

INTRODUCTION TO THE STANDARD MODEL OF PARTICLE PHYSICS

THE FIRST LECTURE

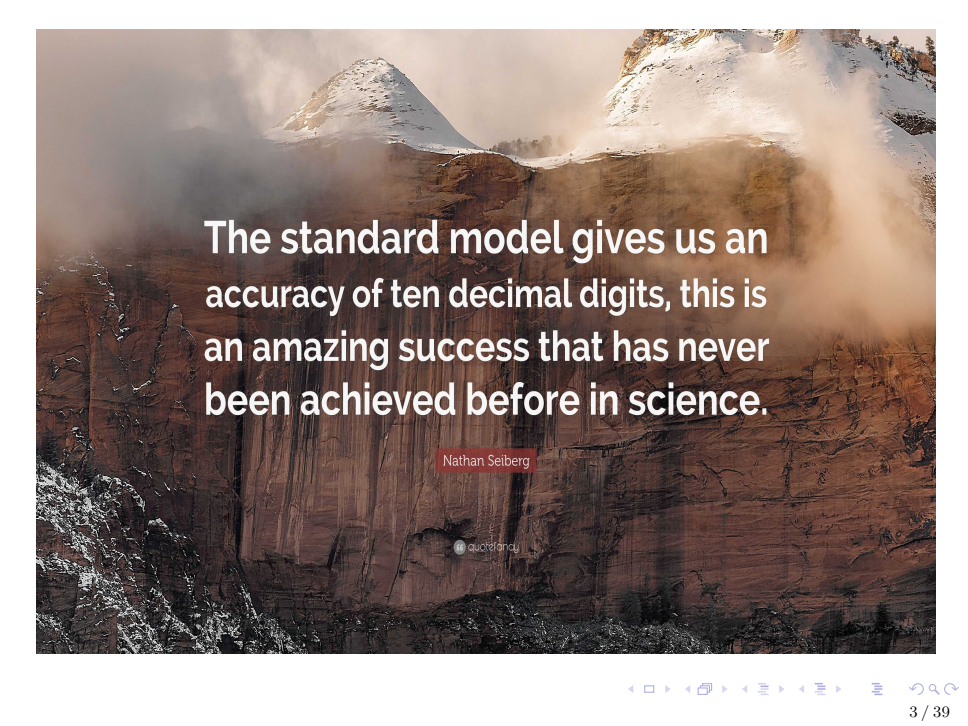
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Vietnam School on Neutrinos, July 15 - 26, 2024

What is the most successful physics model?

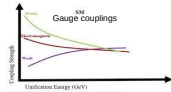


The standard model gives us an accuracy of ten decimal digits, this is an amazing success that has never been achieved before in science.

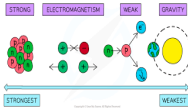
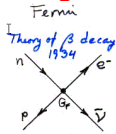
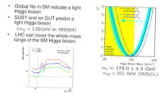
Nathan Seiberg

qurefancy

THE STANDARD MODEL IN THREE HOURS



Higgs Physics



Higgs Mechanism



$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\not{D}\psi + h.c. + \chi_i^\dagger \chi_j + \phi^\dagger + h.c. + |\partial_\mu \phi|^2 - V(\phi)$$

GP in Standard Model
 CKM paradigm provides the framework for CP violation in SM

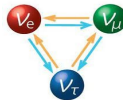
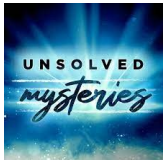
- Observed baryon asymmetry: 6.3×10^{-11}
- Angle of the unitary triangle (measured): $\sim 30^\circ$
- CP violation in CKM paradigm: $\sim 10^{-10}$

Need additional sources

anomaly

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

HADRON



The End

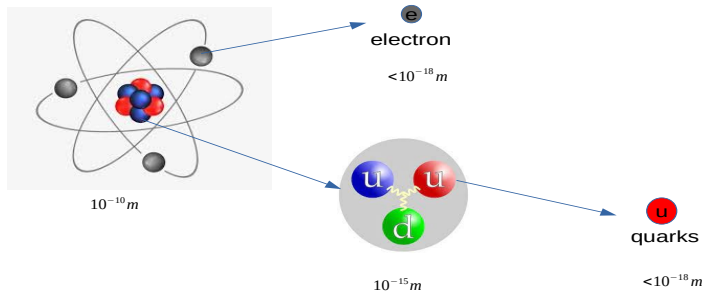
REFERENCES

- Particle Data Group
<https://pdg.lbl.gov/>
- Books:
 - Mathew D. Schwartz "Quantum Field Theory and the Standard Model"
 - Michael E. Peskin "Concepts of Elementary Particle Physics"
 - Michael E. Peskin and Daniel V. Schroeder "An Introduction to Quantum Field Theory"
- Online Lectures:
 - Prof. Leonard Susskind's lectures on the SM
 - Prof. Michael E. Peskin's lectures on youtube
- Discussions with teachers, researchers and friends

