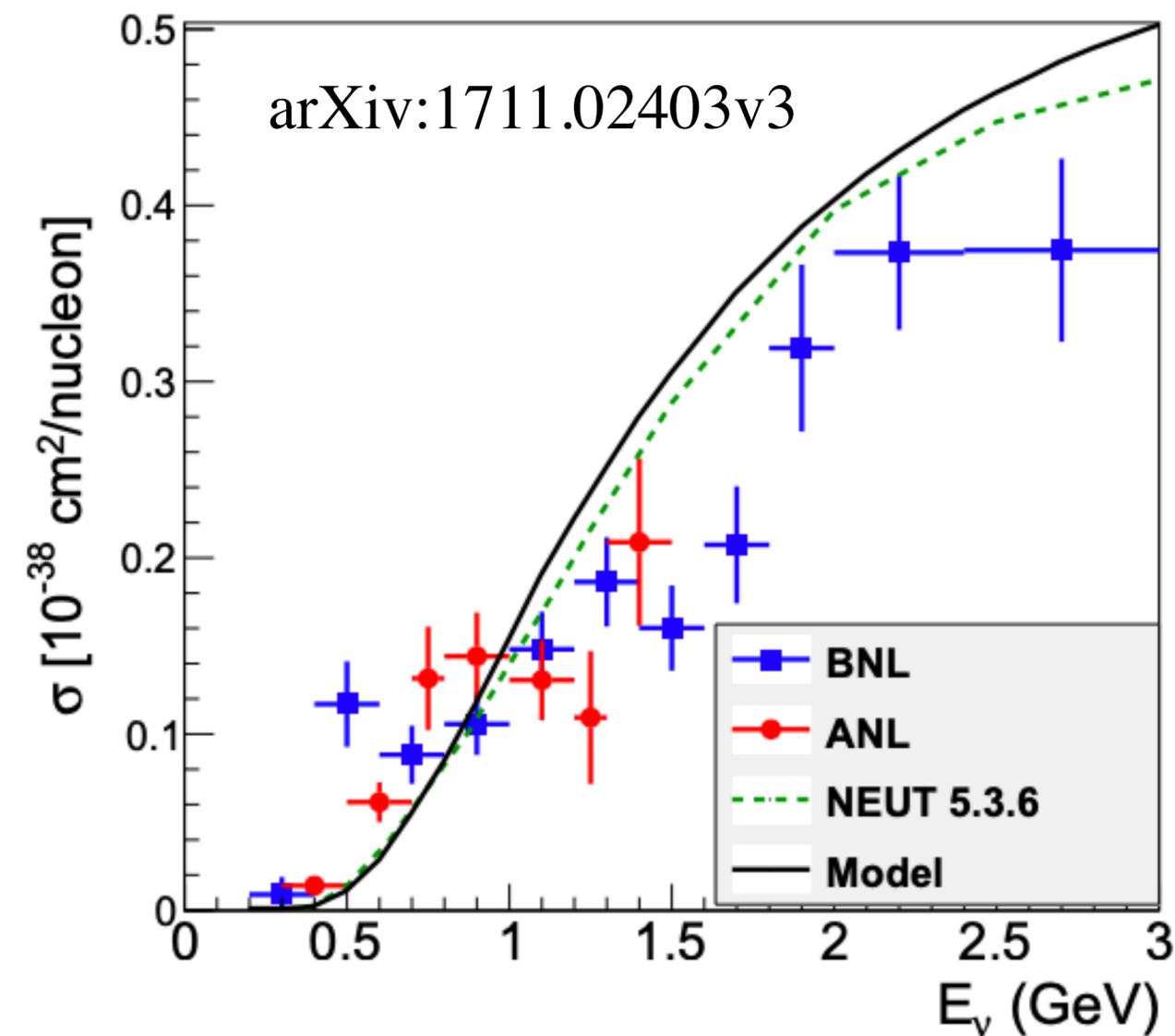
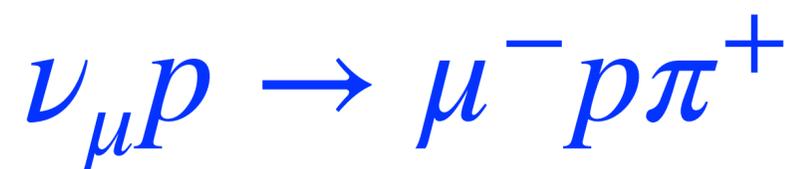
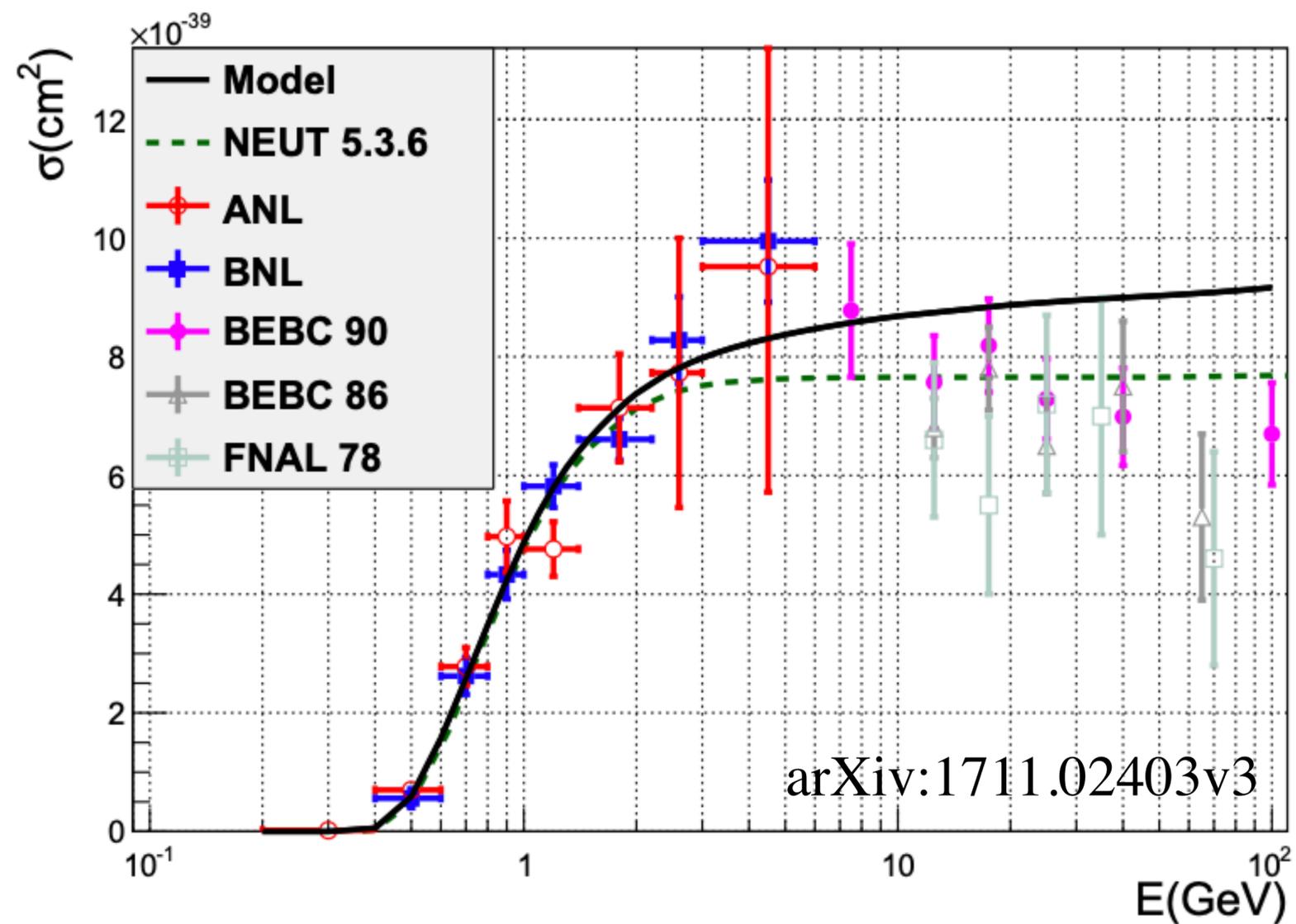
- Main backgrounds of CC0 $\pi$  at T2K:  $\pi$  can mimic  $\mu/e$  signals



- $\pi$  can be absorbed into the nucleus: the event is indistinguishable with a QE event (similar observed final particles).

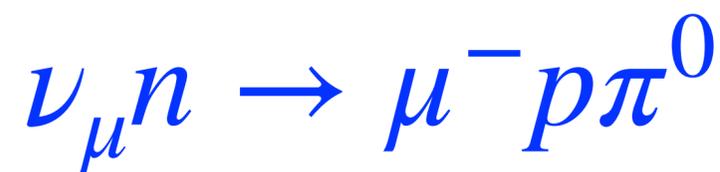
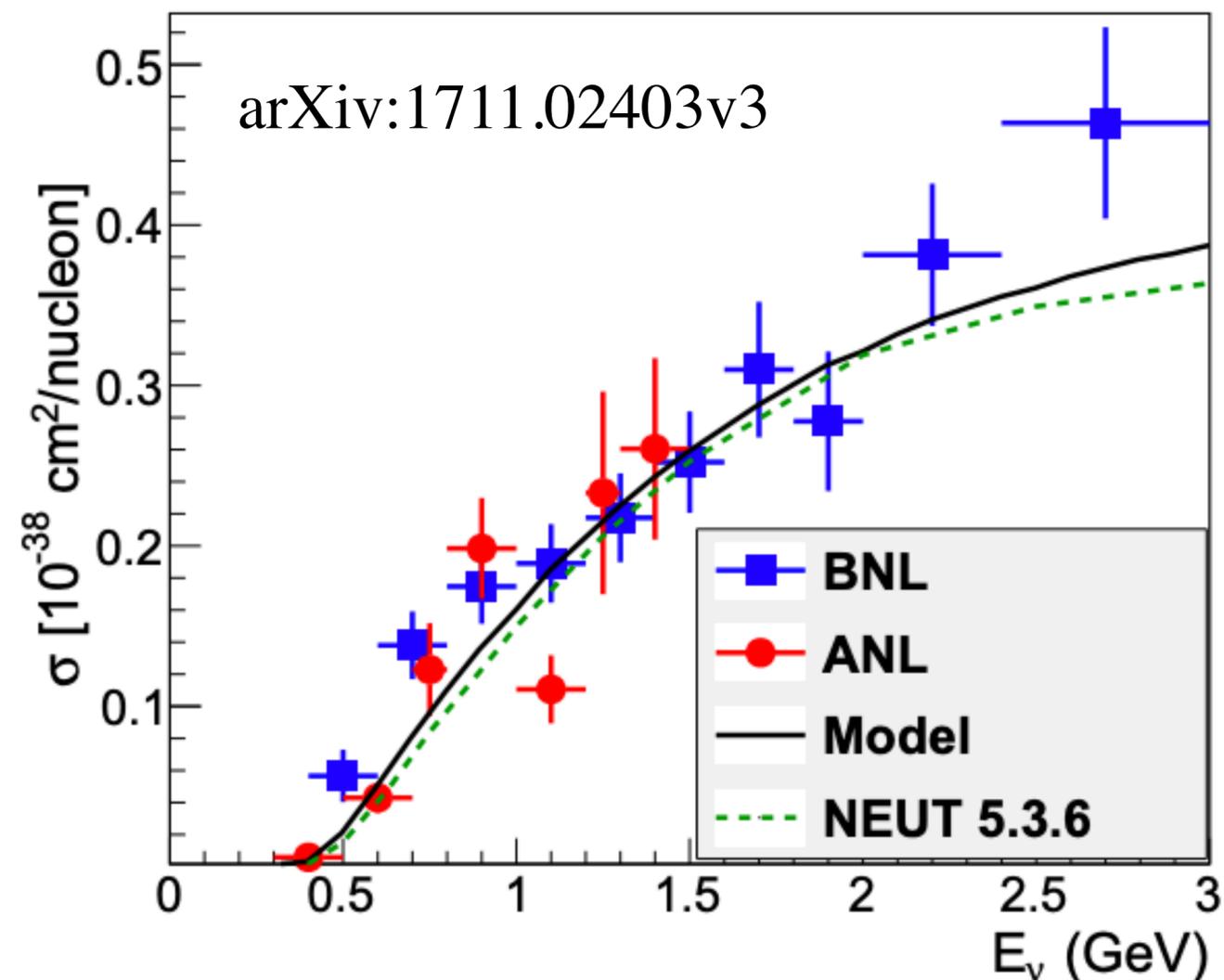
# 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:
  - Resonant meson production (CC1 $\pi$  dominates at low energy)



