Neutrino interactions

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Disclaimers

- To prepare for this lecture, I have borrowed from many sources: books, papers, lectures, internet, … I can not cite them all here.
- Time is limitted, content is unlimitted ! I can not conver all things and go into detail with all contents. If your questions can not be answered in the lecture, discuss later in break or refer to references for more information.
- If you see some complicated formulae and don’t even know any thing about them. Don’t worry, me too !!! 😜

References

- Carlo Giunti and Chung W. Kim. **Fundamentals of Neutrino Physics and Astrophysics**. 2007.
- Mark Thomson. **Modern particle physics**. Cambridge University Press, New York, 2013.
- Rev. Mod. Phys. 84, 1307 (2012) (arXiv:1305.7513v1) **From eV to EeV: Neutrino Cross-Sections Across Energy Scales**

- …

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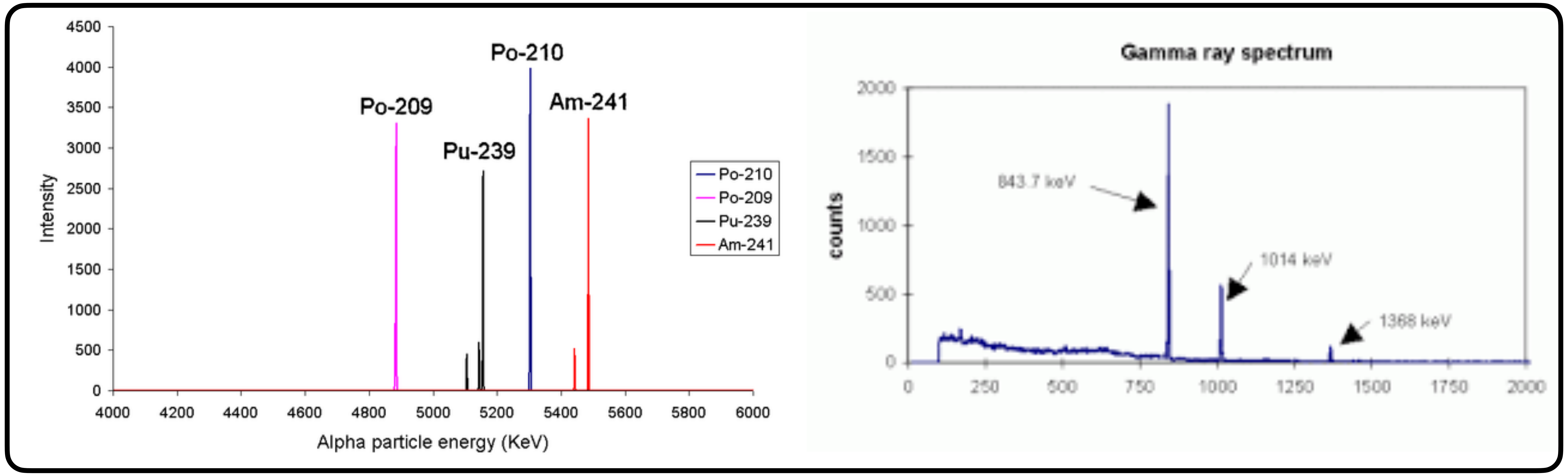
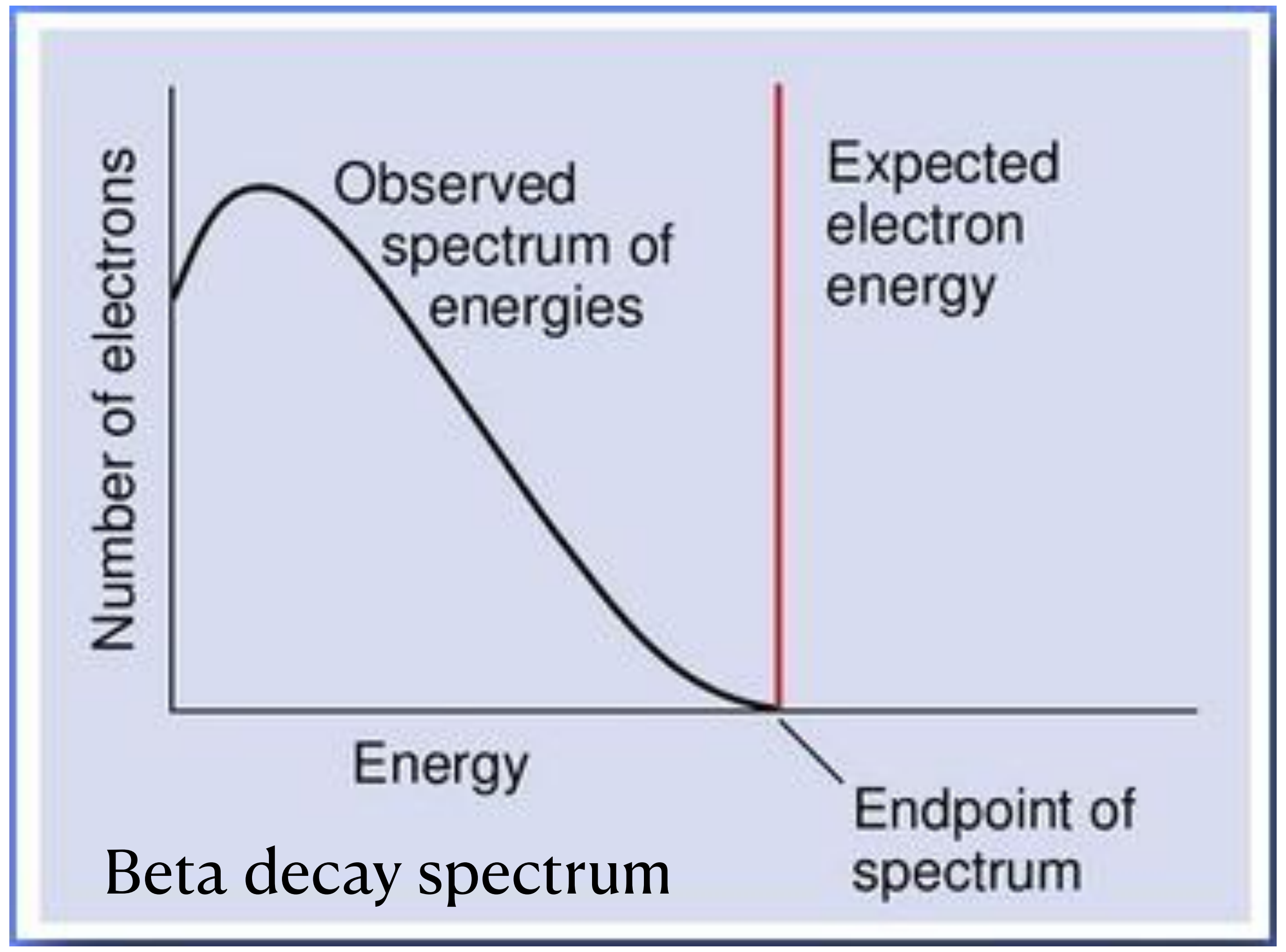
1. A brief history and properties of neutrinos





Beta decay spectrum

- Alpha and gamma decay spectra are discrete
- Two particles detected in the beta decay

$$X_Z^A \rightarrow Y_{Z-1}^A + e^-$$

$$K_e = (m_X - m_Y - m_e)c^2$$

- Beta spectrum was expected to be discrete as alpha and gamma spectra
- **But it was not**

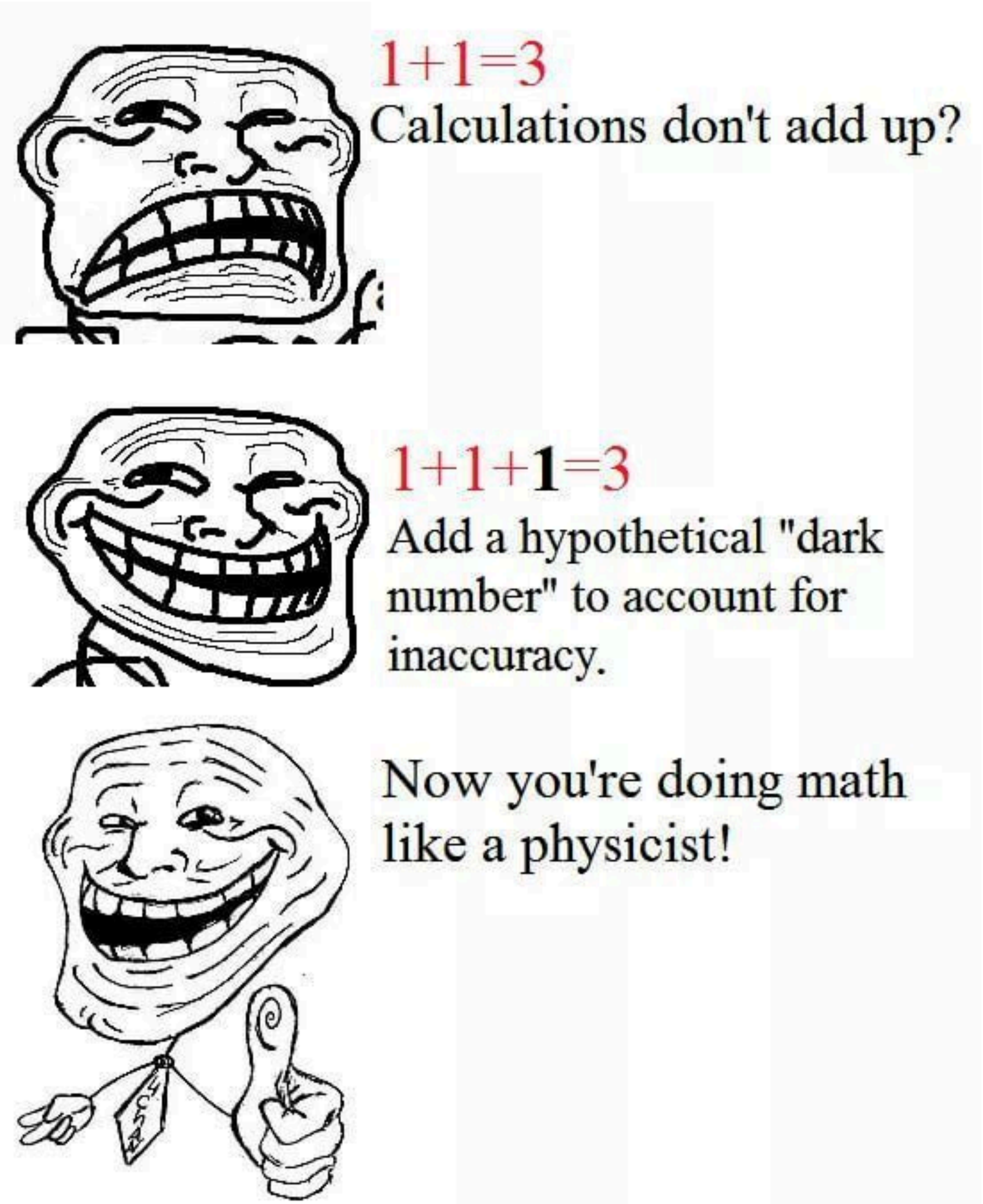
- Beta continuous spectrum: energy is not conserved or what else?
- In 1930, Pauli proposed an extremely light, neutral particle: neutrino (ν)

$$X_Z^A \rightarrow Y_{Z-1}^A + e^- + \bar{\nu}_e$$

$$K_e = (m_X - m_Y - m_e)c^2 - K_{\bar{\nu}_e}$$

- Beta energy now can have any value between 0 and maximum

Tips for doing physics and math

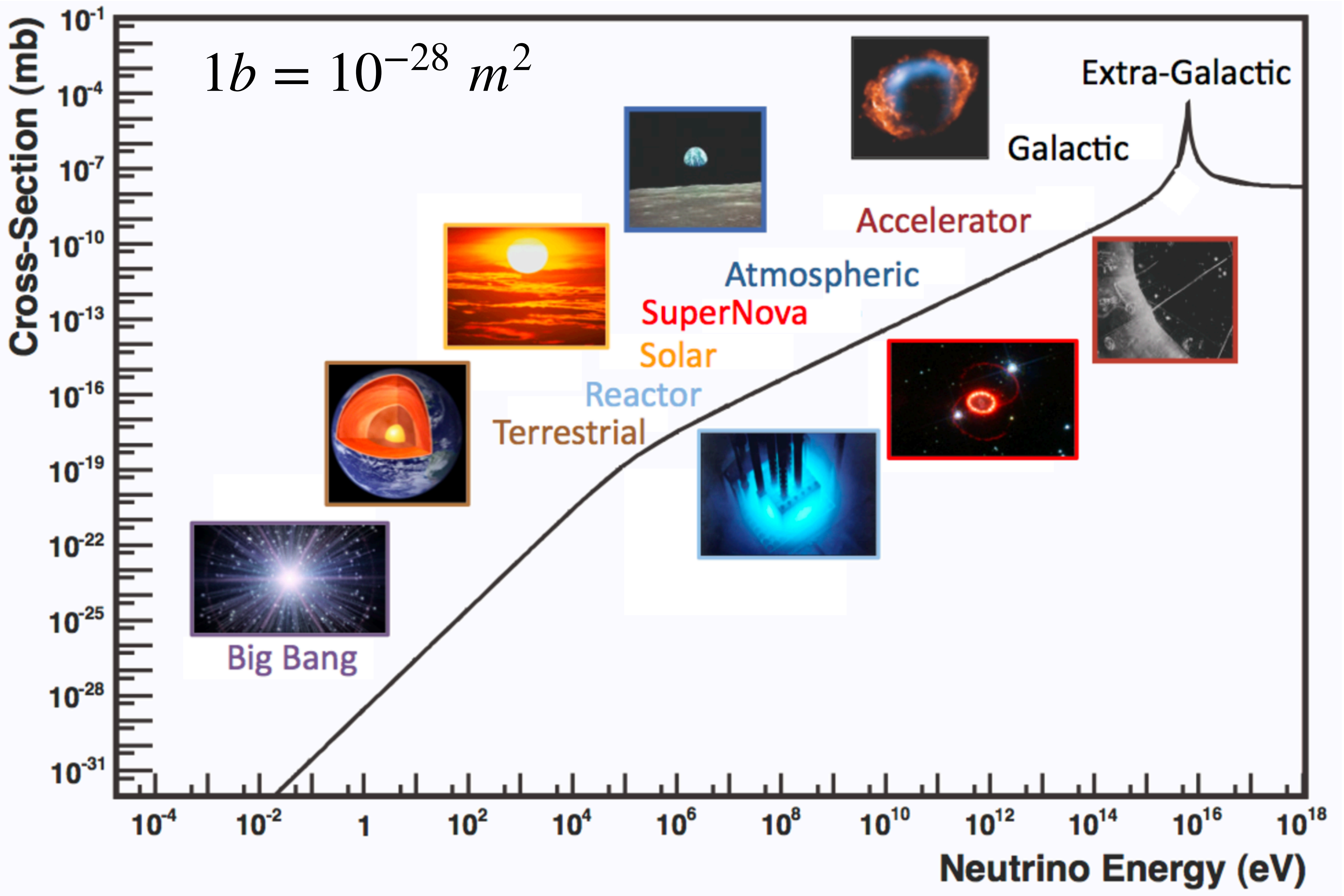


1. A brief history and properties of neutrinos

W. Pauli

"I have done a terrible thing
I have postulated a particle
that cannot be detected"

- Neutrinos:
  - Mass ~ 0: almost no gravitational interaction
  - Neutral: no electromagnetic interaction
  - No strong interaction
  - **Only participate in weak interaction**



- Neutrinos are detectable but they interacts extremmely weak with matter

Extremely small cross section

$$\sigma_\nu \approx 10^{-62}\ m^2 - 10^{-35}\ m^2$$

1. A brief history and properties of neutrinos

- An example to see how weak neutrino-matter interaction is

- ○ Neutrino mean free path: $L = \dfrac{m}{\rho\sigma}$
  - m is mass of nucleon
  - ρ is material density
  - σ is neutrino cross section
- ○ $m_p \approx m_n \approx 1.67 \times 10^{-27} kg$
- ○ $\sigma \approx 10^{-47} m^2$ at 1 MeV
- ○ $\rho_{H_2O} = 1000 kg/m^3$,
  $\rho_{Pb} = 11400 kg/m^3$

- ○ $L_{H_2O} = \dfrac{1.67 \times 10^{-27} kg}{1000 kg/m^3 \times 10^{-47} m^2} = 1.7 \times 10^{17} m$
- ○ $L_{Pb} = \dfrac{1.67 \times 10^{-27} kg}{11400 kg/m^3 \times 10^{-47} m^2} = 1.5 \times 10^{16} m$

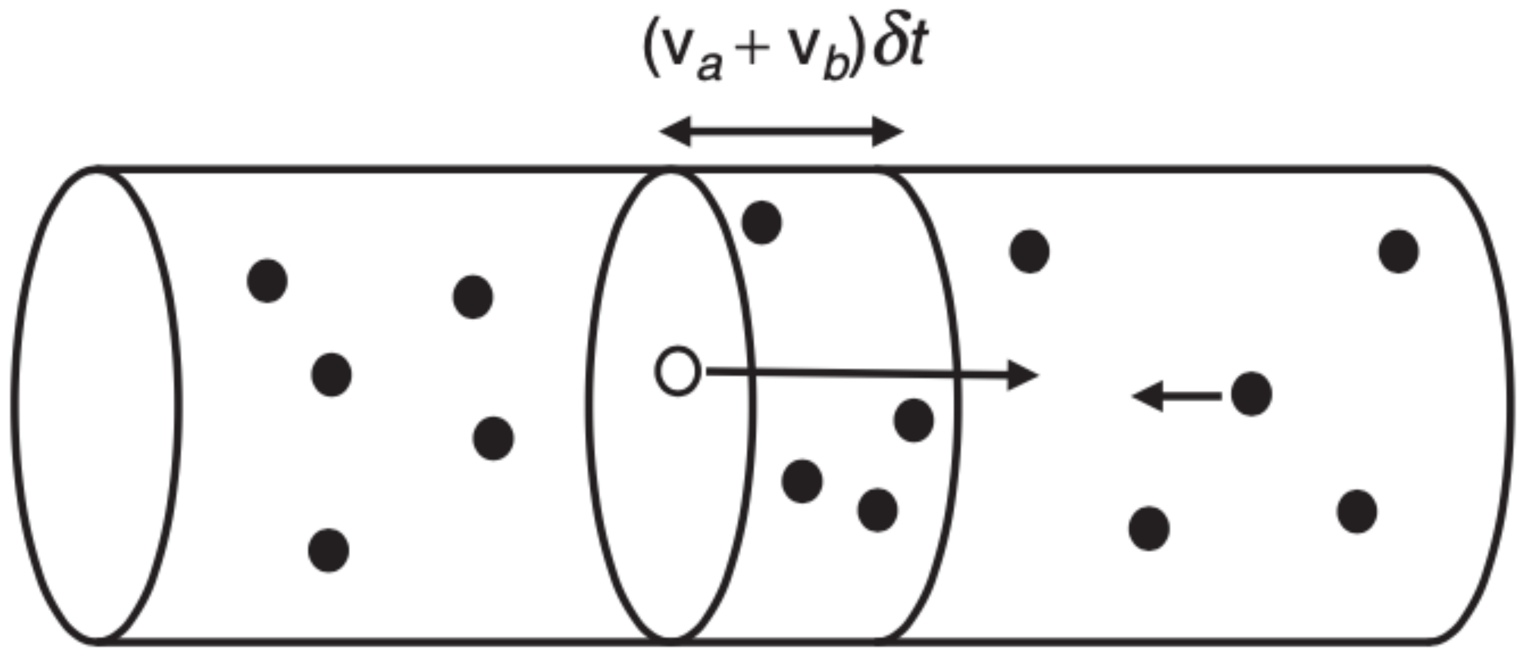
1 light year $\approx 9.5 \times 10^{15} m$

- **Need a block of lead with 1-light-year length to stop neutrinos!!!!**
- **It is a real challenge to study neutrino interactions (to measure cross sections)**

Contents

2. Cross section

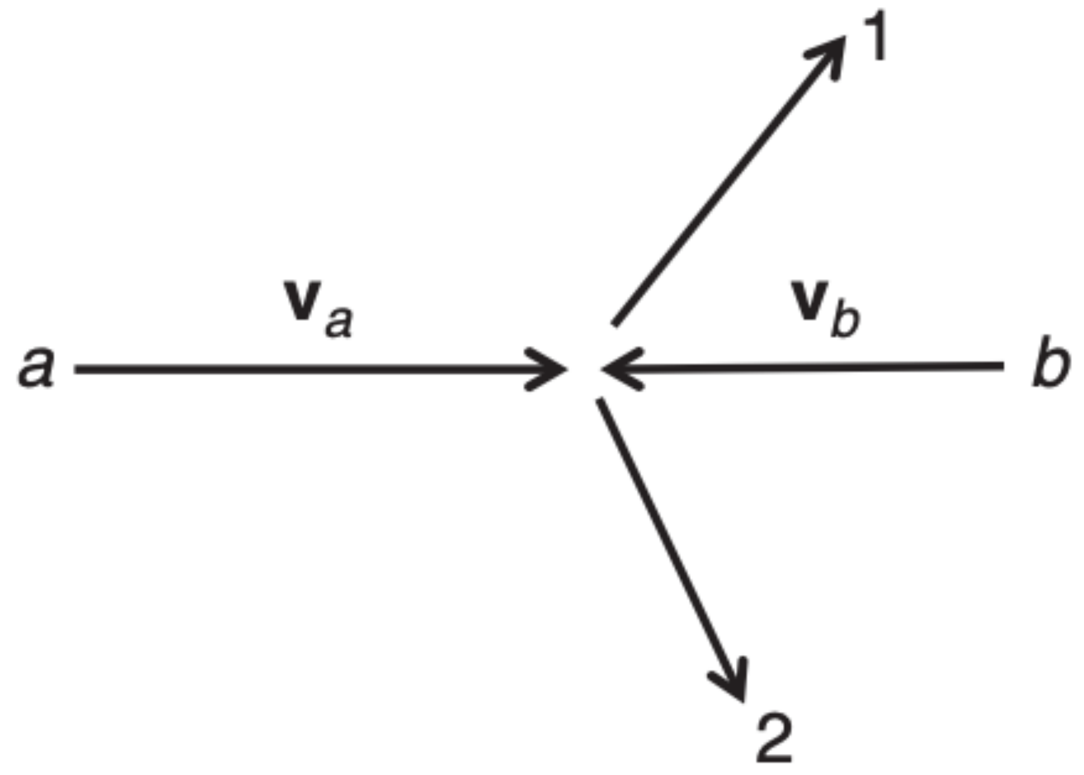
- Consider a beam of particles a with flux ϕ_a, pass through a volume $V = Av\Delta t$ of particle b:
- $v = v_a + v_b$ ($\vec{v_a}, \vec{v_b}$: velocities of a and b)
- $N_a = n_a V$: total number of incident particles in volume V
- $N_b = n_b V$: total number of target particles in volume V
- $\phi_a = \frac{N_a}{A\Delta t} = n_a v$: flux of particle a $\left[\frac{1}{[time]\,.\,[area^2]}\right]$
- $N = \phi_a N_b \sigma$: #interactions = flux x number of target particles x cross section



- **Cross section is a measure of the probability of an interaction to be occured**

$$\sigma = \frac{N}{\phi_a \, . \, N_b} \quad [area^2]$$

2. Cross section



- Consider a scattering: $a + b \rightarrow 1 + 2$
- Cross section is related to the transition rate:

$$\sigma = \frac{\Gamma_{fi}}{\phi_a N_b} = \frac{\Gamma_{fi}}{n_a n_b (v_a + v_b) V}$$

- Normalising the wavefunctions to one particle per unit volume $n_a = n_b = 1/V$

$$\sigma = \frac{\Gamma_{fi} V}{v_a + v_b}$$

- Because the factors of *V* in the expression for the flux will ultimately be cancelled by the corresponding factors from the wavefunction normalisation and phase space (~ 1/*V*), the volume *V* will not appear in the final result

$$\sigma = \frac{\Gamma_{fi}}{v_a + v_b}$$

2. Cross section

- In terms of matrix element M_{fi}

$$\sigma = \frac{(2\pi)^4}{4\sqrt{(p_a\,.\,p_b)^2 - m_a^2 m_b^2}} \int |M_{fi}|^2 \sigma(E_i - E_f)\sigma^3(\overrightarrow{p_i} - \overrightarrow{p_f}) \frac{d^3\overrightarrow{p_1}}{(2\pi)^3 2E_1} \frac{d^3\overrightarrow{p_2}}{(2\pi)^3 2E_2}$$

  - $M_{fi} = \sqrt{2E_a.2E_b.2E_1.2E_2}T_{fi}$, T_{fi} : transition matrix element
  - $E_i = E_a + E_b;\ \ E_f = E_1 + E_2;\ \ \overrightarrow{p_i} = \overrightarrow{p_a} + \overrightarrow{p_b};\ \ \overrightarrow{p_f} = \overrightarrow{p_1} + \overrightarrow{p_2}$
  - Lorentz-invariant flux factor

    $F = 4(v_a + v_b)E_aE_b = 4\sqrt{(p_a\,.\,p_b)^2 - m_a^2 m_b^2}$

- In center-of-mass (COM) frame:

$$\sigma_{COM} = \frac{p_f}{64\pi^2 s p_i} \int |M_{fi}|^2 d\Omega$$

  - $\overrightarrow{p_i} = \overrightarrow{p_a} = -\overrightarrow{p_b}$ and $\overrightarrow{p_f} = \overrightarrow{p_1} = -\overrightarrow{p_2}$ (in COM frame)
  - $s = (E_a + E_b)^2$ (in COM frame)
  - $d\Omega = d(\cos\theta)d\phi$ (θ: polar or zenith angle; ϕ: azimuthal angle)

2. Cross section

- Differential cross section for solid angle distribution

$$\frac{d\sigma}{d\Omega}$$

  - In COM: $\frac{d\sigma_{COM}}{d\Omega} = \frac{p_f}{64\pi^2 s p_i} \int |M_{fi}|^2$
- Differential cross section: $\frac{d\sigma}{dE}, \frac{d\sigma}{dp}, \frac{d\sigma}{dQ^2}, \frac{d\sigma}{d(\cos\theta)}$
- Double differential cross section: $\frac{d^2\sigma}{dEd\Omega}, \frac{d^2\sigma}{d(\cos\theta)d(Q^2)}$

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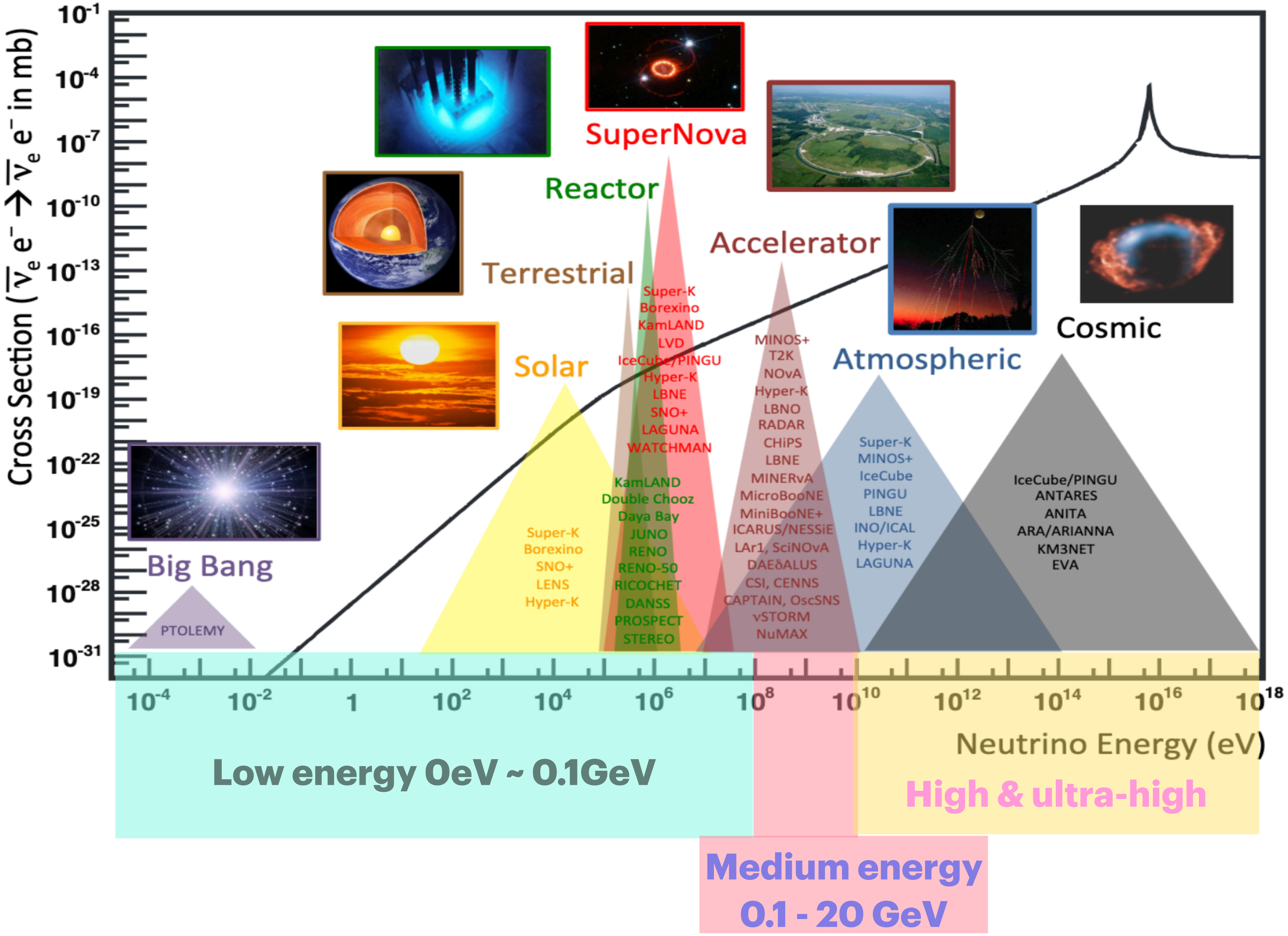
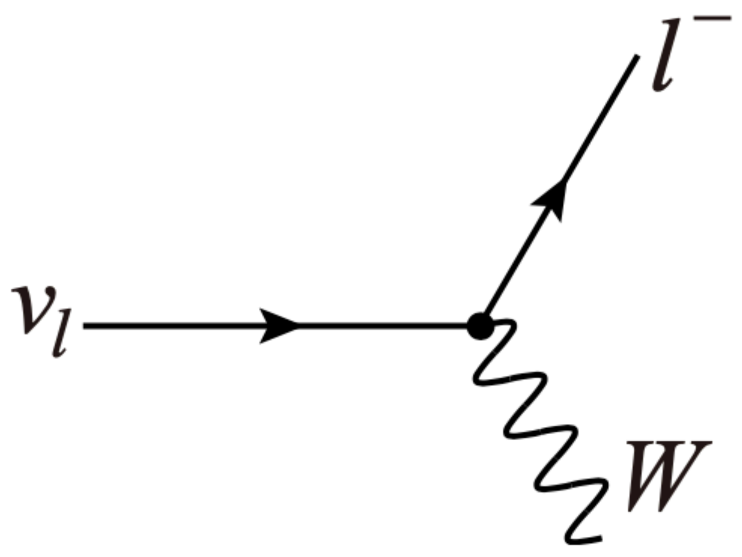
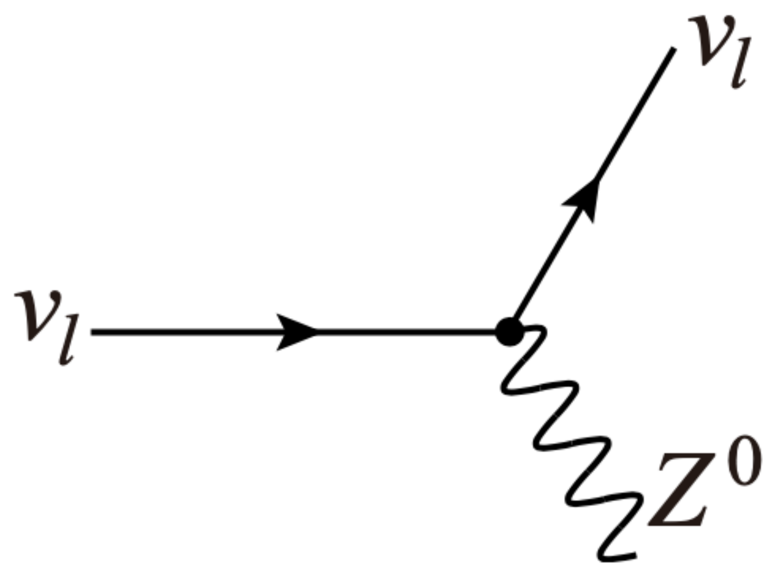
3. Neutrino interactions



- Basically there are 2 types of interactions:
  - Charge current (CC) by exchange $W^{\pm}$ bosons
  - Neutral current (NC) by exchange Z^0 boson





$$j^{\mu}_{CC} = \frac{g_W}{\sqrt{2}} \bar{\psi} \gamma^{\mu} \frac{1}{2}(1 - \gamma^5)\phi,$$

$$j^{\mu}_{NC} = g_Z \bar{\psi} \gamma^{\mu} \frac{1}{2}(g_V - g_A \gamma^5)\psi$$

3.1 Neutrino - electron interactions

- **Neutrino - electron elastic scattering:**

$$\nu_\alpha + e^- \to \nu_\alpha + e^-$$
$$\bar{\nu}_\alpha + e^- \to \bar{\nu}_\alpha + e^-$$

- **Neutrino - electron quasi-elastic scattering:**

$$\nu_\mu + e^- \to \nu_e + \mu^- \quad (E_\nu > 10.92\ GeV)$$
$$\nu_\tau + e^- \to \nu_e + \tau^- \quad (E_\nu > 3089\ GeV)$$

Did you calculate this value as Sanjib's request?

