Lectures on neutrino phenomenology - Part II

Monday - July 18th - VSON 2022

Pasquale Dario Serpico (LAPTh - Annecy, France)

Lecture's Outline

- Motivation: Why focusing on ν mass?
- Some history
- ν oscillations
  - Vacuum (focus on CP violation)
  - Matter

That's how we learned about massive ν nature & still in the process of measuring unknown/poorly known parameters

- Absolute mass scale & ν nature
  - Tritium endpoint
  - Cosmology
  - $0\nu2\beta$
- Conclusions

Two flavour case

Only one angle, plus a possible phase if Majorana (irrelevant for oscillations)

Check that the phase does not show up in *Im J*

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

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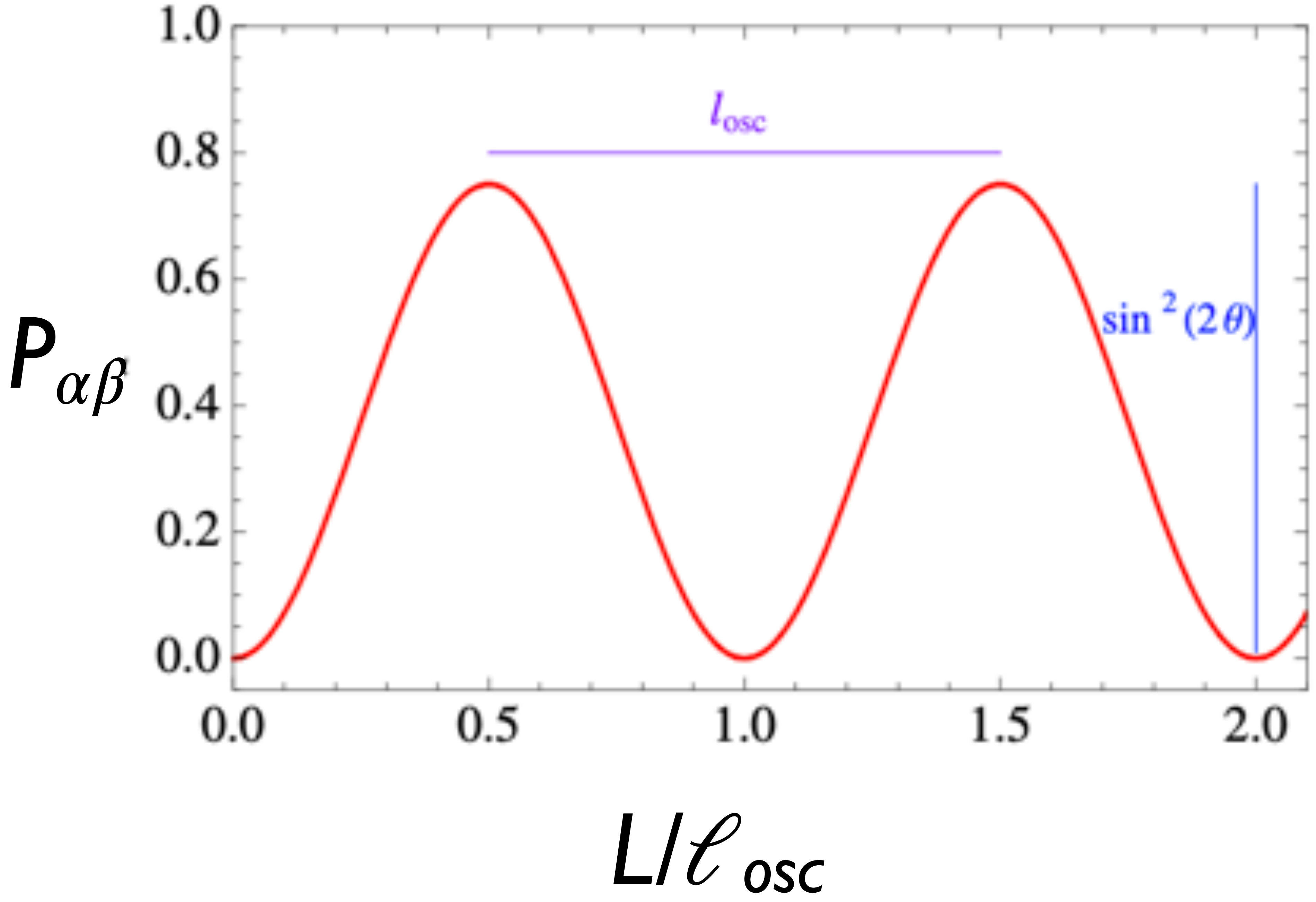
Appearence prob. $$P_{\alpha\to\beta}(L) = |\langle \nu_\beta | \nu_\alpha(L)\rangle|^2 = \sin^2(2\theta)\sin^2\left(\frac{(m_2^2 - m_1^2)L}{4\,E}\right) = \sin^2(2\theta)\sin^2\left(\frac{\pi\,L}{\ell_{\rm osc}}\right),$$

Survival prob. $$P_{\alpha\to\alpha}(L) = 1 - P_{\alpha\to\beta}(L)$$

oscillation length $$\ell_{\rm osc} = \frac{4\pi\,E\,\hbar}{\Delta m^2\,c^3} = 2.48\left(\frac{E}{\rm GeV}\right)\left(\frac{\rm eV^2}{\Delta m^2}\right)\,{\rm km}$$



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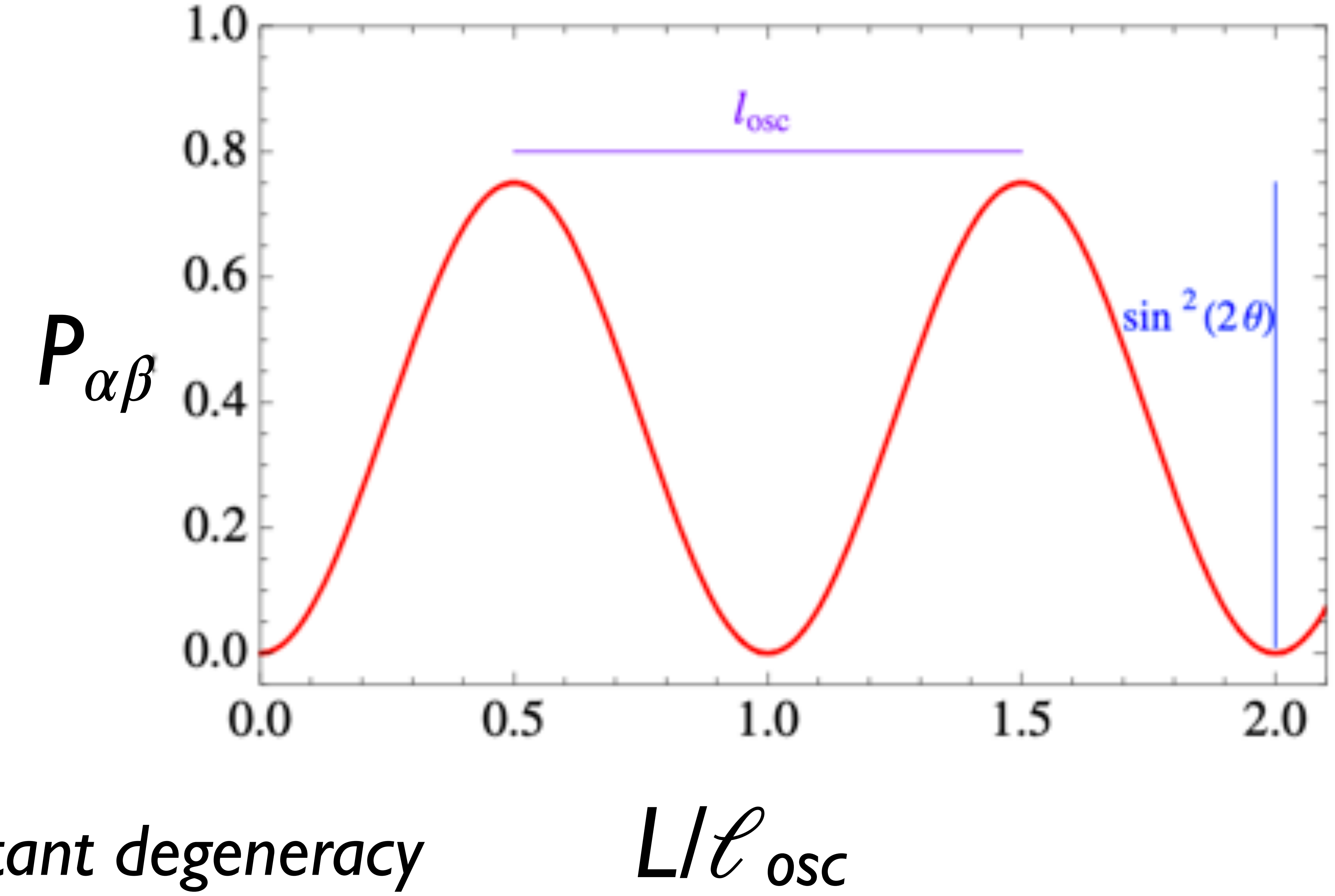

Physical parameter space spanned either with:

$\theta_{ij} \in [0, \pi/2]$ and $\Delta m^2 > 0$ — *Vacuum osc. has octant degeneracy*

or via $\theta_{ij} \in [0, \pi/4]$ and arb. sign for $\Delta m2$ — *Vacuum osc. has degeneracy $\Delta m2 \to -\Delta m2$*

Three flavour case (PMNS matrix)

The mixing matrix can be fully described by 3 mixing angles in the parameter ranges $\theta_{ij} \in [0, \pi/2]$ a phase $\delta_{CP} \in [0, 2\pi[$, and two extra phases if Majorana

Just like for rotations (e.g. *Euler vs. Roll-Pitch-Yaw*), the parametrisation is not unique (observables are!) but there is a *standard* PMNS parameterisation

$$U_{\mathrm{PMNS}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric mixing}} \times \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor mixing}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Solar mixing}} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{pmatrix}$$

The names in the factorisation are related to the fact that "effective" 2x2 mixing is sufficient for a leading order description of phenomena in different settings (and was historically used for first measurements!)

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Manifest that CP-violation in oscillations requires all three angles to be non-zero, and is proportional to $\sin\delta_{CP}$, e.g.

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) - P(\nu_\mu \rightarrow \nu_e) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta \times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

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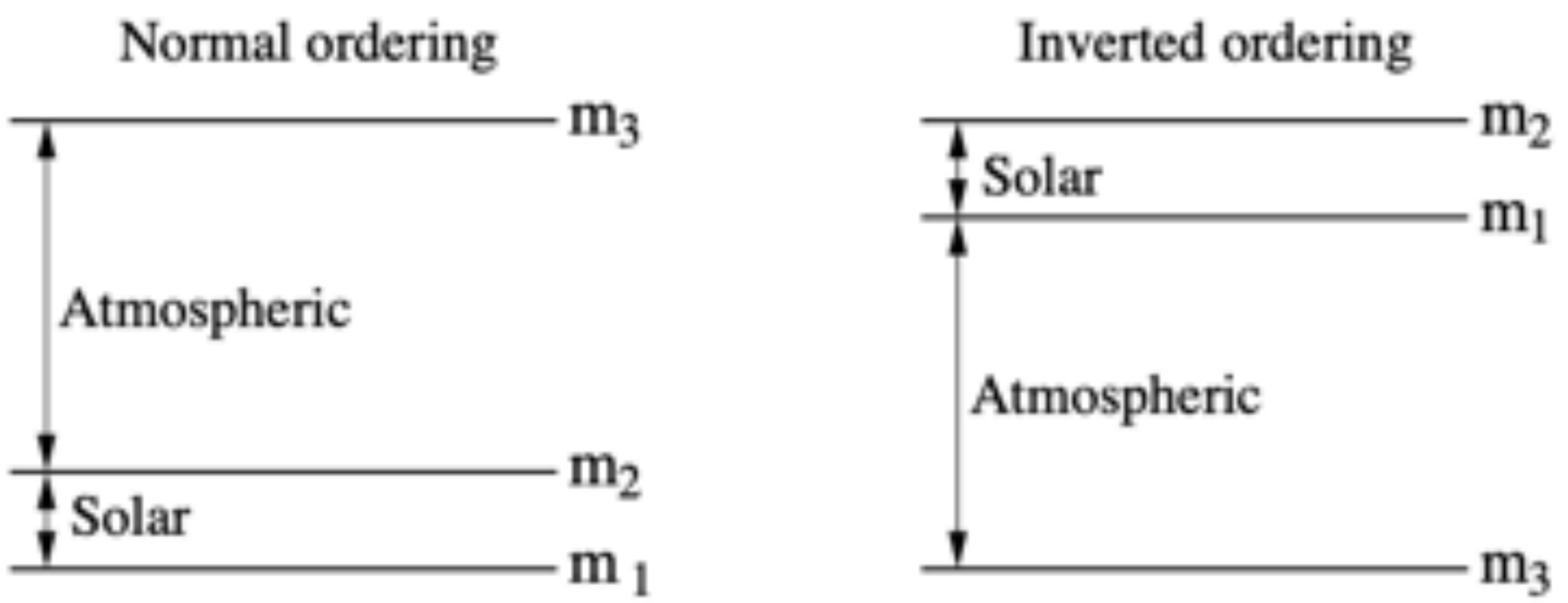
Mass splitting hierarchy: 2-flavour limits of 3-flavour case