Lecture 1

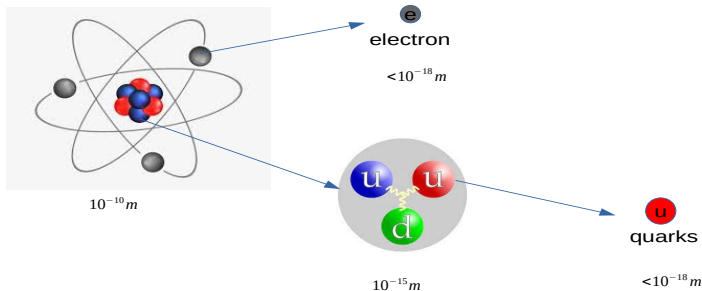
WHAT ARE ELEMENTARY PARTICLES ?

Atom and subatomic particles



WHAT ARE ELEMENTARY PARTICLES ?

Atom and subatomic particles



In natural unit: $\hbar = c = k_B = e_0 = 1$,

$[E] = [p] = \text{eV}$, $[\text{Length}] = [\text{Time}] = \text{eV}^{-1}$

	Atom	nucleon	quark (u,d)	electron
size	500 MeV^{-1}	5 GeV^{-1}	$< 5 \text{ TeV}^{-1}$	$< 5 \text{ TeV}^{-1}$
mass	n GeV	1 GeV	$u(d) \sim 2(5) \text{ MeV}$	0.5 MeV

WHAT ARE ELEMENTARY PARTICLES ?

An elementary particle is a particle with no internal structure. It is considered as a point-like object.

Here are properties of elementary particles

- 1 Electric charge (quantized, ne $n = 0, \pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$)
- 2 Spin (scalar $s = 0$, Fermion $s = \frac{1}{2}$, vector boson $s = 1$)
- 3 Mass (large range: $0 \rightarrow \text{eV} \rightarrow \text{GeV}$)
- 4 life-time (τ , s): stable ($\tau_{\gamma, u, e} \rightarrow \infty$), unstable
 $\tau_{W, Z, t, H} \sim 10^{-25} \text{s}$
- 5 anti particle: carry opposite quantum number e^{-}, e^{+}
- 6 Fundamental Interactions (electromagnetic, weak, strong)
- 7 Some other quantum numbers (color charge, lepton, baryon, strange, CP)

PARTICLE CONTENT OF THE SM

mass →	$\approx 2.3 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 173.07 \text{ GeV}/c^2$	0	$\approx 126 \text{ GeV}/c^2$
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
LEPTONS	$0.511 \text{ MeV}/c^2$	$105.7 \text{ MeV}/c^2$	$1.777 \text{ GeV}/c^2$	$91.2 \text{ GeV}/c^2$	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
	$< 2.2 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 15.5 \text{ MeV}/c^2$	$80.4 \text{ GeV}/c^2$	
	0	0	0	± 1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	
				GAUGE BOSONS	

MASS VS DECAYWIDTH

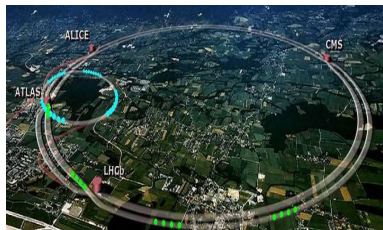
life-time and decay width

$$\tau = \frac{\hbar}{\Gamma}$$

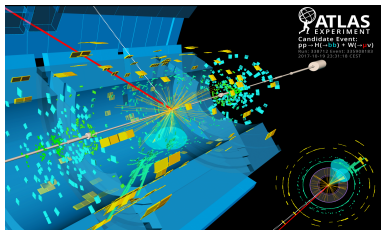


WHERE DO WE OBSERVE ELEMENTARY PARTICLES?