- **Consider process:** $\nu + A \to \sum_X X$
- **A at rest, neglecting neutrino mass, in center of mass frame,** $s = 2E_\nu m_A + m_A^2$ **must greater than** $\left(\sum_X m_X\right)^2$

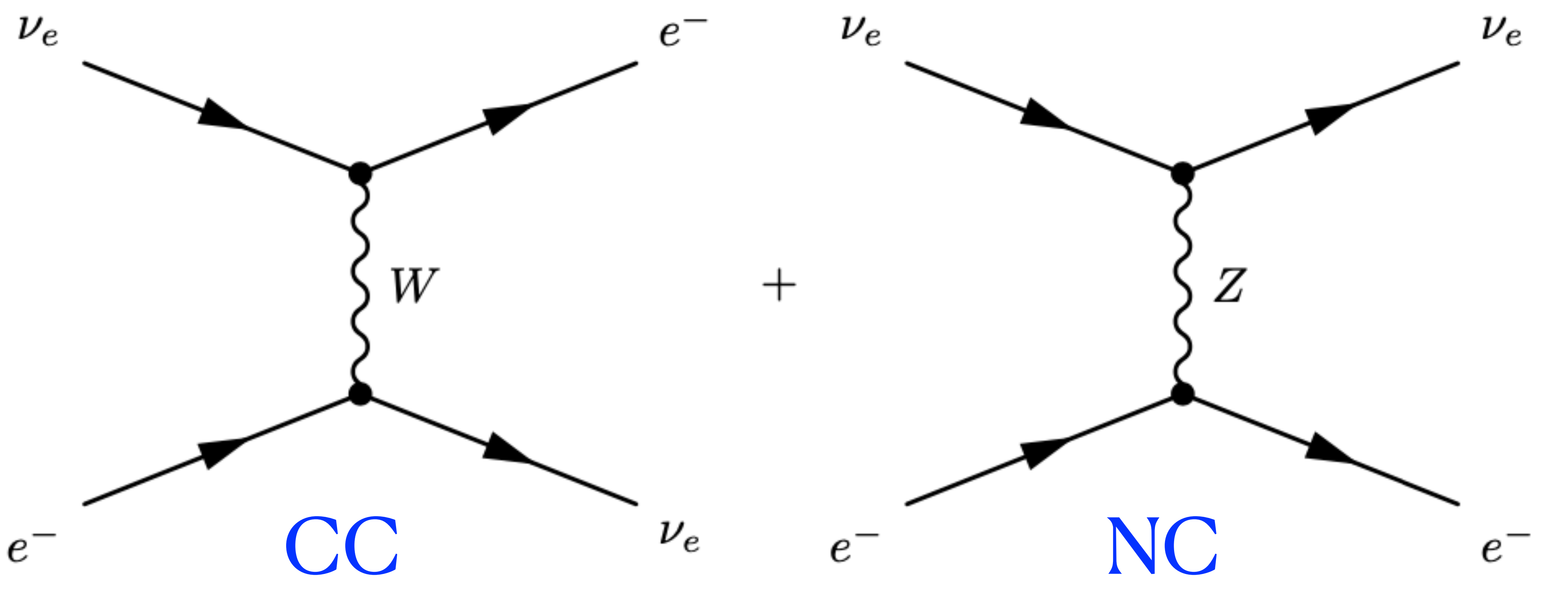
$$E_\nu^{th} = \frac{\left(\sum_X m_X\right)^2}{2m_A} - \frac{m_A}{2}$$

$$m_e = 0.511\ MeV$$
$$m_\mu = 105.66\ MeV$$
$$m_\tau = 1776.9\ MeV$$

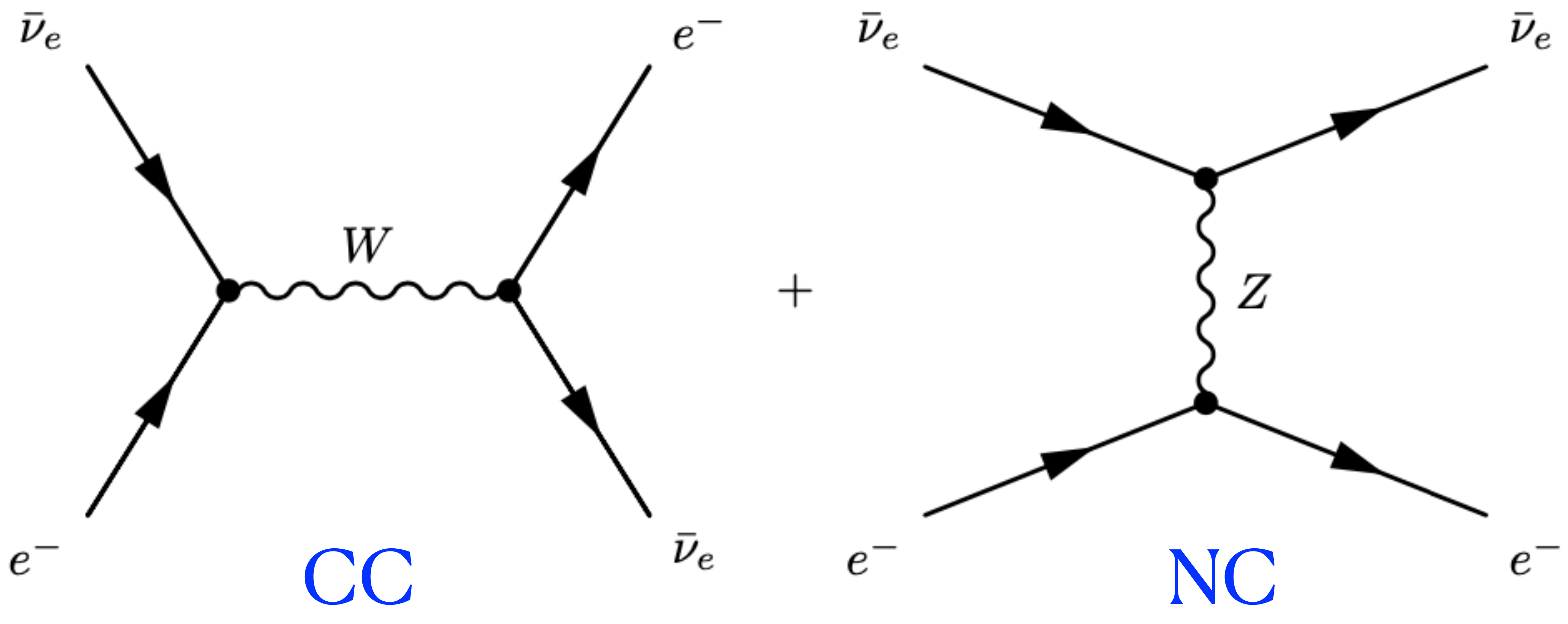
3.1 Neutrino - electron interactions

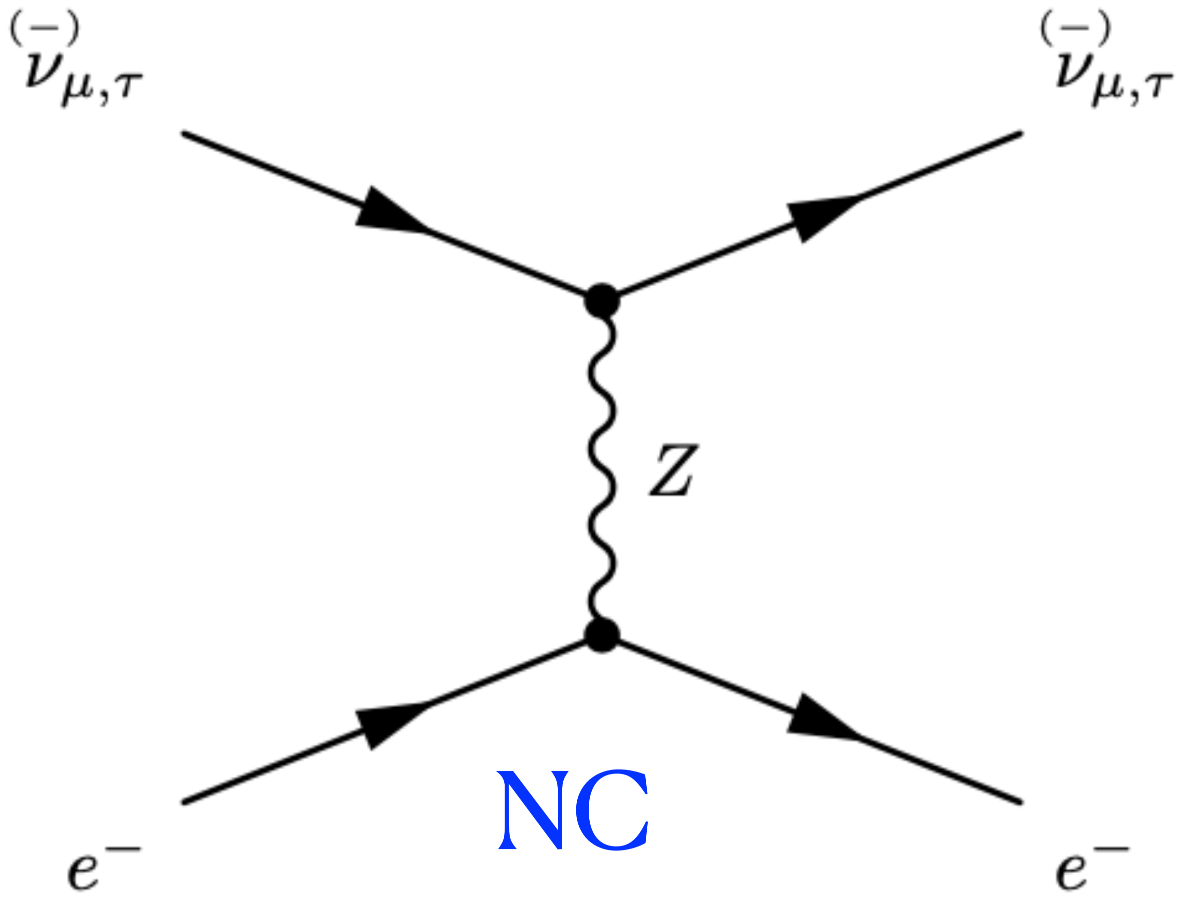
- **Neutrino - electron elastic scattering**

$$\nu_e + e^- \rightarrow \nu_e + e^-$$



$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$





$$\nu_{\mu,\tau} + e^- \rightarrow \nu_{\mu,\tau} + e^-$$

$$\bar{\nu}_{\mu,\tau} + e^- \rightarrow \bar{\nu}_{\mu,\tau} + e^-$$

Only neutral currents

3.1 Neutrino - electron interactions

- **Neutrino - electron elastic scattering**

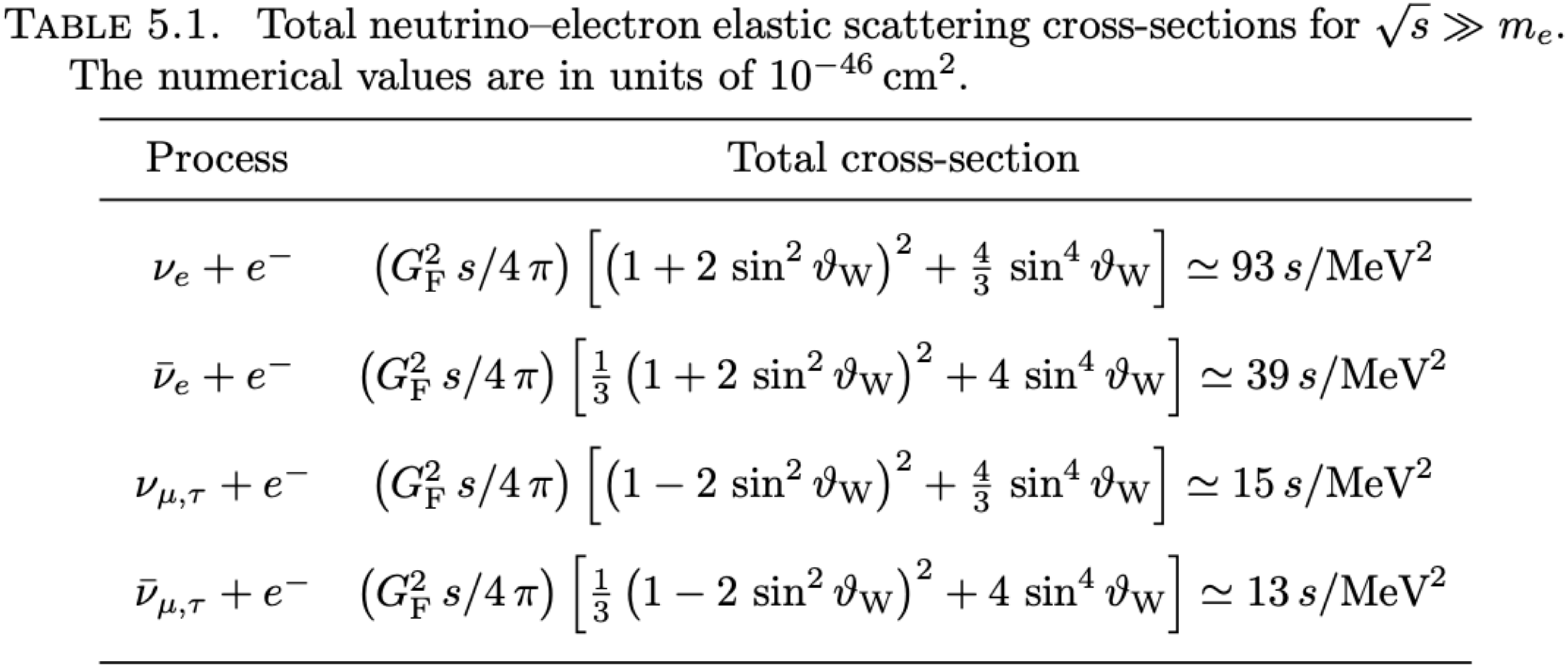


TABLE 5.1. Total neutrino–electron elastic scattering cross-sections for $\sqrt{s} \gg m_e$. The numerical values are in units of $10^{-46}\ \mathrm{cm}^2$.

Process	Total cross-section
$\nu_e + e^-$	$(G_F^2\, s/4\pi)\left[(1+2\sin^2\vartheta_W)^2 + \frac{4}{3}\sin^4\vartheta_W\right] \simeq 93\, s/\mathrm{MeV}^2$
$\bar{\nu}_e + e^-$	$(G_F^2\, s/4\pi)\left[\frac{1}{3}(1+2\sin^2\vartheta_W)^2 + 4\sin^4\vartheta_W\right] \simeq 39\, s/\mathrm{MeV}^2$
$\nu_{\mu,\tau} + e^-$	$(G_F^2\, s/4\pi)\left[(1-2\sin^2\vartheta_W)^2 + \frac{4}{3}\sin^4\vartheta_W\right] \simeq 15\, s/\mathrm{MeV}^2$
$\bar{\nu}_{\mu,\tau} + e^-$	$(G_F^2\, s/4\pi)\left[\frac{1}{3}(1-2\sin^2\vartheta_W)^2 + 4\sin^4\vartheta_W\right] \simeq 13\, s/\mathrm{MeV}^2$

- $G_F = 1.17 \times 10^{-5}\ GeV^2$: Fermi constant
- $s = (E_\nu + E_e)_i^2 = m_e^2 + 2m_eE_\nu$: Mandelstam variable
- $\sin^2\theta_W = 0.2324 \pm 0.0058 \pm 0.0059$ (1994)

3.1 Neutrino - electron interactions

- Neutrino - electron elastic scattering

- Differential cross section

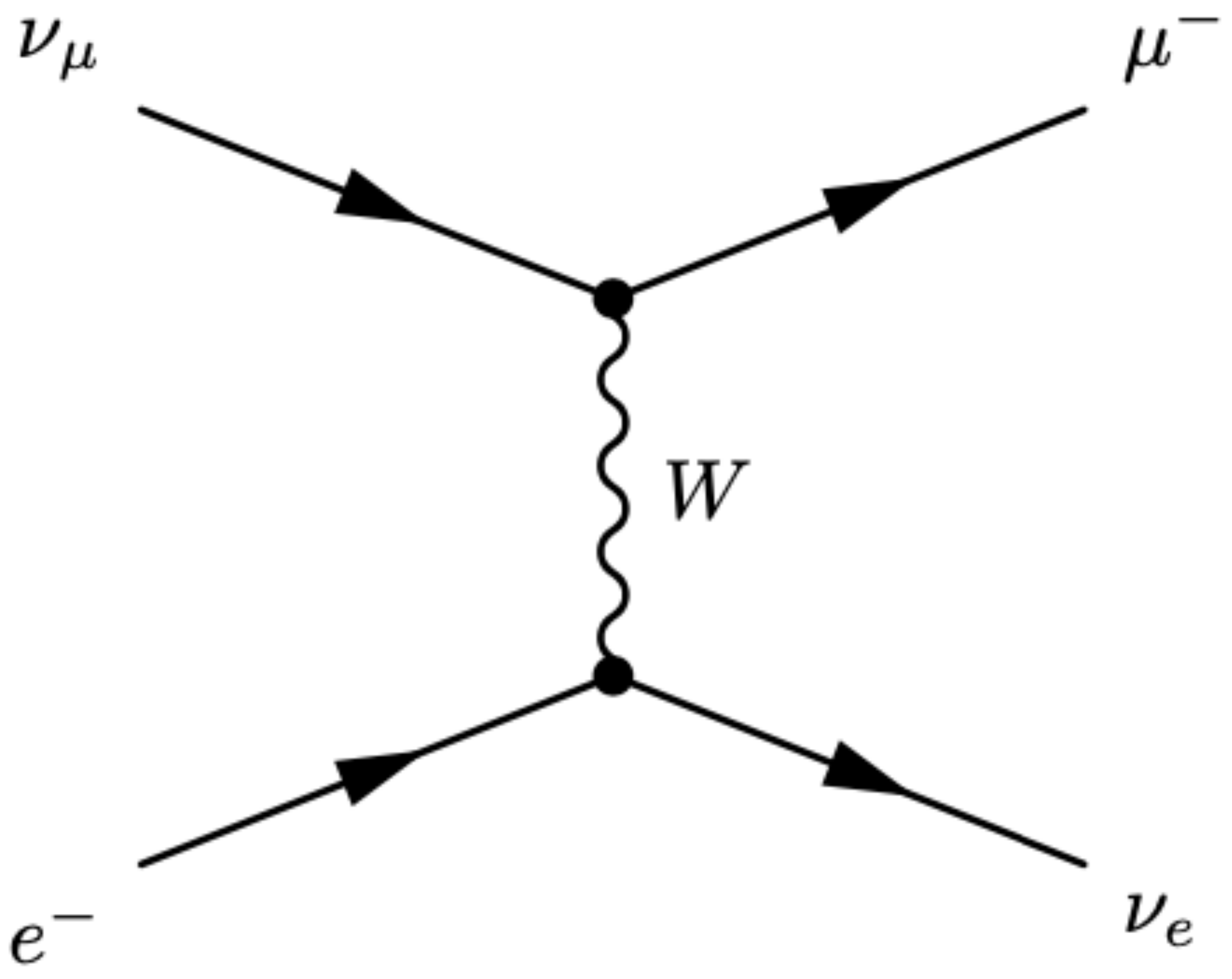
$$\frac{d\sigma}{dQ^2} = \frac{G_F^2}{\pi}\left[g_1^2 + g_2^2\left(1 - \frac{Q^2}{2p_\nu p_e}\right)^2 - g_1 g_2 m_e^2 \frac{Q^2}{2(p_\nu p_e)^2}\right]$$

- $g_1(\nu_e) = g_2(\overline{\nu}_e) = \frac{1}{2} + \sin^2\theta_W \approx 0.73$
- $g_2(\nu_e) = g_1(\overline{\nu}_e) = \sin^2\theta_W \approx 0.23$
- $g_1(\nu_{\mu,\tau}) = g_2(\overline{\nu}_{\mu,\tau}) = -\frac{1}{2} + \sin^2\theta_W \approx -0.27$
- $g_2(\nu_{\mu,\tau}) = g_1(\overline{\nu}_{\mu,\tau}) = \sin^2\theta_W \approx 0.23$

3.1 Neutrino - electron interactions

- Neutrino - electron quasi-elastic scattering

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \qquad E_{\nu_\mu} > 10.92\ GeV$$

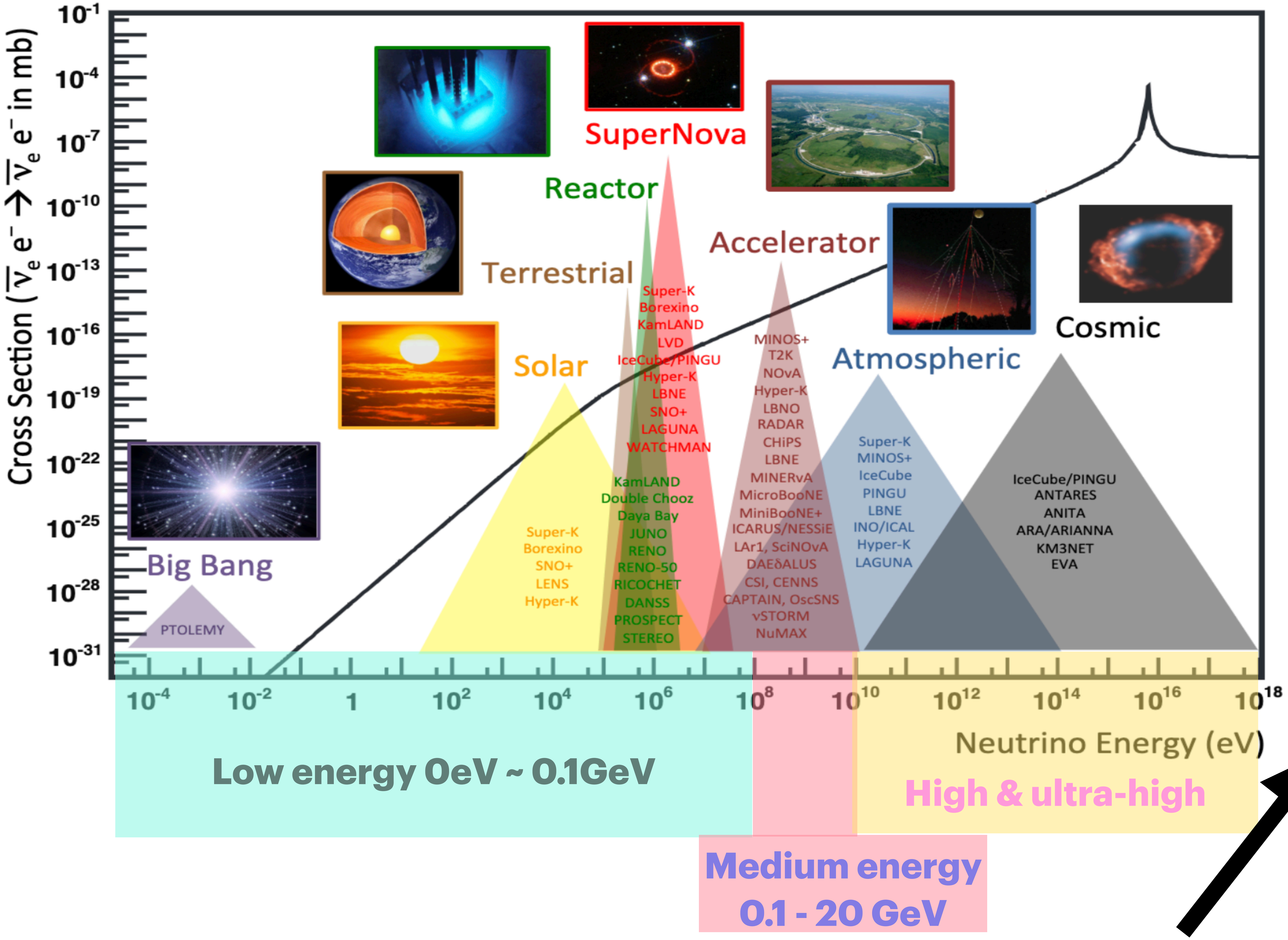


- Total cross section

$$\sigma = \frac{G_F^2 s}{\pi}\left(1 - \frac{m_\mu^2}{s}\right)$$

$$s = (p_{\nu_\mu} + p_e)^2$$

3.2 Neutrino - nucleon interactions



Important for reactor & accelerator neutrino oscillation

- ○ Neutrino interactions at 0-1 MeV energy:
  - Coherent scattering (COH):
  - Neutrino capture on radioative nuclei:
- ○ Neutrino interactions at 1-100 MeV energy:
  - Inverse beta decay
- ○ Neutrino interactions at 0.1-20 GeV energy:
  - Quasi-elastic scattering (QE):
  - Resonant meson production (RES):
  - Coherent pion production (COH):
  - Deep inelastic scattering (DIS):
- ○ Neutrino interactions at high and ultra-high energy (> 20 GeV):
  - Deep inelastic scattering (DIS):

3.2 Neutrino - nucleon interactions

- $G_F = 1.17 \times 10^{-5}\ GeV^2$
- Q: momentum transfer
- R: nuclear radius
- T: recoil kinetic energy
- M_A: target mass
- E_ν: neutrino energy
- $F(Q^2)$: nucleon form factor
- Q_W: weak current term

○ **Neutrino interactions at 0-1MeV energy:**

- **Coherent scattering (COH):**

$\nu_l + A_N^Z \rightarrow \nu_l + A_N^{*Z}$ (QR << 1)

$$\frac{d\sigma}{dT} = \frac{G_F^2}{4\pi} Q_W^2 M_A \left(1 - \frac{M_A T}{2E_\nu^2}\right) F(Q^2)^2$$

- **Neutrino capture on radioative nuclei:**

$\nu_e + A_N^Z \rightarrow e^- + A_{N-1}^{Z+1}$

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 |V_{ud}|^2 F(Z_f, E_e)}{2\pi\beta_\nu} E_e p_e f_V^2(0) \left((1 + \beta_e \beta_\nu \cos\theta) + 3\lambda^2 (1 - \frac{1}{3}\beta_e\beta_\nu \cos\theta) \right)$$

- β_e, β_ν: electron, neutrino velocities
- $E_e, p_e, \cos\theta$: electron energy, momentum, scattering angle
- λ^2: axial-to-vector coupling ratio, $f_V^2(0)$: coupling strength
- $|V_{ud}|^2$: Cabbibo angle
- $F(Z_f, E_e)$: Fermi function

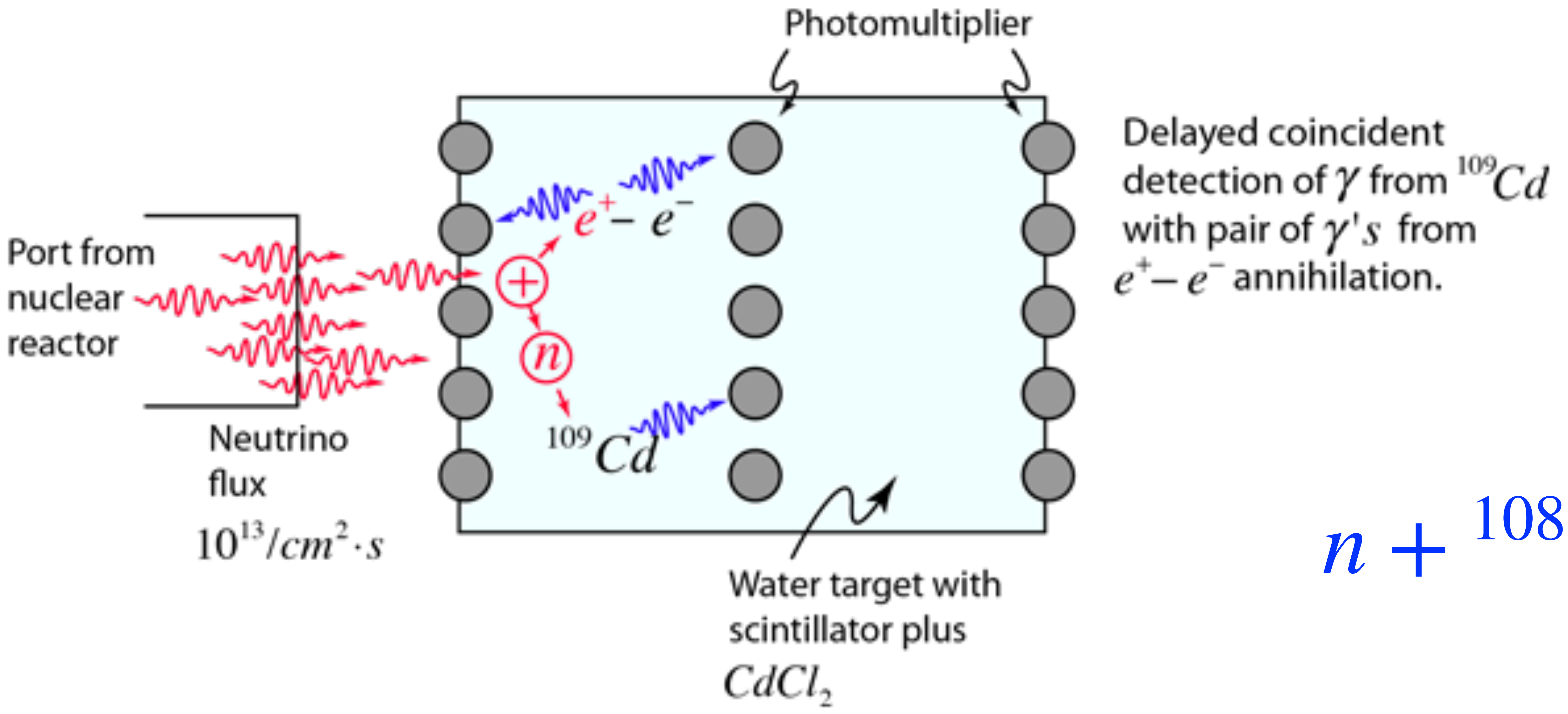
Not experimentally measured yet

3.2 Neutrino - nucleon interactions

- **Neutrino interactions at 1-100MeV energy:**
  - **Inverse beta decay:**

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

- Scattering process used in Reines & Cowan experiment to confirm the existence of neutrino



0.5 MeV promt signals of gamma rays

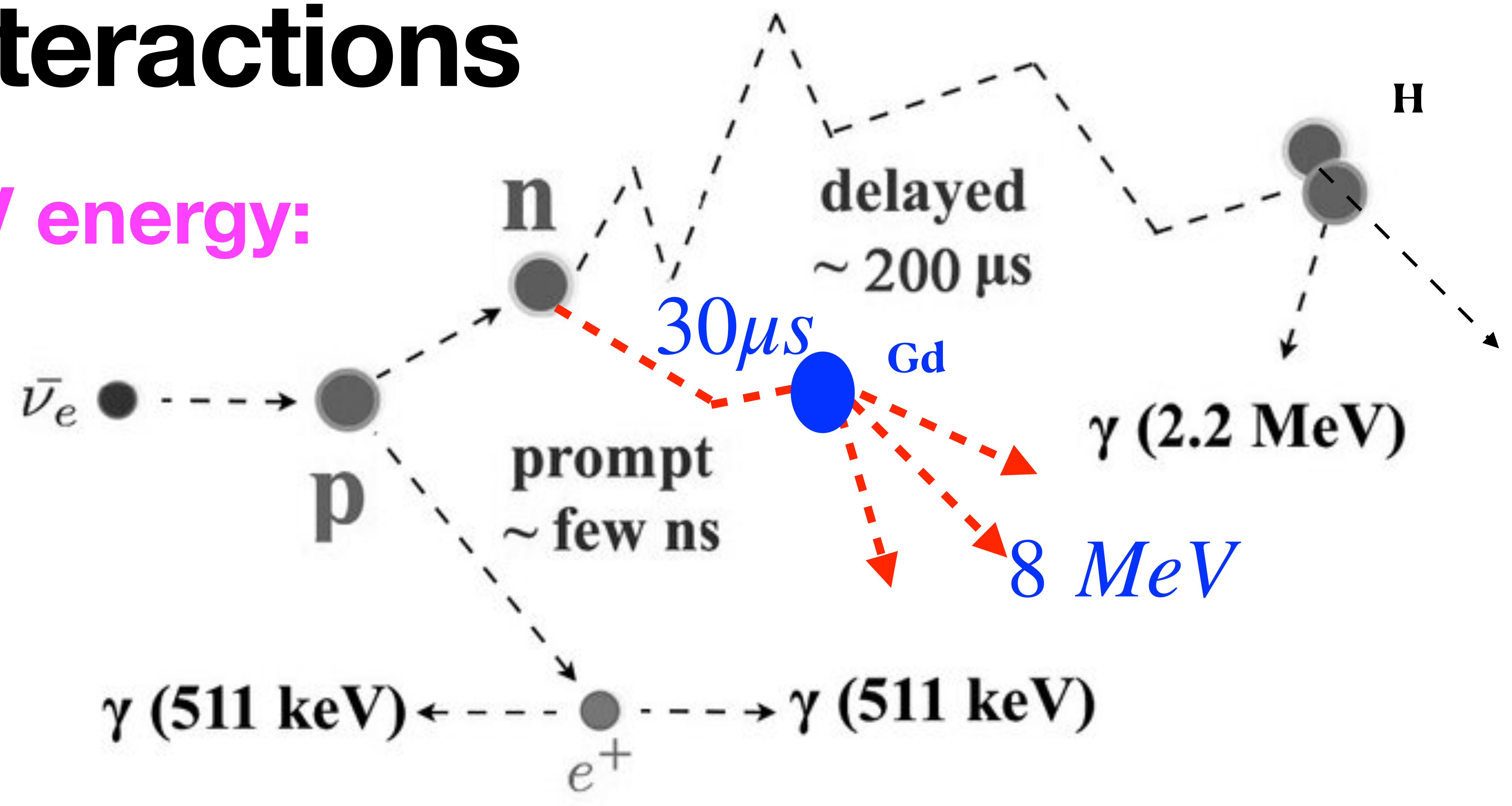
$$e^+ + e^- \rightarrow \gamma + \gamma$$

Delayed gamma signal after $5\mu s$

$$n + {}^{108}Cd \rightarrow {}^{109}Cd^* \rightarrow {}^{109}Cd + \gamma$$

3.2 Neutrino - nucleon interactions

- ○ Neutrino interactions at 1-100MeV energy:
  - **Inverse beta decay:**



$$\bar{\nu}_e + p \rightarrow e^+ + n$$

$$\frac{d\sigma}{d\cos\theta} = \frac{G_F^2 |V_{ud}|^2 E_e p_e}{2\pi}\left(f_V^2(0)(1+\beta_e\cos\theta) + 3f_A^2(0)(1-\frac{1}{3}\beta_e\cos\theta)\right)$$

- f_V: nuclear **vector (Fermi transition)**, f_A: **axial-vector (Gamov-Teller transition)**

- Important for detecting reactor neutrino experiments $(1.806 < E_\nu \leq 10\ MeV)$
- Important for understanding supernova explosion mechanism $(10 < E_\nu \leq 20\ MeV)$

3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

Charge current: W^+ exchange

- **Quasi-elastic scattering (target change but not break up):**
  - CCQE: $\nu_l + n \rightarrow l^- + p$
- **Resonant meson production (target is exited):**
  - CCRES: $\nu_l + N \rightarrow l^- + \Delta(1232) \rightarrow l^- + N' + m$
- **Coherent pion production (target unchange):**
  - CCCOH: $\nu_l + A \rightarrow l^- + A + \pi^+$
- **Deep inelastic scattering (nucleon broken):**
  - CCDIS: $\nu_l + N \rightarrow l^- + X$

Neutral current: Z^0 exchange

- **Elastic scattering (target change but not break up):**
  - NC: $\nu_l + N \rightarrow \nu_l + N'$
- **Resonant meson production (target is exited):**
  - NCRES: $\nu_l + N \rightarrow \nu_l + \Delta(1232) \rightarrow \nu_l + N' + m$
- **Coherent pion production (target unchange):**
  - NCCOH: $\nu_l + A \rightarrow \nu_l + A + \pi^0$
- **Deep inelastic scattering (nucleon broken):**
  - NCDIS: $\nu_l + N \rightarrow \nu_l + X$

Where $l^- = \{e^-, \mu^-, \tau^-\}$; $N = \{p, n\}$; $m = \{\pi, \eta, K\}$; A : nucleus; X : hadrons

3.2 Neutrino - nucleon interactions

○ Anti-neutrino interactions at 0.1-20GeV energy:

Charge current: W^- exchange

- **Quasi-elastic scattering (target change but not break up):**
  - CCQE: $\overline{\nu}_l + p \rightarrow l^+ + n$
- **Resonant meson production (target is exited):**
  - CCRES: $\overline{\nu}_l + N \rightarrow l^+ + \Delta(1232) \rightarrow l^+ + N' + m$
- **Coherent pion production (target unchange):**
  - CCCOH: $\overline{\nu}_l + A \rightarrow l^+ + A + \pi^-$
- **Deep inelastic scattering (nucleon broken):**
  - CCDIS: $\overline{\nu}_l + N \rightarrow l^+ + X$

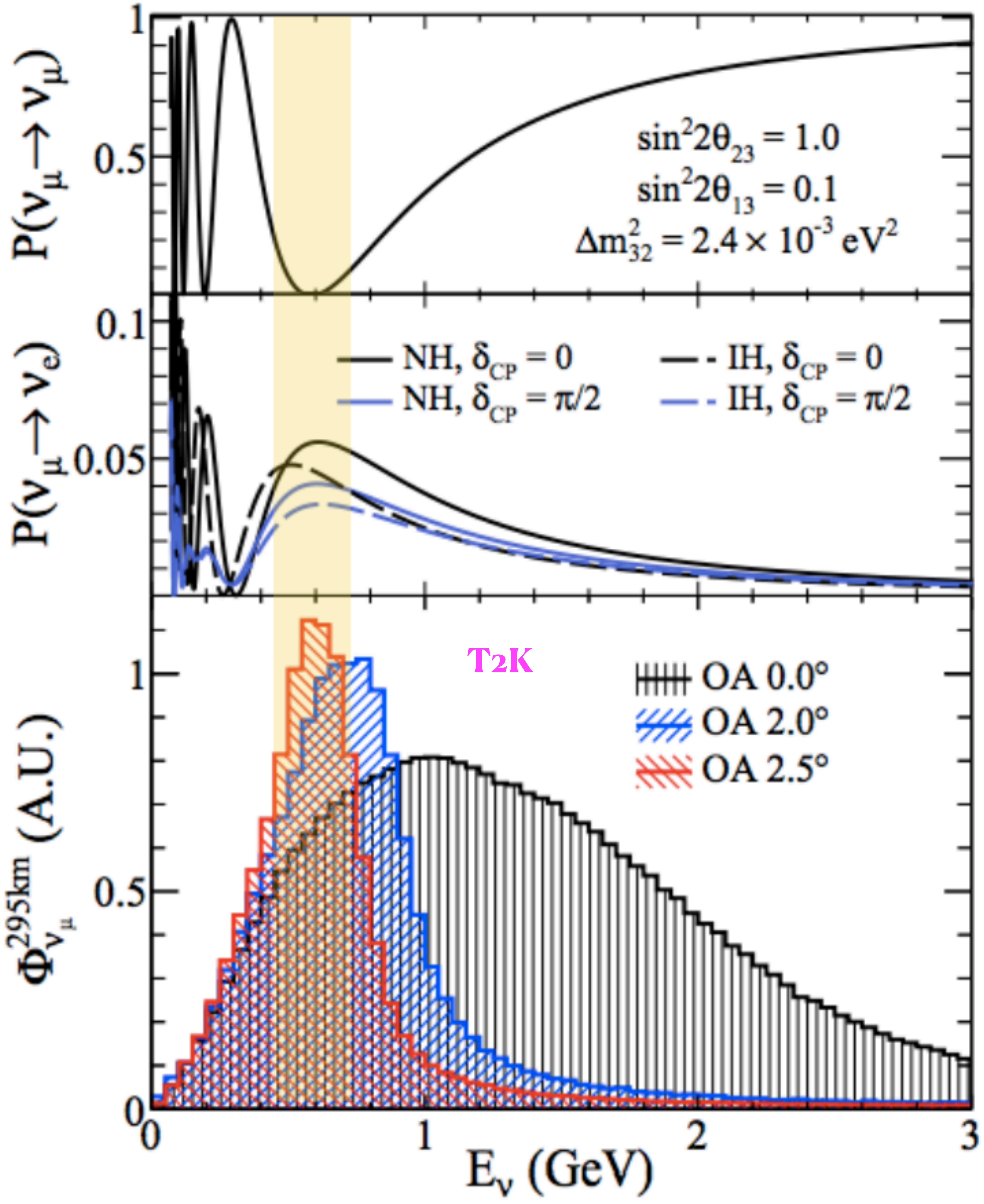
Neutral current: Z^0 exchange

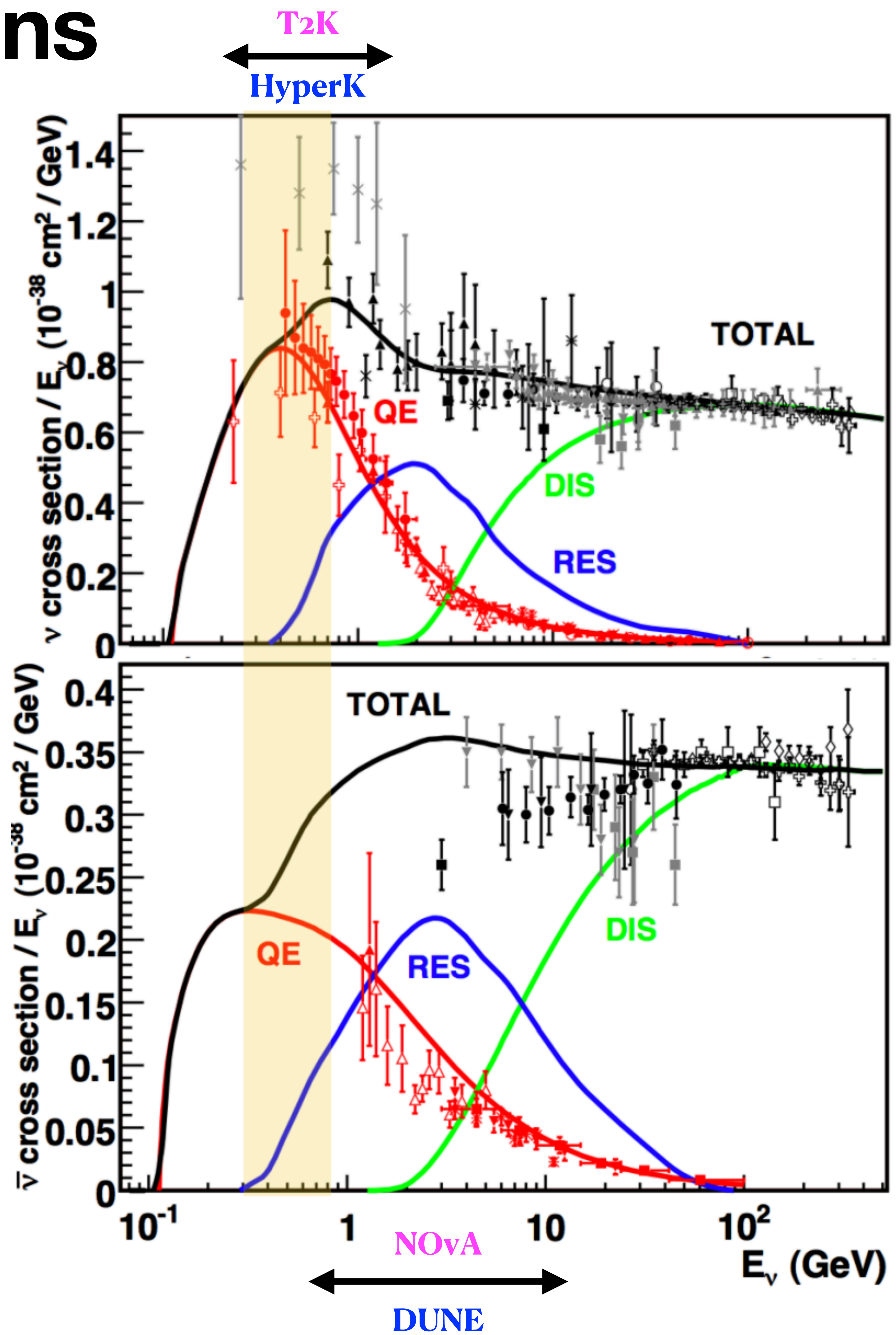
- **Elastic scattering (target change but not break up):**
  - NC: $\overline{\nu}_l + N \rightarrow \overline{\nu}_l + N'$
- **Resonant meson production (target is exited):**
  - NCRES: $\overline{\nu}_l + N \rightarrow \overline{\nu}_l + \Delta(1232) \rightarrow \overline{\nu}_l + N' + m$
- **Coherent pion production (target unchange):**
  - NCCOH: $\overline{\nu}_l + A \rightarrow \overline{\nu}_l + A + \pi^0$
- **Deep inelastic scattering (nucleon broken):**
  - NCDIS: $\overline{\nu}_l + N \rightarrow \overline{\nu}_l + X$