It turns out that is some hierarchy, $\Delta m_{21}{}^2 \ll |\Delta m_{31}{}^2|$

(sign of the former known via solar matter effects, next lecture; latter, not yet known with high confidence)



The general formula

$$P_{\alpha\to\beta}(L) = \delta_{\alpha\beta} - 4\sum_{k>j} \Re J_{kj}^{\alpha\beta} \sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right) + 2\sum_{k>j} \Im J_{kj}^{\alpha\beta} \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)$$

reduces to

$$P_{\alpha\beta} \simeq \delta_{\alpha\beta} - 4(J_{31}^{\alpha\beta} + J_{32}^{\alpha\beta}) \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) - 4J_{21}^{\alpha\beta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- If we select L/E such that arg. 31 ~ $\pi/2$ → $\Delta m_{21}{}^2 \ll |\Delta m_{31}{}^2|$ implies a very small 21-oscillatory part (*atmospheric 2-flavour limit*)
- f we select L/E such that arg. 21~ $\pi/2$ → 31-oscillatory part averages to 1/2 (*solar 2-flavour limit*)

Atmospheric limit

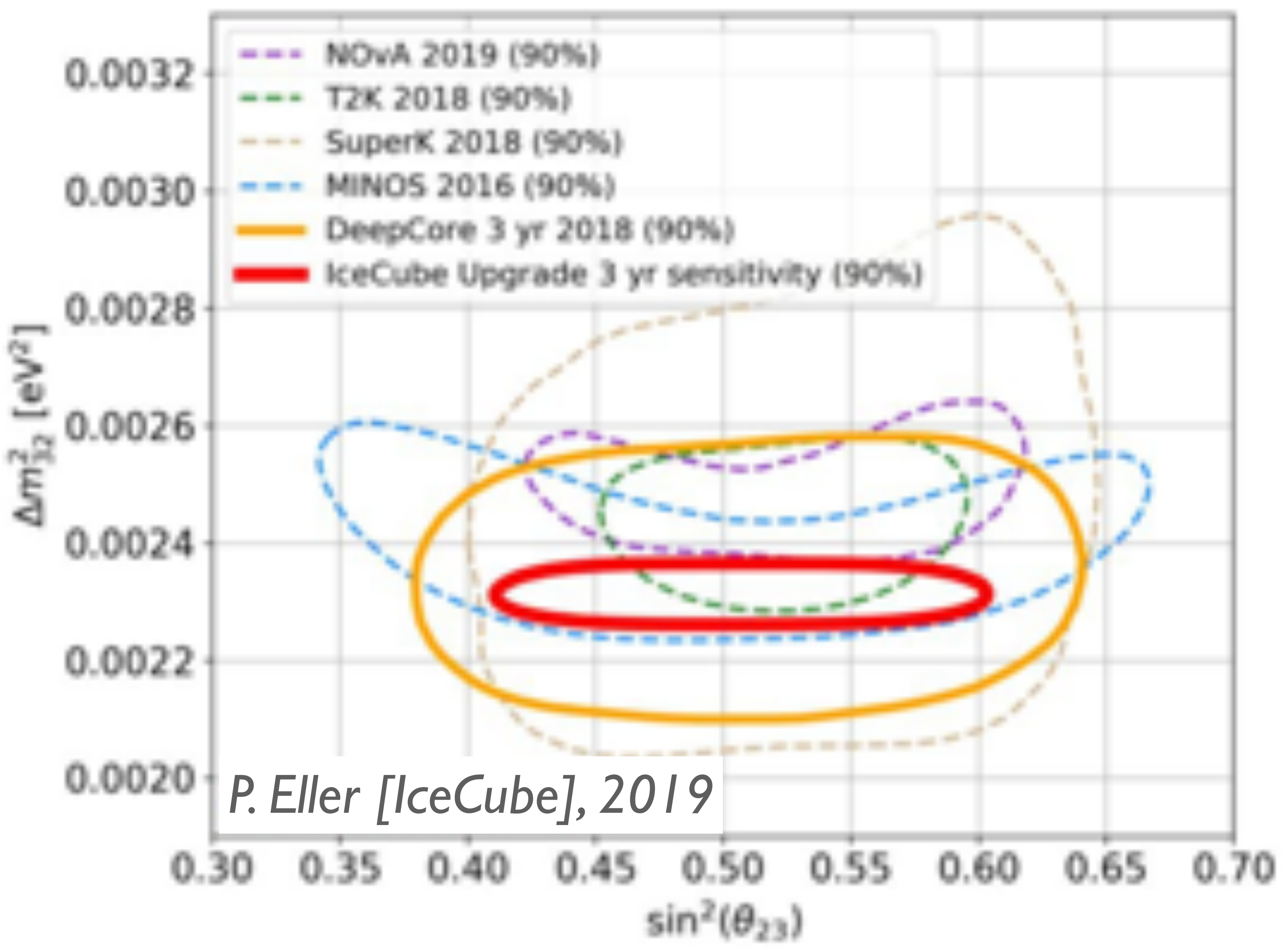
Also 'mild' mixing hierarchy $\theta_{13} << \theta_{12} \lesssim \theta_{23}$.

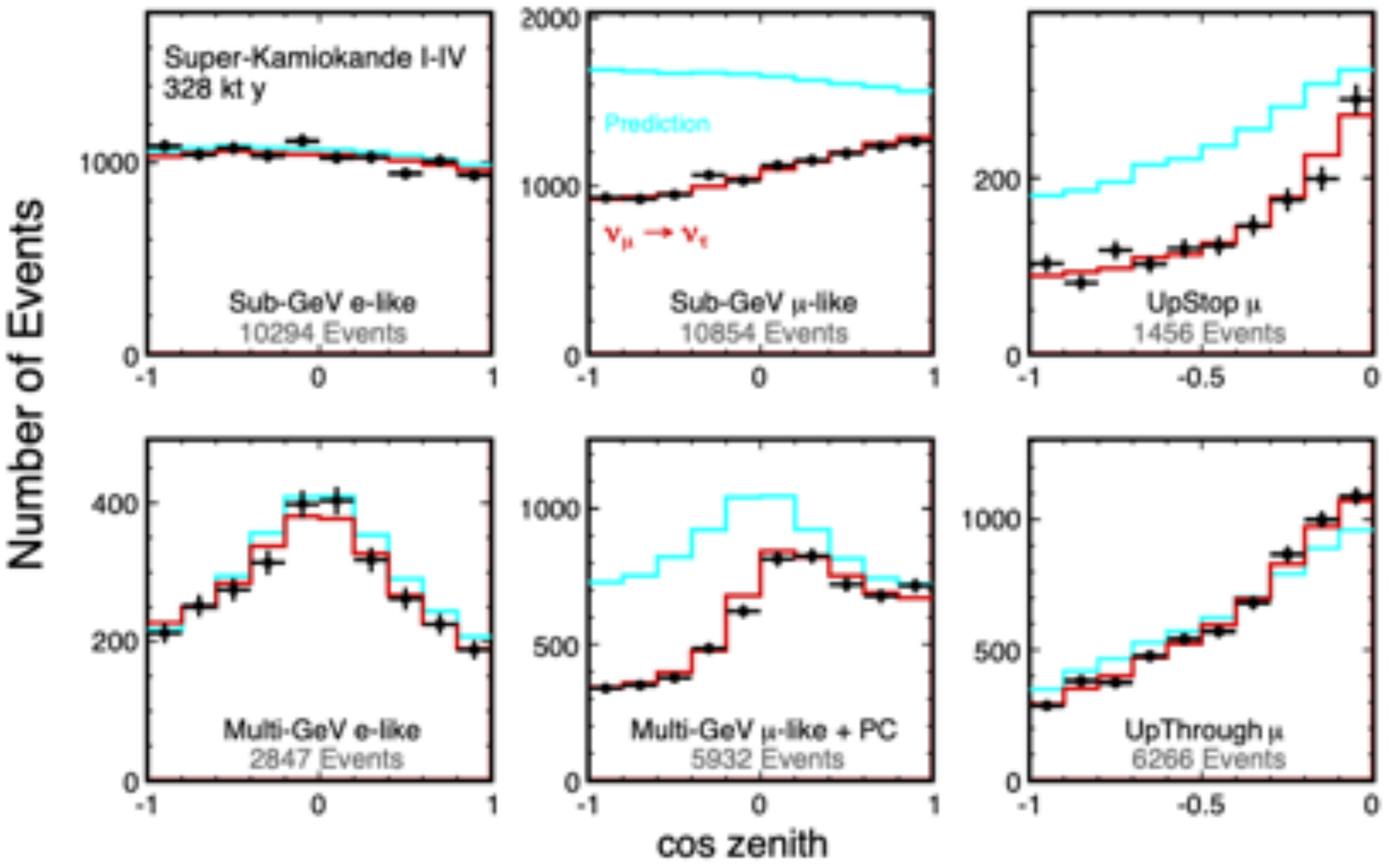
Neglecting terms in θ_{13} we get:

$$P_{ee} \simeq 1 \qquad P_{e\mu} \simeq P_{\mu e} \simeq 0$$

$$P_{\mu\mu} \simeq 1 - \sin^2(2\theta_{23}) \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

Atmospheric experiments use the ν produced (mainly) by pions as secondaries of cosmic ray interactions in the Earth's atmosphere. Similar approximation probed by "conventional ν beams" produced (mainly) by π decays like in the atmosphere



