Basics of Particle Detection (2/2)

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JUL. 18TH 2024

Some textbooks and references

- Reference
  - Mark Thomson, “Modern Particle Physics”
  - Konrad Kleinknecht, “Detectors for particle radiation”
  - PDG https://pdg.lbl.gov/
    - “Passage of Particles Through Matter”
  - …

Outlines

- General concept of particle detection
- Passage of particles through matter
- Detector functionalities
  - Particle identification
  - Calorimeter
  - Tracking
- Application of particle detection

Aim of this short course is to provide foundations/background for other relatively advanced lectures in particle detectors and for hardware training.

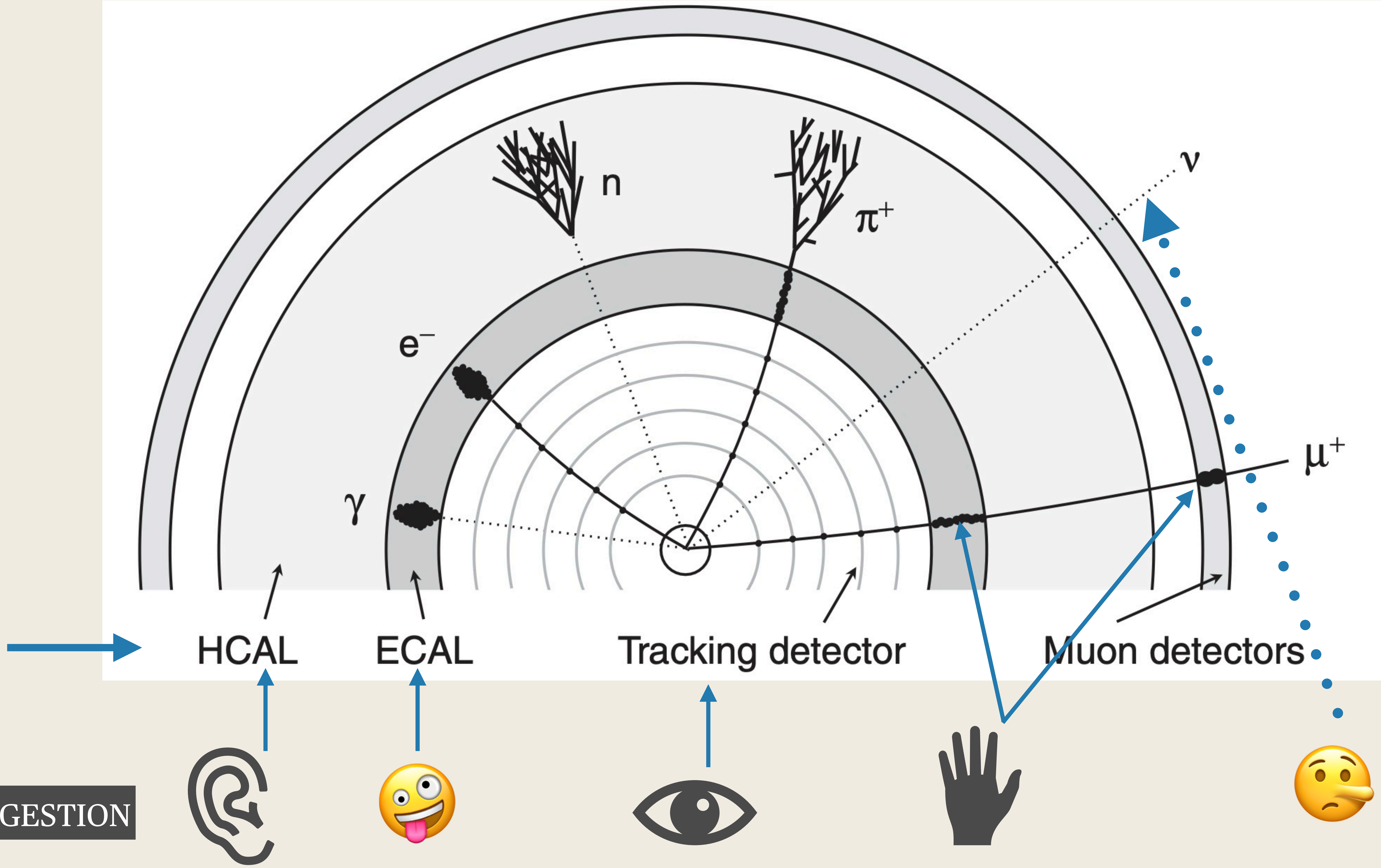
Did you try to ask AI and find any things useful?

- **Can AI suggest neutrino-related questions that have never been asked by human?**
- **Can AI suggest experiments to address the above-posed questions?**

(By the way, AI will be more useful if you have extensive relevant expertise. Otherwise, it is difficult to figure out whether what AI says is good or bad from a scientific perspective.)

Example of particle signatures in the detector

Ignore these technical terms



SOME SUGGESTION

Make things more funny to study

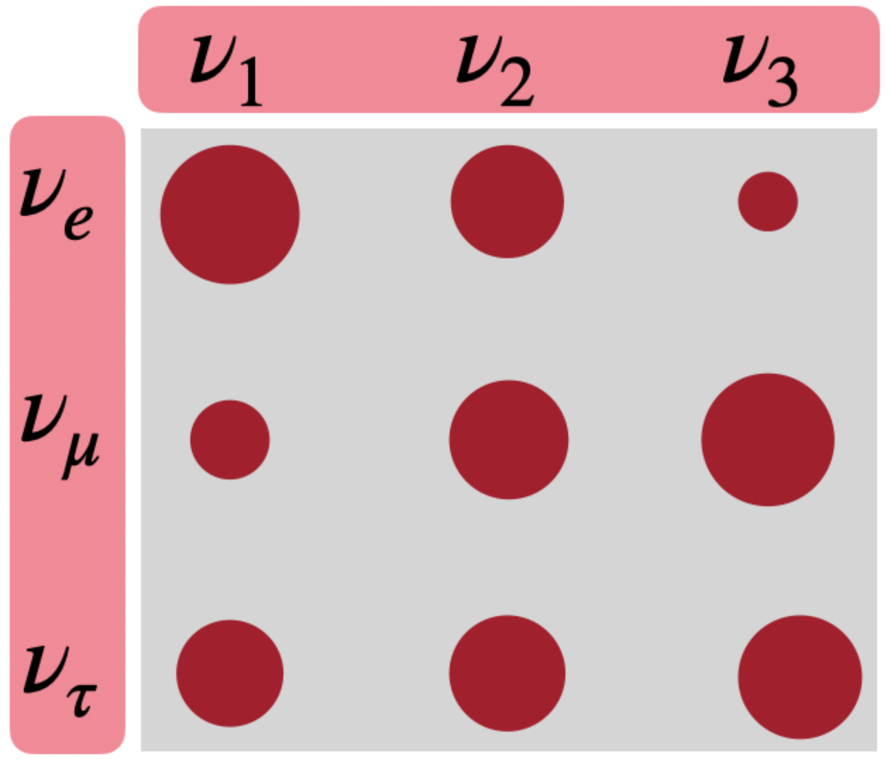
VIETNAMESE FOOD: WE LOVE TO MIX FLAVORS

LIKE LEPTONIC MIXING

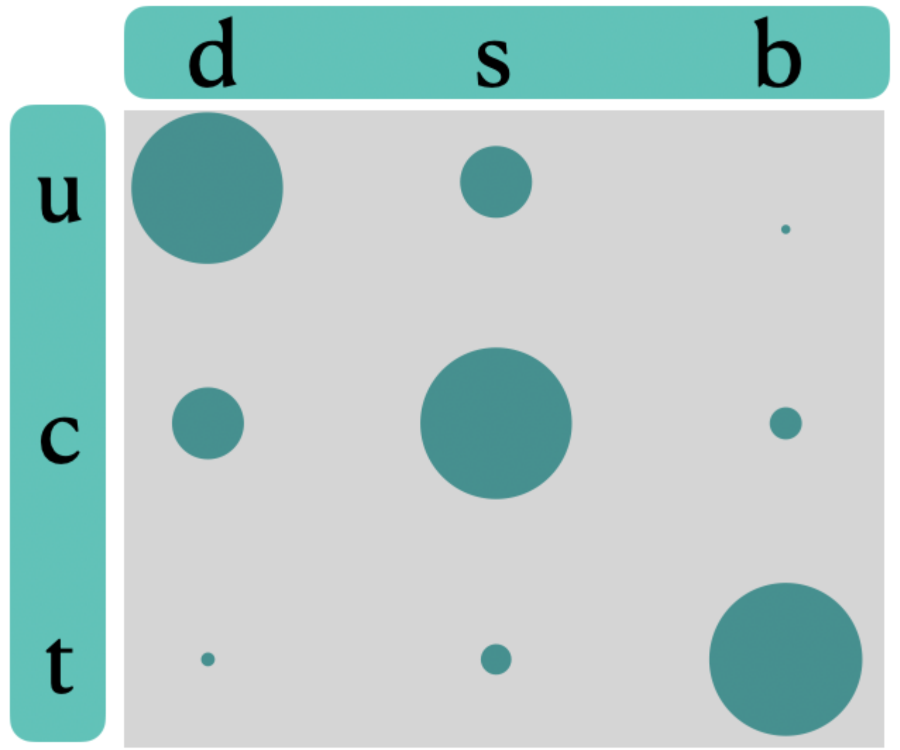
JAPANESE FOOD: FLAVORS ARE MIXED BUT NOT SO STRONG

LIKE QUARK MIXING

How about other countries?



Leptonic PMNS mixing matrix



Quark CKM mixing matrix

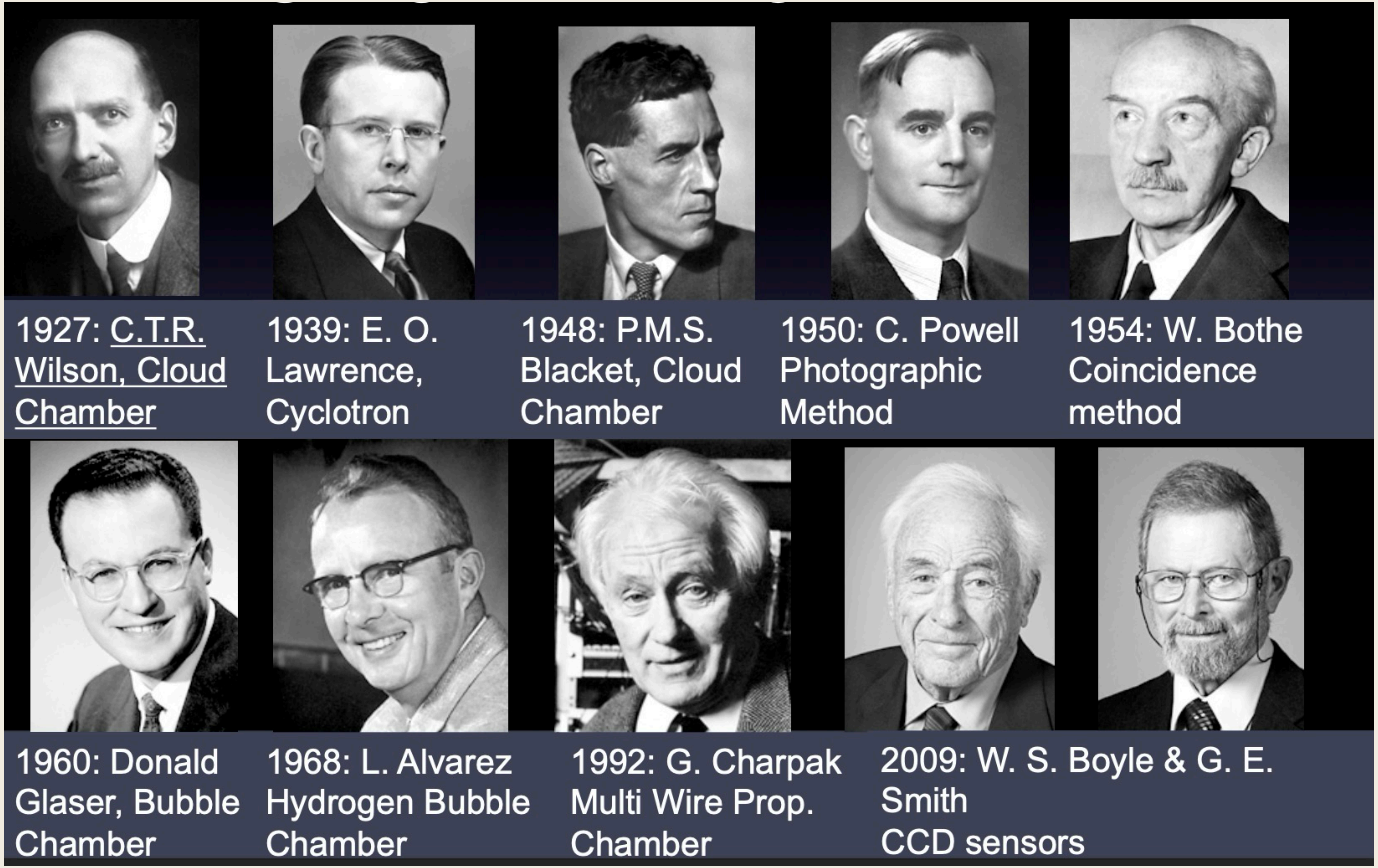
Important point to keep in mind

- We don’t see particle interaction directly, but its end products
- End products are detected via their interaction with active volume in the particle detector
- Any observable effect of particle when interacting with matter can be used as working principle of particle detector

“The real voyage of discovery consists, not in seeking new landscapes, but in having new eyes”

~Marcel Proust~

Nobel prize for instrumentation



1927: C.T.R. Wilson, Cloud Chamber

1939: E. O. Lawrence, Cyclotron

1948: P.M.S. Blacket, Cloud Chamber

1950: C. Powell Photographic Method

1954: W. Bothe Coincidence method

1960: Donald Glaser, Bubble Chamber

1968: L. Alvarez Hydrogen Bubble Chamber

1992: G. Charpak Multi Wire Prop. Chamber

2009: W. S. Boyle & G. E. Smith
CCD sensors

Categorize by detector functionality

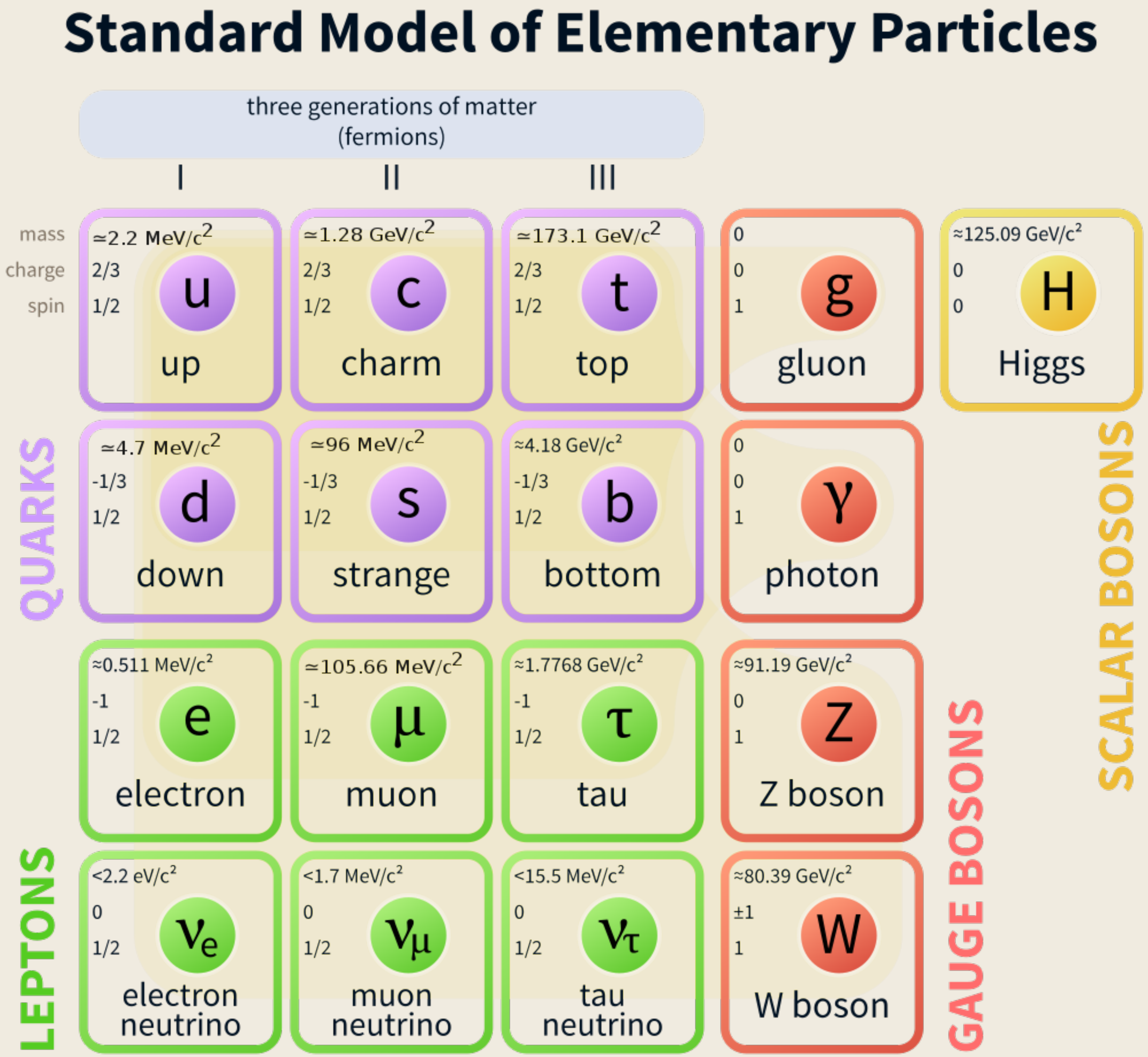
- **Particle identification**
  - Time-of-flight measurement
  - Ionization dE/dx
  - Cherenkov radiation
  - Transition radiation
  - …
- **Measurement of momentum**
  - Energy-loss range of particle
  - Curvature in magnetic field
  - Cherenkov radiation
  - …
- **Measurement of energy**
  - EM calorimeter
  - Hadron calorimeters
  - ….
- **Position or tracking particle trajectory**
  - Multiwire proportional chamber
  - Drift chamber
  - Time projection chamber..
  - Silicon detector
  - Scintillating fibers/cube/bar…
  - …
- **Timing**
  - Fast-response photosensor
  - Fast scintillator
  - …

Detector building d**epends on physical motivation, allowed technology, budge, space, time...**

Particle identification (PID)

What kind of information used to identify particle?

- Mass
- Charge
- Spin
- "Flavor" (also "color")
    - Eg. *Electron neutrino and electron produced in pair*
- Allowed interaction processes and interaction strength
    - Eg. *Leptons (muon, electron, neutrino) do not have strong interaction*
- Prompt decay products (*eg. in case of W, Z, Higgs*)

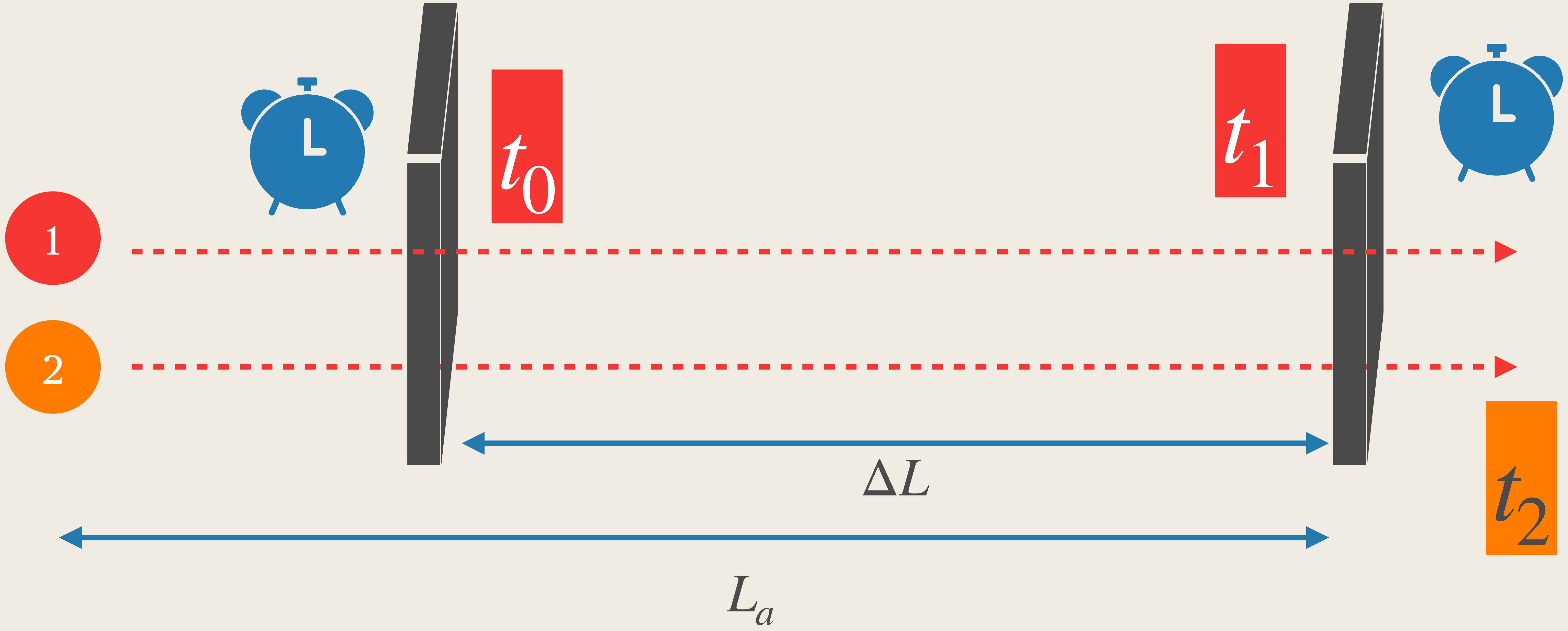


Standard Model of Elementary Particles

three generations of matter (fermions)

	I	II	III		
mass charge spin	$\simeq 2.2\ \mathrm{MeV}/c^2$ 2/3 1/2 u up	$\simeq 1.28\ \mathrm{GeV}/c^2$ 2/3 1/2 c charm	$\simeq 173.1\ \mathrm{GeV}/c^2$ 2/3 1/2 t top	0 0 1 g gluon	$\approx 125.09\ \mathrm{GeV}/c^2$ 0 0 H Higgs
	$\simeq 4.7\ \mathrm{MeV}/c^2$ -1/3 1/2 d down	$\simeq 96\ \mathrm{MeV}/c^2$ -1/3 1/2 s strange	$\approx 4.18\ \mathrm{GeV}/c^2$ -1/3 1/2 b bottom	0 0 1 γ photon	
	$\approx 0.511\ \mathrm{MeV}/c^2$ -1 1/2 e electron	$\simeq 105.66\ \mathrm{MeV}/c^2$ -1 1/2 μ muon	$\approx 1.7768\ \mathrm{GeV}/c^2$ -1 1/2 τ tau	$\approx 91.19\ \mathrm{GeV}/c^2$ 0 1 Z Z boson	
	$<2.2\ \mathrm{eV}/c^2$ 0 1/2 ν_e electron neutrino	$<1.7\ \mathrm{MeV}/c^2$ 0 1/2 ν_μ muon neutrino	$<15.5\ \mathrm{MeV}/c^2$ 0 1/2 ν_τ tau neutrino	$\approx 80.39\ \mathrm{GeV}/c^2$ ±1 1 W W boson	

PID w/ Time-of-flight (TOF)



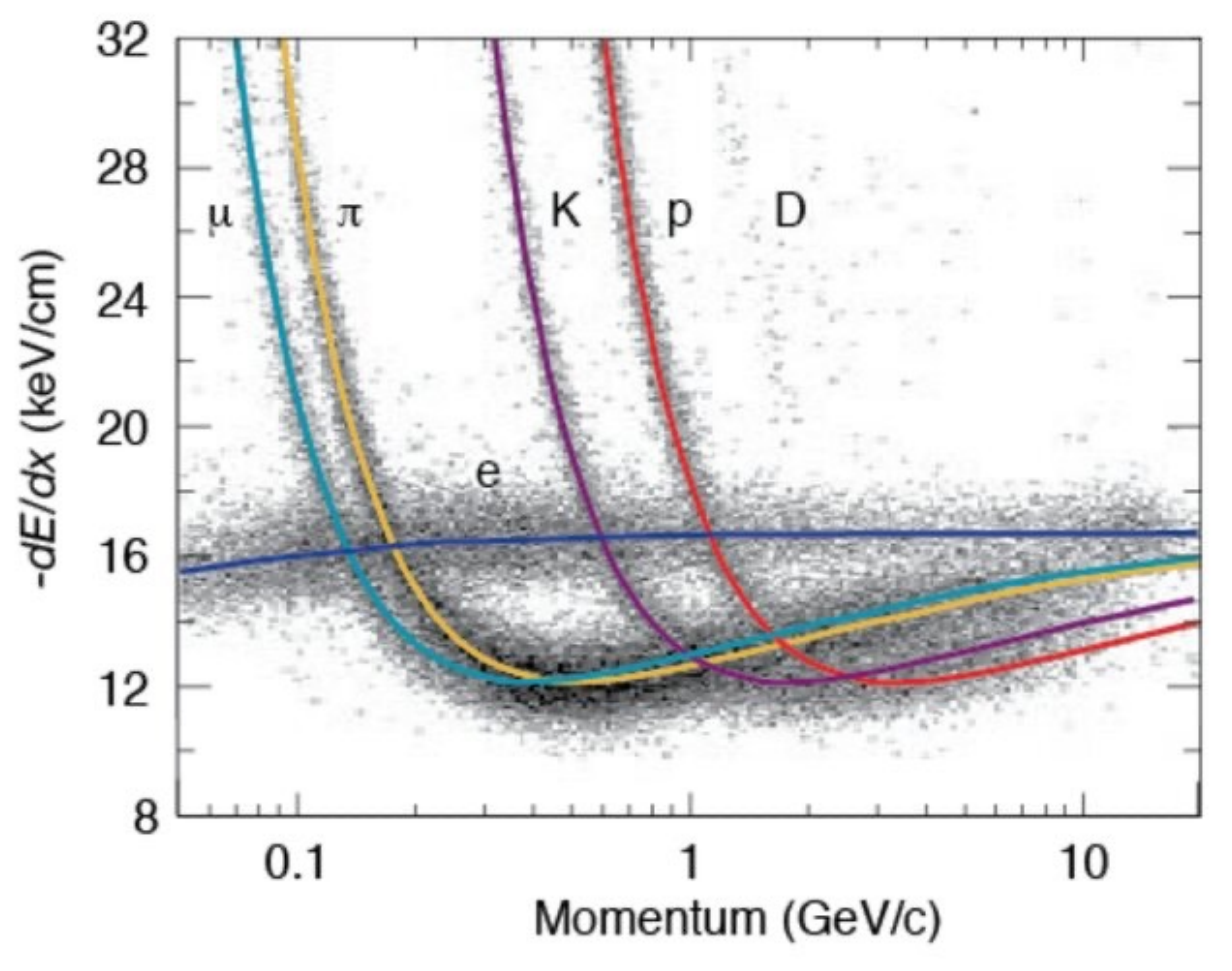
- Absolute TOF: $t_1 - t_0 = \frac{\Delta L}{\beta c} = \frac{\Delta L}{c}\sqrt{1 + \frac{m^2c^2}{p^2}}$
- Relative TOF: $t_2 - t_1 = \frac{\Delta L}{\beta c} = \frac{L_a}{c}\left(\sqrt{1 + \frac{m^2c^2}{p^2}} - \sqrt{1 + \frac{m^2c^2}{p^2}}\right) \approx \frac{L_a c(m_1^2 - m_2^2)}{2p^2}$

m CAN BE MEASURED
IF ONE KNOWS $\Delta t_1, \Delta L, p$

USEFUL CONVERSION: AT SPEED OF LIGHT, PARTICLE PASS 1 METER IN 3.3 NANOSECOND

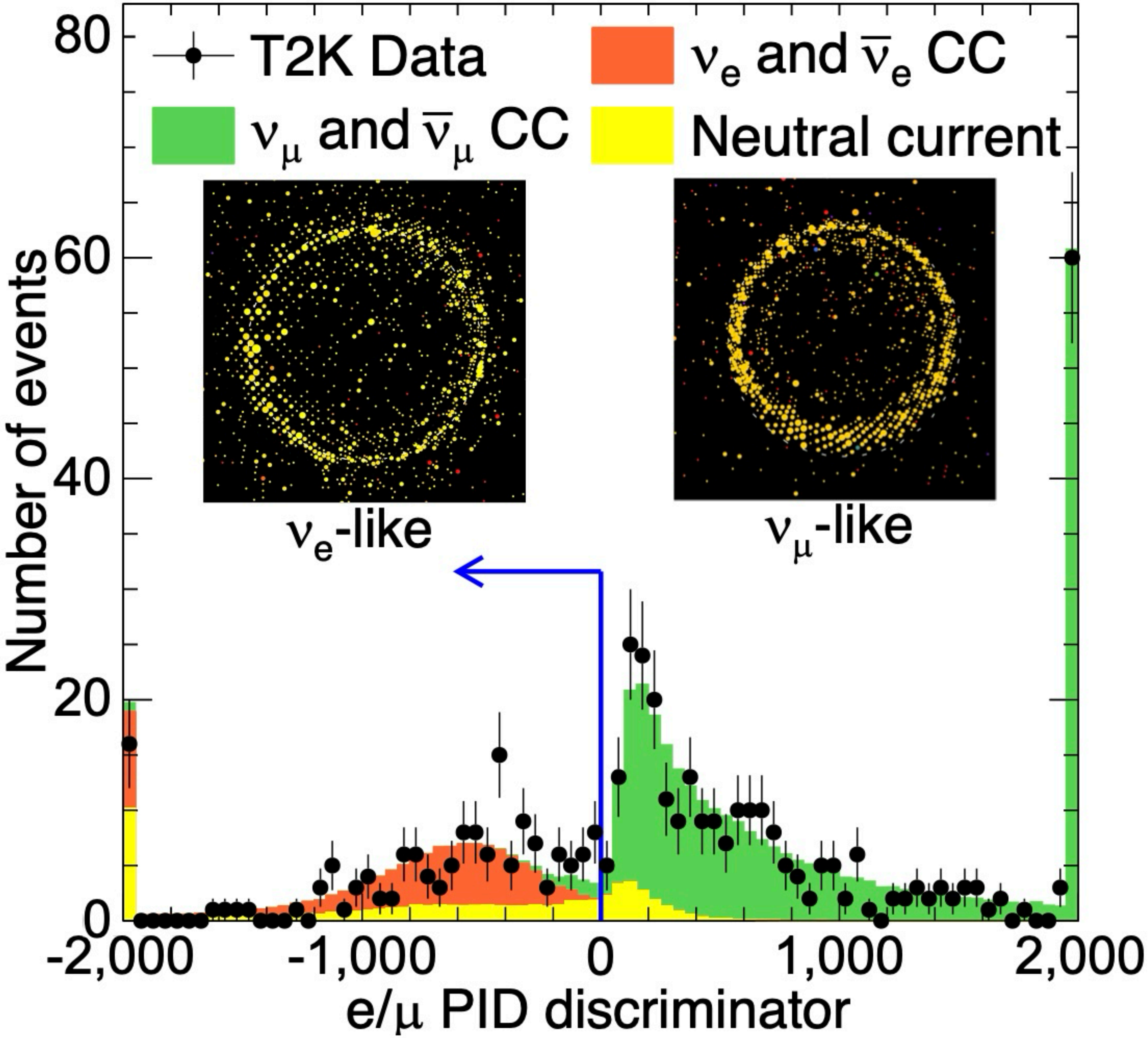
Particle identification using dE/dx

- If one can measure dE/dx simultaneously with momentum, particle can be classified
- Behavior of electron is different due to Bremsstrahlung process