- Natural sources: Sun, explosion, cosmic rays $\rightarrow \nu, \gamma, e, \mu$
- Laboratory:
 - Reactor: e, ν, γ
 - Accelerator: all elementary particles can be produced as long as it has enough energy \rightarrow **High Energy Physics**
Tevatron ($E_{CM} = 2$ TeV), Large Electron-Positron Collider (LEP, $E_{CM} = 209$ GeV)



Large Hadron Collider (LHC)



$E_{CM} = 7, 8, 13, 14$ TeV

HIGH ENERGY PHYSICS

ENERGY MASS RELATION: $E^2 = m^2 + p^2$, $p = m\gamma v$,

$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

LEP: $E_e = 104.5$ GeV

$m_e = 0.5$ MeV

$v_e \sim 0.99999999999885$

LHC: $E_p = 7$ TeV

$m_p = 938$ MeV

$v_p \sim 0.999999999102$

HIGH ENERGY PHYSICS

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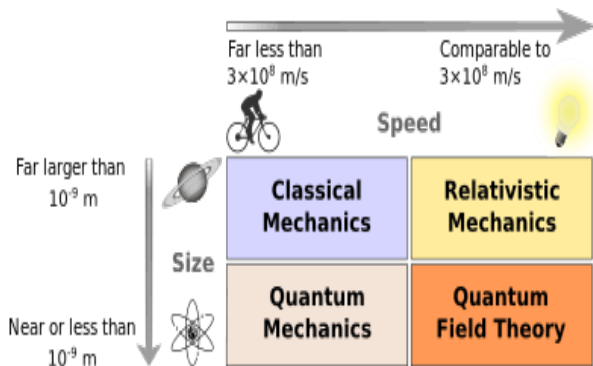
$m_e = 0.5$ MeV

$v_e \sim 0.99999999999885$

LHC: $E_p = 7$ TeV

$m_p = 938$ MeV

$v_p \sim 0.999999999102$



HEP needs QFT
Uncertainty principle

$$\Delta E \cdot \Delta t \geq \hbar/2$$

$$\Delta p \cdot \Delta x \geq \hbar/2$$

consequence:

virtual particle

$$E^2 \neq m^2 + p^2$$

QUANTUM MECHANICS VS QUANTUM FIELD THEORY

QM

- Space and time are separated variables
- Hamiltonian:
$$H = \frac{p^2}{2m} + V(x)$$
- Schoedinger equation:
$$i\hbar \frac{\partial \phi(x)}{\partial t} = H\phi(x)$$
- Wave function: $\phi(x) = \sum_i c_i(E_i, x) e^{-iE_i t/\hbar} |n_i\rangle$
- Conserved quantity:
$$-i\hbar \frac{\partial A}{\partial t} = [H, A] = 0$$

=====

The Legende transformation

$$H = \frac{\partial \mathcal{L}}{\partial(\dot{\phi}(x))} \dot{\phi}(x) - \mathcal{L}$$

Lagrangian: $\mathcal{L}(x(t), \dot{x}(t))$

QFT

- Minkowski space and Lorentz invariance: $\mu, \nu = 0, 1, 2, 3$
$$x^\mu x_\mu = x^\mu g_{\mu\nu} x^\nu = t^2 - \vec{x}^2$$

$$x^\mu = (t, \vec{x}) \equiv (t, x, y, z),$$

$$g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

$$p^\mu = (E, p_x, p_y, p_z)$$

$$\partial^\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$
- Lagrangian: $\mathcal{L}(\phi(x), \partial^\mu \phi(x))$
- Euler-Lagrande equation:
$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} = 0$$
- Conserved current:
$$\phi(x) \rightarrow \phi(x) + \Delta \phi(x)$$

$$\partial^\mu j_\mu = 0, j_\mu = \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi)} \Delta \phi(x)$$

LAGRANGIAN AND FIELD

LAGRANGIAN AT ZERO TEMPERATURE IN MINKOWSKI SPACE (4 DIMENSIONAL SPACE):

- Classical Mechanics: $\mathcal{L} = T - V = \mathcal{L}(\dot{x}(t), x(t))$
- QFT: Lagrangian is a function of **free** fields and their derivatives, $\mathcal{L}(\phi(x), \partial^\mu \phi(x))$, satisfying several requirements:
 - Lorentz invariant, $\partial^\mu \phi(x) \partial_\mu \phi(x), A^\mu(x) A_\mu(x), F^{\mu\nu} F_{\mu\nu}$
 - Mass dimension is 4 since $S = \int dx^4 \mathcal{L}$ is dimensionless
 - Your wished symmetries

FREE FIELD:

it is a operator function of space time and belongs to a representation of Lorentz group (scalar, vector, left (right) handed Weyl spinor, Dirac spinor)

$$\phi(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E}} (a(p) e^{-ip^\mu x_\mu} + b^\dagger(p) e^{+ip^\mu x_\mu})$$

