Introduction to the Standard Model of Particle Physics

The second lecture

Thi Nhung Dao

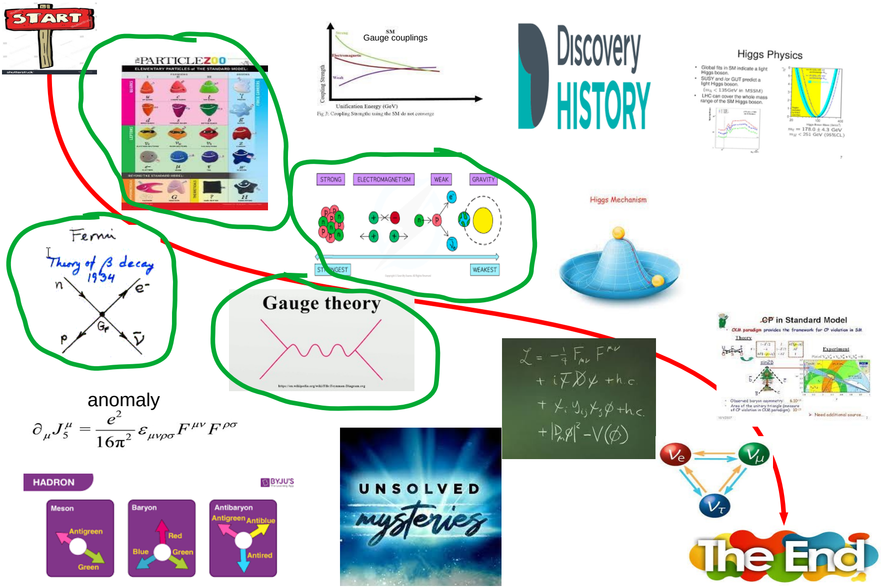
Phenikaa Institute for Advanced Study, Phenikaa University

Vietnam School on Neutrinos July 15 - 26, 2024

Tests about SM

- How many elementary particles are there?
- What are the stable particles?
- What are the most unstable particles?
- What is the theory used for high energy physics?
- How many fundamental interactions?
- What are symmetries used to describle fundamental interactions?

The Standard Model in three hours



Lecture 2

The SM gauge group

- Describing three fundamental interactions: electromagnetic, weak and strong
- Using the gauge group

$$\underbrace{SU(3)_C}_{\textit{strong}} \quad \otimes \quad \underbrace{SU(2)_L \otimes U(1)_Y}_{\textit{electroweak}}$$

The SM gauge group

- Describing three fundamental interactions: electromagnetic, weak and strong
- Using the gauge group

$$\underbrace{SU(3)_C}_{\textcolor{red}{strong}} \quad \otimes \quad \underbrace{SU(2)_L \otimes U(1)_Y}_{\textcolor{blue}{electroweak}}$$

- Gauge fields of $SU(3)_C$ are gluons G^a_μ: $3^2 - 1 = 8$ gluons, $a = 1, \cdots, 8$. Gluons are massless, this group is exact symmetry group $\rightarrow$ no need for gauge symmetry breaking. Quantum Chromodynamics (QCD)

The SM gauge group

- Describing three fundamental interactions: electromagnetic, weak and strong
- Using the gauge group

$$\underbrace{SU(3)_C}_{\text{strong}} \quad \otimes \quad \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electroweak}}$$

- Gauge fields of $SU(3)_C$ are gluons G_μ^a: $3^2 - 1 = 8$ gluons, $a = 1, \cdots, 8$. Gluons are massless, this group is exact symmetry group $\to$ no need for gauge symmetry breaking. Quantum Chromodynamics (QCD)
- Gauge fields of $SU(2)_L \otimes U(1)_Y$ are W_μ^i, B_μ: $2^2 - 1 + 1 = 4$ fields. From W_μ^i, B_μ, $(i = 1, 2, 3)$ how can we create massive $Z, W^\pm$ and one massless photon. This symmetry must be broken

$$SU(2)_L \otimes U(1)_Y \quad \to \quad U(1)_Q$$

$SU(3)_C$: STRONG INTERACTION

- Quarks can participate strong interaction → quarks carry color charge. Each type of quarks belong to the fundamental representation of $SU(3)_C$

$$q = \begin{pmatrix} q_r \\ q_b \\ q_g \end{pmatrix} \quad (1)$$

$q = u, d, c, s, b, t$

- QCD Lagrangian

$$\mathcal{L}_{QCD} = \sum_{q=u,d,c,s,b,t} \bar{q}(i\gamma^\mu D_\mu + m_q)q - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu} \quad (2)$$

where $D_\mu = \partial_\mu - ig_s T^a G^a_\mu$
T^a are generators of the $SU(3)_C$: Gell-Mann matrices
G^a_μ are gluon fields
$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$

- $SU(3)_C$ invariance → three colors of a quark (red, blue, green) must have the same mass.