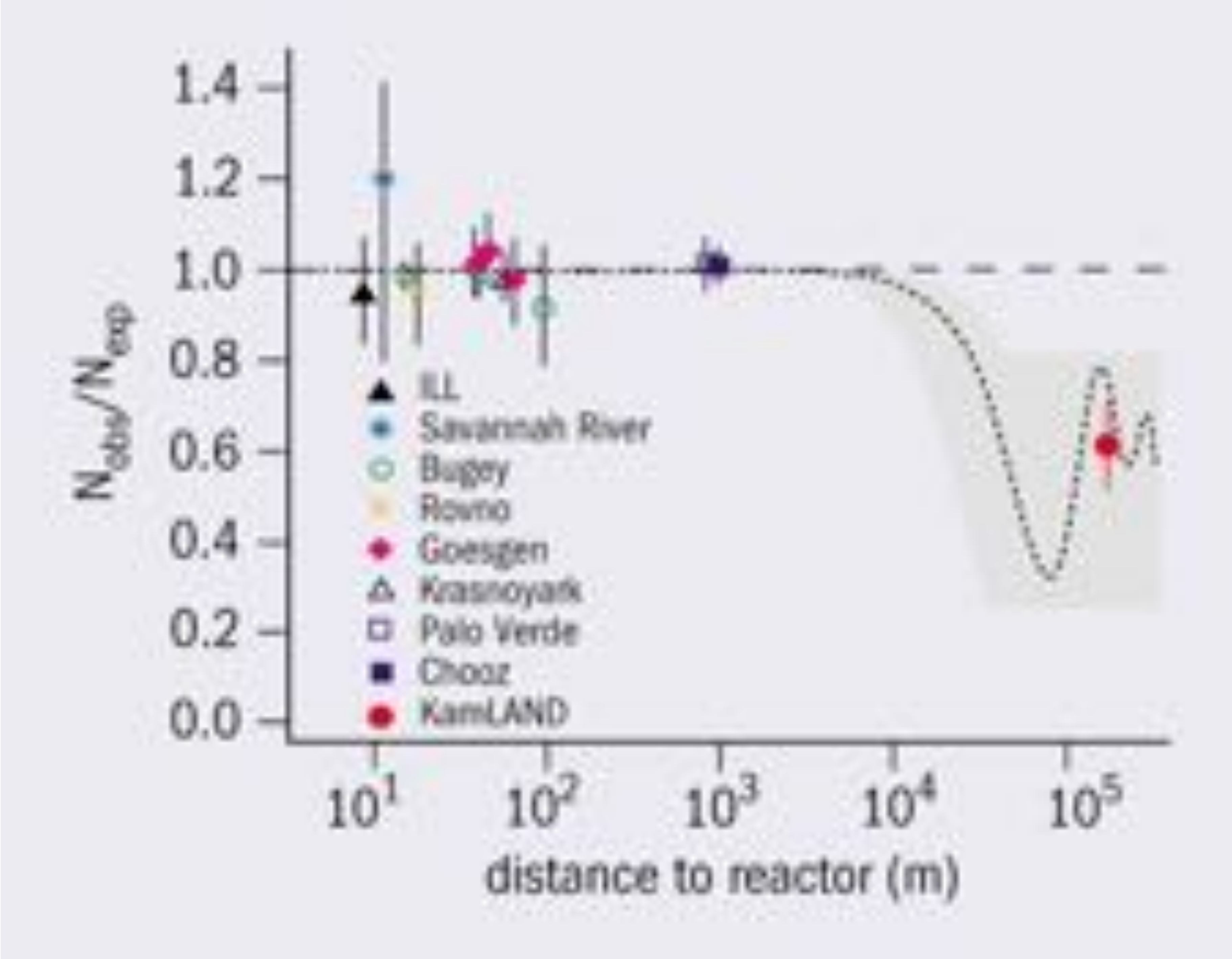


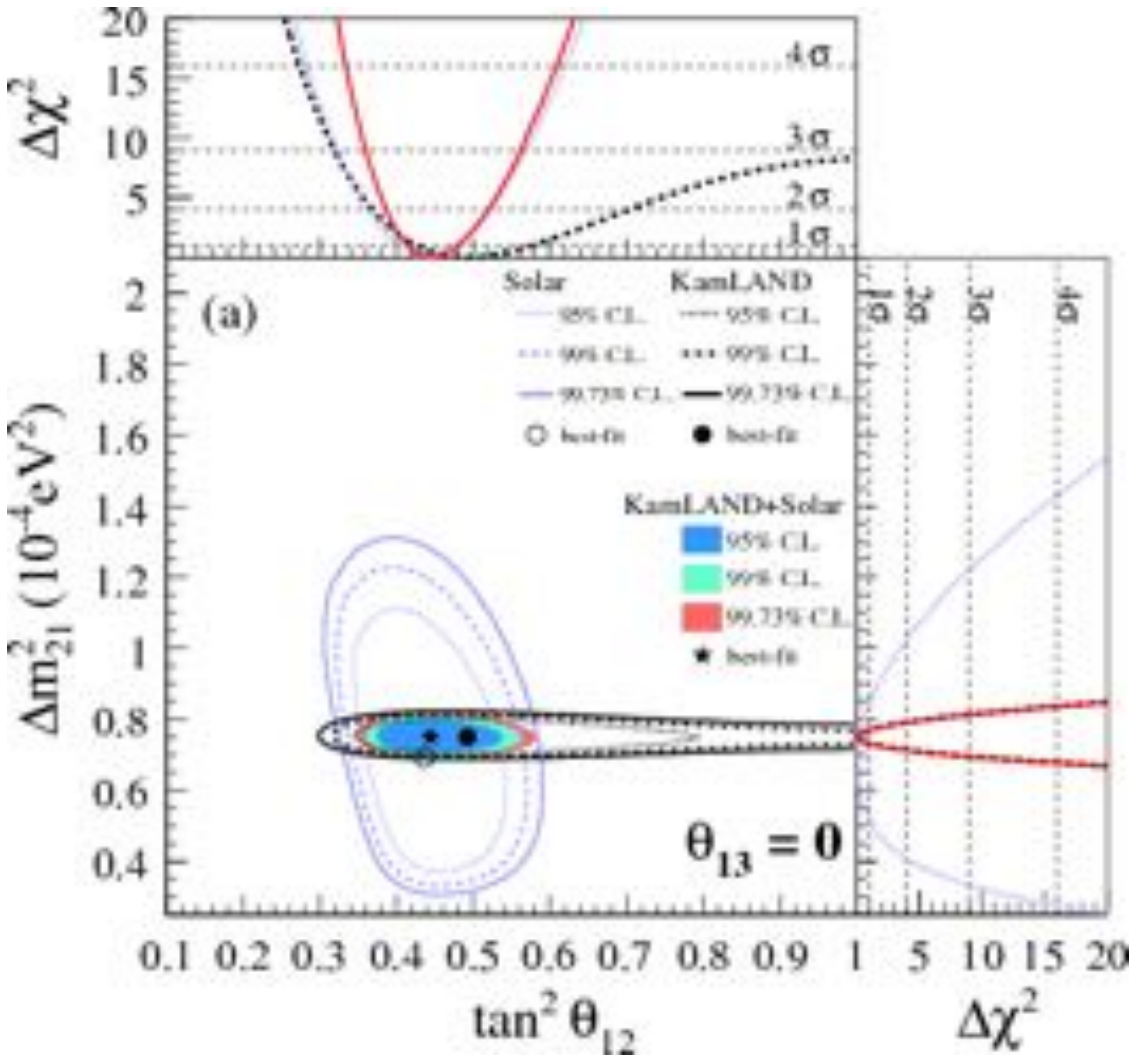
$$\Delta m^2_{32} \simeq \Delta m^2_{31} \simeq 2.4 \times 10^{-3}\,\mathrm{eV}^2 \qquad \sin^2\theta_{23} \simeq 0.5$$

Solar limit

Neglecting terms in θ_{13} we get: $P_{ee} \simeq 1 - \sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$

Although historically probed with solar ν (more on that later), a very long baseline *reactor* experiment (like KamLAND, at hundreds of km) can probe the 'solar parameters' by detecting anti-ν via inverse β decays.



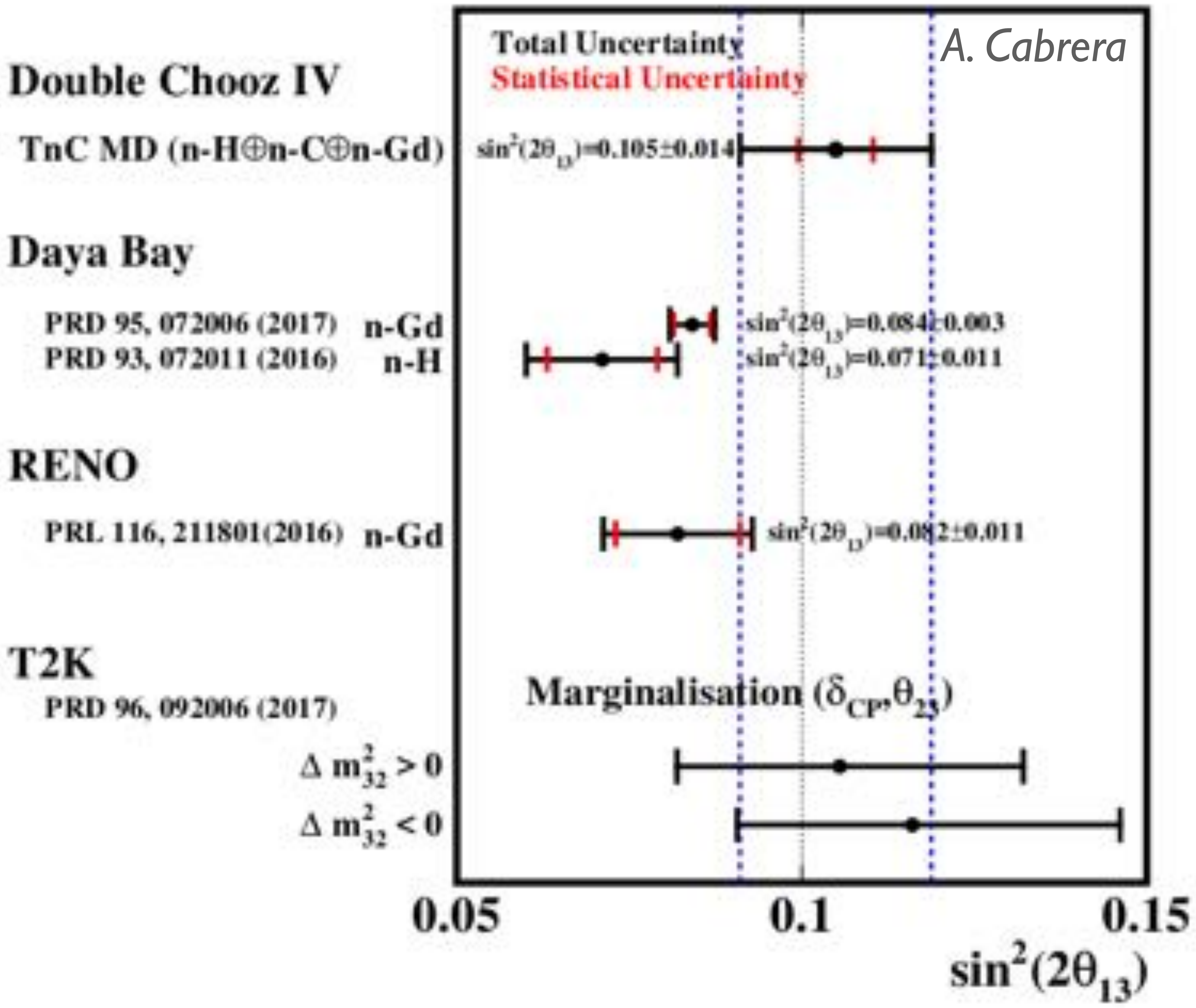


$$\Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\mathrm{eV}^2 \qquad \sin^2\theta_{12} \simeq 0.3$$

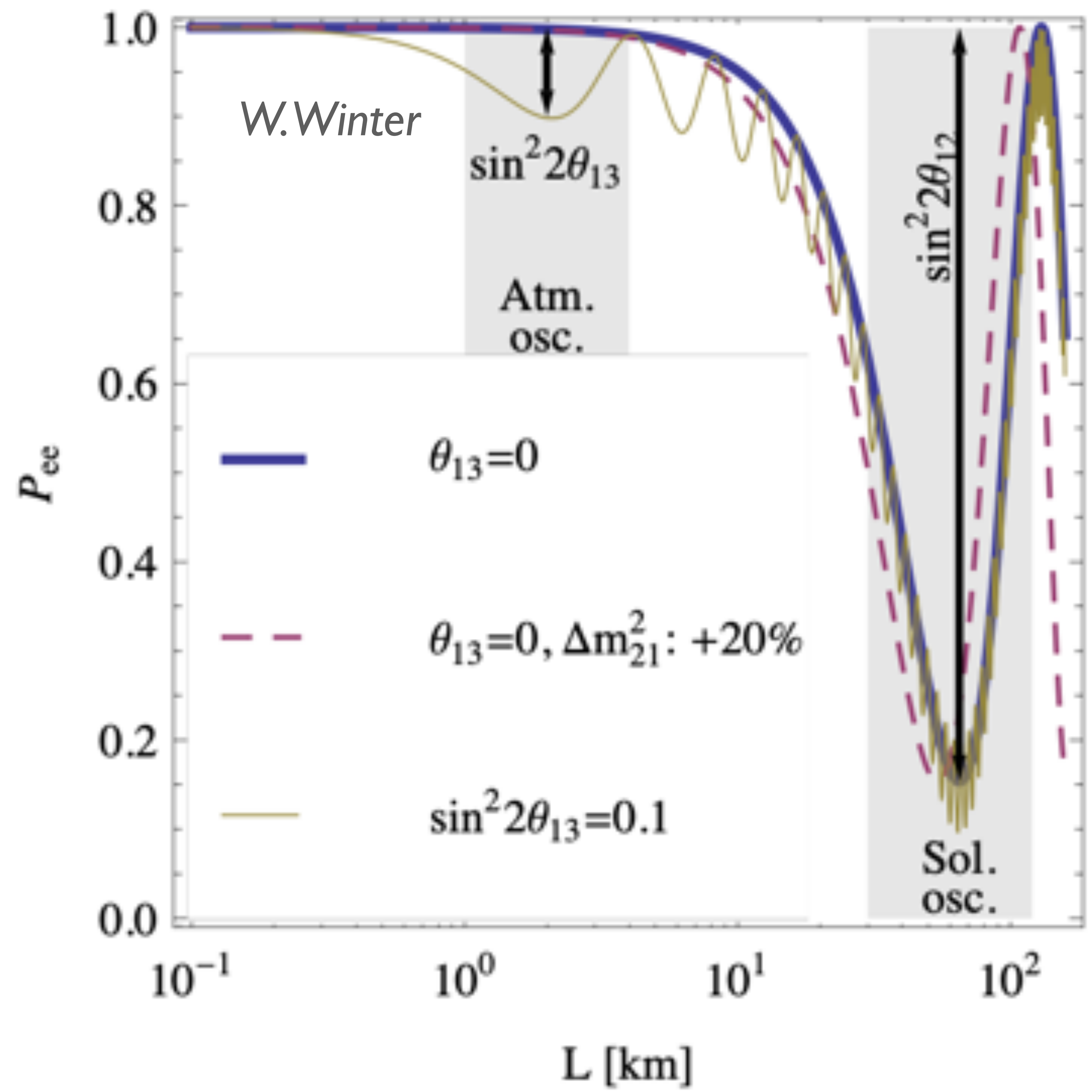
Switching on the ‘reactor angle’