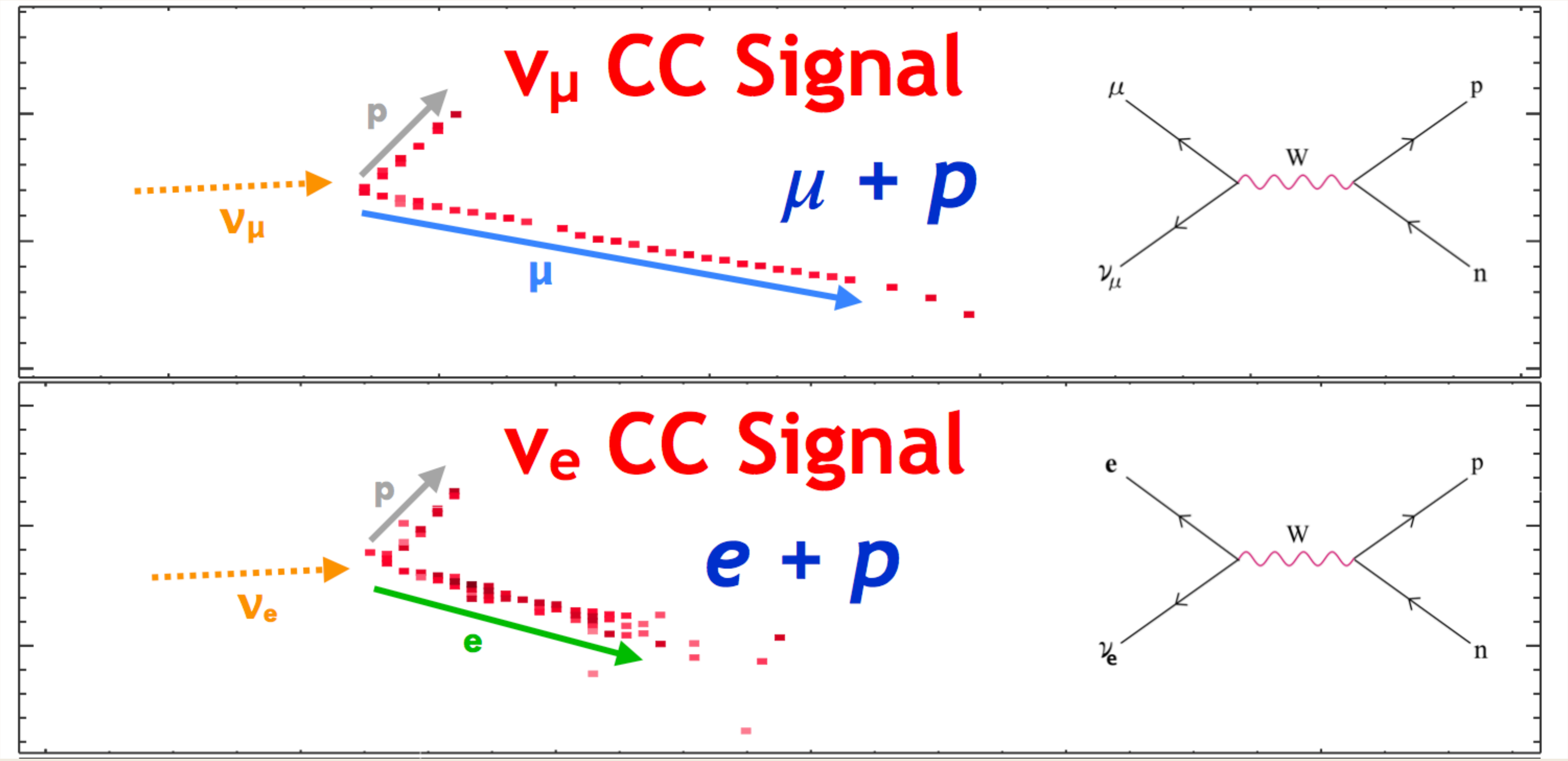
Electromagnetic shower development

- Electron quickly develops electromagnetic shower and give a fuzzy ring pattern than muon
- **The *fuzziness* can be used as particle identity**
- *Also the Cherenkov threshold can be used for particle identity since it depends on particle*



Ranging and Electromagnetic shower development

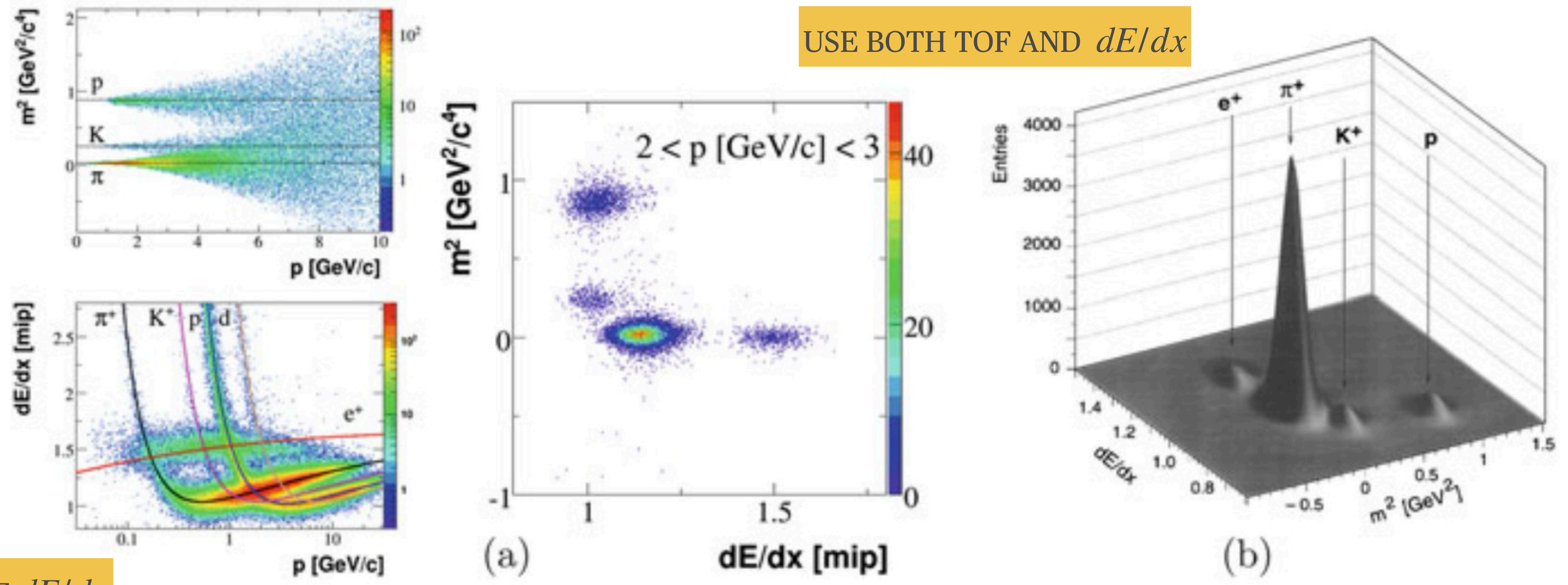
NOvA, scintillator technique



NA49 experiment: Use TOF and dE/dx

USE TOF

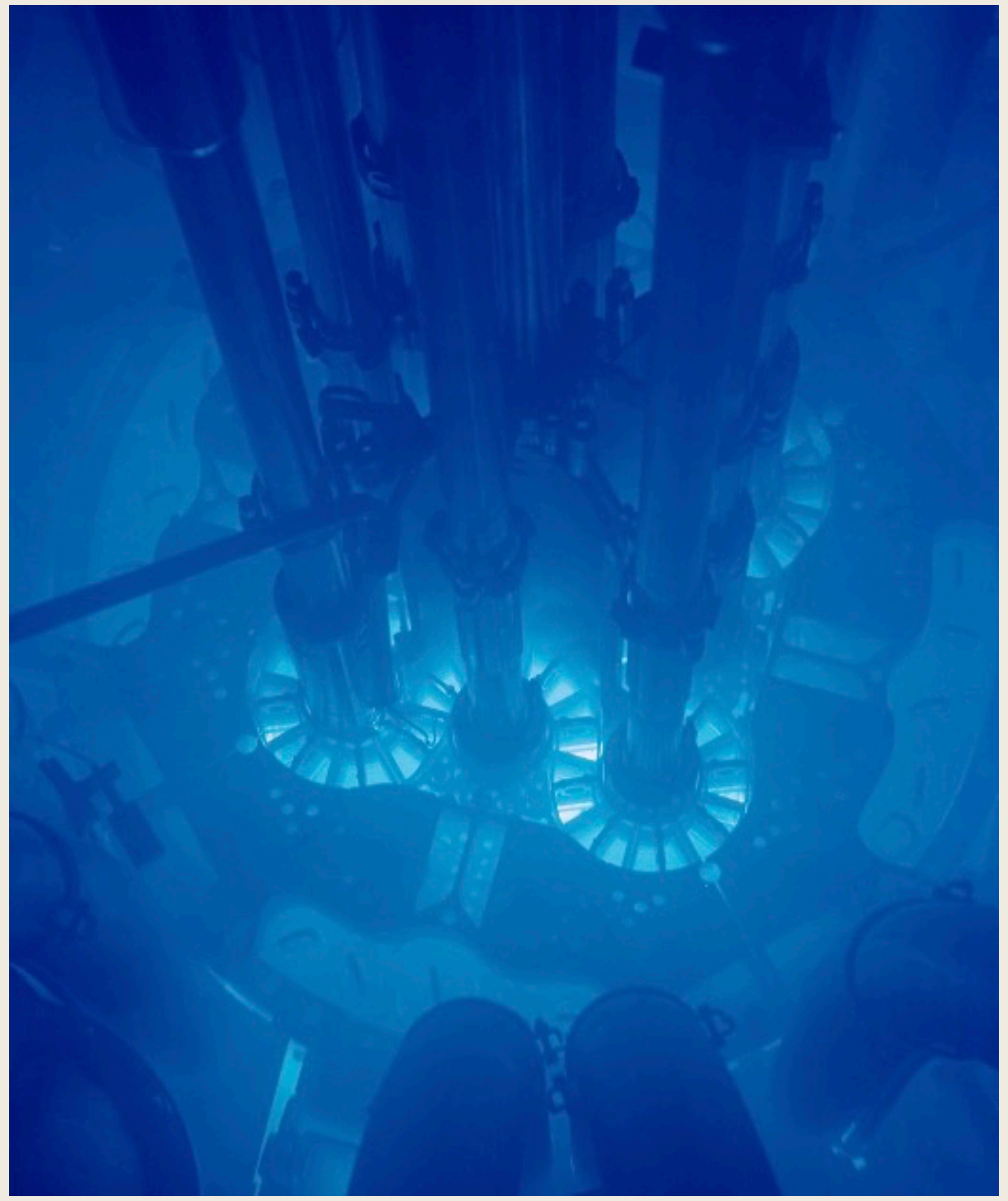
USE BOTH TOF AND dE/dx



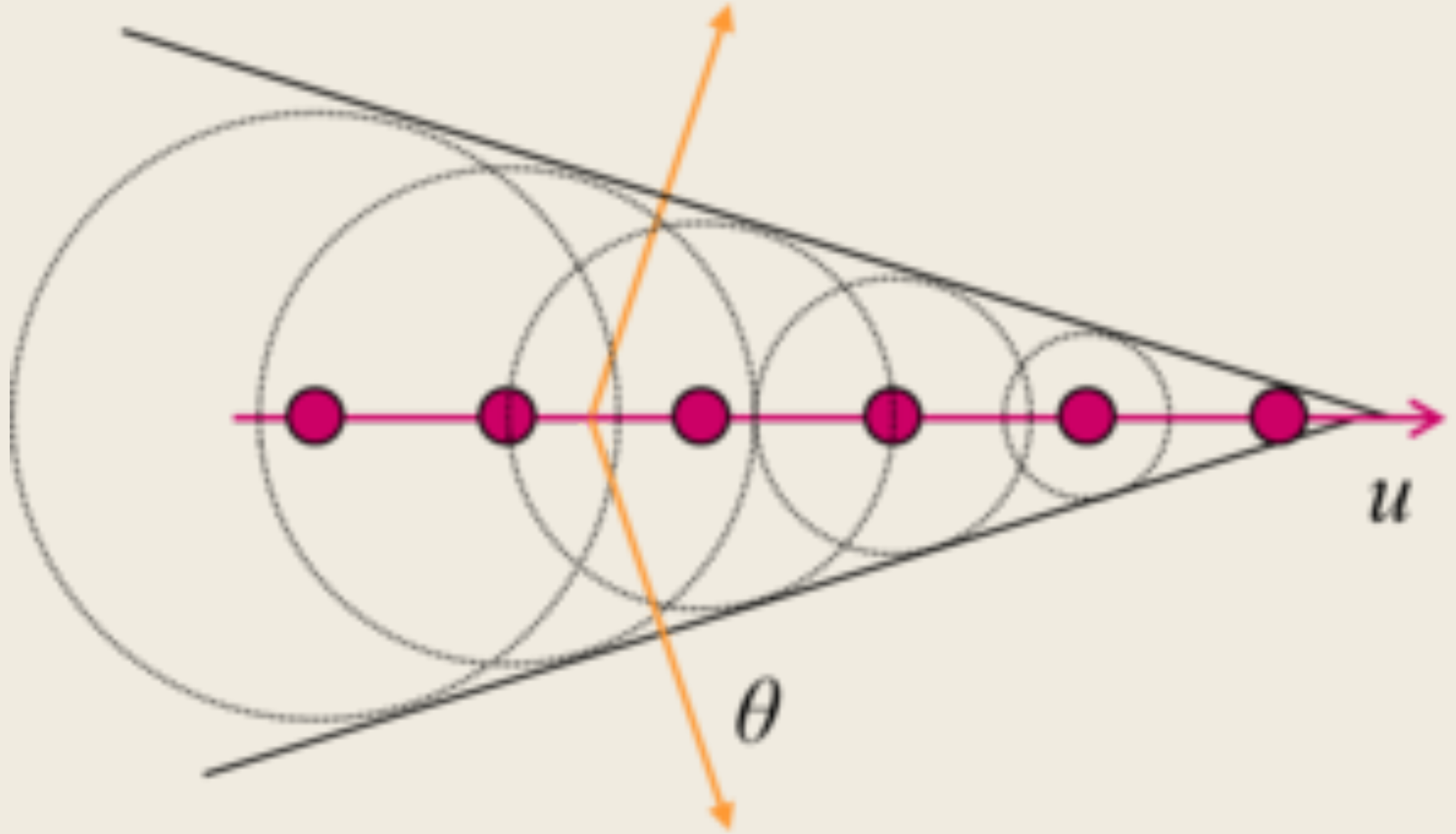
USE dE/dx

Cherenkov radiation for PID

- Discovered by Pavel A. Cherenkov in 1937
- It happens when a charged particle passing through a polarizable medium with speed higher than the speed of light in that medium.
  - $v = \beta c > c/n \rightarrow \beta n > 1$ where n is refractive index
- Mechanism: molecules are polarized in the direction of the charged particle moving
  - Excited molecules emit the light when back to the normal state
  - When particle moves faster than c/n, these emitted light added up constructively (interference) to emit the light at specific angle θ_C

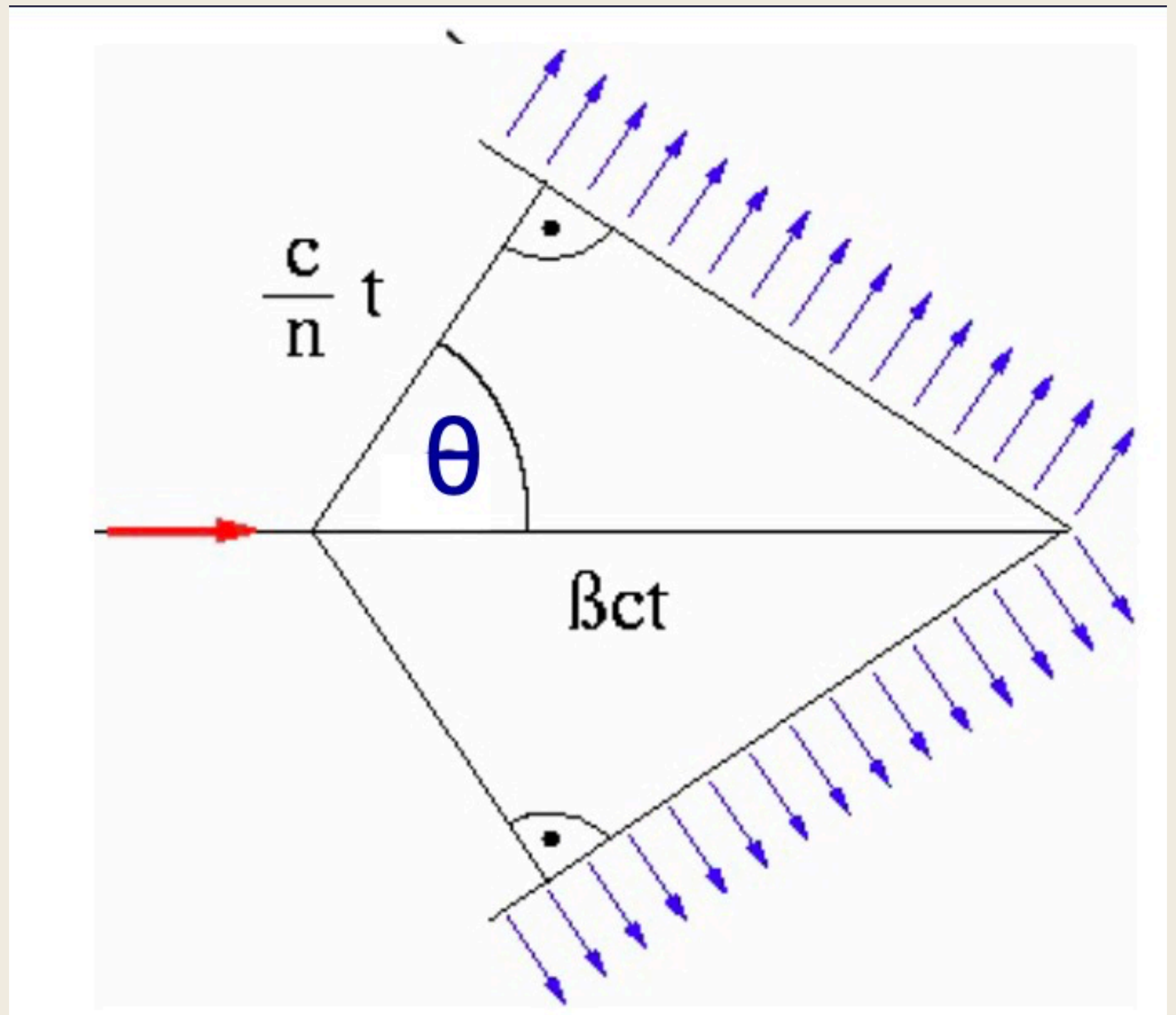


"Blue" water near reactor core



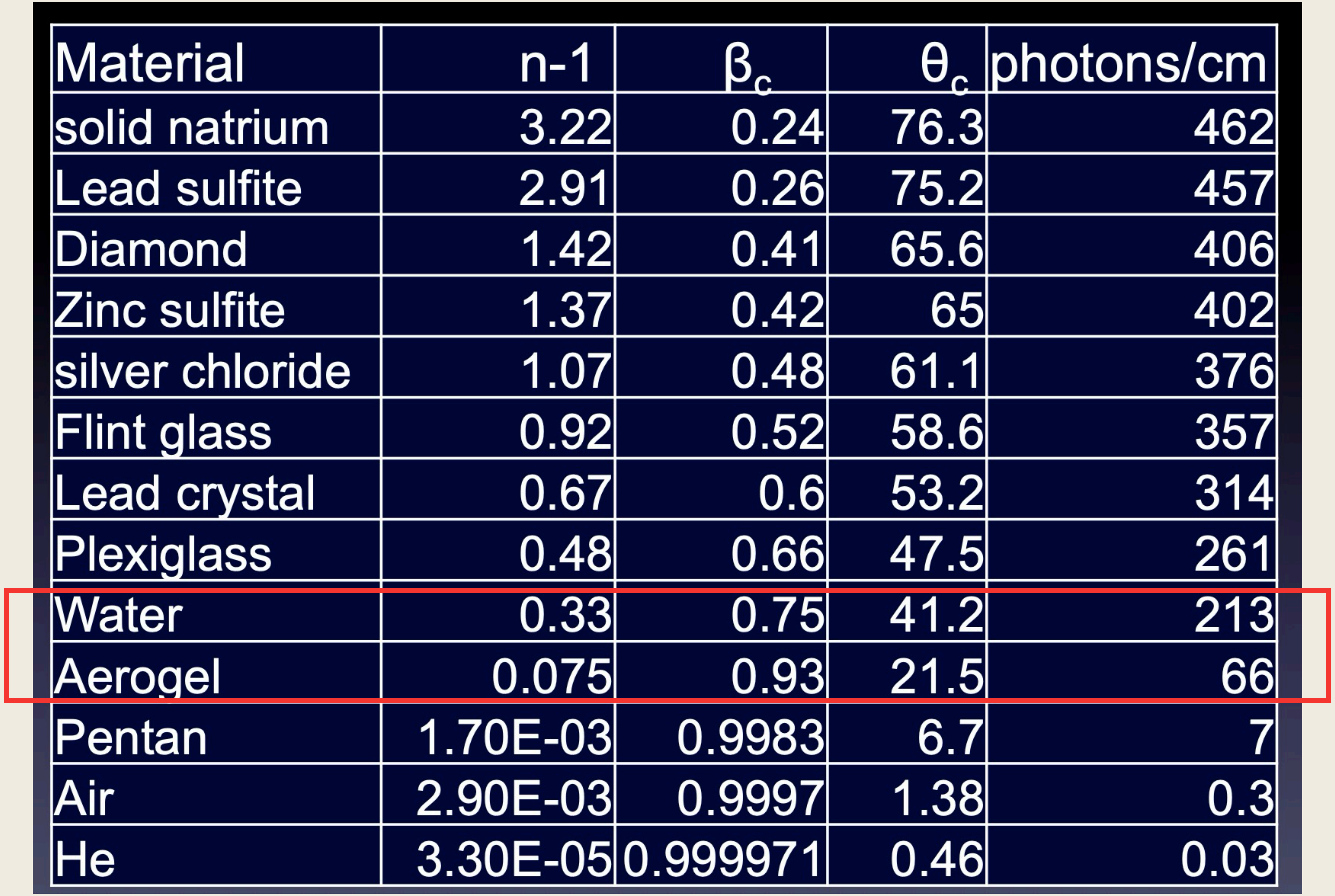
Cherenkov radiation for PID

- Threshold in Cherenkov radiation: $p_{th.} = \frac{mc/n}{\sqrt{1-n^{-2}}}$
  - In the air, threshold for pions is 5GeV/c but for Kaon is 20 GeV/c
  - In water, threshold for electron is 0.77 MeV, for muon is 160 MeV, for tau is 2.7 GeV



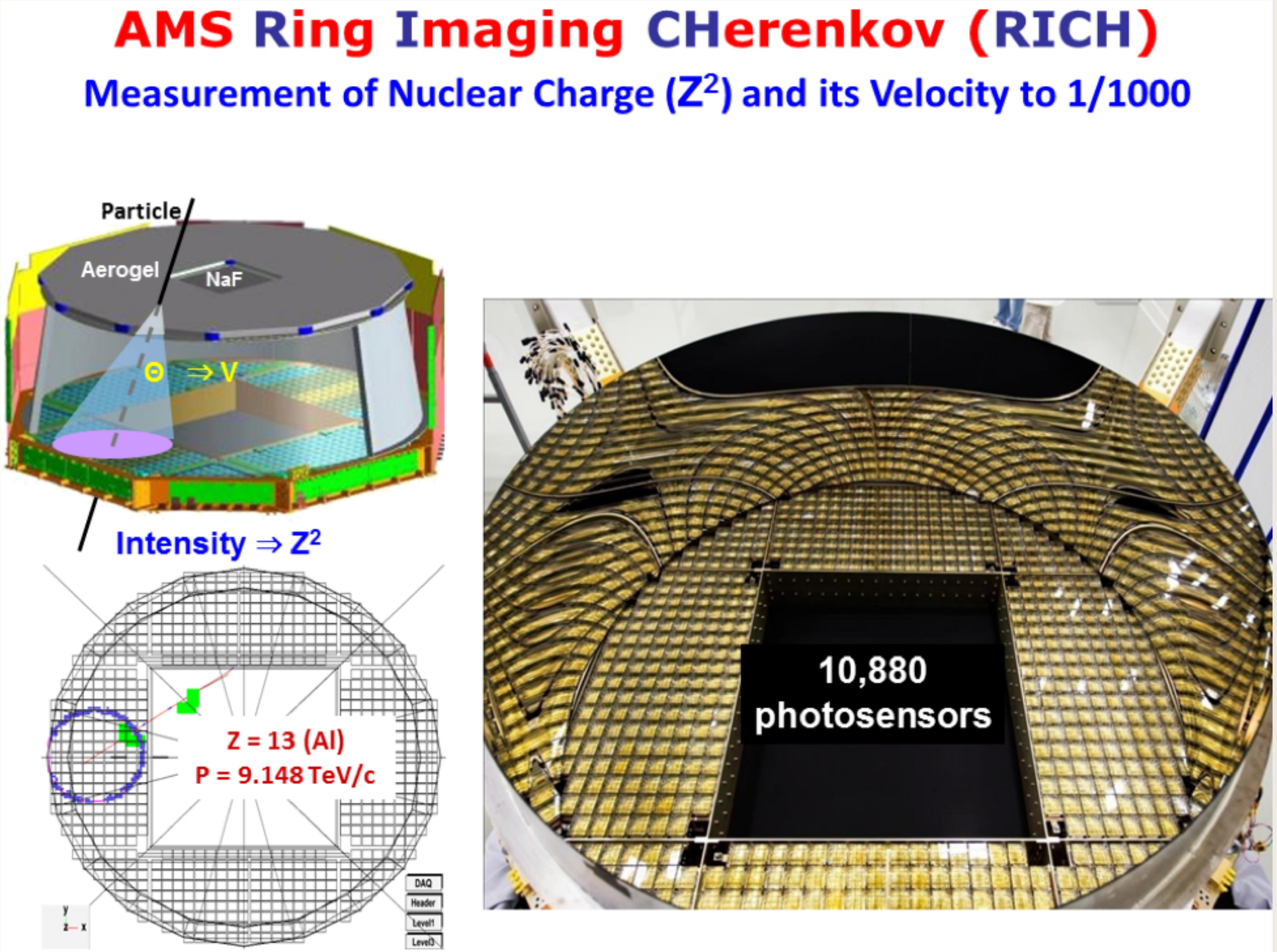
- Cherenkov angle $\cos\theta_C = \frac{1}{\beta n}$, **depend on** β
- Number of photons from Cherenkov radiation at a given wavelength $\frac{\partial^2 N}{\partial x \partial \lambda} = 2\pi\alpha(1 - 1/\beta^2 n^2)1/\lambda^2$
  - More photon at lower wavelength, so Cherenkov mostly at blue and UV range
  - **Also depend on** β

Cherenkov radiation for PID



Material	n-1	β_c	θ_c	photons/cm
solid natrium	3.22	0.24	76.3	462
Lead sulfite	2.91	0.26	75.2	457
Diamond	1.42	0.41	65.6	406
Zinc sulfite	1.37	0.42	65	402
silver chloride	1.07	0.48	61.1	376
Flint glass	0.92	0.52	58.6	357
Lead crystal	0.67	0.6	53.2	314
Plexiglass	0.48	0.66	47.5	261
Water	0.33	0.75	41.2	213
Aerogel	0.075	0.93	21.5	66
Pentan	1.70E-03	0.9983	6.7	7
Air	2.90E-03	0.9997	1.38	0.3
He	3.30E-05	0.999971	0.46	0.03

Cherenkov radiation for PID: RICH exp.