$SU(2)_L \otimes U(1)_Y$: ELECTROWEAK INTERACTION

Group structure:

- $SU(2)$: $2^2 - 1$ generators
  $T^i = \frac{\sigma^i}{2}$ σ^i (i=1,23): Pauli matrices

$$[T^i, T^j] = i\epsilon^{ijk} T^k$$

  ϵ^{ijk} : Levi-Civita tensor with $\epsilon^{123} = 1$
- $U(1)_Y$: 1 generator $Y = Y \times \mathbb{1} \rightarrow$ commutative with all $SU(2)$ generators
- This symmetry must be broken down to

$$SU(2)_L \otimes U(1)_Y \quad \rightarrow \quad U(1)_Q$$

$$T^i, Y \quad \rightarrow \quad Q = aT^3 + bY = \begin{pmatrix} a/2 + bY & 0 \\ 0 & -a/2 + bY \end{pmatrix}$$

Similar to the Gell-Mann-Nishijima formula [Nishijima '53, Gell-Mann '56]

$$Q = I_3 + \frac{1}{2}(B + S)$$

I_3: isospin of quarks and hadrons ($I_3 = 1/2$ up quark, $I_3 = -1/2$ down quark)

Suggestion: $Q = T^3 + \frac{Y}{2}$

How to arrange fermionic fields in the representations of the gauge group?
Look back at the weak interaction

WEAK INTERACTION (3)

Not only charged current interaction

$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}}\bar{u}(x)\gamma^\mu(1-\gamma_5)d(x)W_\mu(x) + \frac{g_2}{2\sqrt{2}}\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu(x)W_\mu(x)$$

But also neutral current interaction

$$\mathcal{L}_{int} = \frac{g_2}{2\cos\theta_W}\bar{f}(x)\gamma^\mu(g_V^f - g_A^f\gamma_5)f(x)Z_\mu(x)$$

f: quark, charged leptons, neutrinos
g_V^f, g_A^f are coefficients depending on electric charge and isospin of fermion f

$SU(2)_L \otimes U(1)_Y$: ELECTROWEAK INTERACTION

How to arrange fields in the representations of the gauge group?
Suggestion: $Q = T^3 + \frac{Y}{2}$

- Only left-handed fermions belong to the fundamental representations of $SU(2)_L$

$$l_L^1 = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}, \quad l_L^2 = \begin{pmatrix} \nu_{\mu,L} \\ \mu_L \end{pmatrix}, \quad l_L^3 = \begin{pmatrix} \nu_{\tau,L} \\ \tau_L \end{pmatrix} \sim Y = -1$$

$$Q_L^1 = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad Q_L^2 = \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \quad Q_L^3 = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim Y = \frac{1}{3}$$

- No right handed fermions interacts with W boson $\rightarrow$ they must be singlet representation of $SU(2)_L$

$$\begin{aligned} e_R, \mu_R, \tau_R, &\quad \sim \quad Y = -2 \\ u_R, c_R, t_R, &\quad \sim \quad Y = \frac{4}{3} \\ d_R, s_R, b_R, &\quad \sim \quad Y = -\frac{2}{3} \end{aligned}$$

- Three gauge fields W_μ^i go with three generators T^i of $SU(2)_L$, 1 gauge field B_μ goes with one generator Y of $U(1)_Y$

$SU(2)_L \otimes U(1)_Y$: Glashow-Weinberg-Salam model ['61, '67, '68]

How to construct Lagrangian

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G$$

The fermionic part of the Lagrangian:

$$\mathcal{L}_F = i\bar{l}_{iL}\gamma^\mu D_\mu l_{iL} + i\bar{e}_{iR}\gamma^\mu D_\mu e_{iR} + i\bar{Q}_{iL}\gamma^\mu D_\mu Q_{iL} + i\bar{u}_{iR}\gamma^\mu D_\mu u_{iR} + i\bar{d}_{iR}\gamma^\mu D_\mu d_{iR},$$

covariant derivatives:

$$D_\mu = \begin{cases} \partial_\mu - ig_2 T^i W^i_\mu - ig_1 YB_\mu, & \text{for } l_{iL} \\ \partial_\mu - ig_1 YB_\mu, & \text{for } l_{iR} \\ \partial_\mu - ig_2 T^i W^i_\mu - ig_1 YB_\mu, & \text{for } Q_{iL} \\ \partial_\mu - ig_1 YB_\mu, & \text{for } u_{iR}, d_{iR}. \end{cases}$$

The gauge part of the Lagrangian:

$$\mathcal{L}_G = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

$$F^i_{\mu\nu} = \partial_\mu W^i_\mu - \partial_\mu W^i_\nu + g_2\epsilon^{ijk}W^j_\mu W^k_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\mu - \partial_\mu B_\nu,$$

How to generate mass for the SM particles???

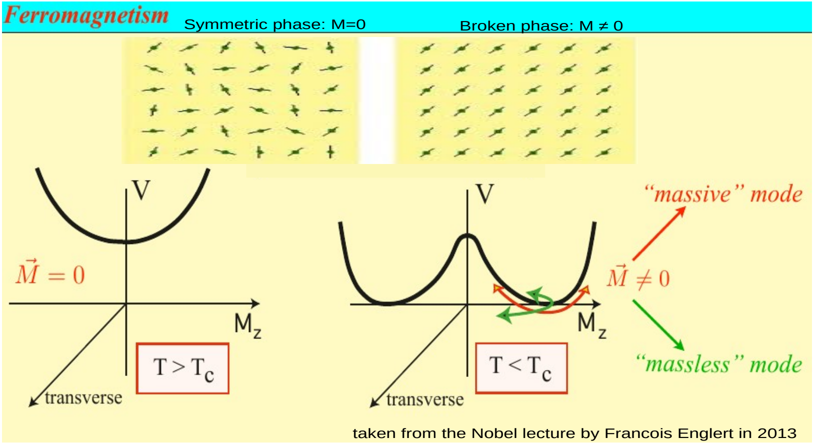
Mass terms: $\bar{Q}^i_L u_{iR}, \quad \bar{Q}^i_L d_{iR}, \quad \bar{l}_{iL} e_{iR}, \quad B^\mu B_\mu, \quad W^{i\mu} W^i_\mu$ → violate $SU(2)_L \otimes U(1)_Y$ gauge symmetry

Gauge symmetry must be broken for at least weak interaction.