# 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:
  - Resonant meson production (CC1 $\pi$  dominates at low energy)



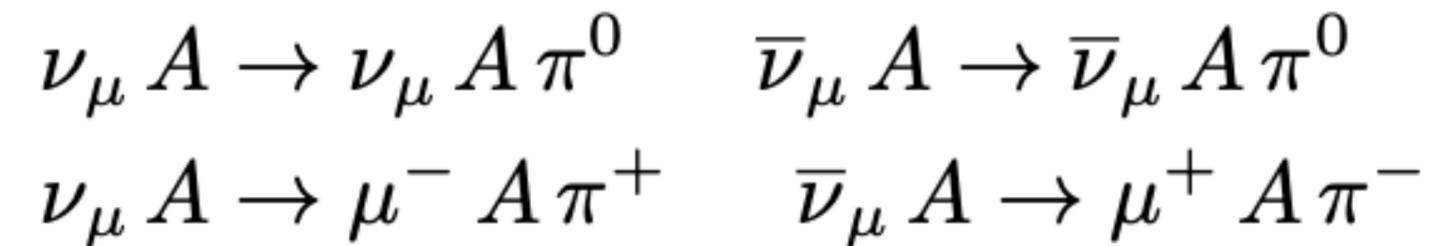
Experiment	Target	NC/CC Ratio	Value	Reference
ANL	$H_2$	$\sigma(\nu_{\mu} p \rightarrow \nu_{\mu} p \pi^0) / \sigma(\nu_{\mu} p \rightarrow \mu^{-} p \pi^+)$	$0.51 \pm 0.25$	(Barish <i>et al.</i> , 1974)
ANL	$H_2$	$\sigma(\nu_{\mu} p \rightarrow \nu_{\mu} p \pi^0) / \sigma(\nu_{\mu} p \rightarrow \mu^{-} p \pi^+)$	$0.09 \pm 0.05^*$	(Derrick <i>et al.</i> , 1981)
ANL	$H_2$	$\sigma(\nu_{\mu} p \rightarrow \nu_{\mu} n \pi^+) / \sigma(\nu_{\mu} p \rightarrow \mu^{-} p \pi^+)$	$0.17 \pm 0.08$	(Barish <i>et al.</i> , 1974)
ANL	$H_2$	$\sigma(\nu_{\mu} p \rightarrow \nu_{\mu} n \pi^+) / \sigma(\nu_{\mu} p \rightarrow \mu^{-} p \pi^+)$	$0.12 \pm 0.04$	(Derrick <i>et al.</i> , 1981)
ANL	$D_2$	$\sigma(\nu_{\mu} n \rightarrow \nu_{\mu} p \pi^-) / \sigma(\nu_{\mu} n \rightarrow \mu^{-} n \pi^+)$	$0.38 \pm 0.11$	(Fogli and Nardulli, 1980)
GGM	$C_3H_8$ $CF_3Br$	$\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} N \pi^0) / 2 \sigma(\nu_{\mu} n \rightarrow \mu^{-} p \pi^0)$	$0.45 \pm 0.08$	(Krenz <i>et al.</i> , 1978a)
CERN PS	$Al$	$\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} N \pi^0) / 2 \sigma(\nu_{\mu} n \rightarrow \mu^{-} p \pi^0)$	$0.40 \pm 0.06$	(Fogli and Nardulli, 1980)
BNL	$Al$	$\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} N \pi^0) / 2 \sigma(\nu_{\mu} n \rightarrow \mu^{-} p \pi^0)$	$0.17 \pm 0.04$	(Lee <i>et al.</i> , 1977)
BNL	$Al$	$\sigma(\nu_{\mu} N \rightarrow \nu_{\mu} N \pi^0) / 2 \sigma(\nu_{\mu} n \rightarrow \mu^{-} p \pi^0)$	$0.25 \pm 0.09^{**}$	(Nienaber, 1988)
ANL	$D_2$	$\sigma(\nu_{\mu} n \rightarrow \nu_{\mu} p \pi^-) / \sigma(\nu_{\mu} p \rightarrow \mu^{-} p \pi^+)$	$0.11 \pm 0.022$	(Derrick <i>et al.</i> , 1981)

- The cross sections of the other channels can be deduced from the above three ones by experimental measurements

## 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:

- Coherent pion production



- Negligible transferred energy
- Coherently produce pion
- No nuclear recoil

$$\left. \frac{d^3\sigma}{dQ^2 dy d|t|} \right|_{Q^2=0} = \frac{G_F^2}{2\pi^2} f_\pi^2 \frac{1-y}{y} \frac{d\sigma(\pi A \rightarrow \pi A)}{d|t|}$$

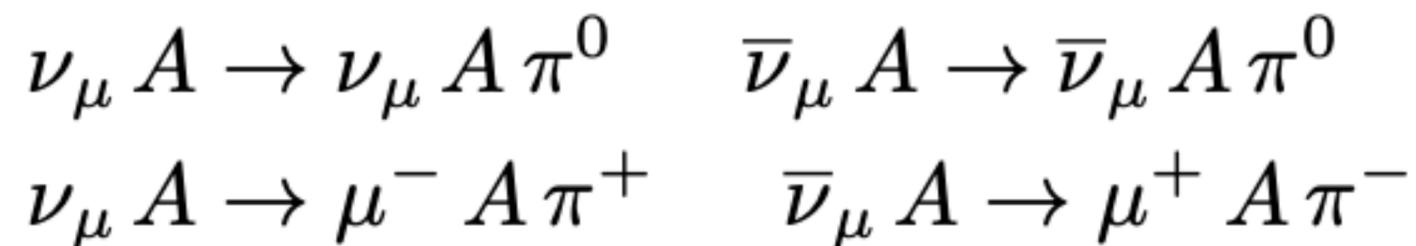
Adler's PCAC theorem

- $y = E_\pi / E_\nu$
- $f_\pi$ : pion decay constant
- $|t|$ : magnitude of the square of the 4-momentum transfer to the nucleus
- $Q^2 = -q^2$ : 4-momentum transferred to hadronic system