$a(p), b(p)$: annihilation operators, $a^\dagger(p), b^\dagger(p)$ creation operators

PARTICLE CLASSIFICATION: BOSON VS FERMION

BOSON

- H, γ, Z, W^\pm, g
- Spin: integer (0,1)
- commutative:
 $\phi(x)\phi(y) - \phi(y)\phi(x) = [\phi(x), \phi(y)] = 0$
- real field:
particle \equiv anti-particle
- complex field:
particle \neq anti-particle

FERMION

- Leptons, quarks
- Spin: half-integer ($\frac{1}{2}$)
- anti-commutative:
 $\psi(x)\psi(y) = -\psi(y)\psi(x)$
 $\{\psi(x), \psi(y)\} = 0$
- only complex field:
particle \neq anti-particle

Higher spin elementary particle: graviton ($s=2$), gravitino ($s=3/2$)
they have not yet been observed.

BOSONIC FIELD

Scalar field: spin = 0, H

- Free Lagrangian: $\phi(x)$ with mass dimension $[\phi(x)] = 1$
 $\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + \frac{m^2}{2} \phi^2$
- Klein-Gordon equation $(\partial^\mu \partial_\mu - m^2) \phi(x) = 0$
- one component field = one degree of freedom (dof)
- $[a(p), a^\dagger(p')] = 2E \delta^3(p - p')$, $[a(p), a(p')] = 0$

$$\phi(x) = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2E}} \left(a(p) e^{-ip^\mu x_\mu} + a^\dagger(p) e^{+ip^\mu x_\mu} \right)$$

one particle state: $|p\rangle = \sqrt{2E} a^\dagger(p) |0\rangle$ $a(p) |0\rangle = 0$

BOSONIC FIELD

Vector field: spin=1, massless γ, g , massive (Z, W^\pm)

- Free Lagrangian: $A^\mu(x)$ with mass dimension $[A^\mu] = 1$
 $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu$ where the field strength tensor
 $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$
- Proca equation: $\partial_\nu F^{\mu\nu} + m^2 A^\mu = 0$
- Four component field > 2 dof (massless), 3 dof (massive) \rightarrow need gauge fixing condition $\partial^\mu A_\mu(x) = 0$
- $\epsilon^\mu(\lambda, p)$: polarization vector $\lambda = \pm 1$ for massless, $\lambda = -1, 0, 1$ for massive

$$A^\mu(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E}} \sum_\lambda \left(\epsilon^\mu(\lambda, p) a(p, \lambda) e^{-ip^\mu x_\mu} + \epsilon^{\mu*}(\lambda, p) a^\dagger(p, \lambda) e^{+ip^\mu x_\mu} \right)$$

$$\sum_\lambda \epsilon^\mu(\lambda, p) \epsilon^\nu(\lambda, p) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m^2}$$

FERMIONIC FIELD

Dirac field: $\text{spin}=\frac{1}{2}$, charged fermions $e, \mu, \tau, u, d, s, c, b, t$

- Free Lagrangian: $\psi(x)$ with mass dimension $[\psi(x)] = \frac{3}{2}$
 $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ where $\bar{\psi} = \psi^\dagger \gamma^0$

Dirac matrix: γ_μ are 4×4 matrices, in chiral representation:

$$\gamma^\mu = \left(\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \left(\begin{array}{cc} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{array} \right) \right) \equiv \left(\begin{array}{cc} 0 & \vec{\sigma}^\mu \\ \sigma^\mu & 0 \end{array} \right)$$

$\sigma^{1,2,3}$ are three 2×2 Pauli matrices

- Dirac equation $(i\gamma^\mu \partial_\mu - m)\psi = 0$
- Dirac field: $\psi(x) \equiv (\psi_1, \psi_2, \psi_3, \psi_4)^T$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3 \sqrt{2E}} \sum_{\lambda=\pm 1} \left(u(\lambda, p) a(p, \lambda) e^{-ip^\mu x_\mu} + v(\lambda, p) b^\dagger(p, \lambda) e^{+ip^\mu x_\mu} \right)$$

$u(\lambda, p), v(\lambda, p)$ are Dirac spinors satisfying $(\gamma^\mu p_\mu - m)u = 0$
and $(\gamma^\mu p_\mu + m)v = 0$

LEFT- AND RIGHT-HANDED WEYL SPINOR

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } [\gamma_5, \gamma_\mu] = 0$$

Let's define left- and right-handed projectors

$$P_L = \frac{1-\gamma_5}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_R = \frac{1+\gamma_5}{2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$P_L P_L = P_L, P_R P_R = P_R, P_L P_R = 0$$

One can construct:

$$\psi_L = P_L \psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ 0 \\ 0 \end{pmatrix}, \quad \psi_R = P_R \psi = \begin{pmatrix} 0 \\ 0 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad \psi = \psi_L + \psi_R$$

ψ_L is left-handed Weyl spinor and ψ_R is right-handed Weyl spinor.