Where $l^- = \{e^-, \mu^-, \tau^-\}$; $N = \{p, n\}; m = \{\pi, \eta, K\}$; A : nucleus; X : hadrons

3.2 Neutrino - nucleon interactions

- **Neutrino interactions at 0.1-20GeV energy:**

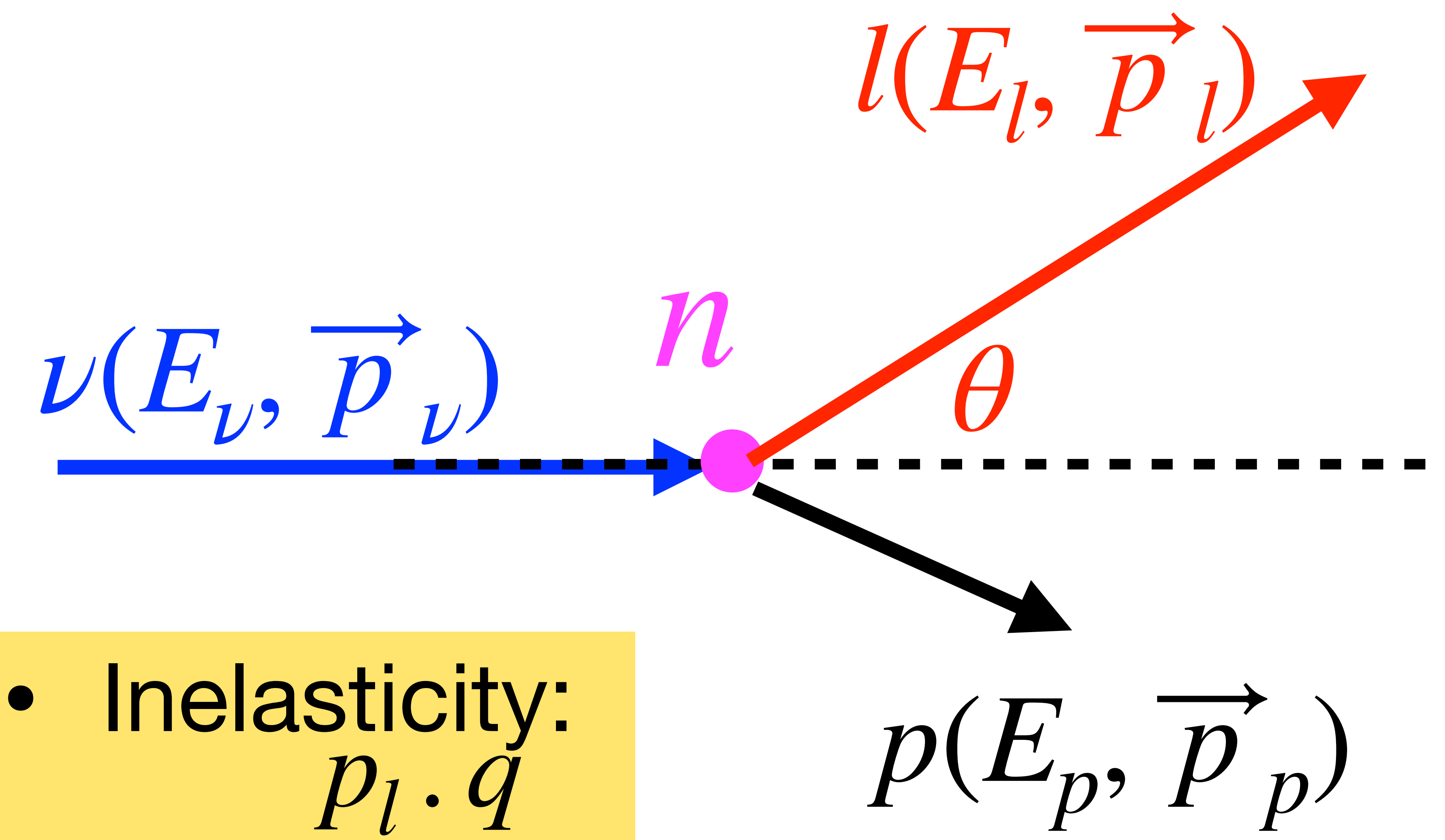




3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

- **Quasi-elastic scattering (QE): scattering variables**
  - CCQE: $\nu_l + n \rightarrow l^- + p$



- Mandelstam variables in lab frame (4-momentum):

$$s = (p_\nu + p_n)^2 = (p_l + p_p)^2 = (p_\nu + p_n)(p_l + p_p)$$
$$t = (p_\nu - p_l)^2 = (p_n - p_p)^2 = (p_\nu - p_l)(p_p - p_n)$$
$$u = (p_\nu - p_p)^2 = (p_l - p_n)^2 = (p_\nu - p_p)(p_l - p_n)$$
$$s + t + u = m_\nu^2 + m_n^2 + m_l^2 + m_p^2$$

- Inelasticity:

$$y = \frac{p_l \,.\, q}{p_l \,.\, p_\nu}$$

- Hadronic invariant mass:

$$W = \sqrt{E_X^2 - p_X^2}$$

- In detail:

$$s = m_N^2 + 2m_N E_\nu$$
$$t = q^2 = -Q^2 = m_l^2 - 2E_\nu(E_l - p_l \cos\theta)$$
$$u = m_N^2 - 2m_N E_\nu + 2E_\nu(E_l - p_l \cos\theta)$$

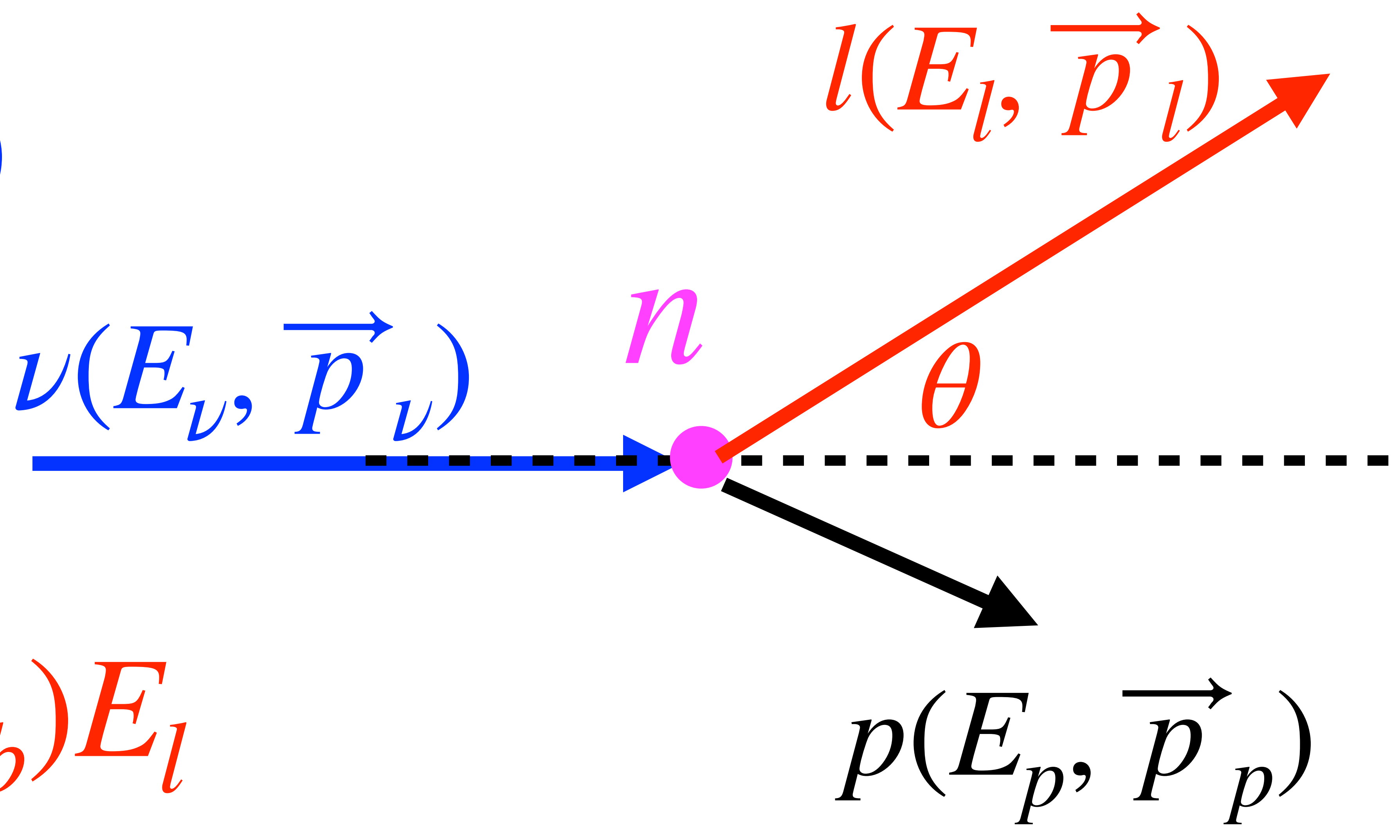
- 4-momentum transferred:

$$Q^2 = -q^2 = 2E_\nu(E_l - p_l \cos\theta) - m_l^2$$

3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

- **Quasi-elastic scattering (QE): (Llewellyn-Smith, 1972)**
  - CCQE: $\nu_l + n \rightarrow l^- + p$
  - Neutrino reconstructed energy (HW: derive it)



$$E_\nu = \frac{m_p^2 - m_l^2 - (m_n - E_b)^2 + 2(m_n - E_b)E_l}{2[(m_n - E_b) - E_l + p_l \cos\theta]}$$

p_l, E_l, θ : out-going lepton momentum, energy, scattered angle

$E_b = 27\ MeV$: neutron binding energy in Oxygen

$m_p,\ m_n,\ m_l$: proton, neutron, lepton masses

- Remove E_b (free neutron) and consider $m_p \approx m_n$: $E_\nu = \dfrac{2m_nE_l - m_l^2}{2(m_n - E_l + p_l\cos\theta)}$

3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

- **Quasi-elastic scattering (QE): (Llewellyn-Smith, 1972)**

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 m_N^2 |V_{ud}|^2}{8\pi(P_\nu . p_{N_i})^2_\nu}\left[A \pm \frac{(s-u)}{m_N^2}B + \frac{(s-u)^2}{m_N^4}C\right]$$

$$A = \frac{(m_l^2+Q^2)}{m_N^2}\Bigg[(1+\eta)F_A^2 - (1-\eta)F_1^2 + \eta(1-\eta)F_2^2 + 4\eta F_1 F_2 \\ -\frac{m_l^2}{4m_N^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - \left(\frac{Q^2}{m_N^2}+4\right)F_P^2\right)\Bigg]$$

$$B = \frac{Q^2}{m_N^2}F_A(F_1+F_2) \qquad C = \frac{1}{4}(F_A^2 + F_1^2 + \eta F_2^2)$$

- (-) +: (anti-) neutrino scattering
- G_F: Fermi constant
- Q^2: squared 4-momentum transfered ($Q^2 = -q^2 > 0$)
- m_N: nucleon mass
- m_l: lepton mass
- E_ν: neutrino energy
- $(s-u) = 4m_N E_\nu - Q^2 - m_l^2$
- F_1, F_2, F_A, F_P: Dirac, Pauli, axial-vector, and preudoscalar form factors
- $\eta = Q^2/4m_N^2$

3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- **Quasi-elastic scattering (QE): (Llewellyn-Smith, 1972)**

- Axial-vector form factor $F_A(Q^2) = \dfrac{g_A}{\left(1 + Q^2/M_A^2\right)^2}$
- Axial mass $M_A = 1.026 \pm 0.021\ GeV$
- Dirac form factor $F_1(Q^2) = F_1^p(Q^2) + F_1^n(Q^2)$
- Pauli form factor $F_2(Q^2) = F_2^p(Q^2) + F_2^n(Q^2)$
- For $\nu_e, \overline{\nu}_e, \nu_\mu, \overline{\nu}_\mu$ scattering, the term related to A can be neglected. In this case, cross sections do not depend on the preudoscalar form factor F_P

$$g_A = F_A(0) = 1.2694 \pm 0.0028$$

$$F_1^p(0) = 1\,, \qquad F_1^n(0) = 0\,,$$

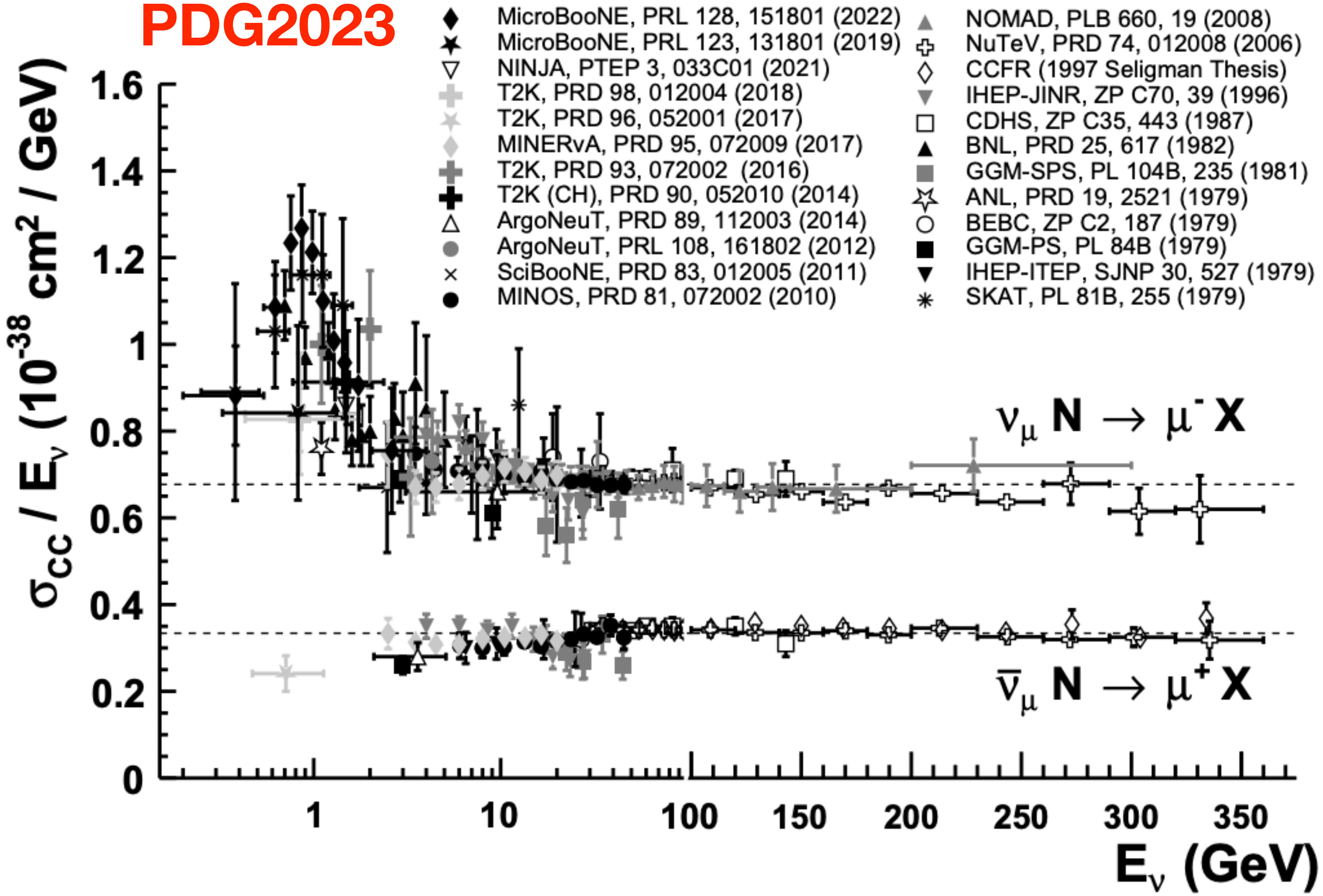
$$F_2^p(0) = \frac{\mu_p}{\mu_{\mathrm{N}}} - 1\,, \qquad F_2^n(0) = \frac{\mu_n}{\mu_{\mathrm{N}}}\,,$$

- Nuclear magneton $\mu_N = e\hbar/2m_p$
- Proton magnetic moment μ_p
- Neutron magnetic moment μ_n

3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- Quasi-elastic scattering (QE): measurement



PDG2023

3.2 Neutrino - nucleon interactions

- **Neutrino interactions at 0.1-20GeV energy:**
- **NC elastic scattering: Similar to formulae of CCQE but without $|V_{ud}|^2$, and different form factors**

$$\frac{d\sigma}{dQ^2} = \frac{G_F^2 m_N^2}{8\pi(P_\nu \cdot p_{N_i})_\nu^2}\left[A \pm \frac{(s-u)}{m_N^2}B + \frac{(s-u)^2}{m_N^4}C\right]$$

(-)+ for (anti-)neutrino scattering

$$F_1(Q^2) = \left(\frac{1}{2} - \sin^2\theta_W\right)\left[\frac{\tau_3(1+\eta(1+\mu_p-\mu_n))}{(1+\eta)(1+Q^2/M_V^2)}\right] - \sin^2\theta_W\left[\frac{1+\eta(1+\mu_p+\mu_n)}{(1+\eta)(1+Q^2/M_V^2)^2}\right] - \frac{F_1^s(Q^2)}{2}$$

$$F_2(Q^2) = \left(\frac{1}{2} - \sin^2\theta_W\right)\frac{\tau_3(\mu_p-\mu_n)}{(1+\eta)\left(1+\frac{Q^2}{M_V^2}\right)^2} - \sin^2\theta_W\frac{\mu_p+\mu_n}{(1+\eta)\left(1+\frac{Q^2}{M_V^2}\right)^2} - \frac{F_2^s(Q^2)}{2}$$

$$F_A(Q^2) = \frac{g_A\tau_3}{2\left(1+\frac{Q^2}{M_A^2}\right)^2} - \frac{F_A^s(Q^2)}{2}$$

- $\tau_3 = +1\ (-1)$ for proton (neutron) scattering
- $F_{1,2}^s(Q^2)$: strange vector form factors
- Strange axial-vector form factor
  (Δs: stragne quark contribution to nucleon spin)

$$F_A^s(Q^2) = \frac{\Delta s}{\left(1+Q^2/M_A^2\right)^2}$$

3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

- **Resonant meson production (CC1π dominates at low energy)**

$$
\begin{array}{ll}
\nu_\mu\, p \to \mu^-\, p\, \pi^+, & \overline{\nu}_\mu\, p \to \mu^+\, p\, \pi^- \\
\nu_\mu\, n \to \mu^-\, p\, \pi^0, & \overline{\nu}_\mu\, p \to \mu^+\, n\, \pi^0 \\
\nu_\mu\, n \to \mu^-\, n\, \pi^+, & \overline{\nu}_\mu\, n \to \mu^+\, n\, \pi^-
\end{array}
$$

$$
\begin{array}{ll}
\nu_\mu\, p \to \nu_\mu\, p\, \pi^0, & \overline{\nu}_\mu\, p \to \overline{\nu}_\mu\, p\, \pi^0 \\
\nu_\mu\, p \to \nu_\mu\, n\, \pi^+, & \overline{\nu}_\mu\, n \to \overline{\nu}_\mu\, n\, \pi^0 \\
\nu_\mu\, n \to \nu_\mu\, n\, \pi^0, & \overline{\nu}_\mu\, n \to \overline{\nu}_\mu\, n\, \pi^0 \\
\nu_\mu\, n \to \nu_\mu\, p\, \pi^-, & \overline{\nu}_\mu\, n \to \overline{\nu}_\mu\, p\, \pi^-
\end{array}
$$

arXiv:1711.02403v3

$$
\frac{d\sigma(\nu N \to l N \pi)}{dk^2 dW d\Omega_\pi} = \frac{G_F^2}{2} \frac{1}{(2\pi)^4} \frac{|\mathbf{q}|}{4} \frac{-k^2}{(k^L)^2} \sum_{\lambda_2,\lambda_1} \Big\{
$$

$$
\left| C_{L_-}\left(\tilde{F}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi)\right) + C_{R_-}\left(\tilde{F}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi)\right) + C_-\left(\tilde{F}^{e_-}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_-}_{\lambda_2\lambda_1}(\theta,\phi)\right) \right|^2
$$

$$
+ \left| C_{L_+}\left(\tilde{F}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_L}_{\lambda_2\lambda_1}(\theta,\phi)\right) + C_{R_+}\left(\tilde{F}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_R}_{\lambda_2\lambda_1}(\theta,\phi)\right) + C_+\left(\tilde{F}^{e_+}_{\lambda_2\lambda_1}(\theta,\phi) - \tilde{G}^{e_+}_{\lambda_2\lambda_1}(\theta,\phi)\right) \right|^2 \Big\}
$$

- For anti-neutrino, swap $C_{L_\pm}$ and $C_{R_\pm}$

3.2 Neutrino - nucleon interactions

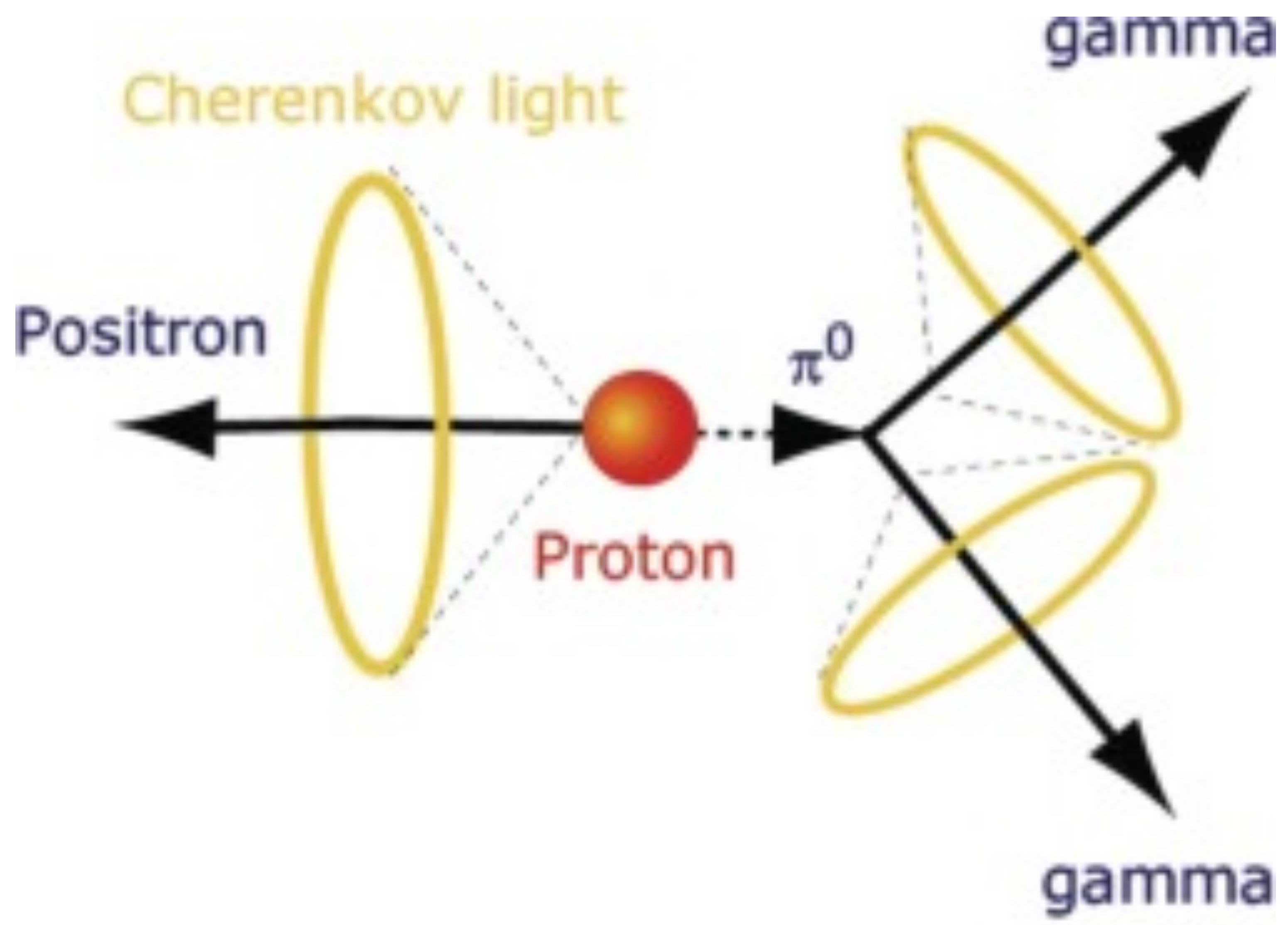
○ **Neutrino interactions at 0.1-20GeV energy:**

- **Resonant meson production (CC1π dominates at low energy)**

$$\begin{aligned} &\nu_\mu p \to \mu^- p \pi^+, && \overline{\nu}_\mu p \to \mu^+ p \pi^- \\ &\nu_\mu n \to \mu^- p \pi^0, && \overline{\nu}_\mu p \to \mu^+ n \pi^0 \\ &\nu_\mu n \to \mu^- n \pi^+, && \overline{\nu}_\mu n \to \mu^+ n \pi^- \end{aligned}$$

$$\begin{aligned} &\nu_\mu p \to \nu_\mu p \pi^0, && \overline{\nu}_\mu p \to \overline{\nu}_\mu p \pi^0 \\ &\nu_\mu p \to \nu_\mu n \pi^+, && \overline{\nu}_\mu n \to \overline{\nu}_\mu n \pi^0 \\ &\nu_\mu n \to \nu_\mu n \pi^0, && \overline{\nu}_\mu n \to \overline{\nu}_\mu n \pi^0 \\ &\nu_\mu n \to \nu_\mu p \pi^-, && \overline{\nu}_\mu n \to \overline{\nu}_\mu p \pi^- \end{aligned}$$

- Main backgrounds of nucleon decay search: same produced particles



$$p \to e^+ + \pi^0$$

3.2 Neutrino - nucleon interactions

- Neutrino interactions at 0.1-20GeV energy:
- **Resonant meson production (CC1π dominates at low energy)**

$$\begin{aligned}
&\nu_\mu p \to \mu^- p \pi^+, &\quad& \overline{\nu}_\mu p \to \mu^+ p \pi^- \\
&\nu_\mu n \to \mu^- p \pi^0, && \overline{\nu}_\mu p \to \mu^+ n \pi^0 \\
&\nu_\mu n \to \mu^- n \pi^+, && \overline{\nu}_\mu n \to \mu^+ n \pi^-
\end{aligned}$$

$$\begin{aligned}
&\nu_\mu p \to \nu_\mu p \pi^0, &\quad& \overline{\nu}_\mu p \to \overline{\nu}_\mu p \pi^0 \\
&\nu_\mu p \to \nu_\mu n \pi^+, && \overline{\nu}_\mu n \to \overline{\nu}_\mu n \pi^0 \\
&\nu_\mu n \to \nu_\mu n \pi^0, && \overline{\nu}_\mu n \to \overline{\nu}_\mu n \pi^0 \\
&\nu_\mu n \to \nu_\mu p \pi^-, && \overline{\nu}_\mu n \to \overline{\nu}_\mu p \pi^-
\end{aligned}$$

- Main backgrounds of $CC0\pi$ at T2K: π can mimic μ/e signals

$$\begin{aligned}
&\pi^+ \to \mu^+ + \nu_\mu \\
&\pi^- \to \mu^- + \overline{\nu}_\mu
\end{aligned} \qquad BR = 0.999877$$

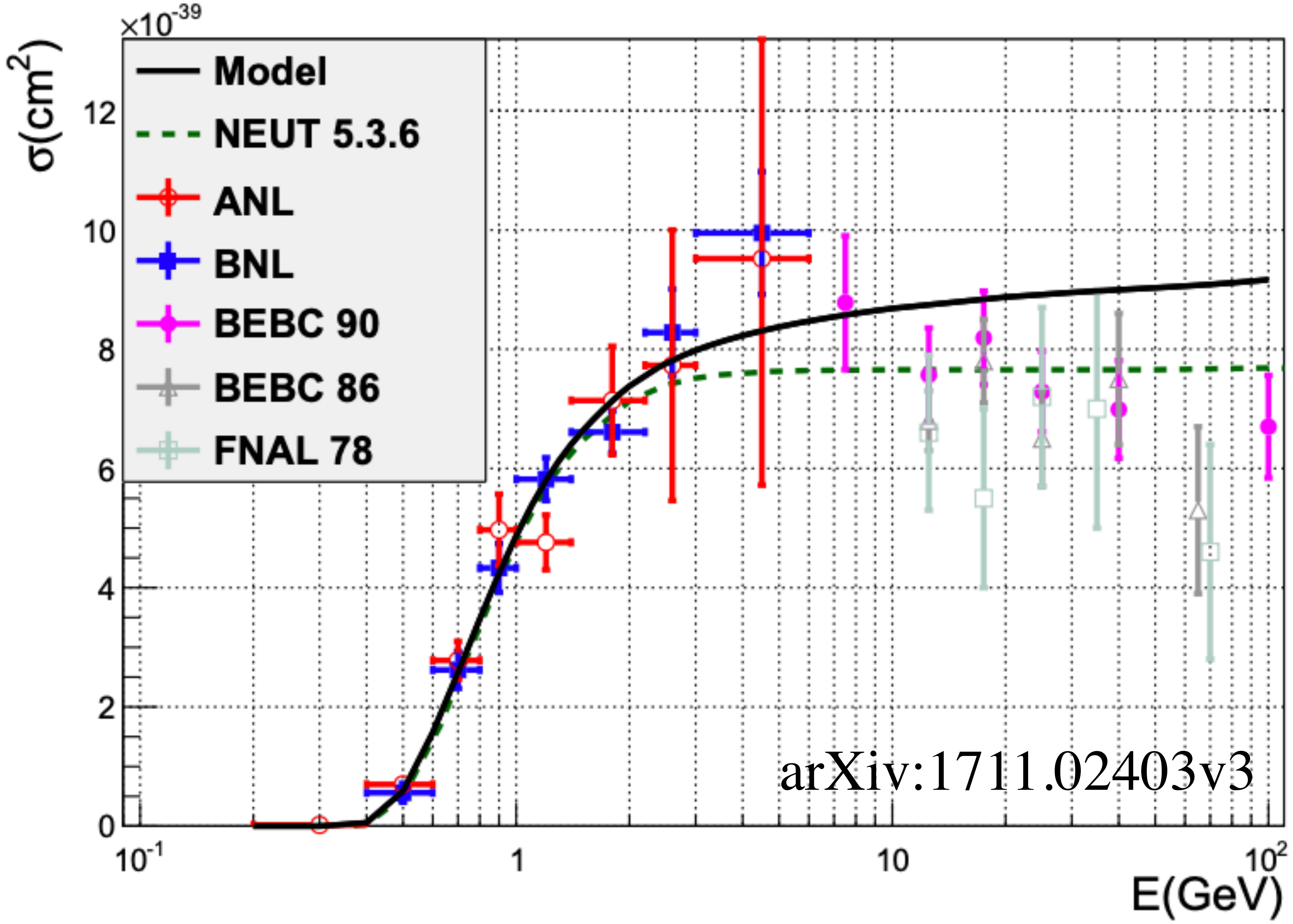
$$\pi^0 \to 2\gamma \qquad BR = 0.98823$$

- π can be absorbed into the nucleus: the event is indistinguishable with a QE event (similar observed final particles).

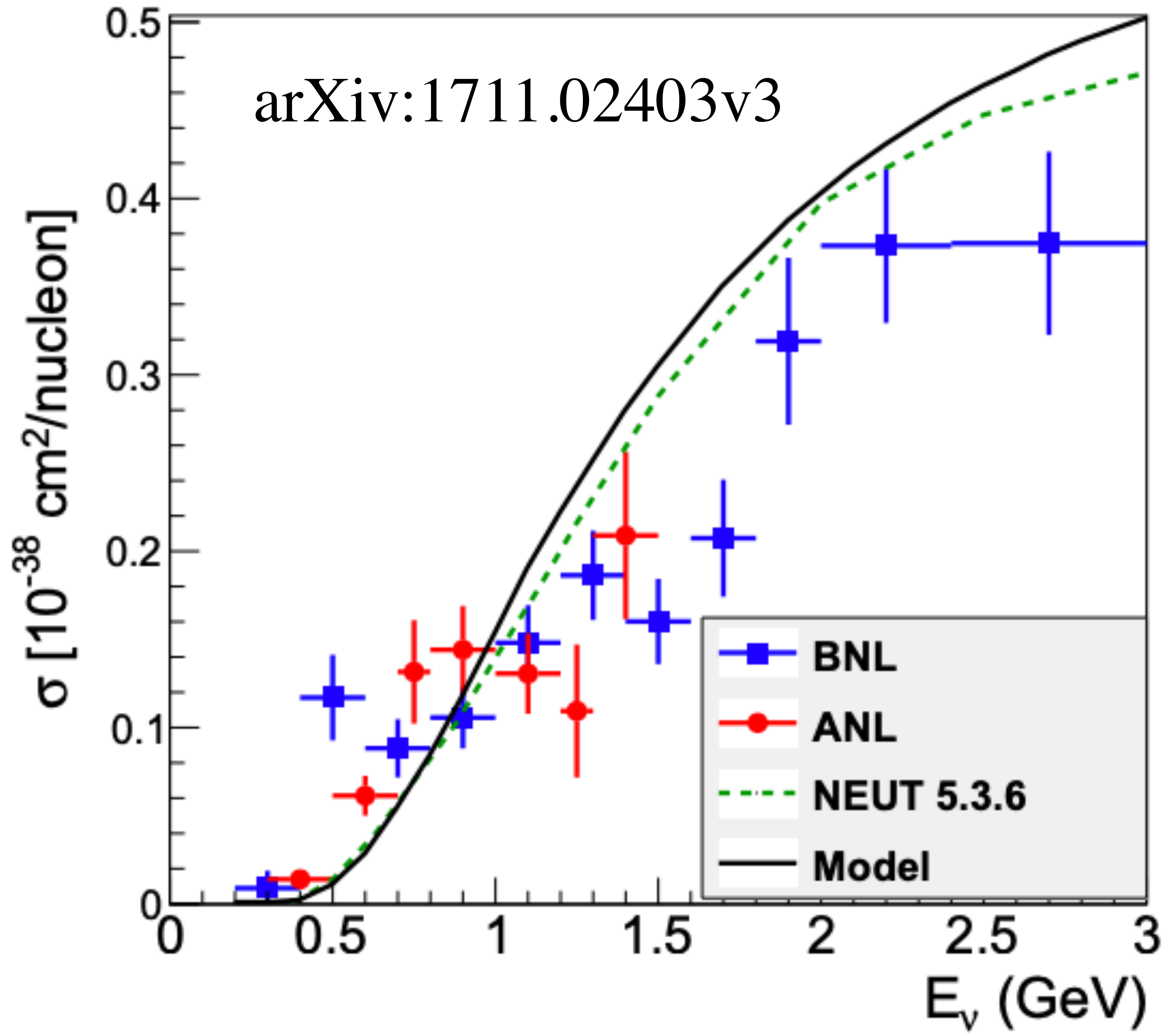
3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- Resonant meson production (CC1π dominates at low energy)