- explicit breaking: add different mass terms for different fermions and for different gauge fields. This loose the predictive feature of the theory.
- spontaneous symmetry breaking?
  Lagrangian is invariant under the symmetry, but the ground state of the theory is not.
  $\rightarrow$ attractive idea

Spontaneous symmetry breaking

Spontaneous symmetry breaking in phase transition [L.D. Landau 1937]
Spontaneous symmetry breaking in field theory [Y. Nambu 1960]
Spontaneous symmetry breaking in the Standard Model [F. Englert, R. Brout 1964, P.W. Higgs 1964]



Spontaneous symmetry breaking in SM

- Require at least one Higgs doublet: simplest case

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim Y = 1$$

- The Higgs Lagrangian:

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \quad V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2,$$

  with μ^2 and λ constants.

Conditions: for a non-zero expectation value ($\langle \Phi \rangle \neq 0$) to break $SU(2)_L \times U(1)_Y \to U(1)_Q$

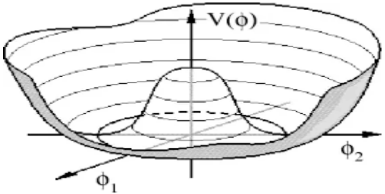
- the potential has to be bounded from below, therefore $\lambda > 0$,
- the potential has an unstable maximum at zero, hence $\mu^2 > 0$,
- the potential has stable minima which are degenerate.

Combining all above requirements, one can show that the vev of the Higgs doublet is

$$\langle\Phi\rangle = \begin{pmatrix} |\langle\phi^+\rangle| \\ |\langle\phi^0\rangle| \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

The upper component carrying electric charge $Q = +1$ cannot have a non-zero expectation value

i$|\langle\phi^0\rangle| = v/\sqrt{2}$ corresponds to a circle on a complex plane



Higgs and gauge boson masses

$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \begin{pmatrix} G^+(x) \\ \left(v + H(x) - iG^0(x)\right)/\sqrt{2} \end{pmatrix}.$$

The Higgs Lagrangian:

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \quad V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2,$$

$$D_\mu = \partial_\mu - i g_2 T^i W^i_\mu - i g_1 Y B_\mu$$

- G^+, G^0 are massless, H has a mass $m_H = \sqrt{2\lambda v^2}$
- Gauge boson masses

$$\begin{aligned}
&\left((g_2 T^i W^i_\mu + g_1 B_\mu) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}\right)^\dagger \left((g_2 T^i W^{i\mu} + g_1 B^\mu) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}\right)) \\
= \quad & \frac{v^2}{2} \begin{pmatrix} g_2 \frac{W^1_\mu - i W^2_\mu}{2} & -g_2 W^3_\mu + g_1 B_\mu \end{pmatrix} \begin{pmatrix} g_2 \frac{W^{1\mu} + i W^{2\mu}}{2} \\ -g_2 W^{3\mu} + g_1 B^\mu \end{pmatrix} \\
= \quad & \frac{g_2 v^2}{4} W^+_\mu W^{-\mu} + \frac{v^2}{2} \begin{pmatrix} W^3_\mu & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}
\end{aligned}$$

$$\frac{g_2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{v^2}{2} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

$$\begin{cases} W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}, \\ Z_\mu = c_W W_\mu^3 - s_W B_\mu, \\ A_\mu = s_W W_\mu^3 + c_W B_\mu, \end{cases}$$

where $c_W = \cos\theta_W$, $s_W = \sin\theta_W$, θ_W is called the weak mixing angle and

$$c_W = \frac{g_2}{\sqrt{g_2^2 + g_1^2}}, \quad s_W = \frac{g_1}{\sqrt{g_2^2 + g_1^2}}$$

The masses of W, Z and A bosons

$$M_W = \frac{g_2 v}{2}, \quad M_Z = \frac{g_2 v}{2 c_W}, \quad M_A = 0$$

The photon remains massless $\rightarrow U(1)_Q$ is the exact gauge symmetry.

Fermion masses

Fermion masses are obtained from Yukawa interactions,

$$\mathcal{L}_Y = -y^e_{ij}\bar{L}^i\tilde{\Phi}e^j_R - y^d_{ij}\bar{Q}^i\tilde{\Phi}d^j_R - y^u_{ij}\bar{Q}^i\Phi u^j_R + \text{ h.c.},$$

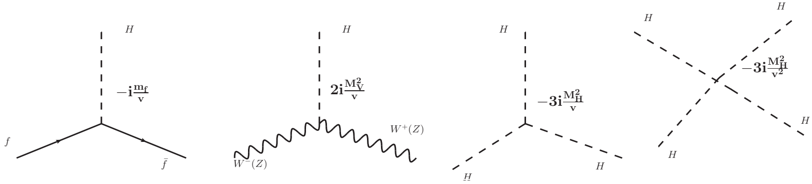
where $y^{e,d,u}_{ij}$ with i,j generation indices are Yukawa couplings, and $\tilde{\Phi} = i\sigma_2\Phi^*$.

$$\Phi(x) = \begin{pmatrix}\phi^+(x)\\ \phi^0(x)\end{pmatrix} = \begin{pmatrix}G^+(x)\\ \left(v + H(x) - iG^0(x)\right)/\sqrt{2}\end{pmatrix}.$$