They are two component spinors.

We define chirality from $\gamma_5 \psi_L = -\psi_L$, $\gamma_5 \psi_R = \psi_R$

Dirac field Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi = i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{\partial} \psi_R + m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

mass term does not respect chirality

MASSLESS FERMIONIC FIELD

LAGRANGIAN

$\mathcal{L} = i\bar{\psi}_L \not{\partial} \psi_L + i\bar{\psi}_R \not{\partial} \psi_R$ Left- and right-handed Weyl spinor are independent. Chirality is Lorentz invariant and from Dirac equation one gets ($i, j, k = 1, 2, 3$)

$$(\gamma^0 |p| - \gamma^i p^i) u(p) = 0 \rightarrow (1 - \frac{\gamma^0 \gamma^i p^i}{|p|}) u(p) = 0$$

Using $\gamma^0 \gamma^i = \gamma^5 \Sigma^i$ where $\Sigma^i = \frac{1}{2} \epsilon^{ijk} \sigma_{jk}$, $\sigma_{jk} = \frac{i}{2} [\gamma_j, \gamma_k]$ then

$$\gamma_5 u(p) = \frac{\vec{\Sigma} \vec{p}}{|p|} u(p) \text{ where } \frac{\vec{\Sigma} \vec{p}}{|p|} \text{ is the helicity operator.}$$

For massless particle helicity is identical to chirality.

NEUTRINO IN THE SM

They are left-handed Weyl spinors and their anti-neutrinos are right-handed Weyl spinors. There is no right-handed neutrinos. As consequence neutrinos are massless in the SM.

Standard Model of Elementary Particles

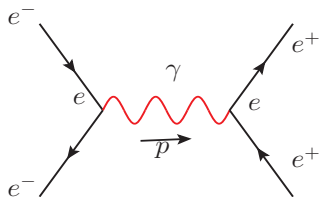
		three generations of matter (fermions)			interactions / force carriers (bosons)	
		I	II	III		
QUARKS	mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
	charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
	spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
		u up	c charm	t top	g gluon	H higgs
		$\approx 4.7 \text{ MeV}/c^2$	$\approx 90 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
		$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		d down	s strange	b bottom	γ photon	
		$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
		-1	-1	-1	0	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		e electron	μ muon	τ tau	Z Z boson	
LEPTONS		$< 1.0 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	
		0	0	0	± 1	
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	
		ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

GAUGE BOSONS
VECTOR BOSONS

SCALAR BOSONS

FUNDAMENTAL INTERACTIONS: ELECTROMAGNETIC

- between particles carrying electric charge
- Quantum view: charged particles interact by exchanging a virtual photon

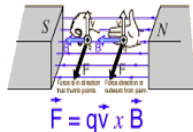


Electric

$$F = \frac{kq_1q_2}{r^2}$$

Diagram showing two positive charges, q_1 and q_2 , with force vectors F pointing away from each other. Below the diagram is the text "Like charges repel".

Magnetic

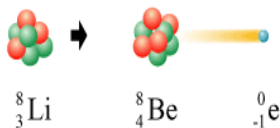


EM interaction can be described by a point-like interaction of charged particle with photon. Lagrangian can be written as

$$\mathcal{L}_{int} = e\bar{\psi}(x)\gamma^\mu\psi(x)A_\mu(x)$$

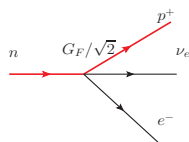
WEAK INTERACTION (1)

- Nuclear beta decay :
 $n \rightarrow p^+ + e^- + \nu_e$
see Prof. Oyama's lecture



- Enrico Fermi proposed (1934)
 $\frac{G_F}{\sqrt{2}}(\bar{p}(x)\gamma_\mu n(x))(\bar{e}(x)\gamma^\mu \nu(x))$
- Wu's experiment (1956) found parity violation in beta decay
see Prof. Agarwalla's lecture
- Sudarshan and Marshak (1957), Feynman and Gell-Mann (1958) suggested

$$\frac{G_F}{\sqrt{2}}(\bar{p}(x)\gamma_\mu(1 - \gamma_5)n(x))(\bar{e}(x)\gamma^\mu(1 - \gamma_5)\nu(x))$$