If relaxing the approximation $\theta_{13} << \theta_{12}, \theta_{23}$ $$P_{ee} = 1 - \sin^2(2\theta_{13})\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) - \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right)$$

choosing a much shorter baseline (few km) than for the solar parameters, reactor experiments can largely ‘isolate’ the effect of θ_{13}



$$\sin^2\theta_{13} \simeq 0.022$$



Nowadays, analyses usually include all the three-flavour parameters and for LB experiments also matter effects

V. Neutrino oscillations in matter

ν oscillations in matter

Consider ν's not in vacuum |0>, but immersed in a 'ordinary matter' background |Ω>, with isotropically distributed *e*, *p*, and *n* (equal numbers of R and L…), density n_i and momentum distribution $f_i(p)$

The EoM for the ν's now contain a ***potential***, i.e. they are now of the form

$$(i\partial\!\!\!/ - m - \gamma^0 V)\nu = 0$$

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The potential can be derived from the part of the Lagrangian describing ν interactions

$$\mathcal{L}^{\rm eff}_{\rm weak} \supset -2\sqrt{2}G_F\Big\{(\bar{\nu}_e\gamma_\mu P_L\nu_e)(\bar{e}\gamma^\mu P_L e) + \Big(\sum_\alpha \bar{\nu}_\alpha\gamma_\mu P_L\nu_\alpha\Big)\Big[\sum_f^{p,n,e}\bar{f}\gamma^\mu(I_3^f P_L - \sin^2\vartheta_W Q_f)f\Big]\Big\}$$

← *Fierz rearrangement of fields*

The potential

Average over the distribution

$$\left\langle \mathcal{L}_{\text{weak}}^{\text{eff}} \right\rangle = -2\sqrt{2}G_F \Big\{ (\bar{\nu}_e \gamma^\mu P_L \nu_e) \int d^3p f_e(p) \langle \Omega | \bar{e} \gamma^\mu P_L e | \Omega \rangle$$

$$+ \frac{1}{2} (\bar{\nu}_\alpha \gamma^\mu P_L \nu_\alpha) \int d^3p \langle \Omega | \Big[f_e(p) \bar{e} \gamma^\mu \left(-P_L + 2s_W^2 \right) e$$

$$+ f_p(p) \bar{p} \gamma^\mu \left(P_L - 2s_W^2 \right) p + f_n(p) \bar{n} \gamma^\mu \left(-P_L \right) n \Big] | \Omega \rangle \Big\}$$

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These operators count the number of L fermions, statistically half of the total

→ factors n_i /2 out of the integration/trace over |Ω>

$$\rightarrow \quad -\sqrt{2} G_F n_e (\bar{\nu}_e \gamma^0 P_L \nu_e) - \frac{G_F}{\sqrt{2}} \left[(1 - 4s^2_W)(n_p - n_e) - n_n \right] (\bar{\nu}_\alpha \gamma^0 P_L \nu_\alpha)$$

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Euler-Lagrange equation finally yield the EoM with the following potential *in flavour space*

$$V = \text{diag}(V_{\text{CC}}+V_{\text{NC}}, V_{\text{NC}}, V_{\text{NC}}). \qquad V_{\text{CC}} = \sqrt{2}G_F n_e \qquad V_{\text{NC}} = \frac{G_F}{\sqrt{2}}\left[(1-4\sin^2\vartheta_W)(n_p-n_e)-n_n\right]$$

Equations of motion

Our results for vacuum evolution equivalent to ν states evolving as

$$i\frac{\partial}{\partial t}\psi = E\psi \simeq \left(p + \frac{m^2}{2p}\right)\psi$$

Getting rid of an overall common phase, now we get in the mass basis

$$i\frac{\partial}{\partial t}\psi_i = \left[\frac{\Delta m_{i1}^2}{2p}\delta_{ij} + \left(UVU^\dagger\right)_{ij}\right]\psi_j$$

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- Note that anti-*ν*'s (*E*<0 solutions to Dirac eq.) the potential has an opposite relative sign to the mass term.

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- In 'exotic' environments (like SN cores, early universe) the potential is more complicated, e.g. the presence of dense ν backgrounds makes the problem non-linear and the physics very rich (but complicated)!

2 flavour case

Remember: p=E, x=t, Opposite sign for anti-ν; terms proportional to identity irrelevant

$$i\frac{\mathrm{d}}{\mathrm{d}x}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \underbrace{\left(-p\mathsf{I} + \frac{1}{2\,p}U\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}U^\dagger + \Delta V\right)}_{G}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} \qquad \Delta V = \begin{pmatrix}\sqrt{2}G_F\,n_e & 0\\ 0 & 0\end{pmatrix}$$

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The propagating states are different from the states previously considered!

They can be found by diagonalising **G**, or any other propagation matrix differing from **G** via a term proportional to **I**. Let us make the 'clever' choice

$$\mathsf{G}_{eff} = \mathsf{G} - \left(-E + \frac{m_1^2 + m_2^2}{4\,E} + \frac{\sqrt{2}}{2}\,G_F\,n_e\right)\mathsf{I} = \frac{\Delta m^2}{4\,E}\begin{pmatrix}-c_{2\theta} + A & s_{2\theta}\\ s_{2\theta} & c_{2\theta} - A\end{pmatrix} \quad \text{where} \quad A \equiv \frac{2\sqrt{2}E\,G_F n_e}{\Delta m^2}$$

A quantifies the relative strength of matter potential to vacuum mixing effects.

We can expect matter effects to be more pronounced with growing energy

Effective mixing parameters

We can further rewrite

$$\frac{\Delta m^2}{4E}\begin{pmatrix} -c_{2\theta}+A & s_{2\theta} \\ s_{2\theta} & c_{2\theta}-A \end{pmatrix} = \frac{\Delta \tilde{m}^2}{4E}\begin{pmatrix} -c_{2\theta_m} & s_{2\theta_m} \\ s_{2\theta_m} & c_{2\theta_m} \end{pmatrix}$$

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Same structure as vacuum mixing case studied before, but for a couple of changes

- The mass value is rescaled as $$\Delta \tilde{m}^2 = \Delta m^2 \sqrt{(\cos(2\theta) - A)^2 + \sin^2(2\theta)}$$
- *Effective* mixing angle given by $$\sin(2\theta_m) = \frac{\sin(2\theta)}{\sqrt{(\cos(2\theta) - A)^2 + \sin^2(2\theta)}} \qquad \cos(2\theta_m) = \frac{\cos(2\theta) - A}{\sqrt{(\cos(2\theta) - A)^2 + \sin^2(2\theta)}}$$

Let's plot and study the effective mixing angle vs. *A* (and impact on effective splitting)

Qualitative discussion of basic matter effects

- Only marginal modification to vacuum when $|A| \ll \cos(2\theta) \leq 1$

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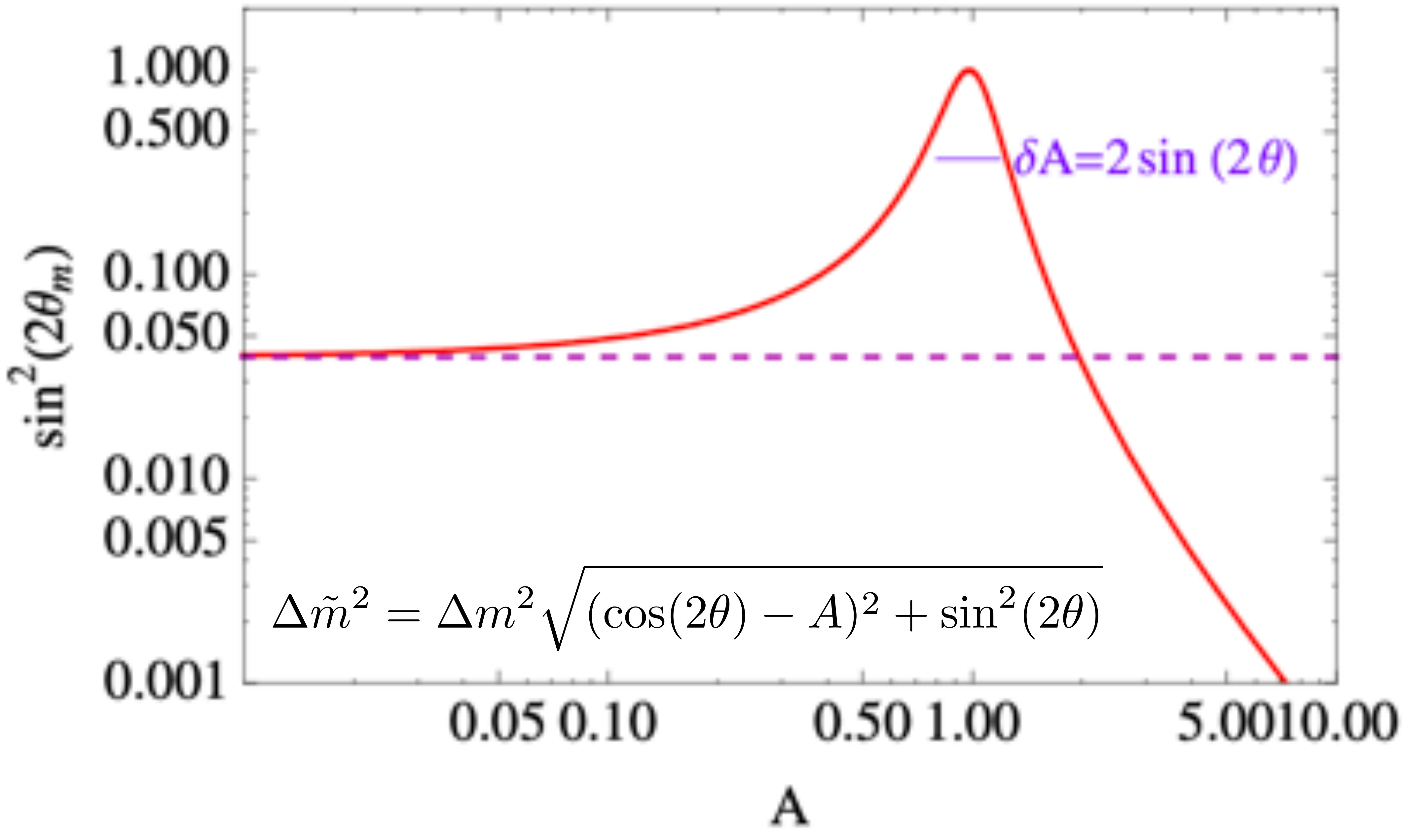