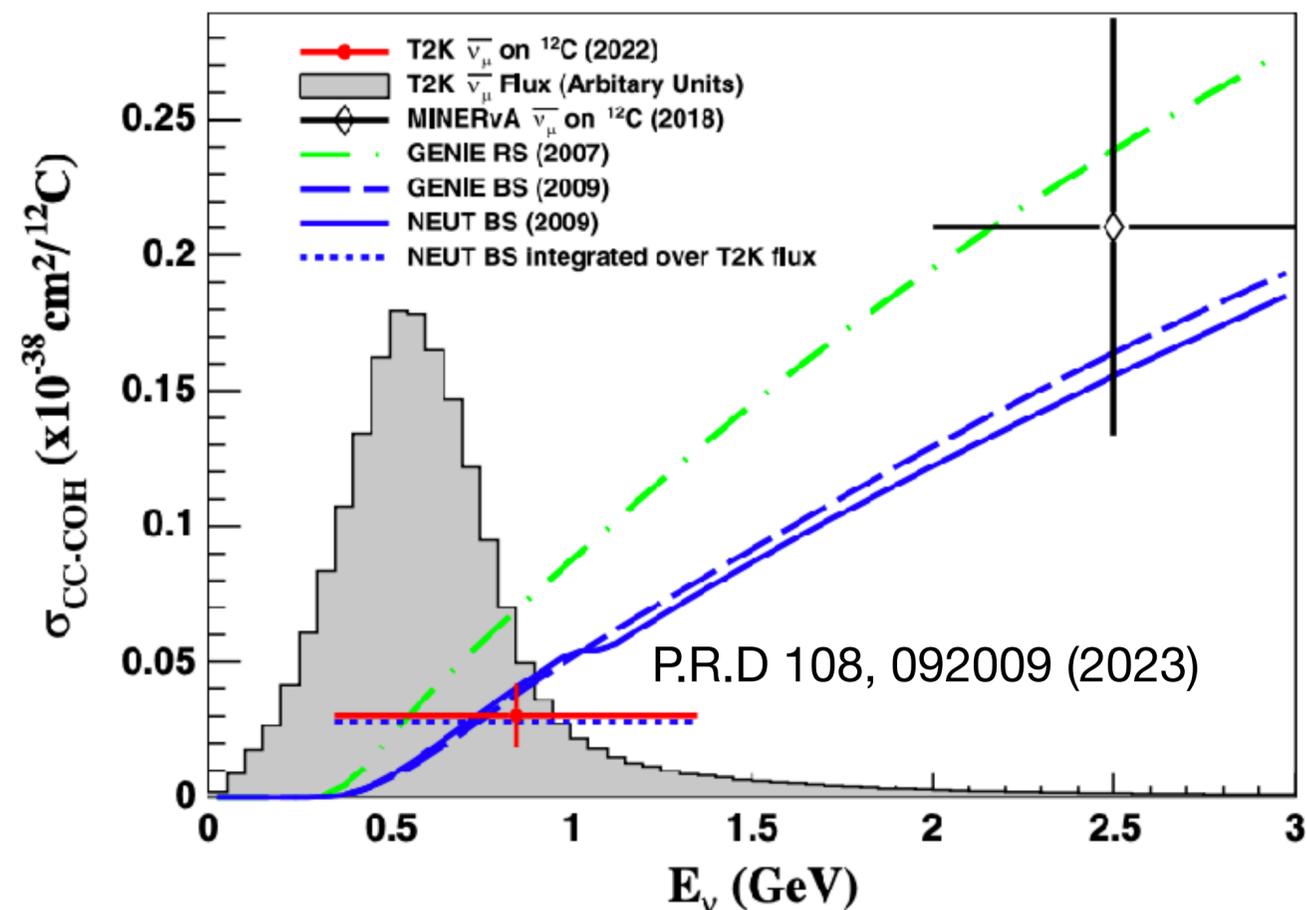
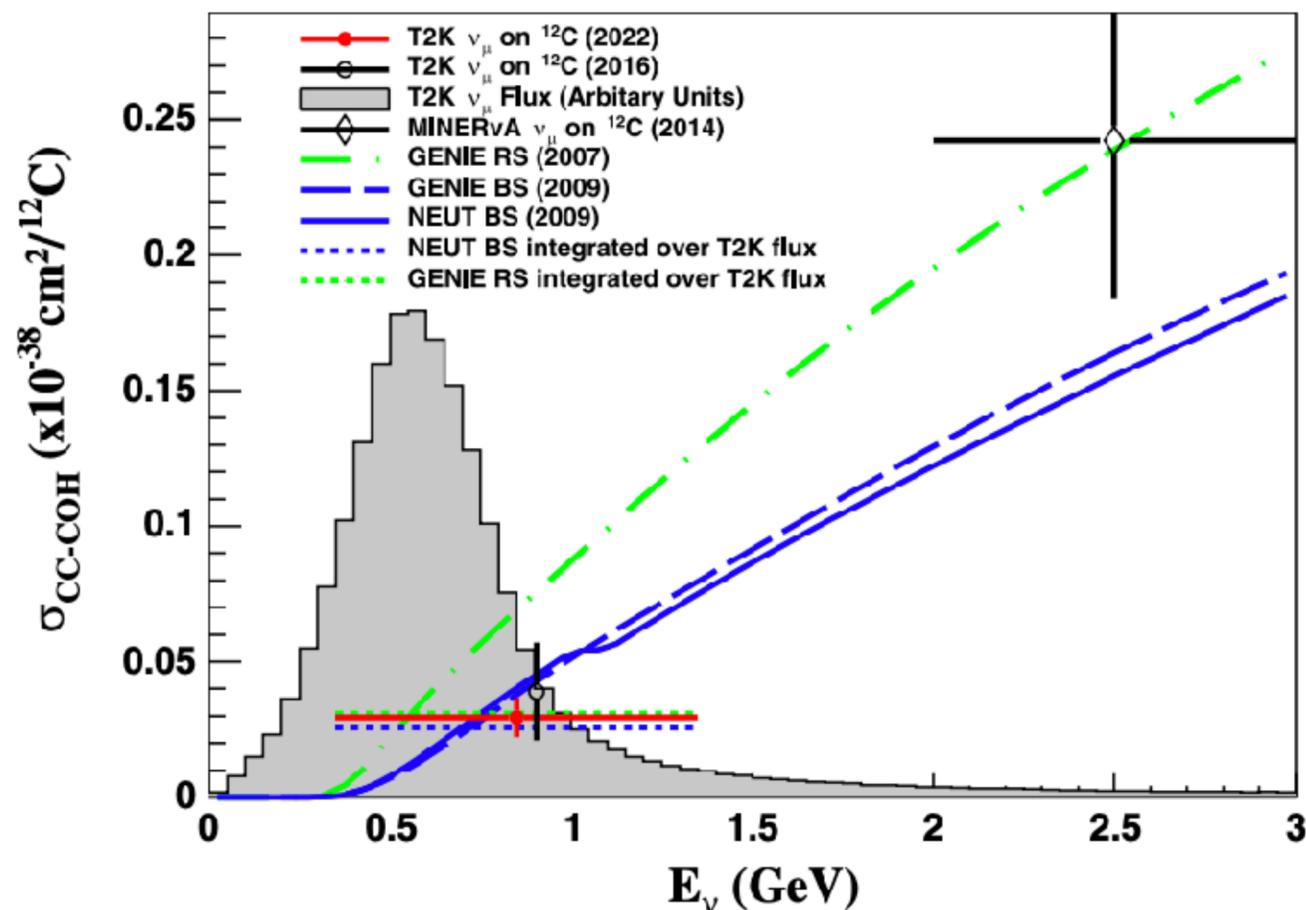
# 3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- Coherent pion production



- Negligible transferred energy
- Coherently produce pion
- No nuclear recoil



- CC COH Xsec measurement on  $_{12}\text{C}$  at T2K for  $\nu_{\mu}$  (left) and  $\bar{\nu}_{\mu}$  (right)

# 3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 20 - 500 GeV energy:

• Deep inelastic scattering (CC)

$$\nu_l + N \rightarrow l^- + X$$

$$\bar{\nu}_l + N \rightarrow l^+ + X$$

(-)+ for (anti-)neutrino scattering

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 m_N E_\nu}{\pi(1 + Q^2/m_{W,Z}^2)^2} \left[ xy^2 F_1(x, Q^2) + (1 - y) F_2(x, Q^2) \pm y(1 - y/2) xF_3(x, Q^2) \right]$$

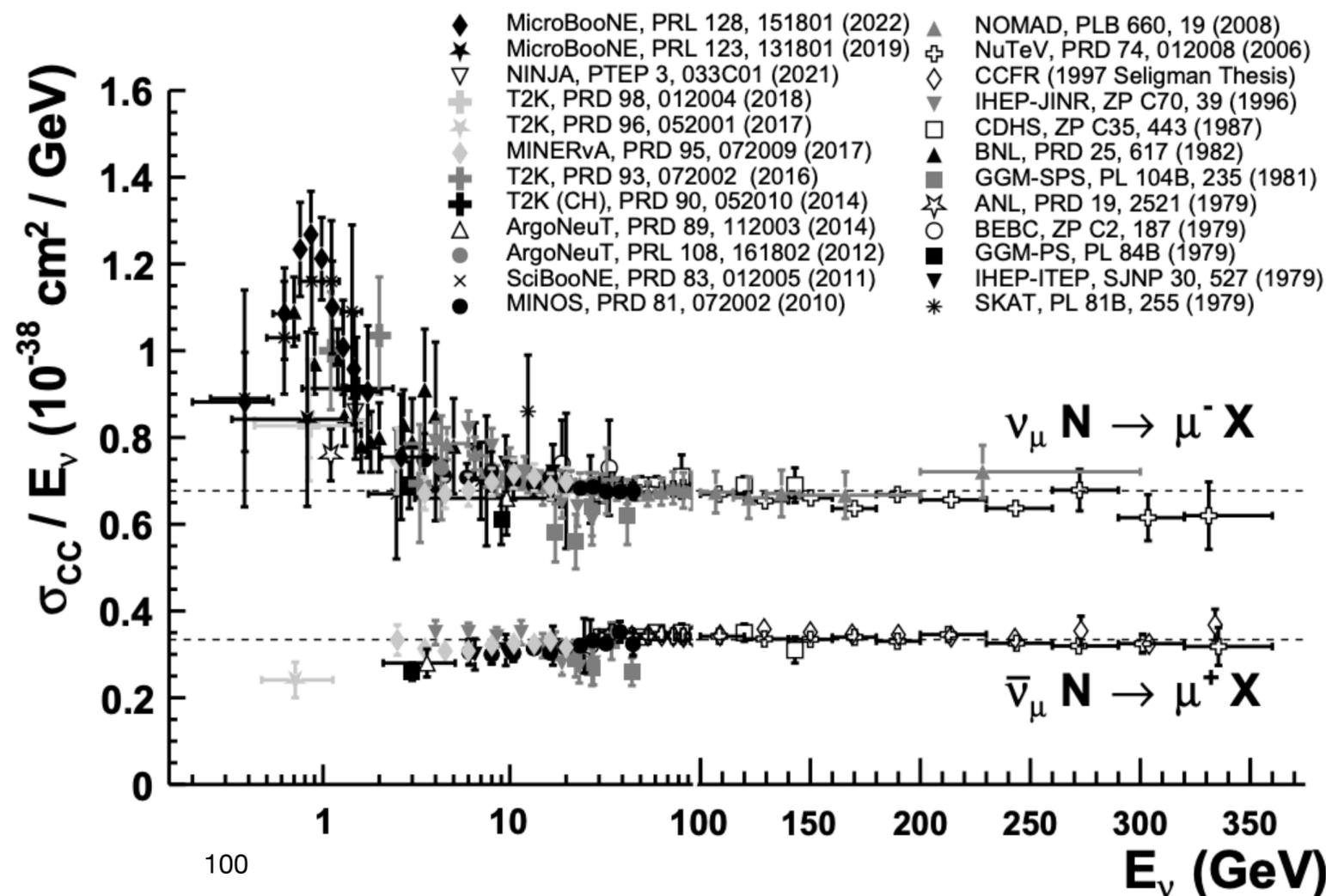
$$x = \frac{Q^2}{2p_N \cdot q}$$

$$y = \frac{p_N \cdot q}{p_N \cdot p_\nu}$$

$$xy = \frac{Q^2}{s - m_N^2}$$

$$s = (p_\nu + p_N)^2 = m_N^2 + 2p_\nu \cdot p_N$$

$$Q^2 = -m_l^2 + 2E_\nu(E_l - p_l \cos \theta)$$



# Abbreviation

- ANL: Argonne National Laboratory
- BNL: Brookhaven National Laboratory
- BEBC: Big European Bubble Chamber
- FNAL: Fermi National Accelerator Laboratory
- Adler's PCAC theorem: partially conserved axial vector current

# Natural unit

$$\hbar = c = 1.$$

$$E^2 = p^2 c^2 + m^2 c^4 \quad \text{becomes} \quad E^2 = p^2 + m^2.$$

**Table 2.1** Relationship between S.I. and natural units.

Quantity	[kg, m, s]	$[\hbar, c, \text{GeV}]$	$\hbar = c = 1$
Energy	$\text{kg m}^2 \text{s}^{-2}$	GeV	GeV
Momentum	$\text{kg m s}^{-1}$	GeV/c	GeV
Mass	kg	$\text{GeV}/c^2$	GeV
Time	s	$(\text{GeV}/\hbar)^{-1}$	$\text{GeV}^{-1}$
Length	m	$(\text{GeV}/\hbar c)^{-1}$	$\text{GeV}^{-1}$
Area	$\text{m}^2$	$(\text{GeV}/\hbar c)^{-2}$	$\text{GeV}^{-2}$