WEAK INTERACTION (2)

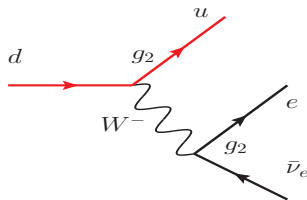
- Proton and neutron consist of quarks (1964, Gell-Mann and Zweig): $p = (uud)$ while $n = (ddu)$

$$\frac{G_F}{\sqrt{2}}(\bar{d}(x)\gamma_\mu(1 - \gamma_5)u(x))(\bar{e}(x)\gamma^\mu(1 - \gamma_5)\nu(x))$$

- The beta decay can be described exchanged by W . It can be written as

$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}}\bar{u}(x)\gamma^\mu(1 - \gamma_5)d(x)W_\mu(x) + \frac{g_2}{2\sqrt{2}}\bar{e}(x)\gamma^\mu(1 - \gamma_5)\nu(x)W_\mu(x)$$

$$\alpha_2 = \frac{g_2^2}{4\pi} \sim 3 \times 10^{-2}$$



WEAK INTERACTION (3)

Not only charged current interaction

$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}} \bar{u}(x) \gamma^\mu (1 - \gamma_5) d(x) W_\mu(x) + \frac{g_2}{2\sqrt{2}} \bar{e}(x) \gamma^\mu (1 - \gamma_5) \nu(x) W_\mu(x)$$

But also neutral current interaction

$$\mathcal{L}_{int} = \frac{g_2}{2 \cos \theta_W} \bar{f}(x) \gamma^\mu (g_V^f - g_A^f \gamma_5) f(x) Z_\mu(x)$$

f : quark, charged leptons, neutrinos

g_V^f, g_A^f are coefficients depending on electric charge and isospin of fermion f

weak neutral current was confirmed in 1973, in a neutrino experiment in the Gargamelle bubble chamber at CERN

W, Z were seen in 1983 at Super Proton Synchrotron at CERN

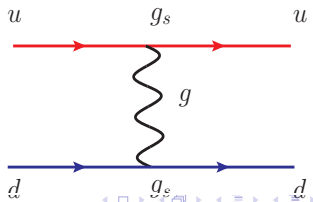
STRONG INTERACTION

- Old: Strong interaction between nucleons to form nuclei
- The quarks model of Gell-Mann and Zweig (1964) to explain the classification of hadrons
- New: all particles carrying color charge participate strong interaction (quarks, gluons)
- Lagrangian: simple guess based on Lorentz invariance

$$\mathcal{L}_{int} = g_s \bar{q} \gamma^\mu q G_\mu + g_s (\partial^\mu G^\nu) G_\mu G_\nu + g_s^2 G_\mu G_\nu G^\mu G^\nu$$

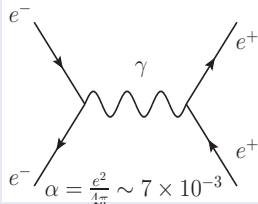
$G_\mu(x)$: gluon field g_s : strong coupling

$$\alpha_s = \frac{g_s^2}{4\pi} \sim 1.2 \times 10^{-1}$$

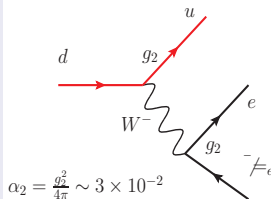


FUNDAMENTAL INTERACTIONS: RECAP

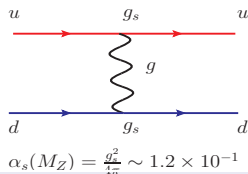
ELECTROMAGNETIC



WEAK



STRONG



GRAVITATION

gravitational force: $F = G \frac{m_1 m_2}{r^2}$

$$G \sim 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$

Construct a dimensionless coupling

$$\alpha_G = \frac{Gm_e^2}{\hbar c} \sim 1.7 \times 10^{-45}$$

$$\alpha_G \ll \alpha < \alpha_2 < \alpha_s$$

SYMMETRY

- makes a theory more predictable
- leads to conservation laws
- makes computation simpler
- easier to convince stubborn physicists

Human-invented symmetry may not be realized in Nature, or may be broken.