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- Only marginal modification to vacuum when $|A| \ll \cos(2\theta) \leq 1$
- Strong suppression of the mixing when $|A| \gg \cos(2\theta)$ always the case when $|A| \gg 1$

flavour states almost matching matter eigenstates, with mass splitting given however by $\Delta\tilde{m}^2 \simeq A\Delta m^2$
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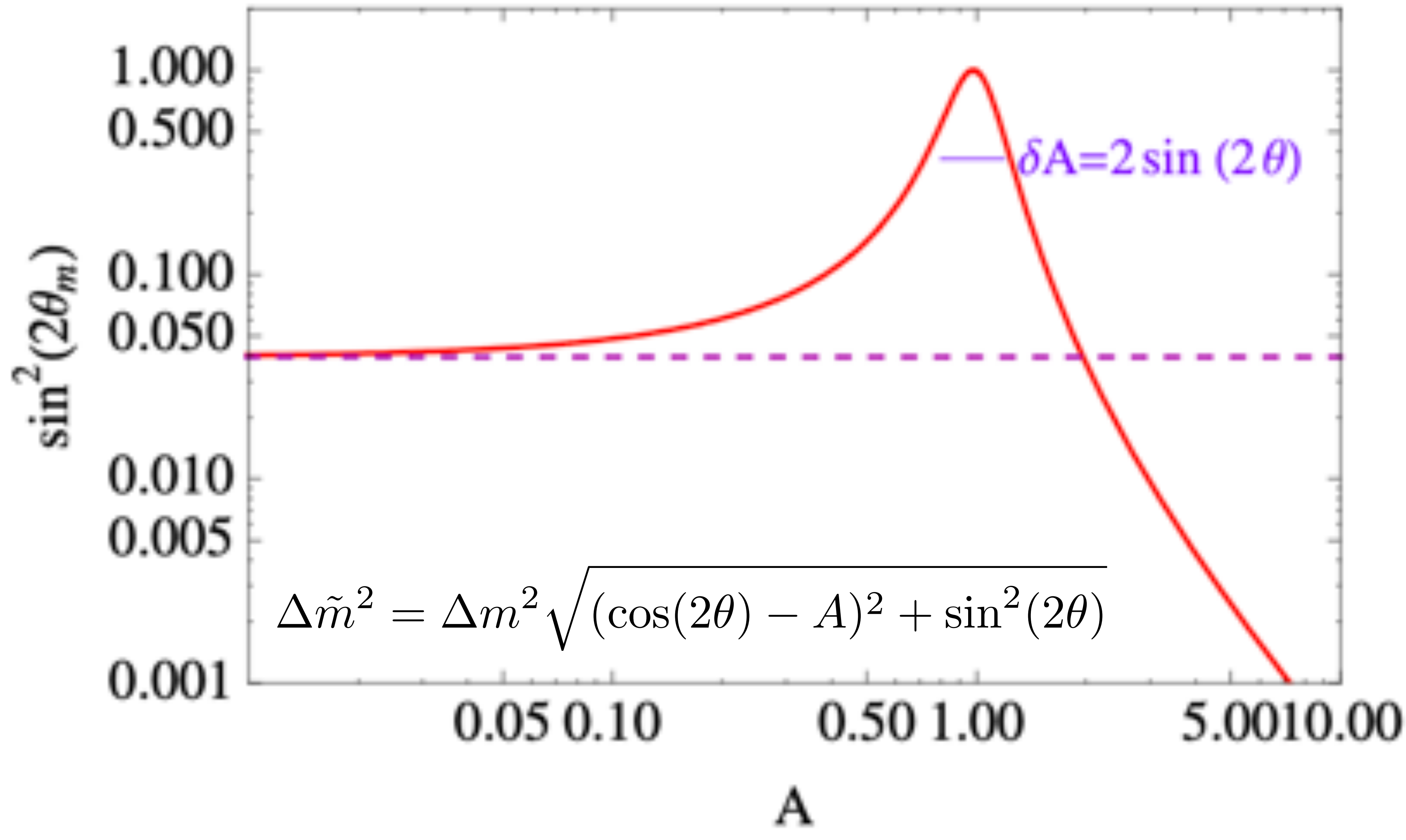

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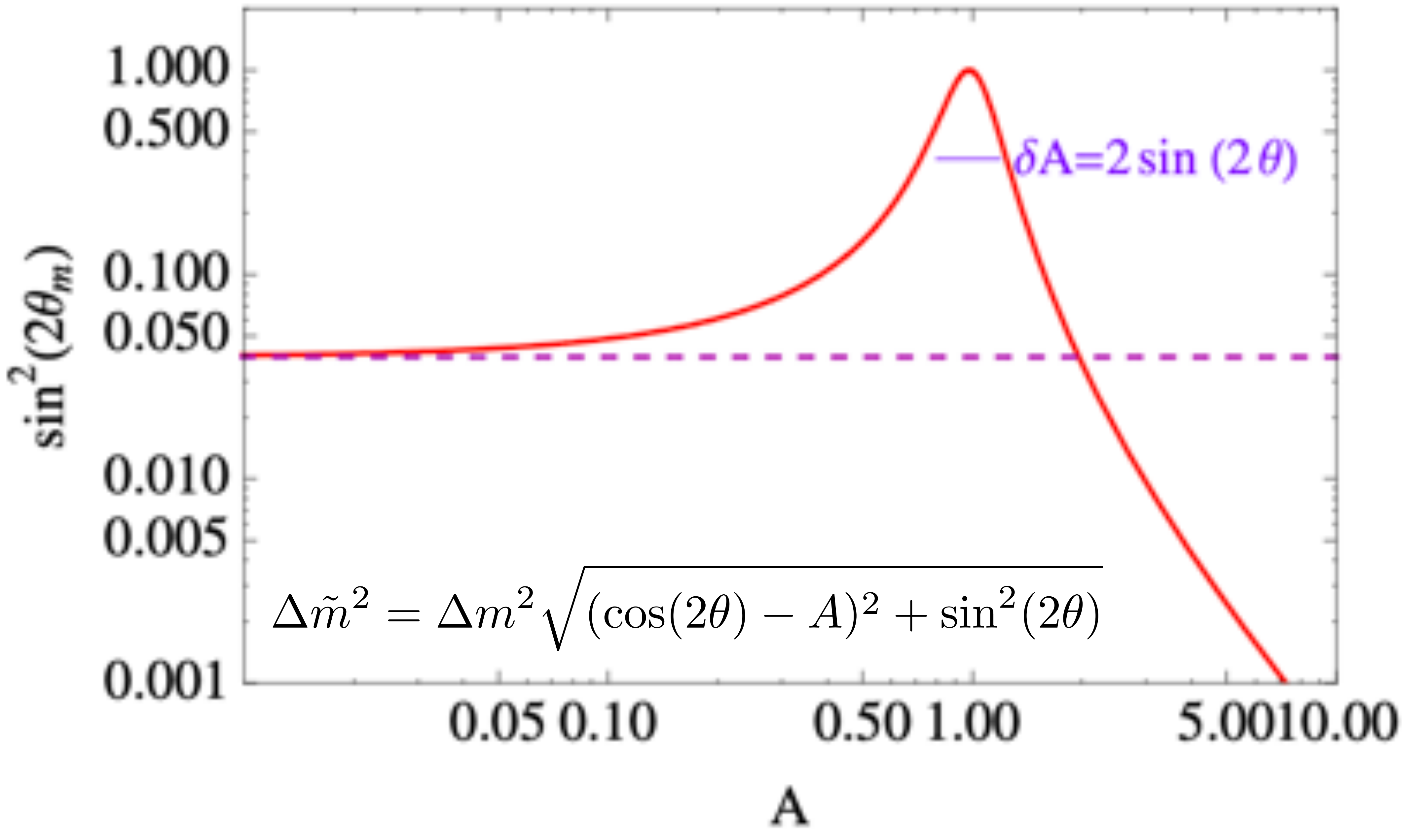
- Resonance when $A \simeq \cos(2\theta)$ within a width $\simeq \sin(2\theta)$

$$\sin(2\theta_m) = \frac{\sin(2\theta)}{\sqrt{(\cos(2\theta) - A)^2 + \sin^2(2\theta)}}$$

States maximally mixed (independent on their vacuum mixing)
and their mass splitting is minimised $\Delta\tilde{m}^2 = \Delta m^2 \sin(2\theta)$

Resonant density $$n_e^{\rm res} = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}$$



Qualitative discussion of basic matter effects

- Only marginal modification to vacuum when $|A| \ll \cos(2\theta) \leq 1$

- Strong suppression of the mixing when $|A| \gg \cos(2\theta)$ always the case when $|A| \gg 1$

flavour states almost matching matter eigenstates, with mass splitting given however by $\Delta\tilde{m}^2 \simeq A\Delta m^2$
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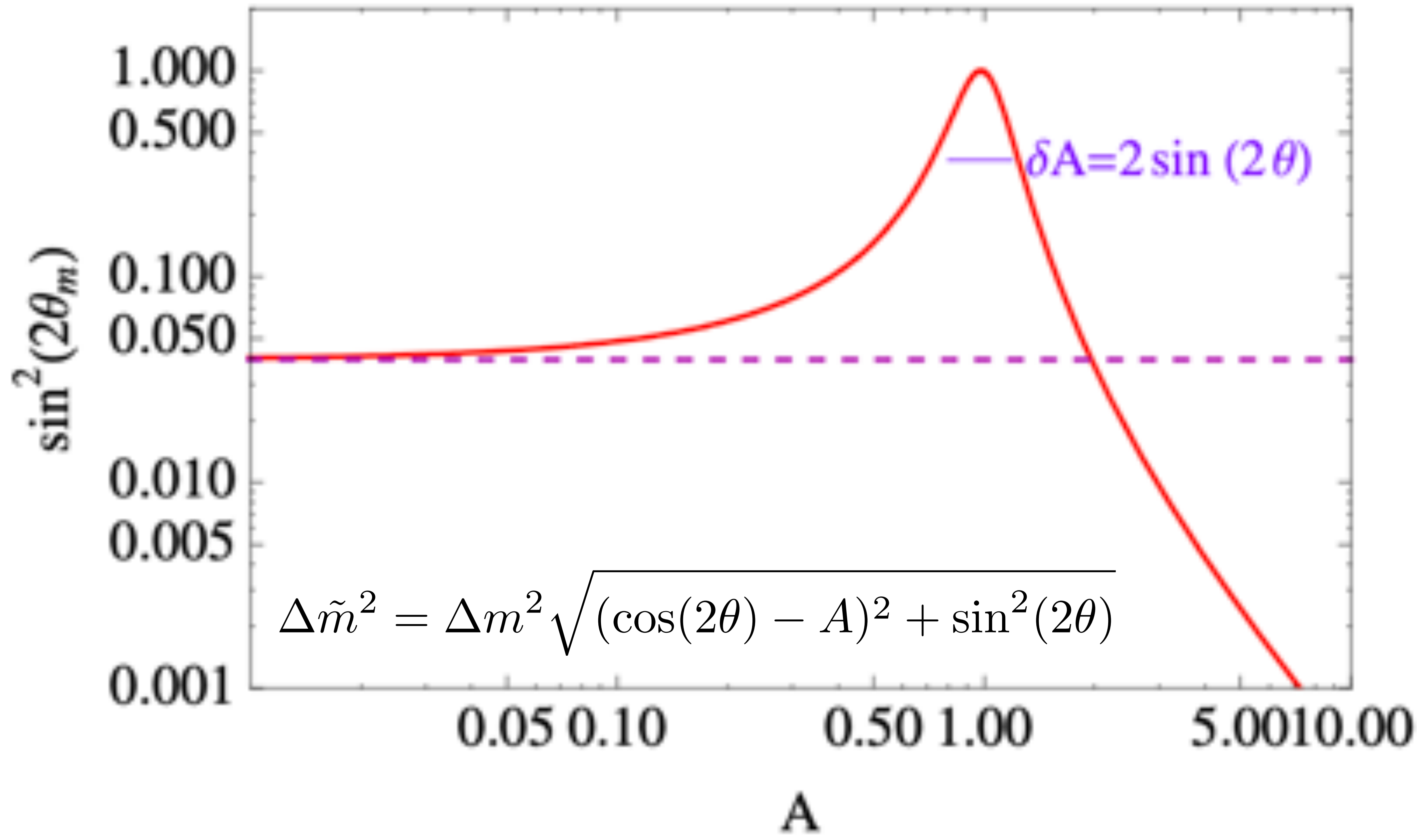

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Note: sign of cos(2θ) switches from + to - when θ goes above π/4. The sign of *A* depends on *ν* vs. anti-*ν*
Depending on the octant of θ, resonance *either* occurs in *ν* or anti-*ν*, not both!