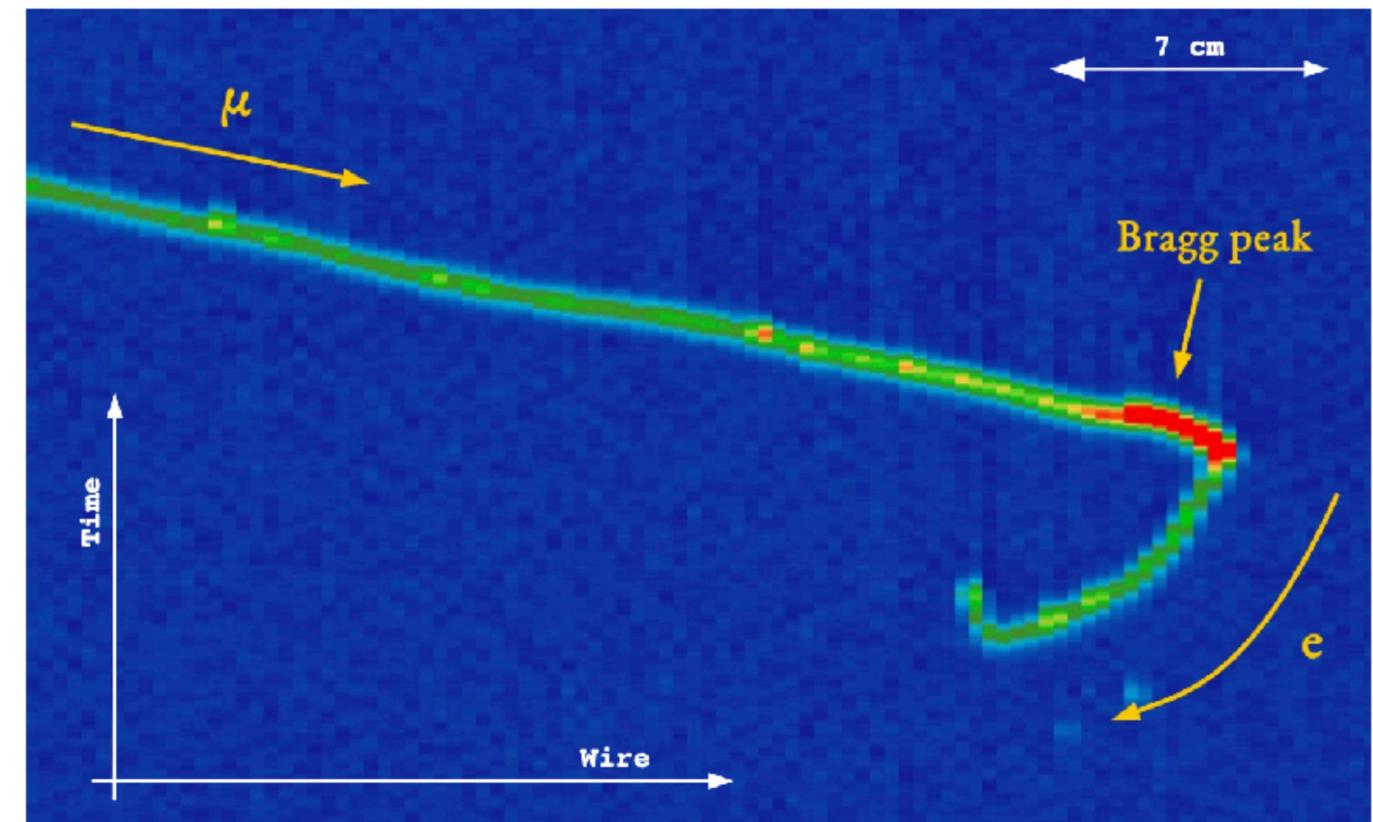
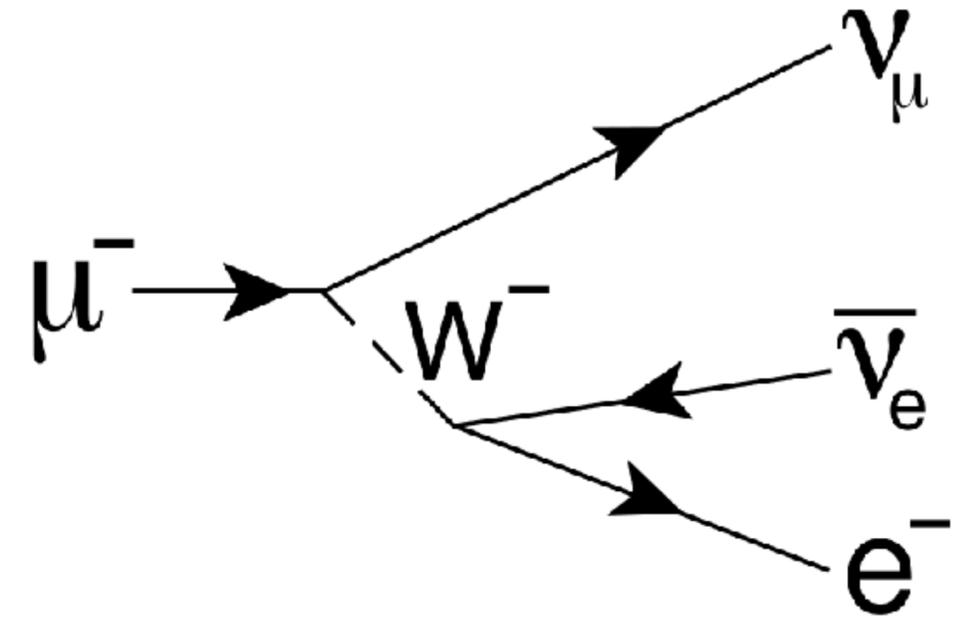
- To convert from natural unit to SI unit, the mass must be multiplied by  $c^2$ , the length must be multiplied by  $\hbar c$ , ...

$$\hbar c = 0.197 \text{ GeV fm},$$

where one femtometre (fm) =  $10^{-15}$  m.

# Michel electron

Experiments soon revealed the manner of muon's decay, into an electron and a pair of neutrinos, and the shape of the electron spectrum was first calculated by Louis Michel in 1950. This is where "Michel electron" comes from. Michel introduced a single parameter  $\rho$  to describe the shape. However, after the non-conservation of parity in weak interactions proposal of Lee and Yang three more parameters ( $\eta$ ,  $\xi$ , and  $\delta$ ) had to be added, and now all four are referred to collectively as "Michel parameters".



# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT

- **NEUT limitations:**
  - Nuclear models are inconsistent between models or steps in the factorisation:
    - Benhar et al. SF can be used for CCQE but no other modes
    - LFG used for FSI nuclear description
  - Benhar et al. SF Pauli blocking uses simple, RFG-like approach
  - Nuclear effects in single pion production are largely ignored
  - Nuclear transparency has no effect on inclusive cross-section

From L. Pickering

# 4. Introduction to NEUT - an event generator

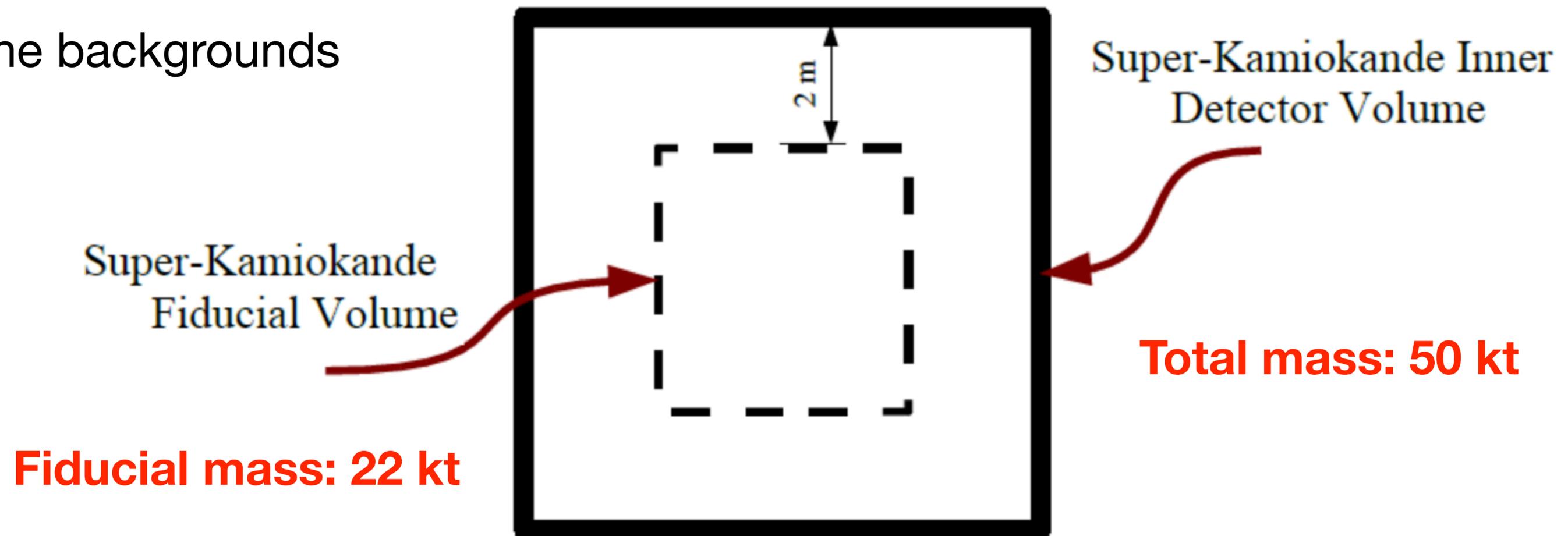
## 4.2: Introduction to NEUT

- **Future plan:**

- 
- Development has begun on NEUT6 - Targeted at HK and final T2K analyses:
    - Significant reorganization of code-base
    - Improved, modern build system
    - Removed dependence on an external CERNLIB2005
    - New TOML-based configuration file
    - Modern C/Fortran interop
    - Automatic C/Fortran interface generation for model integration
  - Aim is to release NEUT6 as open source under the GPL before the end of 2023
    - Will also release the final NEUT5 series release as open source
  - Hope to produce comprehensive data-model comparisons alongside NEUT6 release

# Fiducial volume

- An inner volume of **particle detector** media in which background events are largely excluded
- The well understood region of the detector where the events are accepted for physics analysis
- Reduce the backgrounds