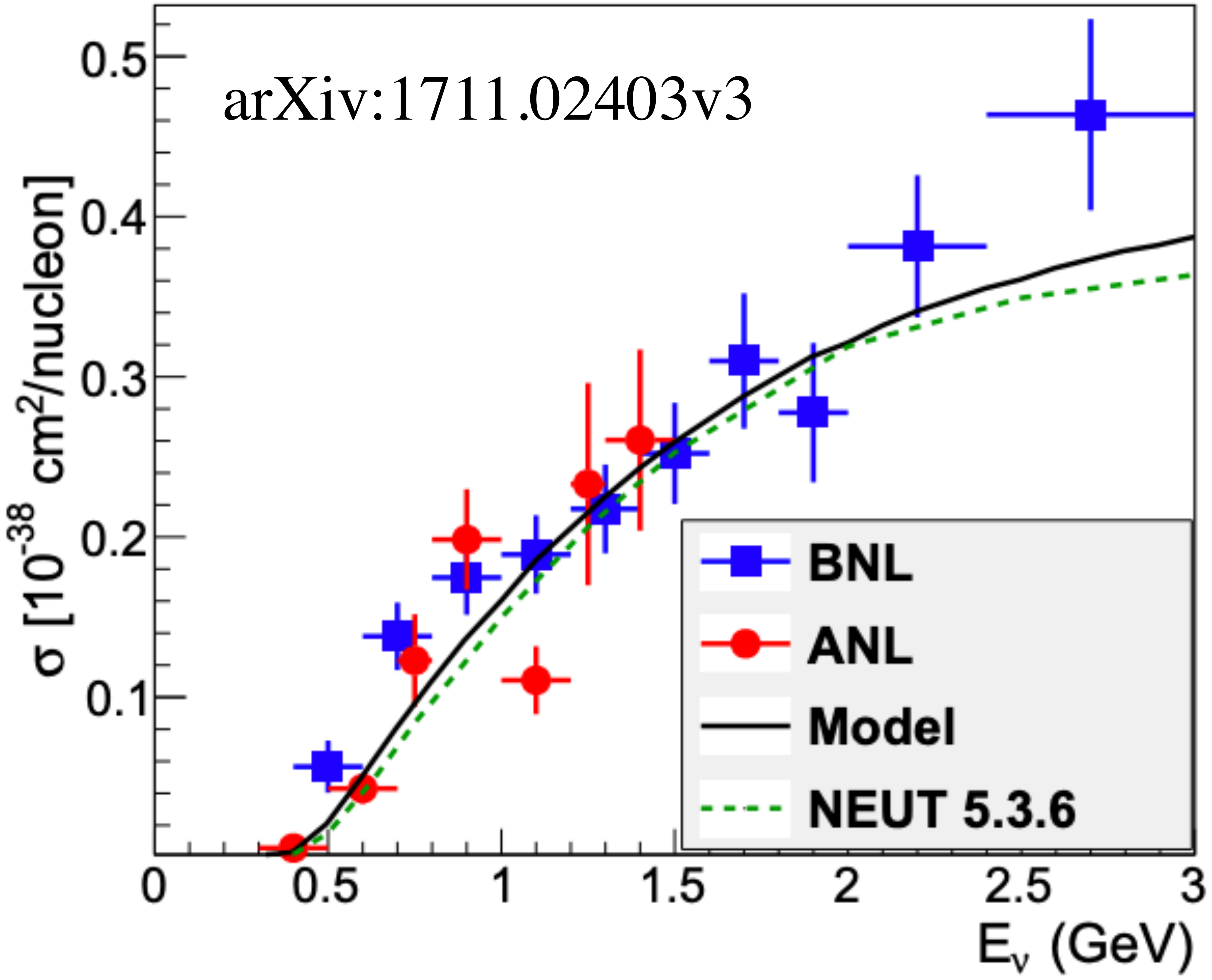
$$\nu_\mu p \rightarrow \mu^- p \pi^+$$



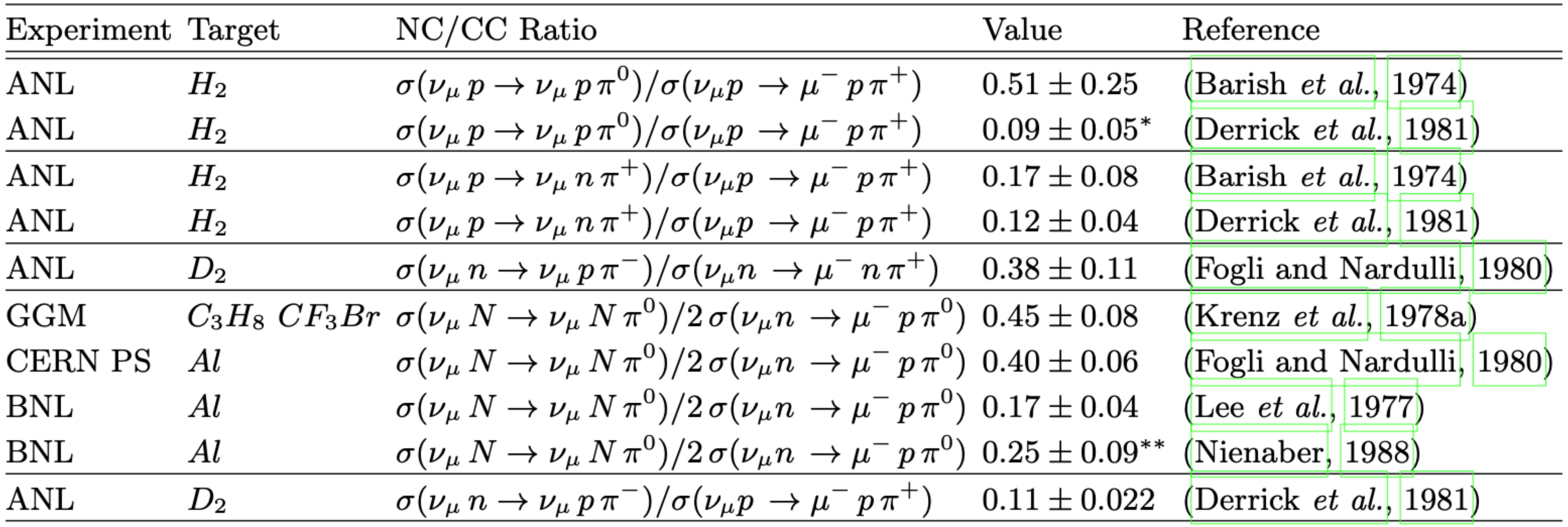
$$\nu_\mu n \rightarrow \mu^- n \pi^+$$

3.2 Neutrino - nucleon interactions

- ○ Neutrino interactions at 0.1-20GeV energy:
- Resonant meson production (CC1π dominates at low energy)


arXiv:1711.02403v3

$$\nu_\mu n \rightarrow \mu^- p\pi^0$$



Experiment	Target	NC/CC Ratio	Value	Reference
ANL	H_2	$\sigma(\nu_\mu\, p \to \nu_\mu\, p\, \pi^0)/\sigma(\nu_\mu p \to \mu^-\, p\, \pi^+)$	0.51 ± 0.25	(Barish *et al.*, 1974)
ANL	H_2	$\sigma(\nu_\mu\, p \to \nu_\mu\, p\, \pi^0)/\sigma(\nu_\mu p \to \mu^-\, p\, \pi^+)$	$0.09 \pm 0.05^*$	(Derrick *et al.*, 1981)
ANL	H_2	$\sigma(\nu_\mu\, p \to \nu_\mu\, n\, \pi^+)/\sigma(\nu_\mu p \to \mu^-\, p\, \pi^+)$	0.17 ± 0.08	(Barish *et al.*, 1974)
ANL	H_2	$\sigma(\nu_\mu\, p \to \nu_\mu\, n\, \pi^+)/\sigma(\nu_\mu p \to \mu^-\, p\, \pi^+)$	0.12 ± 0.04	(Derrick *et al.*, 1981)
ANL	D_2	$\sigma(\nu_\mu\, n \to \nu_\mu\, p\, \pi^-)/\sigma(\nu_\mu n \to \mu^-\, n\, \pi^+)$	0.38 ± 0.11	(Fogli and Nardulli, 1980)
GGM	$C_3H_8\ CF_3Br$	$\sigma(\nu_\mu\, N \to \nu_\mu\, N\, \pi^0)/2\,\sigma(\nu_\mu n \to \mu^-\, p\, \pi^0)$	0.45 ± 0.08	(Krenz *et al.*, 1978a)
CERN PS	Al	$\sigma(\nu_\mu\, N \to \nu_\mu\, N\, \pi^0)/2\,\sigma(\nu_\mu n \to \mu^-\, p\, \pi^0)$	0.40 ± 0.06	(Fogli and Nardulli, 1980)
BNL	Al	$\sigma(\nu_\mu\, N \to \nu_\mu\, N\, \pi^0)/2\,\sigma(\nu_\mu n \to \mu^-\, p\, \pi^0)$	0.17 ± 0.04	(Lee *et al.*, 1977)
BNL	Al	$\sigma(\nu_\mu\, N \to \nu_\mu\, N\, \pi^0)/2\,\sigma(\nu_\mu n \to \mu^-\, p\, \pi^0)$	$0.25 \pm 0.09^{**}$	(Nienaber, 1988)
ANL	D_2	$\sigma(\nu_\mu\, n \to \nu_\mu\, p\, \pi^-)/\sigma(\nu_\mu p \to \mu^-\, p\, \pi^+)$	0.11 ± 0.022	(Derrick *et al.*, 1981)

- The cross sections of the other channels can be deduced from the above three ones by experimental measurements

3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 0.1-20GeV energy:**

- **Coherent pion production**

$$\nu_\mu A \to \nu_\mu A \pi^0 \qquad \overline{\nu}_\mu A \to \overline{\nu}_\mu A \pi^0$$
$$\nu_\mu A \to \mu^- A \pi^+ \qquad \overline{\nu}_\mu A \to \mu^+ A \pi^-$$

- Negligible transferred energy
- Coherently produce pion
- No nuclear recoil

$$\boxed{\left.\frac{d^3\sigma}{dQ^2 dy d|t|}\right|_{Q^2=0} = \frac{G_F^2}{2\pi^2} f_\pi^2 \frac{1-y}{y} \frac{d\sigma(\pi A \to \pi A)}{d|t|}}$$

Adler's PCAC theorem

- $y = E_\pi / E_\nu$
- f_π : pion decay constant
- $|t|$: magnitude of the square of the 4-momentum transfer to the nucleus
- $Q^2 = -q^2$: 4-momentum transferred to hadronic system

3.2 Neutrino - nucleon interactions

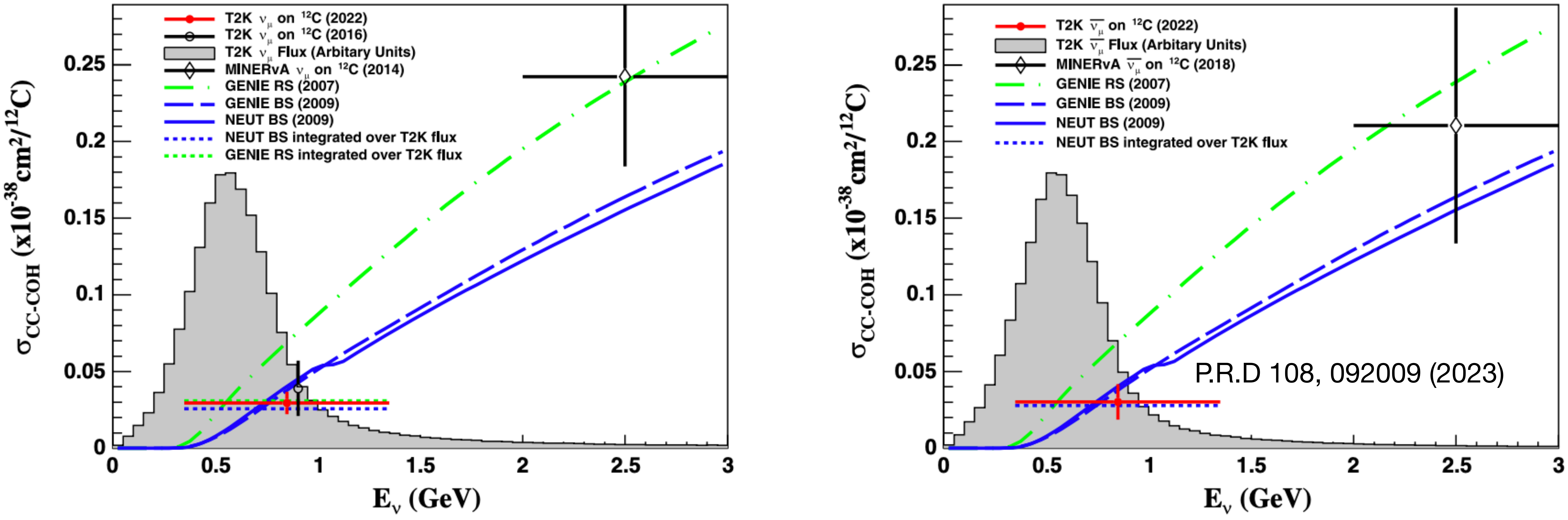
○ **Neutrino interactions at 0.1-20GeV energy:**

- **Coherent pion production**

$$\nu_\mu\, A \rightarrow \nu_\mu\, A\, \pi^0 \qquad \overline{\nu}_\mu\, A \rightarrow \overline{\nu}_\mu\, A\, \pi^0$$

$$\nu_\mu\, A \rightarrow \mu^-\, A\, \pi^+ \qquad \overline{\nu}_\mu\, A \rightarrow \mu^+\, A\, \pi^-$$

- Negligible transferred energy
- Coherently produce pion
- No nuclear recoil



P.R.D 108, 092009 (2023)

- CC COH Xsec measurement on C at T2K for ν_μ (left) and $\overline{\nu}_\mu$ (right)

3.2 Neutrino - nucleon interactions

- ○ **Neutrino interactions at 20 - 500 GeV energy:**
- **Deep inelastic scattering (CC)**

$$\nu_l + N \rightarrow l^- + X \qquad \bar{\nu}_l + N \rightarrow l^+ + X$$

(-)+ for (anti-)neutrino scattering

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 m_N E_\nu}{\pi(1+Q^2/m_{W,Z}^2)^2}\left[xy^2F_1(x,Q^2) + \left(1-y\right)F_2(x,Q^2) \pm y\left(1-y/2\right)xF_3(x,Q^2)\right]$$

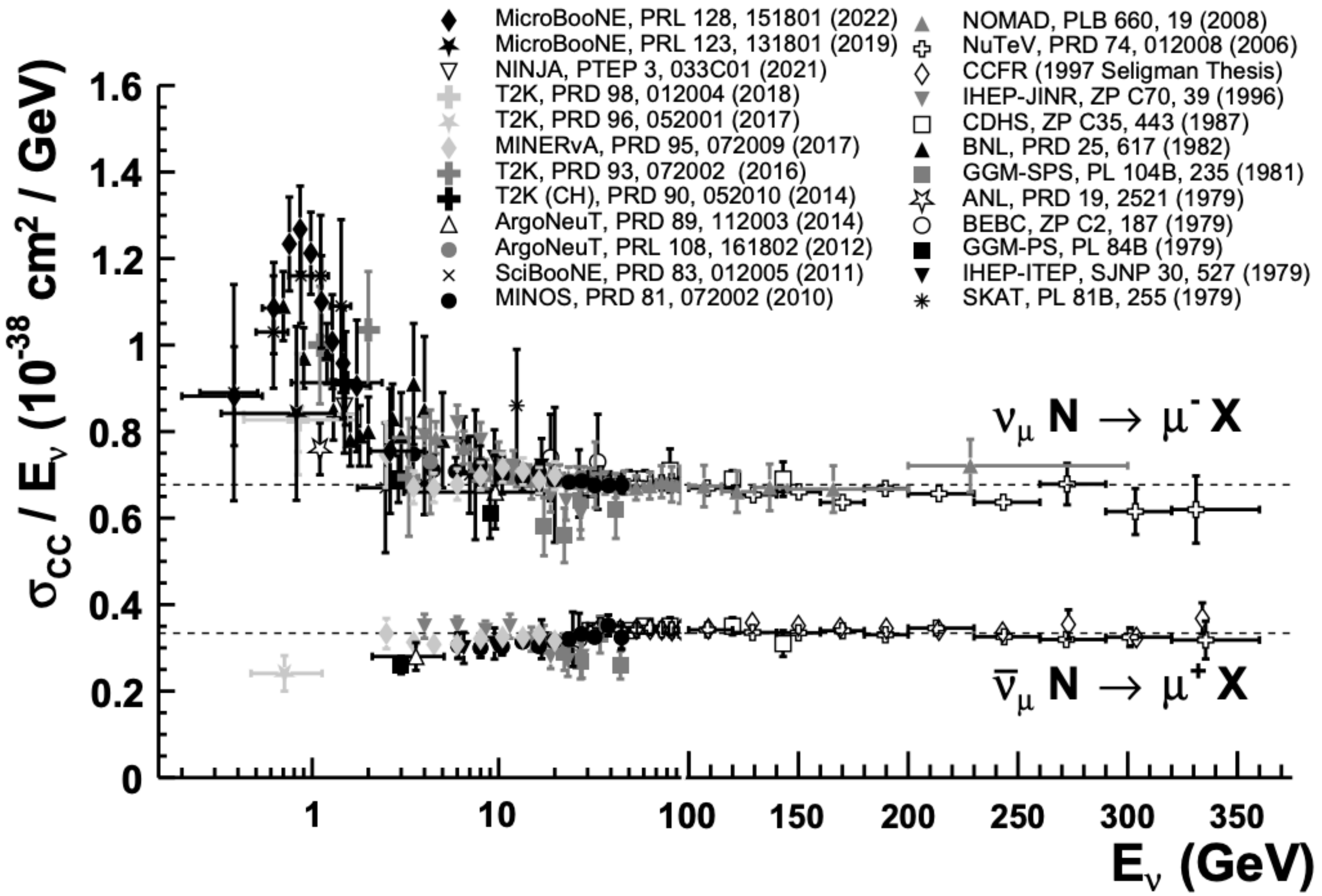
$$x = \frac{Q^2}{2p_N \cdot q}$$

$$y = \frac{p_N \cdot q}{p_N \cdot p_\nu}$$

$$xy = \frac{Q^2}{s - m_N^2}$$

$$s = (p_\nu + p_N)^2 = m_N^2 + 2p_\nu \cdot p_N$$

$$Q^2 = -m_l^2 + 2E_\nu(E_l - p_l\cos\theta)$$



3.3: Cross section measurement

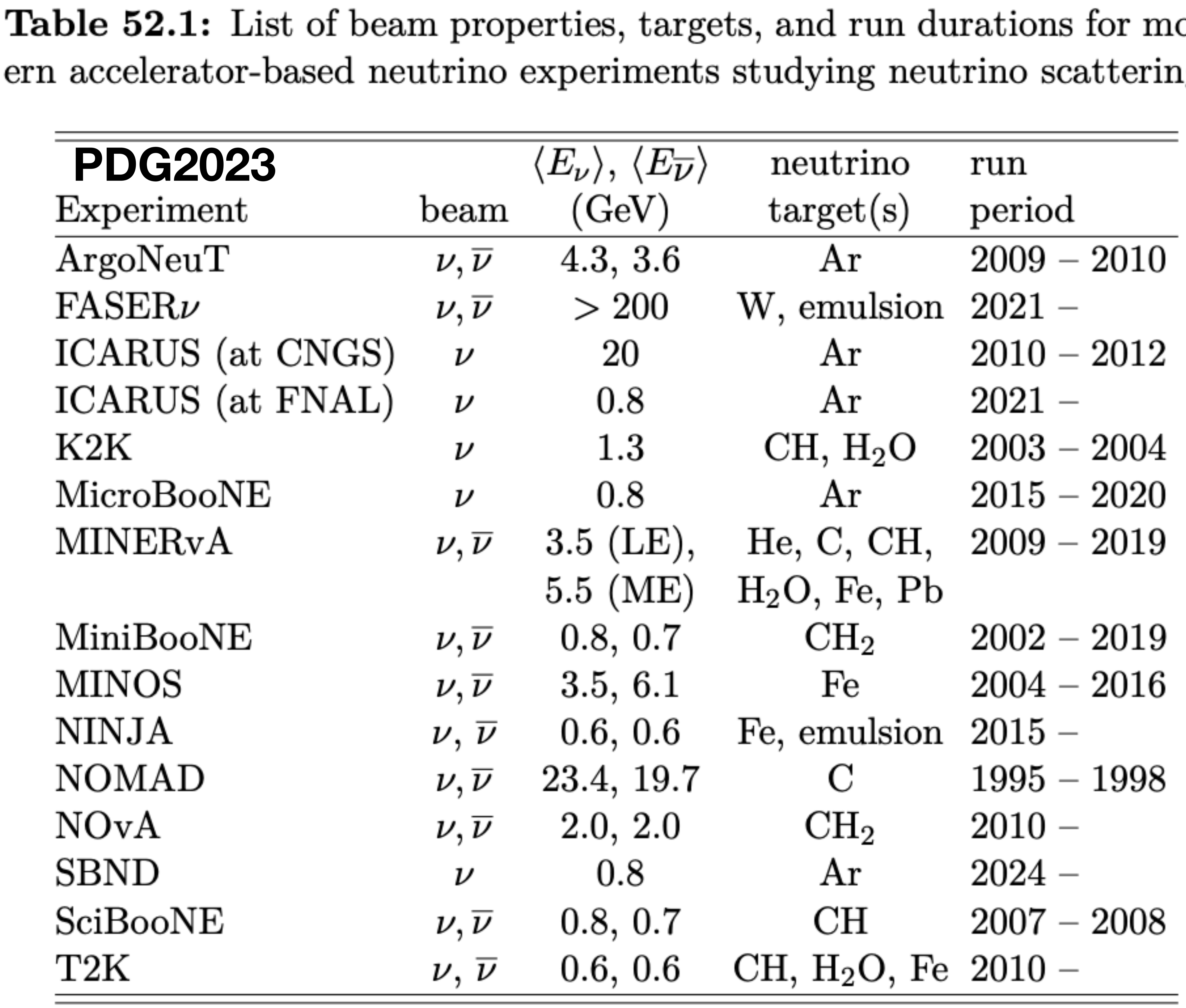


Table 52.1: List of beam properties, targets, and run durations for modern accelerator-based neutrino experiments studying neutrino scattering.

PDG2023

Experiment	beam	$\langle E_\nu \rangle$, $\langle E_{\overline{\nu}} \rangle$ (GeV)	neutrino target(s)	run period
ArgoNeuT	$\nu, \overline{\nu}$	4.3, 3.6	Ar	2009 – 2010
FASERν	$\nu, \overline{\nu}$	> 200	W, emulsion	2021 –
ICARUS (at CNGS)	ν	20	Ar	2010 – 2012
ICARUS (at FNAL)	ν	0.8	Ar	2021 –
K2K	ν	1.3	CH, H_2O	2003 – 2004
MicroBooNE	ν	0.8	Ar	2015 – 2020
MINERvA	$\nu, \overline{\nu}$	3.5 (LE), 5.5 (ME)	He, C, CH, H_2O, Fe, Pb	2009 – 2019
MiniBooNE	$\nu, \overline{\nu}$	0.8, 0.7	CH_2	2002 – 2019
MINOS	$\nu, \overline{\nu}$	3.5, 6.1	Fe	2004 – 2016
NINJA	ν, $\overline{\nu}$	0.6, 0.6	Fe, emulsion	2015 –
NOMAD	$\nu, \overline{\nu}$	23.4, 19.7	C	1995 – 1998
NOvA	$\nu, \overline{\nu}$	2.0, 2.0	CH_2	2010 –
SBND	ν	0.8	Ar	2024 –
SciBooNE	$\nu, \overline{\nu}$	0.8, 0.7	CH	2007 – 2008
T2K	ν, $\overline{\nu}$	0.6, 0.6	CH, H_2O, Fe	2010 –

How do experiments measure Xsec?

3.3: Cross section measurement

- Recall that the **true** number of interactions at the detector:

  $N = \phi_a N_b \sigma$ or in general: $N(E_\nu) = \phi(E_\nu)\,.\,T\,.\,\sigma(E_\nu)$
- Consider an observable **x** that describes the interaction

  $$N(x_{true}) = \int \frac{d\sigma(E_\nu, x_{true})}{dx_{true}} \phi(E_\nu)\,.\,T\,.\,dx_{true}$$
- There is no perfect detector, what you **measured (reconstrued)** is **smeared** from the **true** information $N(x_{recon}) \neq N(x_{true})$. You need to account for this difference and make your measurement closer to the true information as much as possible:
  - Take into account the detector efficiency ϵ
  - Construct a **smearing matrix** **(migration matrix)** $U_{smearing}$ to migrate from **true quantity** to **reconstructed quantity**

  $$N(x_{recon}) = \int U_{smearing} \frac{d\sigma(E_\nu, x_{true})}{dx_{true}} \phi(E_\nu)\,.\,T\,.\,\epsilon\,.\,dx_{true}$$

3.3: Cross section measurement

- Solving for differential cross section:

$$\frac{d\sigma(E_\nu, x_{true})}{dx_{true}} = \frac{U^{-1}_{smearing} N(x_{recon})}{\phi(E_\nu)\,.\,T\,.\,\epsilon\,.\,dx_{true}} \equiv \frac{U_{unfolding} N(x_{recon})}{\phi(E_\nu)\,.\,T\,.\,\epsilon\,.\,dx_{true}}$$

  - Where $U_{unfolding} = U^{-1}_{smearing}$

- In general

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

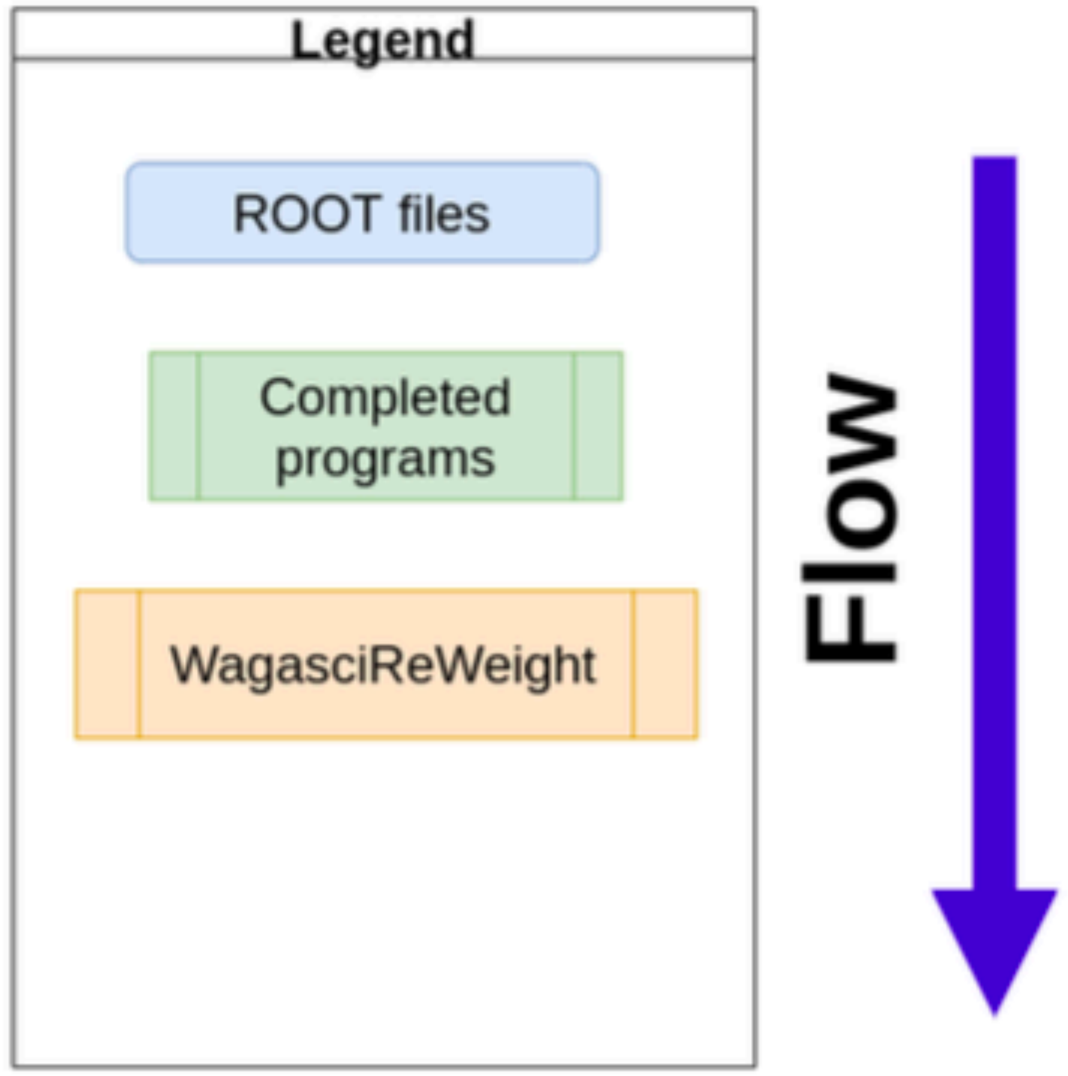
- $U_{i\alpha}$: unfolding matrix (unsmearing matrix)
- $N^{selected}_{data}$: total number of selected events after applying all cuts
- N^{bckg}_{data} : number of backgrounds in selected events
- ϕ : neutrino integrated flux
- T : number of target nucleons
- ϵ : detector efficiency
- Δx : bin width

3.3: Cross section measurement

An example of $\nu_\mu\ CC0\pi$ Xsec measurement at T2K

- Analysis flow:

$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

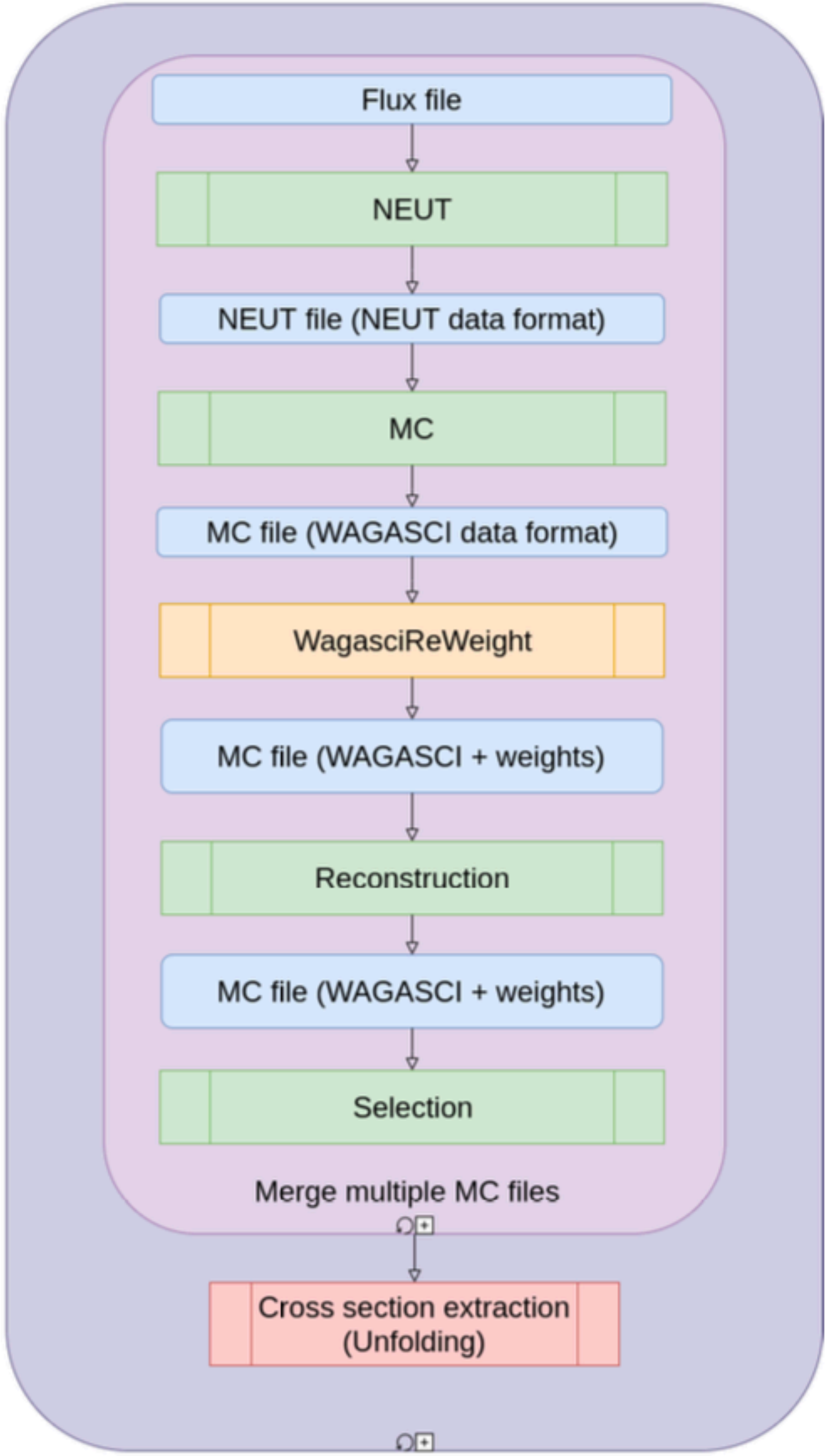


Legend

🟦 ROOT files

🟩 Completed programs

🟧 WagasciReWeight



Flux file

NEUT

NEUT file (NEUT data format)

MC

MC file (WAGASCI data format)

WagasciReWeight

MC file (WAGASCI + weights)

Reconstruction

MC file (WAGASCI + weights)

Selection

Merge multiple MC files

Cross section extraction (Unfolding)

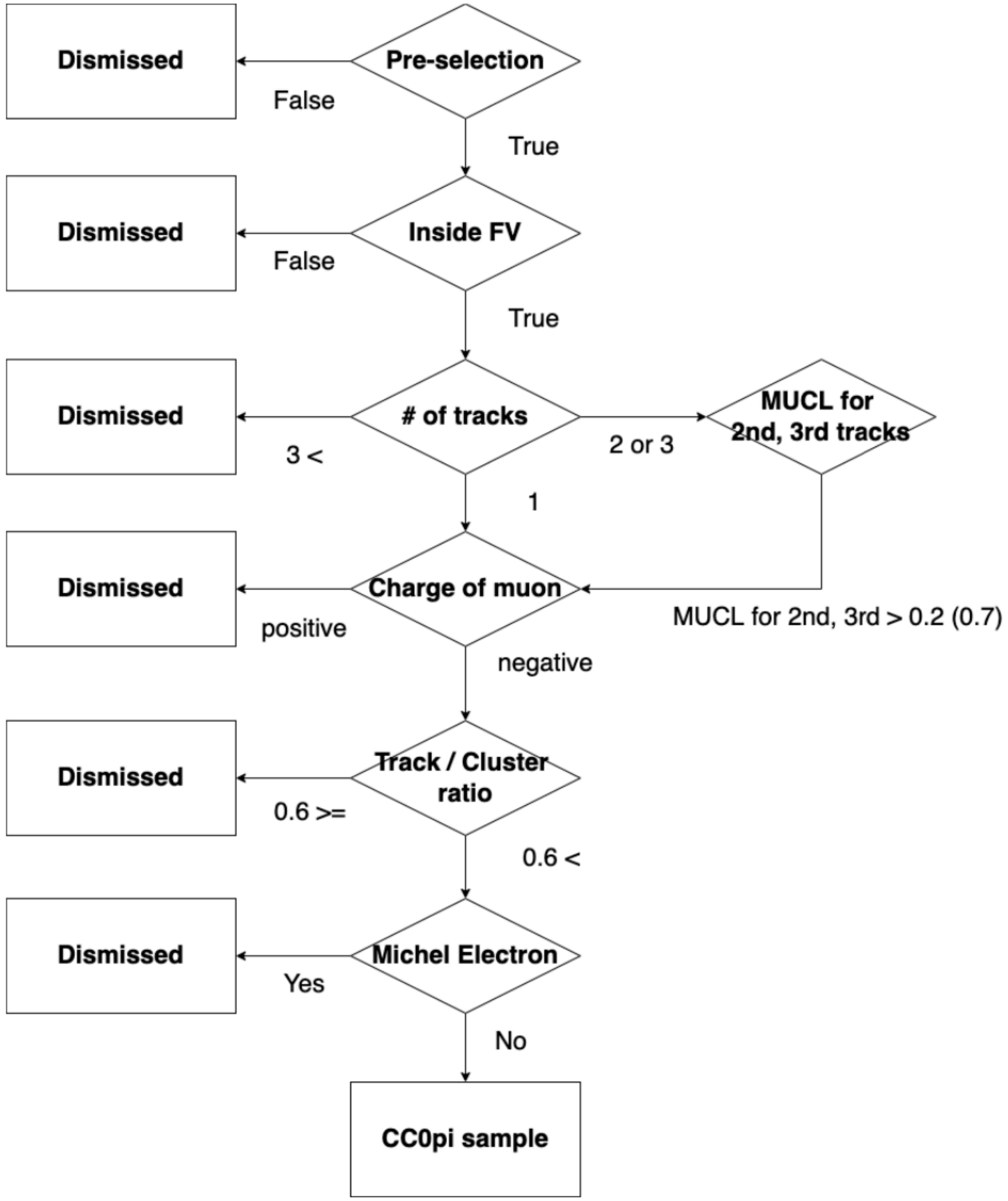
3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut



3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- **Define signal sample ($CC0\pi$)**
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut

$$\nu_\mu + A \rightarrow \mu^- + X + 0\pi^\pm$$

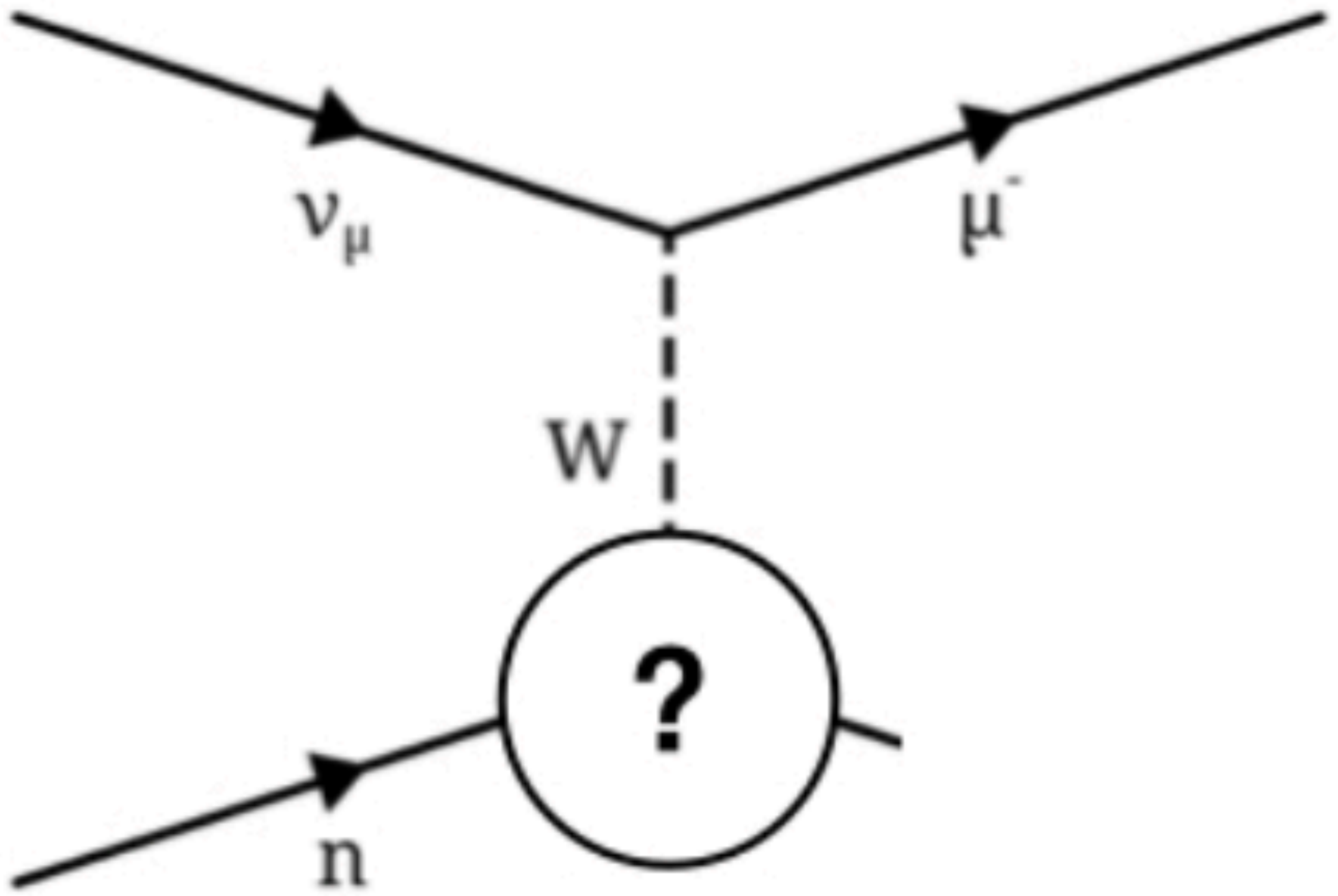


Figure 1: Signal topology for CC$0\pi^\pm$ sample

3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \,.\, T \,.\, \epsilon \,.\, \Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
  - **Pre-selection**
  - Fiducial volume cut
  - Number-of-track cut
  - Muon confidence level cut
  - Particle charge cut
  - Track/cluster ratio cut
  - Michel electron tagging
  - Contained volume cut

- Whole fundamental cuts related to 2D and 3D reconstruction including track matching between detectors

3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \,.\, T \,.\, \epsilon \,.\, \Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - **Fiducial volume cut**
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut

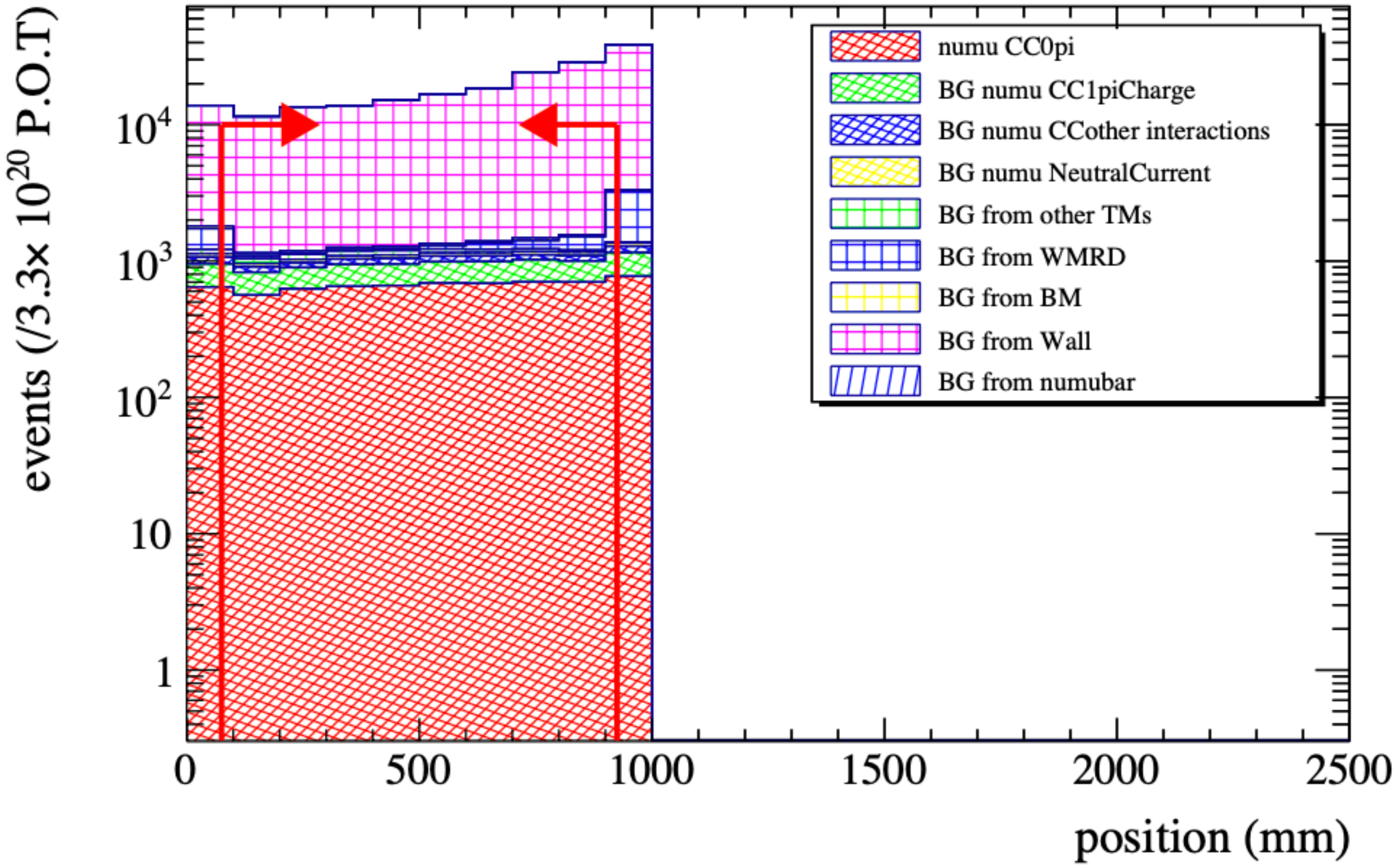


Figure 9: X vertex distribution for H_2O target

What is the fiducial volume?