AMS Ring Imaging CHerenkov (RICH)

Measurement of Nuclear Charge (Z^2) and its Velocity to 1/1000

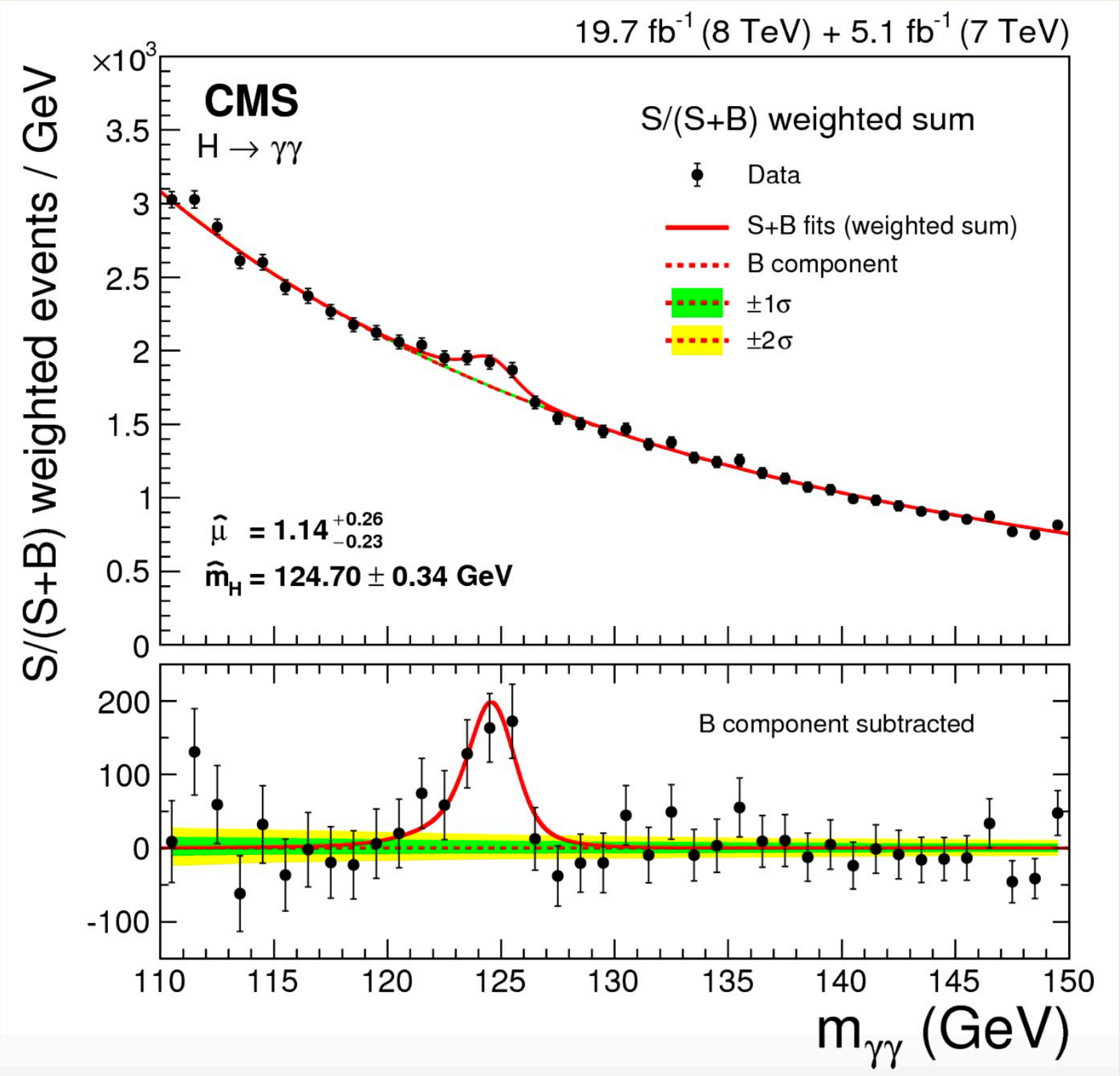
https://ams02.space/detector/ring-imaging-cherenkov-detector-rich

Measurement of energy: Calorimeter

Importance of energy measurement

To determine the energy of particle with some level of precision using the "energy loss phenomena"

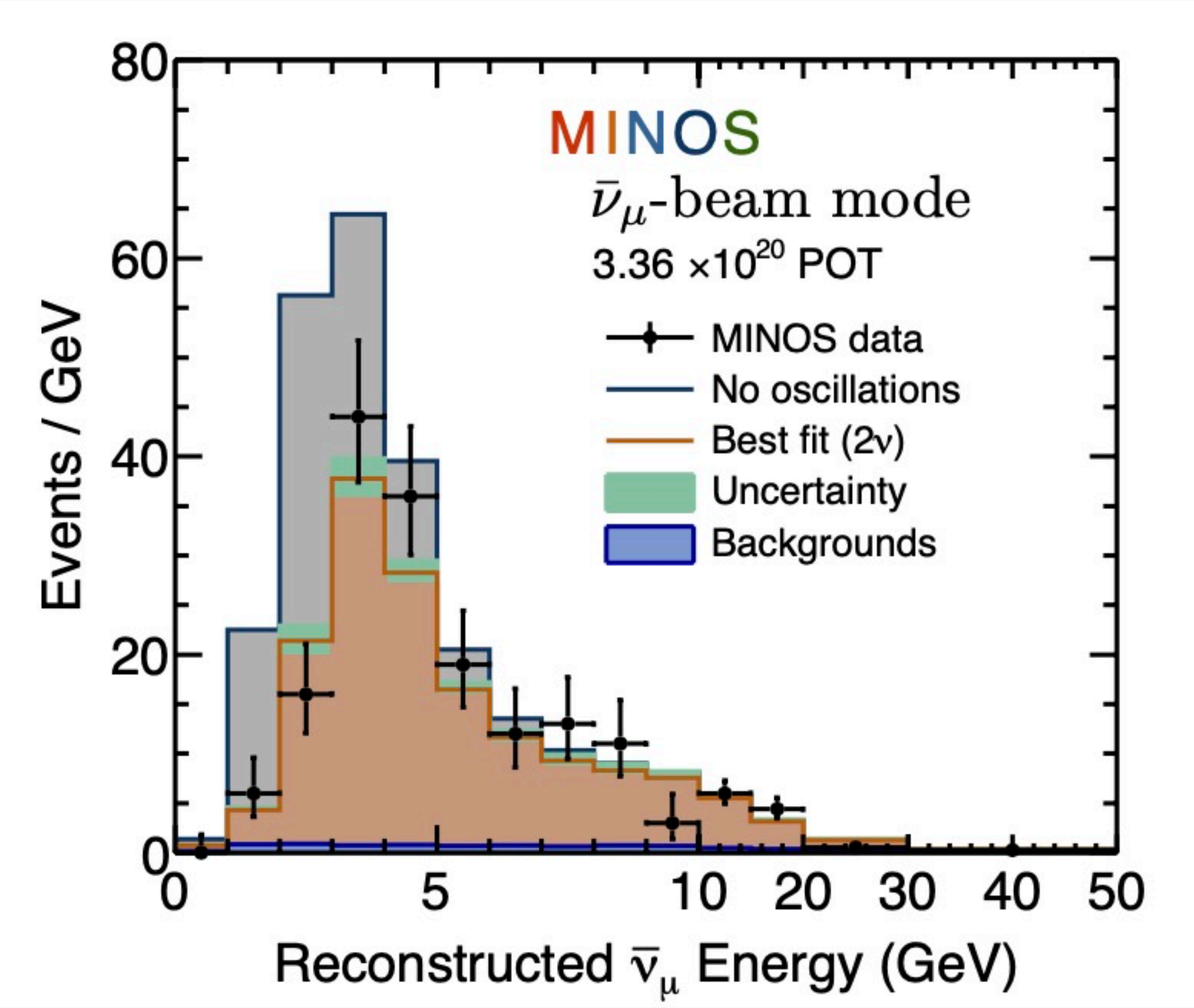
- You can't measure directly but reconstruct from the signal induced of energy loss of particle
- Energy loss is proportional to the particle energy
- For some physics (*eg. Higgs discovery, neutrino oscillation…*), measuring the energy precisely is vital

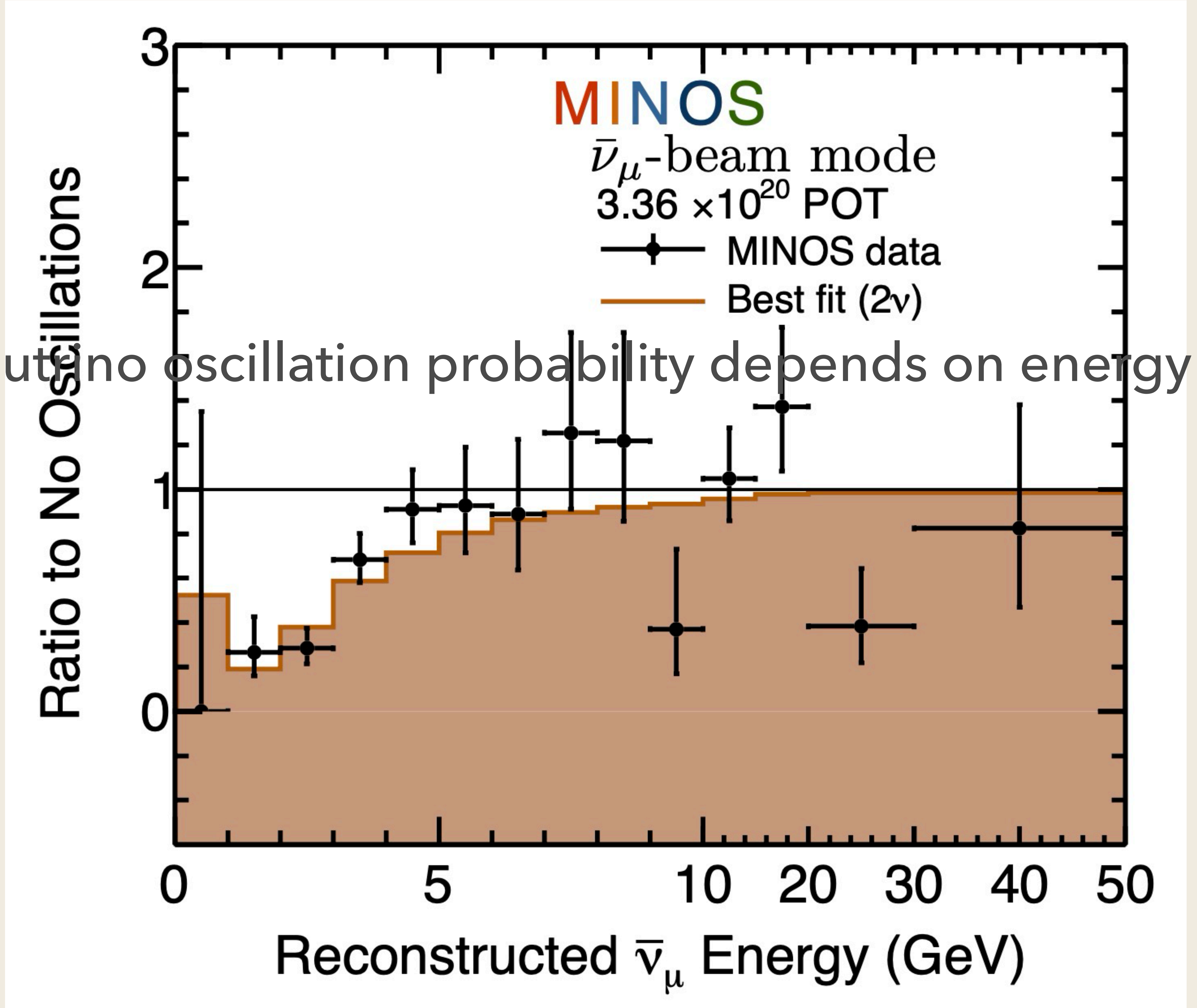


Higgs particle discovery thanks to
the precise measurement of photon energy

Calorimeter for neutrino oscillation measurement

$$P(\nu_\mu \to \nu_\mu) \sim 1 - \sin^2 2\theta_{23} \sin^2 \frac{\Delta m^2_{31} L}{4E_\nu}$$



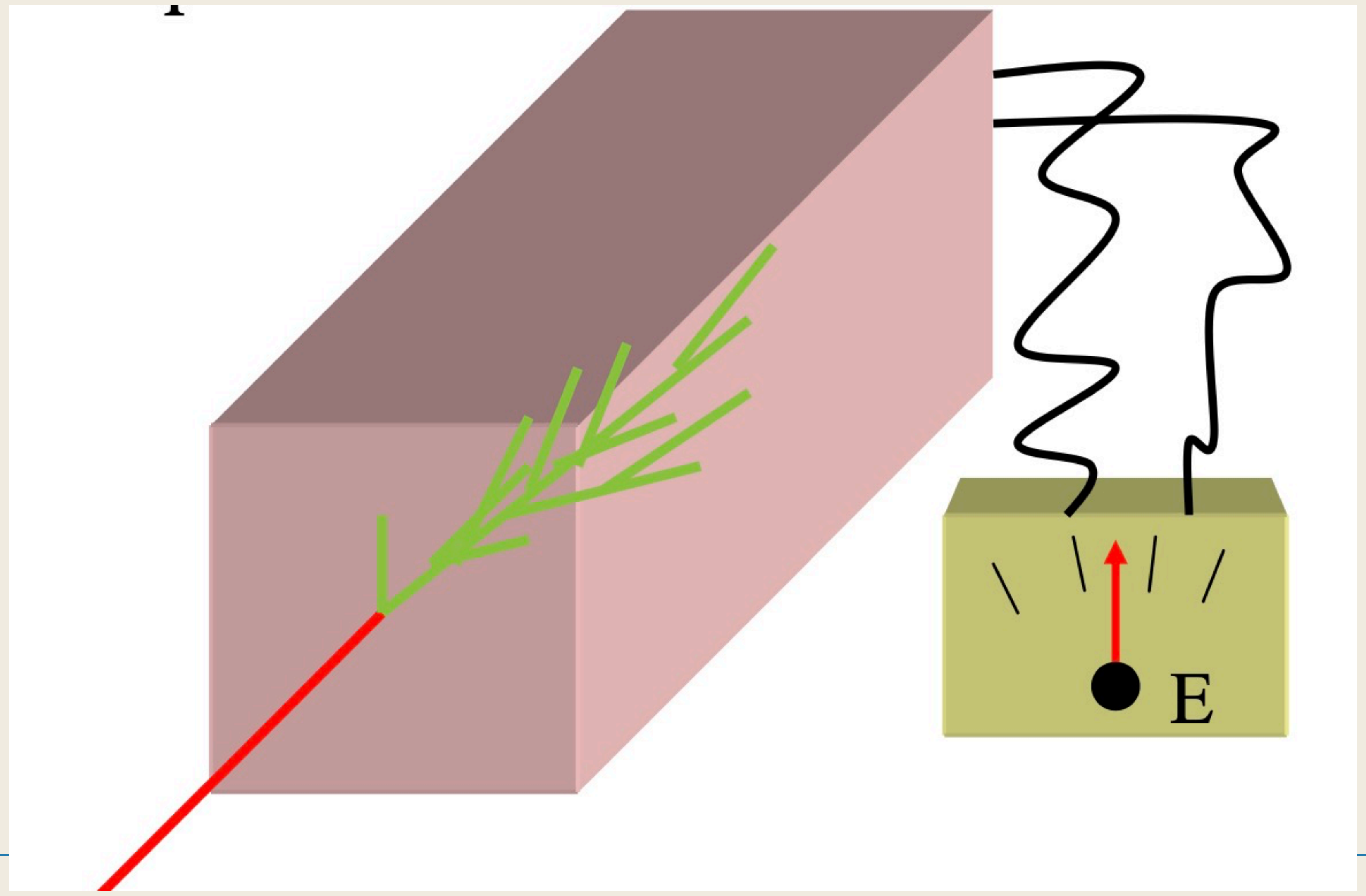


Neutrino oscillation probability depends on energy

Calorimeter: design consideration

Principle: detect particles through total absorption in matter volume of the detector

- **Large enough** and/or **high-density material** to **absorb all of the energy** of to-be-measured particles (*include both charged and neutral particle*)
- R**ecord signal (charge or light collection) to refer to the energy lost**
- Sufficiently **granular** to tell not just how much but where energy was deposited
- **Other practical consideration**: small enough to fit in detector, not too expensive, radiation hardness; fast read-out depending on event rate…



Energy resolution of calorimeter

- Spectrometer performance is characterized by "energy resolution" , typically as σ_E/E
  - Ideally $E \sim N;\ \sigma_E \sim \sqrt{N}$. So $\frac{\sigma_E}{E} \sim \frac{1}{\sqrt{E}}$
  - In reality $\frac{\sigma_E}{E} \sim \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$

STOCHASTIC TERM

CONSTANT TERM

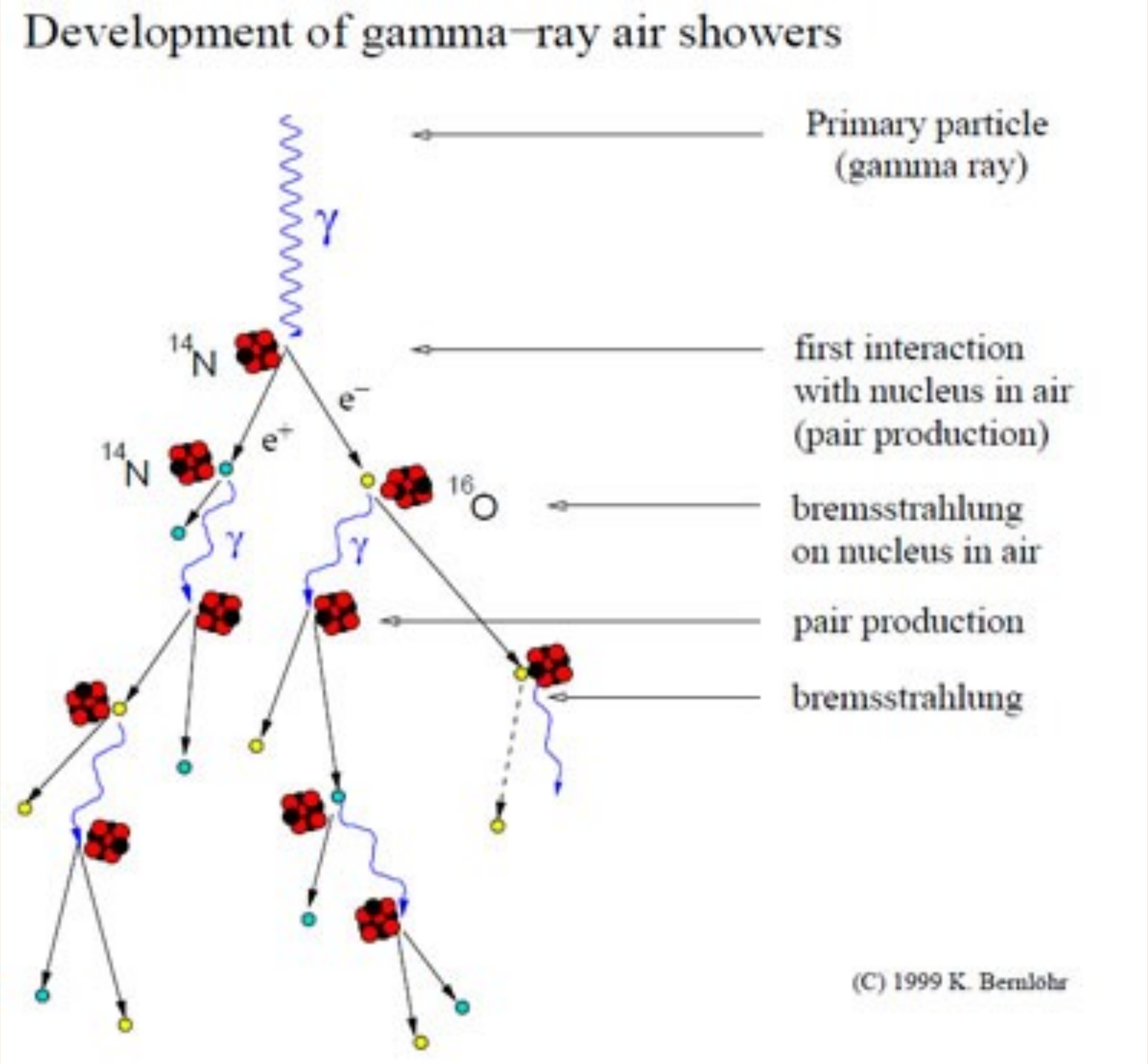
NOISE TERM

- Essential task: calibrate conversion btw. what observed (eg. Scintillation light) to energy

Electromagnetic shower development

- Electromagnetic shower: High-energy photons produce pair of electrons and positrons
- Electrons and positrons radiate photons via Breamsstrahlung when travel through matter, interacting with fields of atoms
- One electron fall below critical energy, more energy loss via ionisation than bremsstrahlung and the shower stops growing

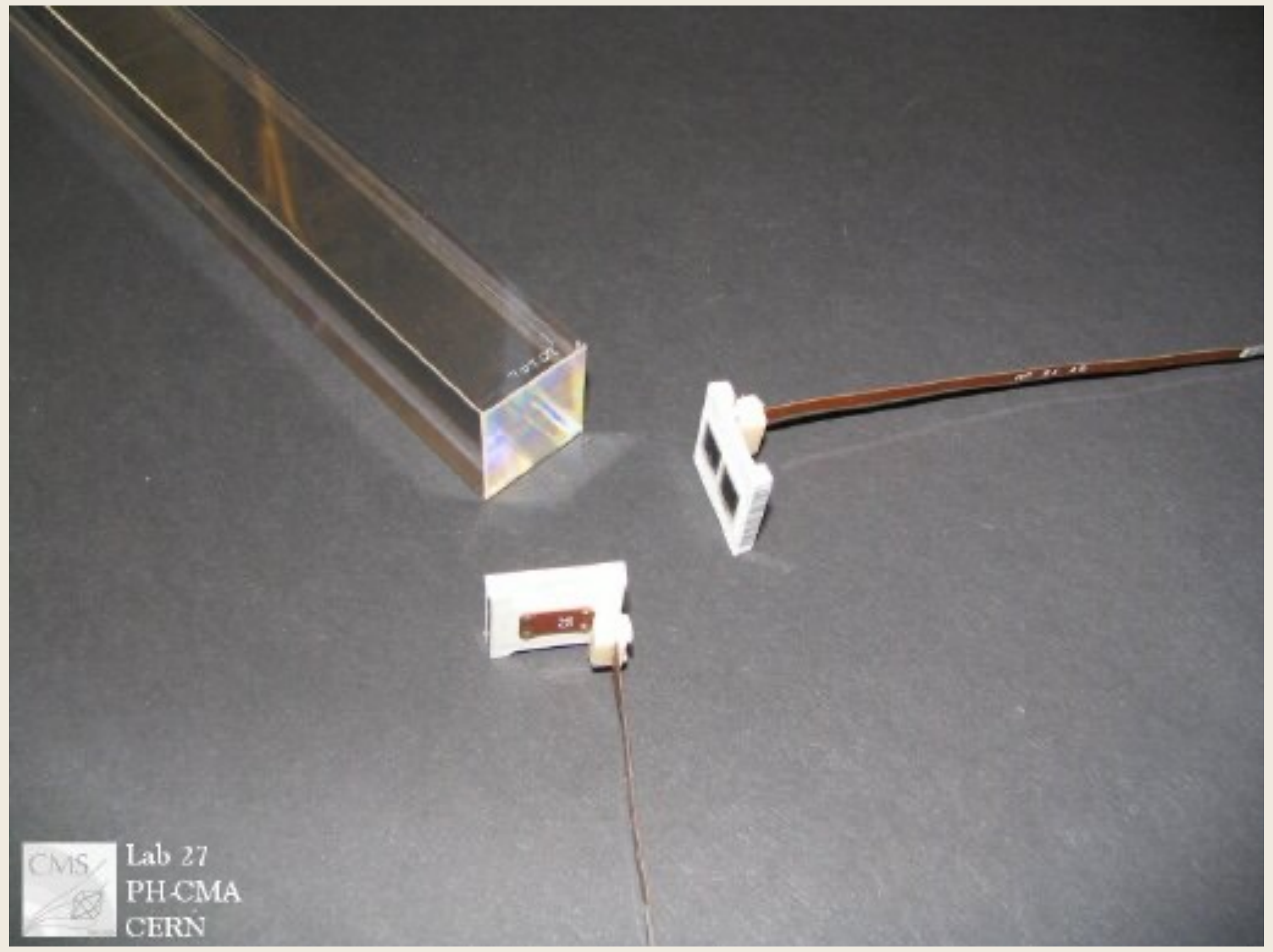
$$\frac{\sigma_E}{E} \sim \frac{3\% - 10\%}{\sqrt{E/\mathrm{GeV}}}.$$



(C) 1999 K. Bernlöhr

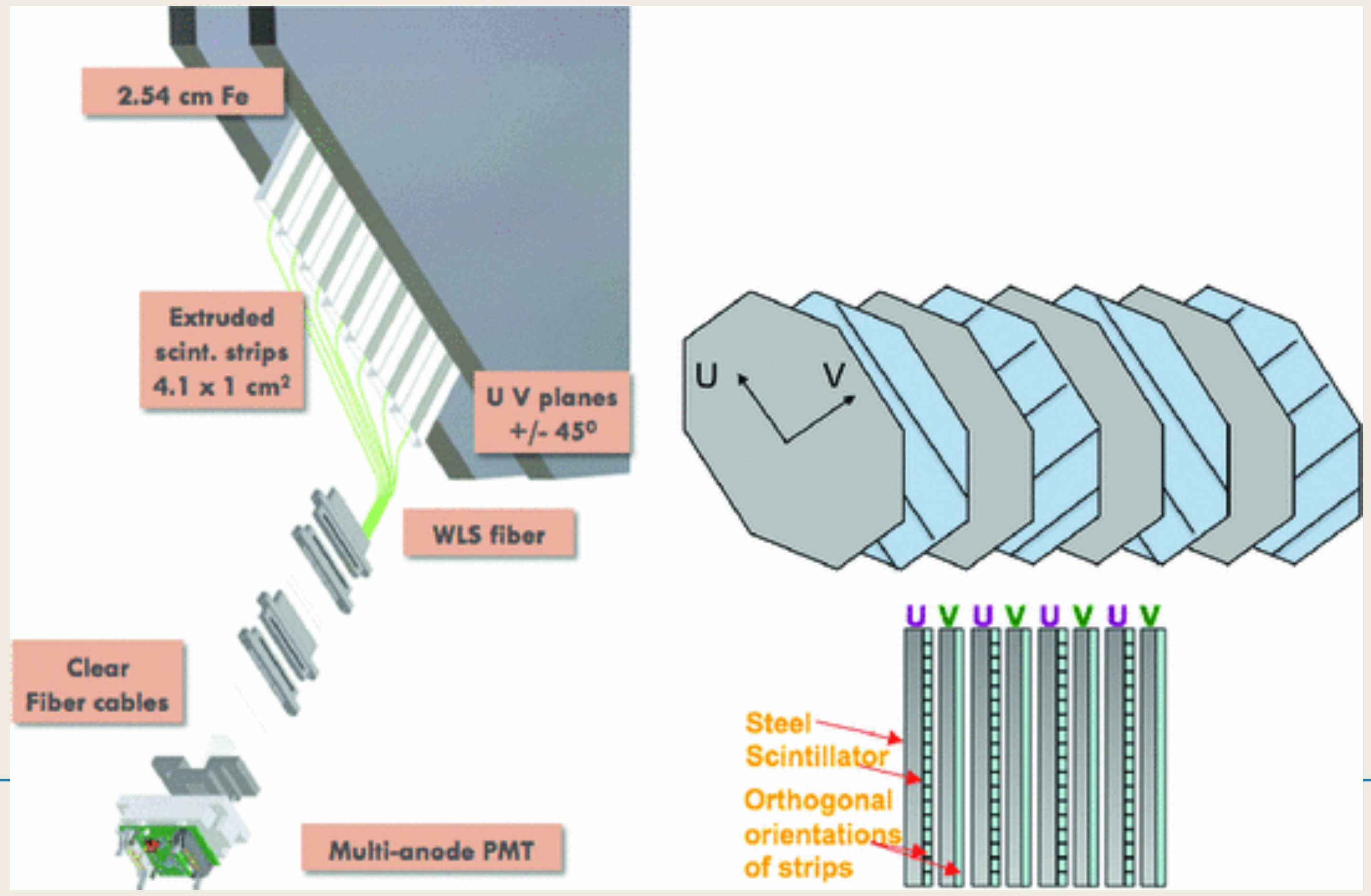
Broadly two types: homogeneous and sampling

- **Homogeneous** type: all active volume, typically with high density transparent materials
  - Pros: excellent energy resolution
  - Cons: expensive; some limit in spatial resolution
- **Sampling**: consists of both passive (high Z materials like iron, tungsten) and active (scintillator) absorbers
  - Pros: cost-effective
  - Cons: not good energy resolution like the homogeneous type



CMS ECAL: USE 80,000 PBWO$_4$ CRYSTAL

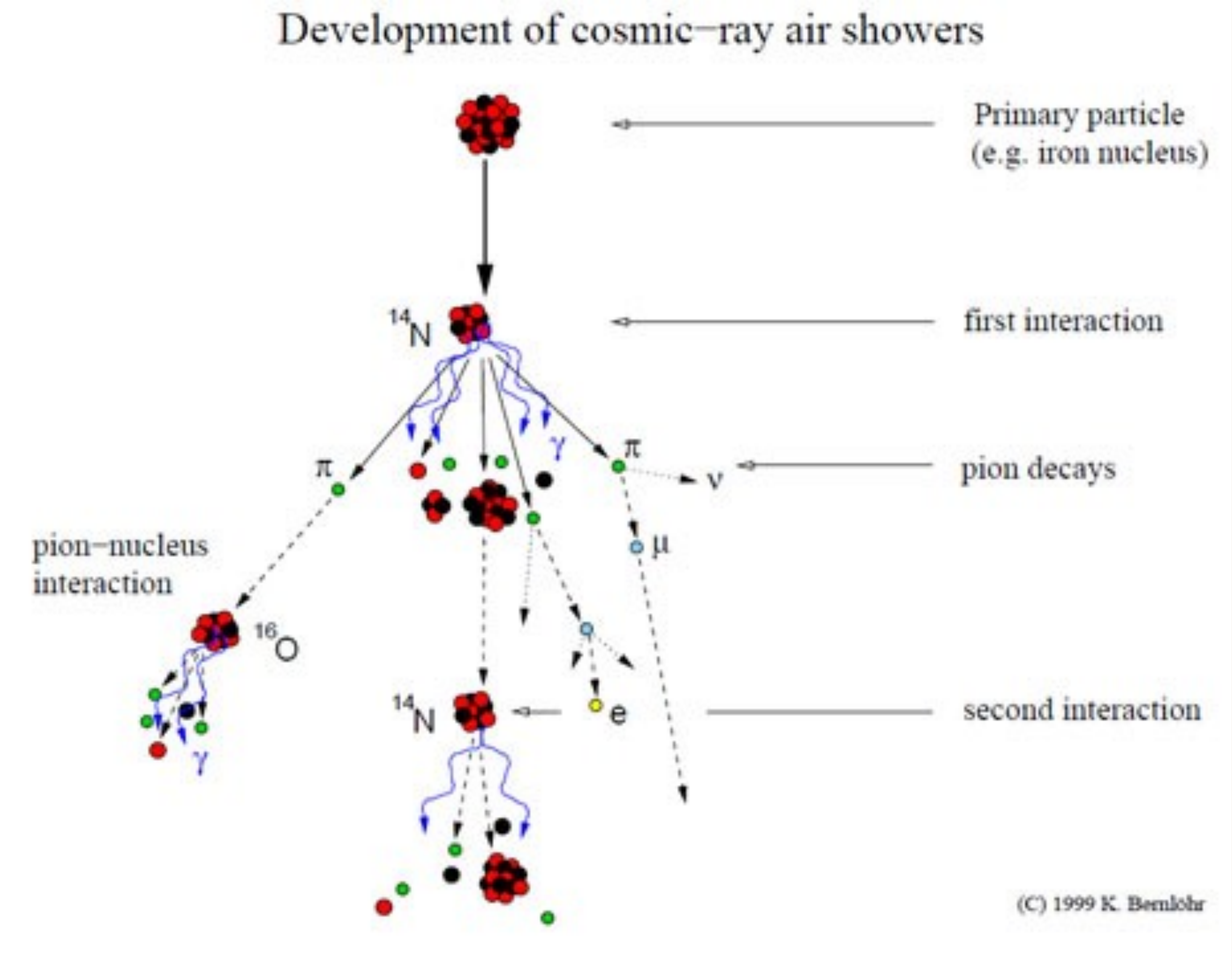
Can achieve $$\frac{\sigma_E}{E} \sim \frac{2.7\,\%}{\sqrt{E}} \oplus 0.55\% \oplus \frac{0.2}{E}$$