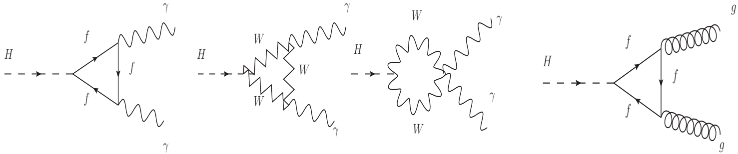
$$\mathcal{L}^f_{\text{mass}} = -\frac{y^e_{ij}v}{\sqrt{2}}\bar{e}^i_L e^j_R - \frac{y^d_{ij}v}{\sqrt{2}}\bar{d}^i_L d^j_R - \frac{y^u_{ij}v}{\sqrt{2}}\bar{u}^i_L u^j_R + \text{ h.c.},$$

Neutrinos are massless since they are purely left-handed in the SM

Have tree-level couplings with only massive particle:



Have loop-induced couplings with photon and gluons:



Donot interact with massless neutrino at all loop order

The Standard Model: recap

Gauge group

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Thee gauge couplings

$$g_s, g_2, g_1$$

left-handed fermions belong to doublets, right-handed fermions belong to singlets of $SU(2)_L$

$$\begin{pmatrix} \nu_{i,L} \\ e^i_L \end{pmatrix}, \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}, e^i_R, u^i_R, d^i_R, \quad i = 1, 2, 3$$

Higgs doublet

$$\Phi(x) = \begin{pmatrix} G^+(x) \\ \left(v + H(x) - iG^0(x)\right)/\sqrt{2} \end{pmatrix}$$

8 gluons and a photon are massless, W, Z have mass

$$M_W = \frac{g_2 v}{2}, \quad M_Z = \frac{g_2 v}{2c_W},$$

The Standard Model: recap

The SM Lagrangian:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}$$

$$\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},$$

$$\mathcal{L}_F = i\bar{l}_{iL}\gamma^\mu D_\mu l_{iL} + i\bar{e}_{iR}\gamma^\mu D_\mu e_{iR} + i\bar{Q}_{iL}\gamma^\mu D_\mu Q_{iL} + i\bar{u}_{iR}\gamma^\mu D_\mu u_{iR} + i\bar{d}_{iR}\gamma^\mu D_\mu d_{iR},$$

$$\mathcal{L}_H = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi), \quad V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2,$$

$$\mathcal{L}_Y = -y^e_{ij}\bar{L}^i\tilde{\Phi}e^j_R - y^d_{ij}\bar{Q}^i\tilde{\Phi}d^j_R - y^u_{ij}\bar{Q}^i\Phi u^j_R + \text{ h.c.},$$

The SM Lagrangian:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_{\text{fix}} + \mathcal{L}_{\text{ghost}}$$

$$\mathcal{L}_{\text{fix}} = -\frac{1}{2\xi_G}(F_G^a)^2 - \frac{1}{2\xi_A}F_A^2 - \frac{1}{2\xi_Z}F_Z^2 - \frac{1}{2\xi_W}F_W^+F_W^-,$$

where

$$F_G^a = \partial_\mu G^{a\mu}, \qquad F_A = \partial_\mu A^\mu,$$
$$F_Z = \partial_\mu Z^\mu - M_Z\xi_Z G^0, \qquad F_W^+ = \partial_\mu W^{+\mu} + iM_W\xi_W G^+,$$

Faddeev-Popov ghost terms ['67]

$$\mathcal{L}_{\text{ghost}} = \bar{c}^\alpha \frac{\delta F_\alpha}{\delta\theta^\beta} c^\beta, \quad \alpha,\beta \in \{G, A, Z, W^\pm\},$$

CP violation in the Standard Model

Charge conjugation: particle $\overset{C}{\rightarrow}$ anti-particle

- scalar field: $\phi \quad \overset{C}{\rightarrow} \quad \phi^c = \eta_c \phi^*$
  In general $\eta_c = e^{i\xi_c}$, $\phi^* \neq \phi$
  If particle $\equiv$ anti-particle: $\eta_c = 1 \rightarrow$ even C-parity, $\eta_c = -1 \rightarrow$ odd C-parity,

- fermion field: $\psi \quad \overset{C}{\rightarrow} \quad \psi^c = e^{i\xi_c}\mathcal{C}\psi^*$ with $\mathcal{C} = -i\gamma^2$

$$\psi_L^c = e^{i\xi_c}\mathcal{C}\psi_L^* = e^{i\xi_c}\mathcal{C}P_L\psi^* = P_R(-i\gamma_2\psi^*) = P_R\psi^c$$

  $\rightarrow$ change chirality

- execises: prove that $\bar{\psi}_1\psi_2 \overset{C}{\rightarrow} \bar{\psi}_2\psi_1, \quad \bar{\psi}_1\gamma^\mu\psi_2 \quad \overset{C}{\rightarrow} \quad -\bar{\psi}_2\gamma^\mu\psi_1$

$$\bar{\psi}_1\gamma^\mu\gamma^5\psi_2 \quad \overset{C}{\rightarrow} \quad \bar{\psi}_2\gamma^\mu\gamma^5\psi_1$$
$$\bar{\psi}_1\gamma^\mu P_L\psi_2 \quad \overset{C}{\rightarrow} \quad -\bar{\psi}_2\gamma^\mu P_R\psi_1$$