3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - **Number-of-track cut**
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut

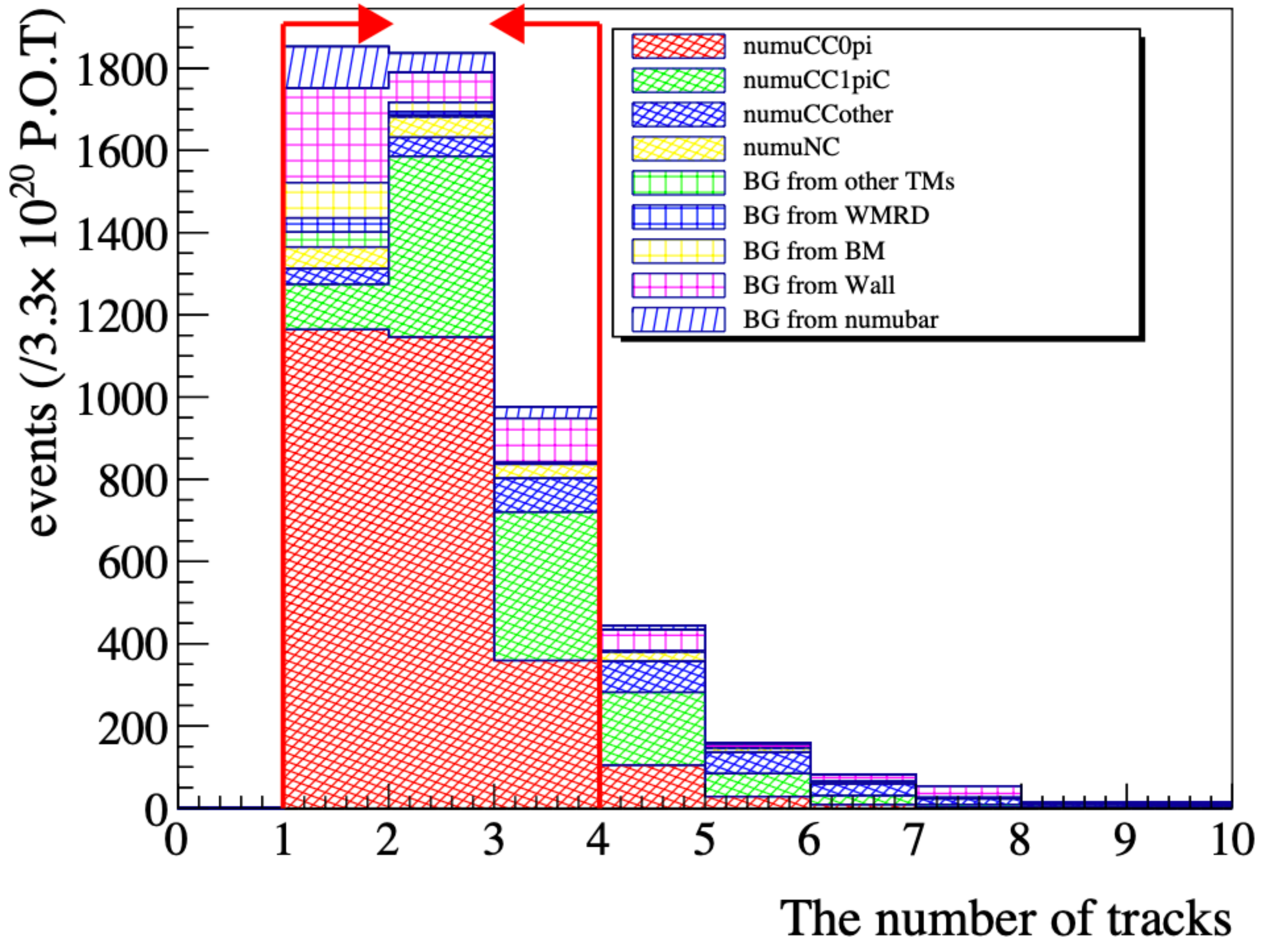


Figure 13: the number of track distribution broken down by interaction topologies for the charged current inclusive sample on water target

$CC0\pi$ typically has 1 or 2 tracks, events with large number of tracks are more likely the backgrounds

3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \,.\, T \,.\, \epsilon \,.\, \Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
  - Pre-selection
  - Fiducial volume cut
  - Number-of-track cut
  - **Muon confidence level cut**
  - Particle charge cut
  - Track/cluster ratio cut
  - Michel electron tagging
  - Contained volume cut

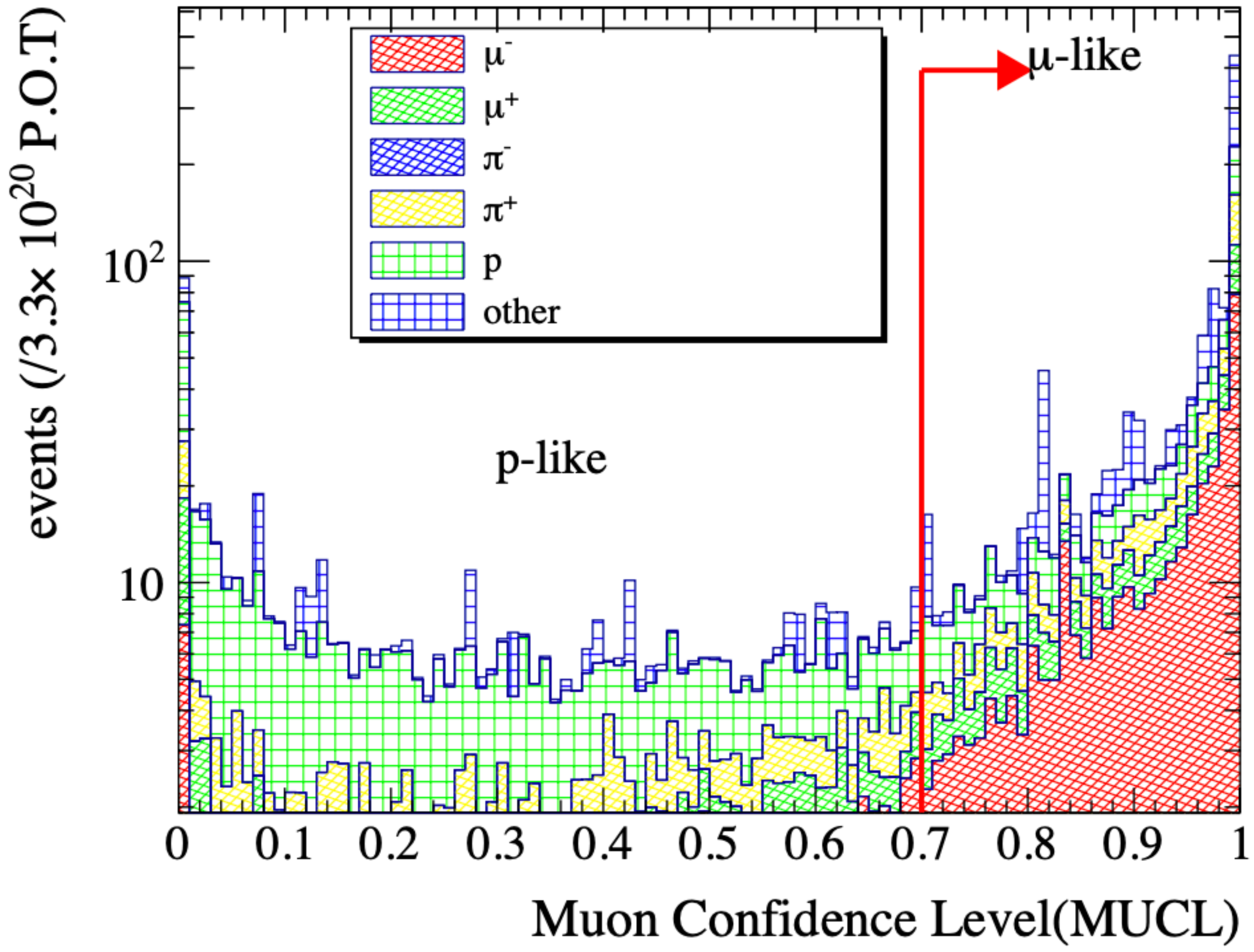


Figure 15: MUCL distribution for UpstreamWAGASCI

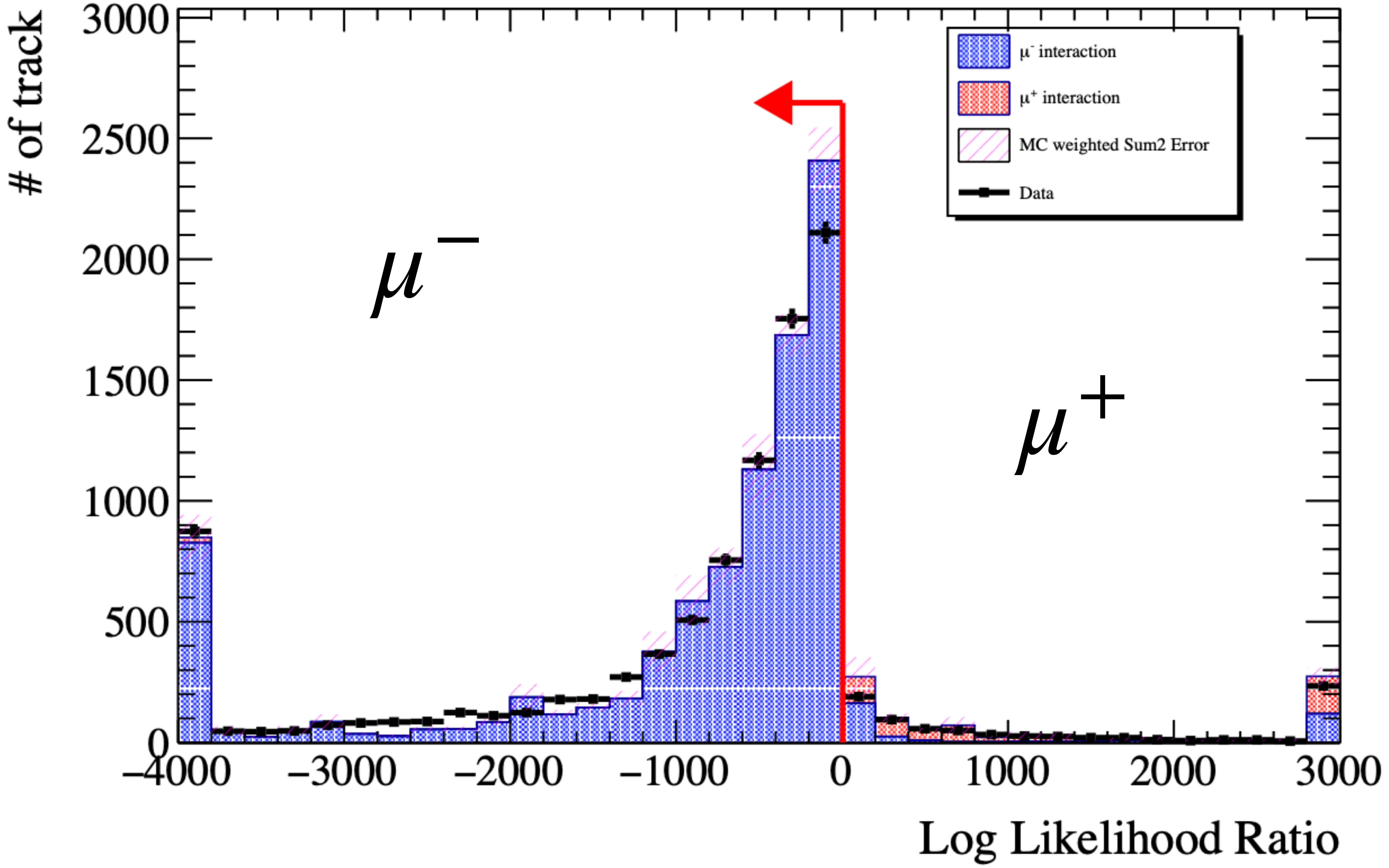
3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \,.\, T \,.\, \epsilon \,.\, \Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - **Particle charge cut**
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut



3.3: Cross section measurement

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - **Track/cluster ratio cut**
    - Michel electron tagging
    - Contained volume cut

- Remove backgrounds from interactions from experimental hall or other subdectors

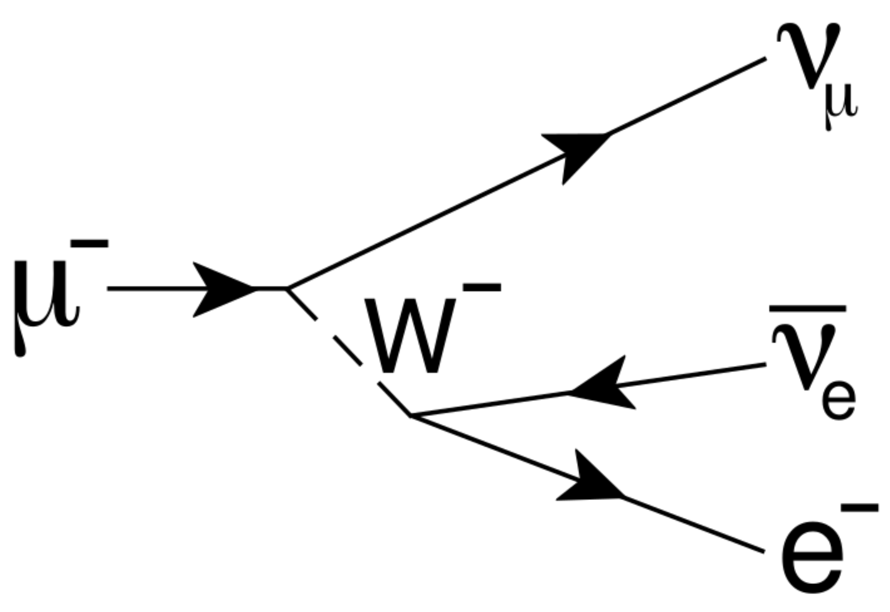
3.3: Cross section measurement

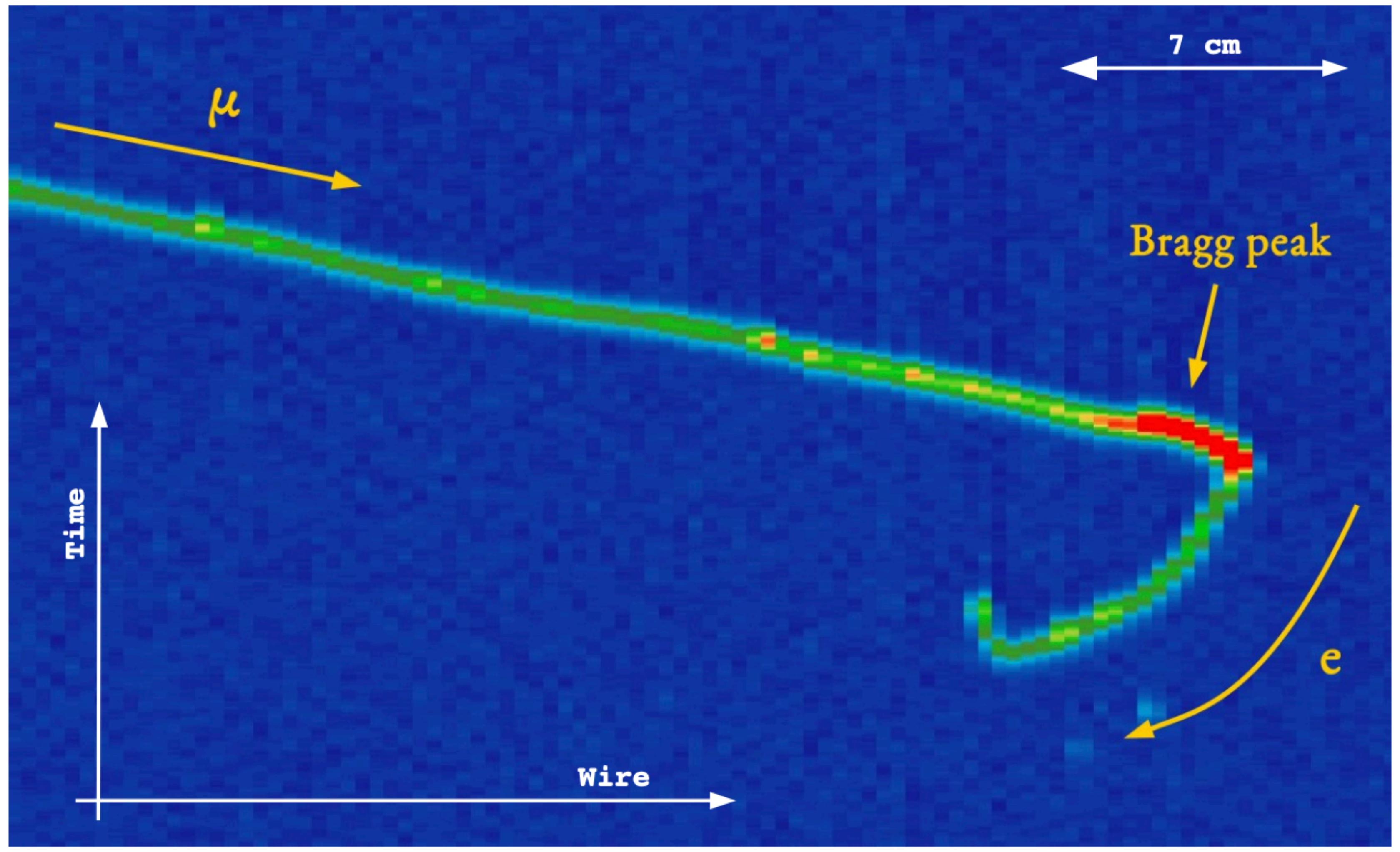
$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \, . \, T \, . \, \epsilon \, . \, \Delta x}$$

An example of $\nu_\mu \; CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - **Michel electron tagging**
    - Contained volume cut





- Remove main backgrounds from Wall MRDs

3.3: Cross section measurement

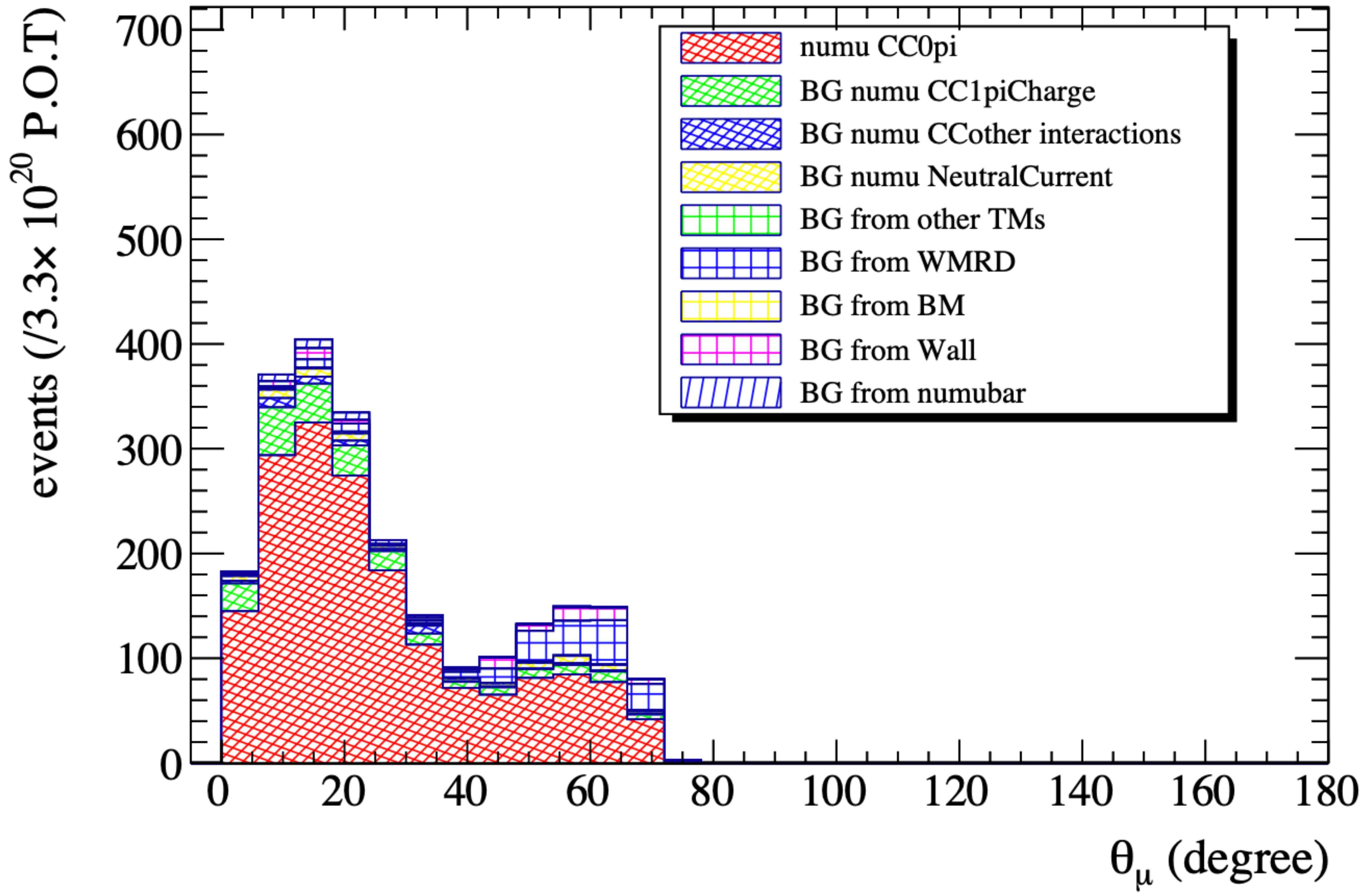
$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \, . \, T \, . \, \epsilon \, . \, \Delta x}$$

An example of $\nu_\mu \; CC0\pi$ Xsec measurement at T2K

○ **Event selection:**

- Define signal sample ($CC0\pi$)
- Define selections (cuts)
    - Pre-selection
    - Fiducial volume cut
    - Number-of-track cut
    - Muon confidence level cut
    - Particle charge cut
    - Track/cluster ratio cut
    - Michel electron tagging
    - Contained volume cut

○ An event which is **not actually a true signal event** can **pass your selection** even after **appying all cuts**, it is called a **background event**



(a) all selection applied

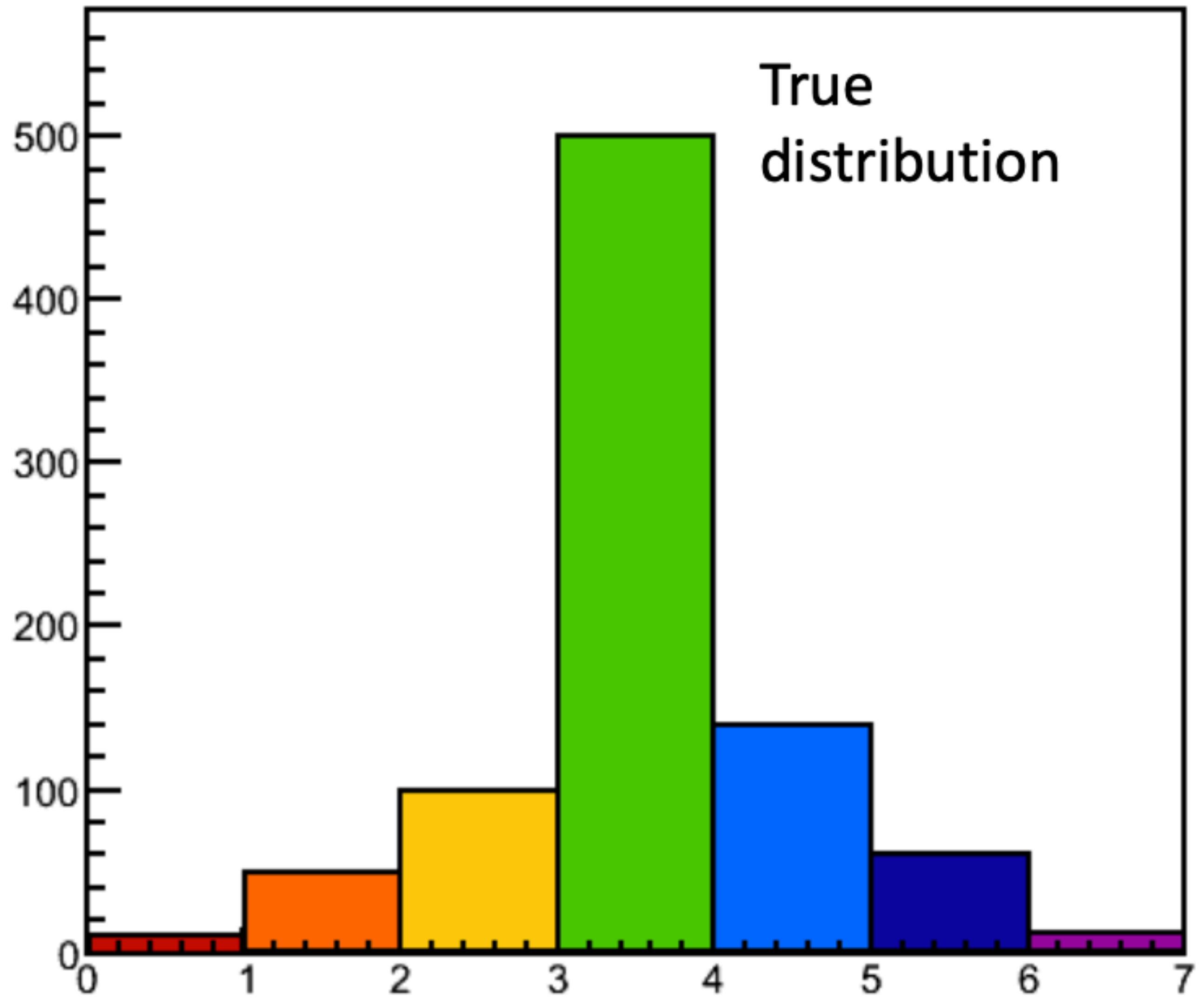
3.3: Cross section measurement

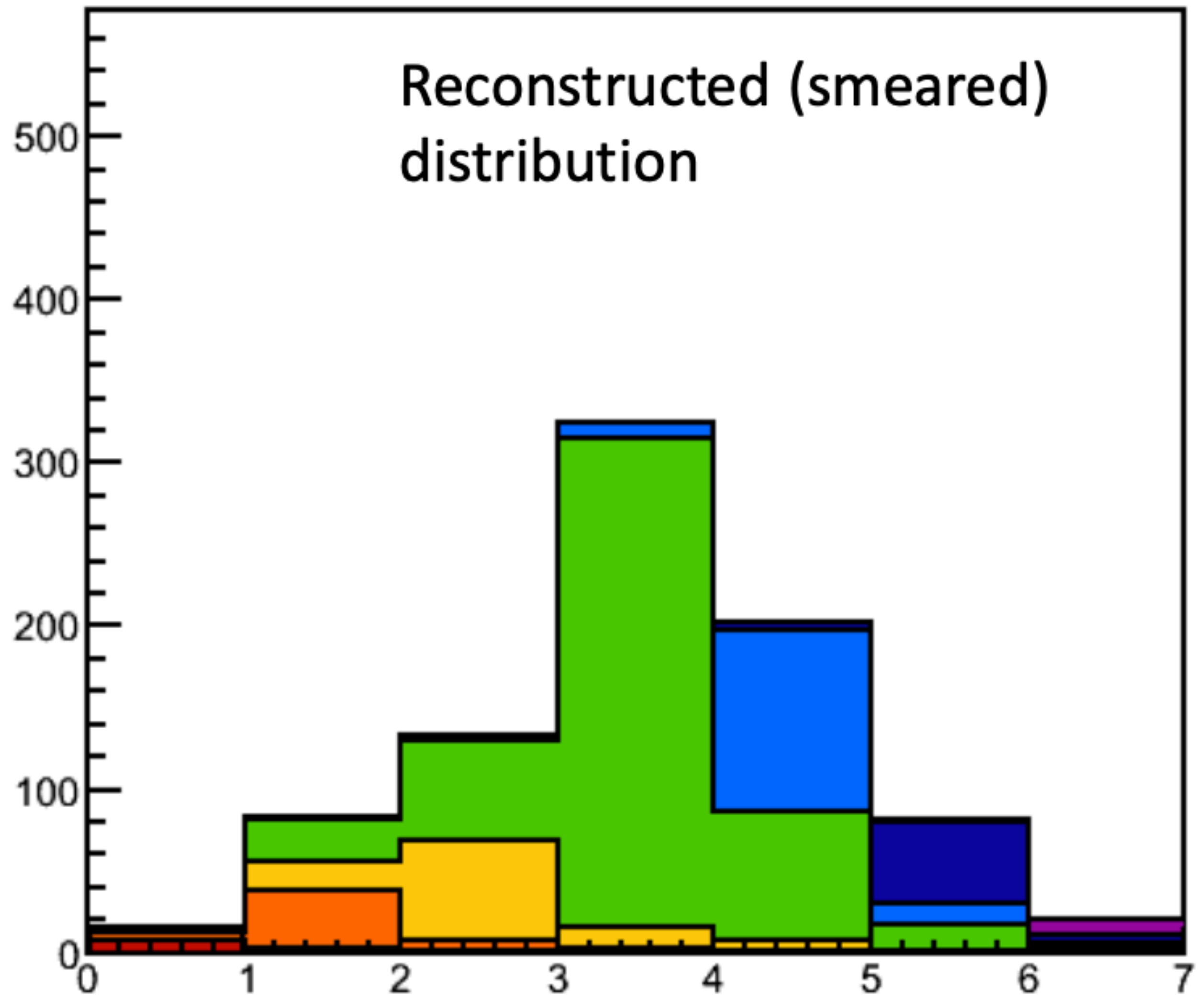
$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi \,.\, T \,.\, \epsilon \,.\, \Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Unfolding:**

- Our reconstruction is not perfect, an event can be reconstructed into a wrong bin
- $U_{i\alpha}$ presents the probability that an event observed in bin i actually happened in bin α
- **We can use our Monte Carlo to construct a smearing matrix (migration matrix) which indicates the fraction of events generated in bin α be mis-reconstructed into bin i**
- If the detector has good resolution, the matrix should be close to diagonal
- Bin width is also important. If the bins are too small compared to our resolution, the matrix may not be diagonal


True distribution


Reconstructed (smeared) distribution

3.3: Cross section measurement

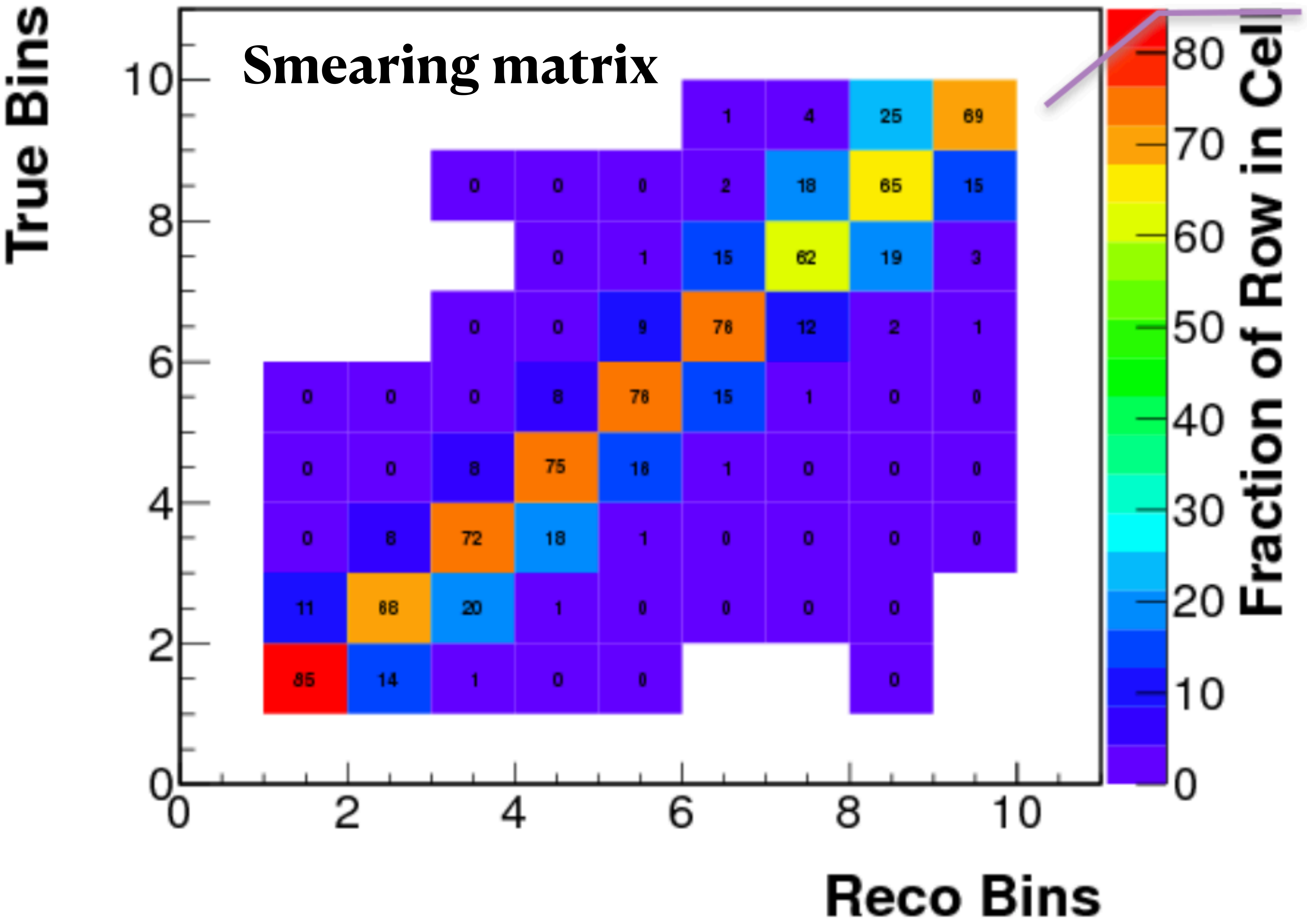
$$\left(\frac{d\sigma}{dx}\right)_{\alpha} = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

○ **Unfolding:**

- **Unfolding matrix** (**unsmearing matrix**) can be obtained by inverting the smearing matrix (migration matrix)

$$U_{unfolding} = U^{-1}_{smearing}$$



Smearing matrix

					1	4	25	69
		0	0	0	2	18	65	15
			0	1	15	62	19	3
		0	0	9	76	12	2	1
0	0	0	8	78	15	1	0	0
0	0	8	75	18	1	0	0	0
0	8	72	18	1	0	0	0	0
11	68	20	1	0	0	0	0	
85	14	1	0	0			0	

