July 20-21, 2023
@VSON2023

Particle and Radiation detectors
- Basic -

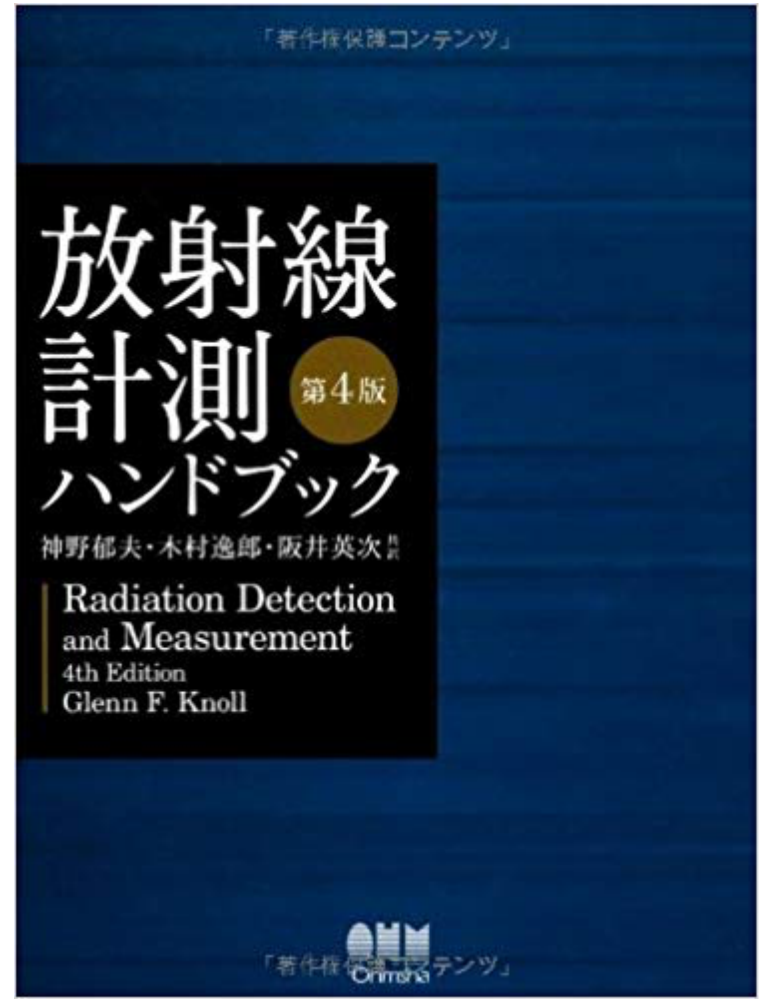
based on my lectures for undergraduate students
in Kyoto University

T. Nakaya (Kyoto University)

Outline

1. Radiation - Nuclear Processes in Radioactive Sources -
2. Passage of Radiation Through Matter
3. Interaction of Photons - X ray and γ ray -
4. Measurement of a charged particle
5. Measurement of a neutral particle (mainly a photon)
6. Measurement of particles -other type of detectors-
7. Trigger and Data Acquisition system
8. High Energy Physics experiments and the data

References



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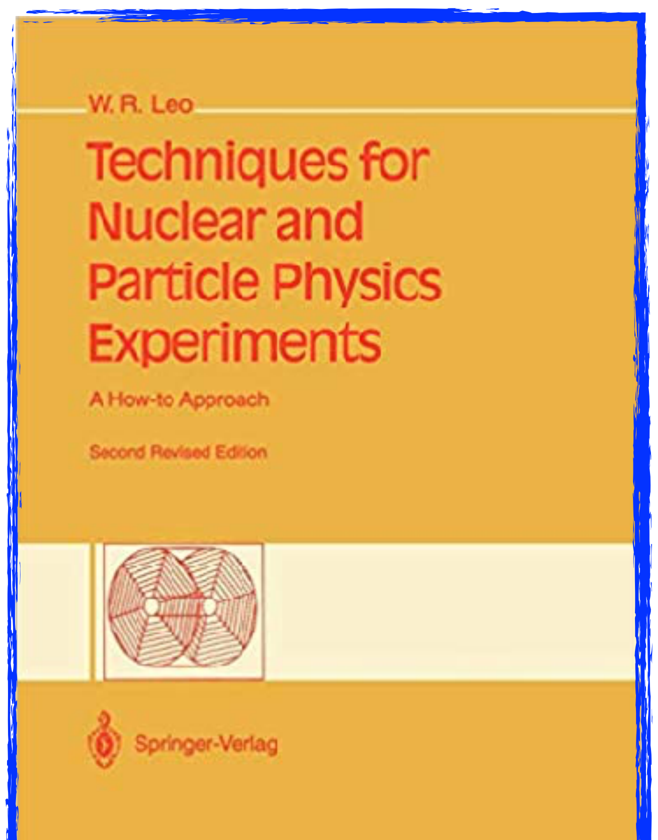

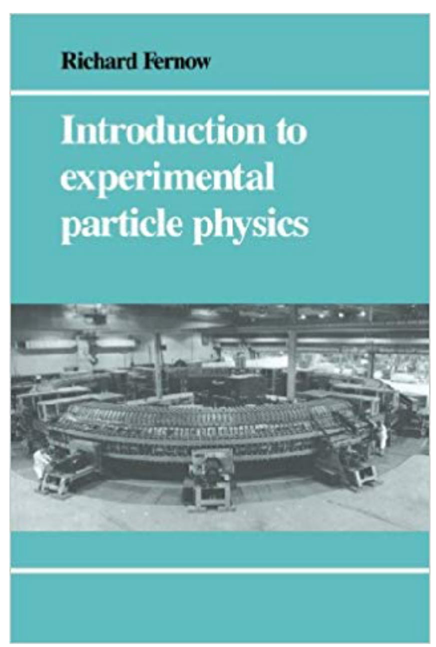

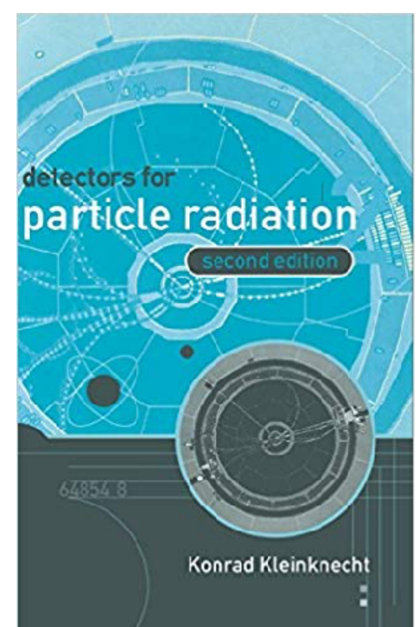

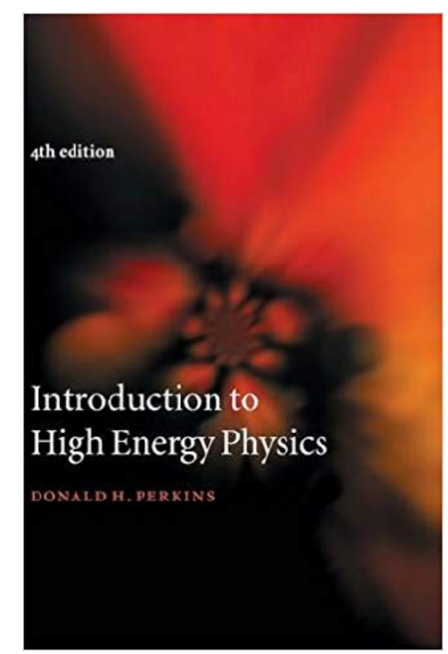

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Radiation

- Nuclear Processes in Radioactive Sources -

1. Radiation

1. Radiation
   - α ray
   - β ray
   - γ ray（X ray)
2. Source Activity Units
   - Becquerel（Bq）：1 disintegrations/sec

$$\frac{dN}{dt} = -\lambda N$$

$$1\ \mathrm{Bq} = 2.703 \times 10^{-11}\ \mathrm{Ci}$$

   - half life time（Nuclear Physics）：time to N/2
   - Life time（Particle Physics）： time to N/e
3. Energy Units
   - eV, (keV, MeV, GeV, TeV, PeV, …)
     - We measure a single particle

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

Discovery of Radiation

- In 1895, Wilhelm Röntgen discovered X ray.

2 | 1 X 線の発見と原子核乾板

図 1.1 レントゲンが最初に撮影したベルタ夫人の手の X 線写真

- Energy of X ray is the function of the wavelength (frequency) of photons
  - E=hν (Plank consonant h: 6.626×10^{-34}J・s or 4.135×10^{-15}eV・s)
  - Wavelength is

$$\lambda = \frac{1.240\times10^{-6}}{E}$$

    - Visible lights from sun
    - Radiation from our body, Electron mass, Proton mass

Where do we find radiation

- Environment
  - β ray（γ ray)
  - α ray
- Cosmic Ray
  - Mainly muon on surface（1μ/100cm²/ec)
    - Interactions of protons with atmosphere Nitrogen and Oxygen produce pions
    - The pions decay to muons and neutrinos.

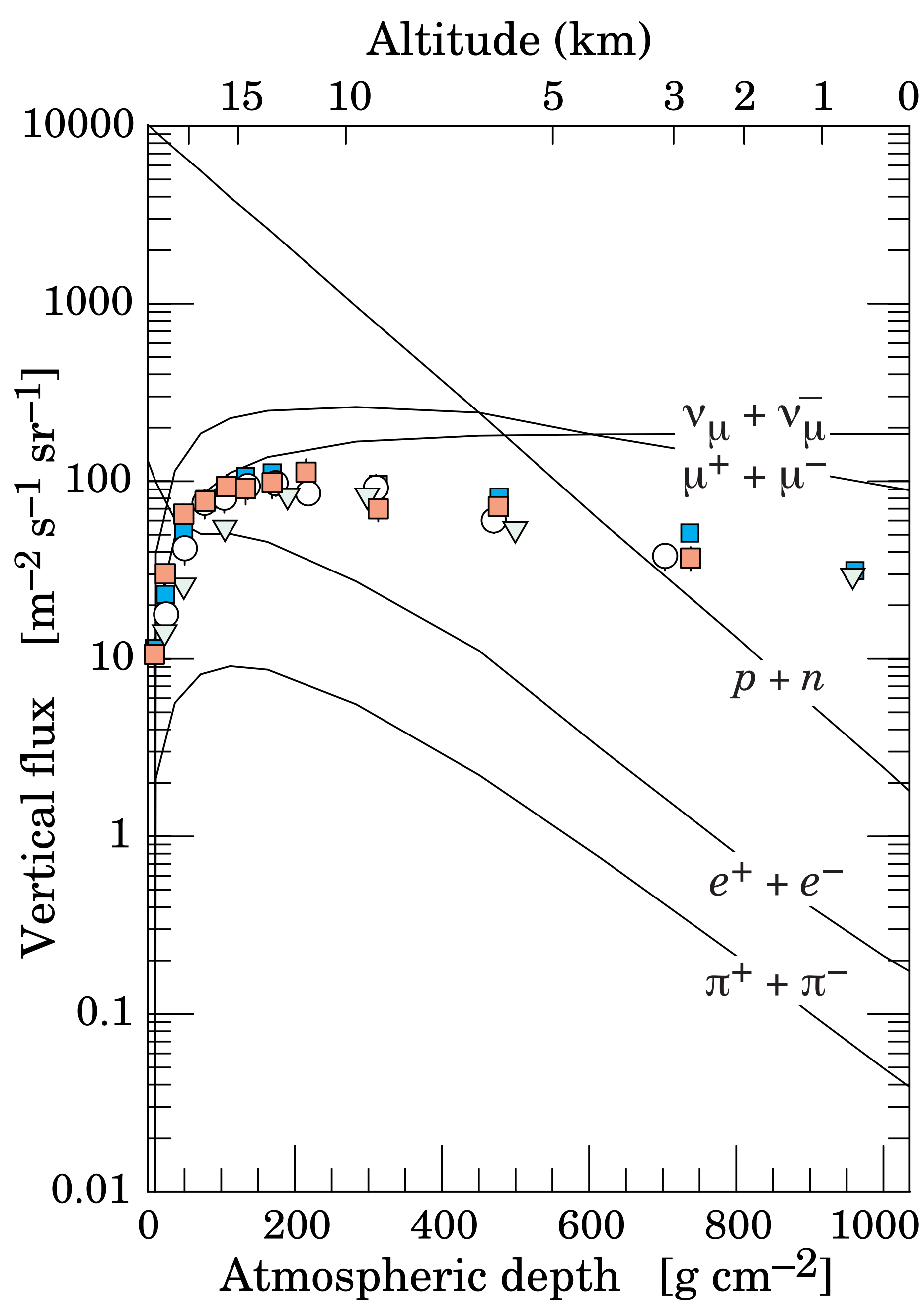
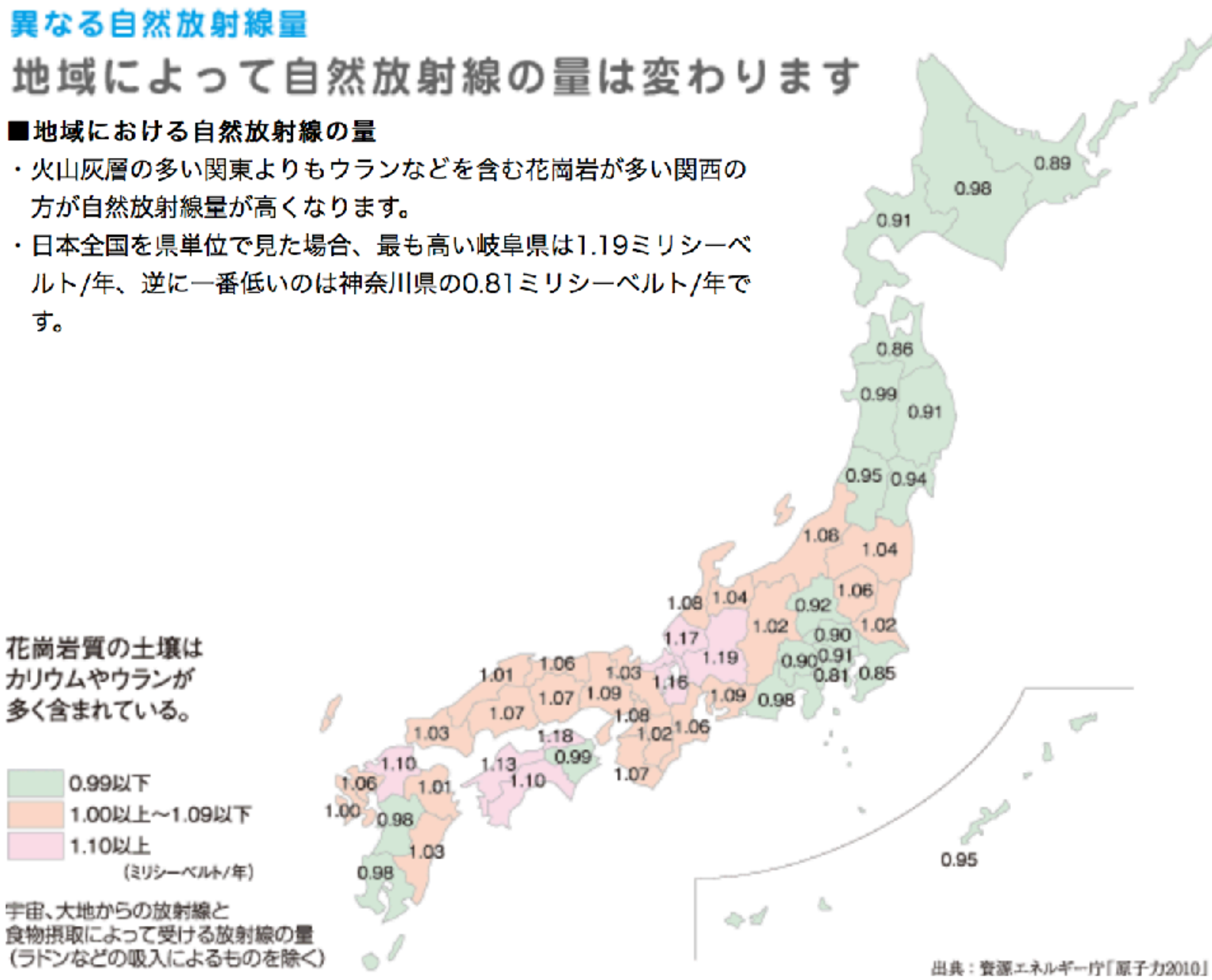
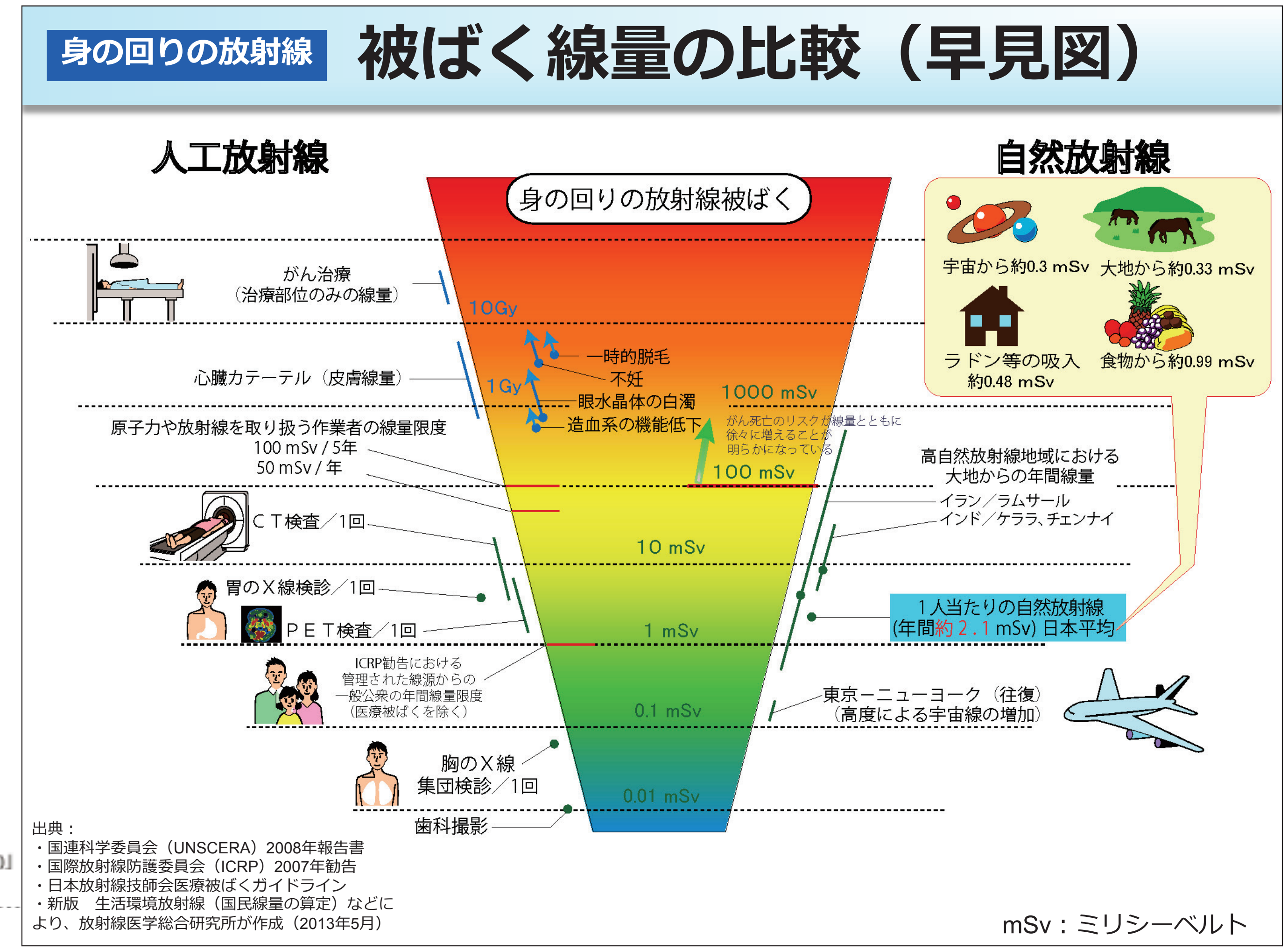


Figure 29.4: Vertical fluxes of cosmic rays in the atmosphere with $E > 1$ GeV estimated from the nucleon flux of Eq. (29.2). The points show measurements of negative muons with $E_\mu > 1$ GeV [41–45].