ν oscillations in matter - varying density

Same as before, but now effective mixing and mass defined instantaneously (or locally)

By rotating into the *instantaneous* (or local) mass basis, obtain a structure as

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_a\\ \nu_b\end{pmatrix} = \begin{pmatrix}\frac{\tilde{m}_a^2(t)}{2E} & i\dot{\theta}(t)\\ i\dot{\theta}(t) & \frac{\tilde{m}_b^2(t)}{2E}\end{pmatrix}\begin{pmatrix}\nu_a\\ \nu_b\end{pmatrix}$$

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Hard to get a general, exact analytical solution. But in limiting cases, we expect that

- If the **off-diagonal term is small, constant density results should apply**, each state evolves independently.
- If the **off-diagonal term is large, states can 'jump' from one to the other** (*Landau-Zener jump equation*)

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$$P_{LZ} = e^{-2\pi\gamma}$$

"Large or small" is gauged with respect to the instantaneous oscillation frequency (or length)

$$\frac{1}{\gamma} = \left|\frac{2\dot{\theta}}{\Delta\tilde{m}^2/(2E)}\right| \ll 1$$

Adiabaticity condition

Application to the Sun, I

Resonant density for 'solar parameters'

$$n_{\rm res} = \frac{\Delta m^2_{21}\cos\theta_{12}}{2\sqrt{2}G_F E} \simeq 10^{26}\left(\frac{\rm MeV}{E}\right){\rm cm}^{-3}$$

Solar profile

$$n_e(r) \simeq n_{\rm core} = 6.5\times 10^{25}{\rm cm}^{-3} \quad r \le r_{\rm core} \simeq 0.1 R_\odot$$

$$n_e(r) \simeq n_{\rm core}\exp\left(-\frac{r-r_{\rm core}}{r_0}\right) \quad r_{\rm core} \le r \le R_\odot$$

$$r_0 \simeq R_\odot/10 \simeq 70000\,{\rm km}$$

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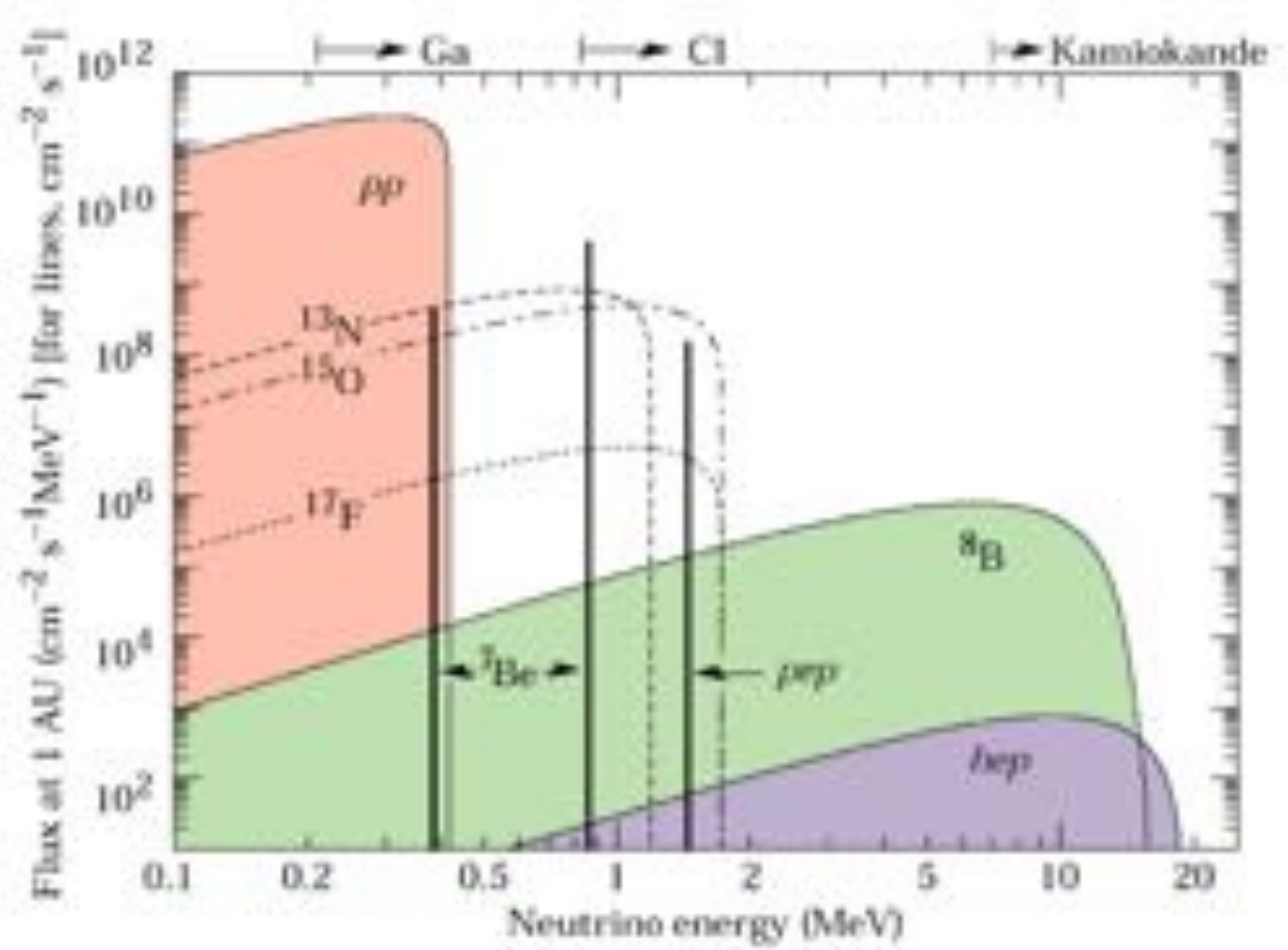

- *High-E* as ^{8}B measured by SK with *E*~10 MeV, should experience *resonance & strong matter effects*
- *Low-E* part below a few MeV like *pp* should not (*quasi-vacuum*)

Application to the Sun, II

The density profile varies 'slowly' compared to neutrino oscillation length

Slowly decreasing $$\frac{1}{n_e}\frac{\mathrm{d}n_e}{\mathrm{d}x} = \frac{1}{r_0} \simeq \frac{10}{R_\odot} \simeq (7\times 10^4\ \mathrm{km})^{-1}$$ To be compared with $$\ell_{\mathrm{osc}} \simeq 25\,\mathrm{km}\frac{E}{\mathrm{MeV}}\frac{10^{-4}\mathrm{eV}^2}{\Delta m^2}$$

Adiabaticity well verified in the Sun, for *E*~MeV and actual solar parameters $\gamma \gg 1$

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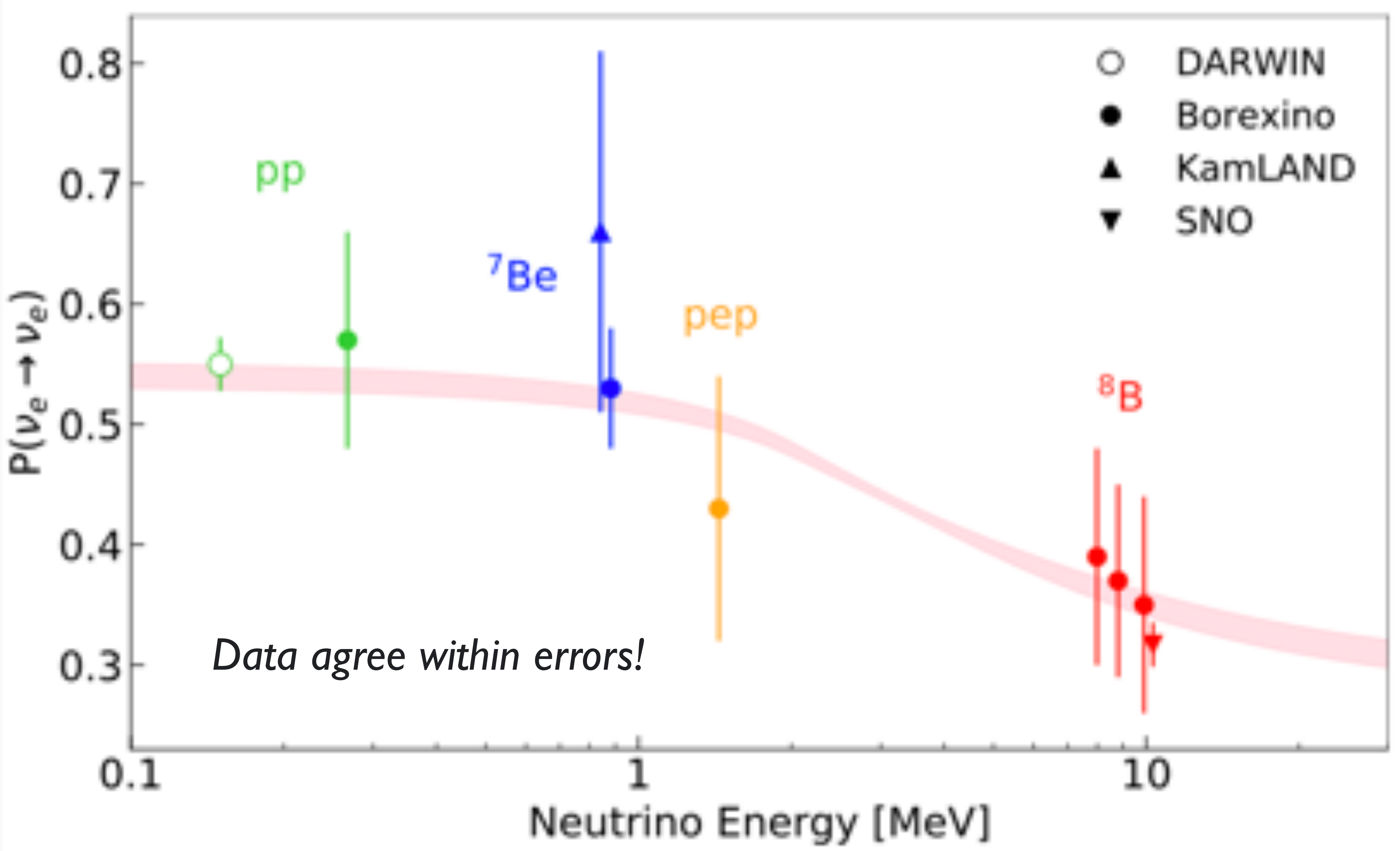

Generalisation to matter effects in 3 flavour case

Similar story, now the effective hamiltonian writes

$$\mathcal{H}_{\text{eff}} = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\Delta m^2_{21}}{2E} & 0 \\ 0 & 0 & \frac{\Delta m^2_{31}}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For LBNE, typically constant density assumption is ok; the impact of matter effects is stronger at higher-*E*
For T2K, $A \approx 0.05$, for NOvA, $A \approx 0.15$, for DUNE, $A \approx 0.21$

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For T2K, *A* ≈ 0.05, for NOvA, *A*≈0.15, for DUNE, *A*≈ 0.21

Typically dealt with numerically (exact analytical formulae anyway not transparent)

For an idea, at leading order, we expect $P_{\mu\to e} \simeq \sin^2(\theta_{23})\sin^2(2\tilde{\theta}_{13})\sin^2\left(\frac{\Delta\tilde{m}^2_{31} L}{4E}\right)$

A conversion probability enhancement is expected for
- ν's $\Delta m^2_{31} > 0$
- anti-ν's $\Delta m^2_{31} < 0$

Effect can be used to determine mass ordering, but it is a nuisance for CP-measurements!