3.3: Cross section measurement

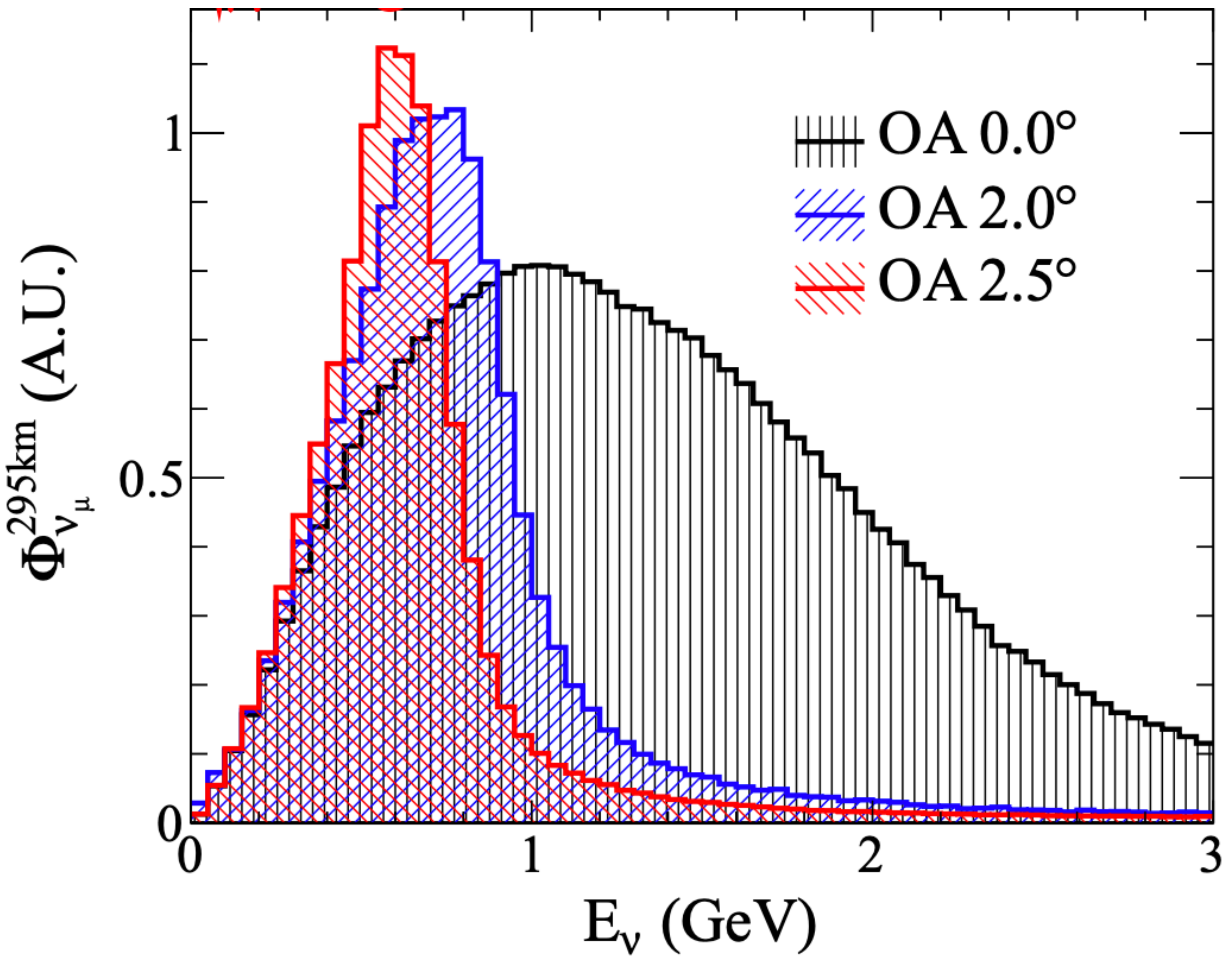
An example of $\nu_\mu\ CC0\pi$ Xsec measurement at T2K

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

○ **Flux:**

- Integrated flux over all neutrino energies

$$\phi = \frac{\int dE_\nu \phi(E_\nu)}{dE_\nu}$$



3.3: Cross section measurement

An example of ν_μ $CC0\pi$ Xsec measurement at T2K

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

- **T: number of nucleons in target**
  - $T = \frac{M}{m}\,.\,N_A * (n_p + n_n)$
  - M: target mass in grams
  - m: atomic mass
  - n_p, n_n: number of protons and neutrons per atom
  - $N_A = 6.02214 \times 10^{23}\ mol^{-1}$: Avogadro's number

- **How many nucleons are there in Super-K detector?**
  - M = 50kt = 5×10^{10} g
  - $m_{H_2O} = 18$
  - $(n_p + n_n)_{H_2O} = (10 + 8) = 18$

$$T = \frac{5 \times 10^{10}}{18} * 6.0214 \times 10^{23} * (10 + 8) = 3 \times 10^{34}$$

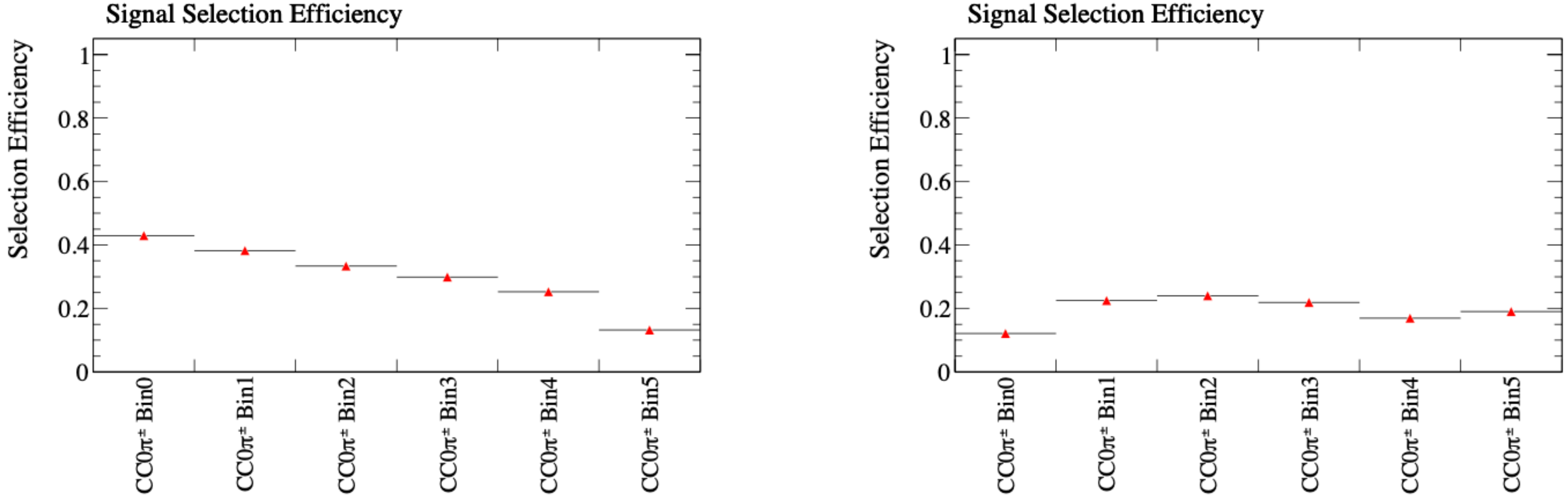
3.3: Cross section measurement

An example of $\nu_\mu\ CC0\pi$ Xsec measurement at T2K

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

- **ϵ: efficiency**

- $$\epsilon = \frac{\text{number of signal events after selection}}{\text{number of signal events in MC}}$$



(a) with respect to μ angle (b) with respect to μ momentum

Figure 37: Selection efficiency in one-dimensional phase space for H_2O target

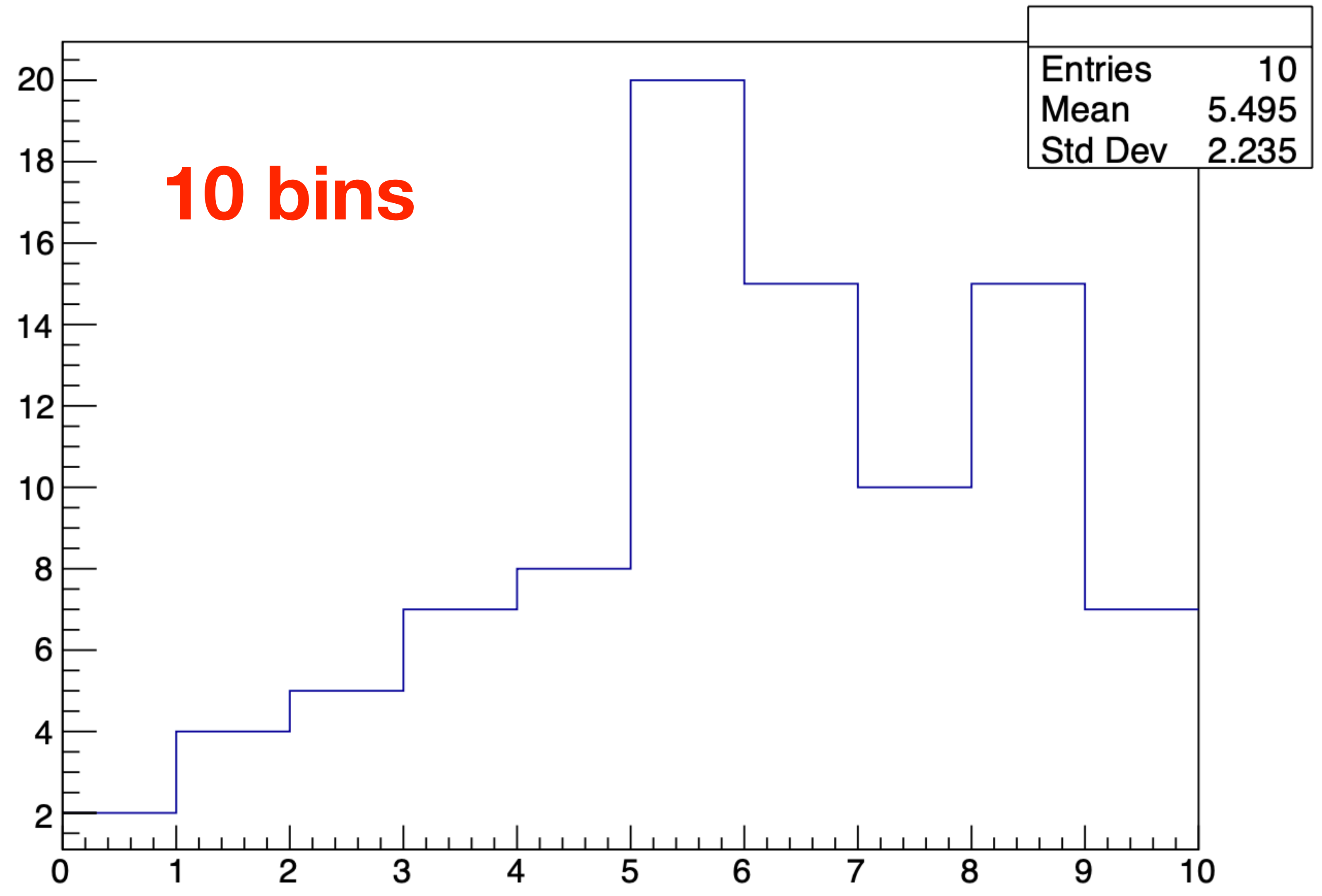
3.3: Cross section measurement

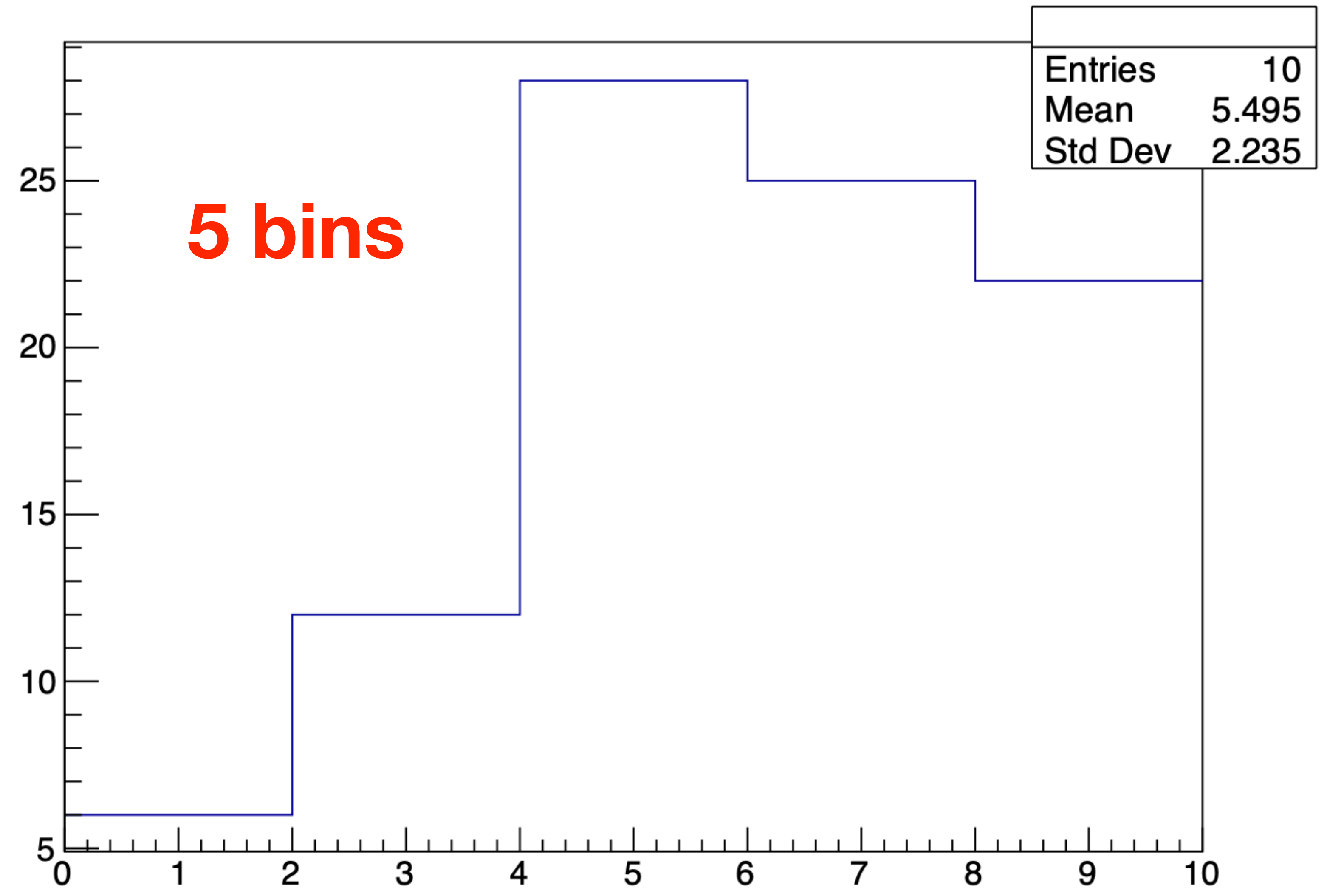
An example of $\nu_\mu\ CC0\pi$ Xsec measurement at T2K

$$\left(\frac{d\sigma}{dx}\right)_\alpha = \frac{\sum_i U_{i\alpha}(N^{selected}_{data,i} - N^{bckg}_{data,i})}{\phi\,.\,T\,.\,\epsilon\,.\,\Delta x}$$

○ **Bin width Δx**

- Bin width is defined based on the number of expected events in each bin
- More bins (smaller bin width) you have, better you can distinguish the features of the distribution
- However, more bins you have, less events in each bin you get (worse statistics)





Contents

3.4: Summary

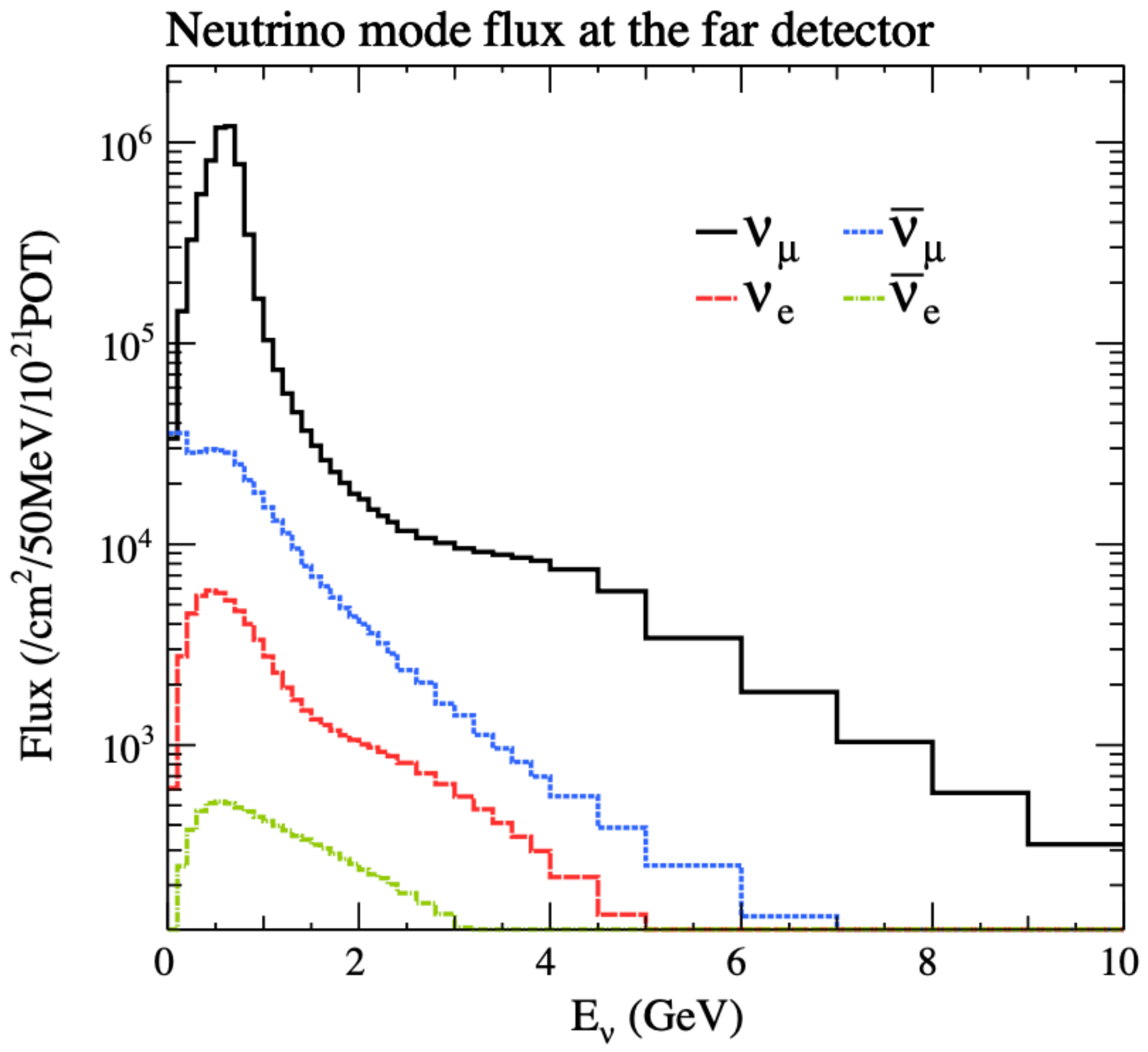
- Three active neutrinos (ν_e, ν_μ, ν_τ) and their anti-particles interact with matter by exchange $W^\pm, Z^0$ via CC or NC
- Active neutrinos only participate in weak interaction with extremely small Xsec
- A neutrino can elastic, quasi-elastic, or inelastic scattering with electron, nucleus, nucleon, or quarks depending on its energy
- Theoretical calculation of Xsec is difficult when dealing with nuclear effects
- Experimental measurement of Xsec is difficult due to tiny Xsec and technical barriers
- Interactions in medium energy region (~GeV) is important because they cover mostly accessible and abundant neutrino sources (solar, reactor, atmospheric, and accelerator)

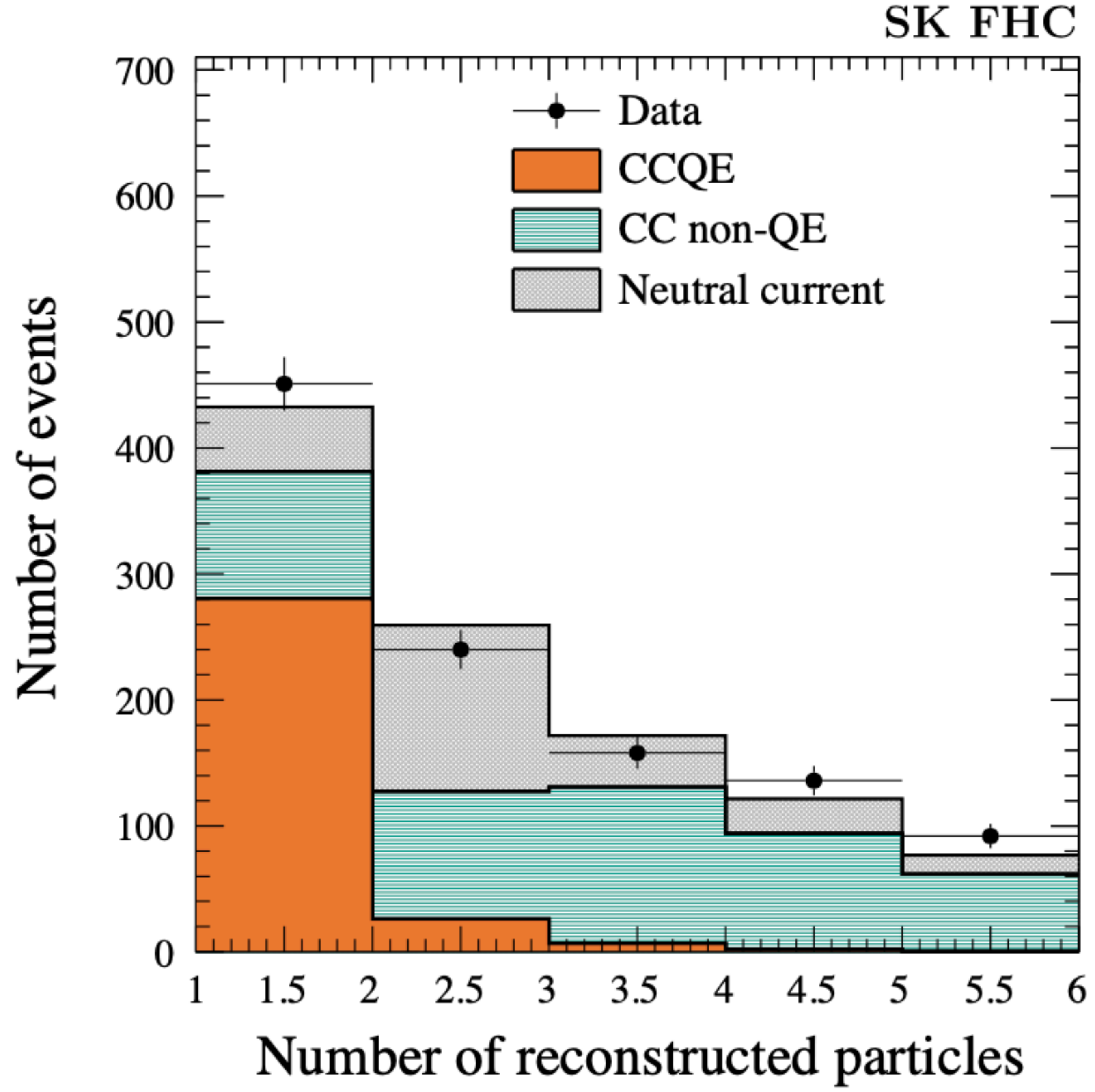
Contents

4. Introduction to NEUT - an event generator

4.1: Neutrino event generators

- Neutrino event generators are MC softwares used for simulating neutrino interactions
- They are bridges connecting between theory and experiment
- They play an important role in oscillation analysis:
  - Predicting neutrino flux
  - Simulate interaction channels
  - Simulate signals and bacgrounds
  - Calculate efficiencies and systematics uncertainties





4. Introduction to NEUT - an event generator

4.1: Neutrino event generators

- How do they work?

☐ Neutrino type
☐ Target
☐ Nu energy or flux
☐ Interaction mode
☐ Other parameters
…

NEUT, GENIE, Neugen, NuWro, GiBUU, …

☐ ROOT file
☐ Kinematics of out-going particles
☐ Other variables
☐ …

From H. Van

4. Introduction to NEUT - an event generator

4.1: Neutrino event generators

- **GENIE:**
  - Developed by international colaboration for neutrino interactions from MeV to PeV $(10^{15}\ eV)$
  - ROOT based neutrino generator written in C++.
  - Well maintainance and open source
  - Recently implemented models of quasielastic and 2p2h interactions
  - Used in many neutrino oscillation experiments such as T2K, NOvA, …

4. Introduction to NEUT - an event generator

4.1: Neutrino event generators

- **GiBUU (Giessen-Boltzmann-Uehling-Uhlenbeck Project):**
    - Provides a unified theory and transport framework in the MeV and GeV energy regimes for elementary reactions on nuclei (electron, photon, neutrino and hadron with nuclei)
    - Provides a full dynamical description of the reaction and delivers the complete final state of an event
    - Not only used as neutrino generator but also in nuclear reactions
    - Written in Fortran and open source

4. Introduction to NEUT - an event generator

4.1: Neutrino event generators

- **NuWro:**
  - Has been developed at the University of Wroclaw since 2006
  - Written in C++
  - Simulate neutrino interactions taking into account beam profile and composition, detailed detector geometry as well as FSI in the nuclear target from 100MeV to TeV

4. Introduction to NEUT - an event generator

4.2: Introduction to NEUT

- **NEUT:**
  - Originally developed for Kamioka experiment to predict neutrino-induced backgrounds for nucleon decay search
  - Has been officially used for Super-K and T2K (currently NEUT 5.4.0)
  - Mainly written in Fortran, not open source yet
  - Primarilly simulate neutrino-nucleus/nucleon interactions in wide range of energy from 100 MeV to few TeV $(10^{12}\ eV)$
  - Incorporates nuclear effects
  - Recently impemented state-of-the-art CCQE and multi-nucleon model, single pion production models, and electron - nucleus scattering

4. Introduction to NEUT - an event generator

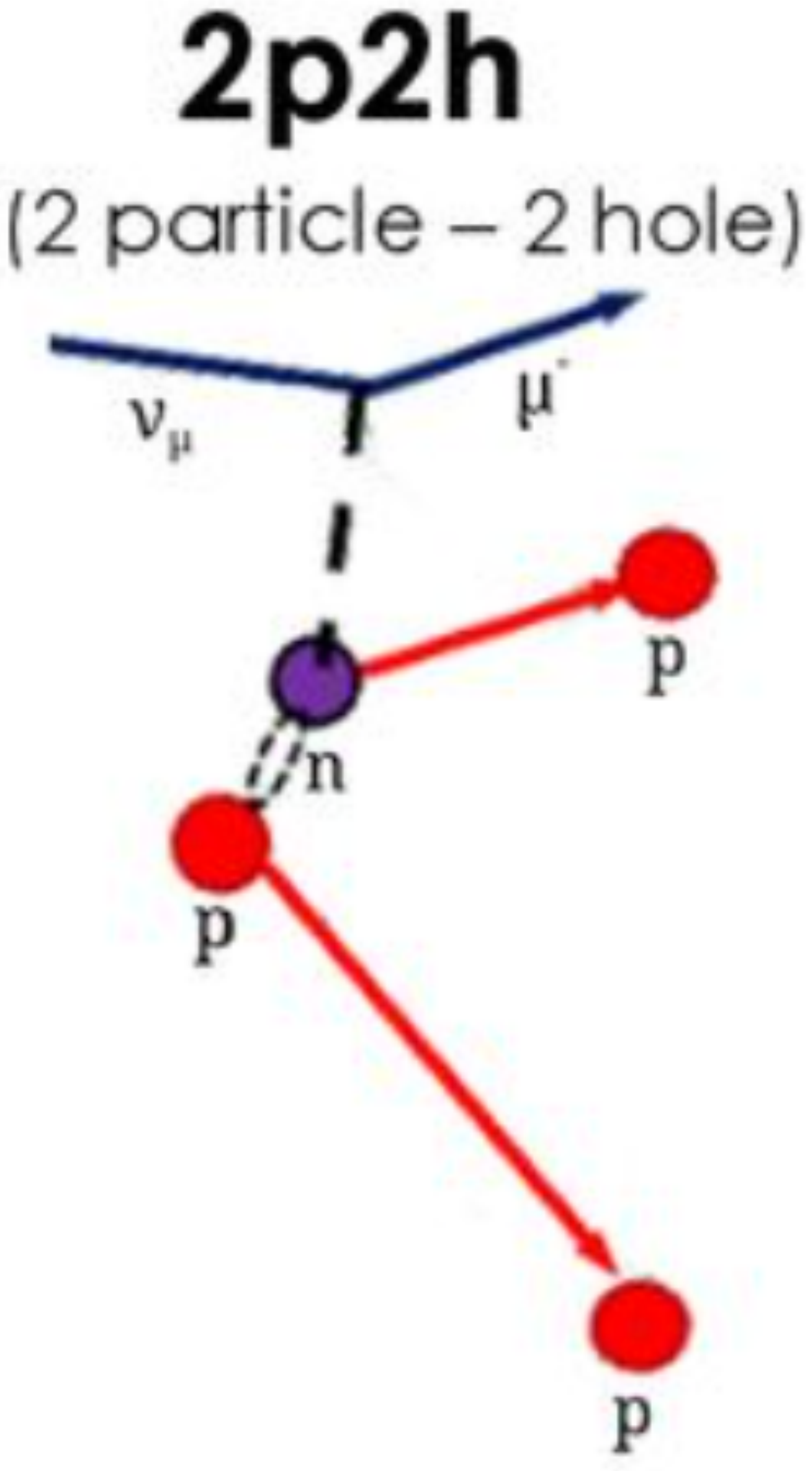
4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **1p1h (CCQE)**
    - Produce single-ring electron-like and muon-like events
    - Contribute 70% of the $1R_{\mu}$ selection at far detector
    - Use Benhar Spectral Function (Nucl. Phys. A 579, 493–517 (1994))
    - Nucleon vector form factors: BBBA05 description (Nucl. Phys. B Proc. Suppl. 159, 127–132 (2006))
    - Nucleon axial mass $M_A^{QE} = 1.03 \pm 0.06\ GeV$

4. Introduction to NEUT - an event generator

4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **CC 2p2h (2-particles 2-holes), NC 2p2h not included yet**
    - Neutrinos interactions with 2 nucleons producing 2 holes
    - Produce single-ring electron-like and muon-like events
    - Contribute 12% of the $1R_\mu$ selection at far detector
    - Use Nieves model (Phys. Lett. B 707, 72–75 (2012))



4. Introduction to NEUT - an event generator

4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **Single pion production**
    - Coherent 1π (Berger–Sehgal model)
    - Resonant and non-resonant 1π (Rein–Sehgal (RS) model) with hadronic invariant mass $1.3 \leq W \leq 2\ GeV$
    - Contribute 13% of the $1R_\mu$ selection at far detector

4. Introduction to NEUT - an event generator

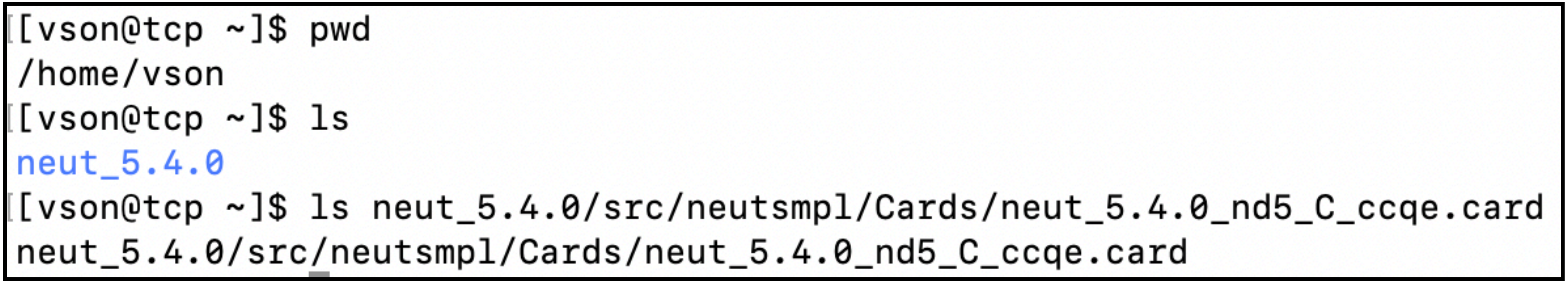
4.2: Introduction to NEUT

- **NEUT interaction models (used at T2K analysis):**
  - **Deep inelastic scattering**
    - Use GRV98 Parton Distribution Functions (PDFs)
    - Bodek–Yang (BY) correction is used for $Q^2 \leq 1.5\ GeV^2$
    - Processes begin where hadronic invariant mass $W > 1.3\ GeV$
    - Pythia 5.72 for $W > 2\ GeV$
    - A custom model interpolating between the Δ(1232) and DIS interactions is employed for $W < 2\ GeV$

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**
- CERNLIB and ROOT are needed for ROOT installtion (**already available on IOP cluster**)
- **Source code: /home/vson/neut_5.4.0**
- **Car file: /home/vson/neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card**

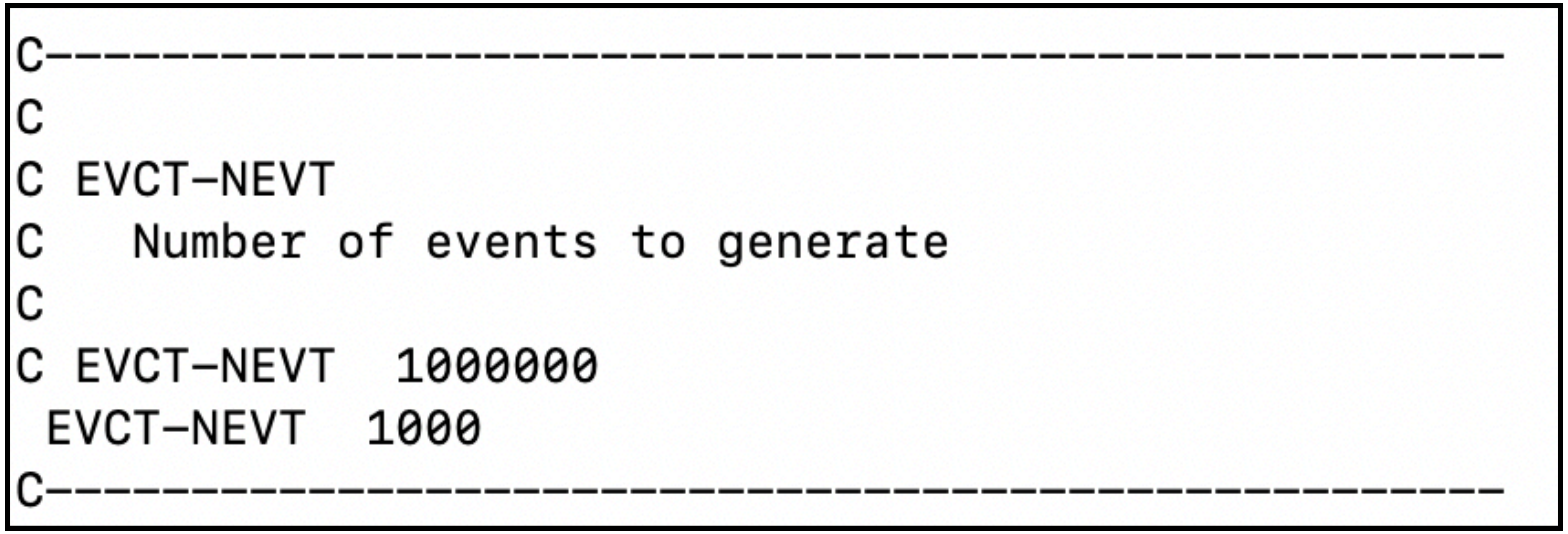


```
[[vson@tcp ~]$ pwd
/home/vson
[[vson@tcp ~]$ ls
neut_5.4.0
[[vson@tcp ~]$ ls neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card
neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card
```

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- Working space is IOP cluster
  - **NEUT input: Card file**
    - Used to specify models, parameters, interaction modes, …
    - Card file: /home/vson/neut_5.4.0/src/neutsmpl/Cards/neut_5.4.0_nd5_C_ccqe.card
    - When setting your card file, check carefully there is no letter **C** at the beginning of the line (**C**: comment out that line)



```
C--------------------------------------------------
C
C EVCT-NEVT
C   Number of events to generate
C
C EVCT-NEVT  1000000
 EVCT-NEVT  1000
C--------------------------------------------------
```

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

1. Log in IOP cluster (2024)

```
$ ssh -XY vson@202.151.162.171
pass: son…
```

2. Create a new folder with your name

$ mkdir **your_name** (in the following steps, replace **tvngoc** by **your_name**)

3. Copy source folder to your working directory you have just created

```
$ cp -r -p neut_5.4.0/ tvngoc/
```

4. Go to your working directory

```
$ cd tvngoc/neut_5.4.0/
$ ls
include  lib  setup_env_neut540.sh  src
```

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

5. Check where you are

```
$ pwd
/home/vson/tvngoc/neut_5.4.0
```

6. Before working with neut, we need to setup environment. Use vim or gedit or other text editors to open and modify the file ***setup_env_neut540.sh*** Change the default directory to your directory, don’t forget to replace tvngoc by your_name

```
$ vi setup_env_neut540.sh
```

7. Press “i” to modify:

`#export NEUT_ROOT=/home/vson/neut_5.4.0`

`export NEUT_ROOT=/home/vson/tvngoc/neut_5.4.0` (same as output when you pwd)

Press “**esc**”, then “**:wq**”, press “**enter**” to save and close the file

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

8. Source the environment file

```
$ source setup_env_neut540.sh
```

9. Go to working directory

```
$ cd /src/neutsmpl
```

10. Clean and make to create binary files

```
$ /bin/csh Cleanneutsmpl.csh (or ./Cleanneutsmpl)
$ /bin/csh Makeneutsmpl.csh (or ./Makeneutsmpl)
```

11. Press “**ls**” then “**enter**” to see if there are “**neutroot2**” binary file created

12. Open card file to see what inside

```
$ vi Cards/neut_5.4.0_nd5_C_ccqe.card
```

Press “**i**” to enter modified mode, press “**esc**” and then “**:wq**” to save and exit.