Particle Data Book





1mSv/y~0.1μSv/h

- シーベルトとは？
  - 吸収線量（Gy：グレイ[J/kg]）をもとに、
    - 線量＝吸収線量×放射線荷重計数×(組織荷重計数)
  - として計算。電子は放射線荷重係数＝１で、以下でも１とする。
- 1Bq/cm^{2}のβ線源（137Cs）がある土の表面に手をかざしたとすると、〜１μSv/h

Table 1.1. Characteristics of nuclear radiations

Type	Origin	Process	Charge	Mass [MeV]	Spectrum (energy)
α-particles	Nucleus	Nuclear decay or reaction	+2	3727.33	Discrete [MeV]
β^--rays	Nucleus	Nuclear decay	−1	0.511	Continuous [keV – MeV)
β^+-rays (positrons)	Nuclear	Nuclear decay	+1	0.511	Continuous [keV – MeV]
γ-rays	Nucleus	Nuclear deexcitation	0	0	Discrete [keV – MeV]
x-rays	Electron cloud	Atomic deexcitation	0	0	Discrete [eV – keV]
Internal conversion electrons	Electron cloud	Nuclear deexcitation	−1	0.511	Discrete [high keV]
Auger electrons	Electron cloud	Atomic deexcitation	−1	0.511	Discrete [eV – keV]
Neutrons	Nucleus	Nuclear reaction	0	939.57	Continuous or discrete [keV – MeV]
Fission fragments	Nucleus	Fission	$\simeq$20	80 – 160	Continuous 30 – 150 MeV

Electron (and positron) source (β ray)

- From the decay of Nucleus (β decay)
    - $n \rightarrow p + e^- + \nu_e$ —

$${}^{A}_{Z}X \rightarrow {}_{Z+1}{}^{A}Y + \beta^- + \overline{\nu}$$

- Similar process
    - β^+ decay
        - $p \rightarrow n + e^+ + \nu_e$
- Internal Conversion：mono-enegetic electron source
    - An excited nucleus after the β decay interacts with an electron in an orbit, and the mono-energetic electron is emitted. An electron (usually a s electron) couples to an excited energy state of the nucleus and take the energy of the nuclear transition directly, without the gamma ray being produced.
- Auger electrons：Lower energy than internal conversion

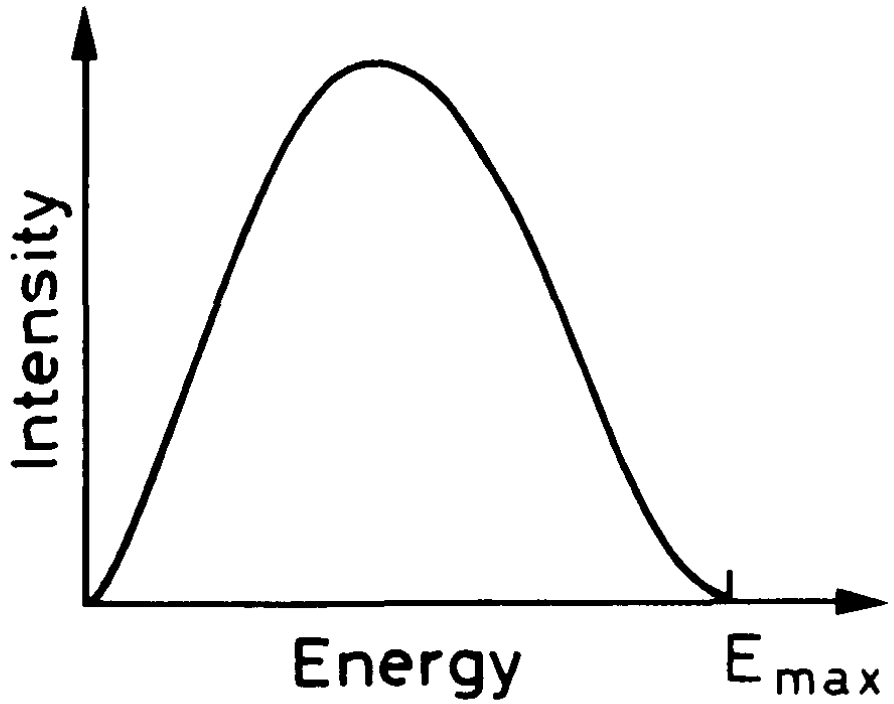


Fig. 1.2. Typical continuous energy spectrum of beta decay electrons

Table 1.3. List of pure β^- emitters

Source	Half-life	E_{max} [MeV]
^{3}H	12.26 yr	0.0186
^{14}C	5730 yr	0.156
^{32}P	14.28 d	1.710
^{33}P	24.4 d	0.248
^{35}S	87.9 d	0.167
^{36}Cl	3.08×10^5 yr	0.714
^{45}Ca	165 d	0.252
^{63}Ni	92 yr	0.067
$^{90}Sr/^{90}Y$	27.7 yr/64 h	0.546/2.27
^{99}Tc	2.12×10^5 yr	0.292
^{147}Pm	2.62 yr	0.224
^{204}Tl	3.81 yr	0.766

Table 1.4. Some internal conversion sources

Source	Energies [keV]
^{207}Bi	480, 967, 1047
^{137}Cs	624
^{113}Sn	365
^{133}Ba	266, 319

Heavy charged particle (mainly α ray)

- α ray（^{4}He nucleus）
  - Stable particle（Why？）
  - Emission from nucleus (α dacay)
- With an accelerator, a proton and other heavy charged particle can be produced

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}\alpha$$

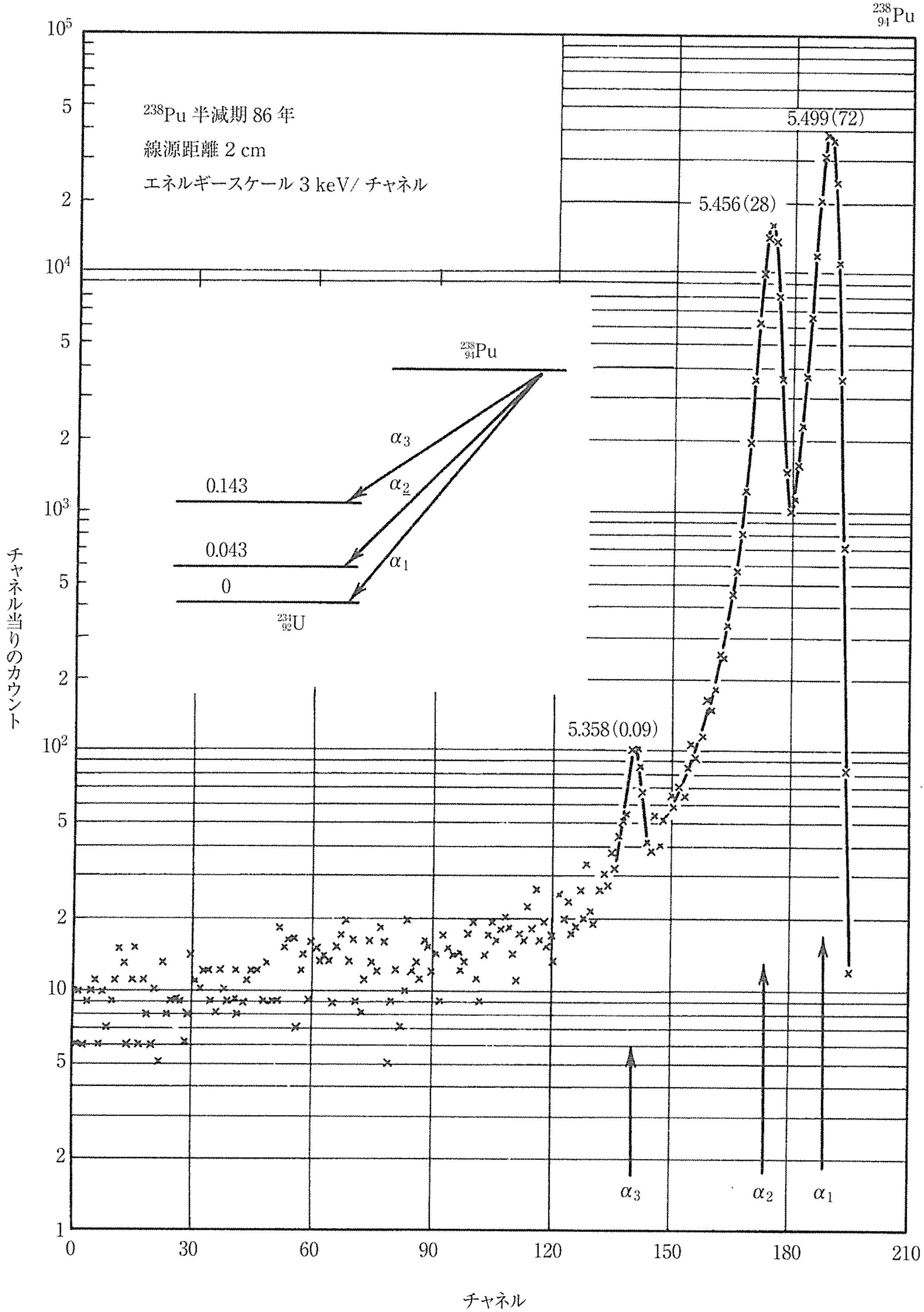


図 1.3　^{238}Pu の崩壊で発生するアルファ粒子の群．パルス波高分布はシリコン表面障壁型検出器によって測定された三つの群を示す．それぞれのピークは MeV で表わされるエネルギーと百分率発生量（カッコ内）により判別される．図中の崩壊図式は MeV 単位で表わされる生成核のエネルギー準位を示す．（スペクトルは Chanda and Deal[2] による）

Table 1.2. Characteristics of some alpha emitters

Isotope	Half-life	Energies [MeV]	Branching
^{241}Am	433 yrs.	5.486	85%
		5.443	12.8%
^{210}Po	138 days	5.305	100%
^{242}Cm	163 days	6.113	74%
		6.070	26%

X ray and γ ray

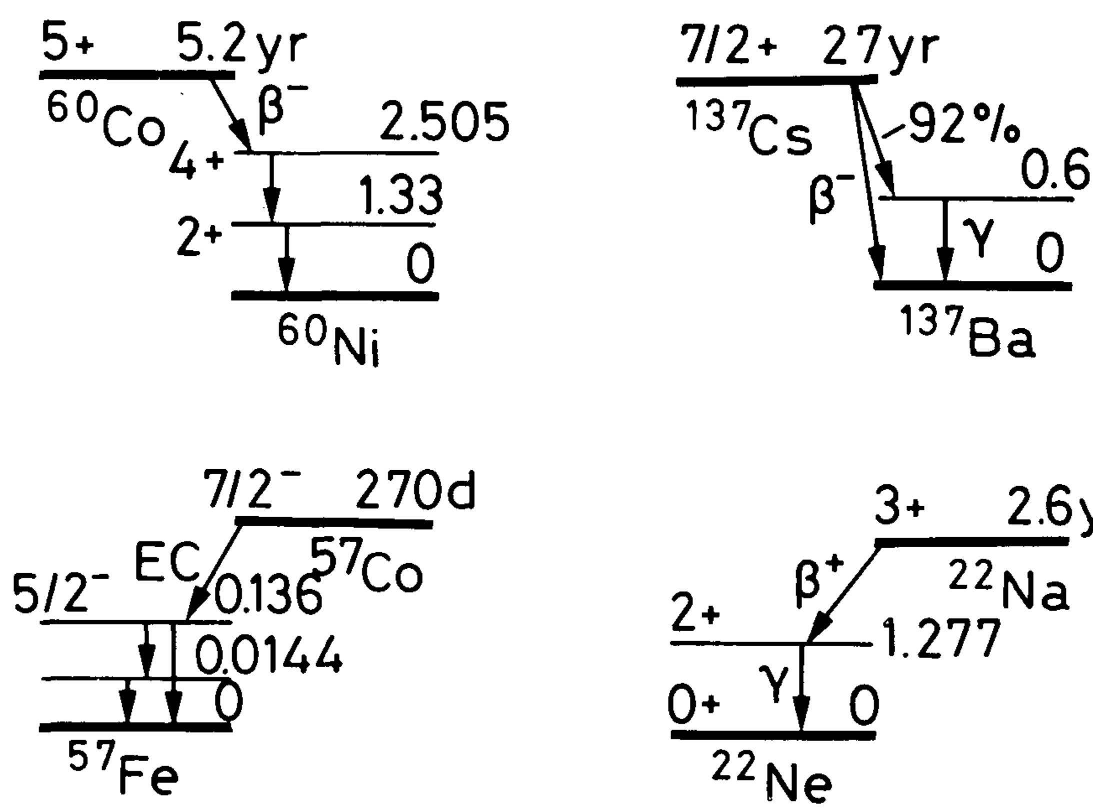
- High energy photons
- γ ray often follows β decay.
- annihilation γ ray (from β^+ decay)
  - $e^+e^- \rightarrow \gamma\gamma$
- γ ray from nuclear reaction
- Bremsstrahlung
- Characteristics X ray
- Synchrotron Radiation

X ray and γ ray

Characteristic X ray

•

γ ray from β decay



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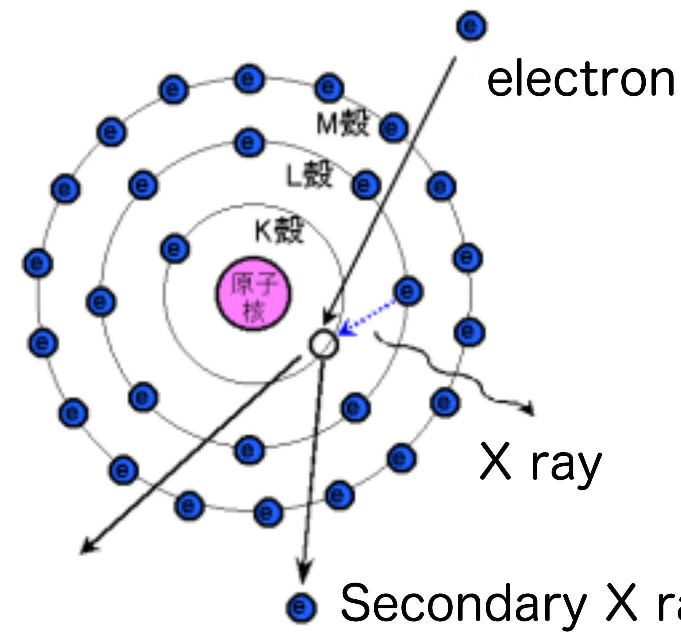
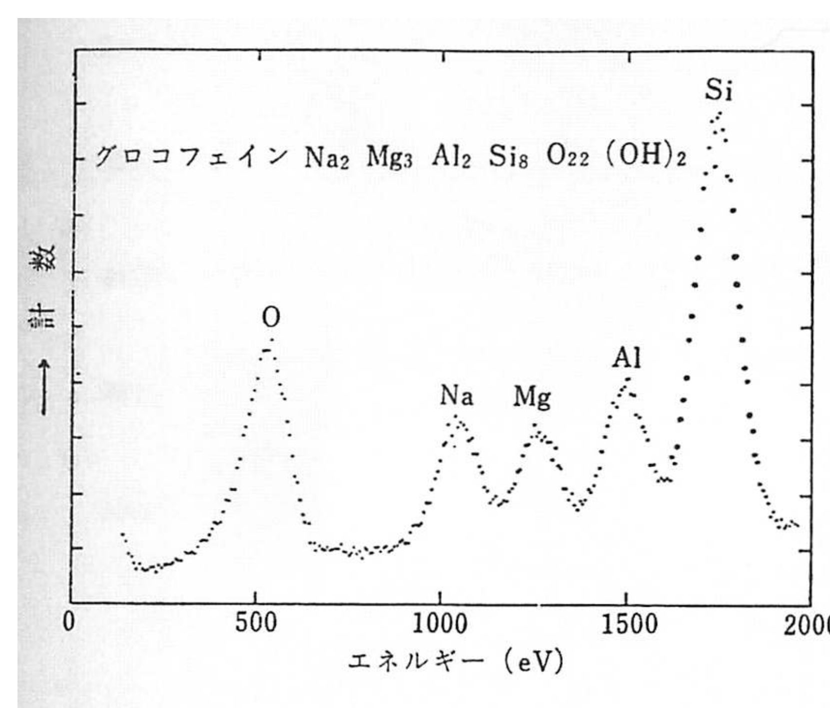
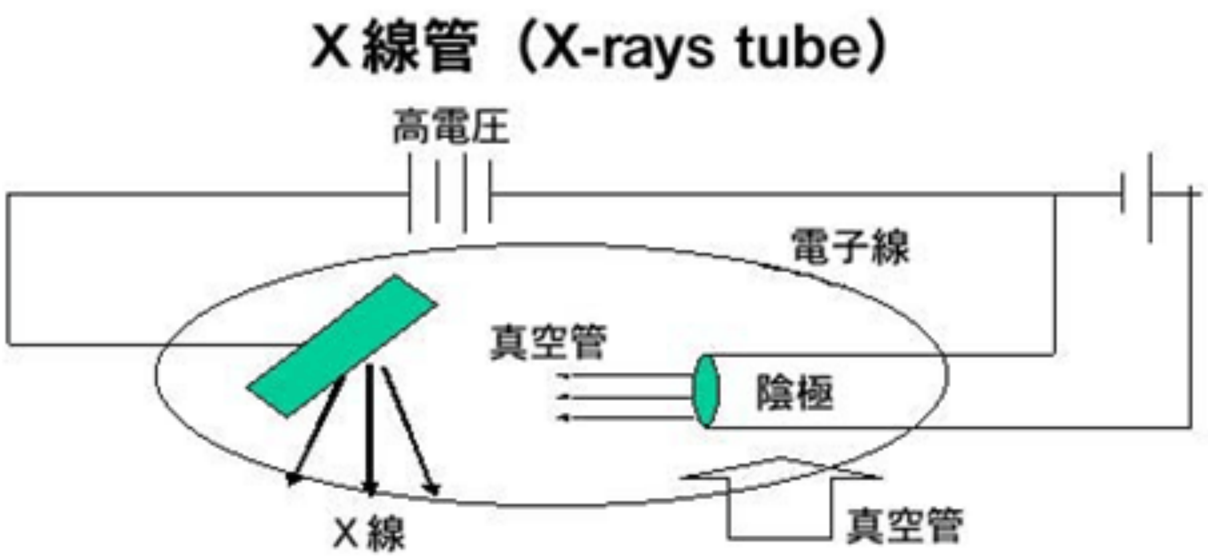