P transformation

Parity transformation: $(t, \vec{x}) \xrightarrow{P} (t, -\vec{x})$

- scalar field: $\phi(t, \vec{x}) \quad \xrightarrow{P} \quad \eta_p \phi(t, -\vec{x})$
  In general $\eta_p = e^{i\xi_p}$. if $\eta_p = 1$ scalar particle has even parity, $\eta_p = -1$ scalar particle has odd parity. Otherwise its parity is not well defined.
- fermion field: $\psi(t, \vec{x}) \quad \xrightarrow{P} \quad e^{i\xi_P} \mathcal{P} \psi(t, -\vec{x})$, $\mathcal{P} = \gamma^0$

  $$\psi_L^P = e^{i\xi_P} \gamma_0 \psi_L(t, -\vec{x}) = e^{i\xi_P} \gamma_0 P_L \psi(t, -\vec{x}) = P_R(e^{i\xi_P} \gamma_0 \psi(t, -\vec{x})) = P_R \psi^P$$

  $\rightarrow$ change chirality
- execises: prove that $\bar{\psi}_1 \psi_2 \xrightarrow{P} \bar{\psi}_1 \psi_2$,

  $$\bar{\psi}_1 \gamma^\mu \psi_2 \quad \xrightarrow{P} \quad (-1)^\mu \bar{\psi}_1 \gamma^\mu \psi_2,$$
  $$\bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2 \quad \xrightarrow{P} \quad -(-1)^\mu \bar{\psi}_1 \gamma^\mu \gamma^5 \psi_2,$$
  $$\bar{\psi}_1 \gamma^\mu P_L \psi_2 \quad \xrightarrow{P} \quad (-1)^\mu \bar{\psi}_1 \gamma^\mu P_R \psi_2,$$

  $\mu = 0$: $(-1)^\mu \equiv 1$
  $\mu = 1, 2, 3$: $(-1)^\mu \equiv -1$

CP transformation

- scalar field: $\phi(t,\vec{x}) \quad \stackrel{CP}{\to} \quad \eta_{CP}\phi^*(t,-\vec{x})$
  particle $\equiv$ anti-particle
  CP even: $\eta_{CP}=1$, $\phi^*(t,-\vec{x})=\phi(t,-\vec{x})$
  CP odd: $\eta_{CP}=-1$, $\phi^*(t,-\vec{x})=-\phi(t,-\vec{x})$
- fermion field: $\psi(t,\vec{x}) \quad \stackrel{CP}{\to} \quad -ie^{i\xi_{CP}}\gamma_2\gamma_0\psi^*(t,-\vec{x})$

$$\psi_L^{CP} = -ie^{i\xi_{CP}}\gamma_2\gamma_0\psi_L^*(t,-\vec{x}) = -ie^{i\xi_{CP}}\gamma_2\gamma_0 P_L\psi^*(t,-\vec{x}) = P_L(-ie^{i\xi_{CP}}\gamma_2\gamma_0\psi^*(t,-\vec{x})) = P_L\psi^{CP}$$

$\to$ donot change chirality

$$\bar{\psi}_1\gamma^\mu P_L\psi_2 \quad \stackrel{CP}{\to} \quad (-1)^\mu\bar{\psi}_2\gamma^\mu P_L\psi_1$$

If a gauge field transform under CP as: $A^\mu \quad \stackrel{CP}{\to} \quad (-1)^\mu A^{*\mu}$ then

$$\bar{\psi}_1\gamma^\mu P_L\psi_2 A_\mu + \text{ h.c.} \quad \stackrel{CP}{\to} \quad \bar{\psi}_2\gamma^\mu P_L\psi_1 A_\mu^* + \text{ h.c.}$$

$\to$ conserved CP

Interactions and C,P symmetries

- QCD and QED are invariant under C and P transformations separately $\rightarrow$ C,P symmetry

$$\mathcal{L}_{QCD} = \sum_{q=u,d,c,s,b,t} \bar{q}(i\gamma^{\mu} D_{\mu} + m_q)q - \frac{1}{4} G^{a\mu\nu} G^{a}_{\mu\nu}$$

$$\mathcal{L}_{QED} = \sum_{f=q,l} \bar{f}(i\gamma^{\mu} D_{\mu} + m_f)f - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

Interactions and C,P symmetries

- QCD and QED are invariant under C and P transformations separately → C,P symmetry

$$\mathcal{L}_{QCD} = \sum_{q=u,d,c,s,b,t} \bar{q}(i\gamma^\mu D_\mu + m_q)q - \frac{1}{4} G^{a\mu\nu} G^a_{\mu\nu}$$

$$\mathcal{L}_{QED} = \sum_{f=q,l} \bar{f}(i\gamma^\mu D_\mu + m_f)f - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

- Weak interaction

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{u}\gamma^\mu(1-\gamma_5)dW^+_\mu + \bar{\nu}\gamma^\mu(1-\gamma_5)eW^+_\mu + \text{ h.c.} \right),$$

$$\mathcal{L}^Z_{NC} = \frac{g_2}{2\cos\theta_W} \left(\bar{u}\gamma^\mu(g_V - g_A\gamma_5)u \right) Z_\mu.$$

violate C, P separately, but conserve CP (one generation here)

Weak Interactions in the SM and CP symmetry

In the flavor basis, three generations mix, (i,j=1,2,3)

$$\mathcal{L}^f_{\text{mass}} = -\frac{y^e_{ij} v}{\sqrt{2}} \bar{e}^i_L e^j_R - \frac{y^d_{ij} v}{\sqrt{2}} \bar{d}^i_L d^j_R - \frac{y^u_{ij} v}{\sqrt{2}} \bar{u}^i_L u^j_R + \text{ h.c.},$$