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Familiar examples:

- Space is isotropic $\rightarrow SO(3)$ rotation group \rightarrow conservation of angular momentum
- Space-time is homogeneous \rightarrow translation group \rightarrow conservation of energy and momentum
- $U(1)_Q$: $\psi_f \rightarrow e^{i\alpha q_f} \psi_f$, q_f is electric charge \rightarrow conservation of electric charge

QUESTION?

Can we use symmetry to build interactions between elementary particles?

UNITARY TRANSFORMATIONS

Important in physics, geometry, informatics

- Preserve probability in quantum mechanics

$$|\langle \psi_2 | U | U \psi_1 \rangle|^2 = |\langle \psi_2 | U^\dagger U | \psi_1 \rangle|^2 = |\langle \psi_2 | \psi_1 \rangle|^2$$

Unitary: $\langle \psi_2 | U^\dagger U | \psi_1 \rangle = \langle \psi_2 | \psi_1 \rangle$

Anti-unitary: $\langle \psi_2 | U^\dagger U | \psi_1 \rangle^* = \langle \psi_2 | \psi_1 \rangle$

- Unitary

$$U^\dagger U = \mathbb{1}$$

- Special Unitary

$$\det U = 1$$

- Matrix representation $SU(N)$: $N \times N$ matrices with

$$U^\dagger U = \mathbb{1} \quad \det U = 1$$

$$U = e^{i\alpha_a T_a}, \quad T_a + T_a^\dagger = 1, \quad \text{Tr}[T_a] = 0,$$

T_a : generators, Hermitian and traceless matrices.

$a = 1, \dots, D$ where D is the number degree of freedom

$$D = N^2 - 1$$

ABELIAN GAUGE SYMMETRY (1)

- $U(1)_Q$ transformation: given a quantum field $\psi(x)$

$$\psi'(x) = e^{i\alpha g Q} \psi(x)$$

α is a continuous parameter $\in \mathbb{R}$, g is parameter characterizing the group, Q is the quantum number carried by the field $\psi(x)$. It acts on field, donot change space-time.

- Abelian/ commutative group:

$$e^{i\alpha_1 g Q} e^{i\alpha_2 g Q} = e^{i(\alpha_1 + \alpha_2) g Q} = e^{i\alpha_2 g Q} e^{i\alpha_1 g Q}$$

- Global transformation: α is independent of space-time

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

where $\bar{\psi} = \psi^\dagger \gamma^0$

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- Assume if there exists a term as $\bar{\psi}_1 \phi \psi_2$ then we have $-Q_{\psi_1} + Q_\phi + Q_{\psi_2} = 0$. It means that Q quantum number is conserved.

ABELIAN GAUGE SYMMETRY (2)

- If α is a function of space-time $\alpha(x) \rightarrow$ local transformation.

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is not invariant anymore.

What should we do to restore symmetry?

- Under $e^{i\alpha(x)gQ}$ transformation

$$\mathcal{L} \rightarrow \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - gQ\bar{\psi}\gamma^\mu\psi\partial_\mu\alpha(x)$$

- If we add one more term in the original Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + gQ\bar{\psi}\gamma^\mu A_\mu\psi \quad (1)$$

and require that the new field A_μ transform as

$$A_\mu \rightarrow A_\mu + \partial_\mu\alpha(x)$$

(1) is invariant under local transformation.

- Rewrite (1) as

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$

with $D_\mu = \partial_\mu - igQA_\mu$: covariant derivative

QUANTUM ELECTRODYNAMICS (QED)

- Lagrangian is invariant under a local transformation. There appears a new vector field A_μ and an interaction term.

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - igQA_\mu)\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

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Identifying g to elementary charge e , Q to electric charge quantum number, $Q = -1$ for electron, $Q = 2/3$ for up-type quark, $Q = -1/3$ for down-type quark, A_μ as photon field, we obtain a theory called **Quantum Electrodynamics (QED)**

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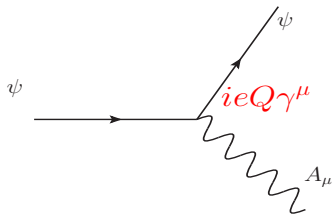
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- Photon is massless since the term $m_A A^\mu A_\mu$ is not gauge invariant.
- Photon is a real vector field.
- There is no self-coupling of photons



NON-ABELIAN GAUGE SYMMETRY (1): YANG-MILLS THEORY (1954)

It is not enough to describe all interactions by only Abelian gauge symmetry, we want to have multiple vector particles appears in the theory.