図3 Characteristic X ray



X ray Spectrum

X ray source とX ray picture



INTPIX4のX線照射試験2

・腹部X線画像

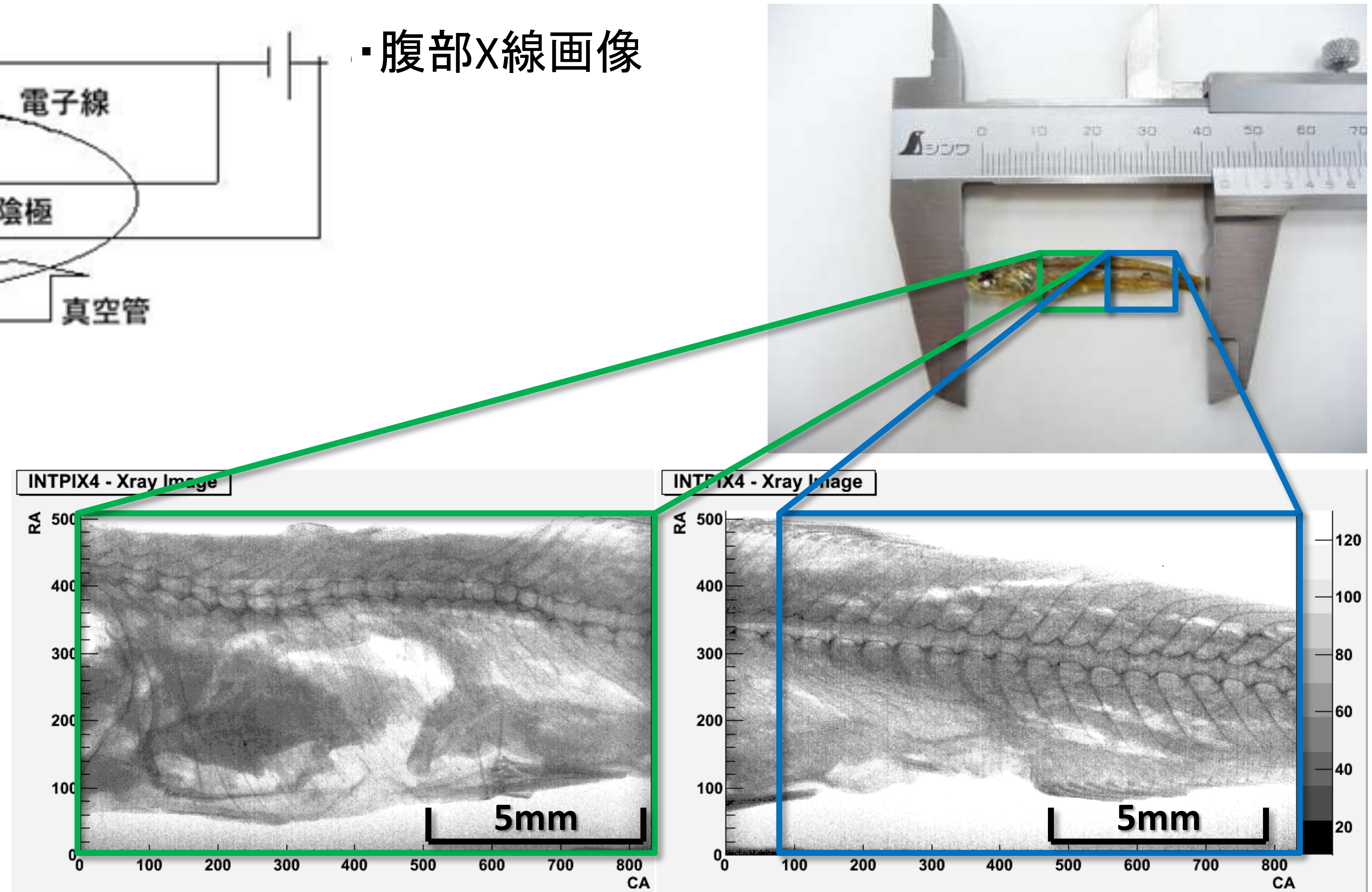


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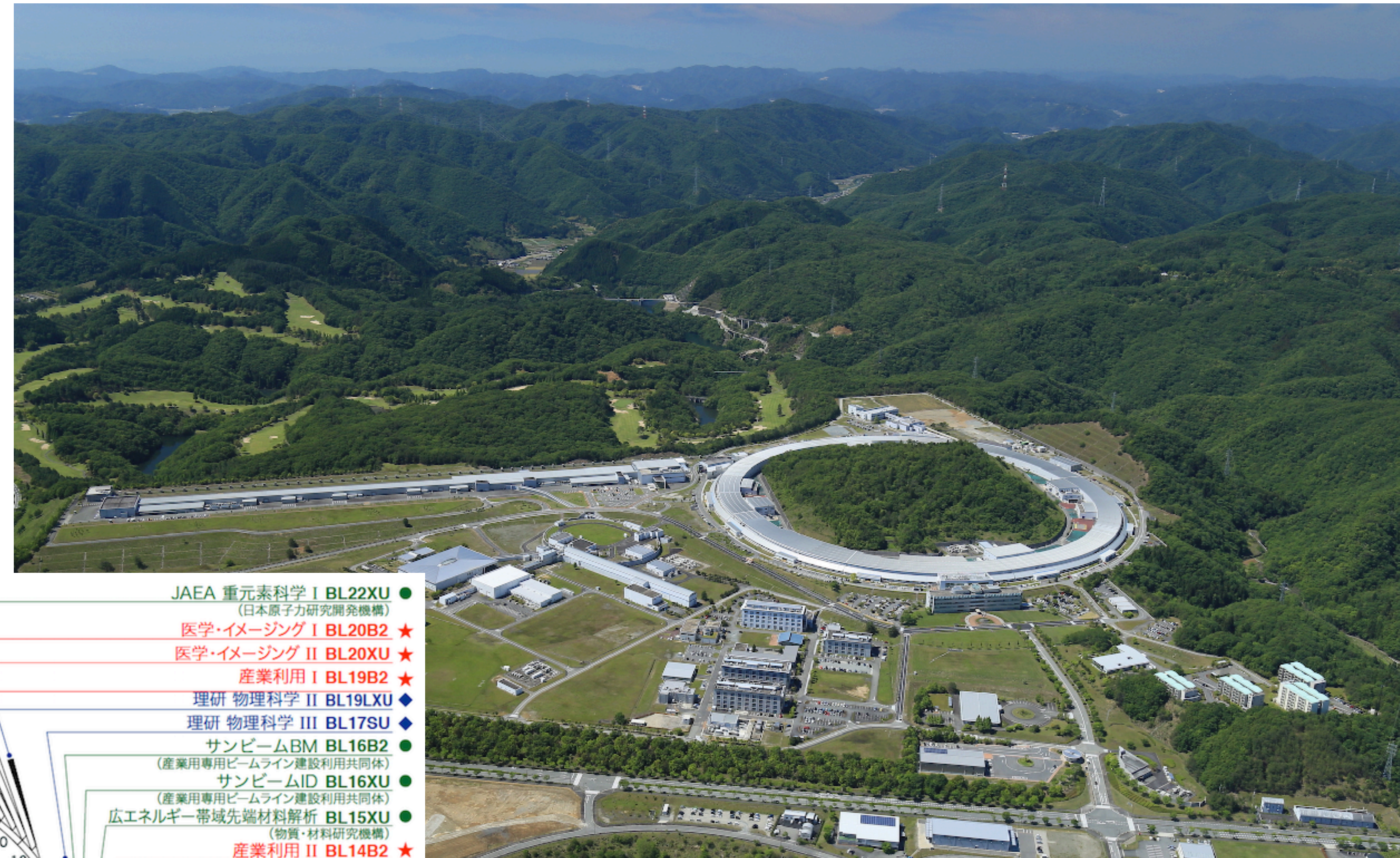
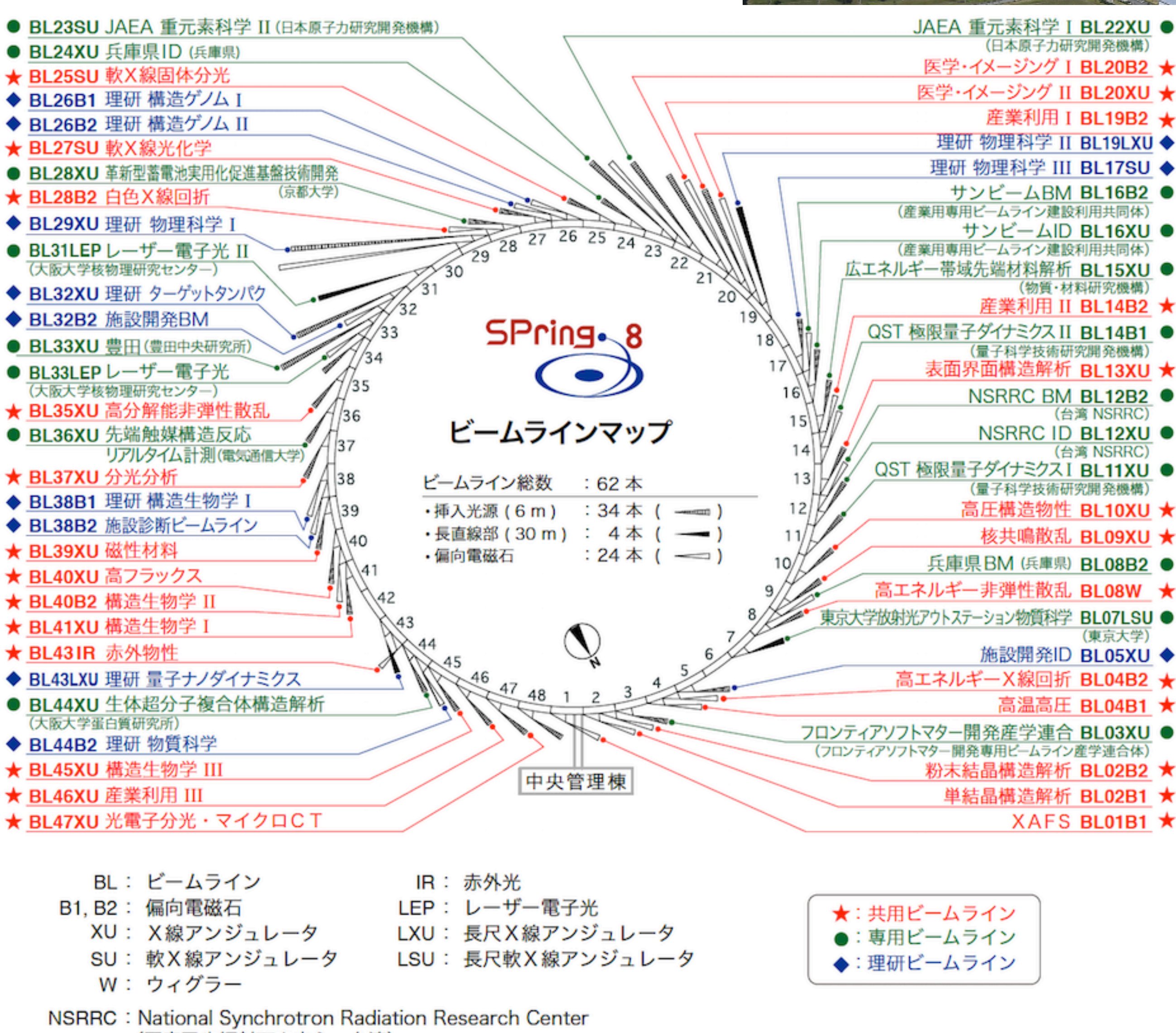
Fig. 煮干しの胸部・腹部X線画像

by Aki Takeda (Kyoto University) at the High Energy Physics Spring School

Synchrotron Radiation





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Passage of Radiation Through Matter

2. Energy Loss of (Heavy) Charged Particle (by Atomic Collisions)

- Path length of radiation
  - How is the scale determined?

- In a typical material (solid or liquid of density ~1 g/cm^3)
  - α ray: 10^{-5} m
  - β ray: 10^{-3} m
  - γ ray: 10^{-1} m
  - Neutron: 10^{-1} m

- In a gas (or atmosphere), the path length is 1000 times longer because of the density difference.

2. Energy Loss of Charged Particle

1. The Bethe-Bloch Formula
   1. Basic of Basics
   2. Energy loss per unit length when a charged particle pass through matters.

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2 m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right], \quad (2.27)$$

with

$$2\pi N_a r_e^2 m_e c^2 = 0.1535 \text{ MeVcm}^2/\text{g}$$

r_e: classical electron radius $= 2.817 \times 10^{-13}$ cm
m_e: electron mass
N_a: Avogadro's number $= 6.022 \times 10^{23}$ mol^{-1}
I: mean excitation potential
Z: atomic number of absorbing material
A: atomic weight of absorbing material
ρ: density of absorbing material
z: charge of incident particle in units of e
$\beta = v/c$ of the incident particle
$\gamma = 1/\sqrt{1-\beta^2}$
δ: density correction
C: shell correction
W_{max}: maximum energy transfer in a single collision.

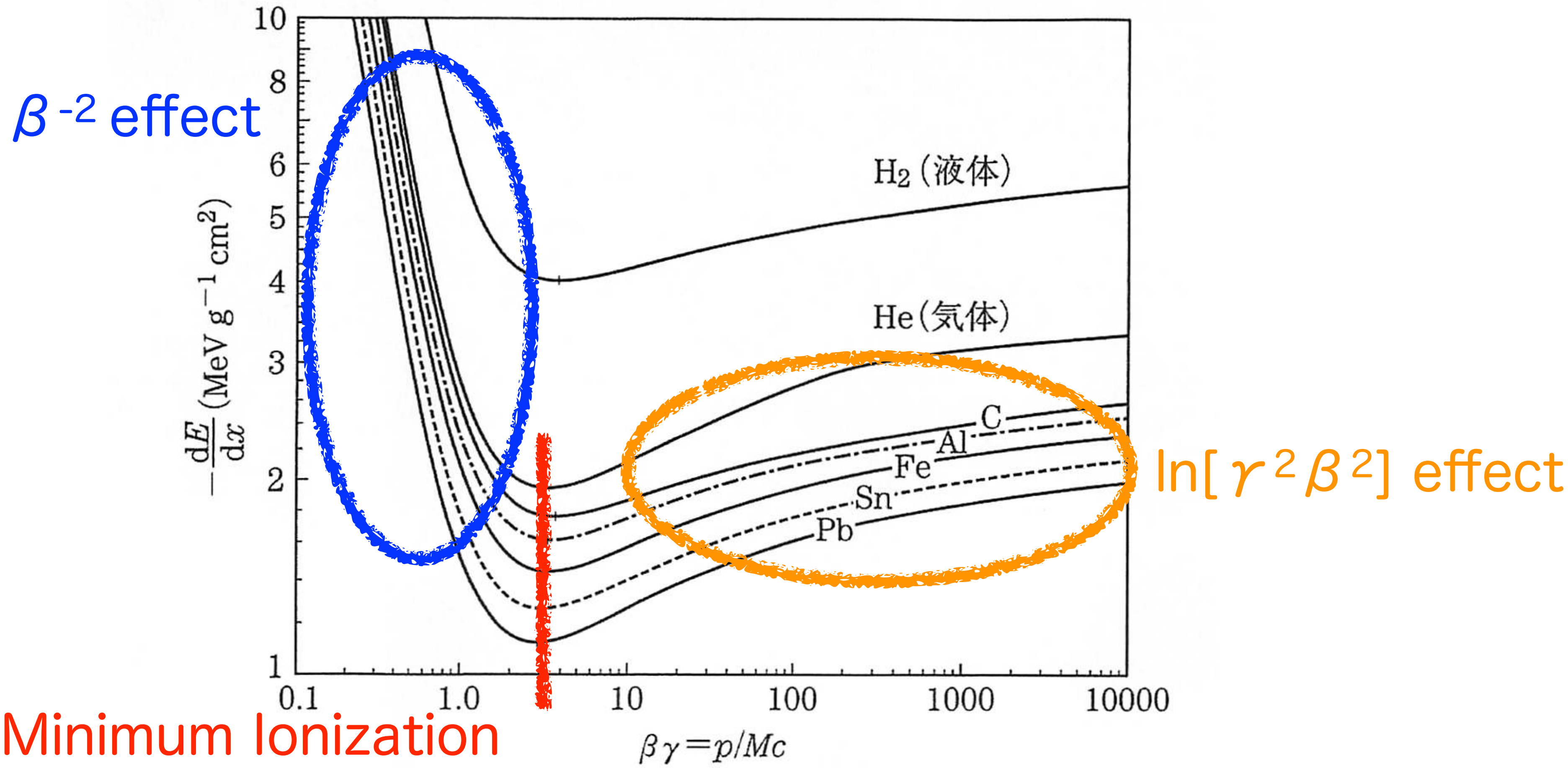


図 A.1 荷電粒子の物質内でのイオン化損失(Review of Particle Physics in Jour. Physics Letters, **G33**(2006)p.259 より)

- In Water（density～1 g/cm^3）, the average energy loss is <u>~2MeV/cm</u>
- A particle with minimum energy loss is often called as MIP (Minimum Ionization Particle)
  - [Q] What is the momentum of a muon to be MIP?

How to derive the Bethe-Bloch Formula

2.2.1 Bohr's Calculation – The Classical Case

Consider a heavy particle with a charge ze, mass M and velocity v passing through some material medium and suppose that there is an atomic electron at some distance b from the particle trajectory (see Fig. 2.2). We assume that the electron is free and initially at rest, and furthermore, that it only moves very slightly during the interaction with the heavy particle so that the electric field acting on the electron may be taken at its initial position. Moreover, after the collision, we assume the incident particle to be essentially undeviated from its original path because of its much larger mass ($M \gg m_e$). This is one reason for separating electrons from heavy particles!