From flavor basis to mass basis: V unitary matrix

$$e^i_R = V^{eR}_{ij} \hat{e}^j_R, \quad e^i_L = V^{eL}_{ij} \hat{e}^j_L, \quad diag(m_e, m_\mu, m_\tau) = V^{eL} \frac{y^e v}{\sqrt{2}} (V^{eR})^\dagger$$

$$u^i_R = V^{uR}_{ij} \hat{u}^j_R, \quad u^i_L = V^{uL}_{ij} \hat{u}^j_L, \quad diag(m_u, m_c, m_t) = V^{uL} \frac{y^u v}{\sqrt{2}} (V^{uR})^\dagger$$

$$d^i_R = V^{dR}_{ij} \hat{d}^j_R, \quad d^i_L = V^{dL}_{ij} \hat{d}^j_L, \quad diag(m_d, m_s, m_b) = V^{dL} \frac{y^d v}{\sqrt{2}} (V^{dR})^\dagger$$

Weak Interactions in the SM and CP symmetry

From flavor basis

$$\mathcal{L}_{CC} = \frac{g_2}{\sqrt{2}} \left(\bar{u}_L \gamma^\mu d_L W_\mu^+ + \bar{\nu}_L \gamma^\mu e_L W_\mu^+ + \text{ h.c.} \right),$$

$$\mathcal{L}_{NC}^Z = \frac{g_2}{\cos\theta_W} \left(\bar{u}_i \gamma^\mu (a_L P_L + a_R P_R) u \right) Z_\mu.$$

to mass basis

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{u}}_L (V^{uL})^\dagger V^{dL} \gamma^\mu \hat{d}_L W_\mu^+ + \bar{\hat{\nu}}_L (V^{\nu L})^\dagger V^{eL} \gamma^\mu \hat{e}_L W_\mu^+ + \text{ h.c.} \right),$$

$$\mathcal{L}_{NC}^Z = \frac{g_2}{2\cos\theta_W} \left(\bar{\hat{u}} \gamma^\mu (a_L P_L (V^{uL})^\dagger V^{uL} + a_R P_R (V^{uR})^\dagger V^{uR}) \hat{u} \right) Z_\mu.$$

We set $(V^{uL})^\dagger V^{dL} = V_{CKM}$, $(V^{uL})^\dagger V^{uL} = (V^{uR})^\dagger V^{uR} = \mathbb{1}$

Since neutrinos are massless in the SM, we can set $(V^{\nu L}) = V^{eL}$, then $(V^{\nu L})^\dagger V^{eL} = \mathbb{1}$

Finally $\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{u}}_L V_{CKM} \gamma^\mu \hat{d}_L W_\mu^+ + \bar{\hat{\nu}}_L \gamma^\mu \hat{e}_L W_\mu^+ + \text{ h.c.} \right),$

$$\mathcal{L}_{NC}^Z = \frac{g_2}{2\cos\theta_W} \left(\bar{\hat{u}} \gamma^\mu (a_L P_L + a_R P_R) \hat{u} \right) Z_\mu.$$

The Cabibbo-Kobayashi-Maskawa matrix (1)

$V_{\text{CKM}} = (V^{uL})^\dagger V^{dL}$: Unitary matrix

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{u}}_L V_{\text{CKM}} \gamma^\mu \hat{d}_L W_\mu^+ + \bar{\hat{d}}_L V_{\text{CKM}}^\dagger \gamma^\mu \hat{u}_L W_\mu^- \right),$$

$\overset{CP}{\longrightarrow}$

$$\mathcal{L}_{CC}^{CP} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{d}}_L V_{\text{CKM}}^T \gamma^\mu \hat{u}_L W_\mu^- + \bar{\hat{u}}_L (V_{\text{CKM}}^\dagger)^T \gamma^\mu \hat{d}_L W_\mu^+ \right),$$

Weak interaction is CP invariant if V_{CKM} is real

$$V_{\text{CKM}} = V_{\text{CKM}}^*$$

The Cabibbo-Kobayashi-Maskawa matrix 1973 (2)

$V_{\text{CKM}} = (V^{uL})^\dagger V^{dL}$: $N \times N$ unitary matrix, N: number of generations

- $N \times N$ unitary matrix is parameterized by N^2 independent parameter

  number of angles: $\frac{N(N-1)}{2}$, number of phases: $\frac{N(N+1)}{2}$
- One can remove phases by redefining fields

$$\hat{d}^i_L \to e^{\alpha^i_d} \hat{d}^i_L, \quad \hat{u}^i_L \to e^{\alpha^i_u} \hat{u}^i_L$$

$$\to \quad V^{ij}_{\text{CKM}} \to e^{i(\alpha^j_d - \alpha^i_u)} V^{ij}_{\text{CKM}}$$

  there are $2N - 1$ independent relative phases which can be used to remove phases of the CKM matrix
- Therefore, number of remaining phases of CKM matrix

$$N_{\text{CP-phases}} = \frac{N(N+1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

N=2 then $N_{\text{CP-phases}} = 0 \to$ no CP violation, (Cabibbo)
N=3 then $N_{\text{CP-phases}} = 1 \to$ CP violation

$\to$ [Nobel price in 2008 for Kobayashi and Maskawa]