The letter “**C**” at the beginning of the line means “comment out” that line.

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

13. Run neut with card file you have just edited. Remember to change the output filename from **output_numu_offaxis_tvngoc.root** to **your_name.root**

```
$ ./neutroot2 Cards/neut_5.4.0_nd5_C_ccqe.card output_numu_offaxis_tvngoc.root
```

Press “**ls**” to if there is a file “**output_numu_offaxis_tvngoc.root**” in the directory

14. Root macro file for analyzing the output. Edit this file if you want to do the exercises.

```
$ vi make_histos_standalone_neut540_ccqe_simple.cc
```

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

15. Run the file to create basic histograms. Remember to change **tvngoc** to **your_name**

```
$ root -b -q
‘make_histos_standalone_neut540_ccqe_simple.cc(“output_numu_offaxis_tvngoc.root","output_numu_offaxis_tvngoc_basichisto.root")'
```

16. Open created root file to check the histograms

```
$ root -l output_numu_offaxis_tvngoc_basichisto.root
root [0]
Attaching file output_numu_offaxis_tvngoc_basichisto.root as _file0...
root [1] .ls
root [2] NEUT_ppro ->Draw();
```

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Summary**

1. **Define input parameters in Card file**
    - **Off-axis muon neutrino mode**
        - Flux: nd5_tuned13av1.1_13anom_run1-7c_fine.root
        - Histogram: enu_nd5_tuned13a_numu
    - **Off-axis muon anti-neutrino mode**
        - Flux: nd5_tuned13av1.1_13anom_run5c-7b_antinumode_fine.root
        - Histogram: enu_nd5_tuned13a_numub
    - **On-axis muon neutrino mode**
        - Flux: nd34_tuned_11bv3.1_250ka.root
        - Histogram: ing3_tune_numu
    - **On-axis muon anti-neutrino mode**
        - Flux: run5c_tune_INGRID_13a_1_1.root
        - Histogram: ing3_tune_numub

- **Change neutrino flavor code accordingly**

4. Introduction to NEUT - an event generator

4.3: Practice with NEUT

- **Working space is IOP cluster**

○ **Summary**

2. Run NEUT to generate events

Name it yourself

```
./neutroot2 directory_to_cardfolder/cardfile.card output_neut.root
```

Same

3. Analyze the output

```
root -b -q
'make_histos_standalone_neut540_ccqe_simple.cc("output_neut.root","
name_your_root_file_basichisto.root")'
```

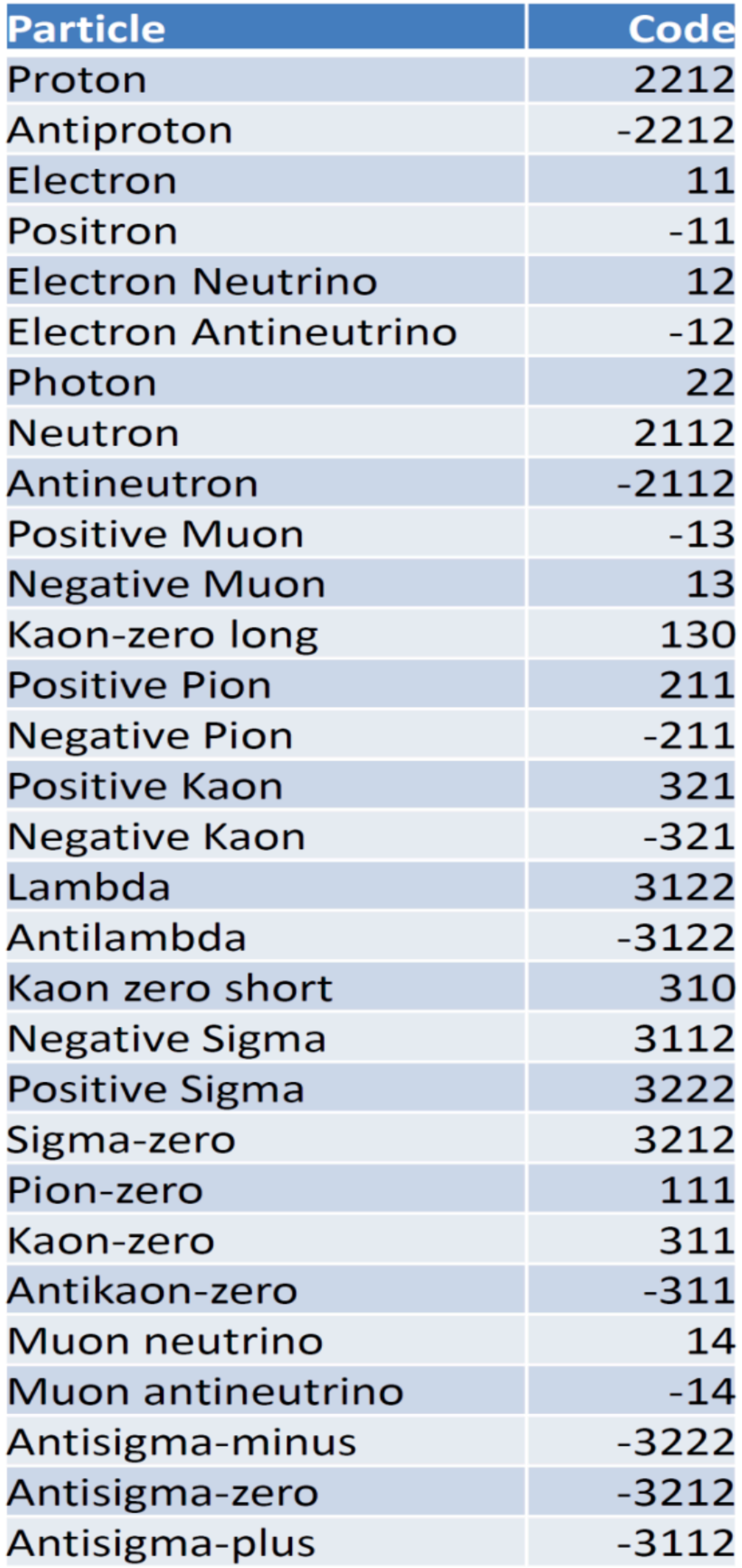
- The macro is at **neut_5.4.0/src/neutsmpl/**

4. Introduction to NEUT - an event generator

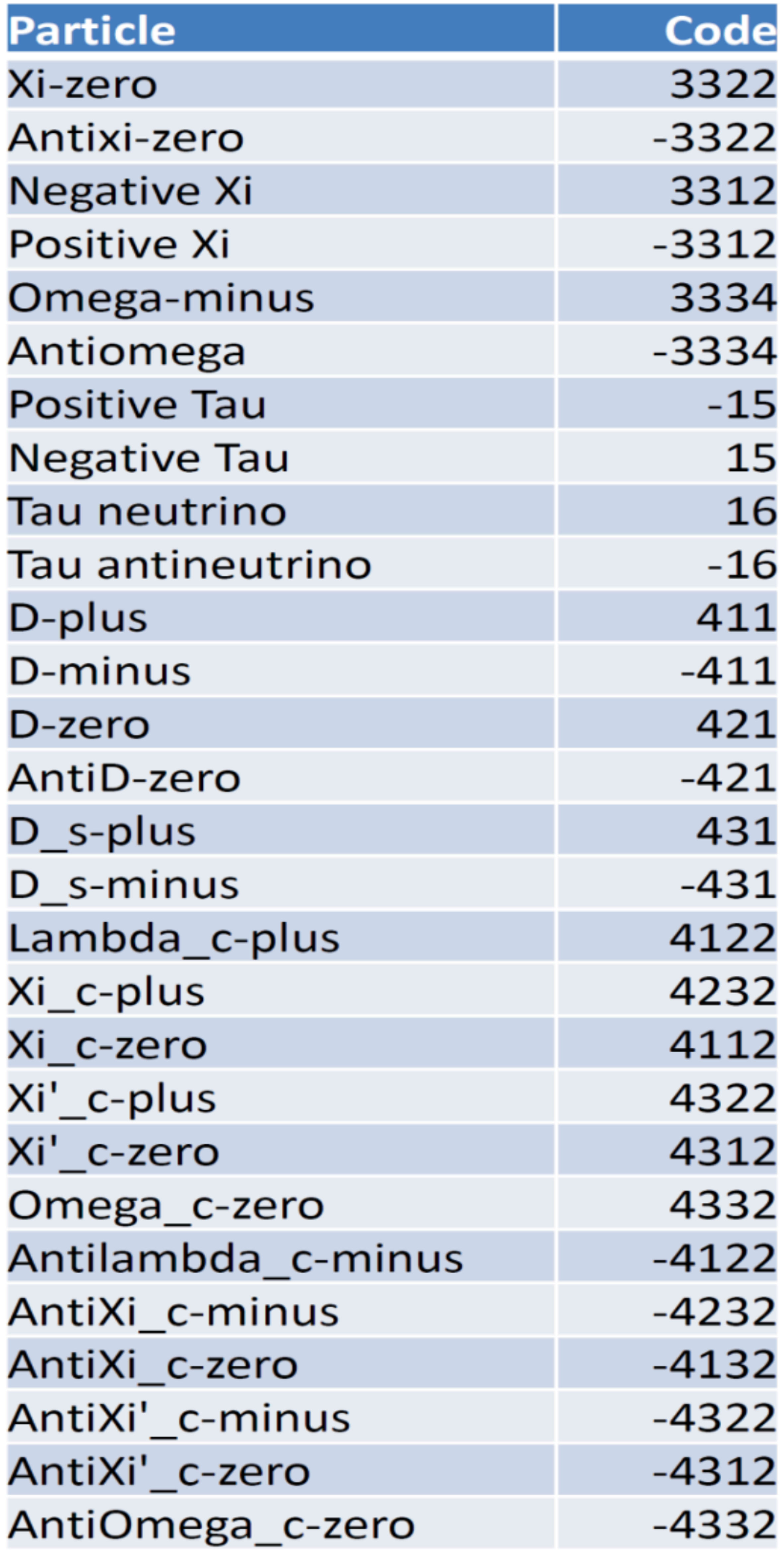
4.4: Exercises

- **Good information**
  - **Particle code (ID)**

```
if(nvect->PartInfo(i)->fPID == 13)
{muon_index=i; cout<<"This is a muon
with the index: "<<muon_index<<endl;}
```



Particle	Code
Proton	2212
Antiproton	-2212
Electron	11
Positron	-11
Electron Neutrino	12
Electron Antineutrino	-12
Photon	22
Neutron	2112
Antineutron	-2112
Positive Muon	-13
Negative Muon	13
Kaon-zero long	130
Positive Pion	211
Negative Pion	-211
Positive Kaon	321
Negative Kaon	-321
Lambda	3122
Antilambda	-3122
Kaon zero short	310
Negative Sigma	3112
Positive Sigma	3222
Sigma-zero	3212
Pion-zero	111
Kaon-zero	311
Antikaon-zero	-311
Muon neutrino	14
Muon antineutrino	-14
Antisigma-minus	-3222
Antisigma-zero	-3212
Antisigma-plus	-3112



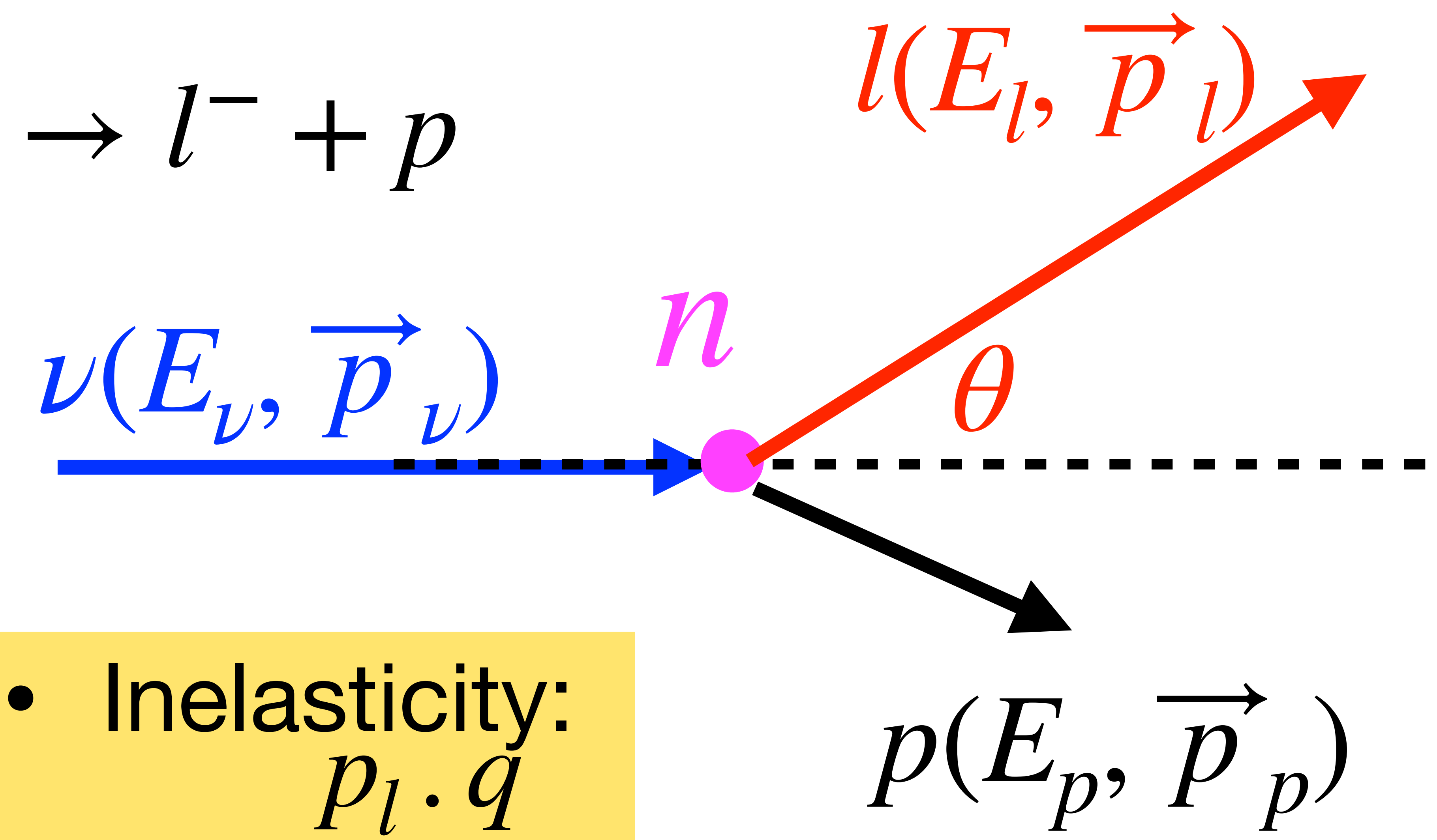
Particle	Code
Xi-zero	3322
Antixi-zero	-3322
Negative Xi	3312
Positive Xi	-3312
Omega-minus	3334
Antiomega	-3334
Positive Tau	-15
Negative Tau	15
Tau neutrino	16
Tau antineutrino	-16
D-plus	411
D-minus	-411
D-zero	421
AntiD-zero	-421
D_s-plus	431
D_s-minus	-431
Lambda_c-plus	4122
Xi_c-plus	4232
Xi_c-zero	4112
Xi'_c-plus	4322
Xi'_c-zero	4312
Omega_c-zero	4332
Antilambda_c-minus	-4122
AntiXi_c-minus	-4232
AntiXi_c-zero	-4132
AntiXi'_c-minus	-4322
AntiXi'_c-zero	-4312
AntiOmega_c-zero	-4332

4. Introduction to NEUT - an event generator

4.4: Exercises

- **Good information**
  - **Kinematic variables**

- CCQE: $\nu_l + n \rightarrow l^- + p$



- Mandelstam variables in lab frame (4-momentum):

$$s = (p_\nu + p_n)^2 = (p_l + p_p)^2 = (p_\nu + p_n)(p_l + p_p)$$
$$t = (p_\nu - p_l)^2 = (p_n - p_p)^2 = (p_\nu - p_l)(p_p - p_n)$$
$$u = (p_\nu - p_p)^2 = (p_l - p_n)^2 = (p_\nu - p_p)(p_l - p_n)$$
$$s + t + u = m_\nu^2 + m_n^2 + m_l^2 + m_p^2$$

- Inelasticity:

$$y = \frac{p_l \,.\, q}{p_l \,.\, p_\nu}$$

- Hadronic invariant mass:

$$W = \sqrt{E_X^2 - p_X^2}$$

- In detail:

$$s = m_N^2 + 2m_N E_\nu$$
$$t = q^2 = -Q^2 = m_l^2 - 2E_\nu(E_l - p_l \cos\theta)$$
$$u = m_N^2 - 2m_N E_\nu + 2E_\nu(E_l - p_l \cos\theta)$$

- 4-momentum transferred: **(Q2rec)**

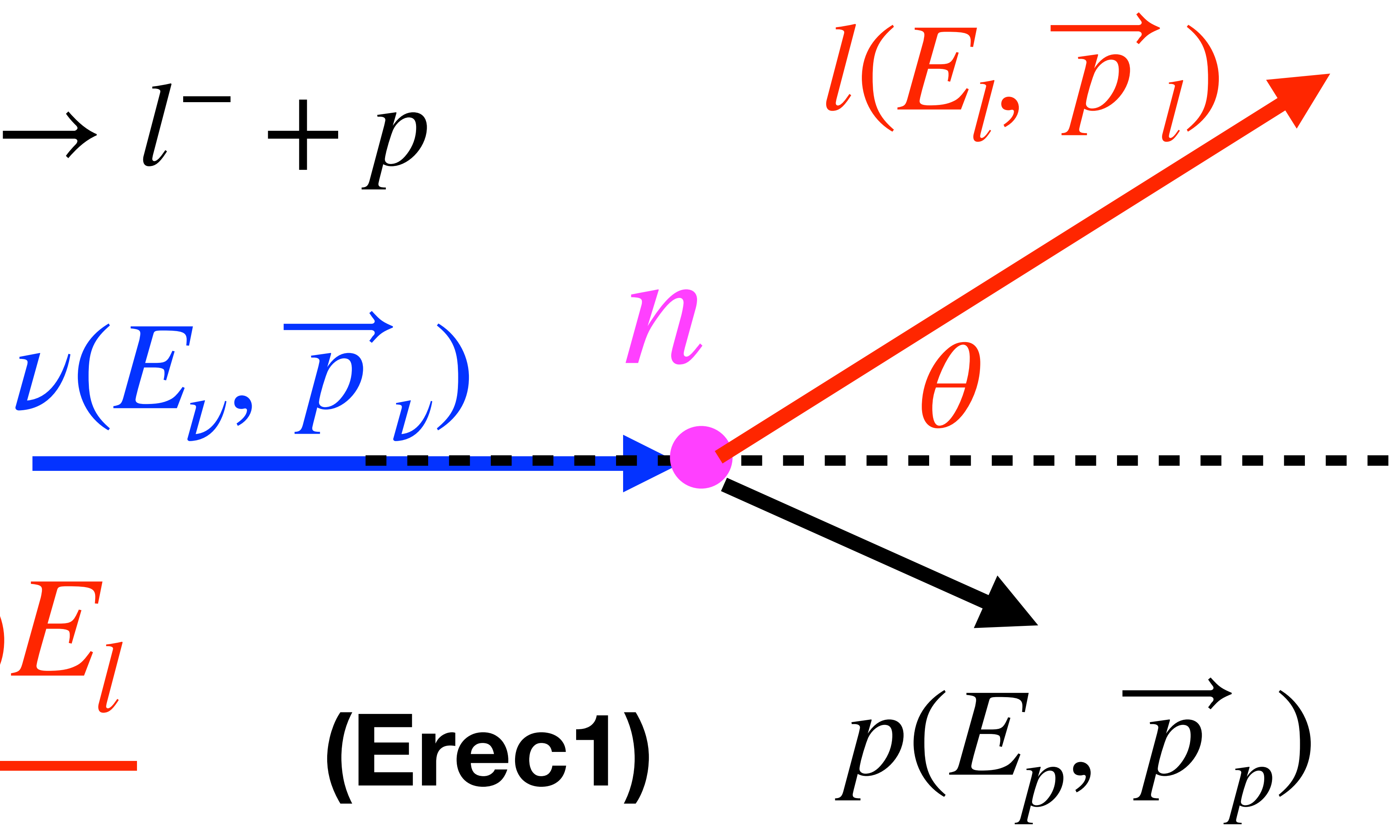
$$Q^2 = -q^2 = 2E_\nu(E_l - p_l \cos\theta) - m_l^2$$

4. Introduction to NEUT - an event generator

4.4: Exercises

- CCQE: $\nu_l + n \rightarrow l^- + p$

- **Good information**
  - **Neutrino reconstructed energy**



$$E_\nu = \frac{m_p^2 - m_l^2 - (m_n - E_b)^2 + 2(m_n - E_b)E_l}{2[(m_n - E_b) - E_l + p_l \cos\theta]}$$ **(Erec1)**

p_l, E_l, θ : out-going lepton momentum, energy, scattered angle

$E_b = 27\ MeV$: neutron binding energy in Oxygen

$m_p,\ m_n,\ m_l$: proton, neutron, lepton masses

- Remove E_b and consider $m_p \approx m_n$: $$E_\nu = \frac{2m_nE_l - m_l^2}{2(m_n - E_l + p_l\cos\theta)}$$ **(Erec2)**

4. Introduction to NEUT - an event generator

4.4: Exercises

○ The following histograms have already created

KEY: TH1D NEUT_mode;1 NEUT mode

KEY: TH1D NEUT_enu;1 Neutrino energy

KEY: TH1D NEUT_pmu;1 muon momentum

KEY: TH1D NEUT_anglemu;1 muon angle

KEY: TH1D NEUT_cosanglemu;1 cos muon angle

KEY: TH2D NEUT_mu_pvscosangle;1 muon momentum vs. cos(#theta)

KEY: TH1D NEUT_enurec;1 (equation (**Enurec2)** in slide 79)

KEY: TH1D flux_numu;1 #nu_{#mu} flux

KEY: TH1D evtrt_numu;1 #nu_{#mu} event rate

4. Introduction to NEUT - an event generator

4.4: Exercises

- **Run NEUT with different inputs from card file**
  - **Off-axis muon neutrino mode**
    - Flux: nd5_tuned13av1.1_13anom_run1-7c_fine.root
    - Histogram: enu_nd5_tuned13a_numu
  - **Off-axis muon anti-neutrino mode**
    - Flux: nd5_tuned13av1.1_13anom_run5c-7b_antinumode_fine.root
    - Histogram: enu_nd5_tuned13a_numub
  - **On-axis muon neutrino mode**
    - Flux: nd34_tuned_11bv3.1_250ka.root
    - Histogram: ing3_tune_numu
  - **On-axis muon anti-neutrino mode**
    - Flux: run5c_tune_INGRID_13a_1_1.root
    - Histogram: ing3_tune_numub

- **Remember to change the output filename accordingly**

4. Introduction to NEUT - an event generator

4.4: Exercises

○ **Produce the following histograms**

- Proton momentum
- Proton angle
- Cosine of proton angle

What is proton ID?

```
if(nvect->PartInfo(i)->fPID == 2212) {proton_index=i;
cout<<"This is a proton with the index:
"<<proton_index<<endl;}
```

- 4 momentum transfer (equation (**Q2rec)** in slide Kinematic variables)
- Neutrino reconstructed energy (equation (**Erec1)** in slide Nu_energy)
- Neutrino true energy (get from neut output), plot in the same canvas to compare with **Erec1** and **Erec2**

4. Introduction to NEUT - an event generator

4.4: Exercises

name

title

#bins xmin xmax

○ **Hints**

- **Define histogram**

TH1D ***NEUT_Q2** = new TH1D(“**NEUT_Q2**","**Q2**", 100, 0., 5.);

- **Define what to fill in your histogram** (4 momentum calculation)

double **Q2 = 2*e_nu*(e_mu-p_mu*cos_mu) - xmmu*xmmu**;

- **Fill histogram**

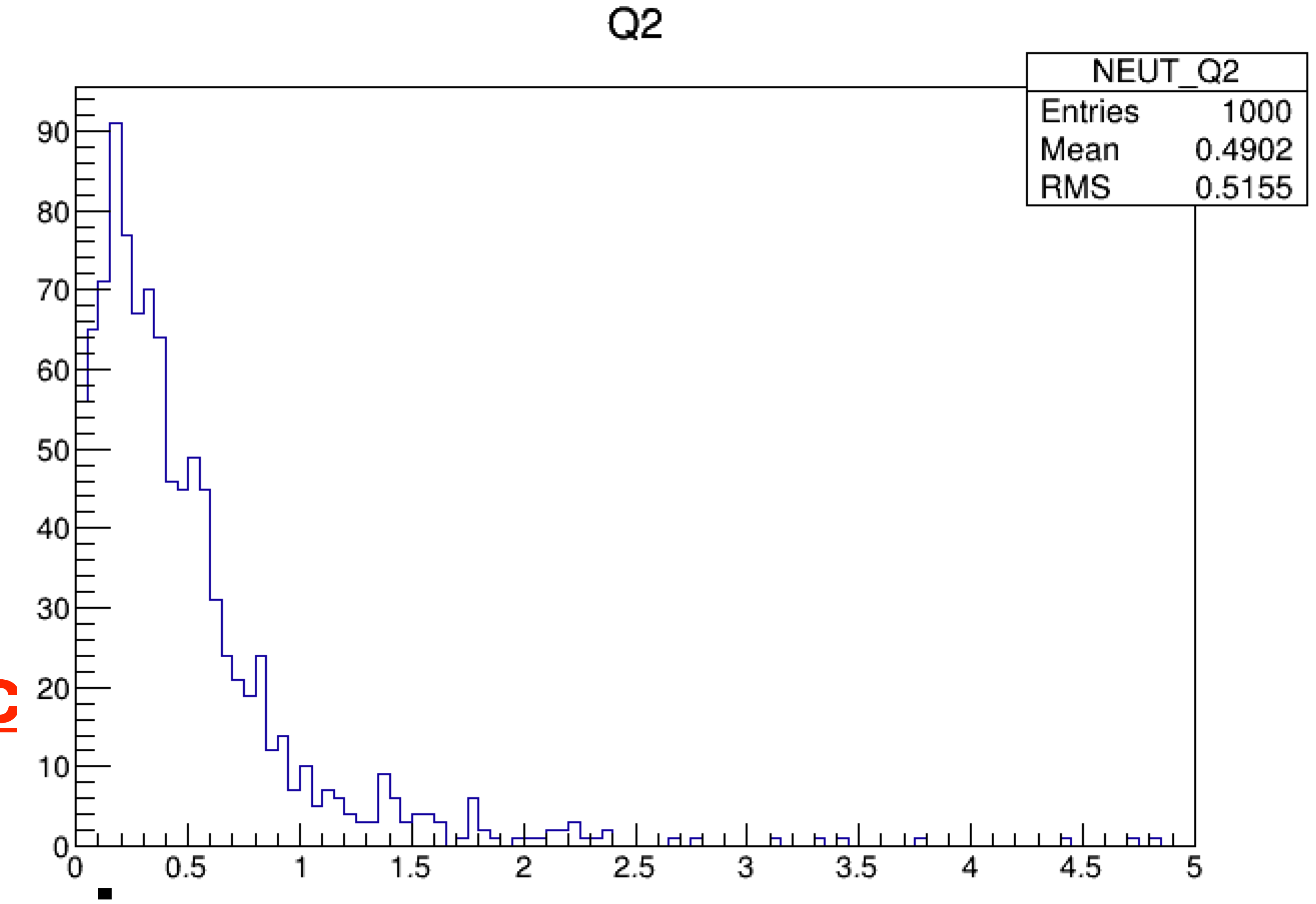
NEUT_Q2->Fill(Q2)

- **Write histogram into root file**

NEUT_Q2->Write();

Equation (Q2rec) in slide kinematic

Should be the same



xmin **xmax**

Backup

Abbreviation

- ANL: Argonne National Laboratory
- BNL: Brookhaven National Laboratory
- BEBC: Big European Bubble Chamber
- FNAL: Fermi National Accelerator Laboratory
- Adler’s PCAC theorem: partially conserved axial vector current

Natural unit

$$\hbar = c = 1.$$

$$E^2 = p^2c^2 + m^2c^4 \quad \text{becomes} \quad E^2 = p^2 + m^2.$$

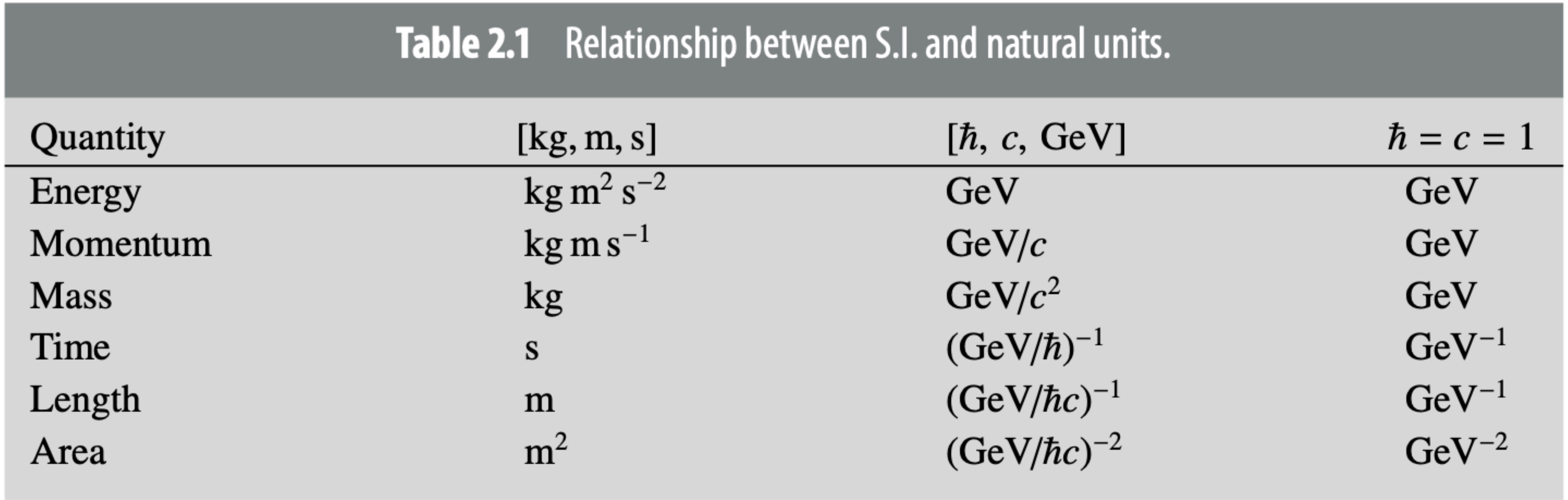


Table 2.1 Relationship between S.I. and natural units.

Quantity	[kg, m, s]	[$\hbar$, c, GeV]	$\hbar = c = 1$
Energy	kg m^2 s^{-2}	GeV	GeV
Momentum	kg m s^{-1}	GeV/c	GeV
Mass	kg	GeV/c^2	GeV
Time	s	(GeV/$\hbar$)$^{-1}$	GeV^{-1}
Length	m	(GeV/$\hbar c$)$^{-1}$	GeV^{-1}
Area	m^2	(GeV/$\hbar c$)$^{-2}$	GeV^{-2}

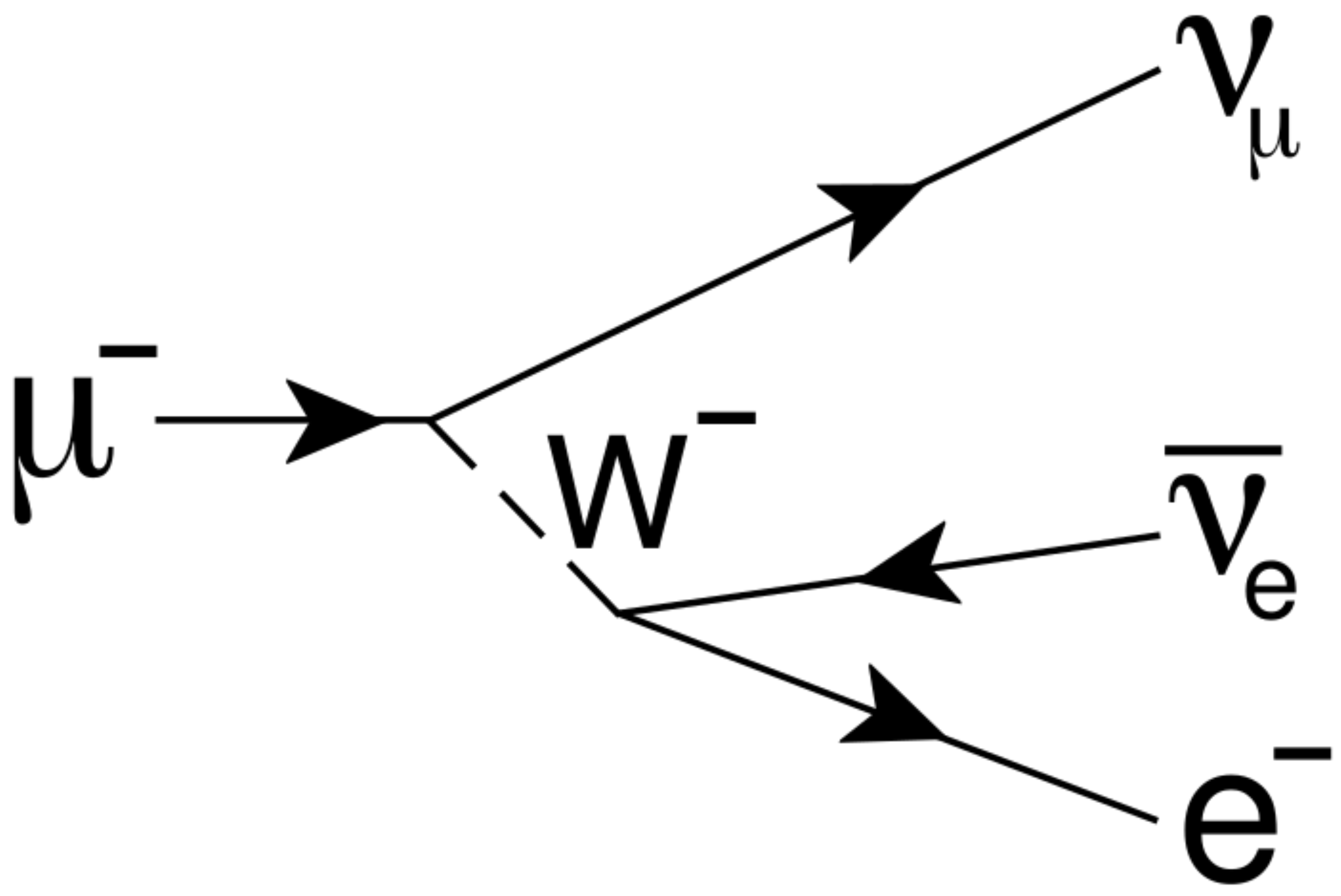
- To convert from natural unit to SI unit, the mass must be multiplied by c^2, the length must be multiplied by $\hbar c$, …

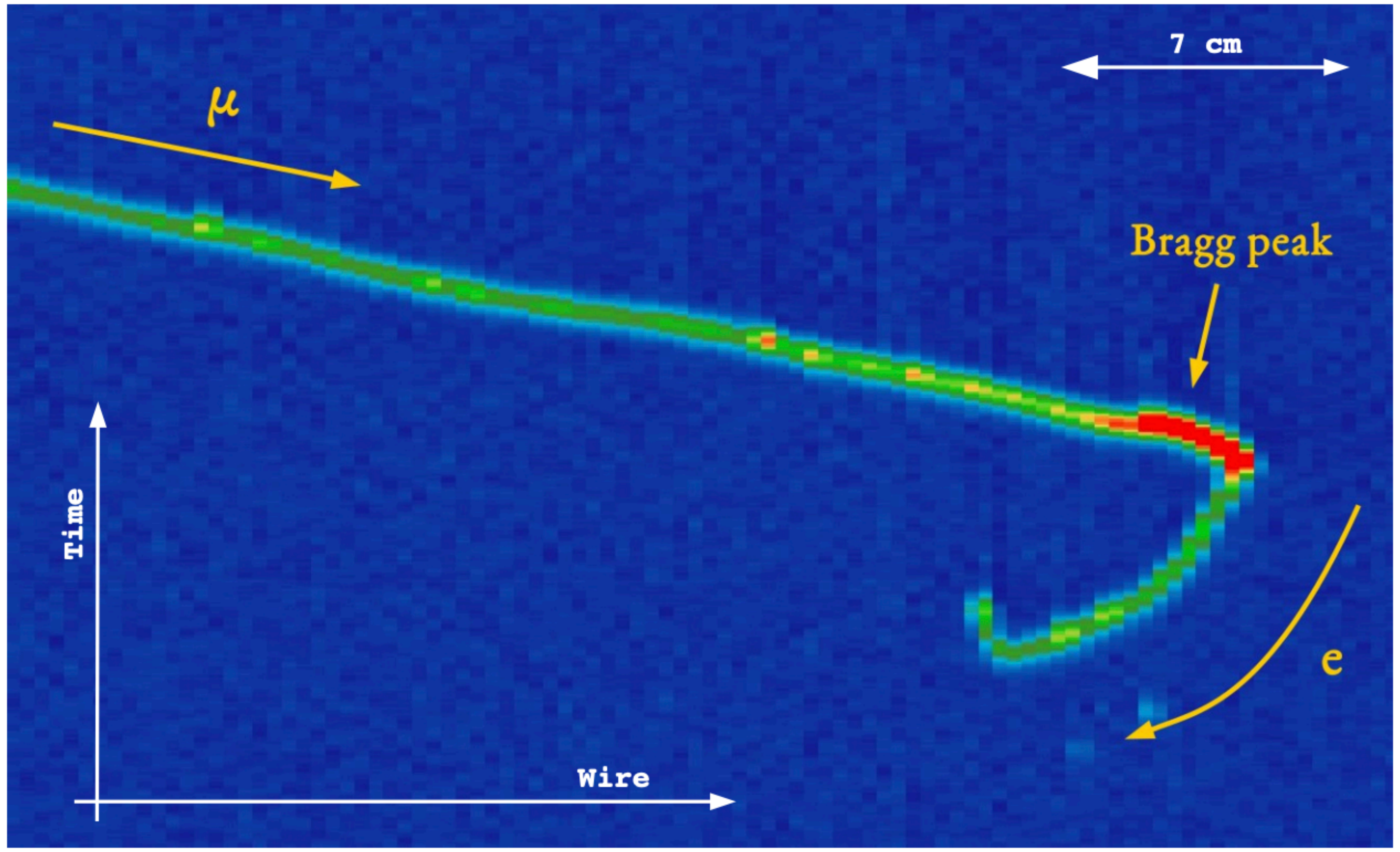
$$\hbar c = 0.197\,\text{GeV fm},$$

where one femtometre (fm) = 10^{-15} m.