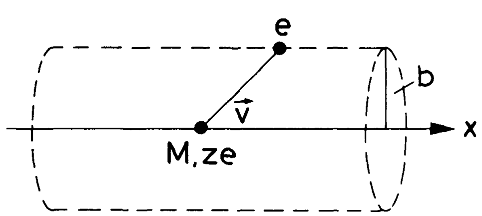


Fig. 2.2. Collision of a heavy charged particle with an atomic electron

Let us now try to calculate the energy gained by the electron by finding the momentum impulse it receives from colliding with the heavy particle. Thus

$$I = \int F\,dt = e\int E_\perp\,dt = e\int E_\perp \frac{dt}{dx}\,dx = e\int E_\perp \frac{dx}{v}\,, \qquad (2.16)$$

where only the component of the electric field $E_\perp$ perpendicular to the particle trajectory enters because of symmetry. To calculate the integral $\int E_\perp\,dx$, we use Gauss' Law over an infinitely long cylinder centered on the particle trajectory and passing through the position of the electron. Then

$$\int E_\perp 2\pi b\,dx = 4\pi ze\,, \qquad \int E_\perp\,dx = \frac{2ze}{b}\,, \qquad (2.17)$$

so that

$$I = \frac{2ze^2}{bv} \qquad (2.18)$$

and the energy gained by the electron is

$$\Delta E(b) = \frac{I^2}{2\,m_e} = \frac{2z^2e^4}{m_e v^2 b^2}\,. \qquad (2.19)$$

If we let N_e be the density of electrons, then the energy lost to all the electrons located at a distance between b and $b+db$ in a thickness dx is

$$-dE(b) = \Delta E(b)\,N_e\,dV = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b}\,dx\,, \qquad (2.20)$$

where the volume element $dV = 2\pi b\,db\,dx$. Continuing in a straight forward manner, one would at this point be tempted to integrate (2.20) from $b=0$ to ∞ to get the total energy loss; however, this is contrary to our original assumptions. For example, collisions at very large b would not take place over a short period of time, so that our impulse calculation would not be valid. As well, for $b=0$, we see that (2.19) gives an infinite energy transfer, so that (2.19) is not valid at small b either. Our integration, therefore, must be made over some limits b_{min} and b_{max} between which (2.19) holds. Thus,

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\frac{b_{max}}{b_{min}}\,. \qquad (2.21)$$

To estimate values for b_{min} and b_{max}, we must make some physical arguments. Classically, the maximum energy transferable is in a head-on collision where the electron obtains an energy of $\frac{1}{2}m_e(2v)^2$. If we take relativity into account, this becomes $2\gamma^2 m_e v^2$, where $\gamma = (1-\beta^2)^{-1/2}$ and $\beta = v/c$. Using (2.19) then, we find

$$\frac{2z^2e^4}{m_e v^2 b_{min}^2} = 2\gamma^2 m v^2, \qquad b_{min} = \frac{ze^2}{\gamma m_e v^2}\,. \qquad (2.22)$$

For b_{max}, we must recall now that the electrons are not free but bound to atoms with some orbital frequency ν. In order for the electron to absorb energy, then, the perturbation caused by the passing particle must take place in a time short compared to the period $\tau = 1/\nu$ of the bound electron, otherwise, the perturbation is adiabatic and no energy is transferred. This is the principle of *adiabatic invariance*. For our collisions the typical interaction time is $t \simeq b/v$, which relativistically becomes $t \Rightarrow t/\gamma = b/(\gamma v)$, so that

$$\frac{b}{\gamma v} \leq \tau = \frac{1}{\bar{\nu}}\,. \qquad (2.23)$$

Since there are several bound electron states with different frequencies ν, we have used here a mean frequency, $\bar{\nu}$, averaged over all bound states. An upper limit for b, then, is

$$b_{max} = \frac{\gamma v}{\bar{\nu}}\,. \qquad (2.24)$$

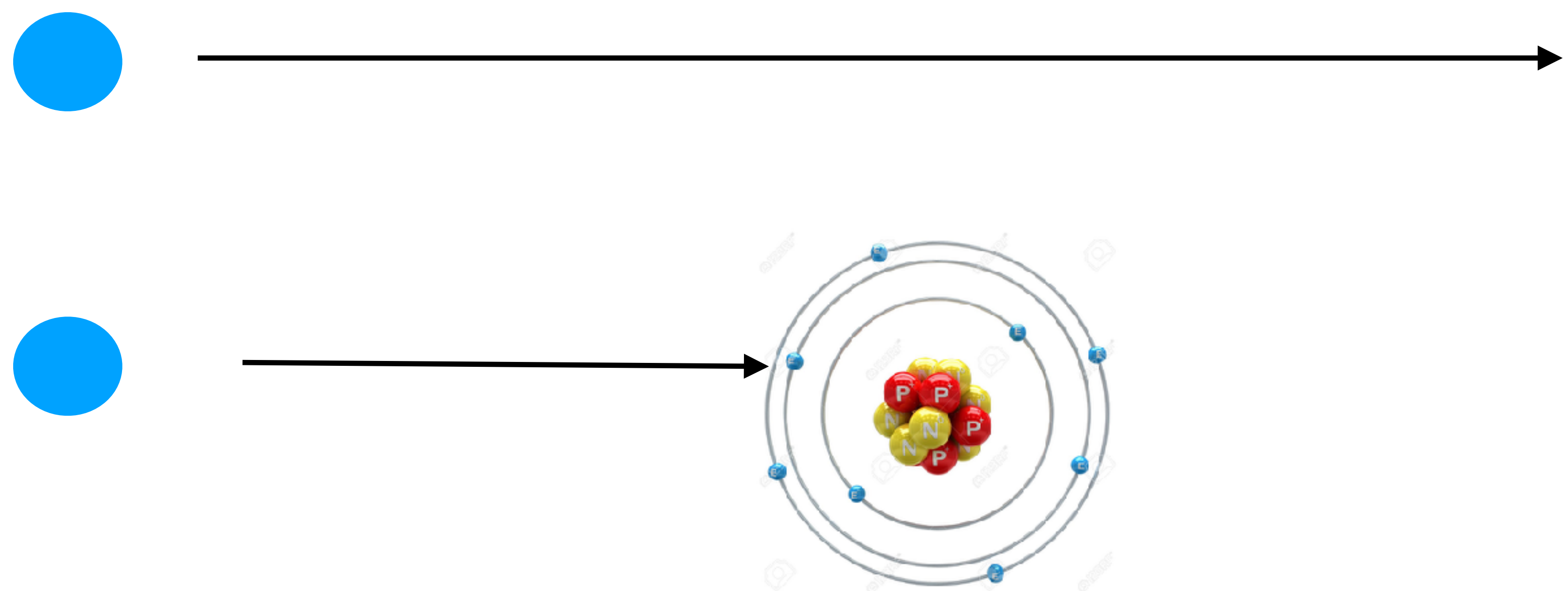
Substituting into (2.21), we find

$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\frac{\gamma^2 m v^3}{ze^2\bar{\nu}}\,. \qquad (2.25)$$

This is essentially Bohr's classical formula. It gives a reasonable description of the energy loss for very heavy particles such as the α-particle or heavier nuclei. However,

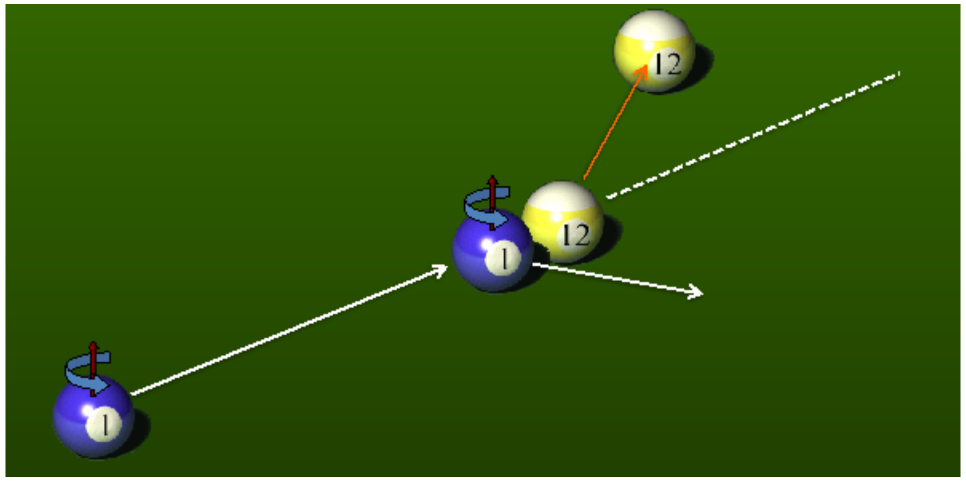
Principle of Bethe-Bloch Formula

- Electromagnetic Interaction
- The charged particle has an interaction with electrons in atoms
  - From a distance, an atom looks neutral that results in no electromagnetic interactions.
  - In a close distance, the charged particle observes
    - electrons (size of electron's orbit = size of the atom): $\sim 10^{-10}$ m
    - nucleus (or proton) : $\sim 10^{-15}$ m
    - [Q] In order to observe atoms, how much energy (how long wavelength) does a photon have? In order to observe nucleus, how much energy does photon have?



What particle is the Bethe-Bloch Formula applicable to?

- Charged Particle (Electromagnetic interaction requires a charge)
  - Electron, Proton, Muon, α particle （He nucleus）, etc..
- Except for an electron!
  - Why is an electron the exception?



From JAEA

- Because the target particle is the electron
  - Electron: same mass as the target particle. It does not go straight because of scattering.
  - Heavy Charged particle: The mass is heavier than that of the target particle (electron), and go straight with less scattering.

Energy observed

- We observe the following energy distribution when a cosmic ray muon pass through scintillator. The distribution is called as the Landau distribution.

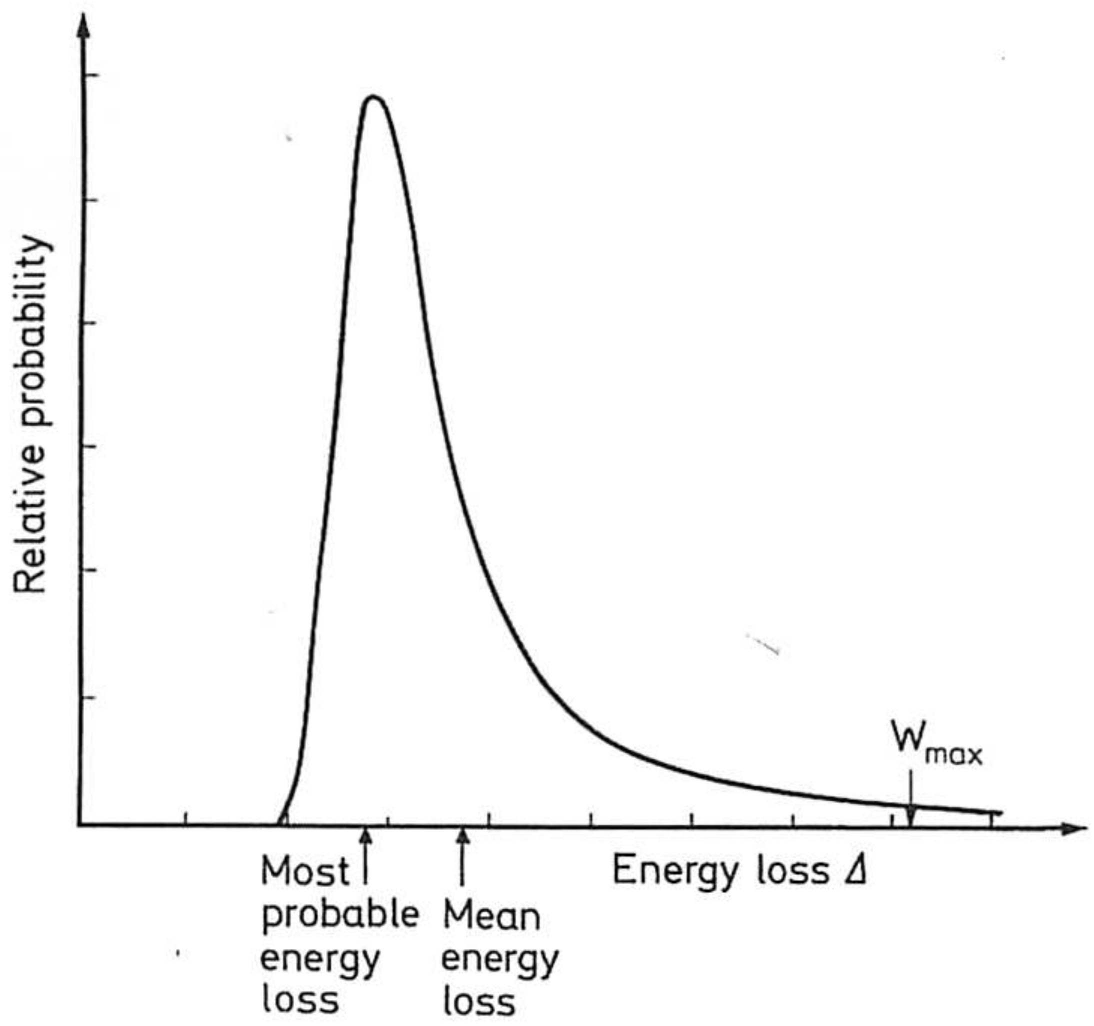


Fig. 2.18. Typical distribution of energy loss in a thin absorber. Note that it is asymmetric with a long high energy tail

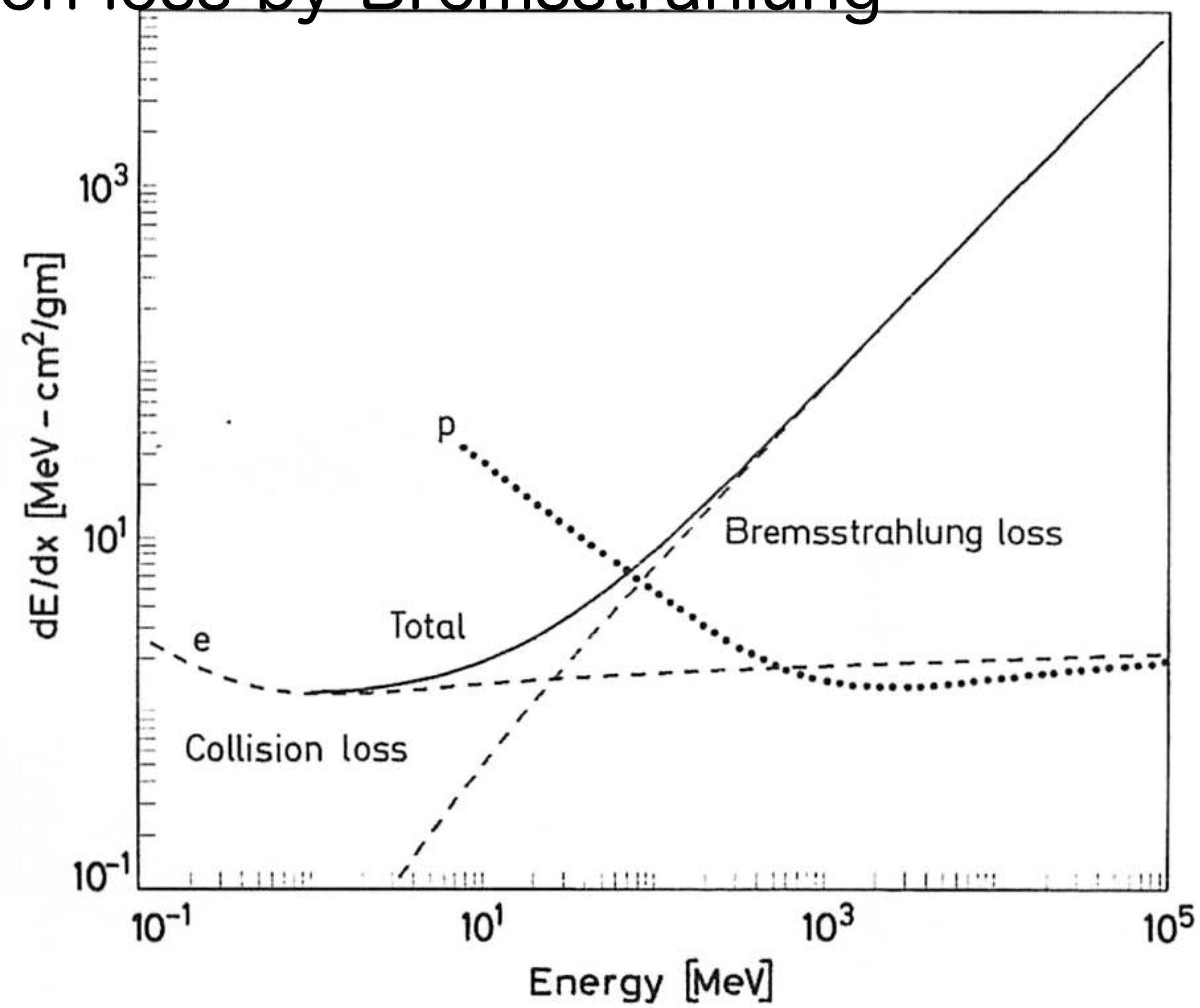
Summary of the Bethe-Bloch Formula

- Electromagnetic Interaction
- A charged particle interacts with electrons in atoms
- Applicable to all type of charged particle except for an electron.
- dE/dx is proportional to β^{-2} in the low energy region.
- In the case of water or plastic (the density ~1g/cm^3), the average energy loss is 2MeV/cm

In the case of electrons

$$\left(\frac{dE}{dx}\right)_{tot} = \left(\frac{dE}{dx}\right)_{rad} + \left(\frac{dE}{dx}\right)_{coll} .$$

- coll: Modified Bethe-Bloch Formula for an electron (An incident and the target particles are identical)
- rad: Radiation loss by Bremsstrahlung



$$\left(\frac{dE}{dx}\right)_{\text{tot}} = \left(\frac{dE}{dx}\right)_{\text{rad}} + \left(\frac{dE}{dx}\right)_{\text{coll}}.$$

原子量 A の物質中での制動放射による平均のエネルギー損失は(A.12)を k で積分して

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\text{rad}} = \frac{4N_0 Z^2 {r_0}^2}{137A} E_0 \left\{ \ln\left(183 Z^{-1/3}\right) + \frac{1}{18} \right\} \tag{A.14}$$

入射粒子が電子の場合には同じ粒子同士の衝突であることを考慮して

$$-\frac{\mathrm{d}E}{\mathrm{d}x} = \frac{2\pi N_0 {r_0}^2 m_{\mathrm{e}} c^2 Z}{A\beta^2} \times \left\{ \ln \frac{m_{\mathrm{e}} c^2 \beta^2 E_{\mathrm{e}}}{2I^2(1-\beta^2)} - \left(2\sqrt{1-\beta^2} - 1 + \beta^2\right) \ln 2 + (1-\beta^2) + \frac{\left(1-\sqrt{1-\beta^2}\right)^2}{8} \right\} \tag{A.8}$$

Radiation length

$$-\left(\frac{\mathrm{d}E}{\mathrm{d}x}\right)_{\mathrm{rad}} \simeq \frac{E_0}{X_0}$$

- Radiation length X_0 : The length when the energy becoming 1/e by radiation.
  - corresponding to the length of electromagnetic interactions.