MINOS (+): STEEL-SCINTILLATOR SANDWICH

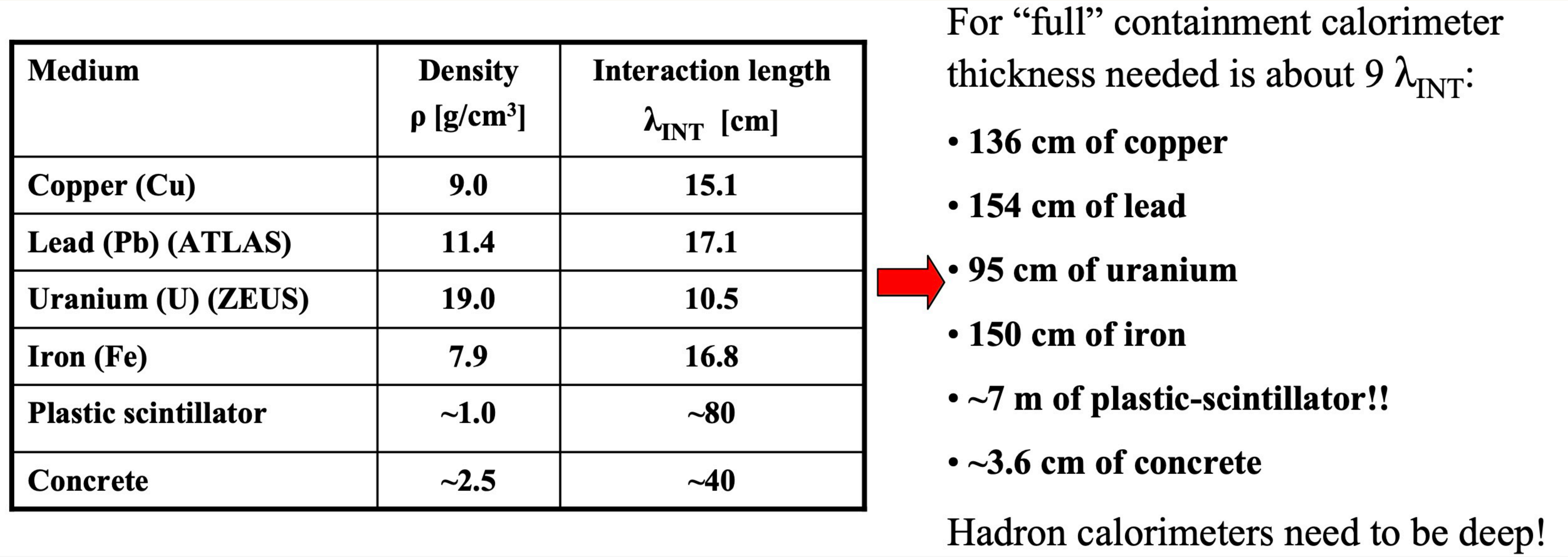
Hadronic shower development

- Driven by inelastic nuclear scattering
- Interaction length $\lambda_{int.} = 35\ [g/cm^2]\ A^{1/3}$, only depend on detector material
  - Eg. In iron, $\rho \approx 7.8\ \ g/cm^3$, A=28, interaction is about X_0= 13cm
  - Calorimeter thickness is about 9 X_0
- Complicated shower with pion, neutron, photon (from neutral pion), neutrino
- Some energy loss can't be observed, eg. Neutrino, nuclear binding energy …
- Typically, hadronic shower energy resolution is worse than EM shower



$$\frac{\sigma_E}{E} \gtrsim \frac{50\%}{\sqrt{E/\mathrm{GeV}}},$$

Hadron calorimeter thickness



Medium	**Density** **ρ [g/cm³]**	**Interaction length** **λ_{INT} [cm]**
Copper (Cu)	**9.0**	**15.1**
Lead (Pb) (ATLAS)	**11.4**	**17.1**
Uranium (U) (ZEUS)	**19.0**	**10.5**
Iron (Fe)	**7.9**	**16.8**
Plastic scintillator	**~1.0**	**~80**
Concrete	**~2.5**	**~40**

For “full” containment calorimeter thickness needed is about 9 λ_{INT}:

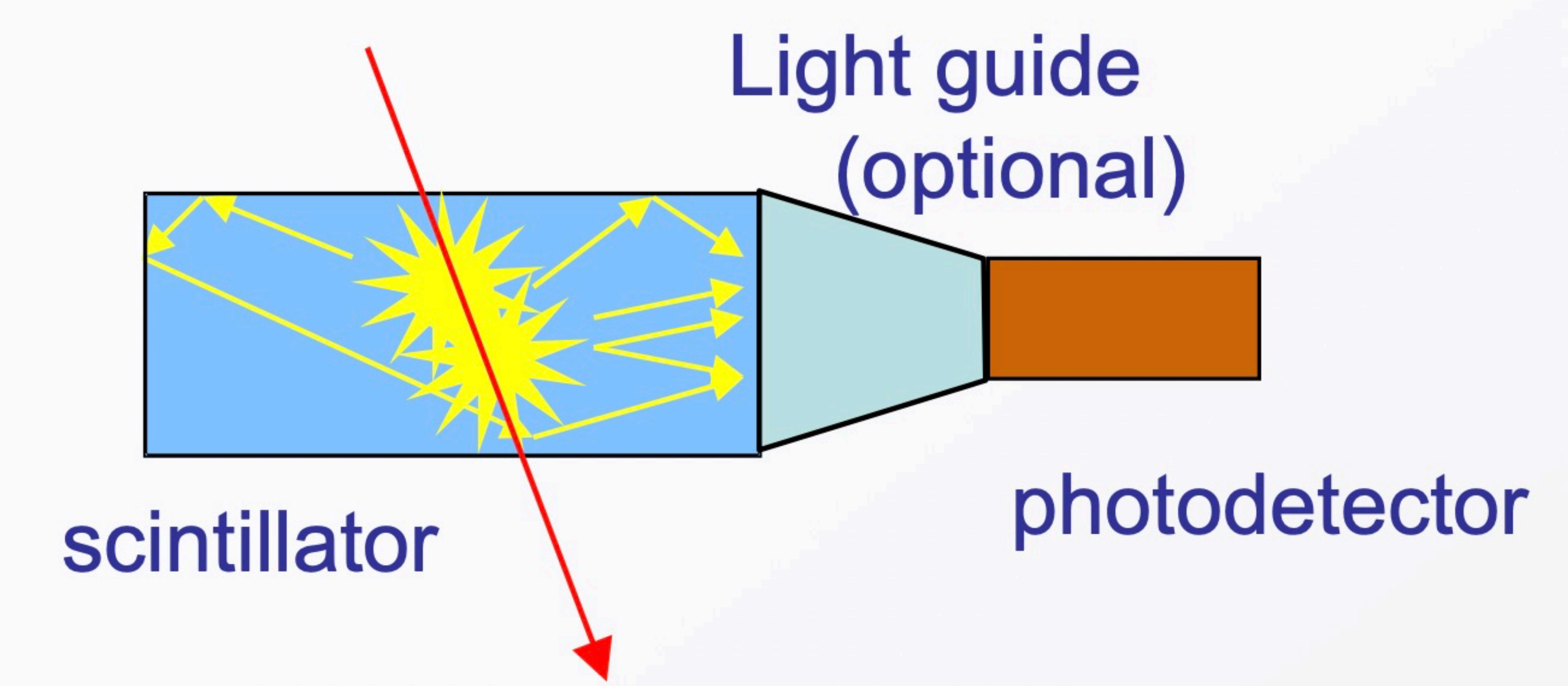
- **136 cm of copper**
- **154 cm of lead**
- **95 cm of uranium**
- **150 cm of iron**
- **~7 m of plastic-scintillator!!**
- **~3.6 cm of concrete**

Hadron calorimeters need to be deep!

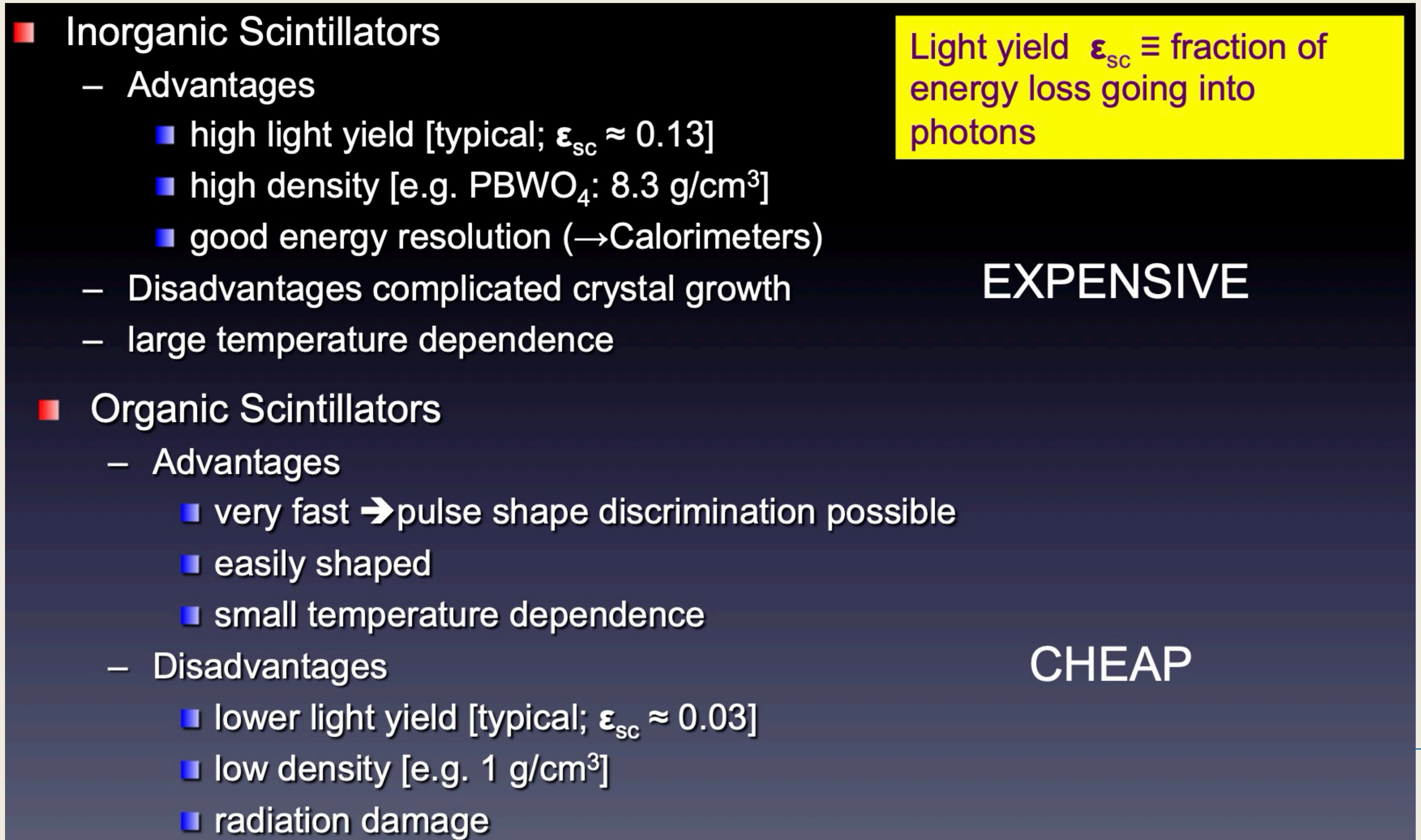
Calorimeter techniques

- Common modern techniques used:
  - Gaseous/liquid Argon ionization
  - Scintillator detector
  - Cherenkov detector
  - …

Scintillation detector



- One of the most common techniques in PN physics
- To detect energy deposited by ionizing particles or photons
- Broadly categorized into: organic scintillator and inorganic scintillator
- Main detection characteristics: linearity to energy; fast response (fast rising time)



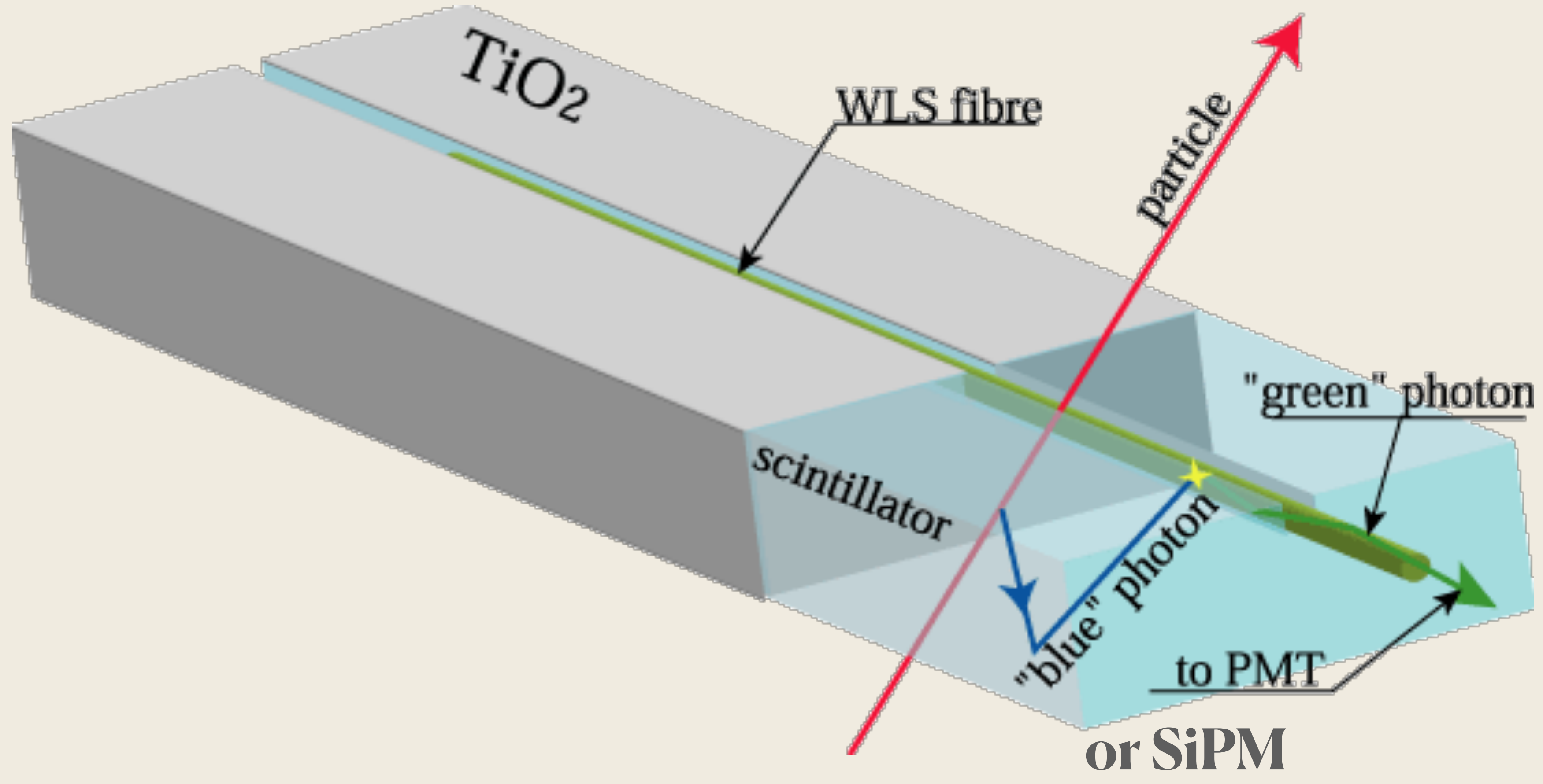
- Inorganic Scintillators
  - Advantages
    - high light yield [typical; $\varepsilon_{sc} \approx 0.13$]
    - high density [e.g. $PBWO_4$: 8.3 g/cm^3]
    - good energy resolution (→Calorimeters)
  - Disadvantages complicated crystal growth
  - large temperature dependence

Light yield ε_{sc} ≡ fraction of energy loss going into photons

EXPENSIVE

- Organic Scintillators
  - Advantages
    - very fast ➔pulse shape discrimination possible
    - easily shaped
    - small temperature dependence
  - Disadvantages
    - lower light yield [typical; $\varepsilon_{sc} \approx 0.03$]
    - low density [e.g. 1 g/cm^3]
    - radiation damage

CHEAP

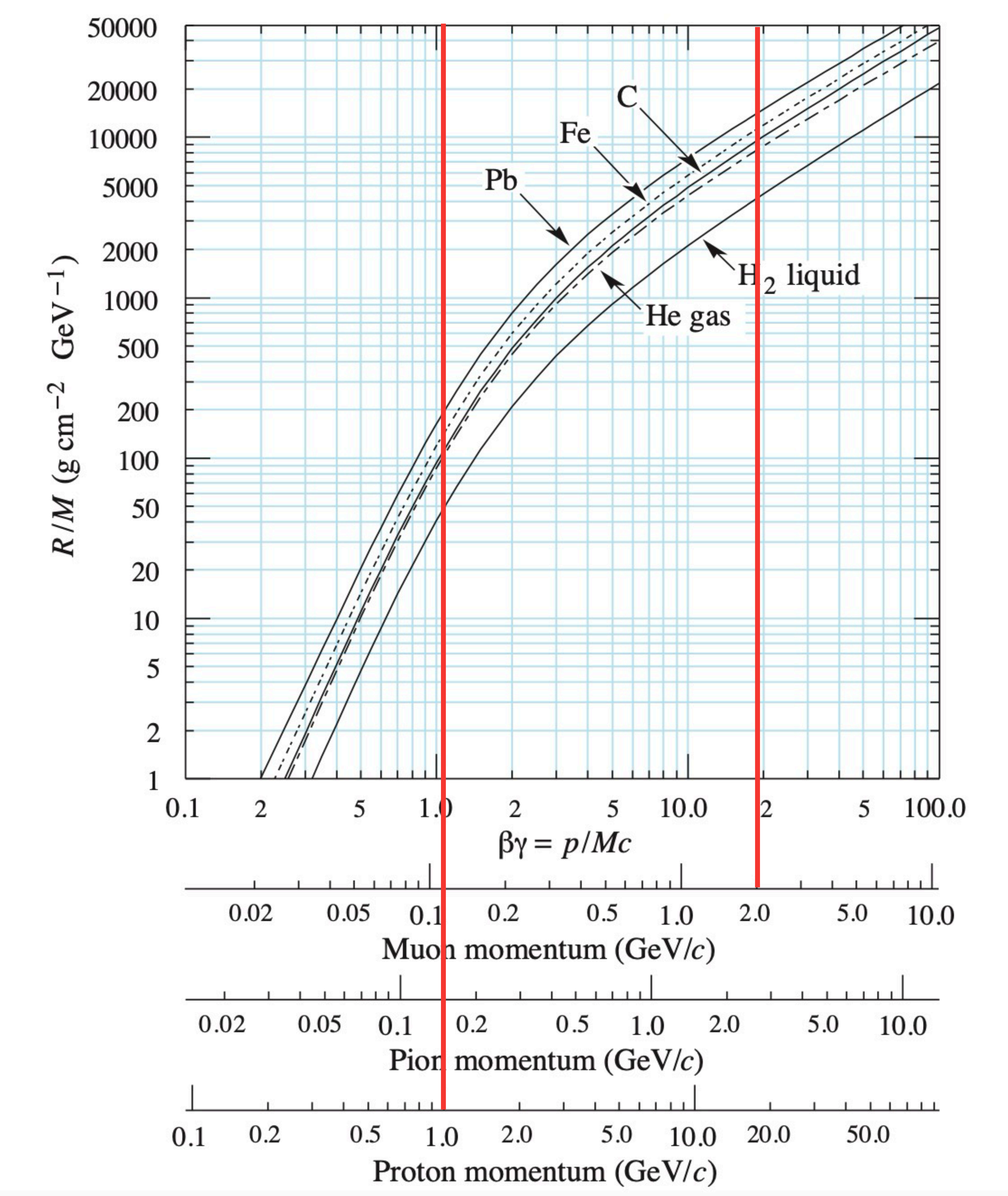


Sometimes need to use with WLS

And normally read out with fast photo-sensor

Momentum measurement

Range of particle slowing down to stop.



- For a particle of mass m and kinetic energy of E_0 entering matter, it loses energy via ionisation and excitation until stop $R(E_0) = \int_{E_0}^{0} \frac{1}{dE/dx} dE$
- Useful for muons below few 100 GeV
- Particle range is proportional to the particle momentum
- Eg:
  - 1GeV proton ($M_p = 0.938\ GeV$) on C (scintillator $\rho = 1.021\ g\,.\,cm^{-3}$), R/M =100$g\,.\,cm^{-2} GeV^{-1}$. So $R = 100 * 0.983/1.021 \approx 100\ cm \equiv 1\ m$
  - 2GeV muon ($M_\mu \approx 0.106\ GeV$) on C, $R/M = 10000\ g\,.\,cm^{-2}\,.\,GeV^{-1}$. So $R = 10000 * 0.106/1.021 \approx 1038\ cm \equiv 10.38\ m$

Momentum measurement w/ curvature in magnetic field

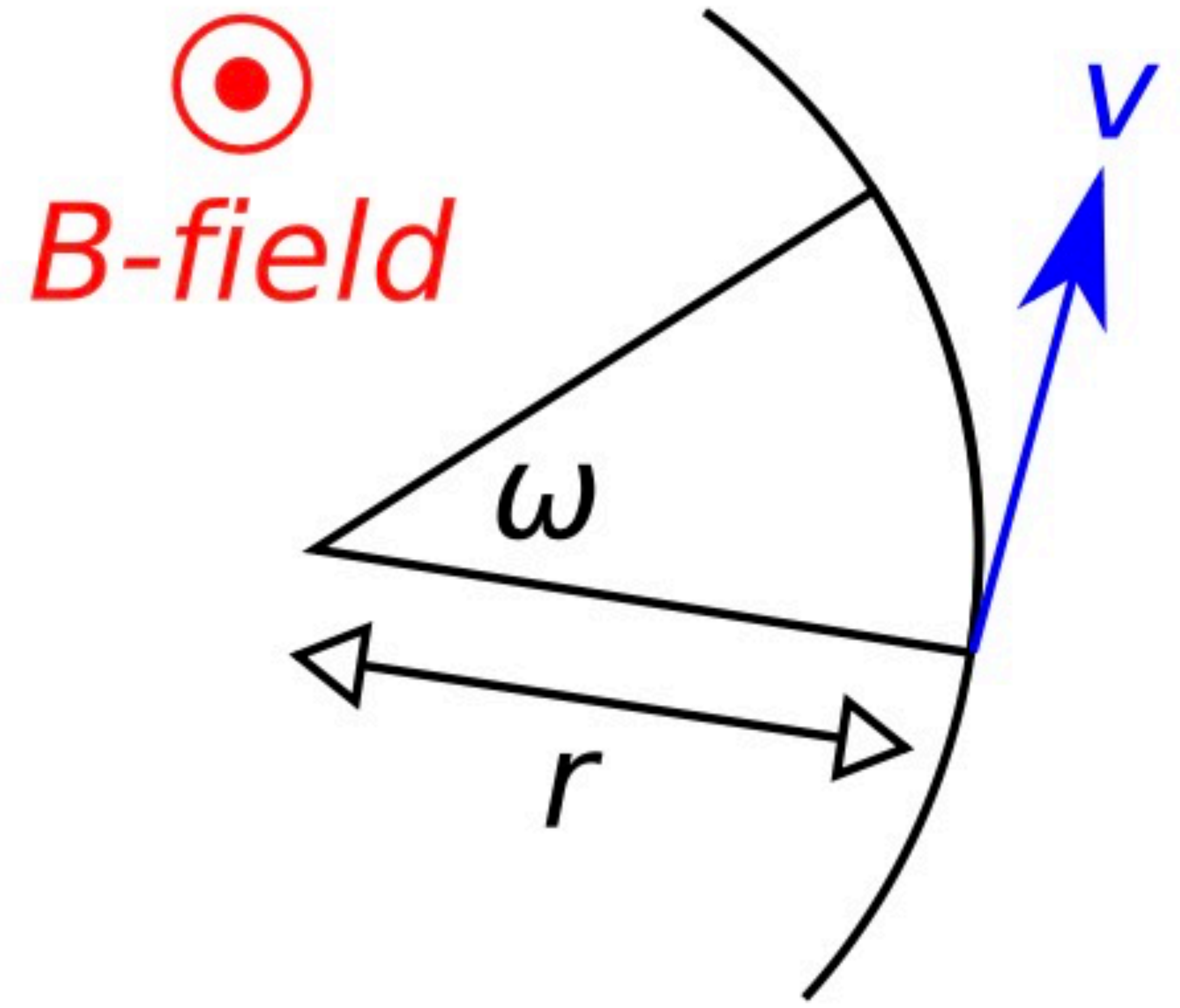
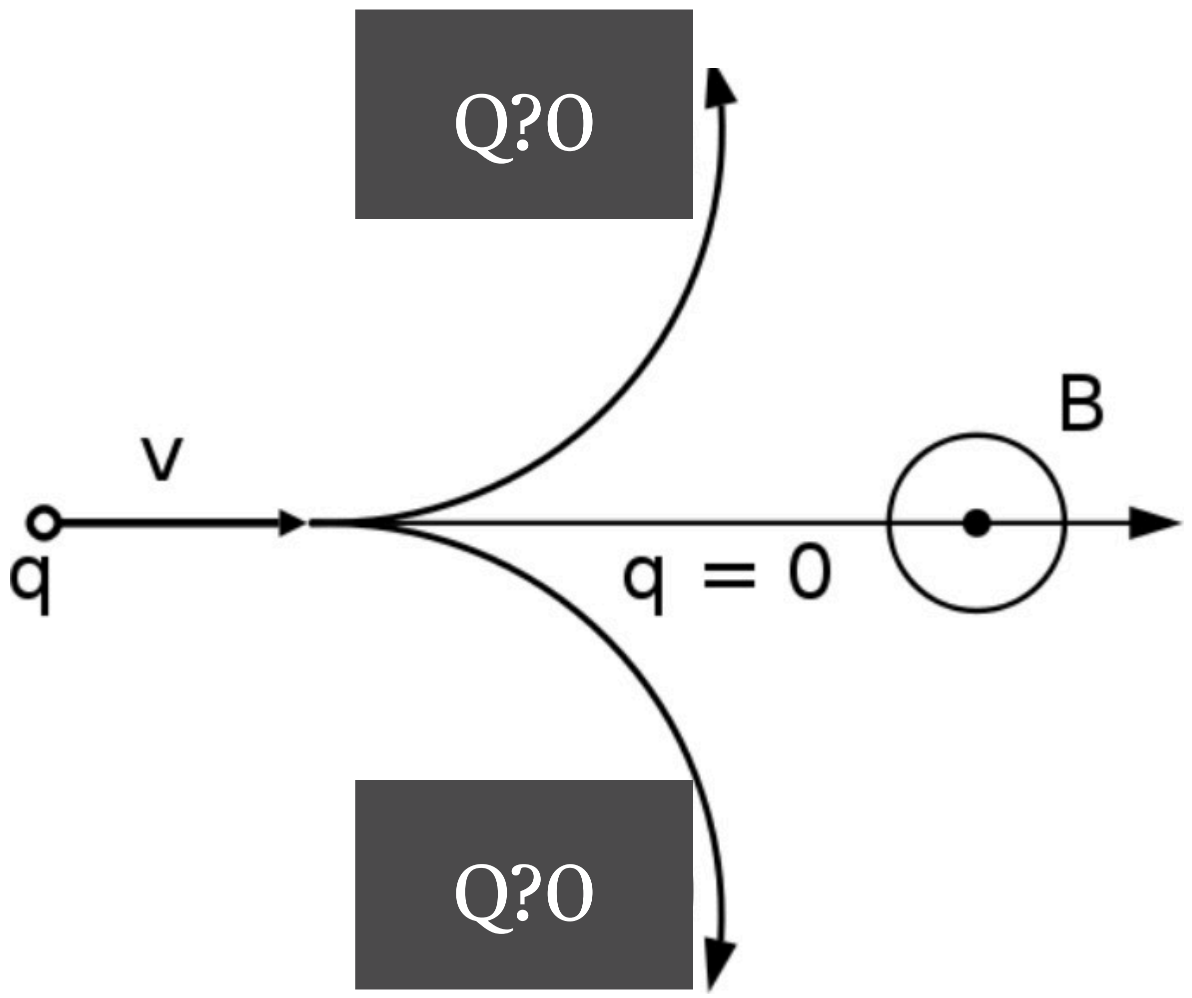


Figure 1: A particle with velocity v entering a magnetic field. In unit time the particle travels a distance v around the circle and turns through an angle ω.