Michel electron

Experiments soon revealed the manner of muon's decay, into an electron and a pair of neutrinos, and the shape of the electron spectrum was first calculated by Louis Michel in 1950. This is where "Michel electron" comes from. Michel introduced a single parameter ρ to describe the shape. However, after the non-conservation of parity in weak interactions proposal of Lee and Yang three more parameters (η, ξ, and δ) had to be added, and now all four are referred to collectively as "Michel parameters".





4. Introduction to NEUT - an event generator

4.2: Introduction to NEUT

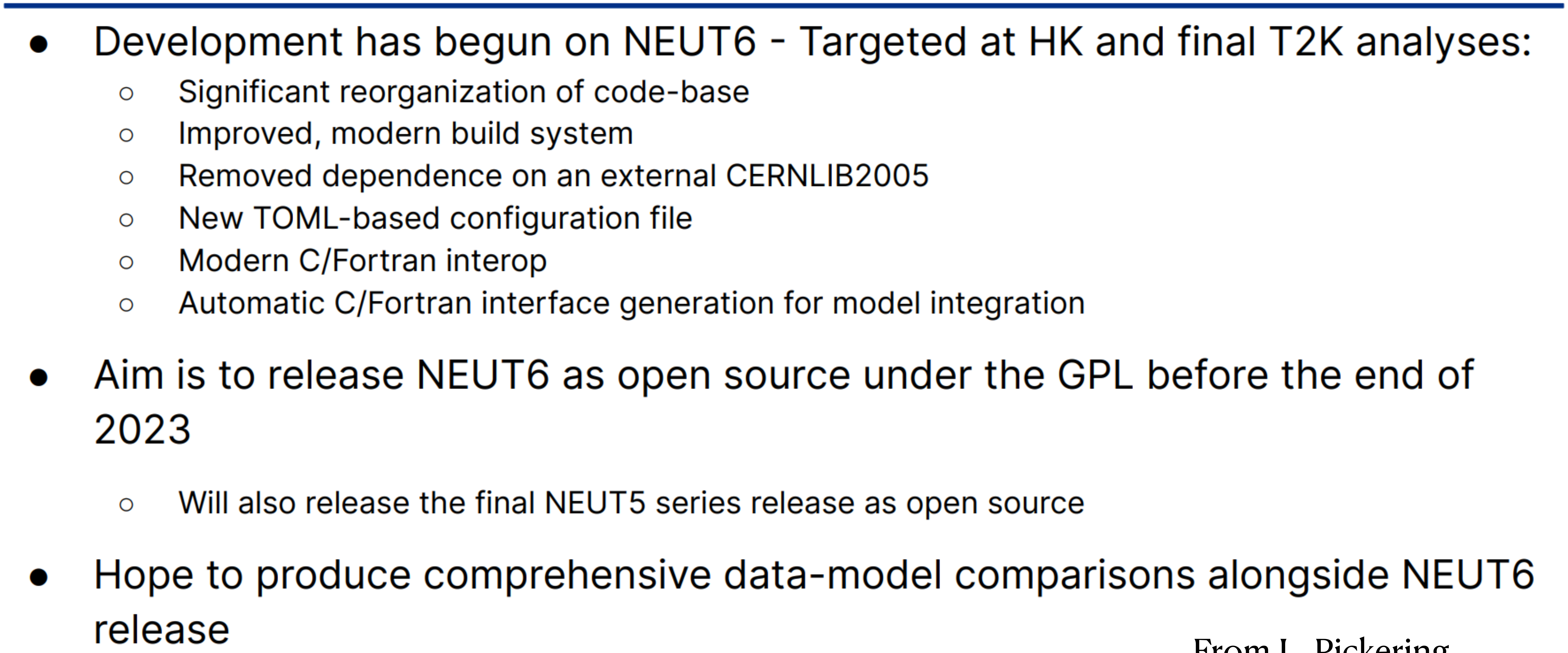
- **NEUT limitations:**
  - Nuclear models are inconsistent between models or steps in the factorisation:
    - Benhar et al. SF can be used for CCQE but no other modes
    - LFG used for FSI nuclear description
  - Benhar et al. SF Pauli blocking uses simple, RFG-like approach
  - Nuclear effects in single pion production are largely ignored
  - Nuclear transparency has no effect on inclusive cross-section

From L. Pickering

4. Introduction to NEUT - an event generator

4.2: Introduction to NEUT

- **Future plan:**

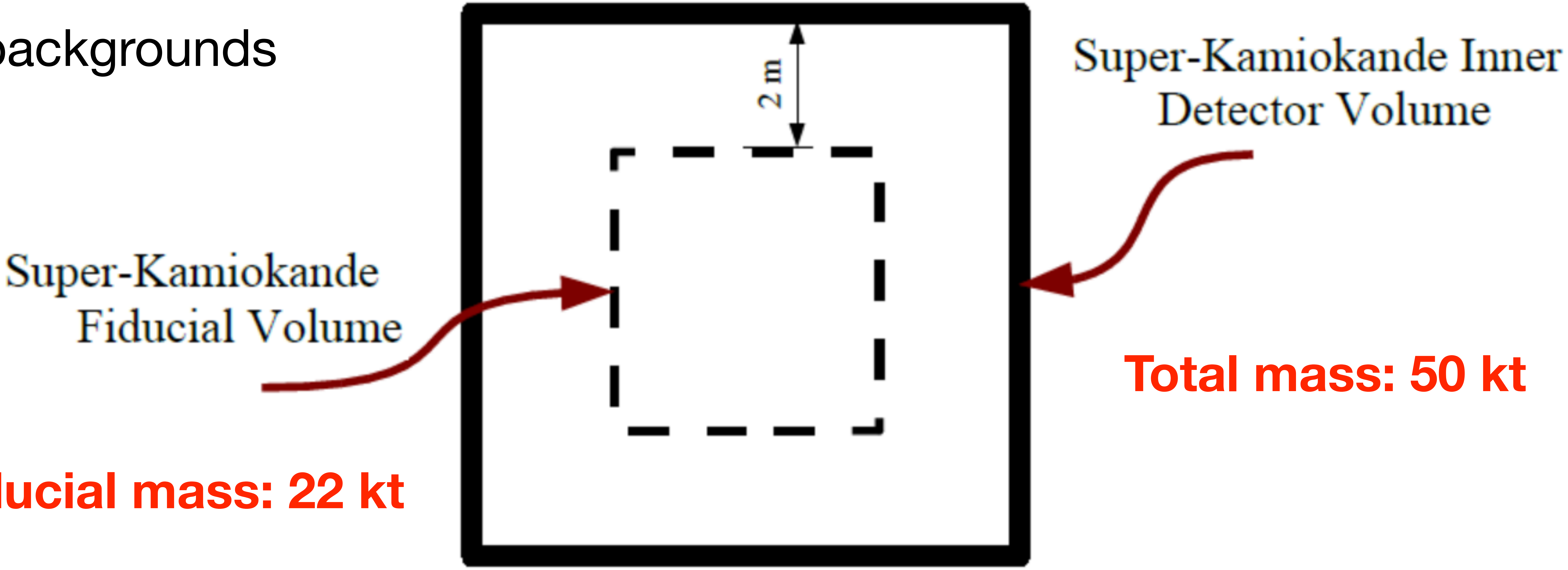


- Development has begun on NEUT6 - Targeted at HK and final T2K analyses:
  - Significant reorganization of code-base
  - Improved, modern build system
  - Removed dependence on an external CERNLIB2005
  - New TOML-based configuration file
  - Modern C/Fortran interop
  - Automatic C/Fortran interface generation for model integration
- Aim is to release NEUT6 as open source under the GPL before the end of 2023
  - Will also release the final NEUT5 series release as open source
- Hope to produce comprehensive data-model comparisons alongside NEUT6 release

From L. Pickering

Fiducial volume

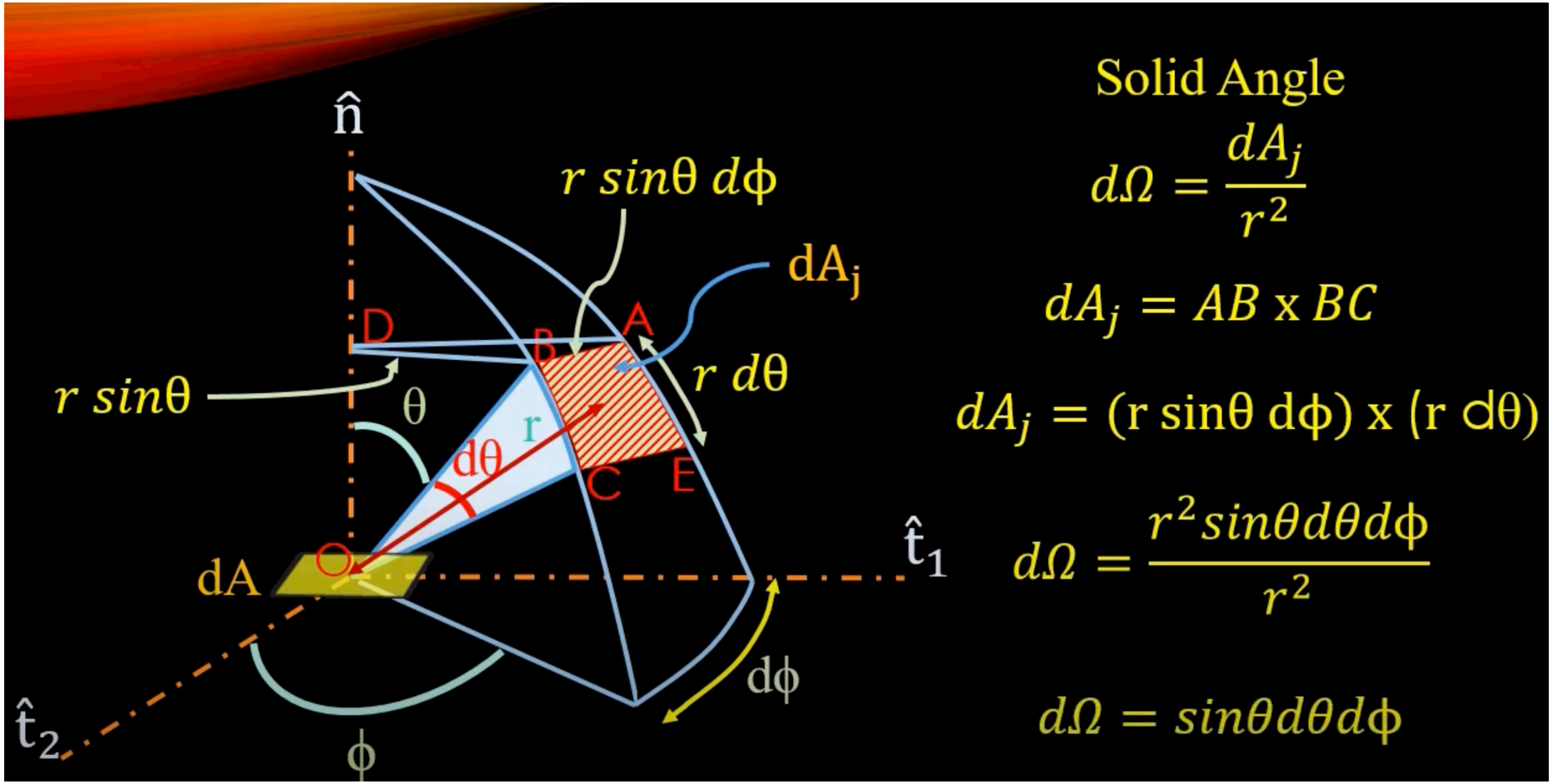
- An inner volume of particle detector media in which background events are largely excluded
- The well understood region of the detector where the events are accepted for physics analysis
- Reduce the backgrounds



Total mass: 50 kt

Fiducial mass: 22 kt

Solid angle



Solid Angle

$$d\Omega = \frac{dA_j}{r^2}$$

$$dA_j = AB \text{ x } BC$$

$$dA_j = (\mathrm{r\ sin\theta\ d\phi}) \text{ x } (\mathrm{r\ d\theta})$$

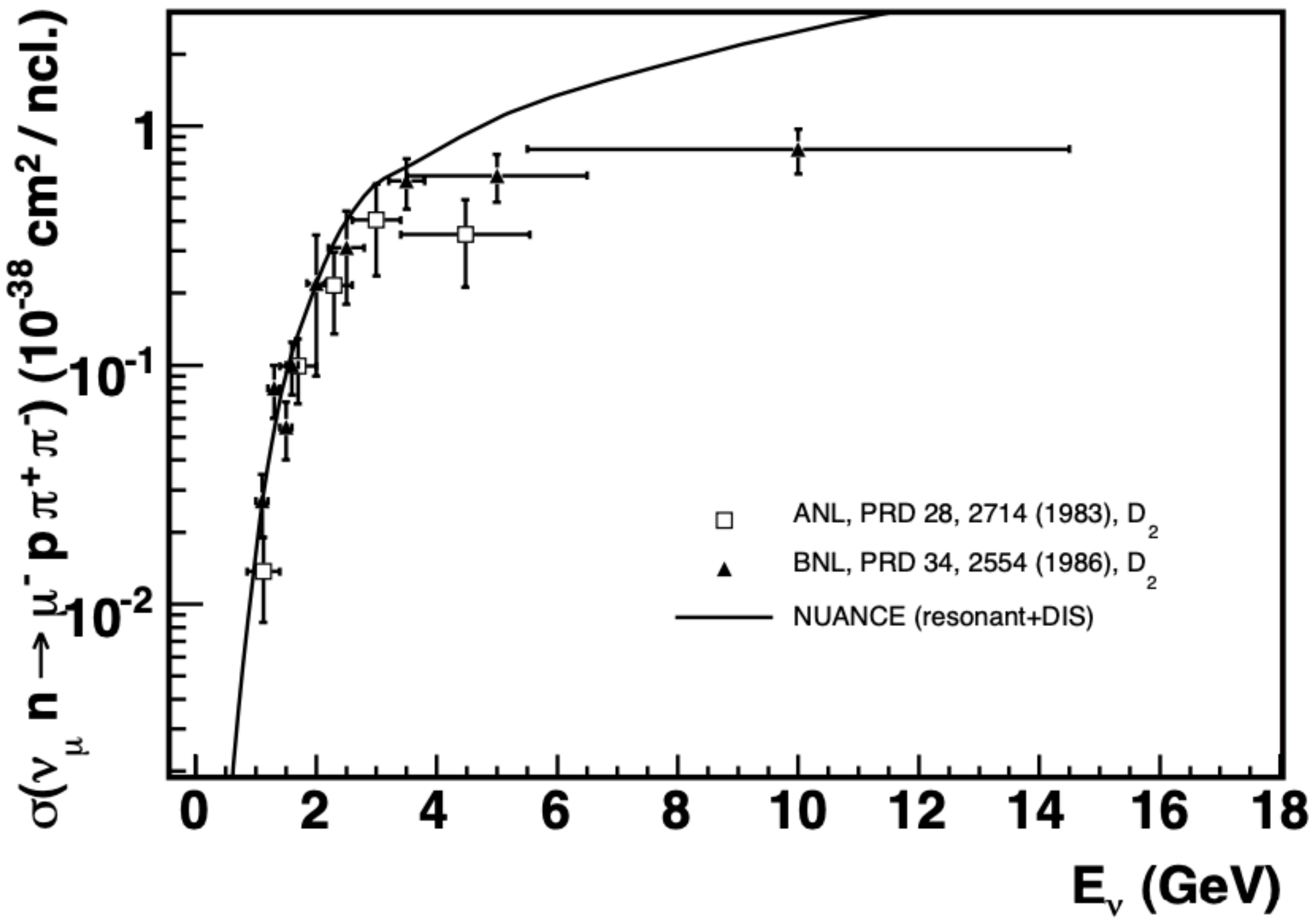
$$d\Omega = \frac{r^2 sin\theta d\theta d\phi}{r^2}$$

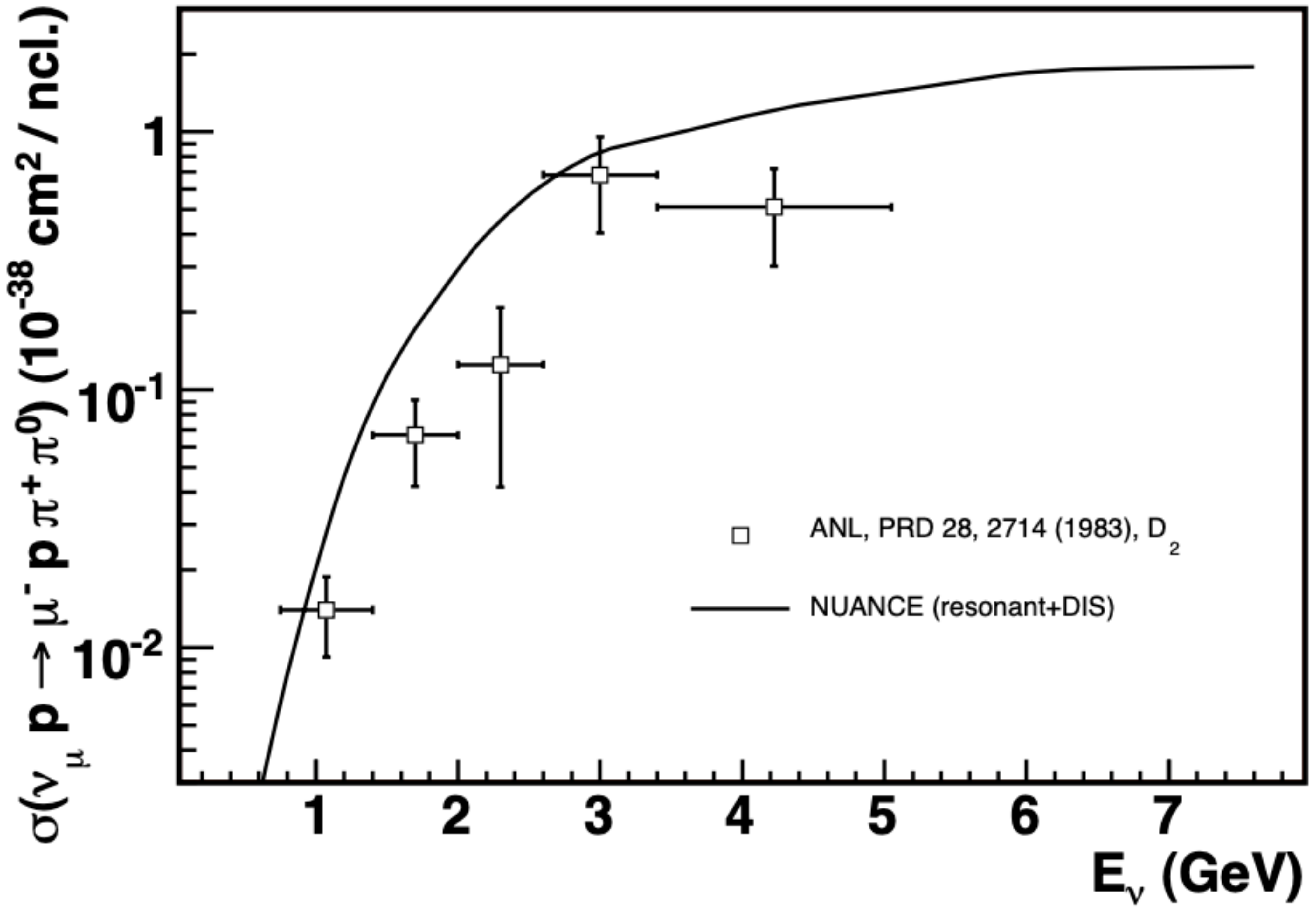
$$d\Omega = sin\theta d\theta d\phi$$

3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- **Multi-pion production**





3.2 Neutrino - nucleon interactions

○ **Neutrino interactions at 500 GeV - 1 EeV ($10^{18}\ eV$) energy:**

- **Ultra-high energy neutrinos**

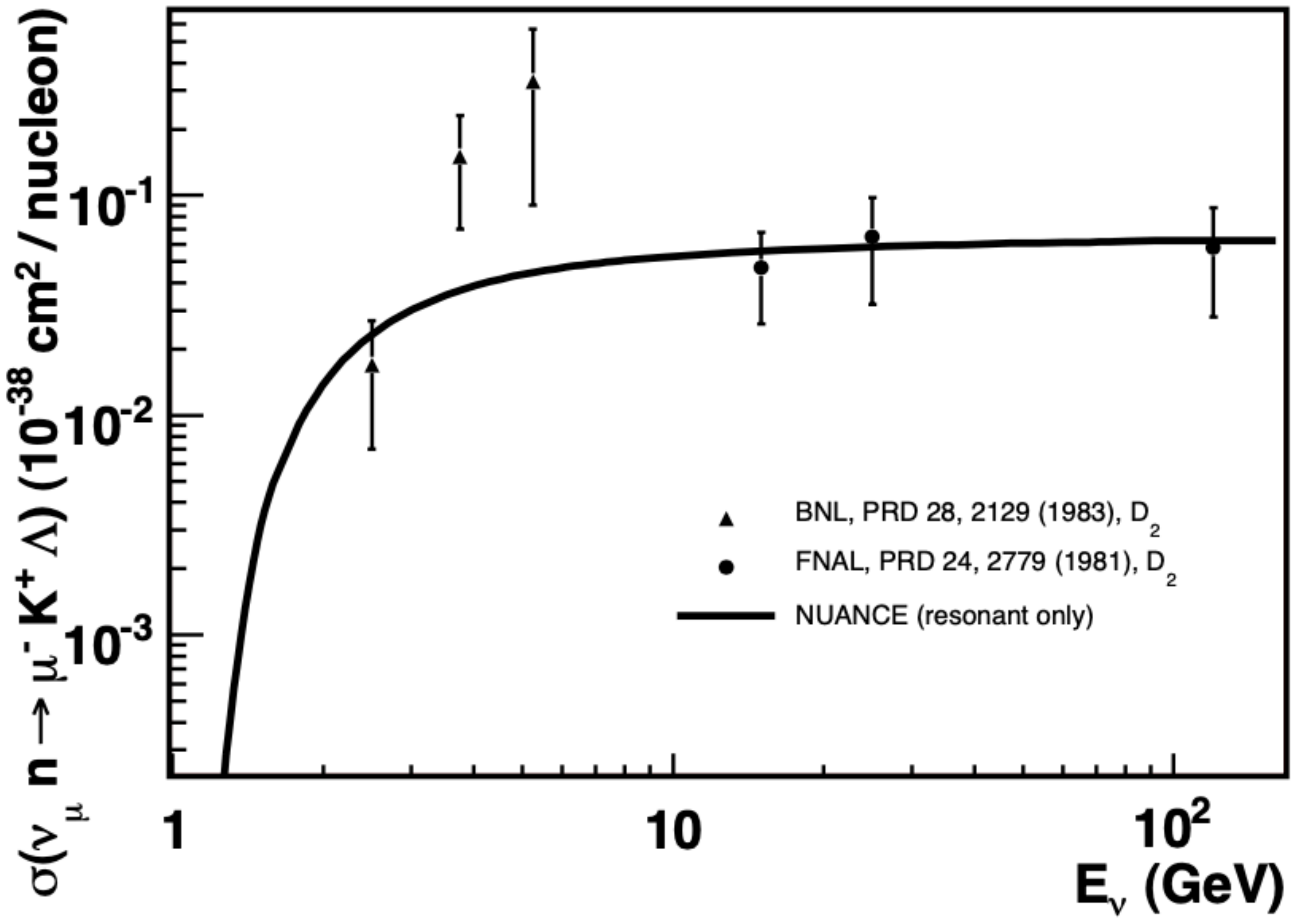
3.2 Neutrino - nucleon interactions

○ Neutrino interactions at 0.1-20GeV energy:

- **Kaon production**

- Potential background of proton decay:
  $p \rightarrow K^+\nu$

CC :	NC :
$\nu_\mu n \rightarrow \mu^- K^+ \Lambda^0$	$\nu_\mu p \rightarrow \nu_\mu K^+ \Lambda^0$
$\nu_\mu p \rightarrow \mu^- K^+ p$	$\nu_\mu n \rightarrow \nu_\mu K^0 \Lambda^0$
$\nu_\mu n \rightarrow \mu^- K^0 p$	$\nu_\mu p \rightarrow \nu_\mu K^+ \Sigma^0$
$\nu_\mu n \rightarrow \mu^- K^+ n$	$\nu_\mu p \rightarrow \nu_\mu K^0 \Sigma^+$
$\nu_\mu p \rightarrow \mu^- K^+ \Sigma^+$	$\nu_\mu n \rightarrow \nu_\mu K^0 \Sigma^0$
$\nu_\mu n \rightarrow \mu^- K^+ \Sigma^0$	$\nu_\mu n \rightarrow \nu_\mu K^+ \Sigma^-$
$\nu_\mu n \rightarrow \mu^- K^0 \Sigma^+$	$\nu_\mu n \rightarrow \nu_\mu K^- \Sigma^+$



Fermi’s golden rule

- Transition rate: $\Gamma_{fi} = 2\pi \,|\,T_{fi}\,|^2\, \rho(E_i) = 2\pi \displaystyle\int |\,T_{fi}\,|^2\, \delta(E_i - E)dn$
- T_{fi}: transition matrix element
- $\rho(E_i) = \left|\dfrac{dn}{dE}\right|_{E_i} = \displaystyle\int \dfrac{dn}{dE}\delta(E_i - E)dE$: density of states
- $dn = (2\pi)^3 \dfrac{d^3\overrightarrow{p_1}}{(2\pi)^3}\dfrac{d^3\overrightarrow{p_2}}{(2\pi)^3}\delta^3\left(\overrightarrow{p_a} + \overrightarrow{p_b} - \overrightarrow{p_1} - \overrightarrow{p_2}\right)$: number of accessible states in the energy range $E \rightarrow E + dE$
- Lorentz invariant matrix element: $M_{fi} = \sqrt{2E_a.2E_b.2E_1.2E_2}\,T_{fi}$
- $\Gamma_{fi} = \dfrac{(2\pi)^4}{4E_aE_b}\displaystyle\int |\,M_{fi}\,|^2\, \delta(E_a + E_b - E_1 - E_2)\delta^3(\overrightarrow{p_a} + \overrightarrow{p_b} - \overrightarrow{p_1} - \overrightarrow{p_2})\dfrac{d^3\overrightarrow{p_1}}{(2\pi)^3 2E_1}\dfrac{d^3\overrightarrow{p_2}}{(2\pi)^3 2E_2}$

3.1 Neutrino - electron interactions

- Neutrino - electron quasi-elastic scattering

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \qquad E_{\nu_\mu} > 10.92\ GeV$$

Did you calculate this value as Sanjib's request?

- Calculate threshold:

$$\nu + A \rightarrow \sum_X X$$

- A at rest, neglecting neutrino mass, in center of mass frame, $s = 2E_\nu m_A + m_A^2$ must greater than $\left(\sum_X m_X\right)^2$

$$E_\nu^{th} = \frac{\left(\sum_X m_X\right)^2}{2m_A} - \frac{m_A}{2}$$

$$m_e = 0.511\ MeV$$

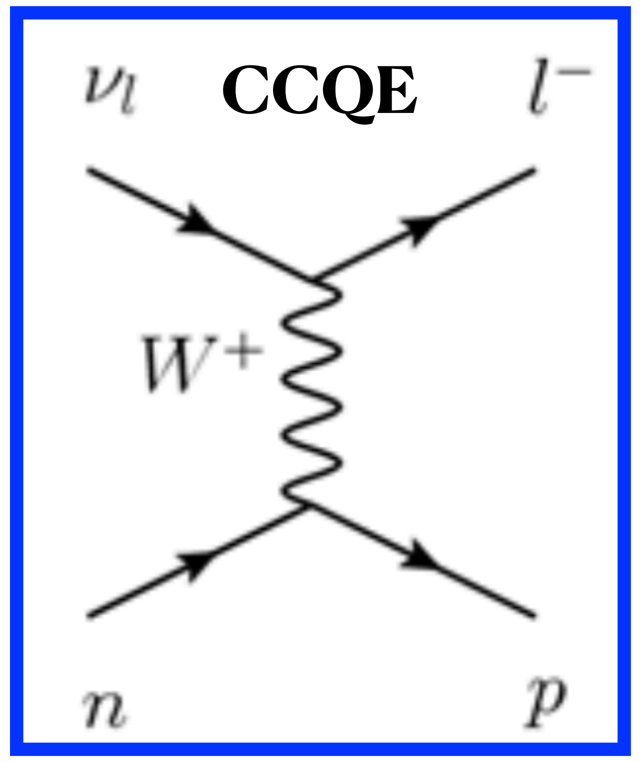
$$m_\mu = 105.66\ MeV$$

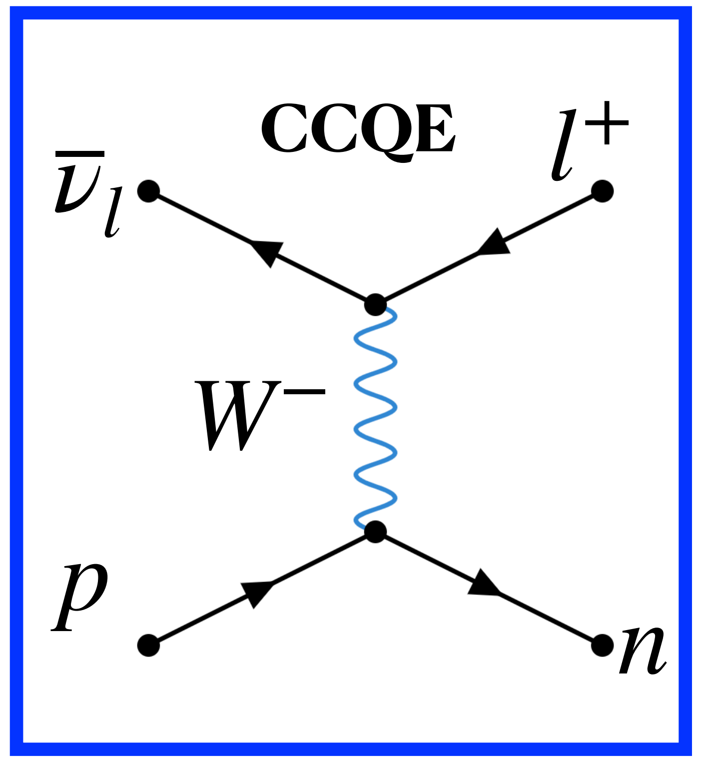
3. Neutrino energies and interactions

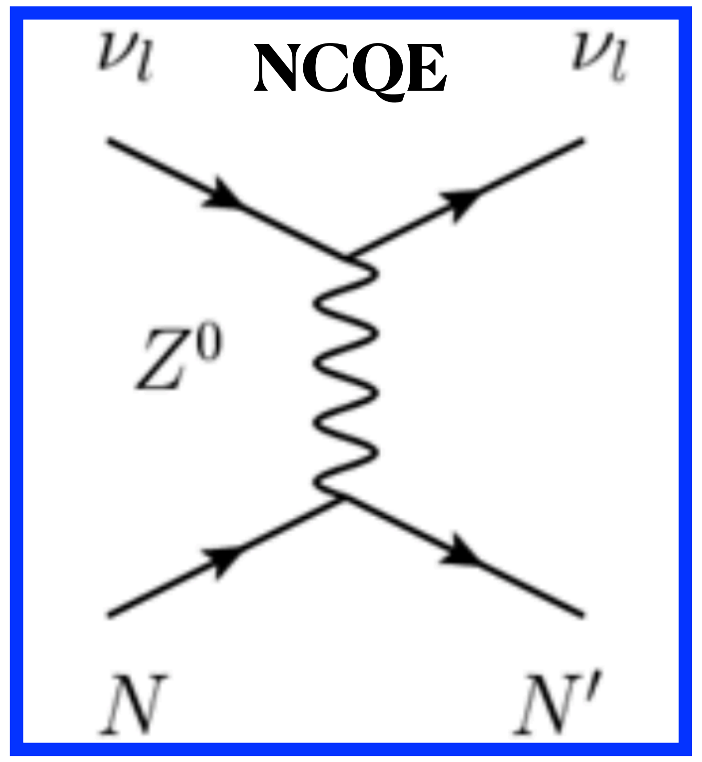
○ Neutrino interactions at 0.1-20GeV energy:

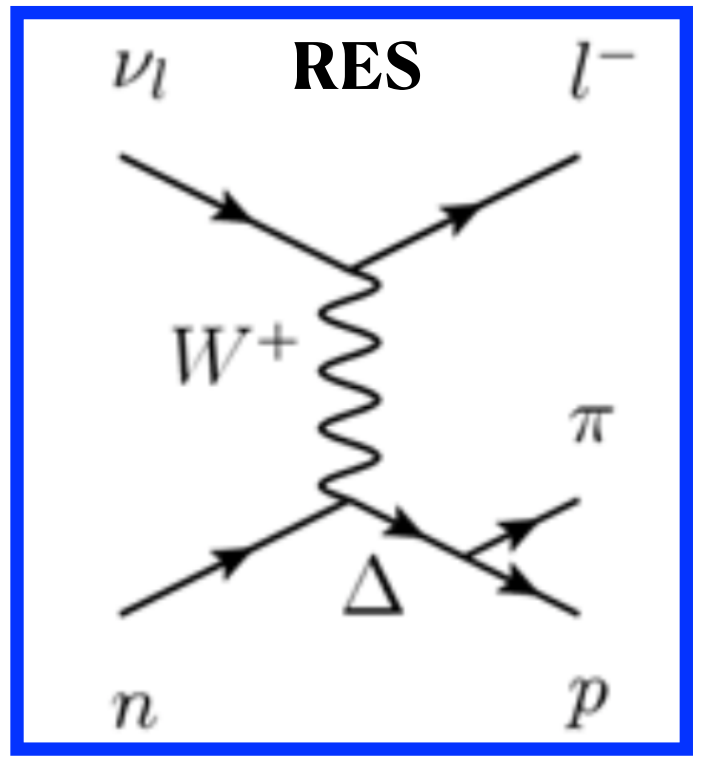
- **Quasi-elastic scattering (QE):**
  - CCQE: $\nu_l + n \rightarrow l^- + p$
  - NC: $\nu_l + N \rightarrow \nu_l + N'$
- **Resonant meson production (RES):**
  - CC RES: $\nu_l + N \rightarrow l^- + N' + m$
  - NC RES: $\nu_l + N \rightarrow \nu_l + N' + m$
- **Coherent pion production (COH):**
  - CC COH: $\nu_l + A \rightarrow l^- + A + \pi^+$
  - NC COH: $\nu_l + A \rightarrow \nu_l + A + \pi^0$
- **Deep inelastic scattering (DIS):**
  - CC DIS: $\nu_l + N \rightarrow l^- + N' +$ hadrons
  - NC DIS: $\nu_l + N \rightarrow \nu_l + N' +$ hadrons

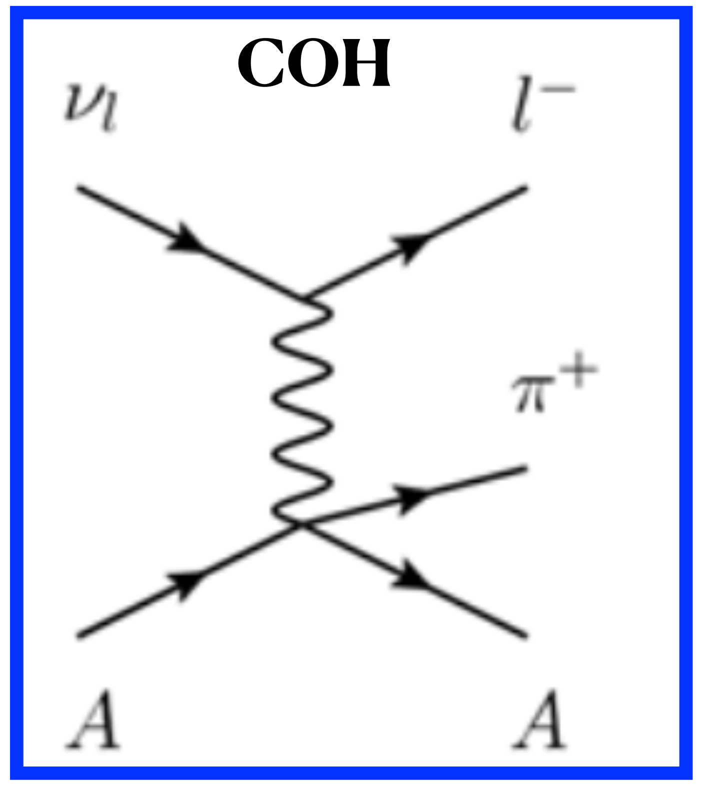
Where $l^- = \{e^-, \mu^-, \tau^-\};\ N = \{p, n\};$
$m = \{\pi, \eta, K\};\ A$: nucleus

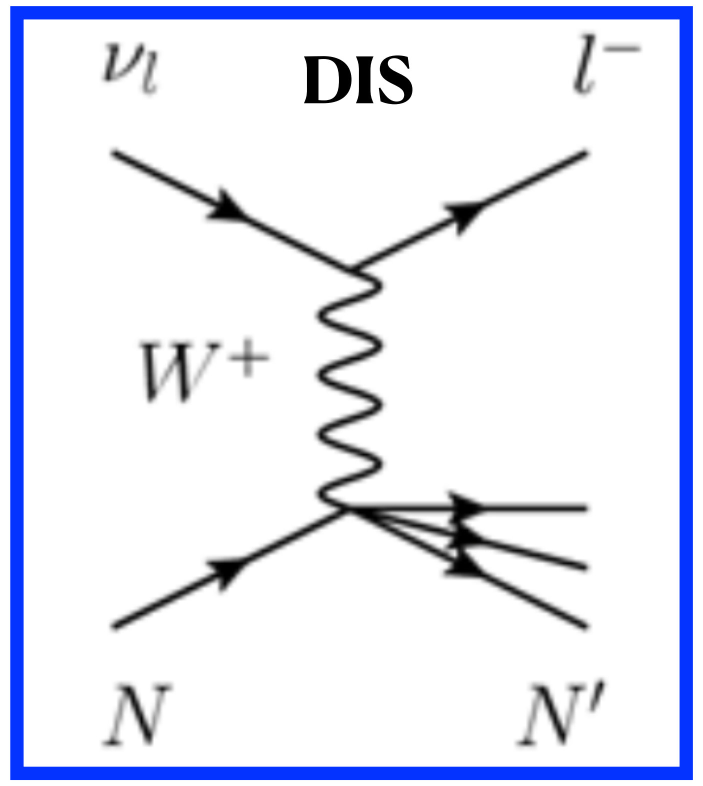












- There are similar channels for anti-neutrino
- For RES at low E, $CC1\pi$ & $NC1\pi$ dominates
- We will focus on neutrino interactions at this energy range

Neutrino energies and interactions