# Solid angle

Solid Angle

$$d\Omega = \frac{dA_j}{r^2}$$

$$dA_j = AB \times BC$$

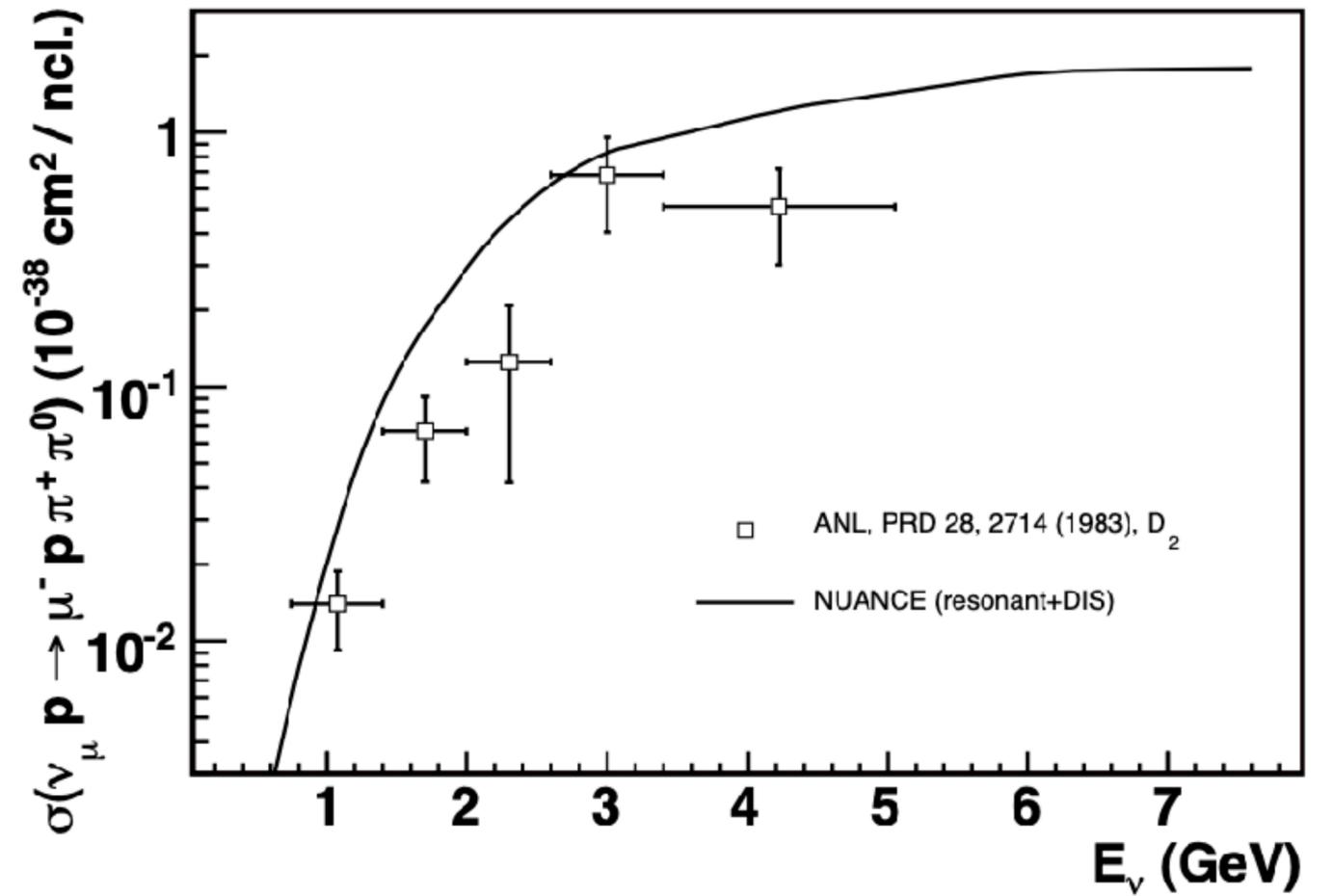
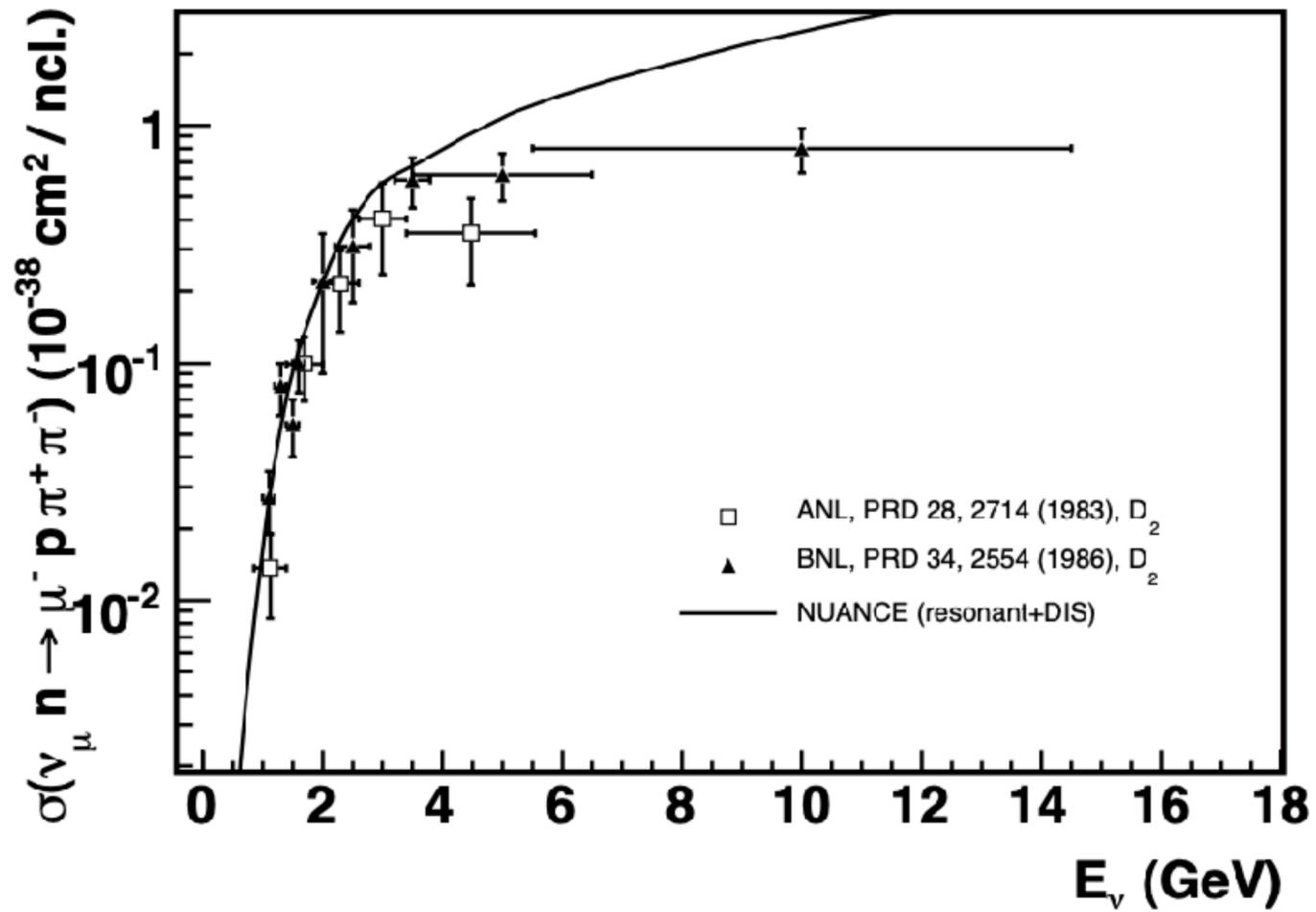
$$dA_j = (r \sin\theta d\phi) \times (r d\theta)$$

$$d\Omega = \frac{r^2 \sin\theta d\theta d\phi}{r^2}$$

$$d\Omega = \sin\theta d\theta d\phi$$

# 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:
  - Multi-pion production



## 3.2 Neutrino - nucleon interactions

- Neutrino interactions at 500 GeV - 1 EeV ( $10^{18}$  eV) energy:
  - Ultra-high energy neutrinos

# 3.2 Neutrino - nucleon interactions

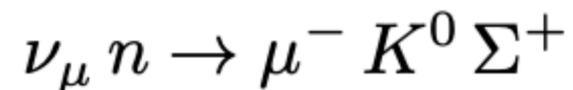
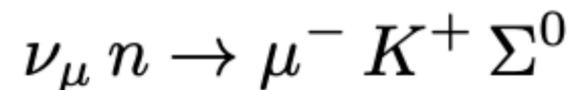
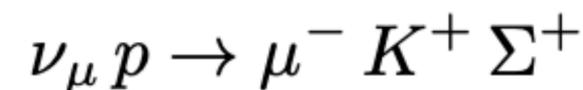
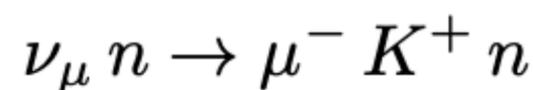
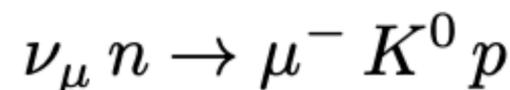
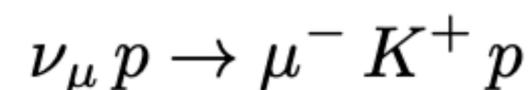
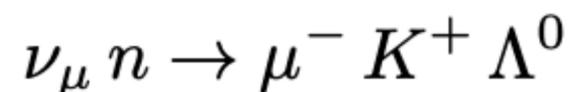
- Neutrino interactions at 0.1-20GeV energy:

- Kaon production

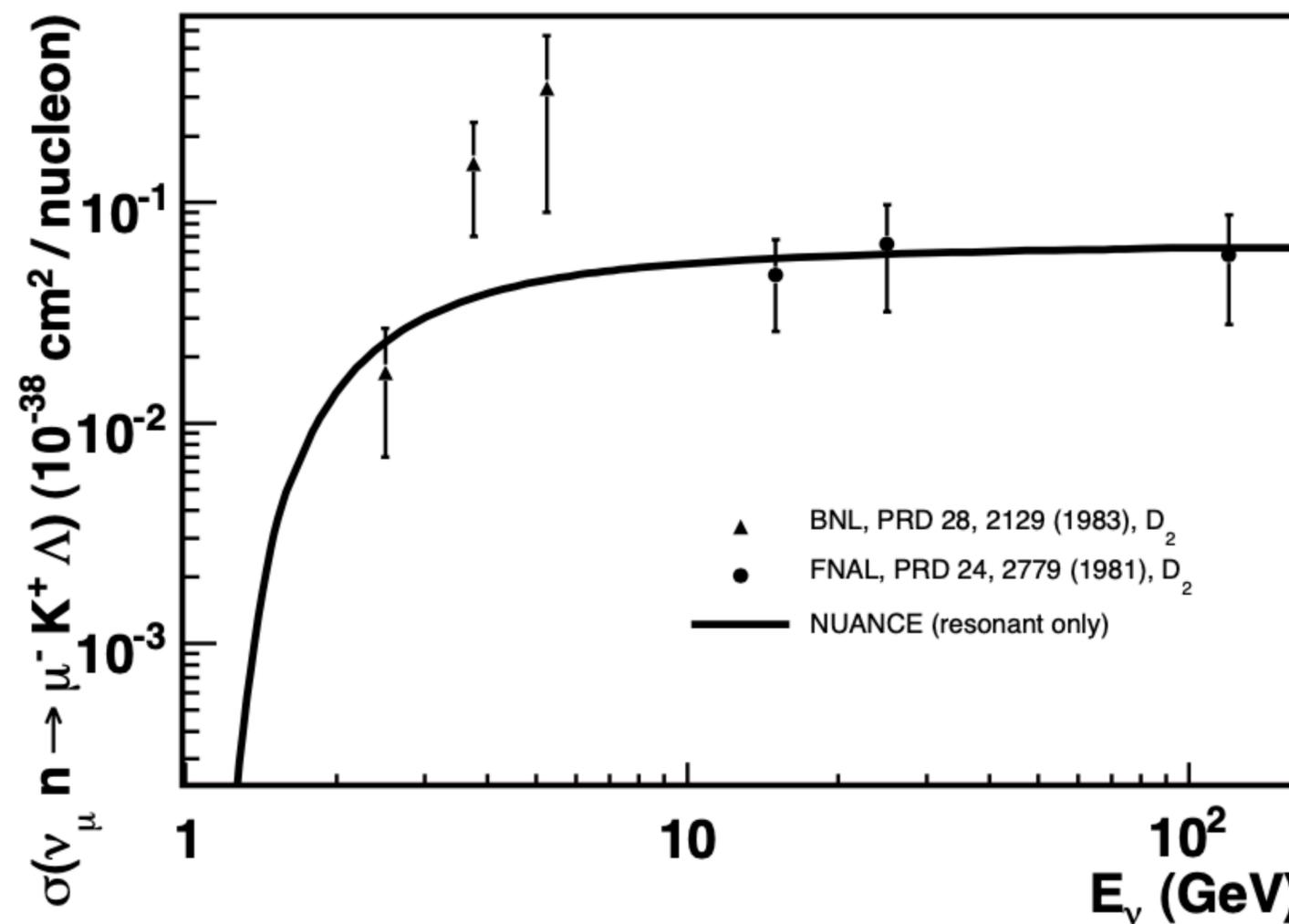
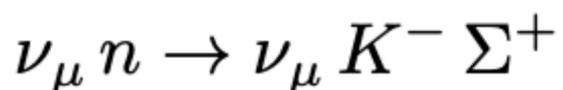
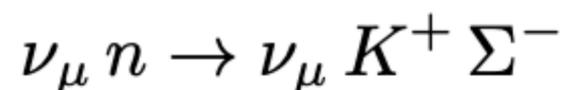
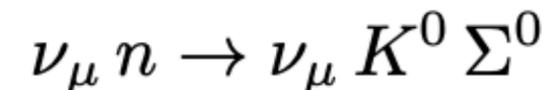
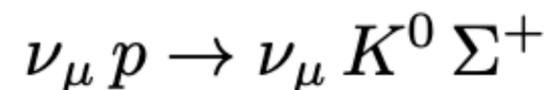
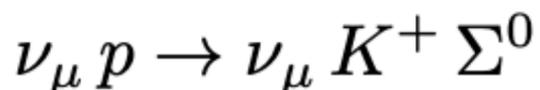
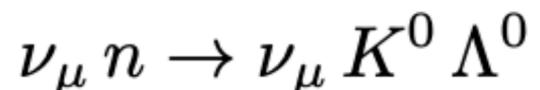
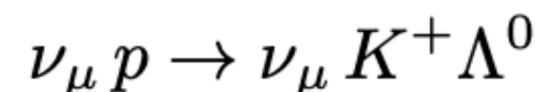
- Potential background of proton decay:



CC :



NC :



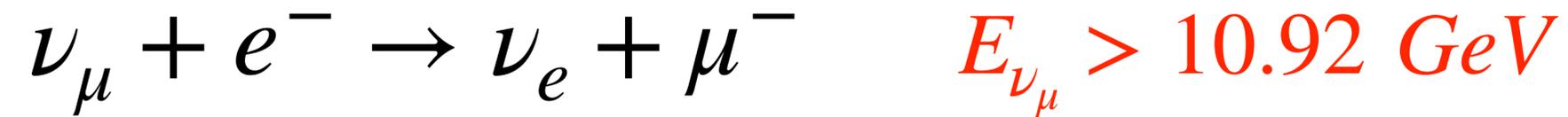
# Fermi's golden rule

- Transition rate:  $\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_i) = 2\pi \int |T_{fi}|^2 \delta(E_i - E) dn$
- $T_{fi}$ : transition matrix element
- $\rho(E_i) = \left. \left| \frac{dn}{dE} \right| \right|_{E_i} = \int \frac{dn}{dE} \delta(E_i - E) dE$ : density of states
- $dn = (2\pi)^3 \frac{d^3 \vec{p}_1}{(2\pi)^3} \frac{d^3 \vec{p}_2}{(2\pi)^3} \delta^3(\vec{p}_a + \vec{p}_b - \vec{p}_1 - \vec{p}_2)$ : number of accessible states in the energy range  $E \rightarrow E + dE$
- Lorentz invariant matrix element:  

$$M_{fi} = \sqrt{2E_a \cdot 2E_b \cdot 2E_1 \cdot 2E_2} T_{fi}$$
- $$\Gamma_{fi} = \frac{(2\pi)^4}{4E_a E_b} \int |M_{fi}|^2 \delta(E_a + E_b - E_1 - E_2) \delta^3(\vec{p}_a + \vec{p}_b - \vec{p}_1 - \vec{p}_2) \frac{d^3 \vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3 \vec{p}_2}{(2\pi)^3 2E_2}$$

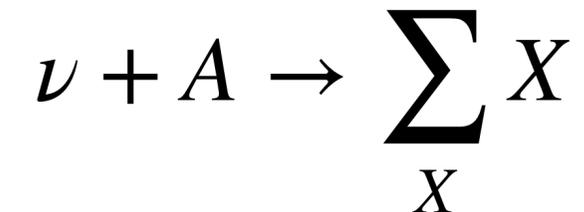
# 3.1 Neutrino - electron interactions

- Neutrino - electron quasi-elastic scattering



Did you calculate this value as Sanjib's request?

- Calculate threshold:



- A at rest, neglecting neutrino mass, in center of mass frame,  $s = 2E_{\nu}m_A + m_A^2$  must

greater than  $\left(\sum_X m_X\right)^2$

$$E_{\nu}^{th} = \frac{\left(\sum_X m_X\right)^2}{2m_A} - \frac{m_A}{2}$$

$$m_e = 0.511 \text{ MeV}$$

$$m_{\mu} = 105.66 \text{ MeV}$$

# 3. Neutrino energies and interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

• **Quasi-elastic scattering (QE):**

- CCQE:  $\nu_l + n \rightarrow l^- + p$
- NC:  $\nu_l + N \rightarrow \nu_l + N'$

• **Resonant meson production (RES):**

- CC RES:  $\nu_l + N \rightarrow l^- + N' + m$
- NC RES:  $\nu_l + N \rightarrow \nu_l + N' + m$

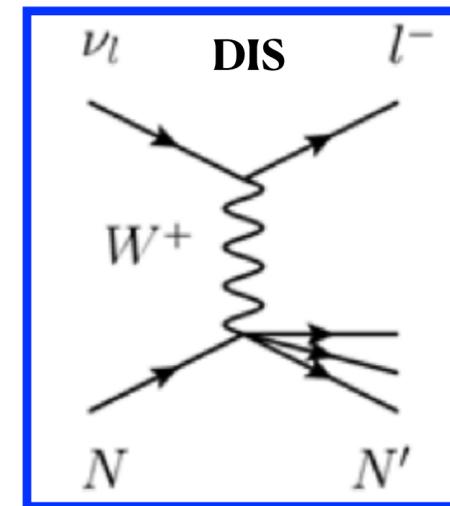
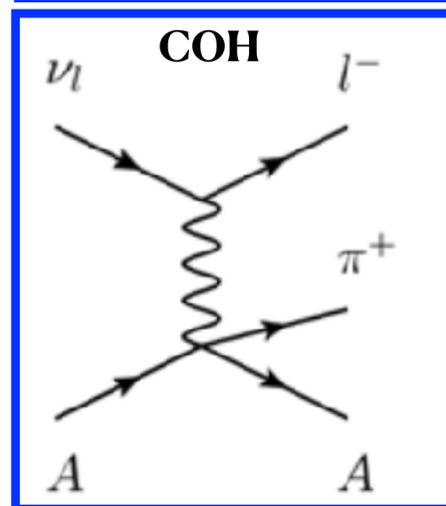
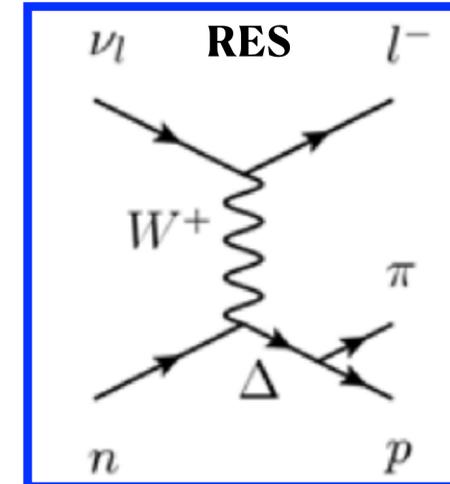
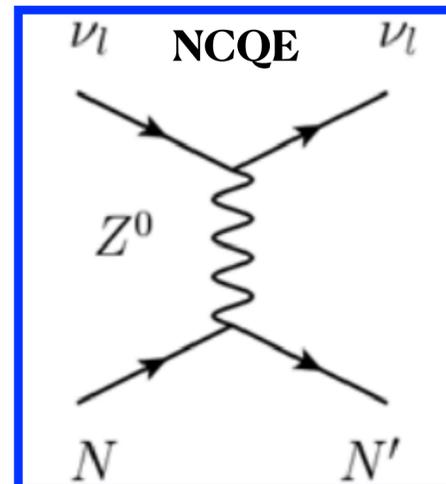
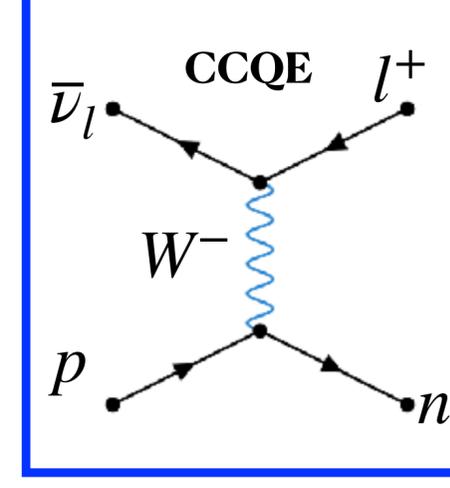
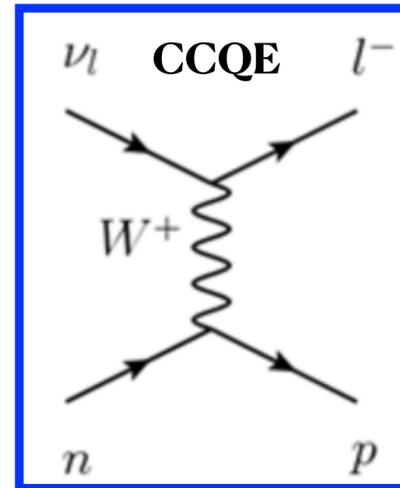
• **Coherent pion production (COH):**