$$F = qvB$$

$$ma = qvB$$

$$m\left(\frac{v^2}{r}\right) = qvB$$

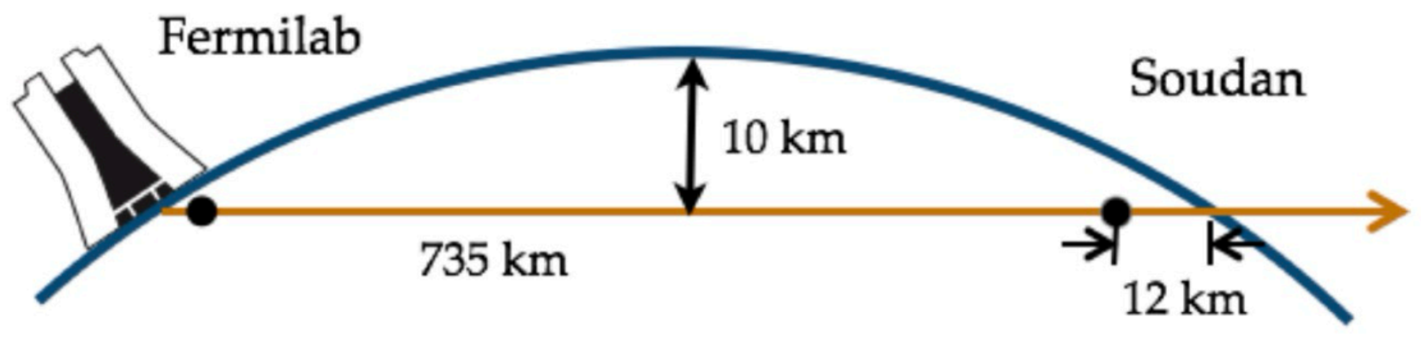
$$\frac{mv}{r} = qB$$

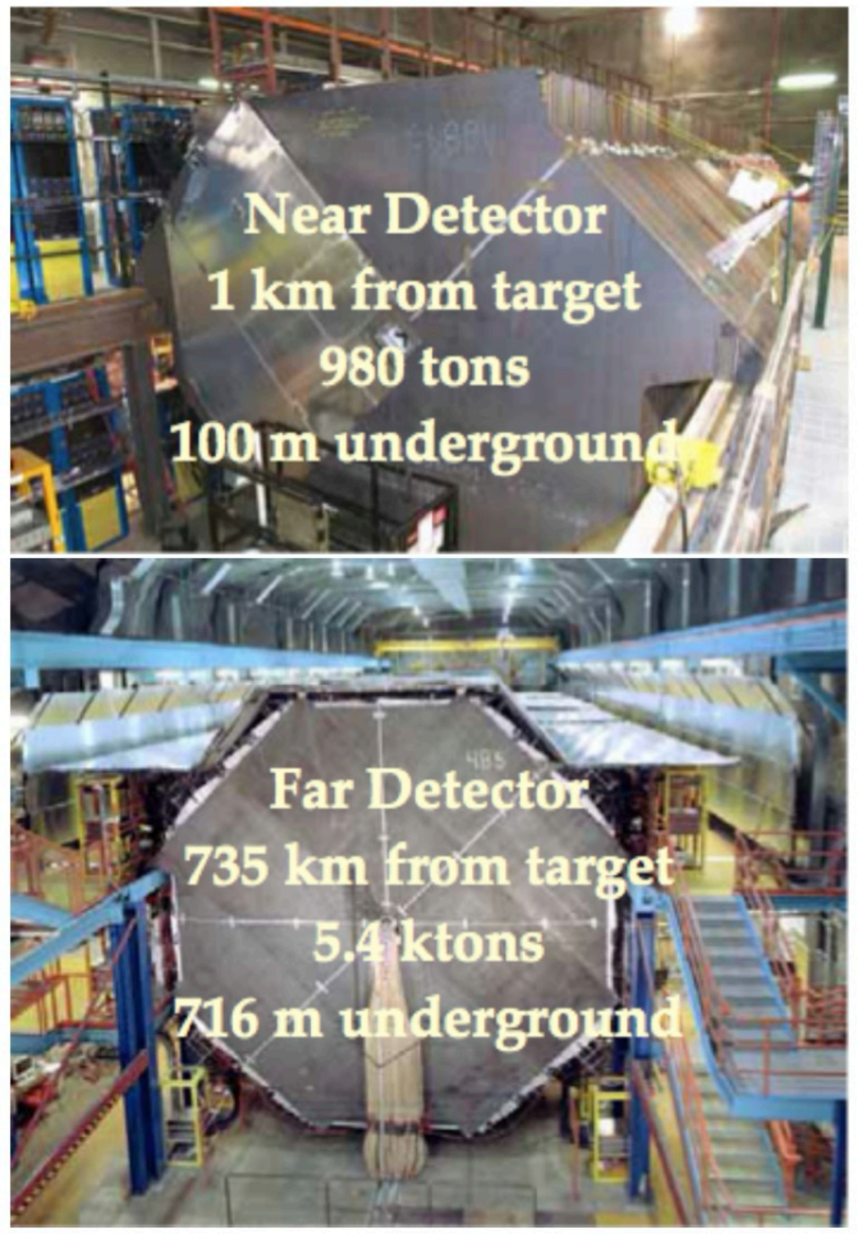
$$r = \frac{mv}{qB} = \frac{p}{qB}$$

$$r \propto p$$

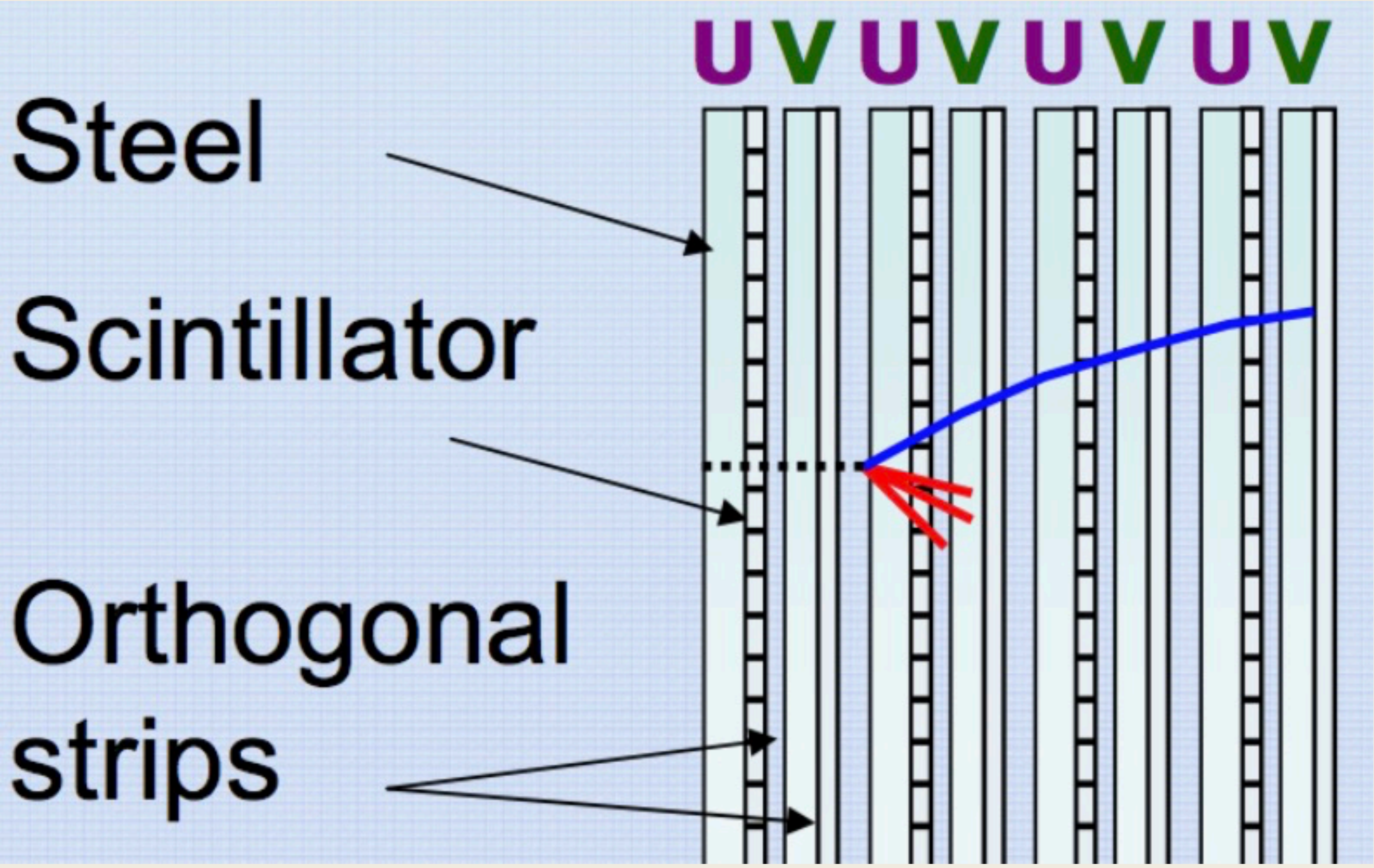
- Lorentz force, left-hand rule
- Radius of the curvature is proportional to the particle momentum
- If β can be measured independently, mass of particle can be inferred
- Direction curvature bend also can be used to distinguish positive and negative charge

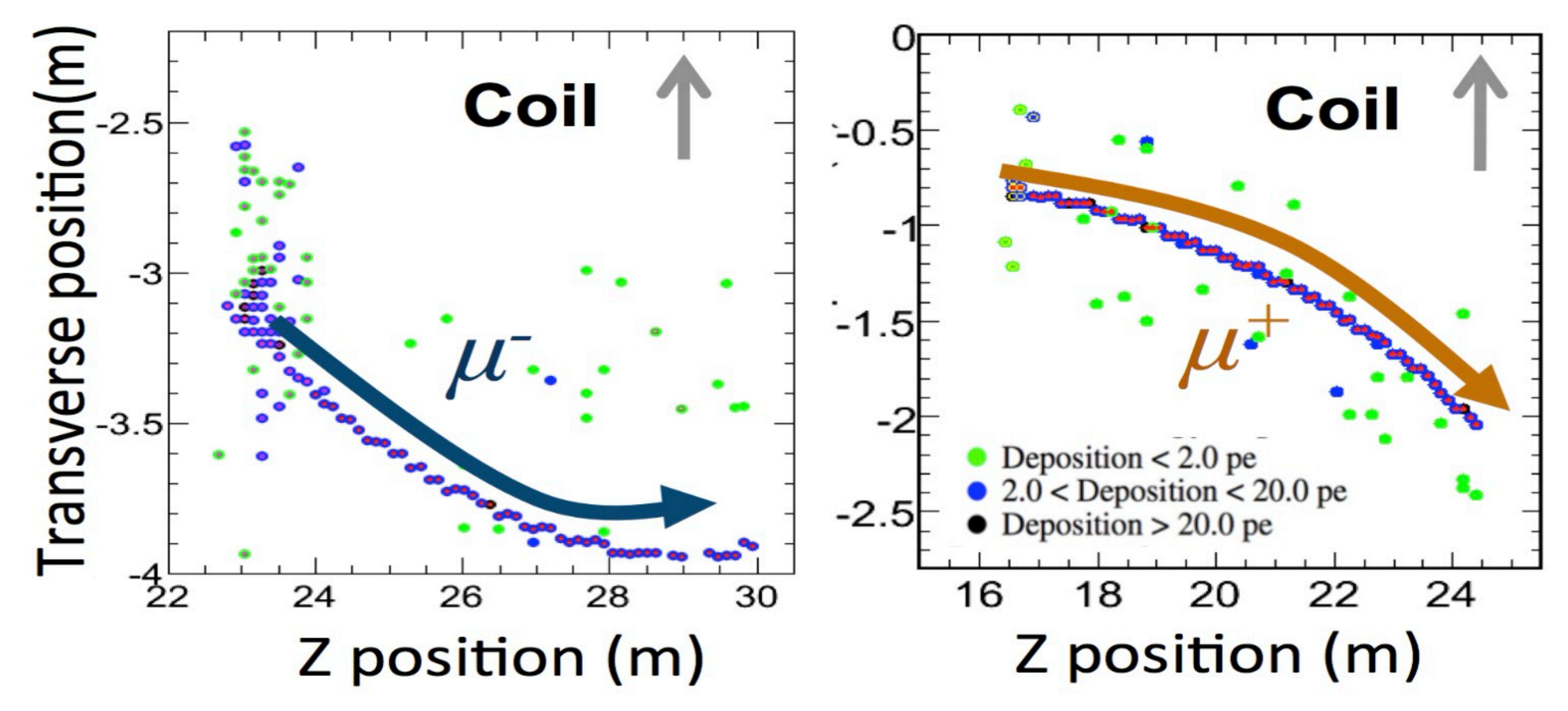
Eg: Tracking with MINOS experiment





- NuMI high intensity neutrino beam
- Near Detector at Fermilab, IL
- Far Detector at Soudan, MN
- Two-detector design to mitigate systematic uncertainties





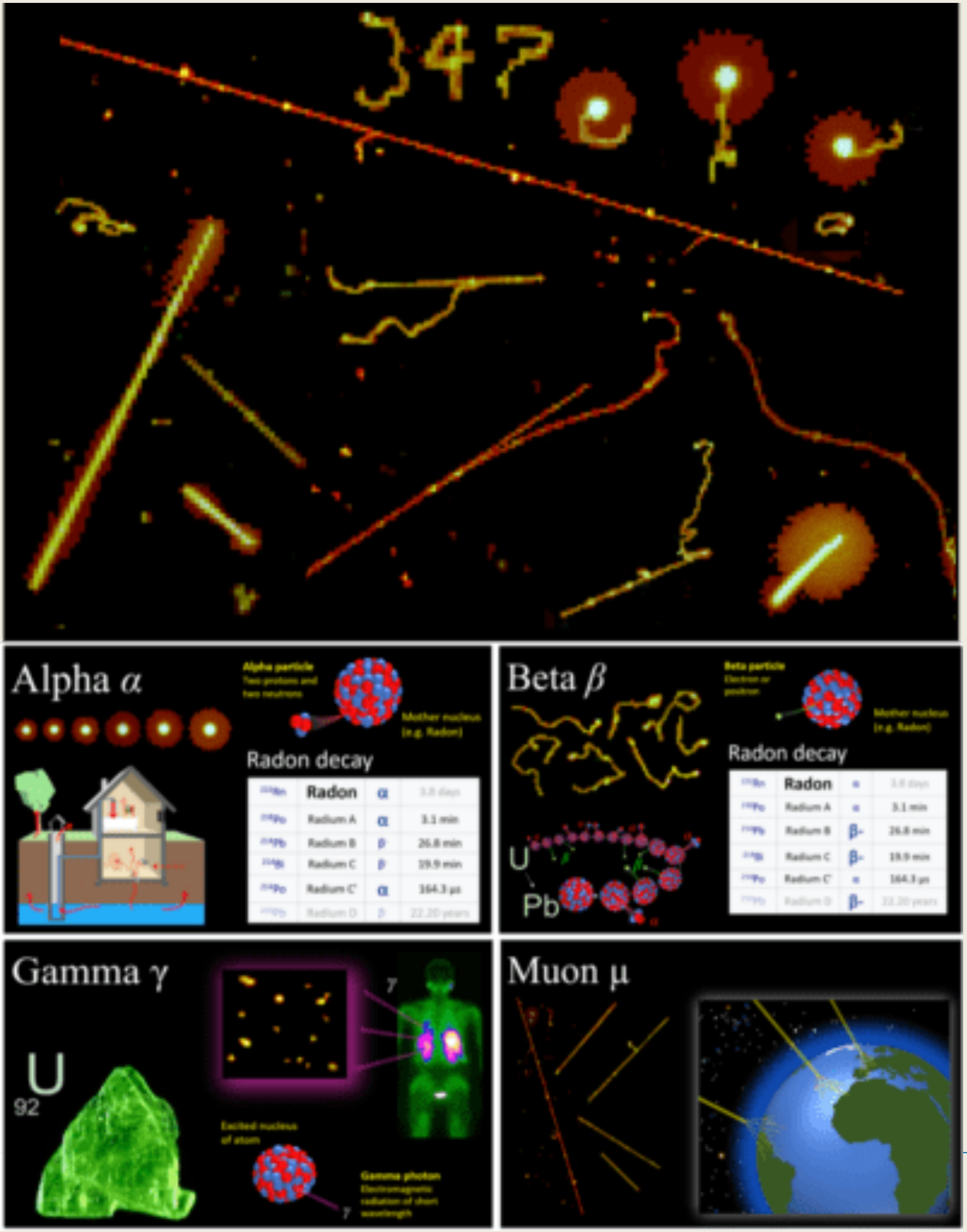
Tracking detector

Tracking detector

To reveals the path, or “track”, of a (charged) particle by providing

- Spatial information
- Temporal information
- *While tracking, the detector also provide (partly) information of particle energy via measuring of the energy loss*
- *Sometimes, tracking detector can be designed to measure some particle properties, eg. charge, and/or momentum.*

Eg. State-of-art of radiation tracking to the classroom



A transferred technology by CERN, being use by NASA

(Sometimes in the future, we hope to have this!)

https://advacam.com/application/education/

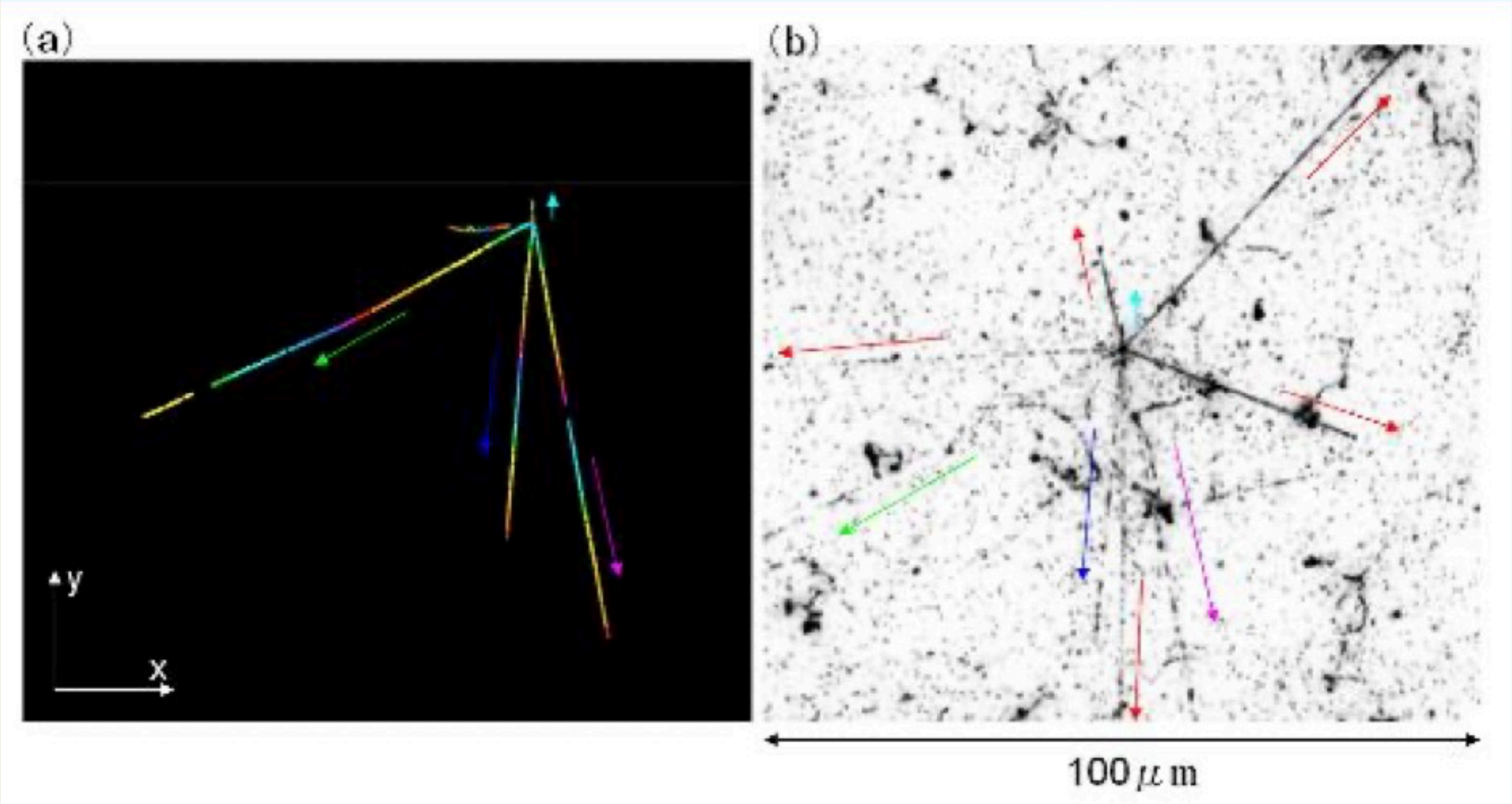
Principle for tracking

Charged particle passing through matter

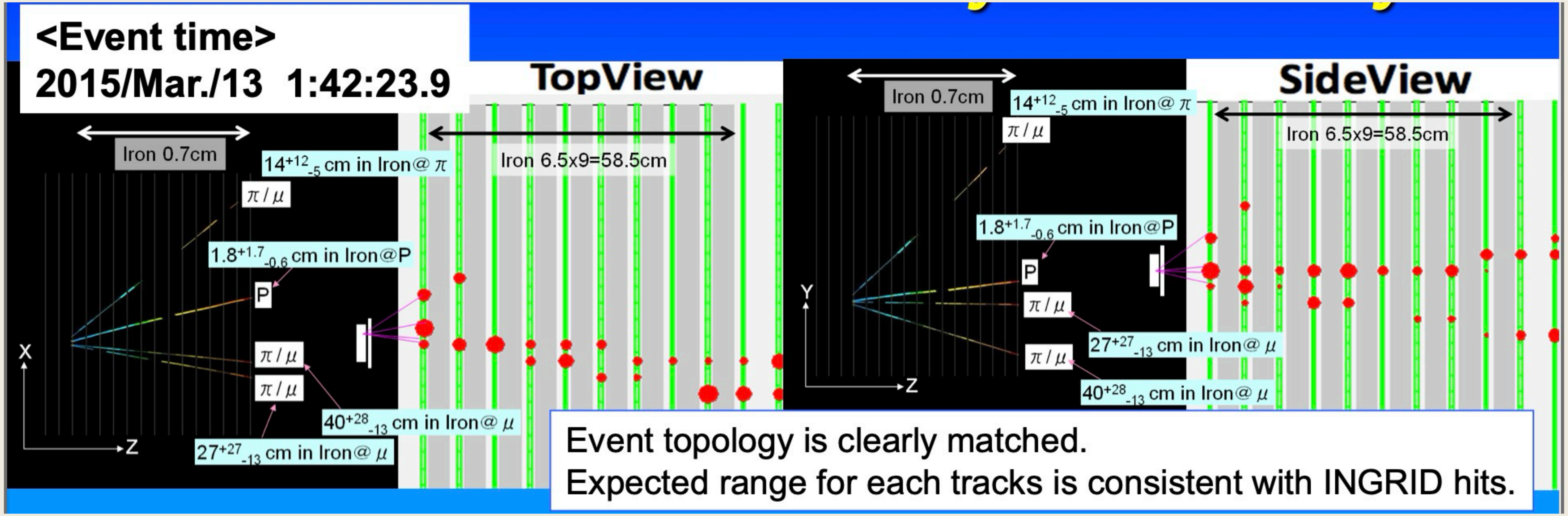
- Create **electron-ion pairs** in the gas
- **Electron-hole pairs** in semiconductor
- **Scintillation light** from excited molecules (liquid/plastic scintillators)
- **Cherenkov light** in the transparent materials
- …

Understand these fundamental processes is critical for designing a tracking system.

Eg: Tracking with NINJA (Emulsion technique)

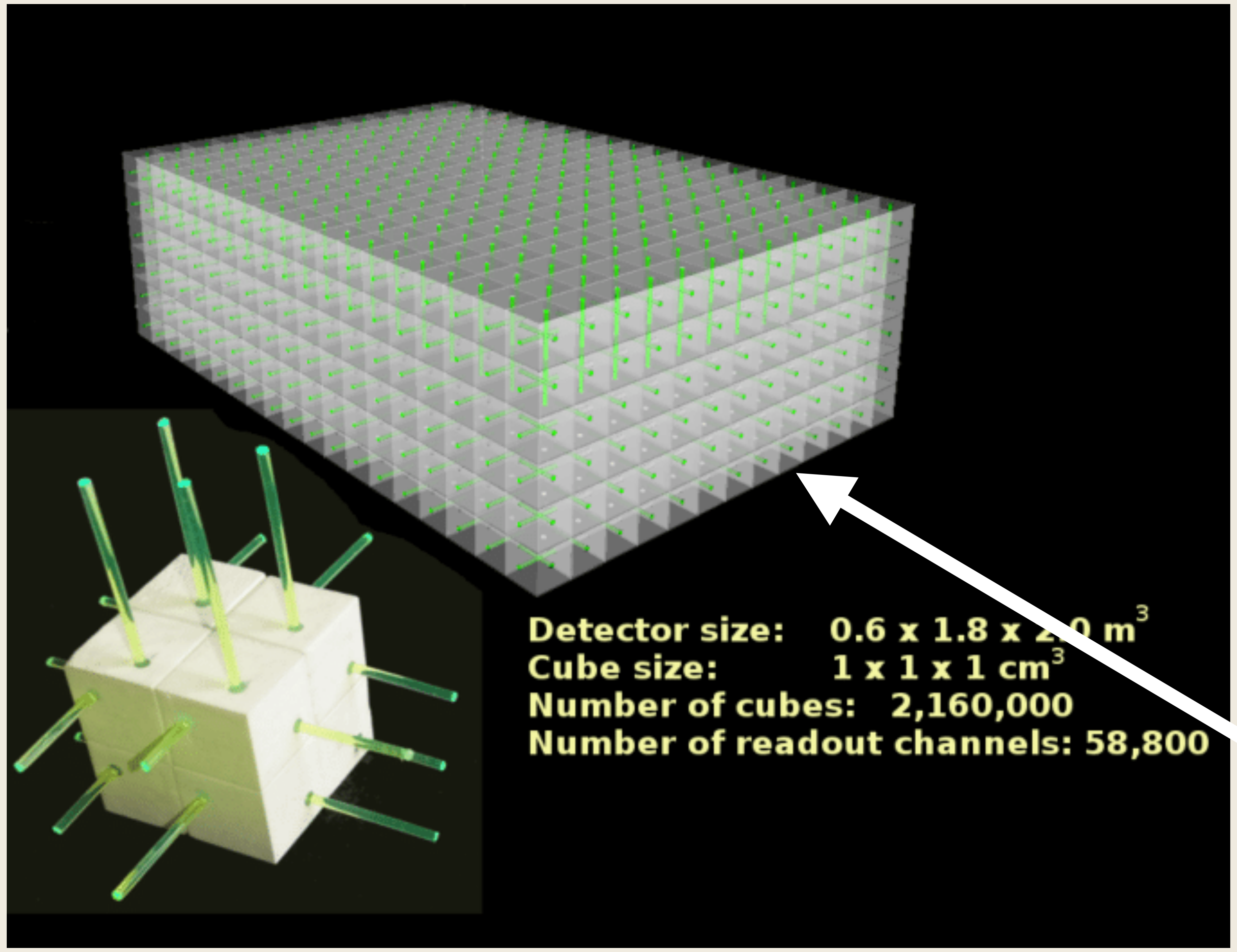


In hybrid w/ other detector

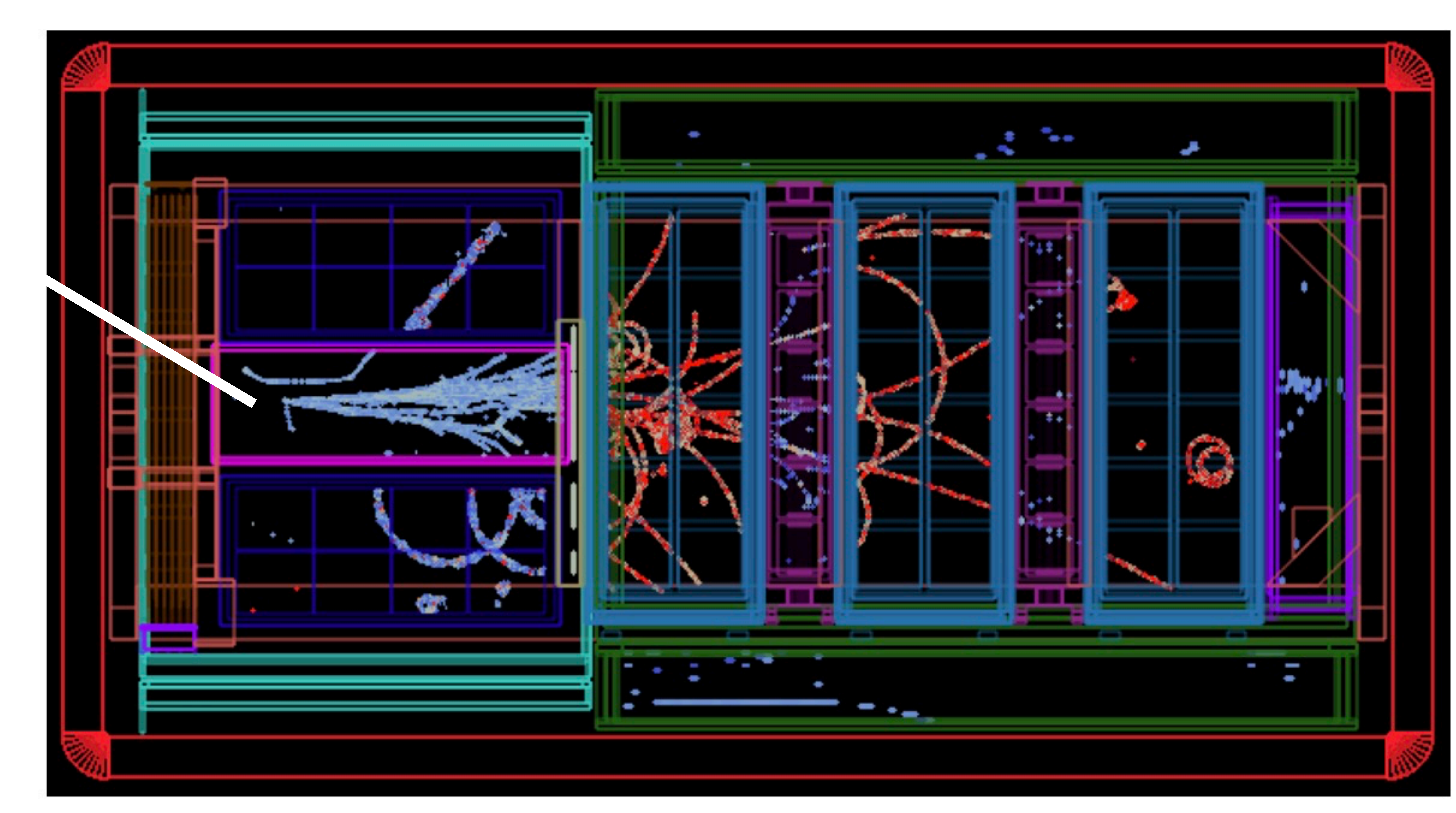

<Event time>
2015/Mar./13 1:42:23.9

Event topology is clearly matched.
Expected range for each tracks is consistent with INGRID hits.