Table 2.3. Radiation lengths for various absorbers

Material	[gm/cm^2]	[cm]
Air	36.20	30050
H_2O	36.08	36.1
NaI	9.49	2.59
Polystyrene	43.80	42.9
Pb	6.37	0.56
Cu	12.86	1.43
Al	24.01	8.9
Fe	13.84	1.76
BGO	7.98	1.12
BaF_2	9.91	2.05
Scint.	43.8	42.4

Characters of Interaction

- When a charged particle with the mass M and the kinematic energy E interacts with an electron with the mass m_e, the maximum energy loss by one collision is $4Em_e/M$. In the case of an alpha particle with energy of 5 MeV, $m_e/M \sim 0.511 MeV/4000 MeV \sim 1/10000$ and the maximum energy by one collision is $4*5/10000 \sim 2$ keV.
- When the charged particle interacts the electron,
  - The atom is excited or ionized,
    - The electron is kicked out from the atom and moves as the secondary particle if the electron energy is high enough.
      - called "delta ray"

Stopping power

- The “dE/dx” is sometimes called as “linear stopping power”.
- It depends on the type of the particle
    - In the experiment, the character is used for the identification of the particle.

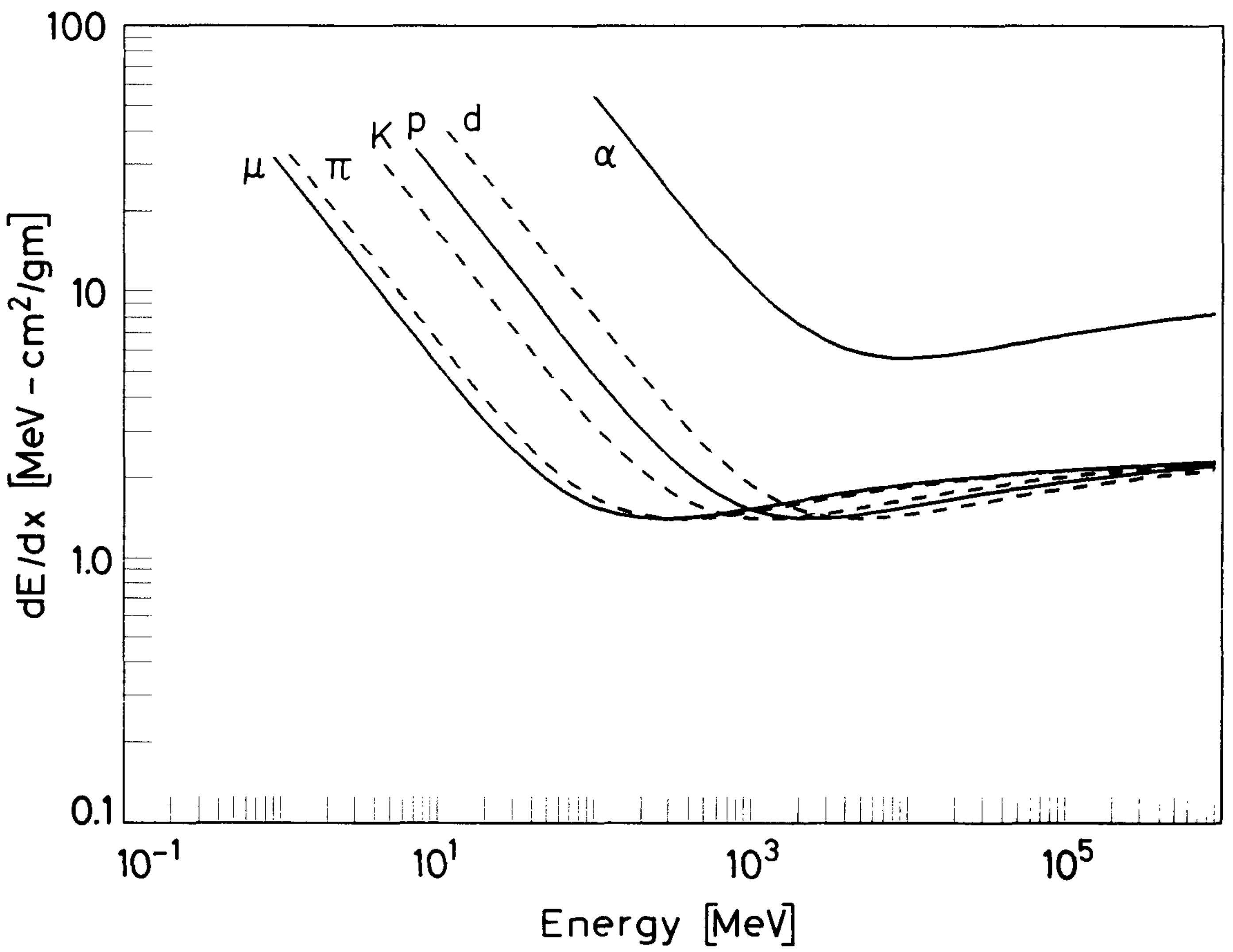


Fig. 2.4. The stopping power dE/dx as function of energy for different particles

Bragg Curve

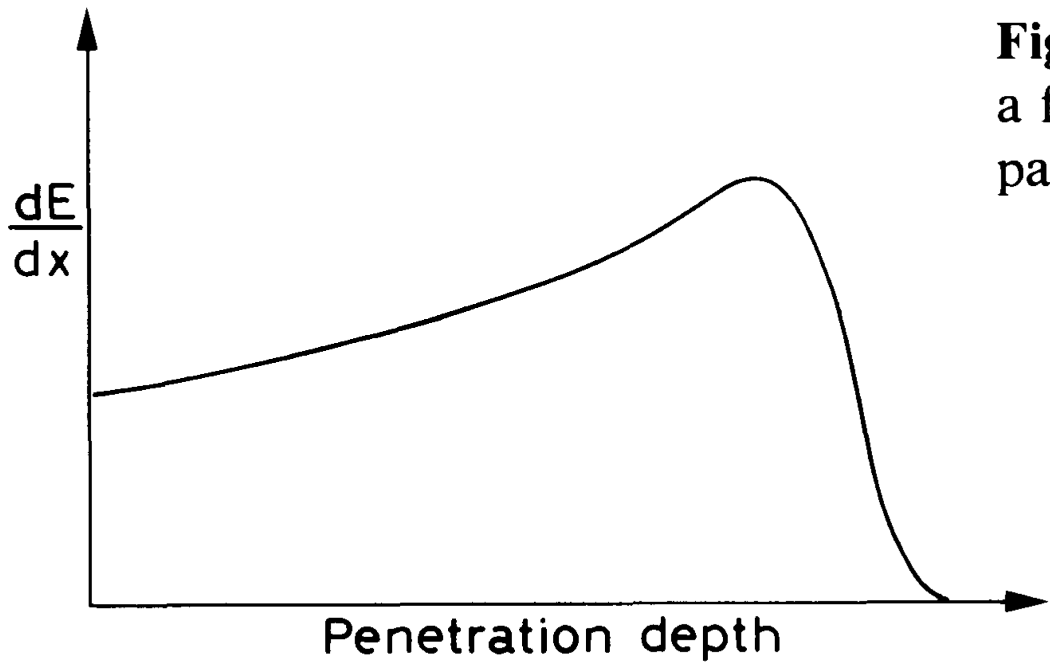
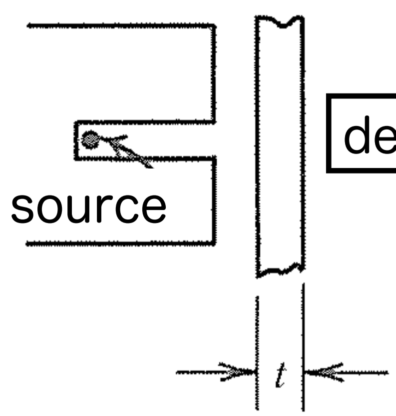
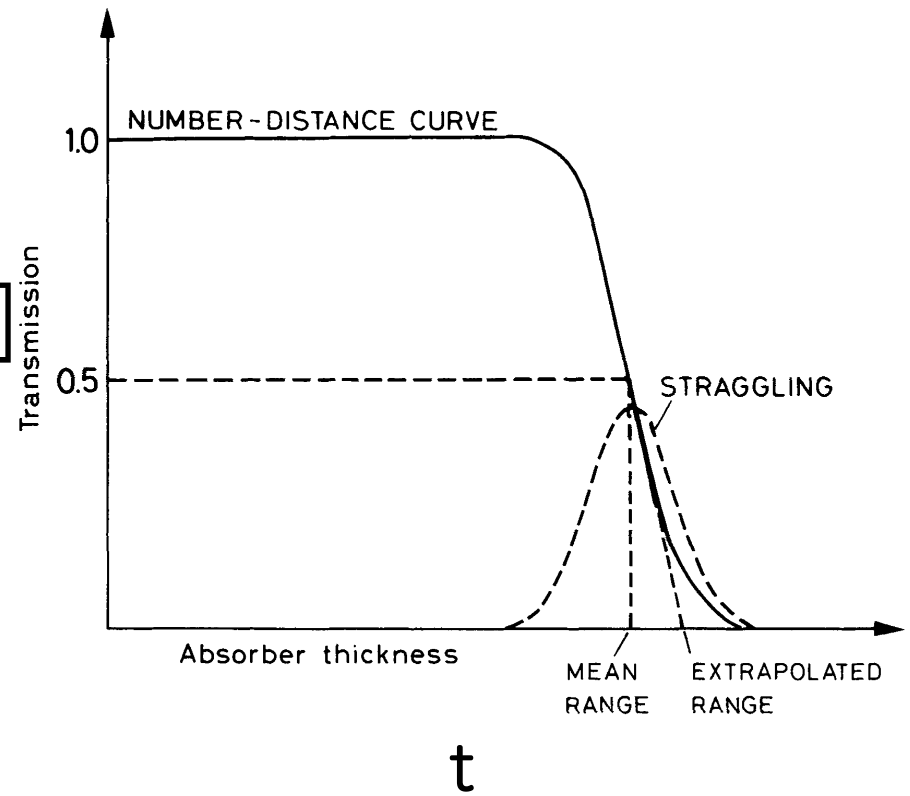


Fig. 2.5. A typical Bragg curve showing the variation of *dE/dx* as a function of the penetration depth of the particle in matter. The particle is more ionizing towards the end of its path

- Around the maximum penetration depth (when a particle is stopping), the dE/dx becomes maximum. Why?
  - This character is often used for the cancer treatment by radiation of heavy charged particles (proton, alpha or nucleus). Since the heavy charged particles deposit the large amount of energy when stopping, they kill cancel cells around the stopping point efficiently.

Range





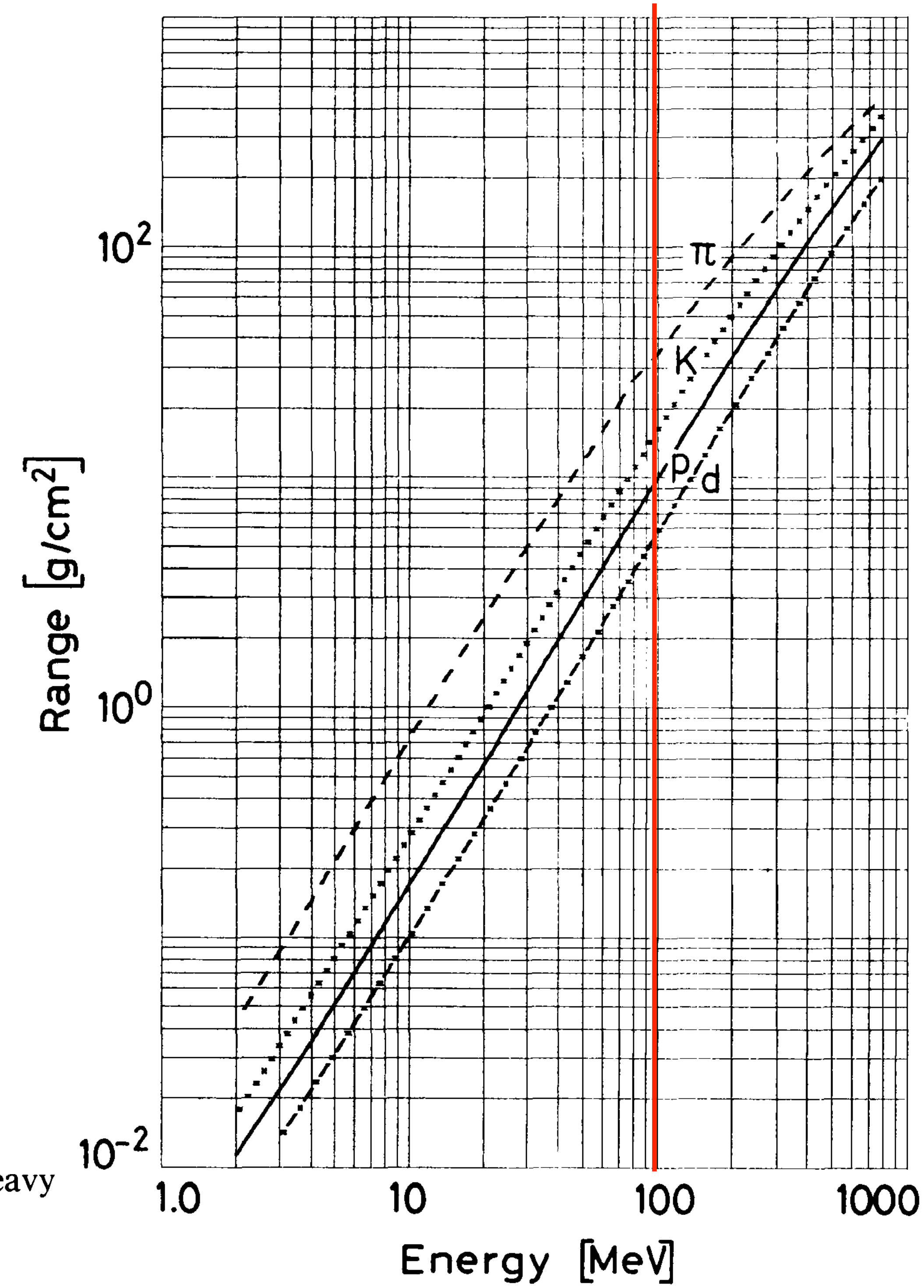
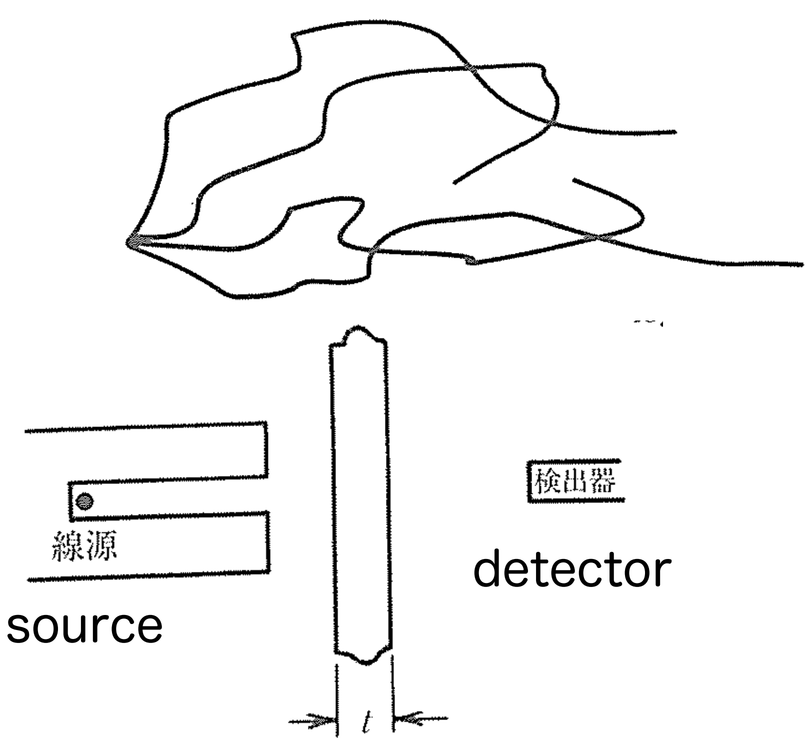


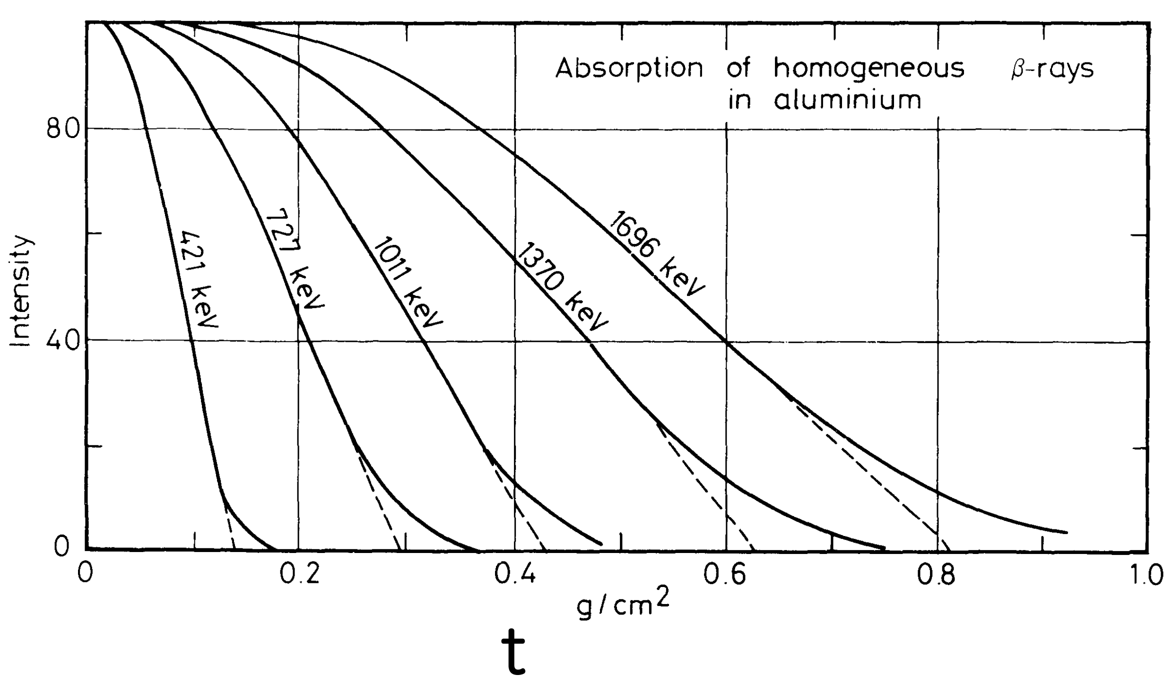
Fig. 2.8. Calculated range curves of different heavy particles in aluminium