An application to long baseline experiments

This means that the so-called '***CP asymmetry parameter***'

$$a_{\rm CP} \equiv \frac{P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}}}{P_{\alpha\beta} + P_{\bar{\alpha}\bar{\beta}}}$$

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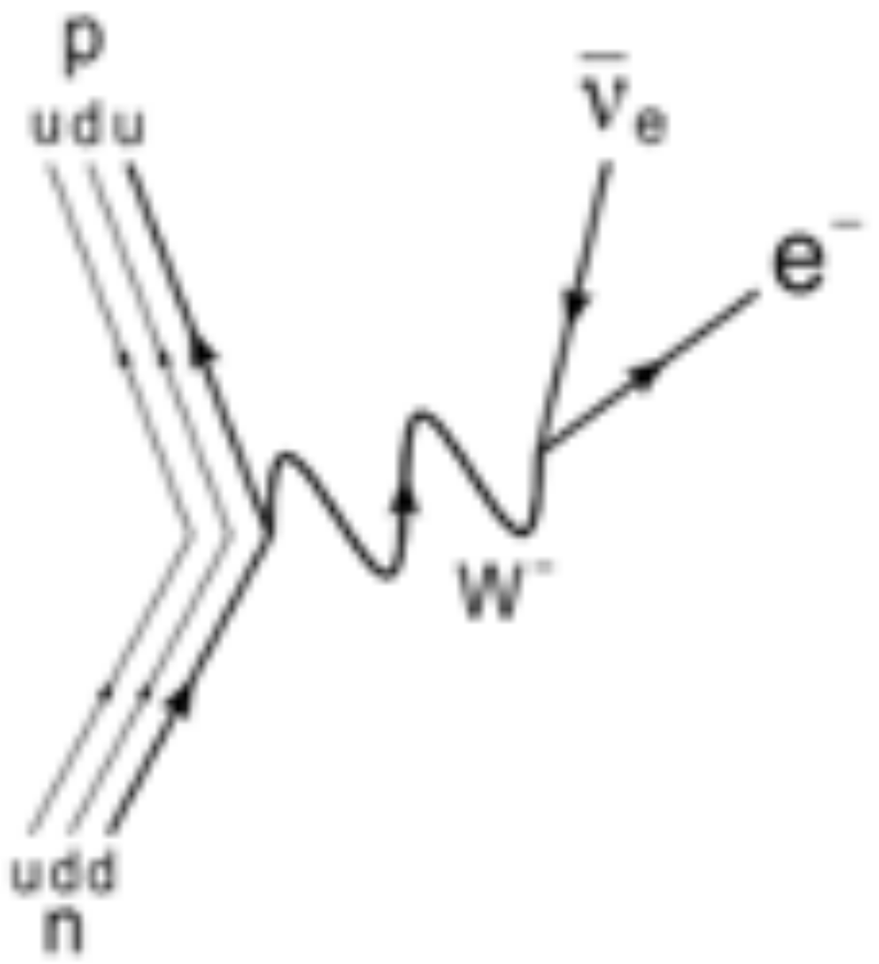
Overcoming that requires e.g. using enough E-resolution, different baselines, different oscillation channels…

VI. The quest for the absolute mass scale

Currently pursued in three ways:

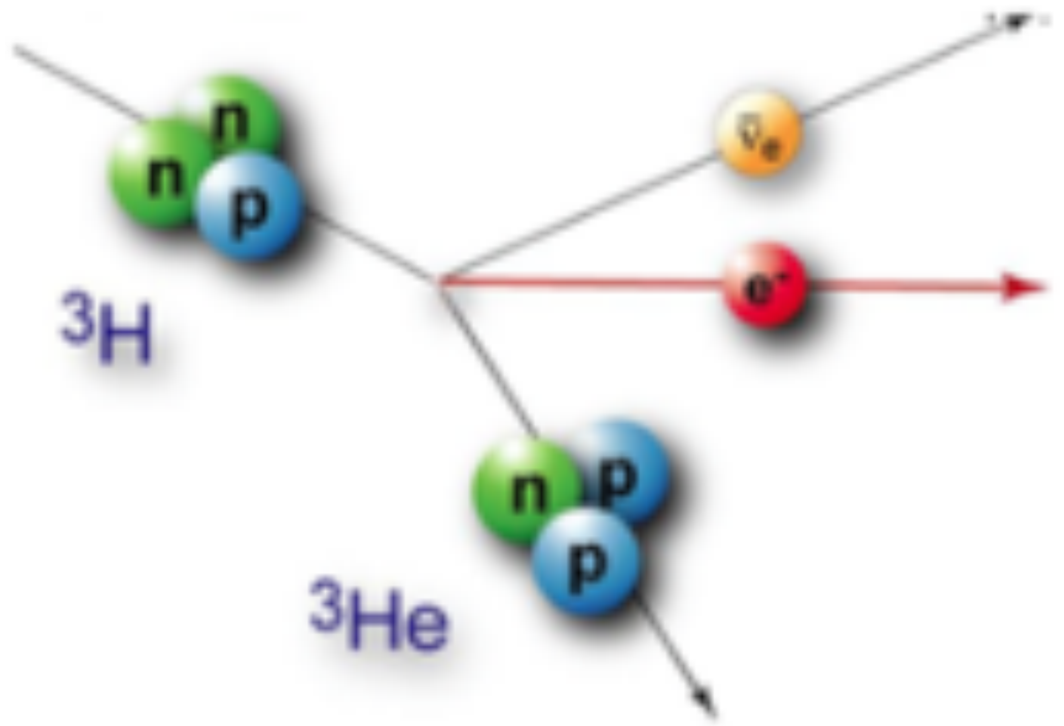
- Kinematical measurement in particle processes
- Gravitational measurement
- Via observation of processes only allowed by a finite mass

Beta decay endpoint $_{Z}X \rightarrow \, _{Z\pm1}X' + e + \nu_e$



Most widely studied:

$^3H \rightarrow \, ^3He + e^- + \overline{\nu}_e$



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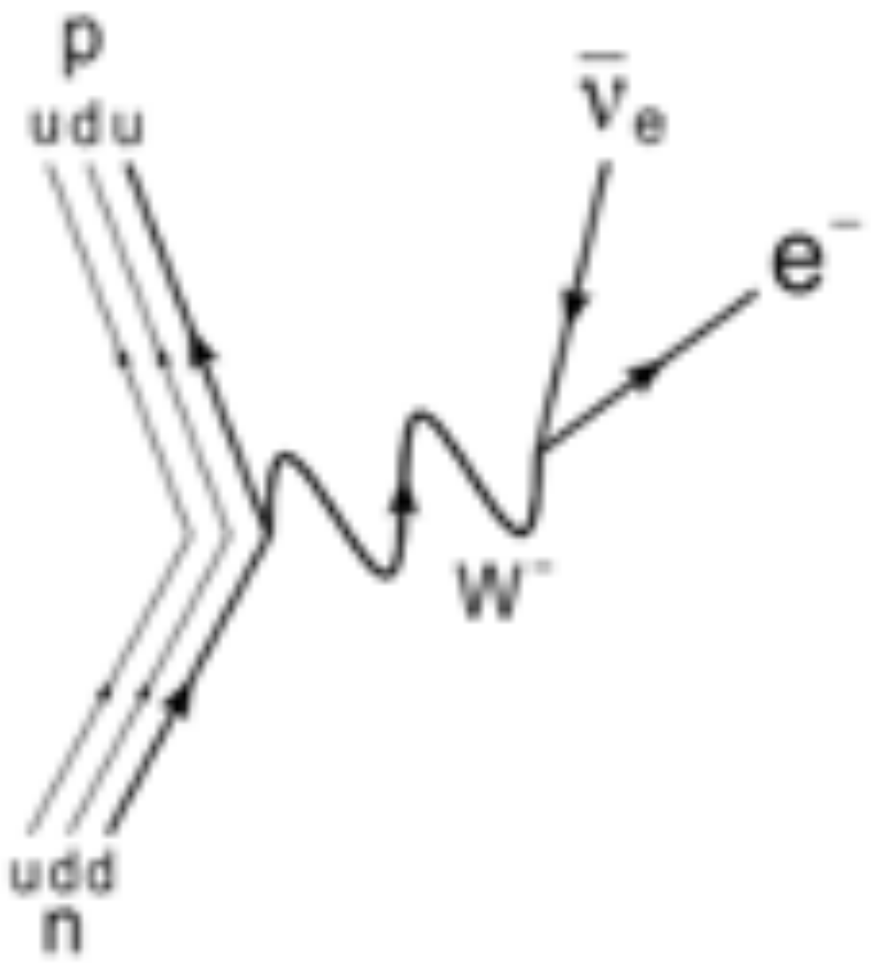

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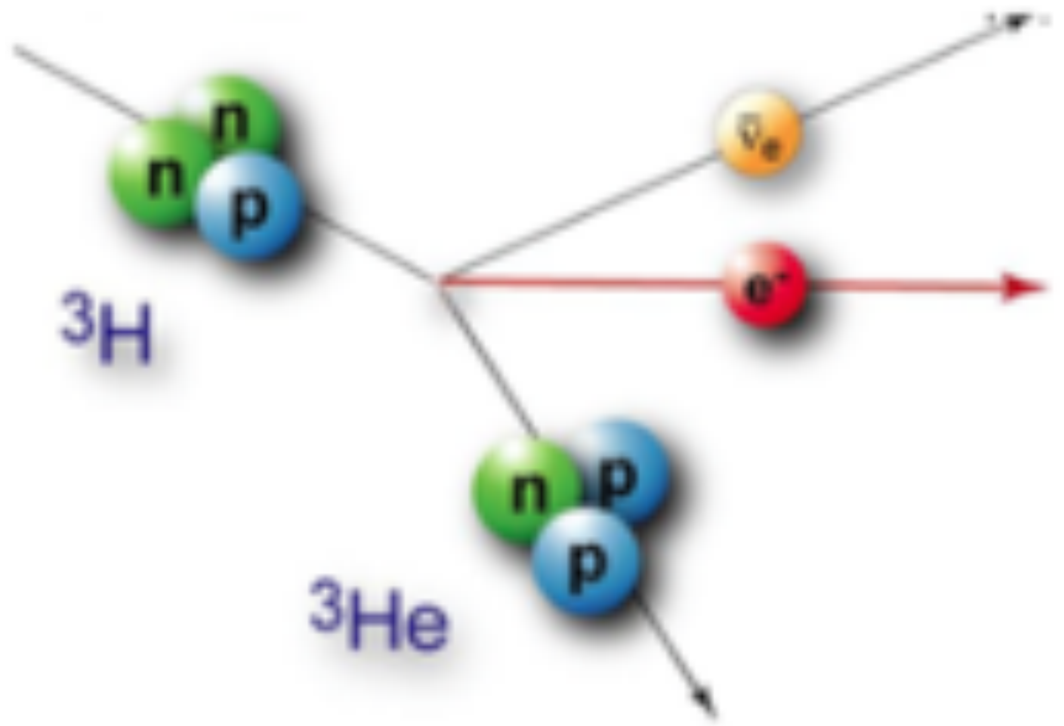

If neglecting recoil daughter nucleus (carries less than 0.05% of the reaction Q-value)

(excitations of X' should be included if present!)

$$\Gamma \propto \int |\mathcal{M}|^2 \mathrm{d}f_e \mathrm{d}f_\nu$$

where

$$\mathrm{d}f_i = \frac{p^2 \mathrm{d}p \mathrm{d}\Omega}{(2\pi)^3} = \frac{p p_0 \mathrm{d}p_0 \mathrm{d}\Omega}{(2\pi)^3}$$

$$K_e^{\max} = Q - K_{\mathrm{rec}} - K_{\mathrm{ex}} \qquad \epsilon_\nu = K_e^{\max} - K_e$$