The Unitary triangle

Wolfenstein parameterization exploiting observed hierarchy of matrix elements

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

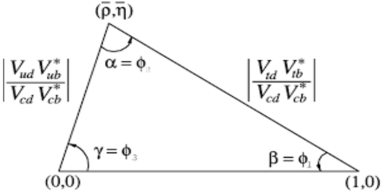
Unitary:

$$\sum_j (V_{\text{CKM}})_{ji}(V^*_{\text{CKM}})_{jk} = \delta_{ik}$$

- Rows (3) are orthogonal, as are the columns (3) $\rightarrow$ six unitary triangles

$$\underbrace{V_{ud}V^*_{ub}}_{A\lambda^3(\bar\rho+i\bar\eta)} + \underbrace{V_{cd}V^*_{cb}}_{-A\lambda^3} + \underbrace{V_{td}V^*_{tb}}_{A\lambda^3(1-\bar\rho-i\bar\eta)} = 0$$

$$\rightarrow \underbrace{\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}}_{-(\bar\rho+i\bar\eta)} + 1 + \underbrace{\frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}}}_{-(1-\bar\rho-i\bar\eta)} = 0$$



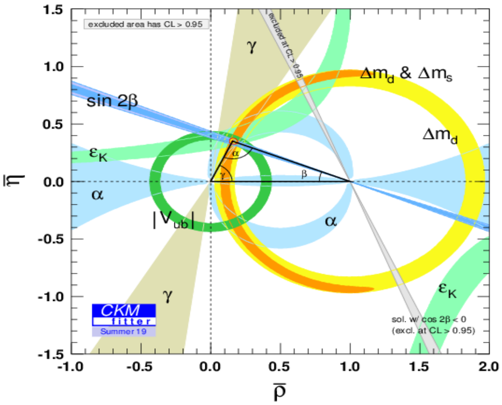
- Six unitary triangles have different shapes but the same area

$$J_{Jarlskog} \equiv 2(\text{area}) = \text{Im}(V_{ud}V^*_{ub}V^*_{td}V_{tb}) \sim A\lambda^3\bar\eta$$

The CKMfitter global fit results '19

Observables	A	λ	$\bar{\rho}$	$\bar{\eta}$
Central $\pm 1\,\sigma$	$0.8235^{+0.0056}_{-0.0145}$	$0.224837^{+0.000251}_{-0.000060}$	$0.1569^{+0.0102}_{-0.0061}$	$0.3499^{+0.0079}_{-0.0065}$

$J_{Jarlskog} = 3.06^{+0.071}_{-0.079} \times 10^{-5}$



Other parameterization of the CKM matrix

$$\begin{aligned} V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

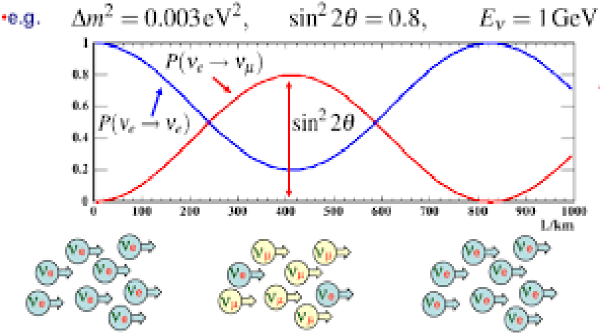
Relation to the Wolfenstein parameterization

$$s_{12} = \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta)$$

One can estimate: $\theta_{12} \sim 13^o$, $\theta_{23} \sim 2.36^o$, $\theta_{13} \sim 0.2^o$ and $\delta \sim 69^o$

CP violation in the lepton sector

In the SM, neutrino are massless $\rightarrow$ no CP violation in the lepton sector.
BUT!!! Neutrino oscillation indicates that at least two neutrinos are massive.



$\rightarrow$ Neutrinos as signature of physics beyond the SM

CP violation in the lepton sector

- We can not set $(V^{\nu L}) = V^{eL}$, and $(V^{\nu L})^\dagger V^{eL} \neq \mathbb{1}$
- $V^{\nu L} \equiv U_{PMNS}$ and $V^{eL} = \mathbb{1}$

$$\begin{pmatrix} \nu_L^e \\ \nu_L^\mu \\ \nu_L^\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \hat{\nu}_L^1 \\ \hat{\nu}_L^2 \\ \hat{\nu}_L^3 \end{pmatrix}$$

$\hat{\nu}^1, \hat{\nu}^2, \hat{\nu}^3$ are mass eigenstates, ν^e, ν^μ, ν^τ flavor eigenstates (or interaction eigenstates)

- The charged current for the lepton sector

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{\nu}}_L U_{PMNS}^\dagger \gamma^\mu \hat{e}_L W_\mu^+ + \text{ h.c.} \right),$$

How many independent parameters to determine the U_{PMNS} unitary matrix?

The Pontecorvo-Maki-Nakagawa-Sakata matrix 1962

- If neutrino has Dirac mass, right-handed neutrino ν_R exists but it is singlet and donot interact with other particles $\rightarrow$ donot observe them

$$\bar{\nu}_L^i \frac{h_{ij}^{\nu} v}{\sqrt{2}} \nu_R^j$$

The U_{PMNS} is similar to the $V_{\rm CKM}$ matrix. It is defined by three angles and one complex phase

- If neutrino donot have Dirac mass, but Majorana mass

Example: $d = 5$ Weinberg operator: $c_{ij}^{d=5} \left(\overline{L_{iL}^c} \tilde{\Phi}^*\right) \left(\tilde{\Phi}^\dagger L_{jL}\right)$

where $L_{jL} = \begin{pmatrix} \nu_L^j \\ e_L^j \end{pmatrix}$ and $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $\tilde{\Phi} = i\sigma^2 \Phi$

$$\rightarrow \quad \overline{\nu_{iL}^c} \frac{c_{ij}^{d=5} v^2}{2} \nu_{jL}$$

$\rightarrow$ cannot rephase neutrino fields, but only 3 charged lepton fields
$\rightarrow$ number of complex phases$= 6$ -3 $= 3$.