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- $SU(N)$, $N \geq 2$: a set of fermionic fields belong to a fundamental representation of $SU(N)$.

$$\Sigma = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} \rightarrow e^{ig_N \alpha_a(x) T^a} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$$

$a = 1, \dots, N^2 - 1$, T^a are $N \times N$ Hermitian matrices, generators of $SU(N)$ which obey the group algebra

$$[T^a, T^b] = i f^{abc} T^c \quad f^{abc} \text{ group structure constant}$$

NON-ABELIAN GAUGE SYMMETRY (2)

- Lagrangian of the set of fermionic fields

$$\mathcal{L} = i\bar{\Sigma}\gamma^\mu D_\mu \Sigma - \bar{\Sigma} M \Sigma$$

M is $N \times N$ mass matrix.

- Under $SU(N)$ local transformation

$$\mathcal{L} \rightarrow i\bar{\Sigma} U^\dagger \gamma^\mu D'_\mu U \Sigma - \bar{\Sigma} U^\dagger M U \Sigma$$

It is invariant if

$$\begin{aligned} D'_\mu &= U D_\mu U^\dagger \\ U^\dagger M U &= M \end{aligned}$$

$M = m\mathbb{1}$ means that all fields in the multiplet must have the same mass.

NON-ABELIAN GAUGE SYMMETRY (3)

- Using the similarity with Abelian case, we set

$$D_\mu = \partial_\mu - ig_N T^a A_\mu^a, \quad a = 1, \dots, N^2 - 1$$

The requirement $D'_\mu = U D_\mu U^\dagger$ means that

$$\begin{aligned} T.A'_\mu &= UT.A_\mu U^\dagger + \frac{i}{g_N} U \partial_\mu U^\dagger \\ &= T.A_\mu + T.\partial_\mu \phi + i[T.\partial_\mu \phi, T.A_\mu] + \dots \end{aligned}$$

- How to construct field strength tensor?

$$[D_\mu, D_\nu] = -ig_N F_{\mu\nu}^a T^a$$

$$\text{then } F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_N f^{abc} A_\mu^b A_\nu^c$$

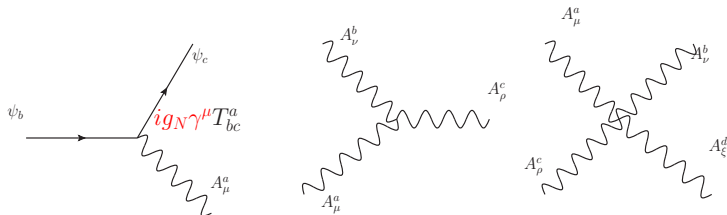
NON-ABELIAN GAUGE SYMMETRY (4)

- Lagrangian of the set of fermionic fields and new vector fields (gauge fields)

$$\mathcal{L} = i\bar{\Sigma}\gamma^\mu D_\mu \Sigma - m\bar{\Sigma}\Sigma - \frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_N f^{abc} A_\mu^b A_\nu^c \quad D_\mu = \partial_\mu - ig_N T^a A_\mu^a$$

- all fields in the multiplet must have the same mass, gauge fields are massless
- number of new gauge fields $N^2 - 1$, they belong to the adjoint representation of the $SU(N)$
- trilinear and quartic gauge couplings exist



- Gauge fields are real vector fields and massless

DISCUSSIONS ON GAUGE SYMMETRY

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- Interaction terms appear naturally with a common coupling for each group

DISCUSSIONS ON GAUGE SYMMETRY

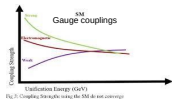
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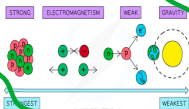
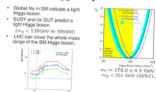
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Is that enough to describe the electromagnetic, weak and strong interactions?

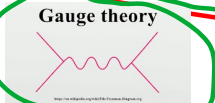
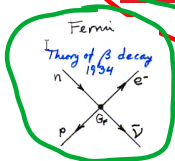
SUMMARY OF LECTURE 1



Higgs Physics



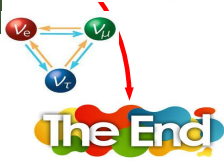
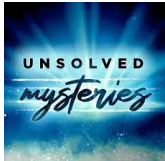
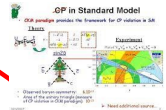
Higgs Mechanism



anomaly

$$\partial_\mu J_5^\mu = \frac{e^2}{16\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + h.c. + \sum_i \bar{\psi}_i \psi_i + \phi^\dagger + h.c. + |\partial_\mu \phi|^2 - V(\phi)$$



The End