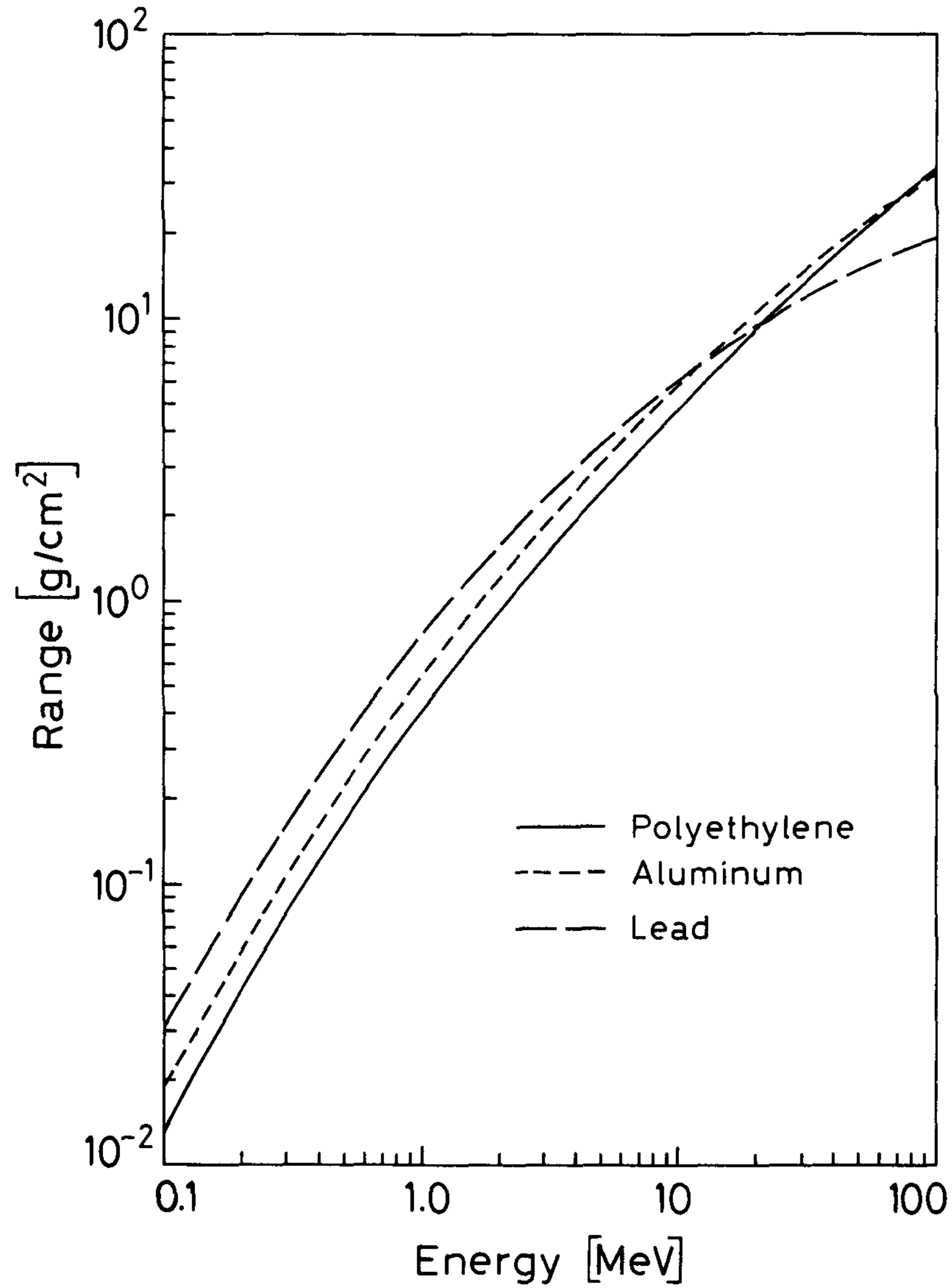
- Range: the number of particles becomes half after they pass the length of range .
- For a proton with energy of 100 MeV, the range is several cm or so in aluminium.
  - [Q] What is the range of a pion with 100 MeV?

Range of electrons

- Electrons do not go straight.







Range of electrons: Back Scattering

- Electrons do not go straight.
  - sometimes scattered backward.

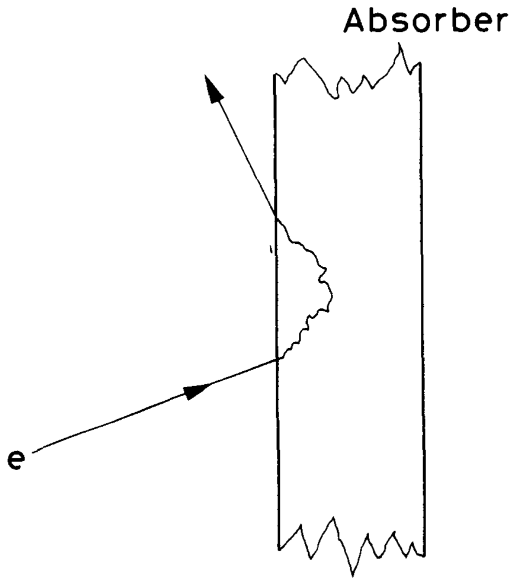


Fig. 2.16. Backscattering of electrons due to large angle multiple scatterings

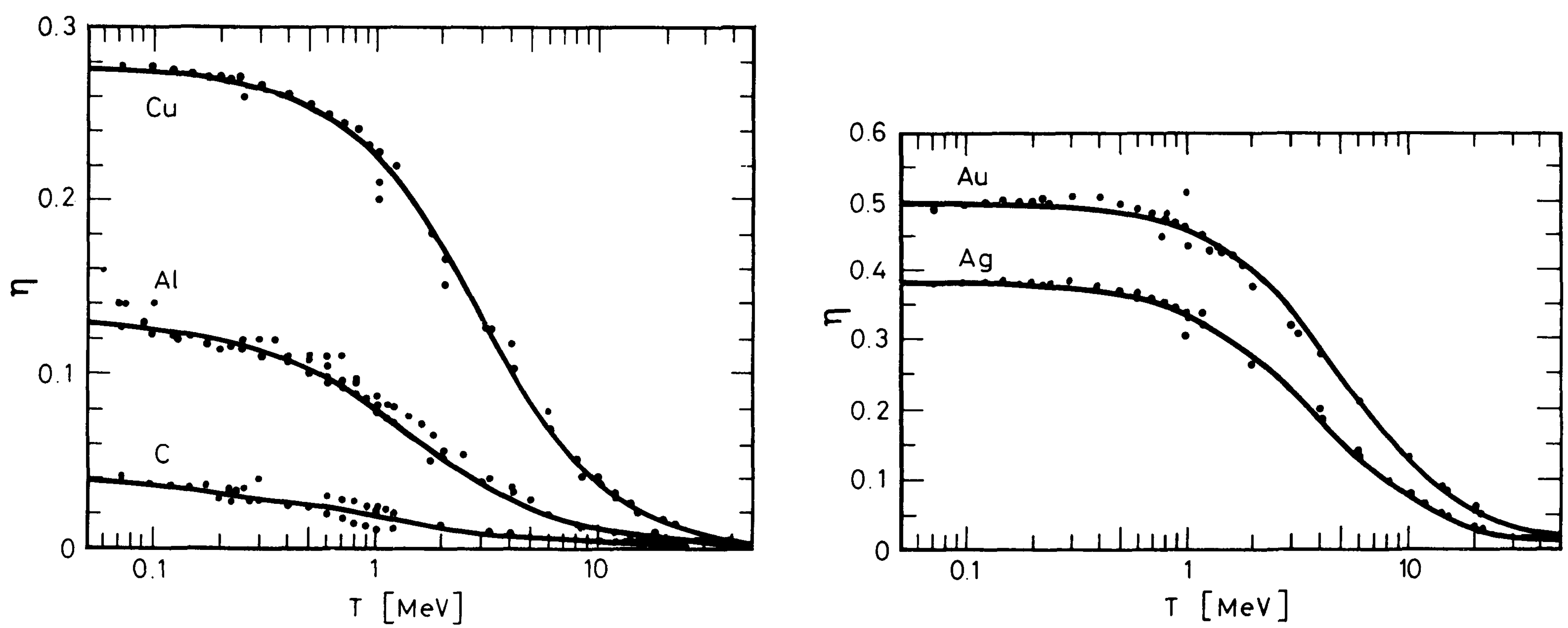


Fig. 2.17. Some measured electron backscattering coefficients for various materials. The electrons are perpendicularly incident on the surface of the sample (from *Tabata* et al. [2.24])

Interactions of a positron (e^+)

- Same as an electron when it is moving.
- When the positron stops, it forms a positronium (e^+e^-) with an electron and annihilates to photons.
  - $e^+ + e^- \rightarrow \gamma\gamma$ or $\gamma\gamma\gamma$
    - $\gamma\gamma$ from para-positronium (Spin 0)
    - $\gamma\gamma\gamma$ from ortho-positronium (Spin 1)
      - [Q] What is the spin of photon?
      - [Q] What is the lifetime of positroniums？

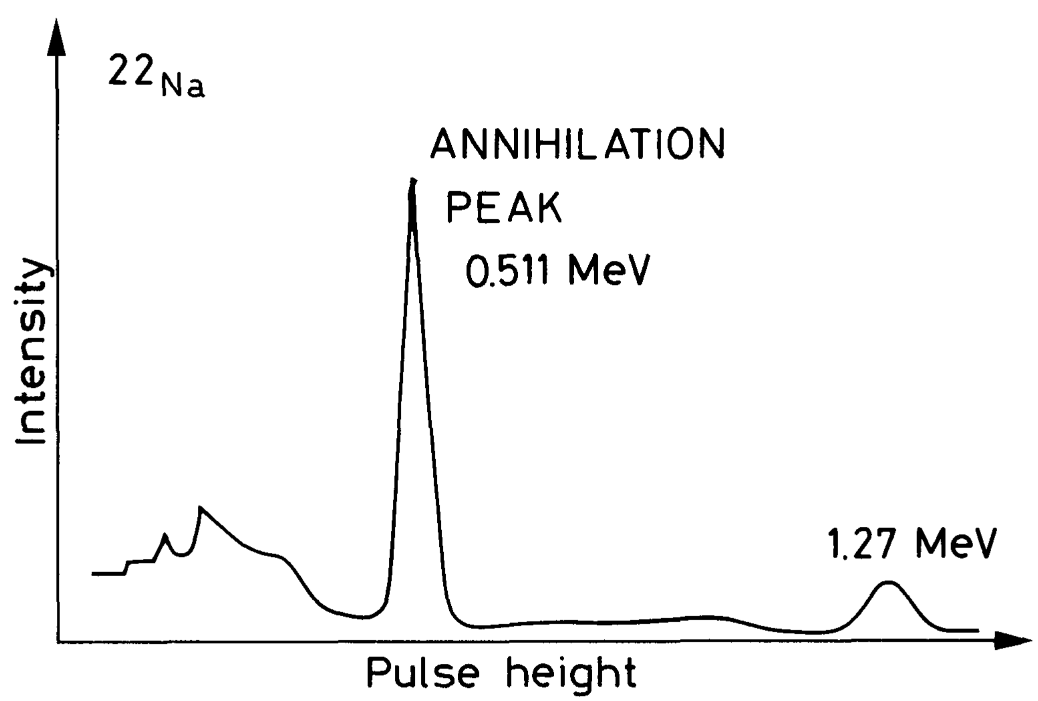


Fig. 1.4. Gamma-ray spectrum of a ^{22}Na source as observed with a NaI detector. Because of positron annihilation in the detector and the source itself, a peak at 511 keV is observed corresponding to the detection of one of the annihilation photons

Interaction of Photons
— X ray and γ ray —

3. Interaction of Photons

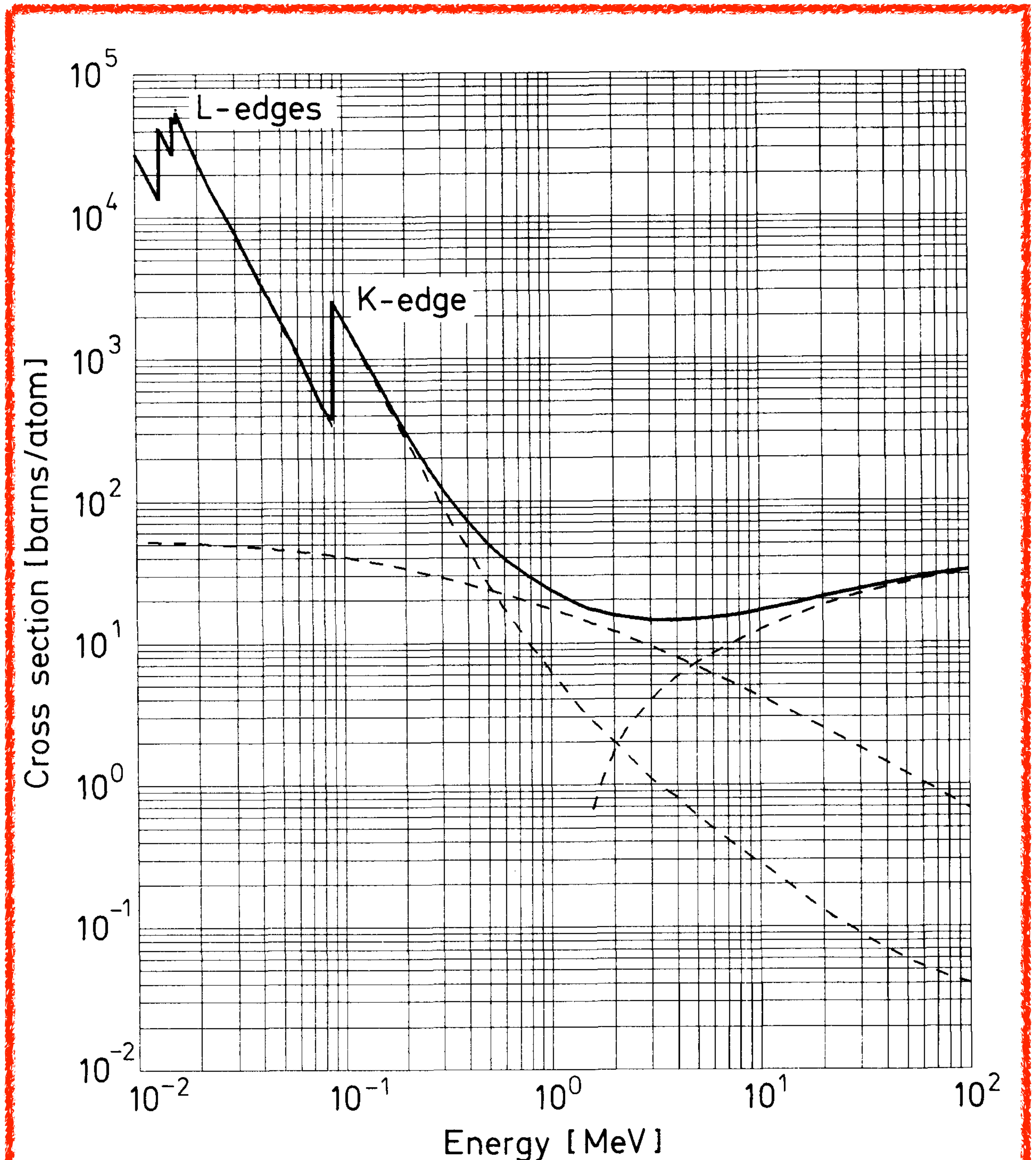


Fig. 2.29. Total photon absorption cross section for lead

- Important Plots

- Remember:
  - In a typical material (solid or liquid of density ~1 g/cm³)
    - α ray: 10^{-5} m
    - β ray: 10^{-3} m
    - γ ray: 10^{-1} m
    - Neutron: 10^{-1} m

Interactions of Photons

(1) Photoelectric Effect [in low energy]

(2) Compton Scattering [in medium energy]

(3) Pair production [in high energy]

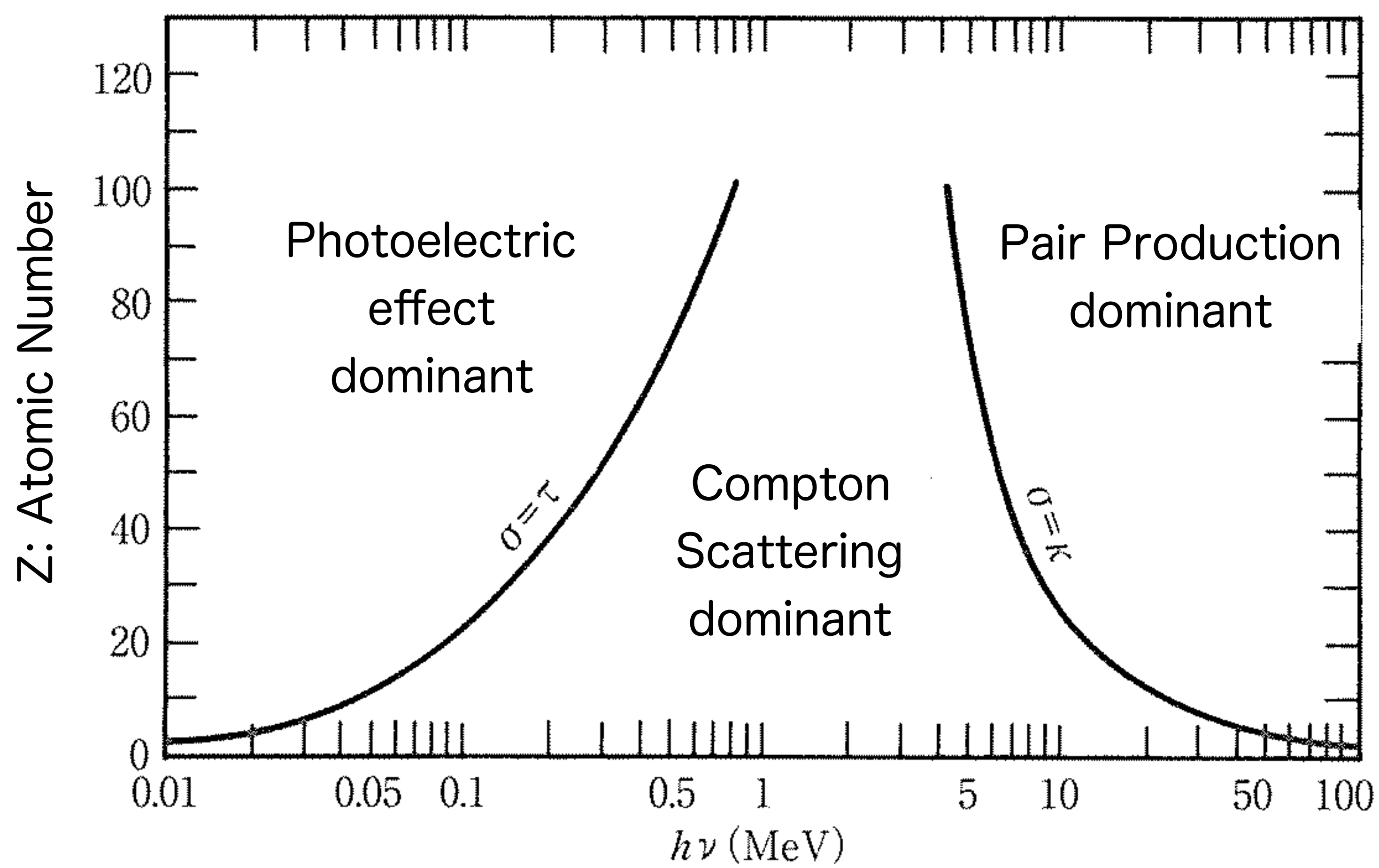
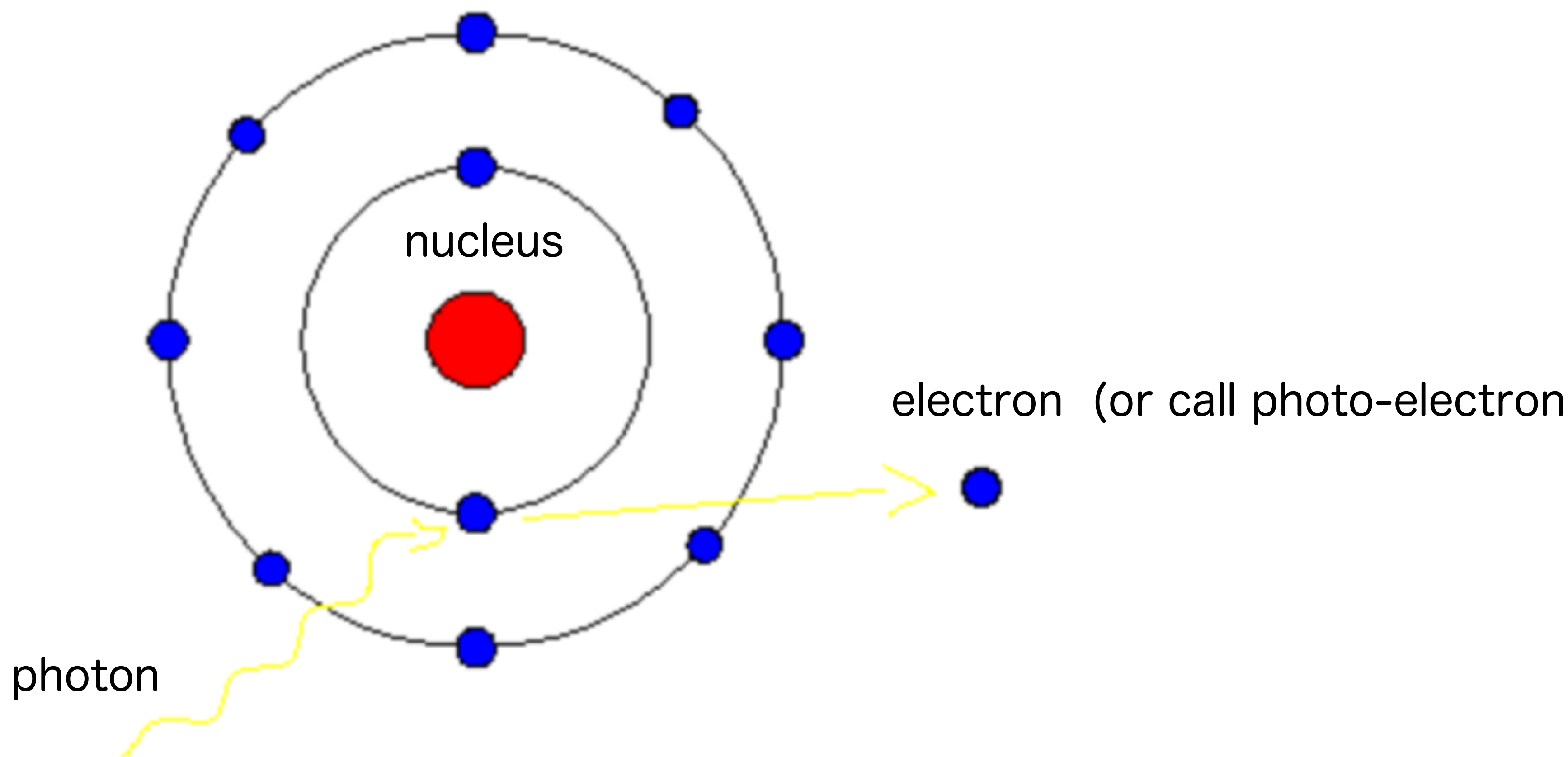