The U_{PMNS} is defined by three angles and 3 complex phase (1 Dirac phase and two Majorana phases)

Standard parameterization of the U_{PMNS} matrix

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This matrix can be determined from neutrino oscillation measurements.

Nufit 2020 results

NuFIT 5.3 (2024)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.66^{+0.73}_{-0.70}$	$31.60 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.407 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00058}$	$0.02029 \to 0.02391$	$0.02219^{+0.00059}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+41}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.511^{+0.027}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.024}$	$-2.581 \to -2.409$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 9.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.344$
	$\theta_{12}/^\circ$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$	$33.67^{+0.73}_{-0.71}$	$31.61 \to 35.94$
	$\sin^2\theta_{23}$	$0.454^{+0.019}_{-0.016}$	$0.411 \to 0.606$	$0.568^{+0.016}_{-0.021}$	$0.412 \to 0.611$
	$\theta_{23}/^\circ$	$42.3^{+1.1}_{-0.9}$	$39.9 \to 51.1$	$48.9^{+0.9}_{-1.2}$	$39.9 \to 51.4$
	$\sin^2\theta_{13}$	$0.02224^{+0.00056}_{-0.00057}$	$0.02047 \to 0.02397$	$0.02222^{+0.00069}_{-0.00057}$	$0.02049 \to 0.02420$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.13}_{-0.11}$	$8.23 \to 8.95$
	$\delta_{CP}/^\circ$	232^{+39}_{-25}	$139 \to 350$	273^{+24}_{-26}	$195 \to 342$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.81 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.505^{+0.024}_{-0.026}$	$+2.426 \to +2.586$	$-2.487^{+0.027}_{-0.024}$	$-2.566 \to -2.407$

CP violation in the SM: recap

CP is violated in the weak interaction both for the quark and lepton sector as consequence of flavor basis $\neq$ mass basis and three generations.

- The charged current for the quark sector

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{u}}_L V_{\text{CKM}} \gamma^\mu \hat{d}_L W^+_\mu + \text{ h.c.} \right),$$

  V_{CKM} (3 angles, 1 CP phase) is well determined from weak interacion in hardon physics. $\theta_{12} \sim 13$, $\theta_{23} \sim 2.36$, $\theta_{13} \sim 0.2$ and $\delta \sim 69$

- The charged current for the lepton sector

$$\mathcal{L}_{CC} = \frac{g_2}{2\sqrt{2}} \left(\bar{\hat{\nu}}_L U^\dagger_{PMNS} \gamma^\mu \hat{e}_L W^+_\mu + \text{ h.c.} \right),$$

  $U^\dagger_{PMNS}$ (3 angles, 1 or 3 complex phases) can only determined from neutrino oscillation measurements. $\theta_{12} \sim 33.44$, $\theta_{23} \sim 49.2$, $\theta_{13} \sim 8.15$ and $\delta_{CP} \sim 197^{+27}_{-24}$
  $\rightarrow$ maximal mixing ??? many unsolved problems in the neutrino sector

Why neutrino mass is tiny

Majorana mass: $d = 5$ Weinberg operator

$$c_{ij}^{d=5} \left(\overline{L_{iL}^{c}}\tilde{\Phi}^{*}\right)\left(\tilde{\Phi}^{\dagger}L_{jL}\right)$$

$c_{ij}^{d=5} = \frac{c_{ij}}{\Lambda}$: If $\Lambda \gg v$, Λ can be mass of new particle outside the SM spectrum, then the mass of the neutrinos is suppressed by a large scale $\rightarrow$ Seesaw mechanism

Why neutrino mass is tiny

Majorana mass: $d = 5$ Weinberg operator

$$c_{ij}^{d=5}\left(\overline{L_{iL}^{c}}\tilde{\Phi}^{*}\right)\left(\tilde{\Phi}^{\dagger}L_{jL}\right)$$

$c_{ij}^{d=5} = \frac{c_{ij}}{\Lambda}$: If $\Lambda \gg v$, Λ can be mass of new particle outside the SM spectrum, then the mass of the neutrinos is suppressed by a large scale $\rightarrow$ Seesaw mechanism

$m_\nu \sim \frac{v^2}{2\Lambda} < 1\,\text{eV}$
$v = 246$ GeV,
$\Lambda \sim 10^{14}$ GeV

Three seesaw models

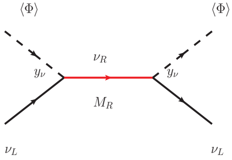
How to generate Weinberg operator from tree-level diagrams?

$$c_{ij}^{d=5}\left(\overline{L_{iL}^{c}}\tilde{\Phi}^{*}\right)\left(\tilde{\Phi}^{\dagger}L_{jL}\right)$$

- New field must be added to the SM.
- Mass of new field must be large
- Its couplings to the neutrino should be small

Type I

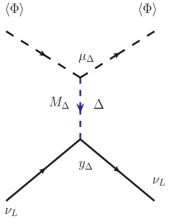
Fermion Singlet



$$m_\nu = y_\nu^T \frac{1}{M_R} y_\nu v^2$$

Type II

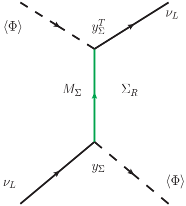
Scalar Triplet



$$m_\nu = y_\Delta^T \frac{\mu_\Delta}{M_\Delta^2} y_\nu v^2$$

Type III

Fermion Triplet



$$m_\nu = y_\Sigma^T \frac{1}{M_\Sigma} y_\Sigma v^2$$

Standard Model Lecture 1+2: summary