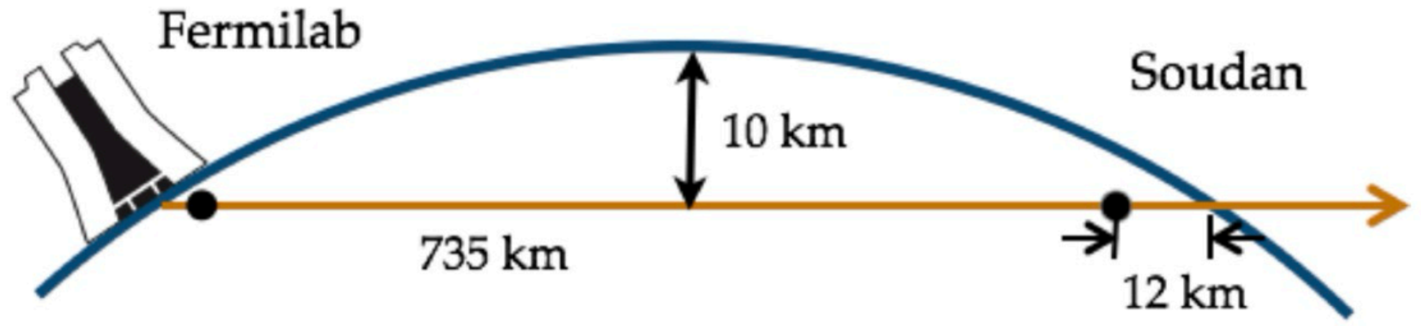
Eg: Tracking with Super-FGD



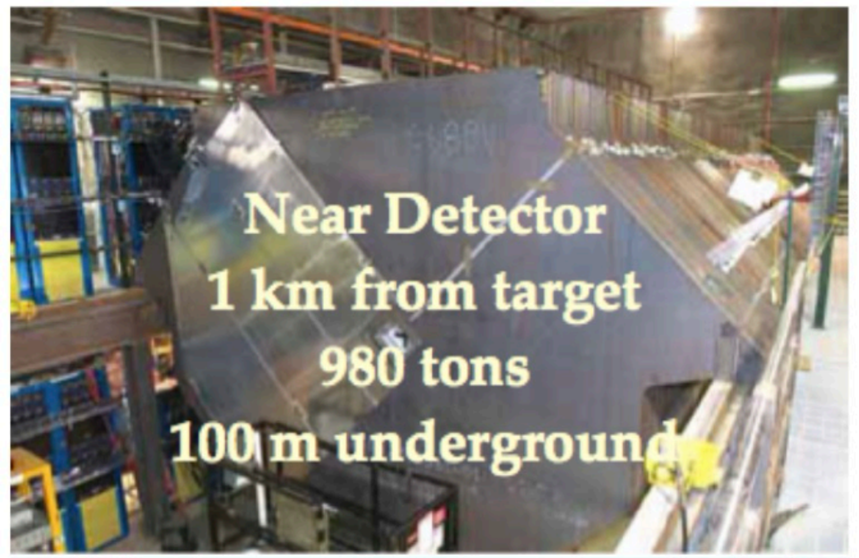
SCINTILLATION CUBES

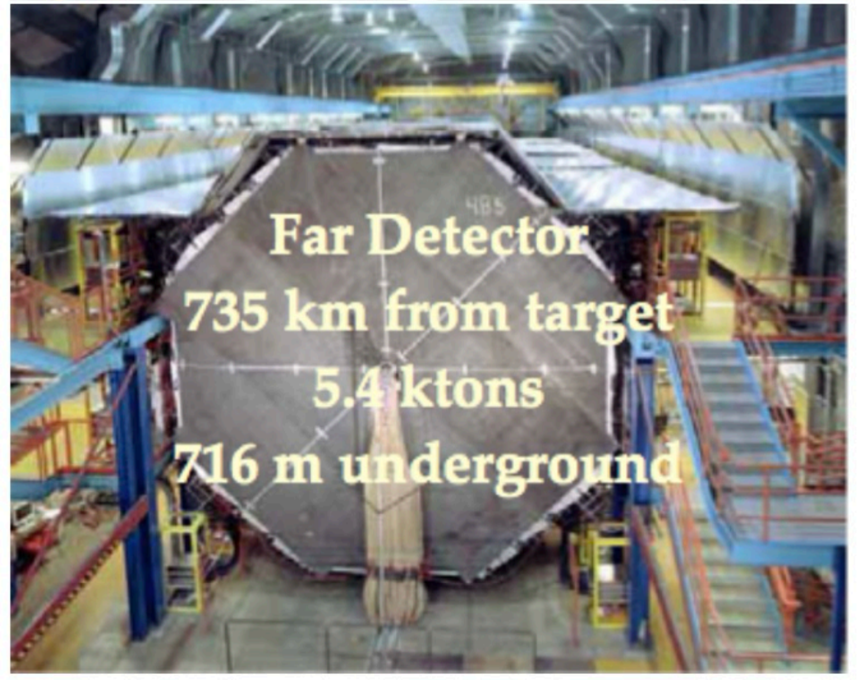


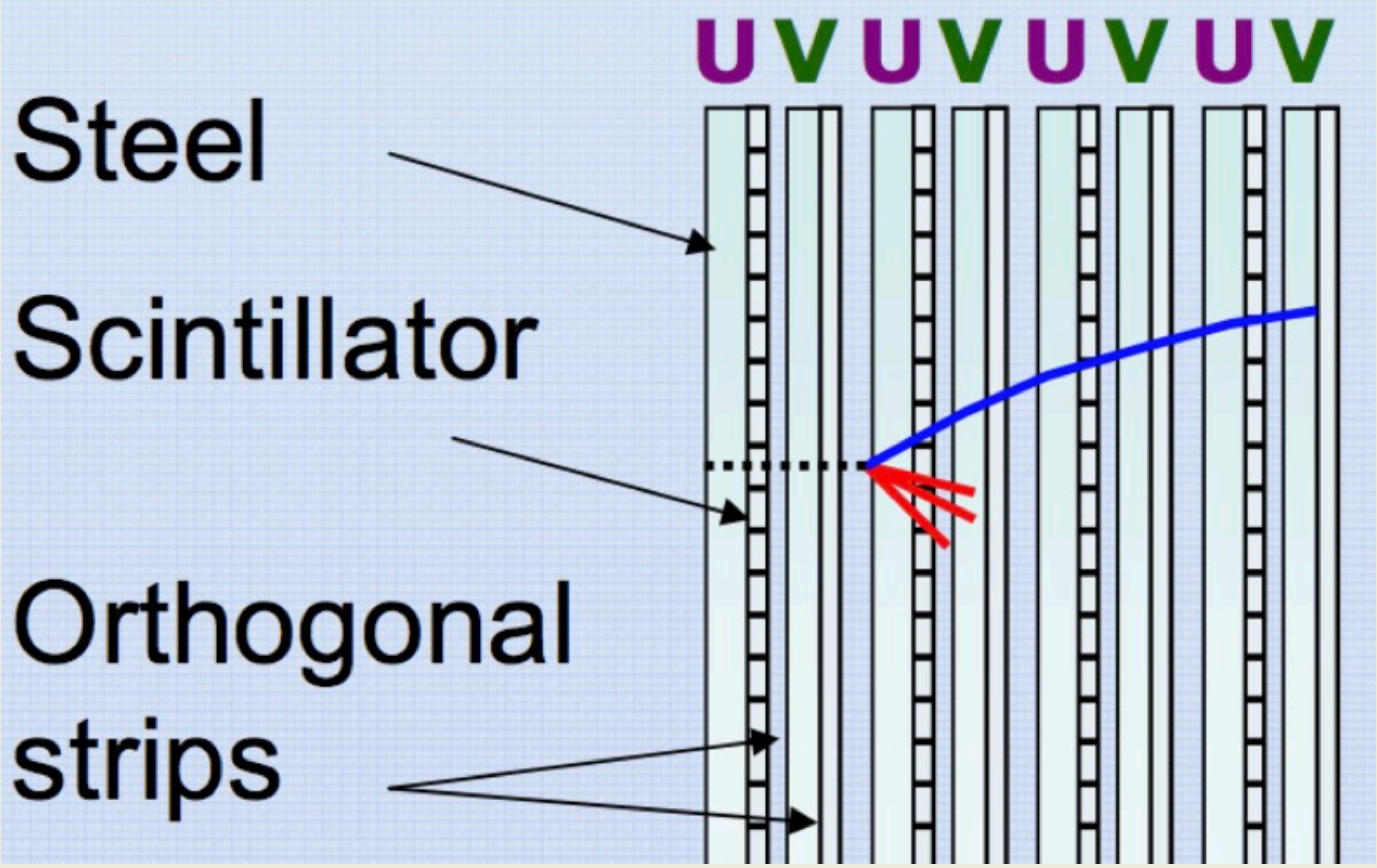
Eg: Tracking with charge identification

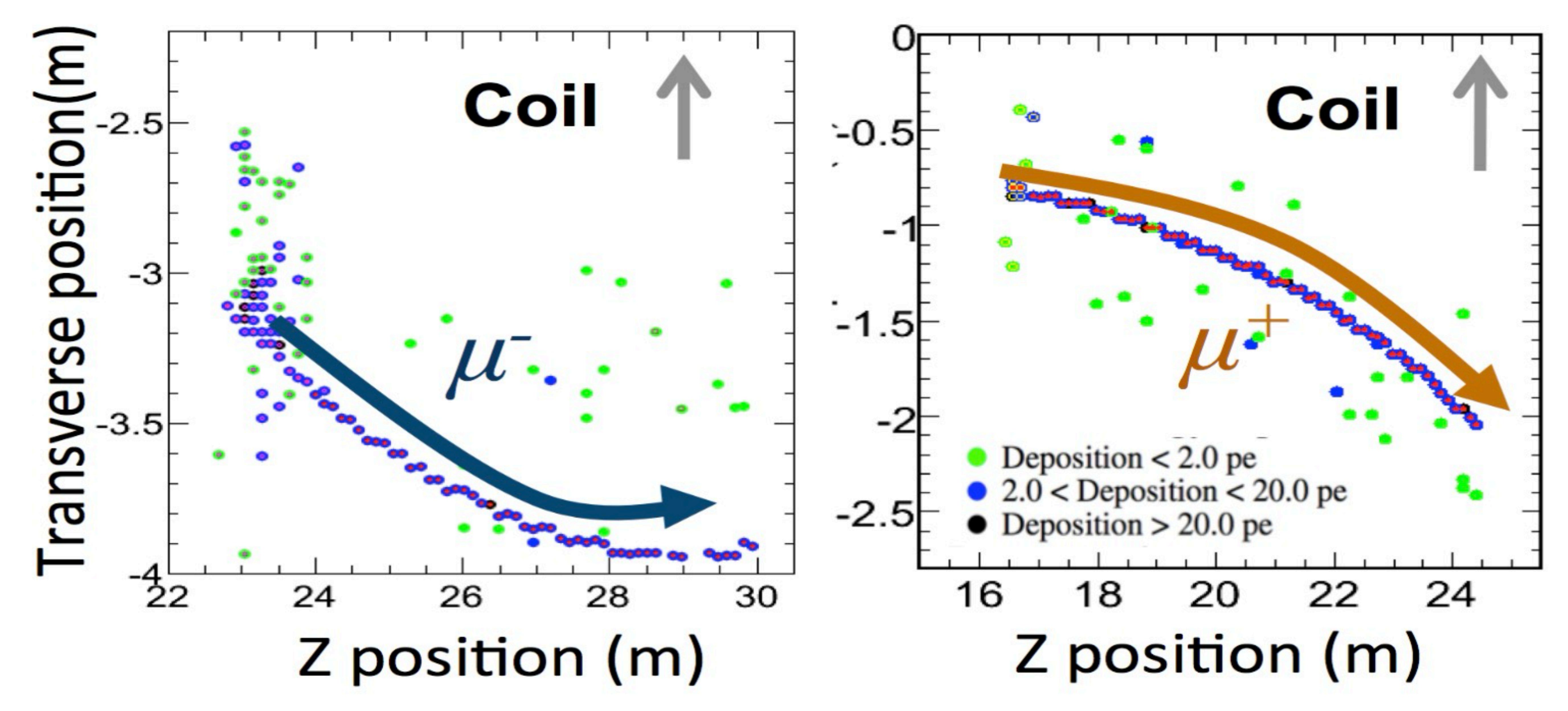


- ✧ NuMI high intensity neutrino beam
- ✧ Near Detector at Fermilab, IL
- ✧ Far Detector at Soudan, MN
- ✧ Two-detector design to mitigate systematic uncertainties



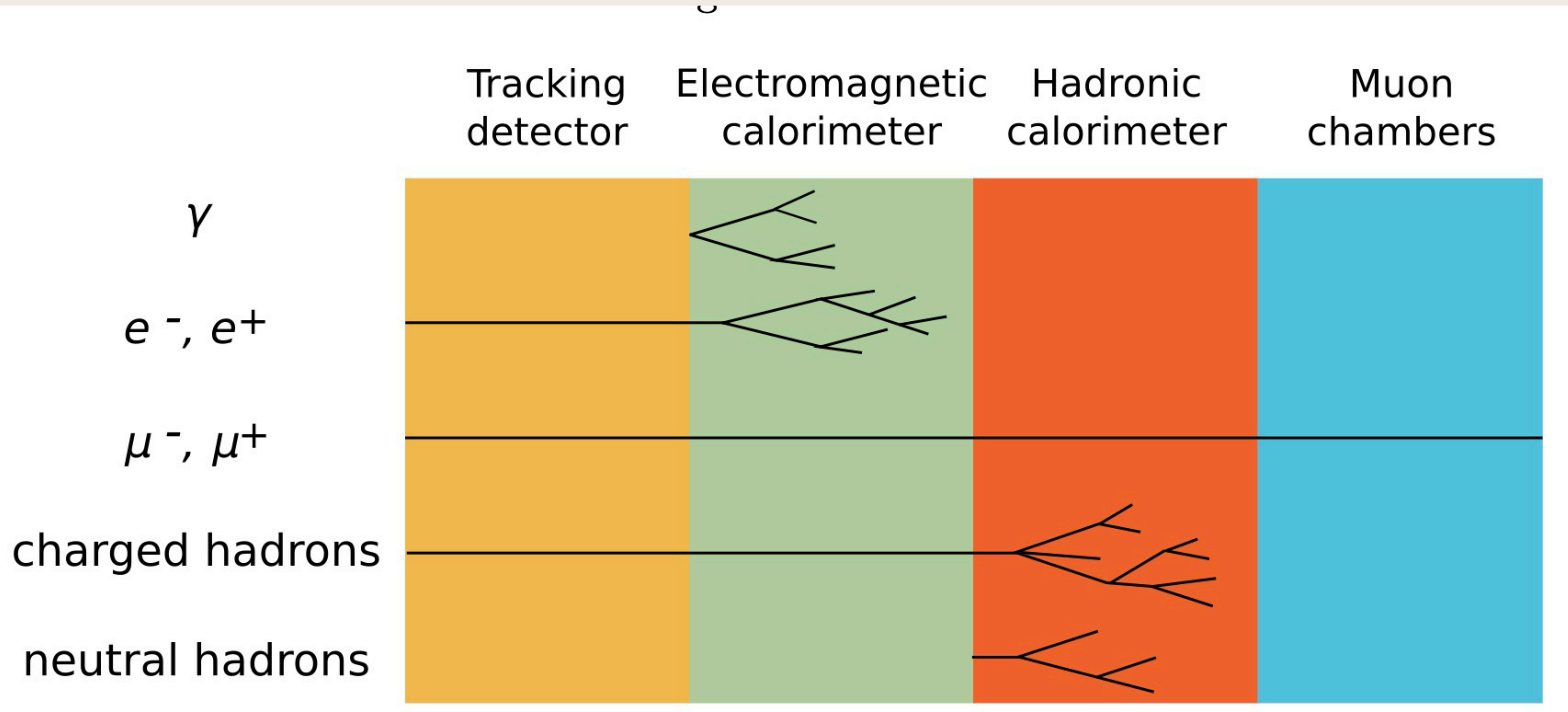






Example of State-of-art of particle detector

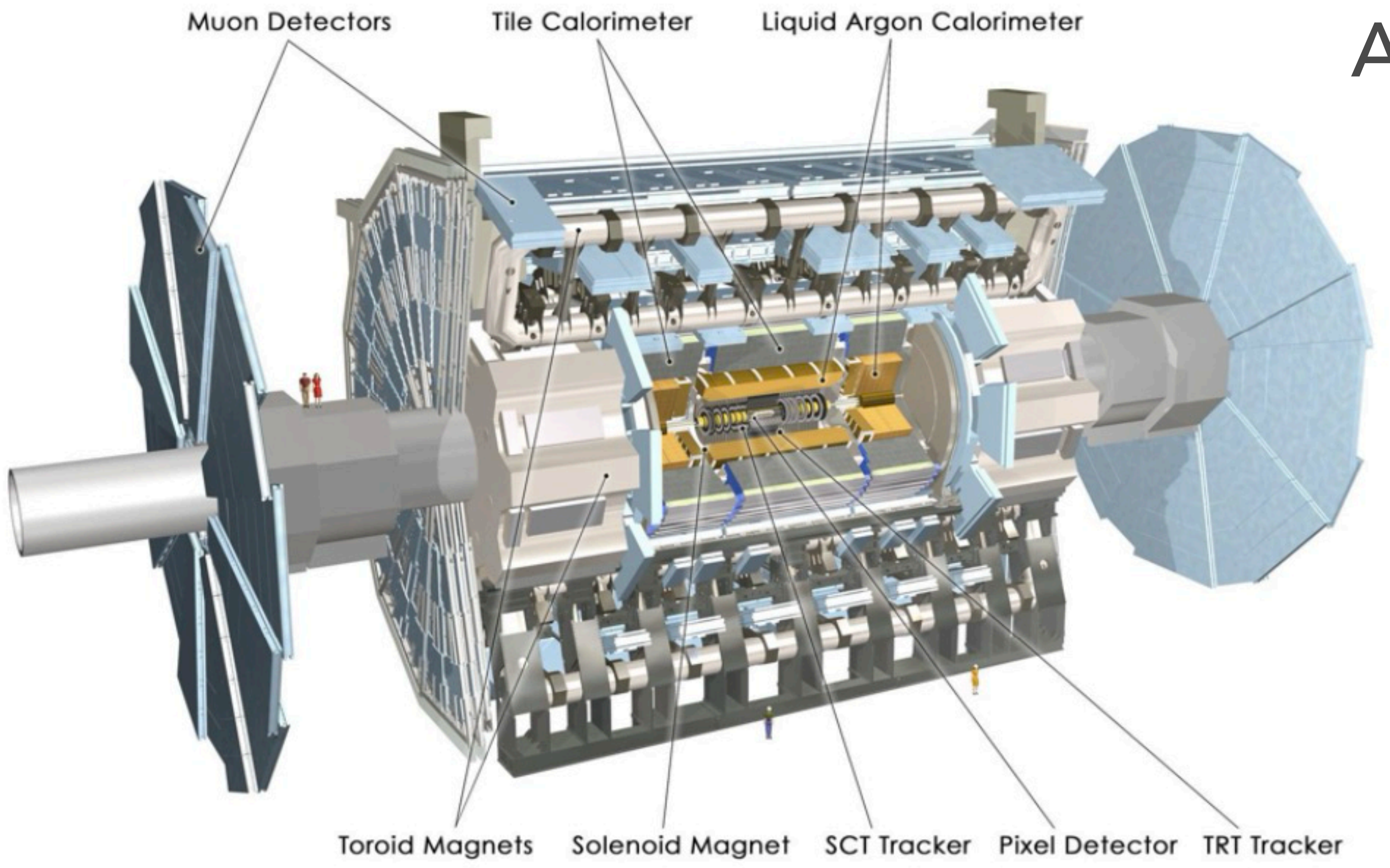
State-of-art of particle detection

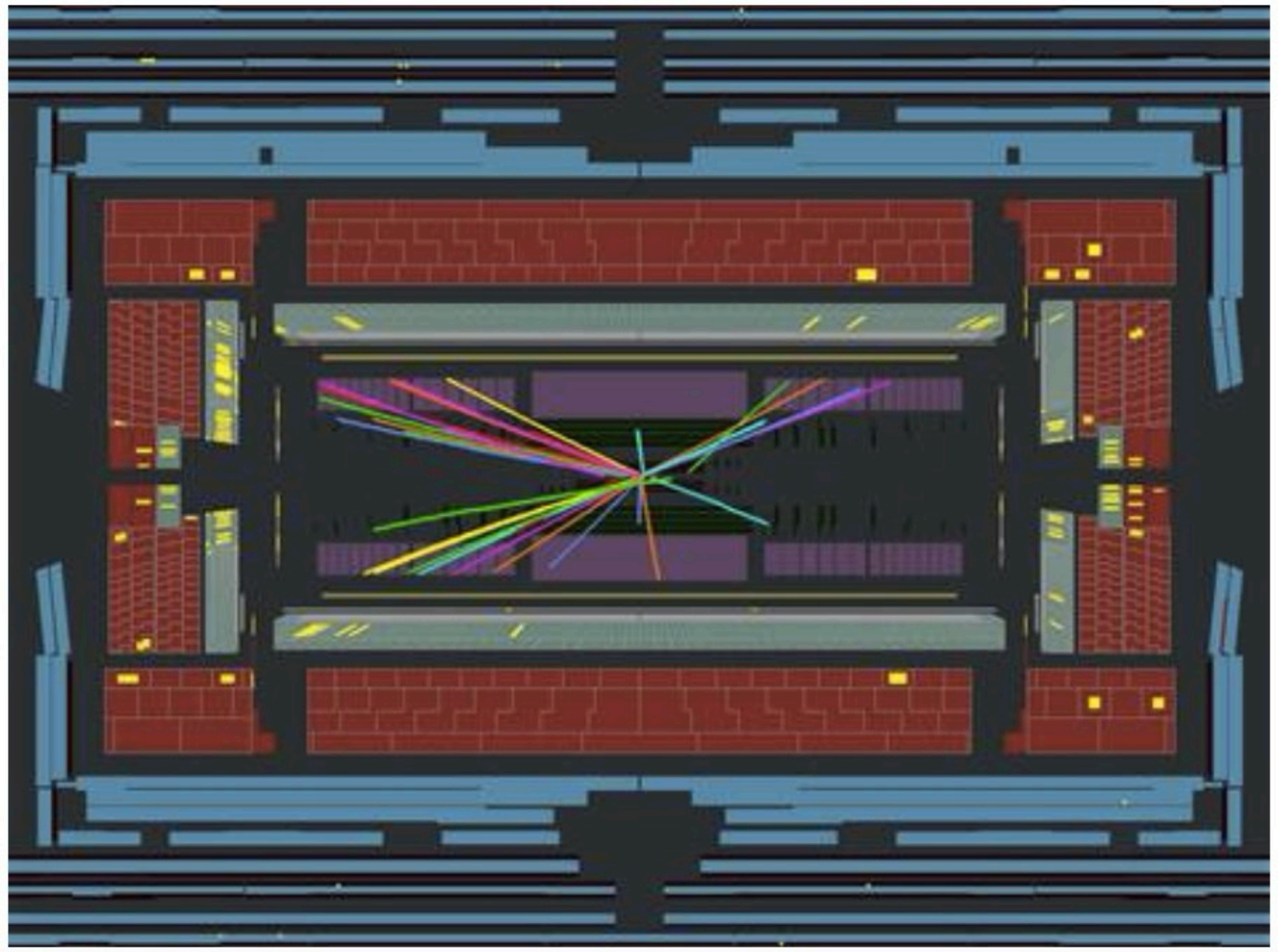


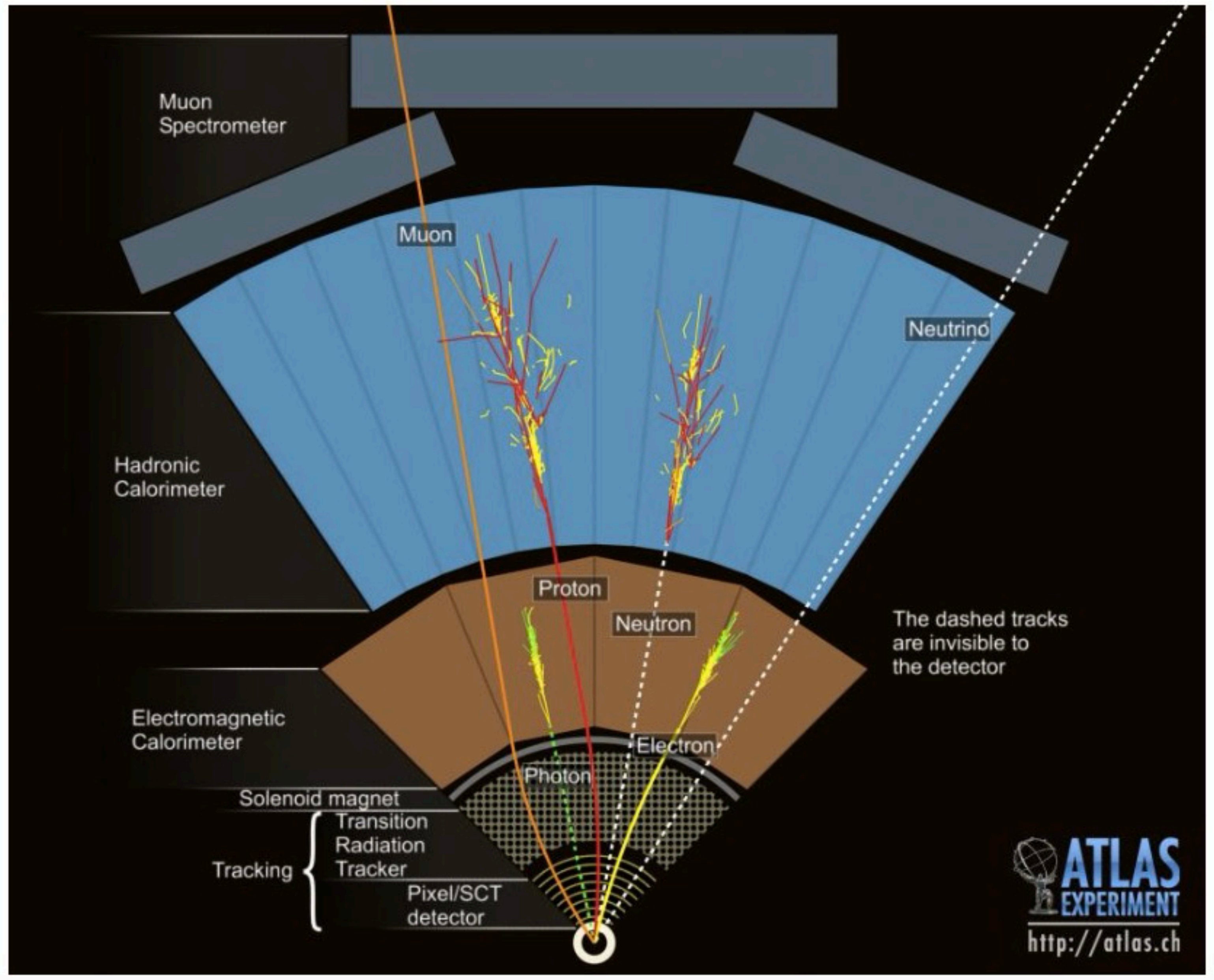
To put all sub-detector together for maximizing the physics potential output with other (*cost, space, radiation, power…*) consideration.

State-of-art of particle detection

ATLAST experiment







Application of particle detectors

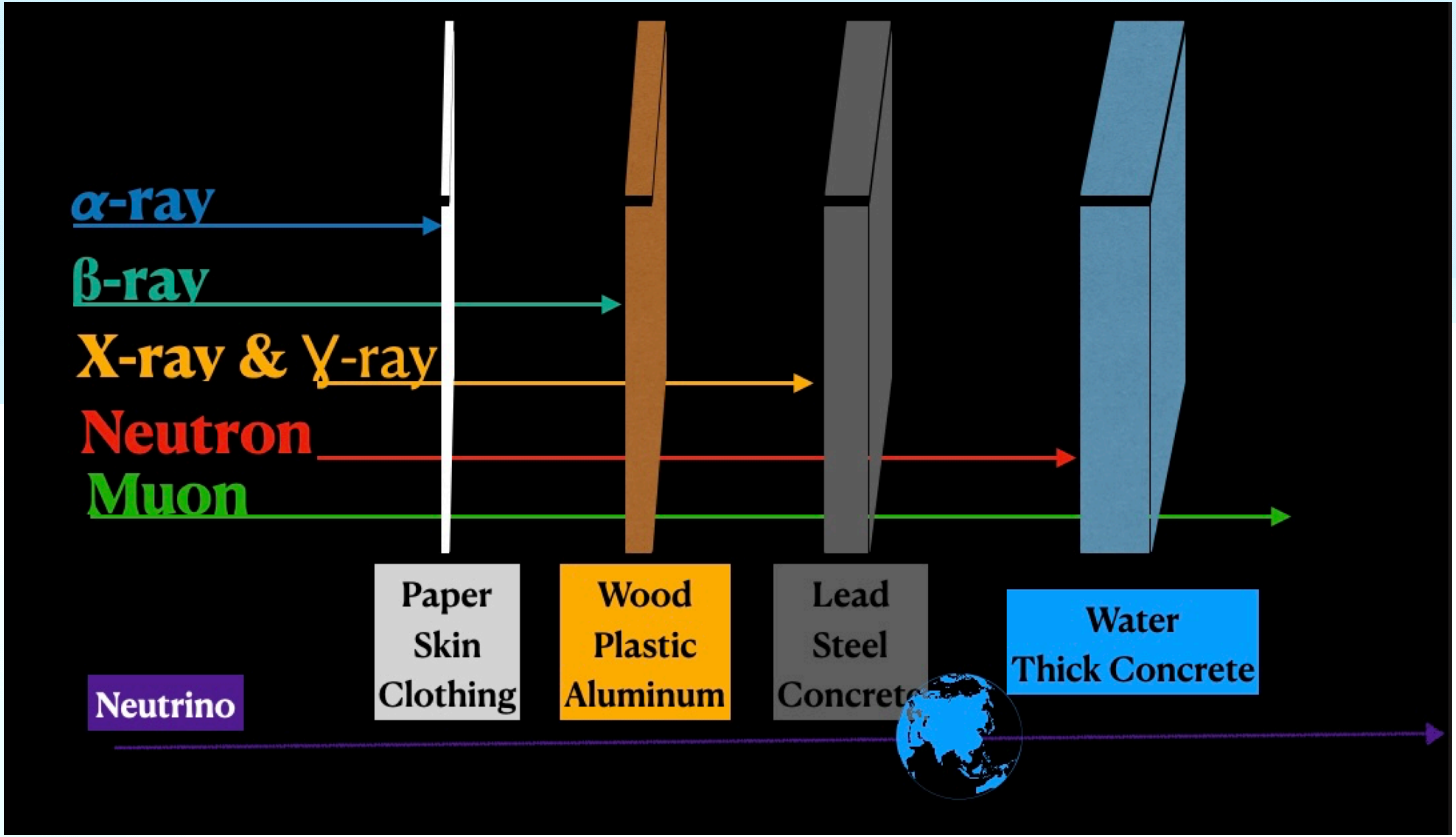
(Just few highlights)

Interplay between science and technology

- **Transistor** *was not invented by people who wanted to build computers but by physicists who dealing with counting the nuclear particles*
- **Nuclear power** *was not developed by one who seek for new power but Curies, Rutherford, and Fermi*
- **Electron discovery** *(credited for by J.J. Thomson) was not for electronic industry but to understand the basics of atoms.*
- **Induction coils** *in motor cars and other vast application, not invented by who want to make motor transport but the principle of induction was discovered by M. Faraday*
- **Communication with electromagnetic waves,** *founded by H. Hertz who wanted to emphasized the beauty of physics*
- **Global positioning system (GPS**) *can't function well if not including the General Relativity*
- **Development of new materials and molecules** *benefits from precise mathematical techniques used in particle physics*
- **Word Wide Web** was *first invented by particle physicists to share information quickly and effectively around the world.*
- **Cancer therapy, drug development** thanks to the **particle accelerators**
- **Behind of almost all photodetectors is photoelectric effect, which is discovered by Hertz in 1887 and modeled by Einstein in 1905**
- **Photodetectors developed and improved for particle physics** *drive industrial application such as x-ray, medical scanning*
- **Help for national security**, *eg. cargo scanning, looking insides of the nuclear reactor*
- …

IN RESPONSE, ADVANCEMENTS IN TECHNOLOGY ARE CRITICAL FOR BASIC RESEARCH.