図 2.20
三つの主要なガンマ線相互作用が支配的な領域．２本の線は二つの効果がちょうど等しくなるような Z と $h\nu$ の値を示している．
(R. D. Evans, *The Atomic Nucleus* (1955) より引用)

(1) Photoelectric Effect

- A photon is absorbed by an atom, and an electron (called a photo-electron) is emitted.
  - This interaction occurs between a photon and an atom.
  - The probability that the electron in the K-shell (in the most inner orbit) is emitted is high.
  - The probability is high when the atomic number Z is large.



Energy of photo-electron

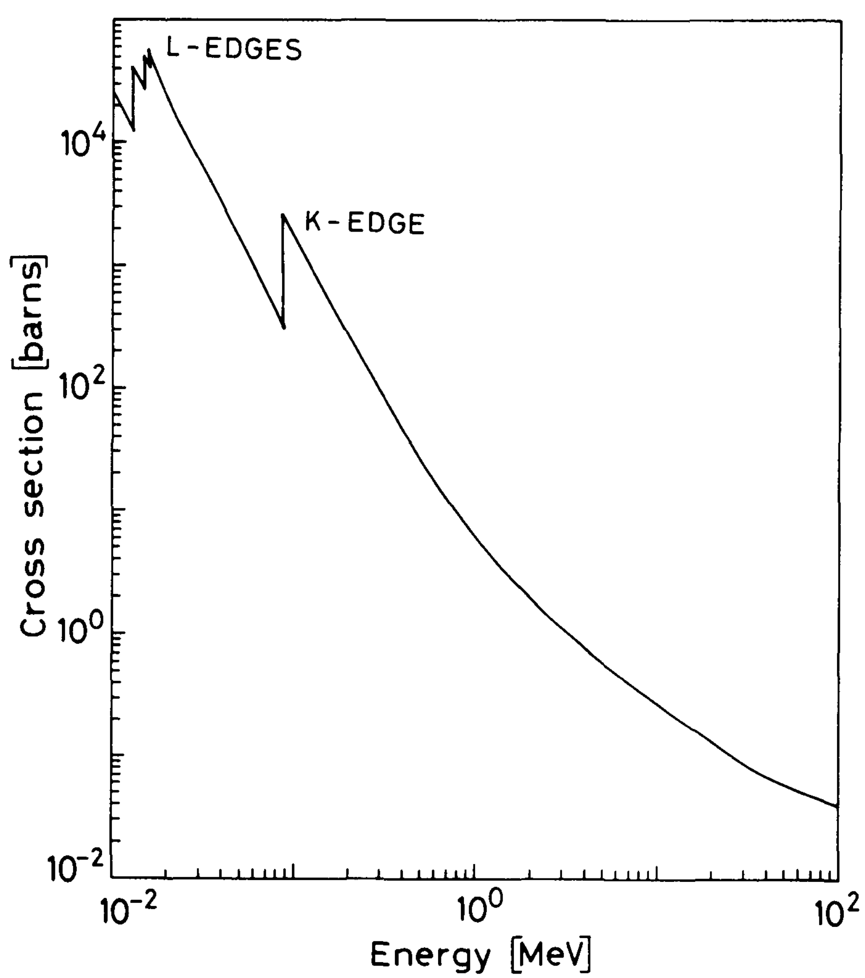
$$E_{e^-} = h\nu - E_b \qquad (2.15)$$

$h\nu$ (=E_γ) : Photon Energy
E_b : Electron binding energy

Interaction Probability

$$\tau \cong 定数 \times \frac{Z^n}{E_\gamma^{3.5}} \qquad (2.16)$$

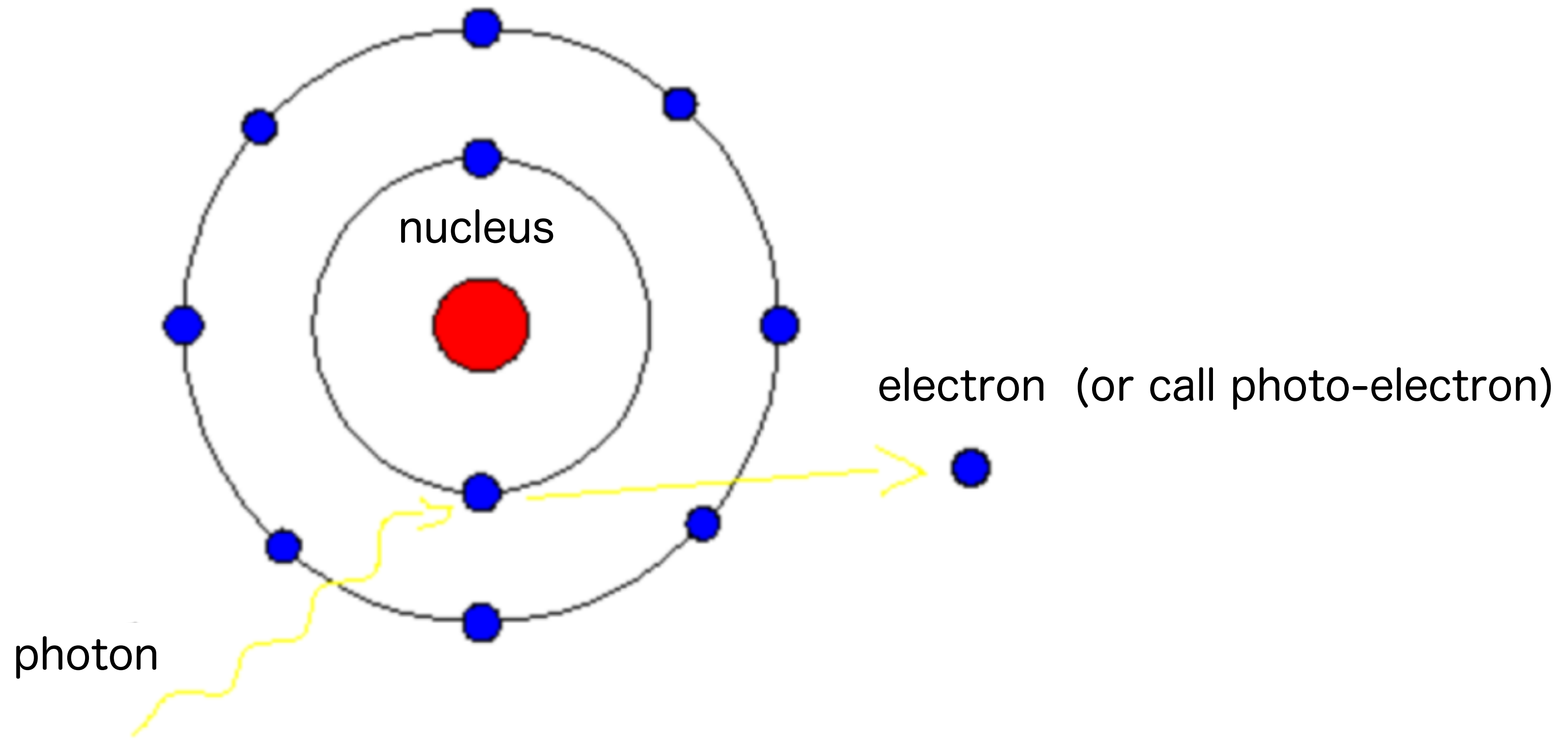
n=4〜5



- [Q]
  - Doesn't this process occur with a free electron?
  - Why does this process often happen with the K-shell electron?
  - Why does the probability become lower when the photon energy is higher?

Process following photoelectric effect

- After the electron (in the K-shell) is emitted in the photoelectric effect, a characteristic X ray is often emitted by de-excitation of the electron from the L-shell (outer orbit).
  - (Example: Energy) $E_{X\text{-}ray} = E_{L\text{-}electron} - E_{K\text{-}electron}$
- Instead of the characteristic X ray, the energy is transferred to the L-shell electron and the L-shell electron is emitted (called Auger electron).



(2) Compton Scattering

- A photon (X ray, γ ray) is scattered by (almost free) electron.

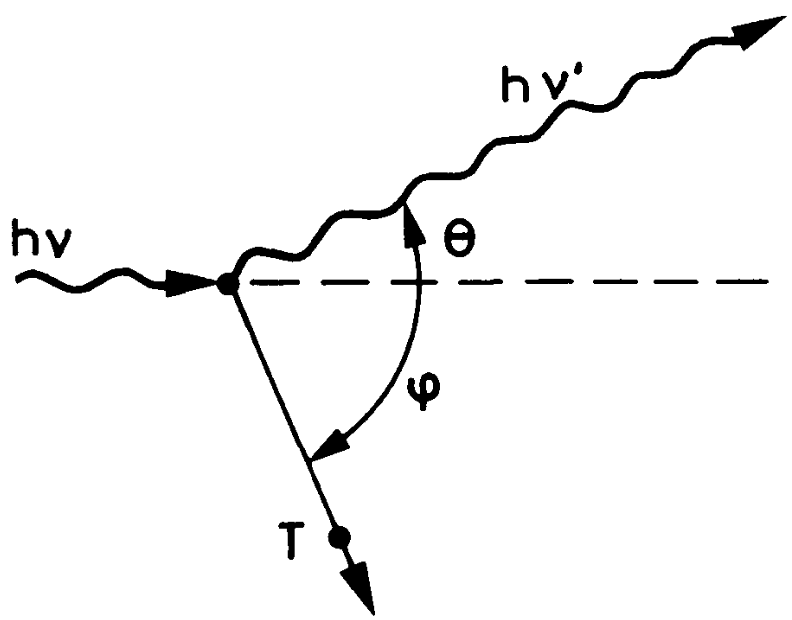


Fig. 2.22. Kinematics of Compton scattering

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos\theta)} \qquad (2.17)$$

3.2　コンプトン散乱

光の粒子性を示すもう一つの例がコンプトン散乱である。X 線（もしくは γ 線）の散乱を観測すると

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta) \tag{3.2}$$

の関係があった。

この関係式を光を粒子として説明しよう。特殊相対論によれば、運動量 p、質量 m の粒子のエネルギー E は

$$E = \sqrt{(pc)^2 + (mc^2)^2} \tag{3.3}$$

で与えられる。

最初、電子は止まっているとする。初期状態の電子のエネルギーは mc^2、運動量は 0、光子のエネルギーは $h\nu$、運動量は $h\nu/c$（式 3.3）より $E = pc$ となる。終状態の電子の運動量の大きさを p'、光のエネルギーを $h\nu'$ とする。

エネルギー保存則

$$h\nu + mc^2 = \sqrt{(p'c)^2 + (mc^2)^2} + h\nu' \tag{3.4}$$

運動量保存則

$$(\text{入射方向})\quad \frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p'\cos\phi \tag{3.5}$$

$$(\text{垂直方向})\quad 0 = -\frac{h\nu'}{c}\sin\theta + p'\sin\phi \tag{3.6}$$

式 3.4 より、

$$(pc)^2 = \{h(\nu-\nu') + mc^2\}^2 - (mc^2)^2 \tag{3.7}$$

式 3.5 と 3.6 より、

$$(pc)^2[\cos^2\phi + \sin^2\phi] = (h\nu'\sin\theta)^2 + (h\nu - h\nu'\cos\theta)^2 \tag{3.8}$$

式 3.7 と 3.8 より、

$$\{h(\nu-\nu') + mc^2\}^2 - (mc^2)^2 = (h\nu'\sin\theta)^2 + (h\nu - h\nu'\cos\theta)^2 \tag{3.9}$$

整理すると、

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1-\cos\theta)} \tag{3.10}$$

$\lambda = \frac{c}{\nu}$ なので、式 3.2 が導ける。

$$\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta)$$

ここで、

$$\lambda_{comp} \equiv \frac{h}{mc} \simeq 2.4\times 10^{-10}\quad \text{cm} \tag{3.11}$$

をコンプトン波長と定義する。コンプトン波長は光子のエネルギーで $E = h\frac{c}{\lambda} = \frac{1240\text{eV・nm}}{2.4\times10^{-3}\text{nm}} \sim$ 500 keV で電子の静止質量程度であり、高エネルギーの X 線や γ 線で見られる現象であることが分かる。

- Another calculation with Special relativity

4 元運動量を次のように定義する。

$$p^\mu = (p^0, p^1, p^2, p^3) = (p^0, \vec{p}) = (\frac{E}{c}, \vec{p})$$

粒子 a と粒子 b においてその内積は

$$p_a \cdot p_b = \sum_{\mu=0}^{3} p_{a\mu} p_b^\mu = p_a^0 p_b^0 - \vec{p_a} \cdot \vec{p_b}$$

となる。同一粒子における内積（その運動量の大きさ）は

$$p \cdot p = \sum_{\mu=0}^{3} p_\mu p^\mu = p^0 p^0 - \vec{p} \cdot \vec{p} = \frac{E^2}{c^2} - |\vec{p}|^2 = m^2 c^2$$

となる。

この関係式を使って、コンプトン散乱を計算する。入射光子、標的電子、散乱光子、散乱電子の 4 元運動量を $(\frac{E}{c}, \vec{p})$、$(mc, 0)$、$(\frac{E'}{c}, \vec{p'})$、$(\frac{E_e}{c}, \vec{p_e})$ とすると、4 元運動量の保存則より

$$(\frac{E}{c}, \vec{p}) + (mc, 0) = (\frac{E'}{c}, \vec{p'}) + (\frac{E_e}{c}, \vec{p_e})$$

$$(\frac{E}{c}, \vec{p}) - (\frac{E'}{c}, \vec{p'}) + (mc, 0) = (\frac{E_e}{c}, \vec{p_e})$$

光子の質量はゼロ（$p \cdot p = 0$、$|\vec{p}| = \frac{E}{c}$）を考慮して両辺を 2 乗すると、

$$2mc(\frac{E}{c} - \frac{E'}{c}) - 2\frac{E}{c}\frac{E'}{c} + 2|\vec{p}||\vec{p'}|\cos\theta + m^2c^2 = m^2c^2$$

$$E'(\frac{E}{c^2} + m - \frac{E}{c^2}\cos\theta) = mE$$

$$E' = \frac{E}{1 + \frac{E}{mc^2}(1 - \cos\theta)}$$

$E = h\nu(E' = h\nu')$ とすると、

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)}$$

The cross section for Compton scattering was one of the first to be calculated using quantum electrodynamics and is known as the *Klein-Nishina* formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2}\,\frac{1}{[1+\gamma(1-\cos\theta)]^2}\left(1+\cos^2\theta+\frac{\gamma^2(1-\cos\theta)^2}{1+\gamma(1-\cos\theta)}\right), \tag{2.107}$$