- CC COH:  $\nu_l + A \rightarrow l^- + A + \pi^+$
- NC COH:  $\nu_l + A \rightarrow \nu_l + A + \pi^0$

• **Deep inelastic scattering (DIS):**

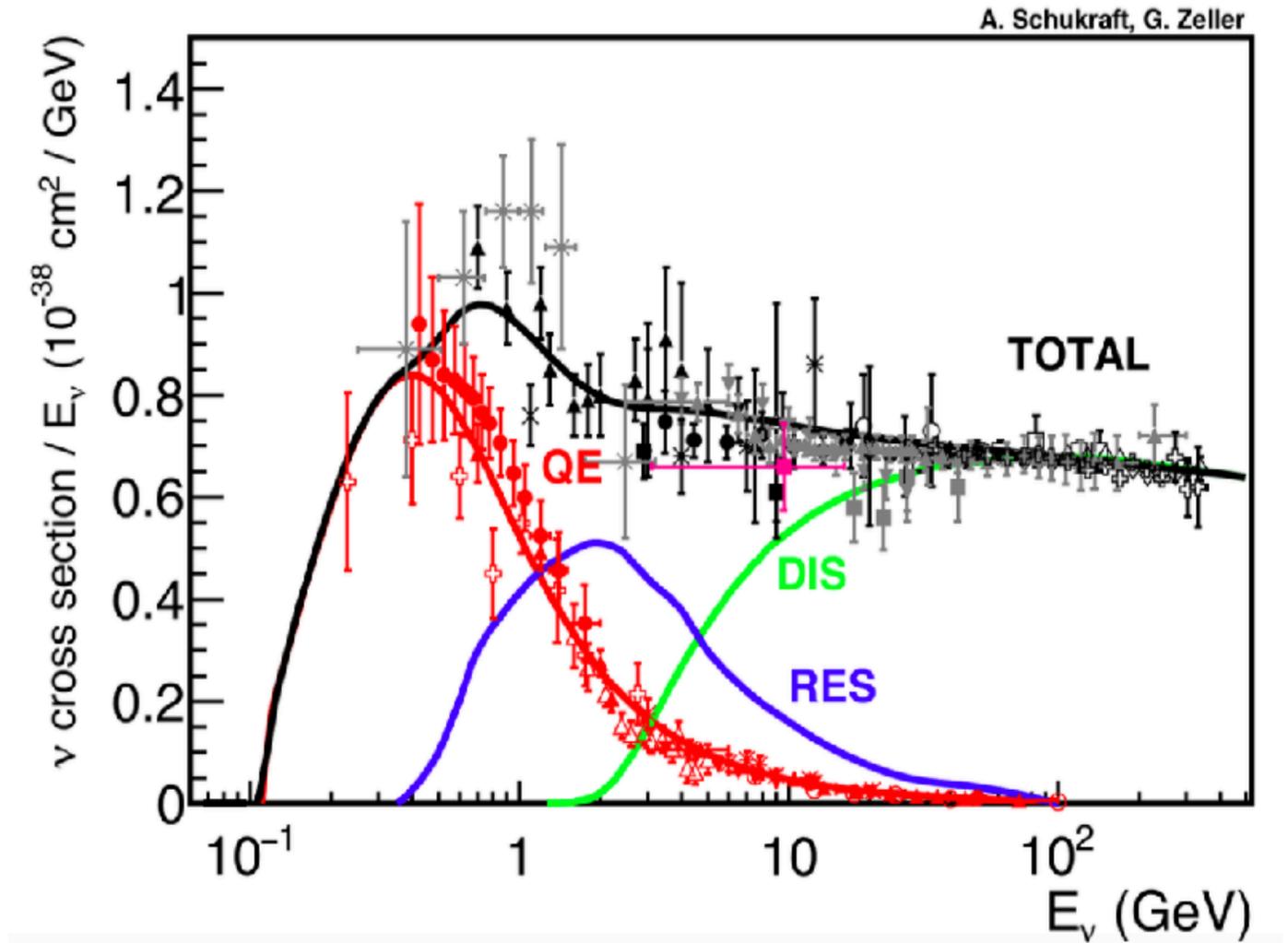
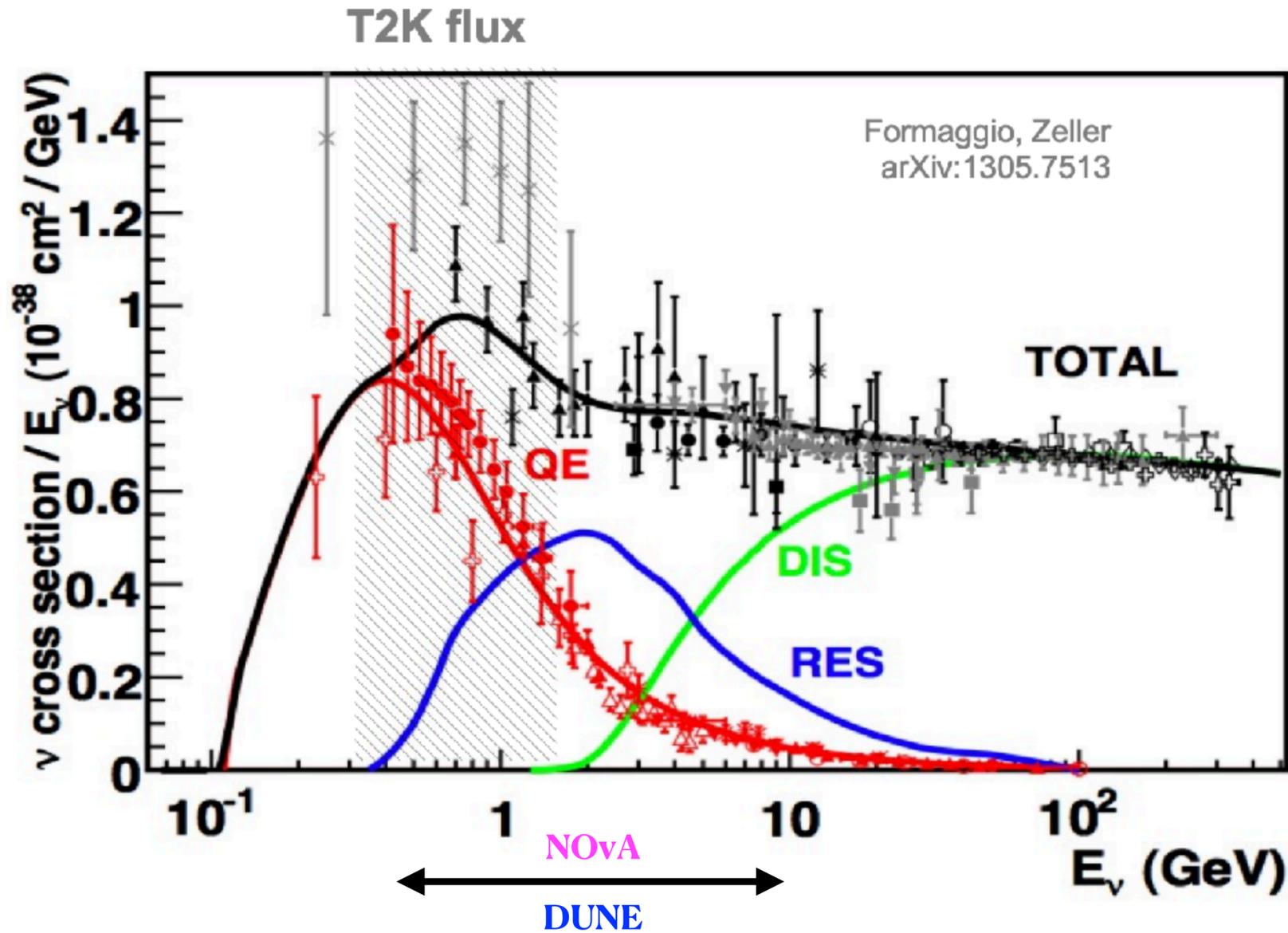
- CC DIS:  $\nu_l + N \rightarrow l^- + N' + \text{hadrons}$
- NC DIS:  $\nu_l + N \rightarrow \nu_l + N' + \text{hadrons}$

Where  $l^- = \{e^-, \mu^-, \tau^-\}$ ;  $N = \{p, n\}$ ;  
 $m = \{\pi, \eta, K\}$ ;  $A$  : nucleus



- There are similar channels for anti-neutrino
- For RES at low E,  $CC1\pi$  &  $NC1\pi$  dominates
- We will focus on neutrino interactions at this energy range

# Neutrino energies and interactions



# 4. Introduction to NEUT - an event generator

## 4.1: Neutrino event generators

- **GENIE:**

- Developed by international collaboration for neutrino interactions from MeV to PeV ( $10^{15}$  eV)
- ROOT based neutrino generator written in C++.
- Well maintained and open source
- Recently implemented models of quasielastic and 2p2h interactions
- Used in many neutrino oscillation experiments such as T2K, NOvA, ...

# 4. Introduction to NEUT - an event generator

## 4.1: Neutrino event generators

- **GiBUU (Giessen-Boltzmann-Uehling-Uhlenbeck Project):**
  - Provides a unified theory and transport framework in the MeV and GeV energy regimes for elementary reactions on nuclei (electron, photon, neutrino and hadron with nuclei)
  - Provides a full dynamical description of the reaction and delivers the complete final state of an event
  - Not only used as neutrino generator but also in nuclear reactions
  - Written in Fortran and open source

# 4. Introduction to NEUT - an event generator

## 4.1: Neutrino event generators

- **NuWro:**

- Has been developed at the University of Wroclaw since 2006
- Written in C++
- Simulate neutrino interactions taking into account beam profile and composition, detailed detector geometry as well as FSI in the nuclear target from 100MeV to TeV

# Form factor

- Form factor is a function that describes how the internal structure of a particle affects the way it interacts with other particles.
- Form factor modifies the interaction strength depending on:
  - The momentum transfer ( $q^2$ )
  - The type of current involved (vector, axial-vector, ...)