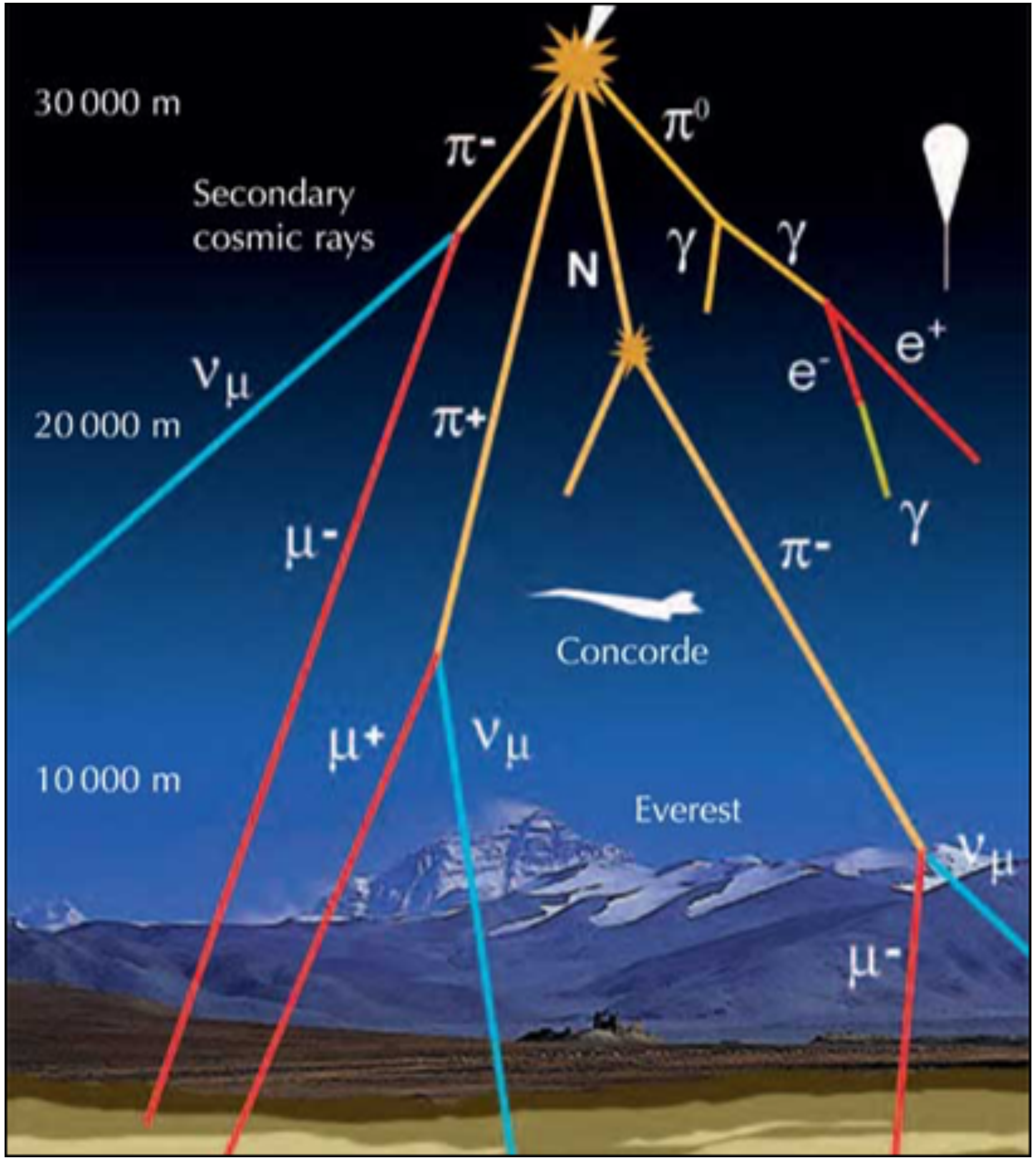
Penetration of particle through matter



High energy astrophysical particles (eg. hydrogen & helium from the Sun) interact with the Earth's atmosphere ➔produce vast amount of muons, brother of electrons (almost identical except the mass)

~ 1 muon/cm²/s

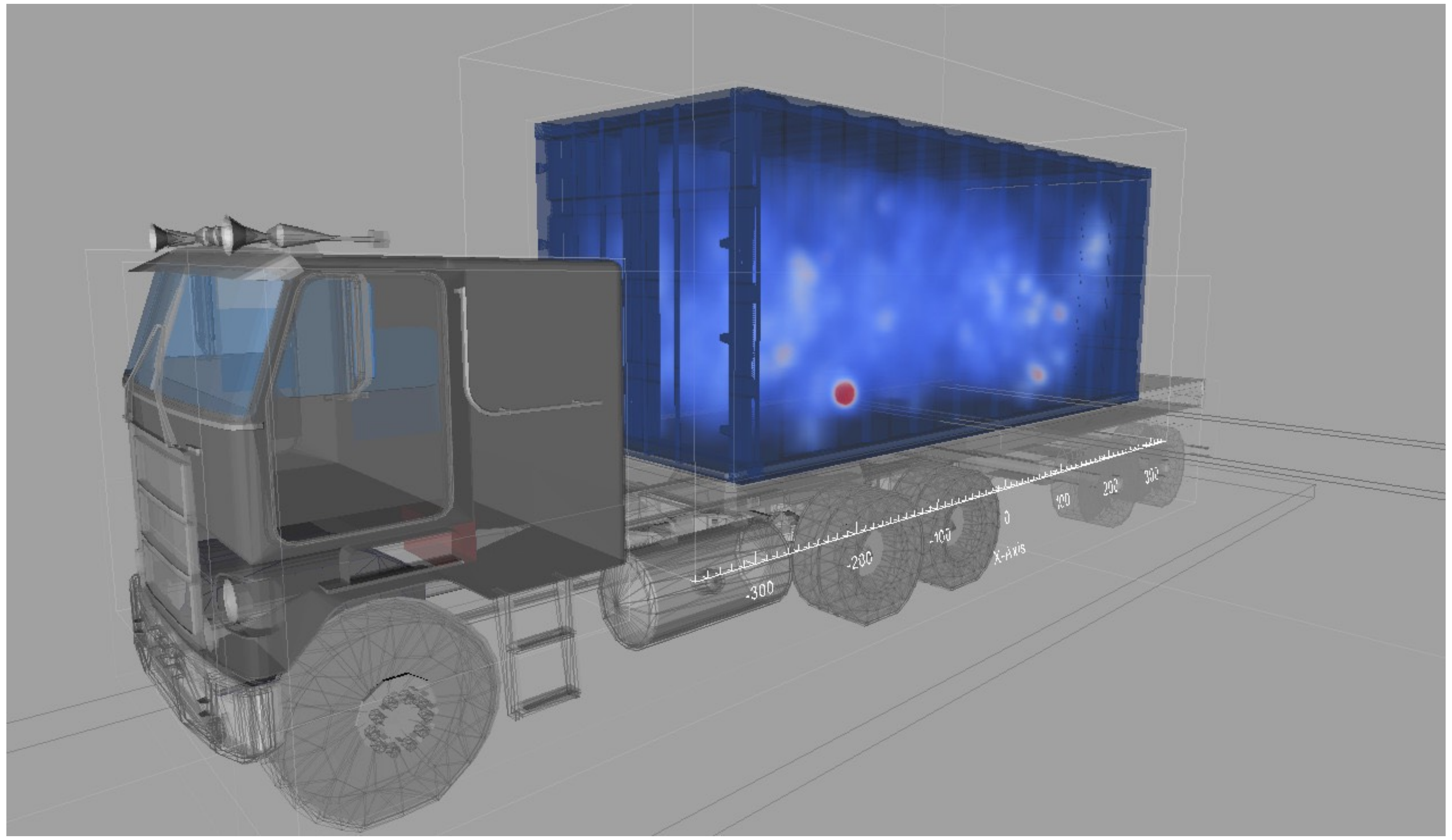
but you can't see them with your eyes



Application of muon and its detection

https://www.nature.com/articles/nature24647

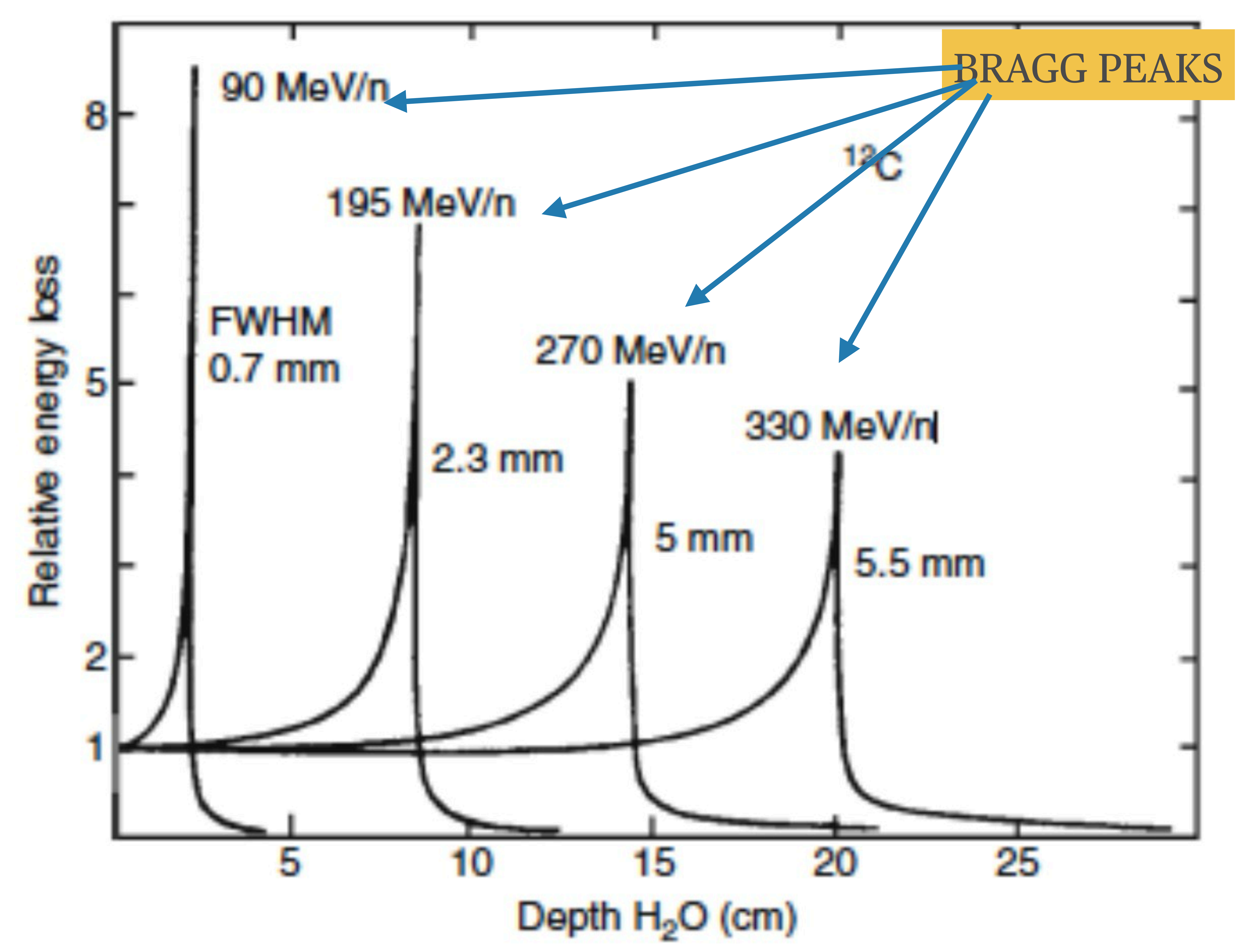
Scan pyramid

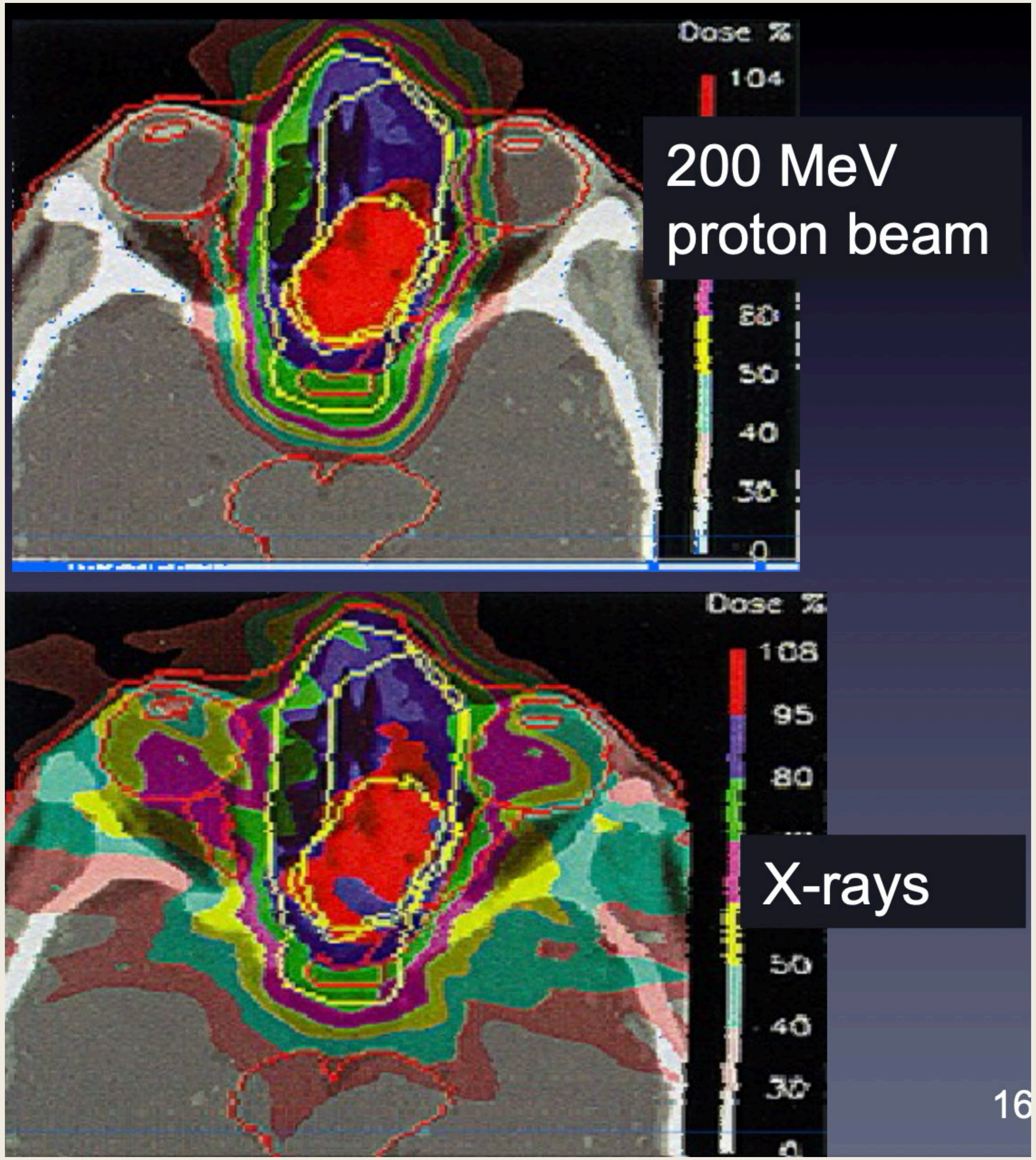


Homeland/ cross-border security

Proton therapy

Understand proton behaviour in matter is critical for proton therapy for cancer treatment





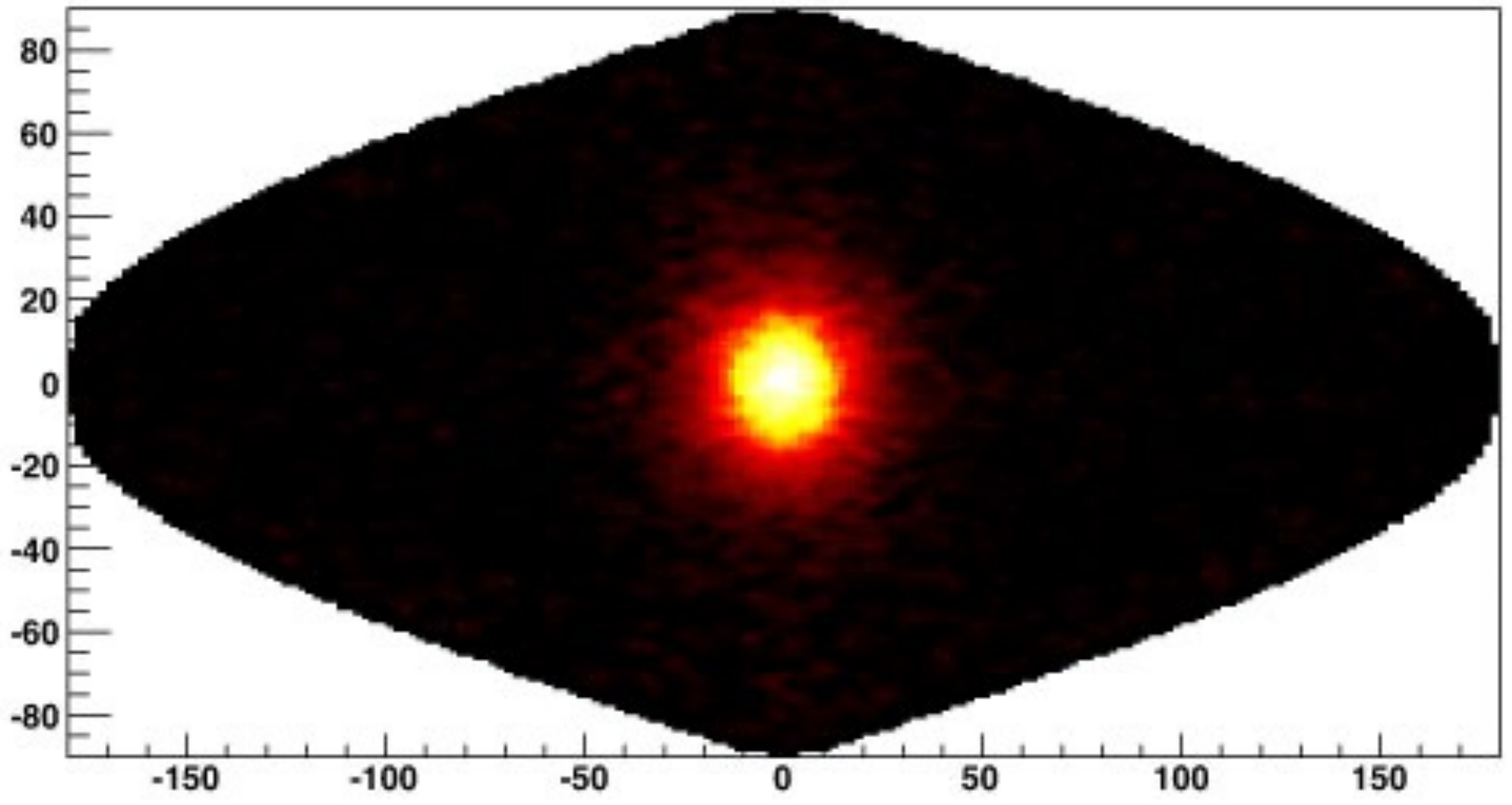
COMPARE PROTON THERAPY VS. X-RAYS TREATMENT

Can neutrino be a practical thing?

> *"**I don't say that the neutrino is going to be a practical thing**, but it has been a time-honored pattern that science leads, and then technology comes along, and then, put together, **these things make an enormous difference in how we live**"*
> ——**Fredrick Reines**, Nobel prize winner, co-discover of the neutrino, NYT 1997

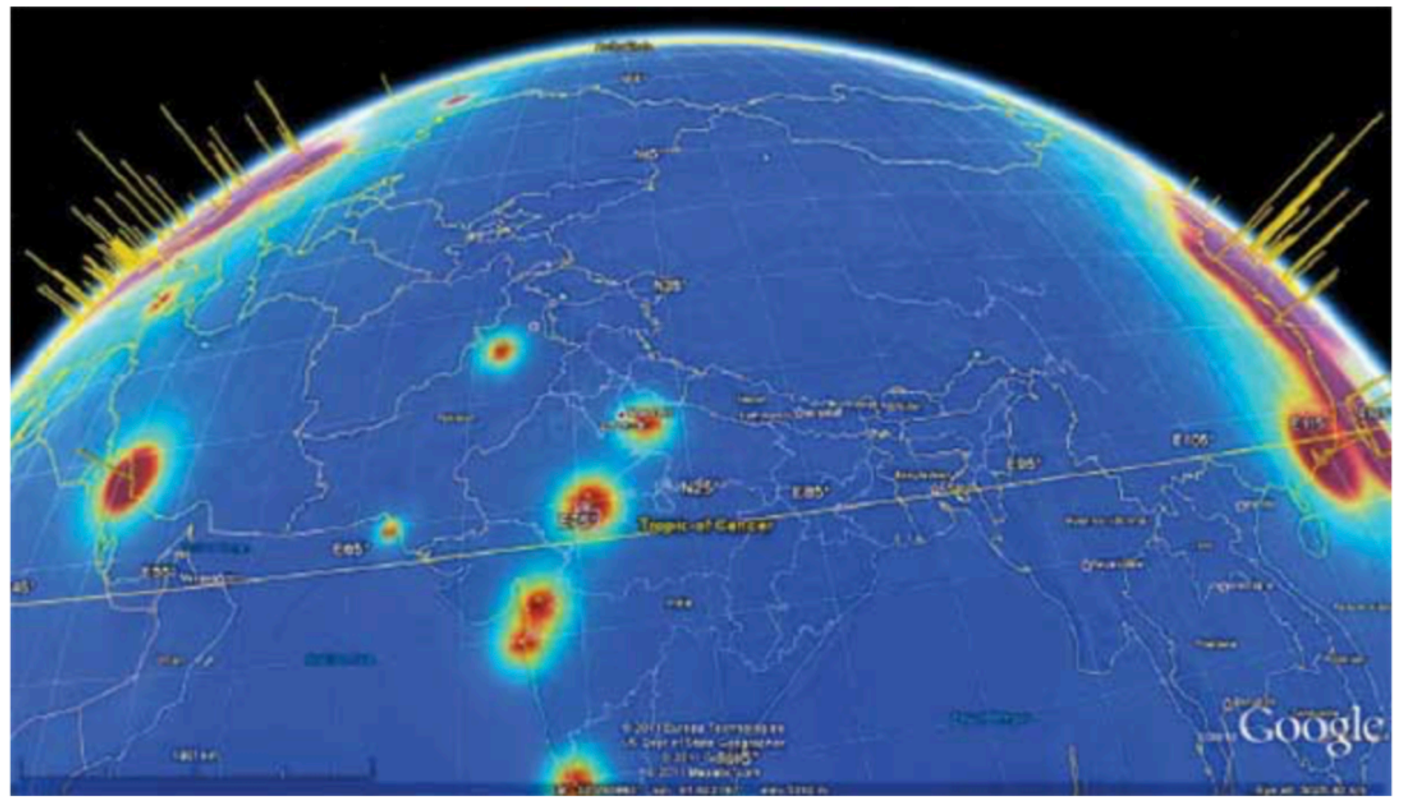
- Look inside of the Sun
- Tomography of the Earth
- Monitor the reactors
- Astrophysical messenger from the extragalactic
- (May less (?) practical at present) source, non-destructive light-speed communication, neutrino energy

Sun pictured with neutrino by Super-K



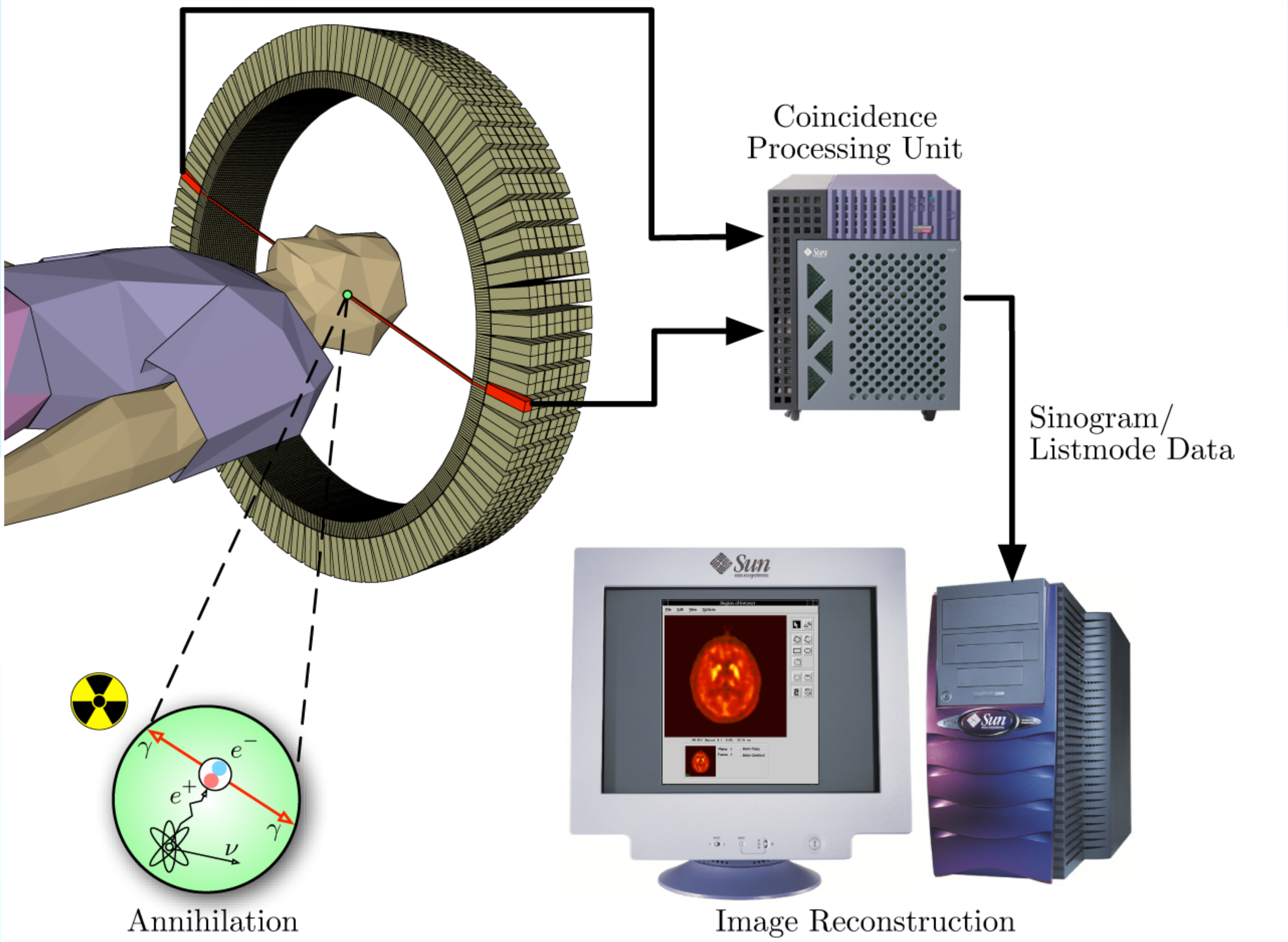
Neutrinos and National Security

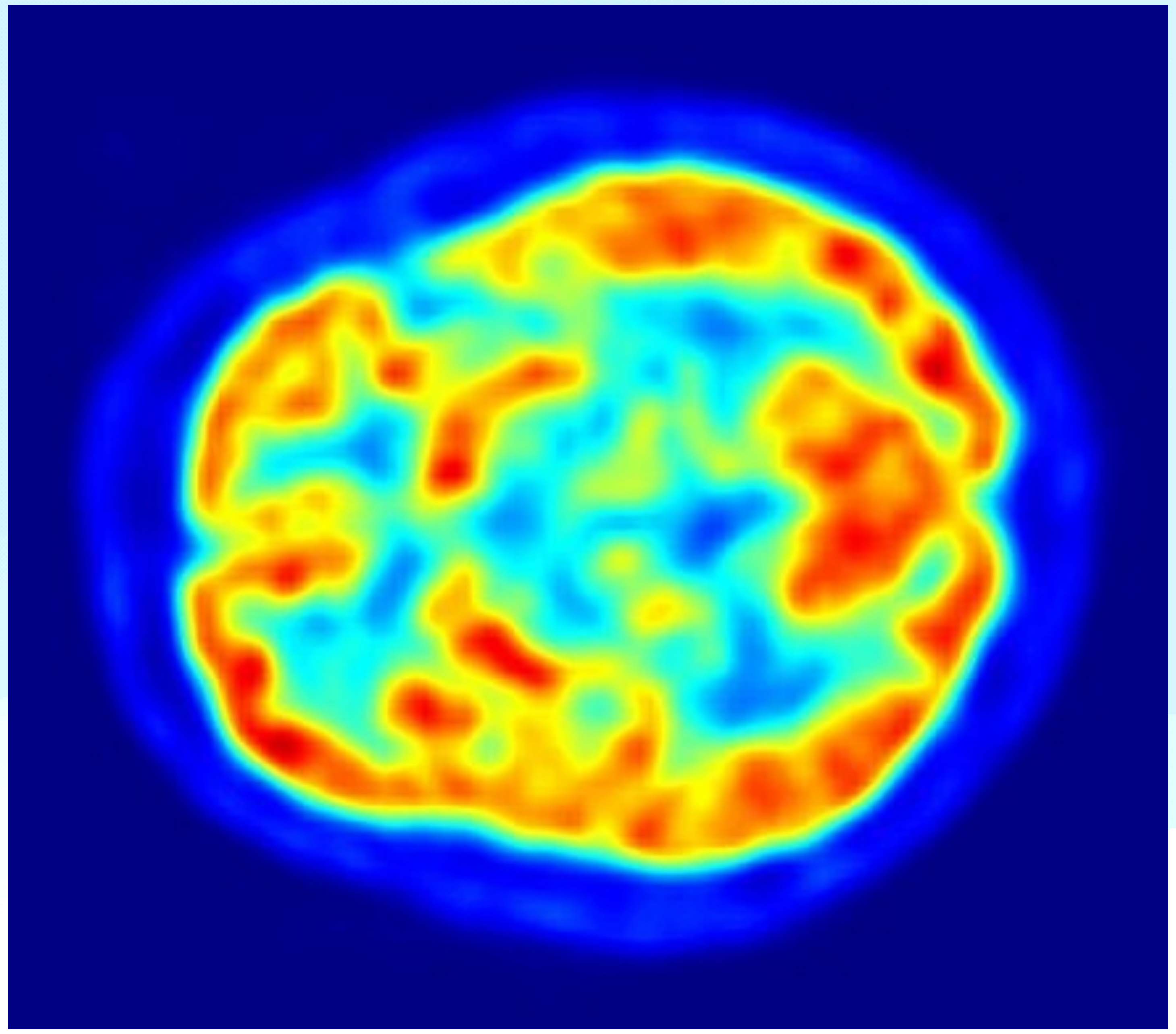
By Michael Lucibella



Global map of reactor neutrino emission.
Photo courtesy of Glenn Jocher and John Learned, University of Hawaii

Positron emission tomography (PET)



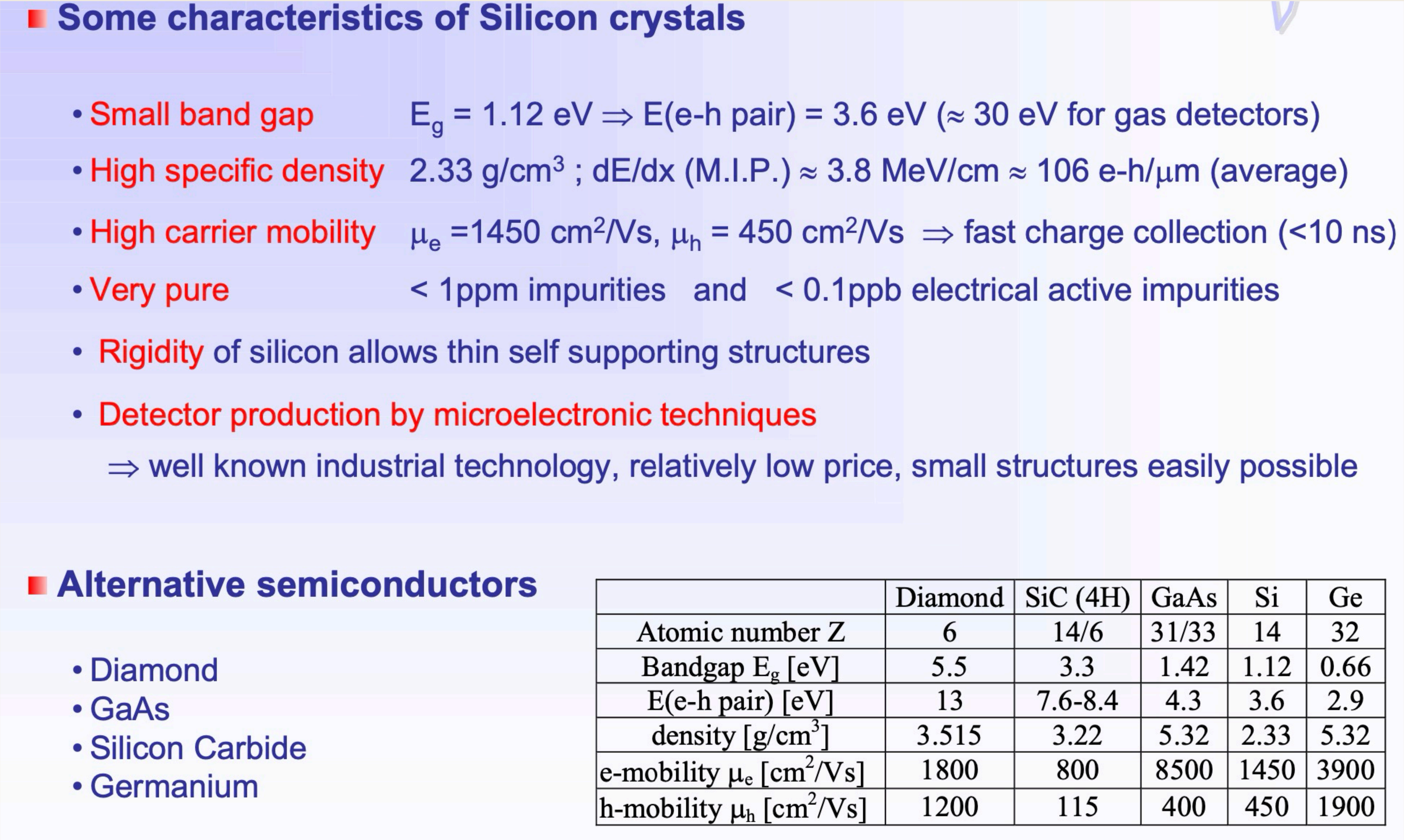


Why is particle detector development essential and so innovative?