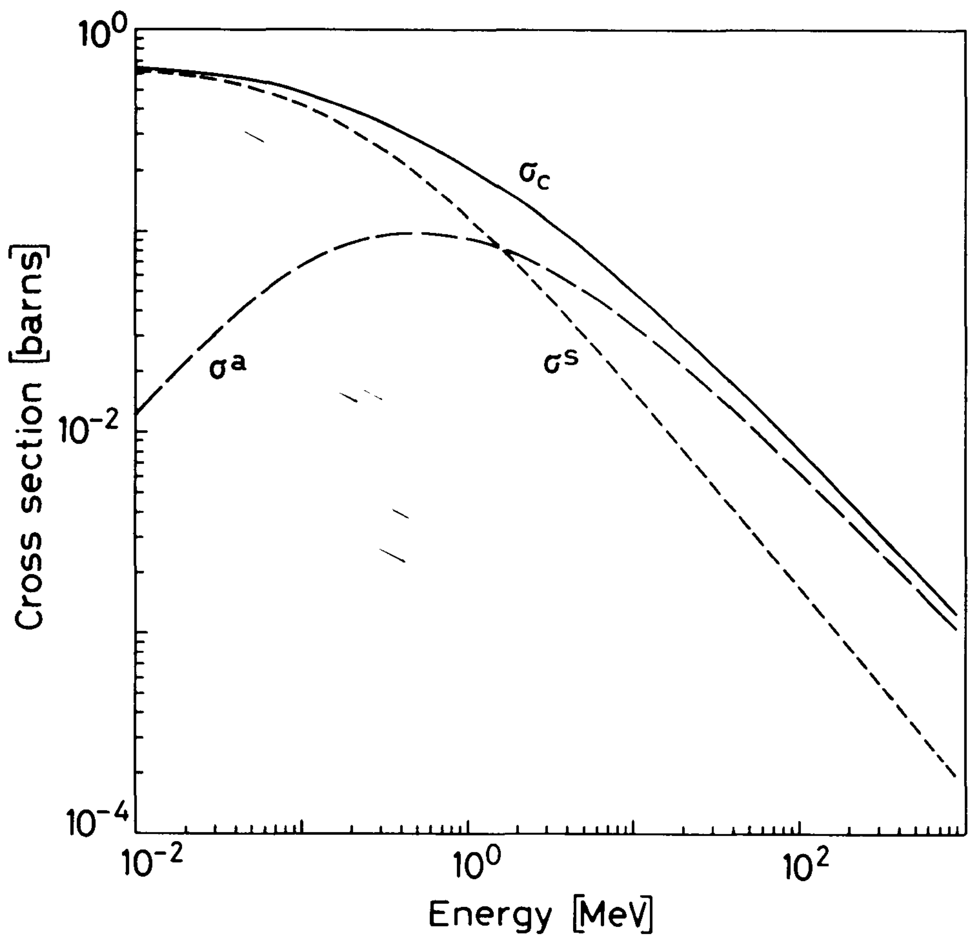


Fig. 2.23. Total Compton scattering cross sections

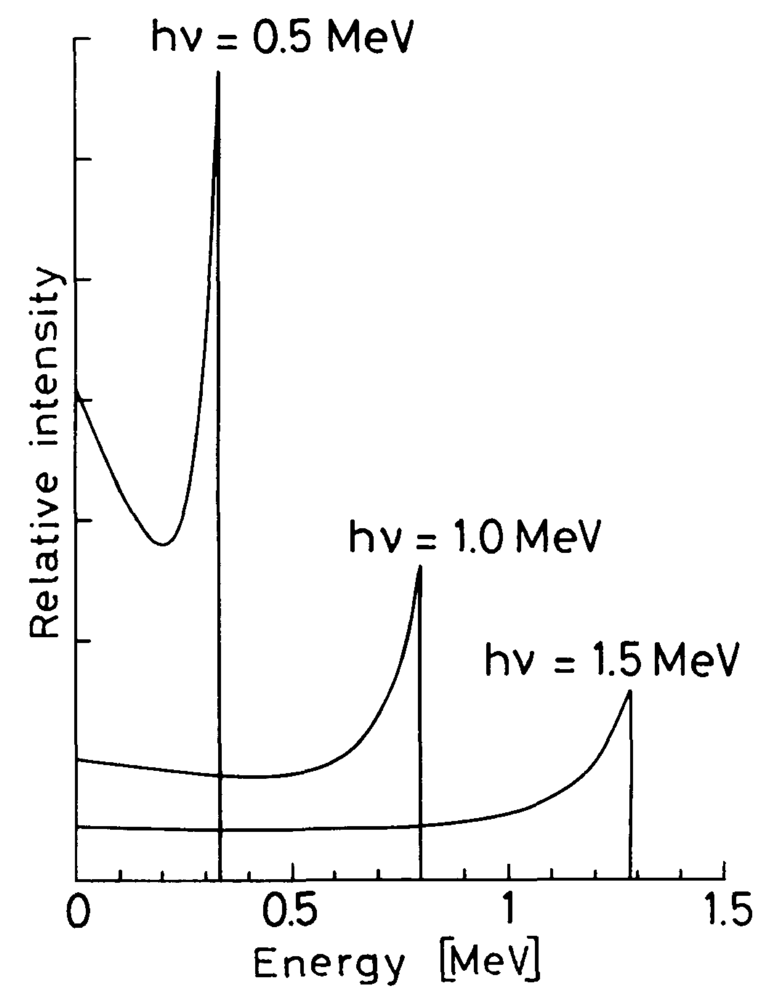


Fig. 2.24. Energy distribution of Compton recoil electrons. The sharp drop at the maximum recoil energy is known as the *Compton edge*

(3) Pair Production

- If the photon energy is higher than $2m_e$, a pair production of e^+e^- becomes possible. In experiments of high energy (~1GeV or higher), it is the dominant process.
- In the pair production
  - $\gamma + \gamma^* \rightarrow e^+ + e^-$
    - γ^* : Electric field of nucleus (virtual photon)。To satisfy the energy and momentum conservation, γ^* is necessary.

$$\frac{d\sigma_{\mathrm{pair}}(E_-)}{dE_-} = \frac{4Z^2 {r_0}^2}{137} \cdot \frac{{E_+}^2 + {E_-}^2 + 2E_+E_-/3}{k^3} \times \left(\ln \frac{2E_+E_-}{m_e c^2 k} - \frac{1}{2} \right) \qquad (\mathrm{A.24})$$

- k: energy of incident photon, :E_: energy of electron, E_+: energy of positron
- The formula is derived in QED.

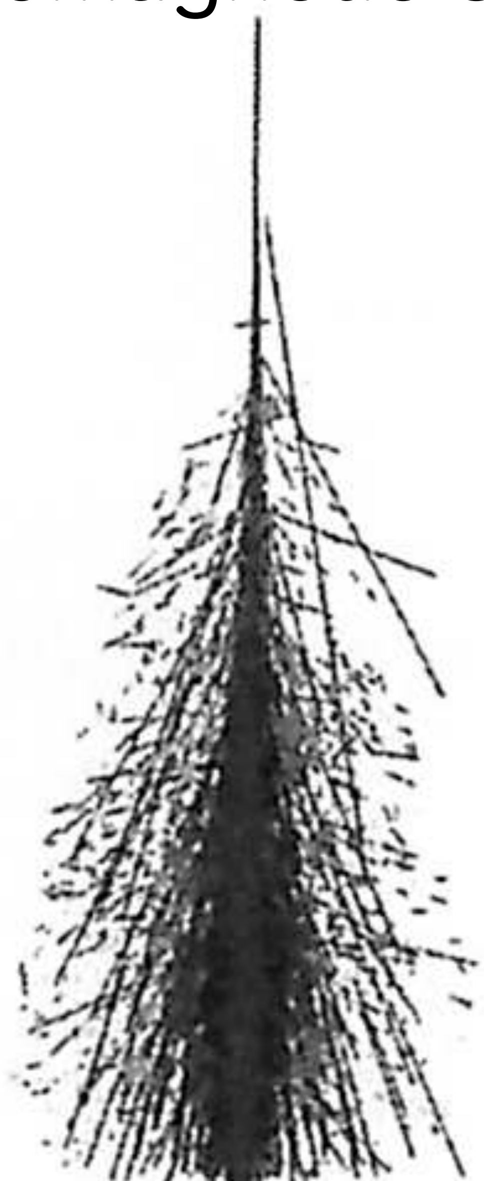
- The cross section of pair-production is related with the radiation length X_0.

$$\sigma_{\text{pair}} \approx \frac{(7/9)A}{(N_0 X_0)} \quad \text{(A.26)}$$

  - (N_0：Avogadro number、A：Atomic weight)
- The interaction length X_{pair} is formulated as
  - $X_{pair} = \dfrac{1}{\sigma_{\text{pair}} \cdot \left(\dfrac{N_0}{A}\right)}$
- and, $X_{pair} = 9/7 X_0$

(4) Electro-magnetic shower

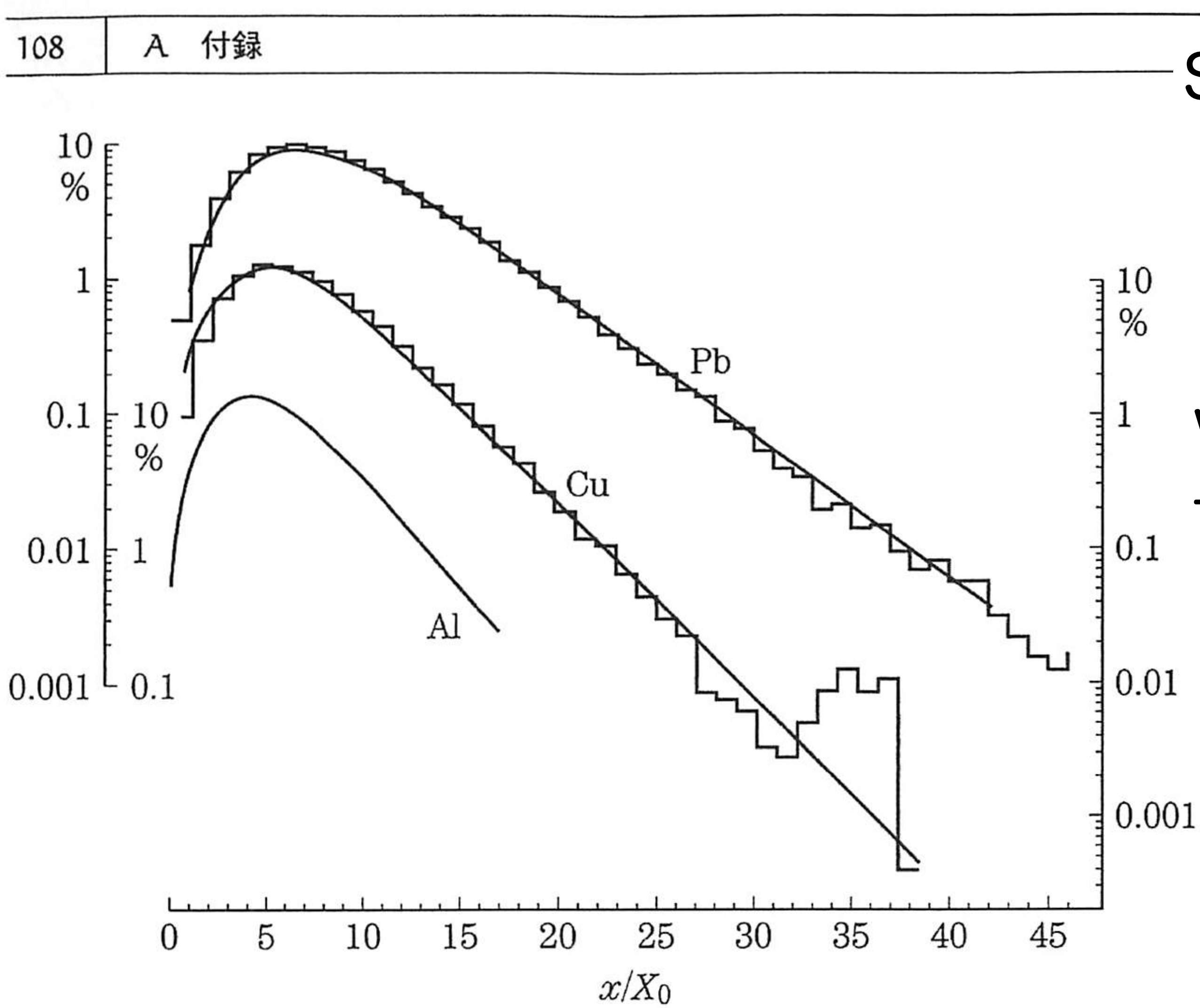
- When a high energy photon (~GeV or higher) is incident, sequential precesses of pair-production and bremsstrahlung of electrons and positrons happen, that results in electromagnetic shower.
-



Electromagnetic shower

(4) Electro-magnetic shower development

- In electromagnetic shower, many particles (photons, electrons, and positrons) are produced when their energy is high enough. Once the energy becomes lower, the production of particles is decreasing. There is the depth (called the shower maximum) where the number of particles becomes maximum.



Fraction of energy deposit as a function of the shower depth x/X_0

Shower maximum X_{max}

$$\frac{X_{\rm max}}{X_0} \simeq \ln\left(\frac{E_0}{E_c}\right) - t_0 \qquad \text{(A.30)}$$

where t_0 = 1.1 for an electron and =0.3 for a photon

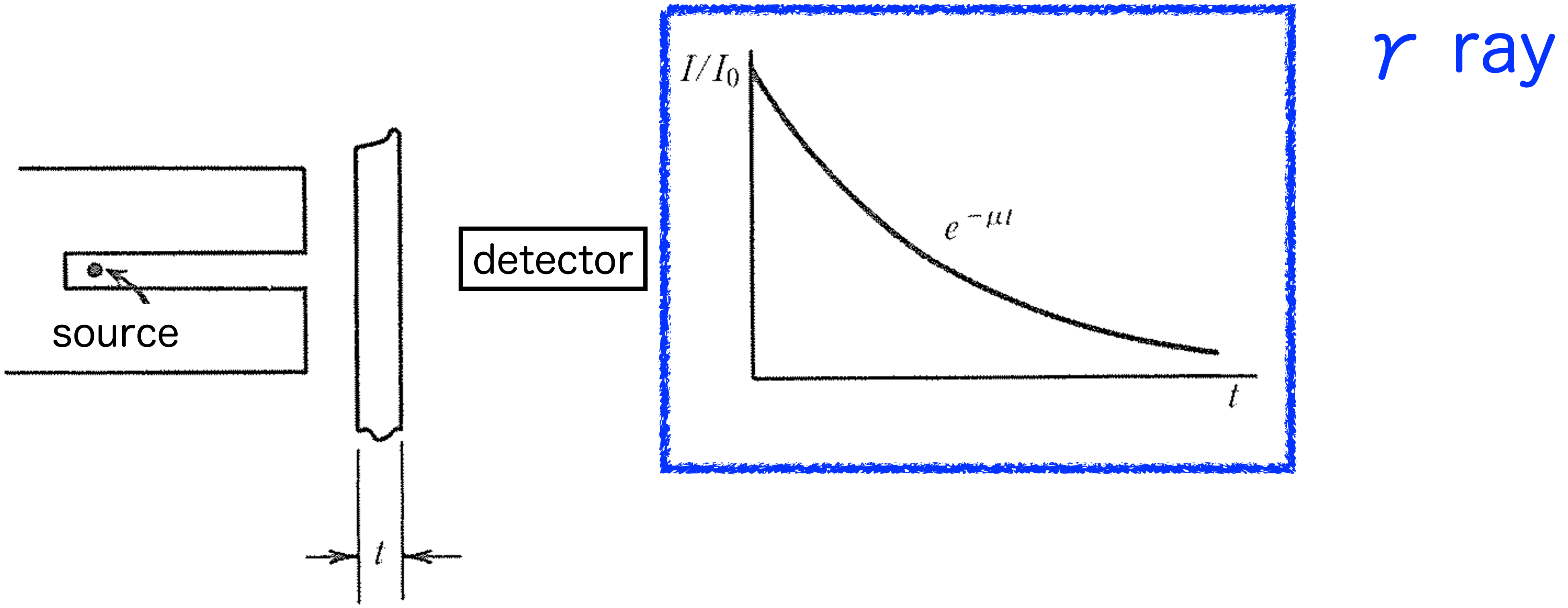
(Ref.) Mollere radius

- express the transverse distribution of the shower
- Define the Mollere radius where the 90% energy deposit is contained.

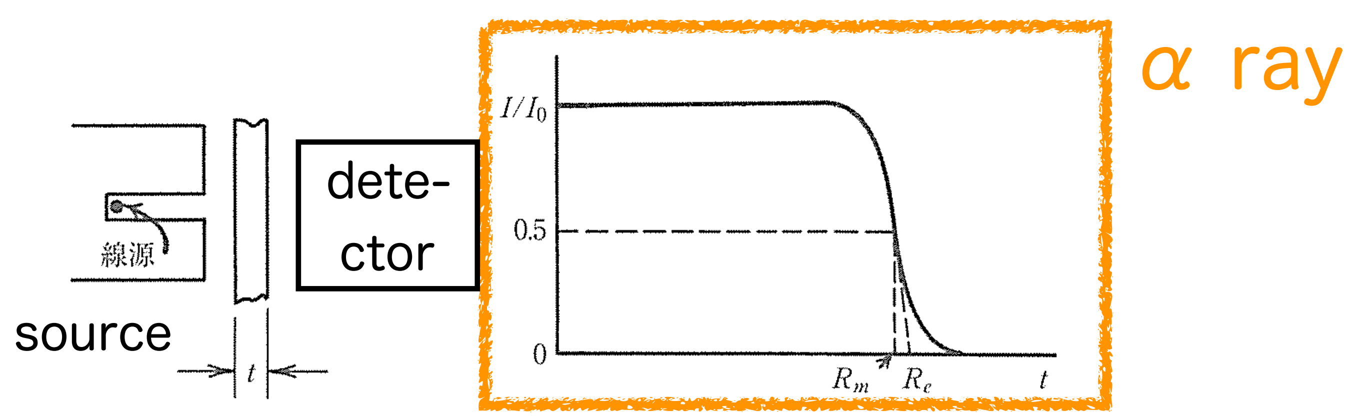
$$R_{\rm M} \simeq 21{\rm MeV}\cdot\frac{X_0}{E_c}$$

Attenuation of γ ray

- Range of γ ray is very different from that of a charged particle.



Why?

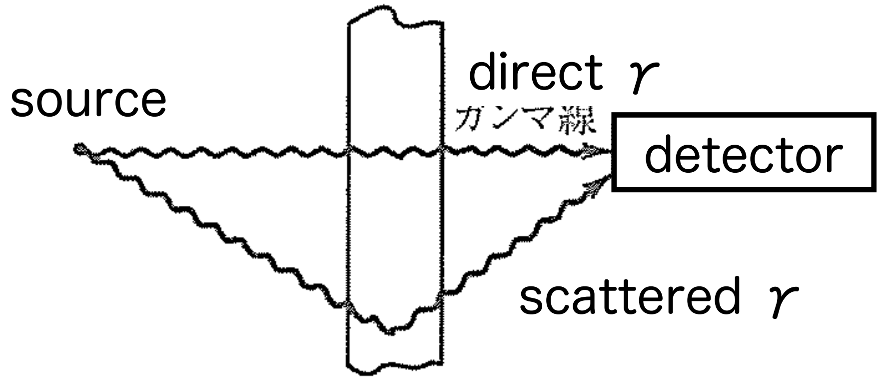


Attenuation of γ ray

- Energy loss of γ ray is a discrete process.
  - Energy loss of a charged particle is a continuous process.
- Attenuation length λ (coefficient μ) of γ ray is divined as follows.

$$\frac{I}{I_0} = e^{-\mu t}$$

$$\lambda = \frac{\int_0^\infty x e^{-\mu x} dx}{\int_0^\infty e^{-\mu x} dx} = \frac{1}{\mu}$$



(Info) In the measurement of the attenuation length, we must consider the effect of scattered photon.