Type	Symbol	Meaning
Vector form factors	$F_1(q^2), F_2(q^2)$	Related to charge and magnetism
Axial form factor	$F_A(q^2)$	Related to spin and axial currents
Pseudoscalar form factor	$F_P(q^2)$	Small at low energy

# Axial mass

- In NEUT, dipole form of axial form factor is used  $F_A^2 = \frac{g_A}{\left(1 + \frac{q^2}{M_A^2}\right)^2}$
- $g_A \approx 1.27$ : axial vector coupling constant
- $q^2 = -Q^2$ : squared momentum transfer
- $M_A$ : Axial mass ( $\sim 1$  GeV)
- $M_A$  controls how quickly the form factor falls off with increasing energy transfer.
- Larger  $M_A \Rightarrow$  slower fall-off  $\Rightarrow$  larger cross section at higher  $q^2$
- In T2K,  $M_A$  is tuned ( $=1.03$  GeV) to match data (compensates for missing nuclear effects (like 2p2h) in simpler models).

# Nuclear models in neutrino - nucleus interactions

Feature / Model	Global Fermi Gas (FG)	Local Fermi Gas (LFG)	Spectral Function (SF)	RPA Corrections
<b>Description</b>	Uniform distribution of nucleons in momentum space (constant density)	Fermi momentum varies with position in the nucleus	Realistic distribution of momentum + binding energy	Collective nuclear excitations (long-range)
<b>Momentum Distribution</b>	Sharp step function (up to $k_F$ )	Smooth, position-dependent $k_F(r)$	Realistic tail (including high-momentum nucleons)	Built on top of FG/LFG or SF
<b>Binding Energy</b>	Constant average (~25–30 MeV)	Local binding varies with density	Distributed via $S(p, E)$	Included in base model
<b>Short-Range Correlations</b>	✗ No	✗ No	✓ Yes (via high- $p$ , high- $E$ )	✗ No (handles long-range only)
<b>Long-Range Nuclear Effects</b>	✗ No	✗ No	✗ No	✓ Yes (collective excitations, screening)
<b>Energy Conservation</b>	Approximate (fixed binding)	Improved with local binding	Accurate via integration over $E$	Improved via self-consistent treatment
<b>Complexity (computation)</b>	★ Very simple	★★ Simple	★★★★ Complex	★★★★ Complex
<b>Used in experiments</b>	GENIE, NEUT (older versions), MiniBooNE	T2K, NOvA	MINERvA, T2K, DUNE (more recently)	T2K, MINERvA, DUNE (especially at low $Q^2$ )
<b>Energy range</b>	~0.1–few GeV	~0.1–few GeV	~0.1–few GeV	<~1 GeV mostly (for low- $Q^2$ corrections)
<b>Key strength</b>	Simplicity	Spatial realism	Realistic physics of bound nucleons	Describes nuclear collectivity, screening
<b>Key weakness</b>	Too simplistic	Still ignores nucleon correlations	Harder to implement, needs data input	Adds complexity, depends on model tuning

# 4. Introduction to NEUT - an event generator

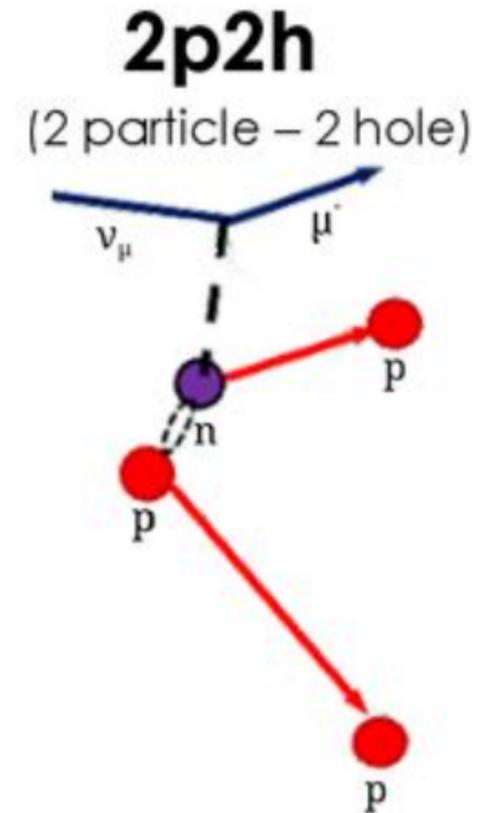
## 4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **1p1h (CCQE)**
    - Produce single-ring electron-like and muon-like events
    - Contribute 70% of the  $1R_\mu$  selection at far detector
    - Use Benhar Spectral Function (Nucl. Phys. A 579, 493–517 (1994))
    - Nucleon vector form factors: BBBA05 description (Nucl. Phys. B Proc. Suppl. 159, 127–132 (2006)) => [Detailed description in Physics of Neutrino Interactions \(10.4.5\)](#)
    - Nucleon axial mass  $M_A^{QE} = 1.03 \pm 0.06 \text{ GeV}$

# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **CC 2p2h (2-particles 2-holes), NC 2p2h not included yet**
    - Neutrinos interactions with 2 nucleons producing 2 holes
    - Produce single-ring electron-like and muon-like events
    - Contribute 12% of the  $1R_{\mu}$  selection at far detector
    - Use Nieves model (Phys. Lett. B 707, 72–75 (2012))
  - **MEC** (Meson Exchange Currents): In most basic models, a neutrino interacts with **one nucleon** (quasi-elastic scattering). But nuclei are **strongly correlated systems** where nucleons interact via mesons (like pions). In MEC: The neutrino couples to a meson (usually a pion) that is being exchanged between two nucleons.
  - This means the neutrino effectively interacts with a **nucleon pair**, not just a single nucleon. MEC is the primary source of what's called **2p2h interactions**:



# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **Single pion production**
    - Coherent  $1\pi$  (Berger–Sehgal model)
    - Resonant and non-resonant  $1\pi$  (Rein–Sehgal (RS) model) with hadronic invariant mass  $1.3 \leq W \leq 2 \text{ GeV}$
    - Contribute 13% of the  $1R_\mu$  selection at far detector

# 4. Introduction to NEUT - an event generator

## 4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **Deep inelastic scattering**
    - Use GRV98 Parton Distribution Functions (PDFs)
    - Bodek–Yang (BY) correction is used for  $Q^2 \leq 1.5 \text{ GeV}^2$
    - Processes begin where hadronic invariant mass  $W > 1.3 \text{ GeV}$
    - Pythia 5.72 for  $W > 2 \text{ GeV}$
    - A custom model interpolating between the  $\Delta(1232)$  and DIS interactions is employed for  $W < 2 \text{ GeV}$

# 3. Neutrino interactions: Quasi-elastic scattering (QE):

- Useful variables

- Mandelstam variables in lab frame (4-momentum):

$$s = (p_\nu + p_n)^2 : \text{total energy squared}$$

$$t = (p_\nu - p_l)^2 : \text{momentum transferred}$$

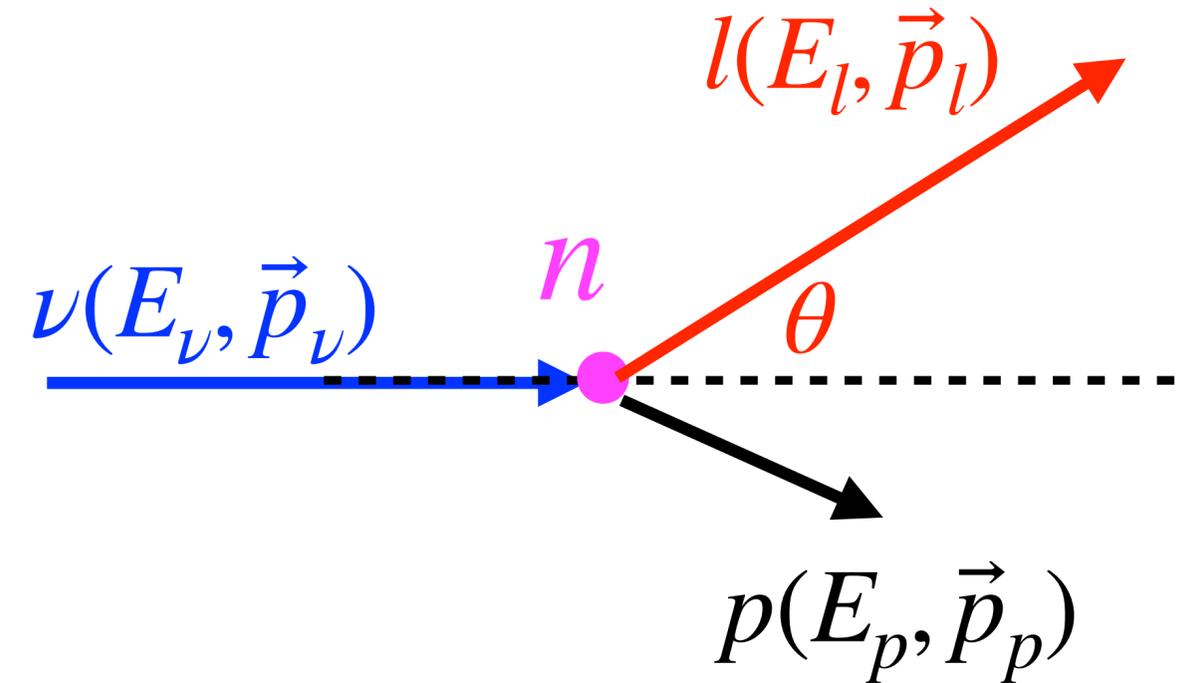
$$u = (p_\nu - p_p)^2 : \text{momentum transferred}$$

- In detail:

$$s = m_N^2 + 2m_N E_\nu$$

$$t = q^2 = -Q^2 = m_l^2 - 2E_\nu(E_l - p_l \cos \theta)$$

$$u = m_N^2 - 2m_N E_\nu + 2E_\nu(E_l - p_l \cos \theta)$$



- Hadronic invariant mass:

$$m_X = \sqrt{E_X^2 - p_X^2}$$

# RES vs. COH

## Summary Table:

Feature	Resonant Production	Coherent Production
Interaction with	Single nucleon	Whole nucleus
Nuclear breakup	Possible	No breakup (nucleus intact)
Resonance involved	Yes (e.g., $\Delta(1232)$ )	No
Final state particles	Lepton + nucleon + pion	Lepton + pion + nucleus
Pion direction	Wide angle	Forward-focused
Energy transfer	High	Low
Cross section	Larger	Smaller