Physicists always want to explore more. To go beyond, they need to build the best detector ever with their physics backgrounds.

Backup

Why silicon?



Some characteristics of Silicon crystals

- Small band gap E_g = 1.12 eV ⇒ E(e-h pair) = 3.6 eV (≈ 30 eV for gas detectors)
- High specific density 2.33 g/cm^3 ; dE/dx (M.I.P.) ≈ 3.8 MeV/cm ≈ 106 e-h/μm (average)
- High carrier mobility μ_e =1450 cm^2/Vs, μ_h = 450 cm^2/Vs ⇒ fast charge collection (<10 ns)
- Very pure < 1ppm impurities and < 0.1ppb electrical active impurities
- Rigidity of silicon allows thin self supporting structures
- Detector production by microelectronic techniques

  ⇒ well known industrial technology, relatively low price, small structures easily possible

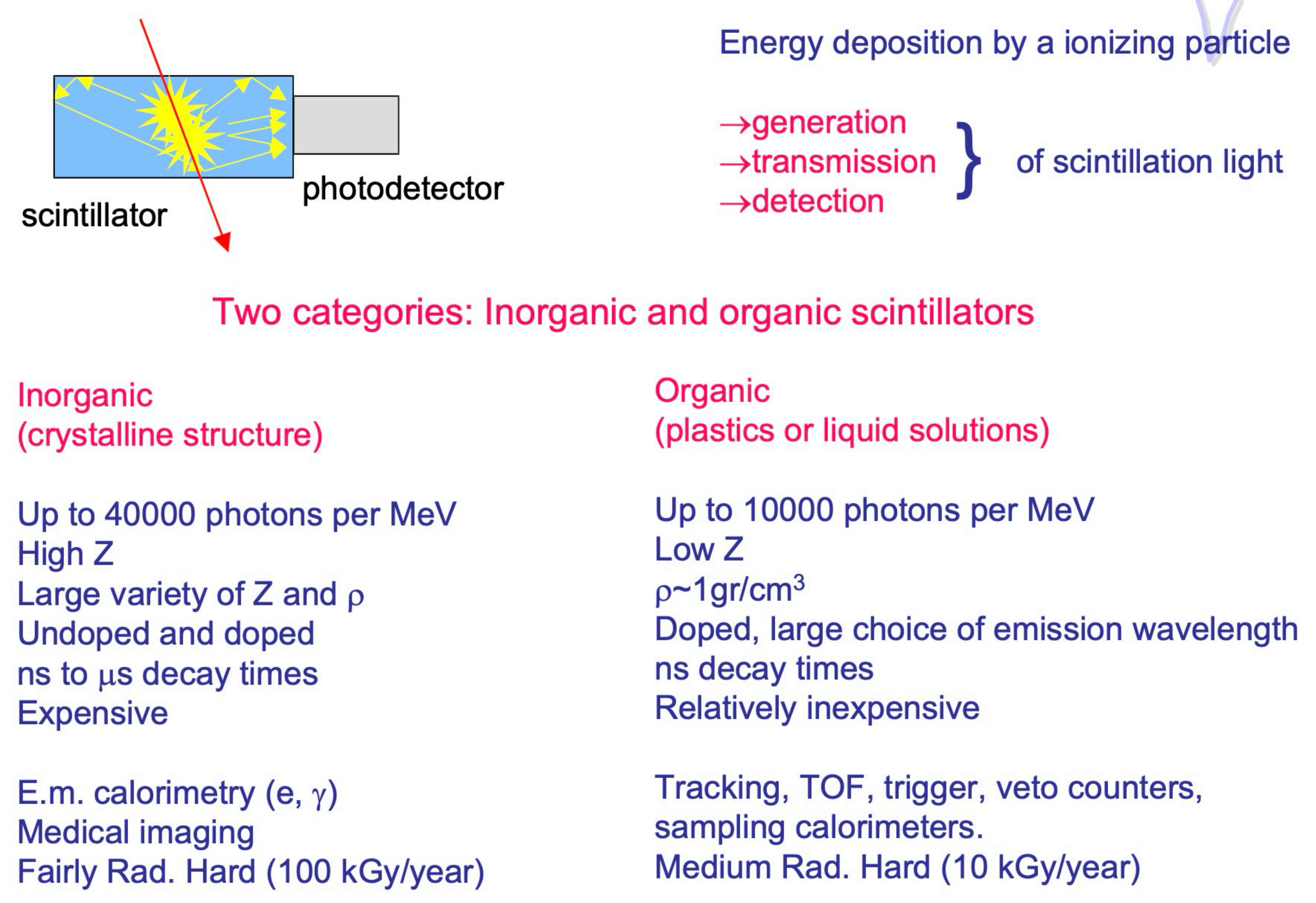
Alternative semiconductors

- Diamond
- GaAs
- Silicon Carbide
- Germanium

	Diamond	SiC (4H)	GaAs	Si	Ge
Atomic number Z	6	14/6	31/33	14	32
Bandgap E_g [eV]	5.5	3.3	1.42	1.12	0.66
E(e-h pair) [eV]	13	7.6-8.4	4.3	3.6	2.9
density [g/cm^3]	3.515	3.22	5.32	2.33	5.32
e-mobility μ_e [cm^2/Vs]	1800	800	8500	1450	3900
h-mobility μ_h [cm^2/Vs]	1200	115	400	450	1900

cern_particleDet_CAT2005_all.pdf

Scintillator



Energy deposition by a ionizing particle

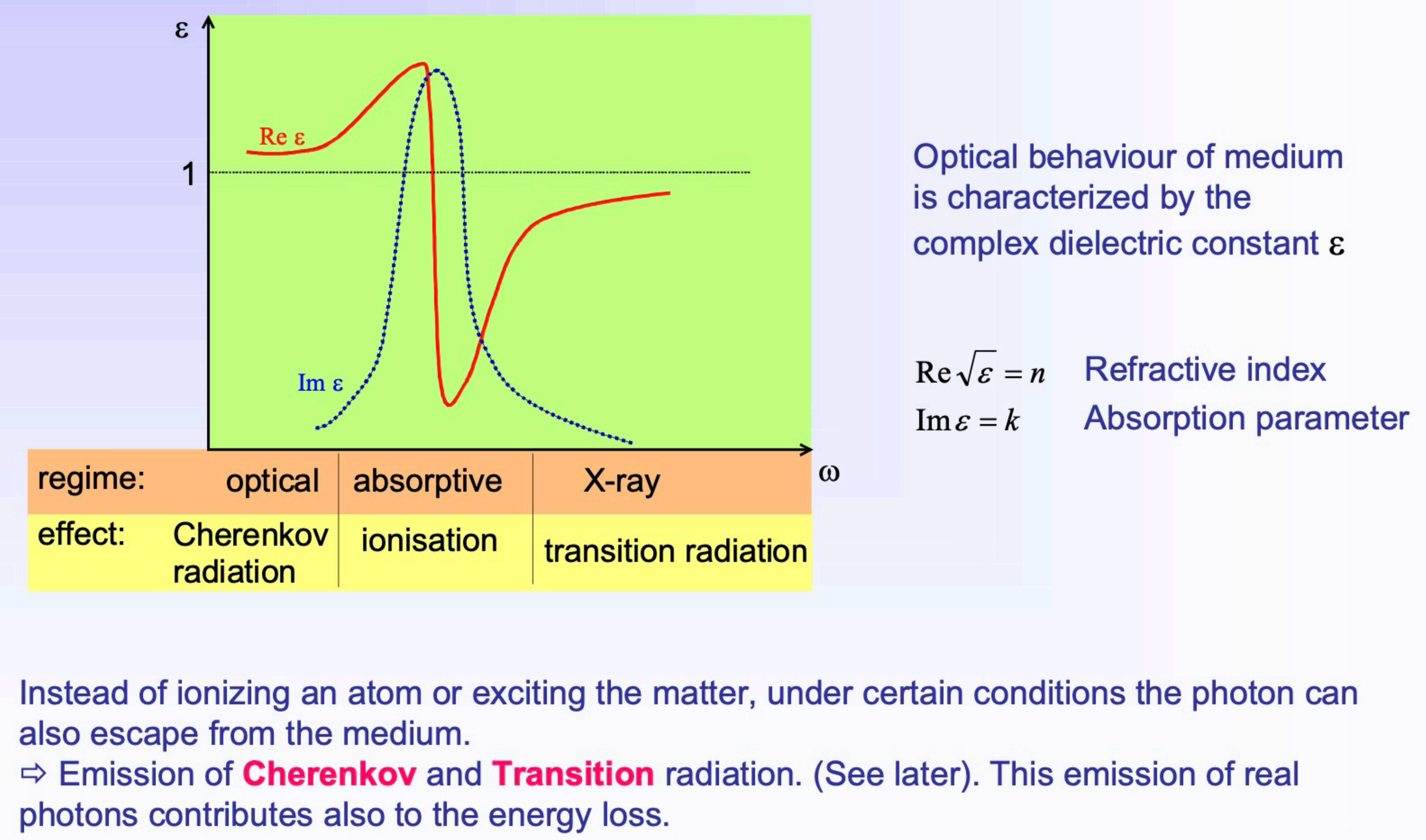
→generation
→transmission
→detection
} of scintillation light

Two categories: Inorganic and organic scintillators

Inorganic (crystalline structure)	Organic (plastics or liquid solutions)
Up to 40000 photons per MeV	Up to 10000 photons per MeV
High Z	Low Z
Large variety of Z and ρ	$\rho \sim 1$gr/cm^3
Undoped and doped	Doped, large choice of emission wavelength
ns to μs decay times	ns decay times
Expensive	Relatively inexpensive
E.m. calorimetry (e, γ) Medical imaging Fairly Rad. Hard (100 kGy/year)	Tracking, TOF, trigger, veto counters, sampling calorimeters. Medium Rad. Hard (10 kGy/year)

cern_particleDet_CAT2005_all.pdf

Optical behavior of medium



regime:	optical	absorptive	X-ray
effect:	Cherenkov radiation	ionisation	transition radiation

Optical behaviour of medium is characterized by the complex dielectric constant ε

$\mathrm{Re}\sqrt{\varepsilon} = n$ Refractive index

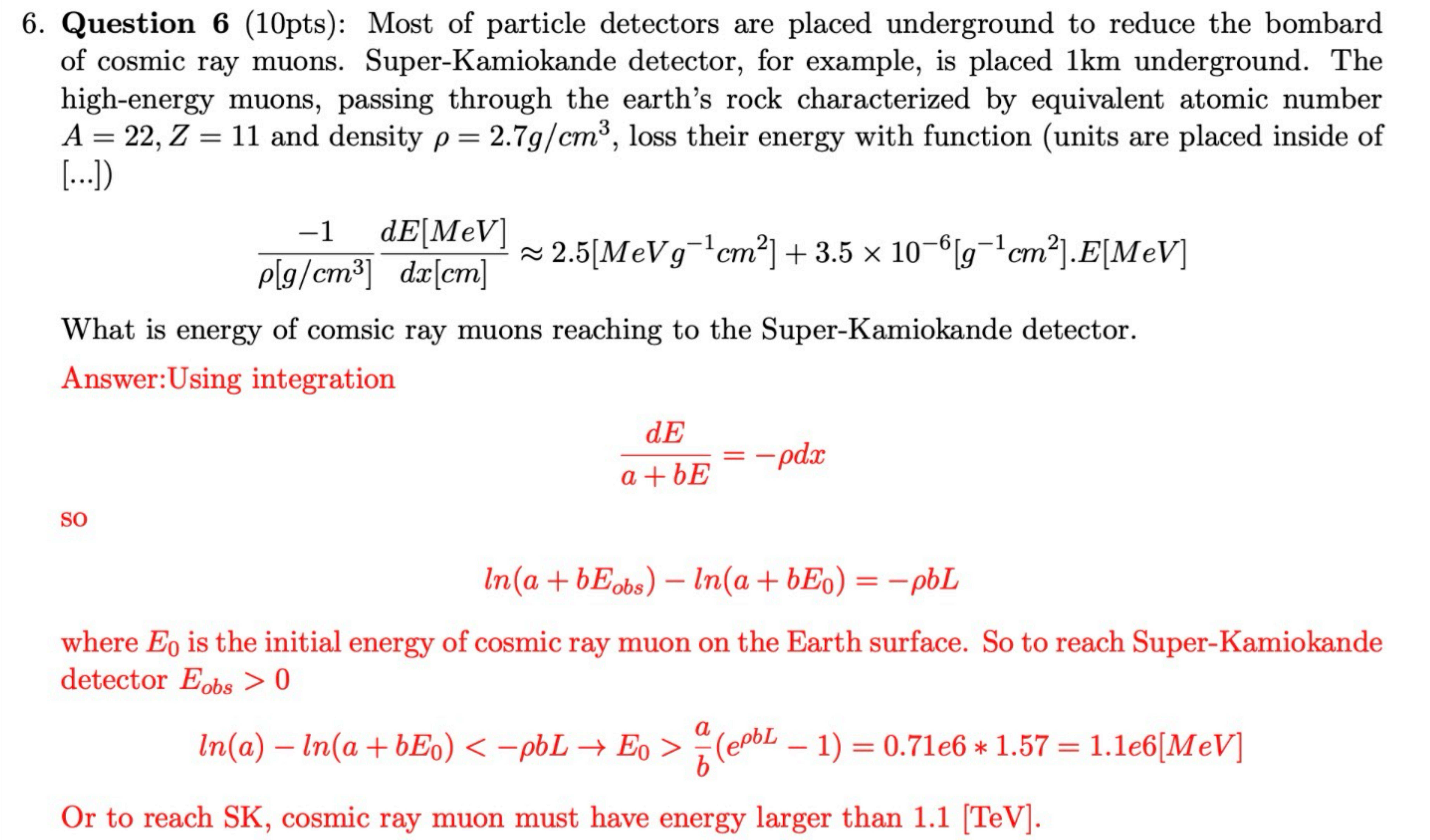
$\mathrm{Im}\,\varepsilon = k$ Absorption parameter

Instead of ionizing an atom or exciting the matter, under certain conditions the photon can also escape from the medium.

⇨ Emission of **Cherenkov** and **Transition** radiation. (See later). This emission of real photons contributes also to the energy loss.

cern_particleDet_CAT2005_all.pdf

Cosmic-ray reaching Super-K



6. **Question 6** (10pts): Most of particle detectors are placed underground to reduce the bombard of cosmic ray muons. Super-Kamiokande detector, for example, is placed 1km underground. The high-energy muons, passing through the earth's rock characterized by equivalent atomic number $A = 22, Z = 11$ and density $\rho = 2.7g/cm^3$, loss their energy with function (units are placed inside of [...])

$$\frac{-1}{\rho[g/cm^3]}\frac{dE[MeV]}{dx[cm]} \approx 2.5[MeVg^{-1}cm^2] + 3.5 \times 10^{-6}[g^{-1}cm^2].E[MeV]$$

What is energy of comsic ray muons reaching to the Super-Kamiokande detector.

Answer:Using integration

$$\frac{dE}{a+bE} = -\rho dx$$

so

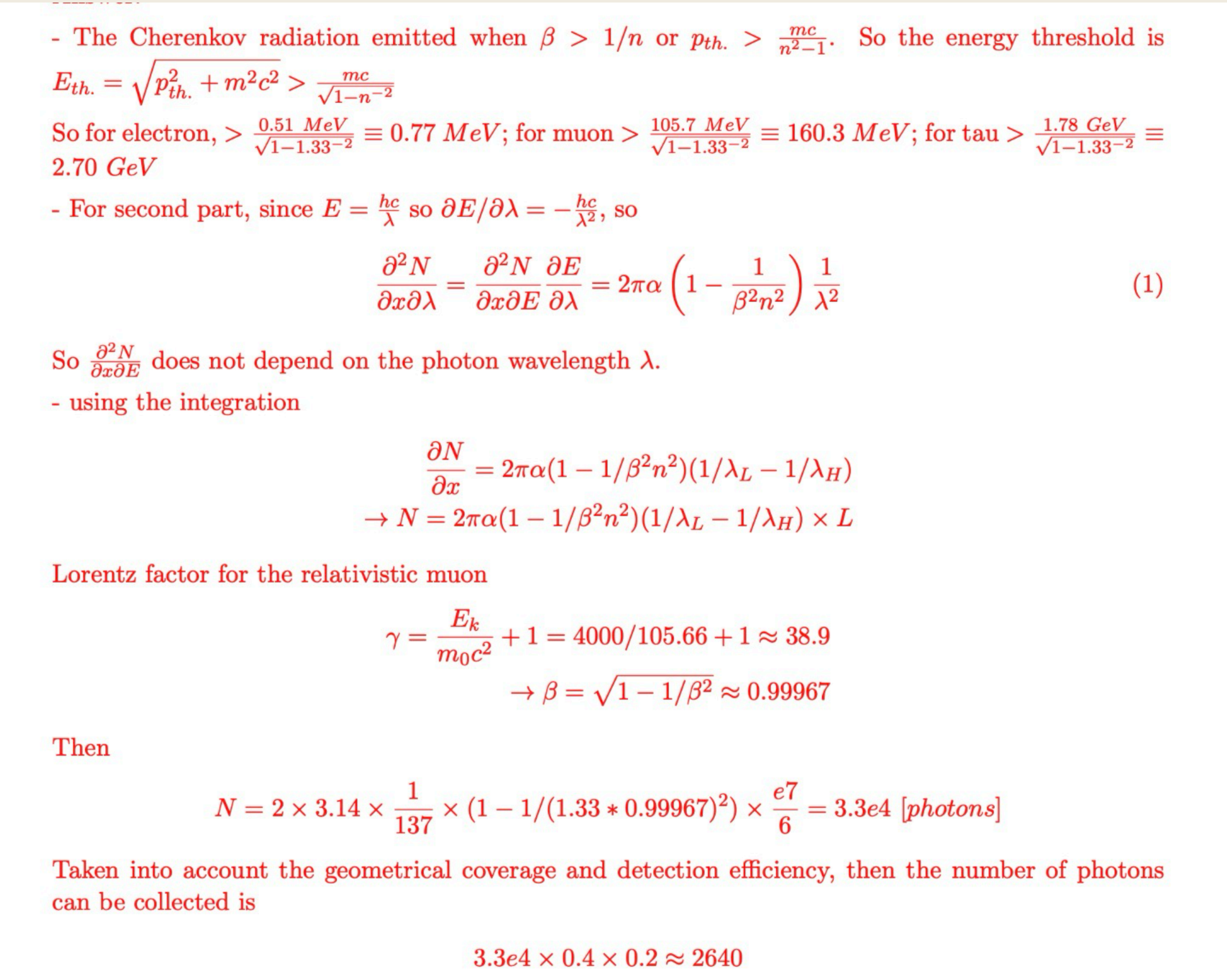
$$ln(a + bE_{obs}) - ln(a + bE_0) = -\rho bL$$

where E_0 is the initial energy of cosmic ray muon on the Earth surface. So to reach Super-Kamiokande detector $E_{obs} > 0$

$$ln(a) - ln(a + bE_0) < -\rho bL \rightarrow E_0 > \frac{a}{b}(e^{\rho bL} - 1) = 0.71e6 * 1.57 = 1.1e6[MeV]$$

Or to reach SK, cosmic ray muon must have energy larger than 1.1 [TeV].

Cherenkov threshold and No. Of photons



- The Cherenkov radiation emitted when $\beta > 1/n$ or $p_{th.} > \frac{mc}{n^2-1}$. So the energy threshold is $E_{th.} = \sqrt{p_{th.}^2 + m^2c^2} > \frac{mc}{\sqrt{1-n^{-2}}}$

So for electron, $> \frac{0.51\ MeV}{\sqrt{1-1.33^{-2}}} \equiv 0.77\ MeV$; for muon $> \frac{105.7\ MeV}{\sqrt{1-1.33^{-2}}} \equiv 160.3\ MeV$; for tau $> \frac{1.78\ GeV}{\sqrt{1-1.33^{-2}}} \equiv 2.70\ GeV$

- For second part, since $E = \frac{hc}{\lambda}$ so $\partial E/\partial\lambda = -\frac{hc}{\lambda^2}$, so

$$\frac{\partial^2 N}{\partial x \partial \lambda} = \frac{\partial^2 N}{\partial x \partial E}\frac{\partial E}{\partial \lambda} = 2\pi\alpha\left(1 - \frac{1}{\beta^2 n^2}\right)\frac{1}{\lambda^2} \tag{1}$$

So $\frac{\partial^2 N}{\partial x \partial E}$ does not depend on the photon wavelength λ.

- using the integration

$$\frac{\partial N}{\partial x} = 2\pi\alpha(1 - 1/\beta^2n^2)(1/\lambda_L - 1/\lambda_H)$$
$$\rightarrow N = 2\pi\alpha(1 - 1/\beta^2n^2)(1/\lambda_L - 1/\lambda_H) \times L$$

Lorentz factor for the relativistic muon

$$\gamma = \frac{E_k}{m_0c^2} + 1 = 4000/105.66 + 1 \approx 38.9$$
$$\rightarrow \beta = \sqrt{1 - 1/\beta^2} \approx 0.99967$$

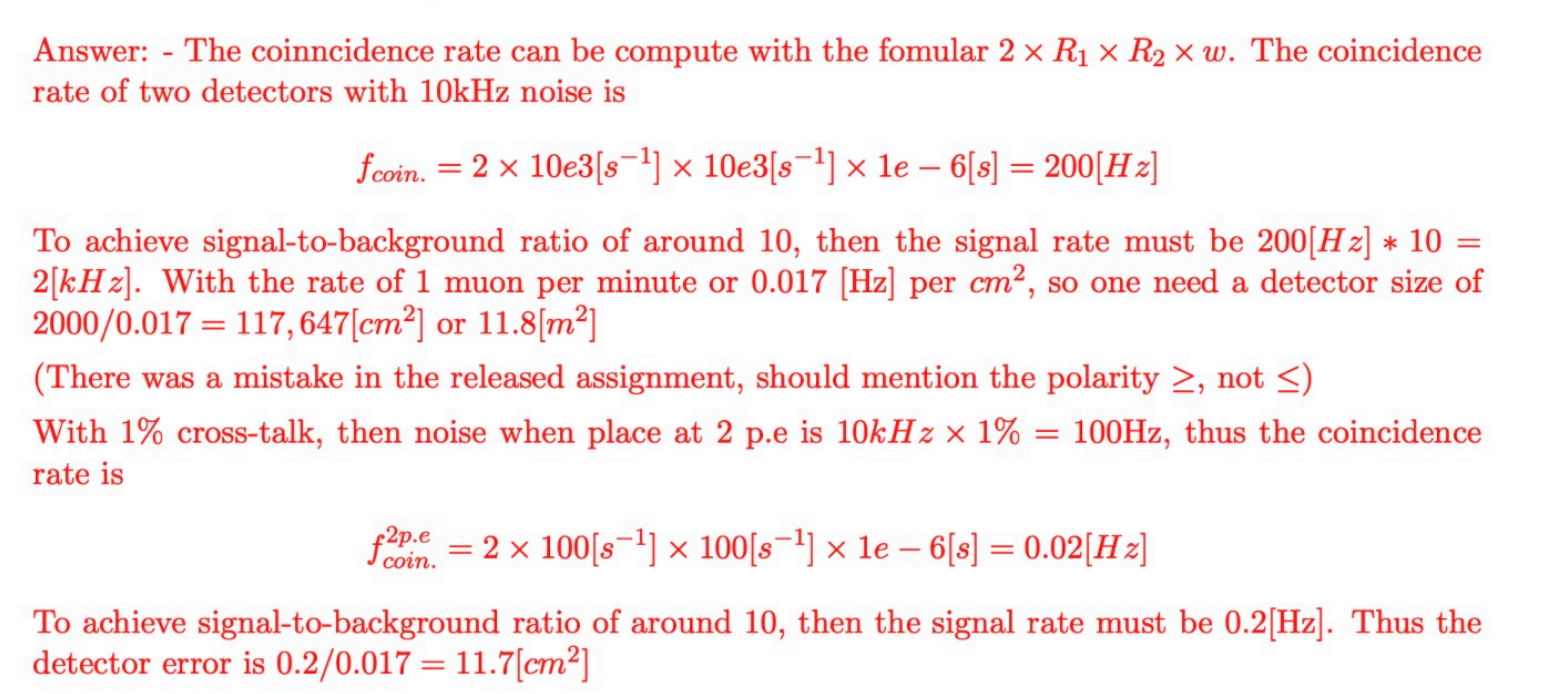
Then

$$N = 2 \times 3.14 \times \frac{1}{137} \times (1 - 1/(1.33 * 0.99967)^2) \times \frac{e7}{6} = 3.3e4\ [photons]$$

Taken into account the geometrical coverage and detection efficiency, then the number of photons can be collected is

$$3.3e4 \times 0.4 \times 0.2 \approx 2640$$

Coincidence rate



Answer: - The coinncidence rate can be compute with the fomular $2 \times R_1 \times R_2 \times w$. The coincidence rate of two detectors with 10kHz noise is

$$f_{coin.} = 2 \times 10e3[s^{-1}] \times 10e3[s^{-1}] \times 1e-6[s] = 200[Hz]$$

To achieve signal-to-background ratio of around 10, then the signal rate must be $200[Hz] * 10 = 2[kHz]$. With the rate of 1 muon per minute or 0.017 [Hz] per cm^2, so one need a detector size of $2000/0.017 = 117,647[cm^2]$ or $11.8[m^2]$

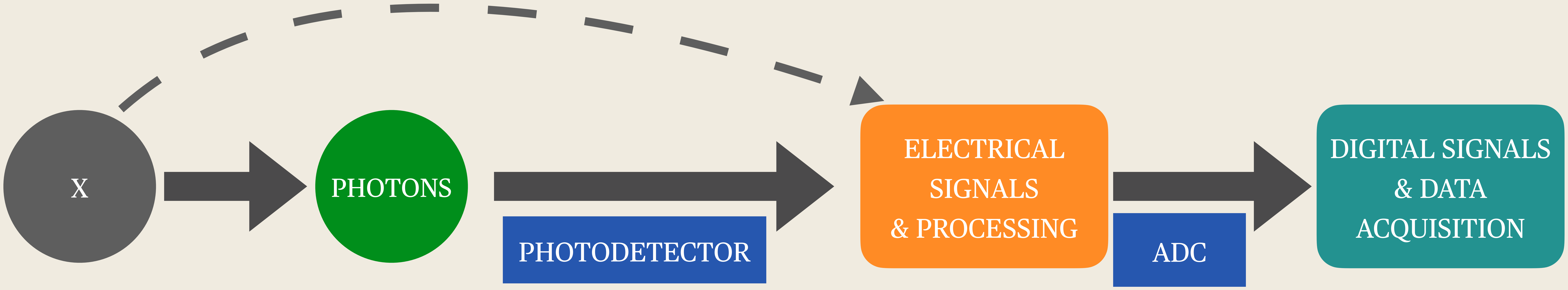
(There was a mistake in the released assignment, should mention the polarity $\geq$, not $\leq$)

With 1% cross-talk, then noise when place at 2 p.e is $10kHz \times 1\% =$ 100Hz, thus the coincidence rate is

$$f^{2p.e}_{coin.} = 2 \times 100[s^{-1}] \times 100[s^{-1}] \times 1e-6[s] = 0.02[Hz]$$

To achieve signal-to-background ratio of around 10, then the signal rate must be 0.2[Hz]. Thus the detector error is $0.2/0.017 = 11.7[cm^2]$

General principle of modern PN detector



- Turn invisible things to visible things (*accessible to human perception*)
- (*Modern detector*) be **electrical** in nature, i.e at *some points the information is converted into electrical impulses and treated with electronic devices*
- NO detector can be sensitive to all types of radiations at all energies ➜ selection of the appropriate photosensor