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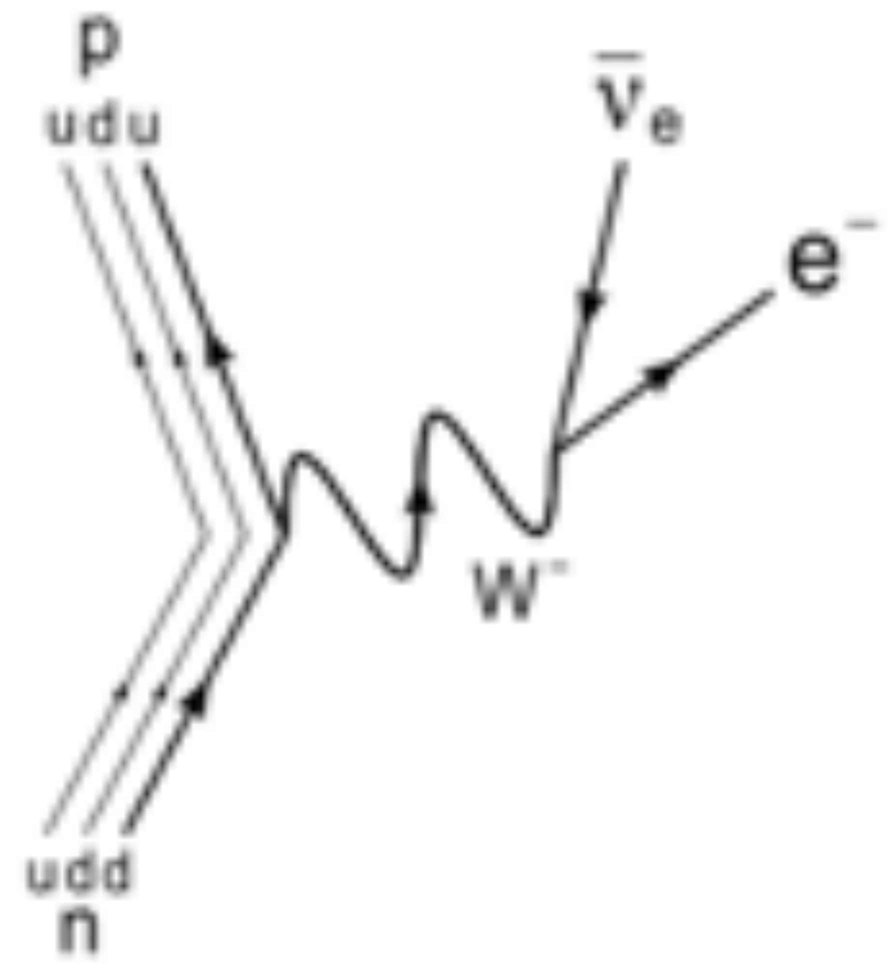

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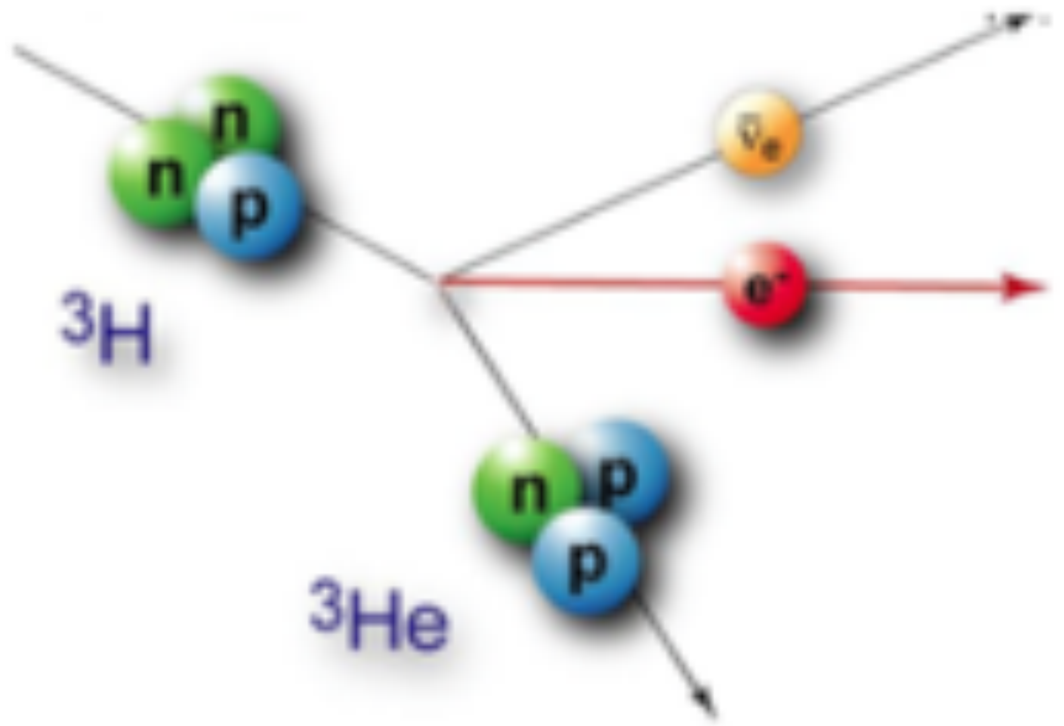

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Observable:

Electron spectrum

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}K_e} \propto |\mathcal{M}|^2 p_e \,(K_e + m_e)\epsilon_\nu \sqrt{\epsilon_\nu^2 - m_\nu^2}$$

Typically studied in detail via spectrometers (e.g. KATRIN) or calorimeters (e.g. MARE)

Tritium decay endpoint $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

Traditionally one defines a (Fermi-)Kurie function $\mathcal{K}(K_e) \propto \sqrt{\frac{\mathrm{d}\Gamma}{\mathrm{d}K_e}}$

Which has a different behaviour near the endpoint, depending on the ν mass

$m_\nu = 0$	$m_\nu \neq 0$
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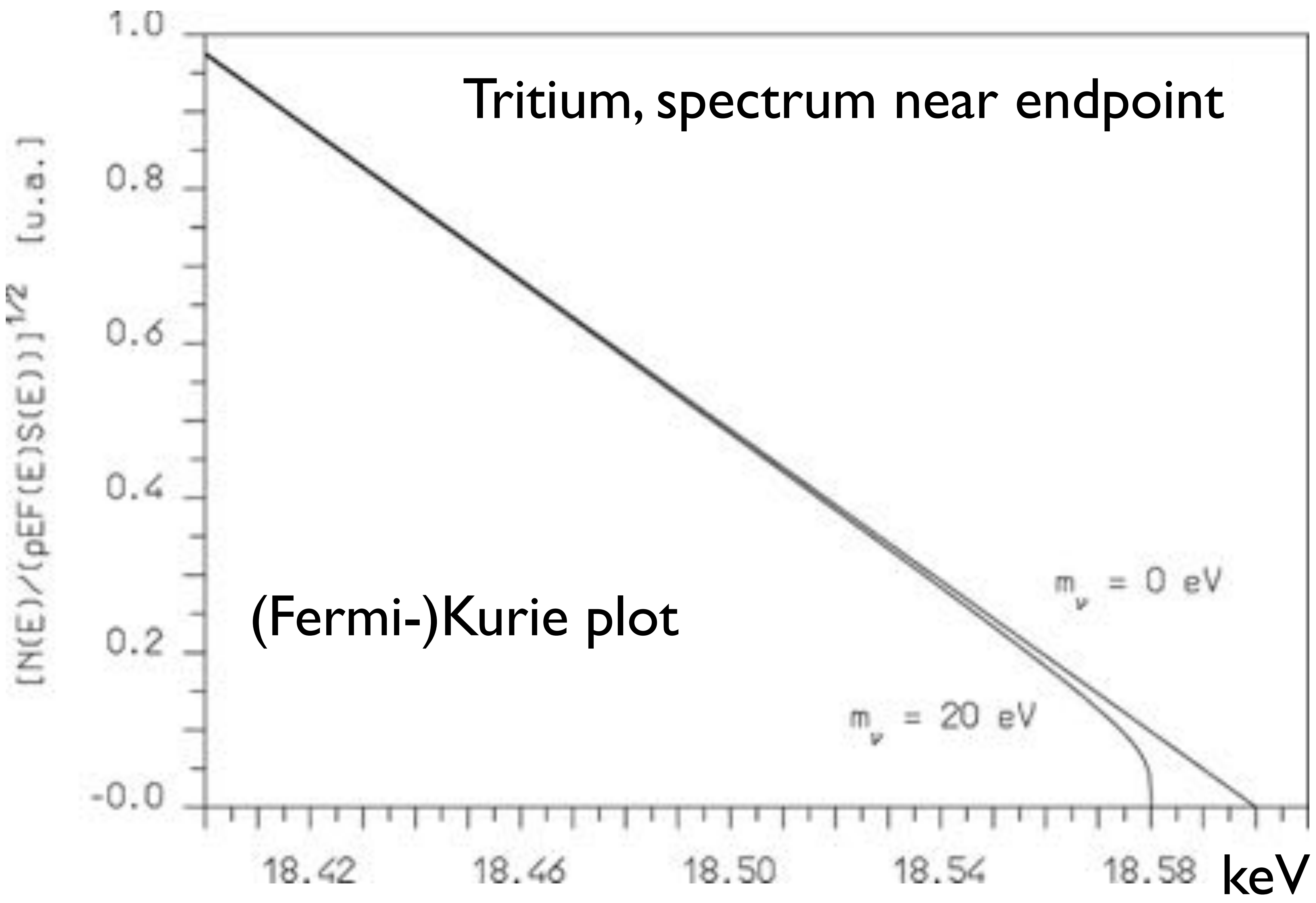

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Effective neutrino mass in β-decays

Different ν's masses (& different *X'* excitations, if any) contribute each with corresponding probabilities (incoherent sum)

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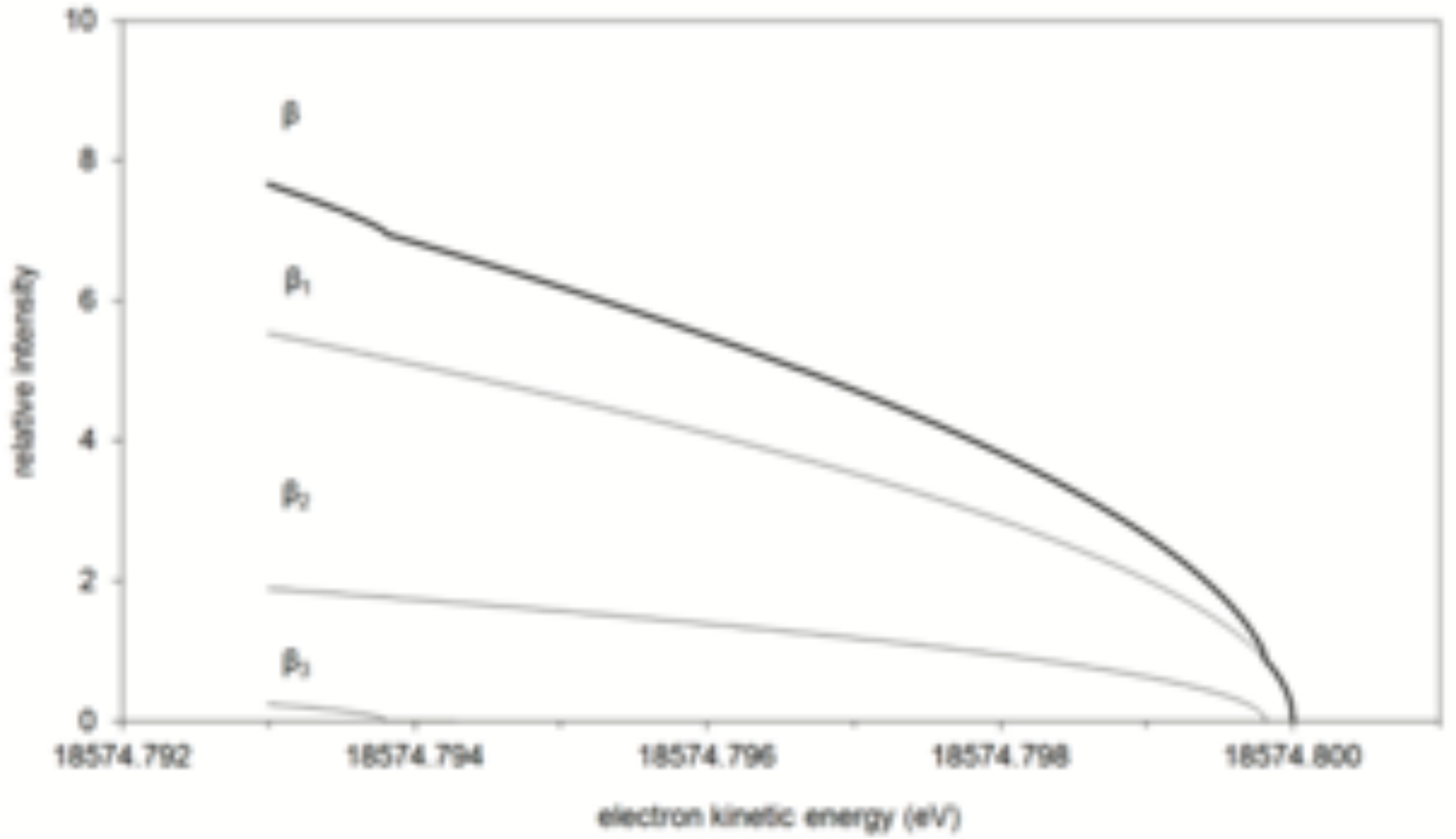

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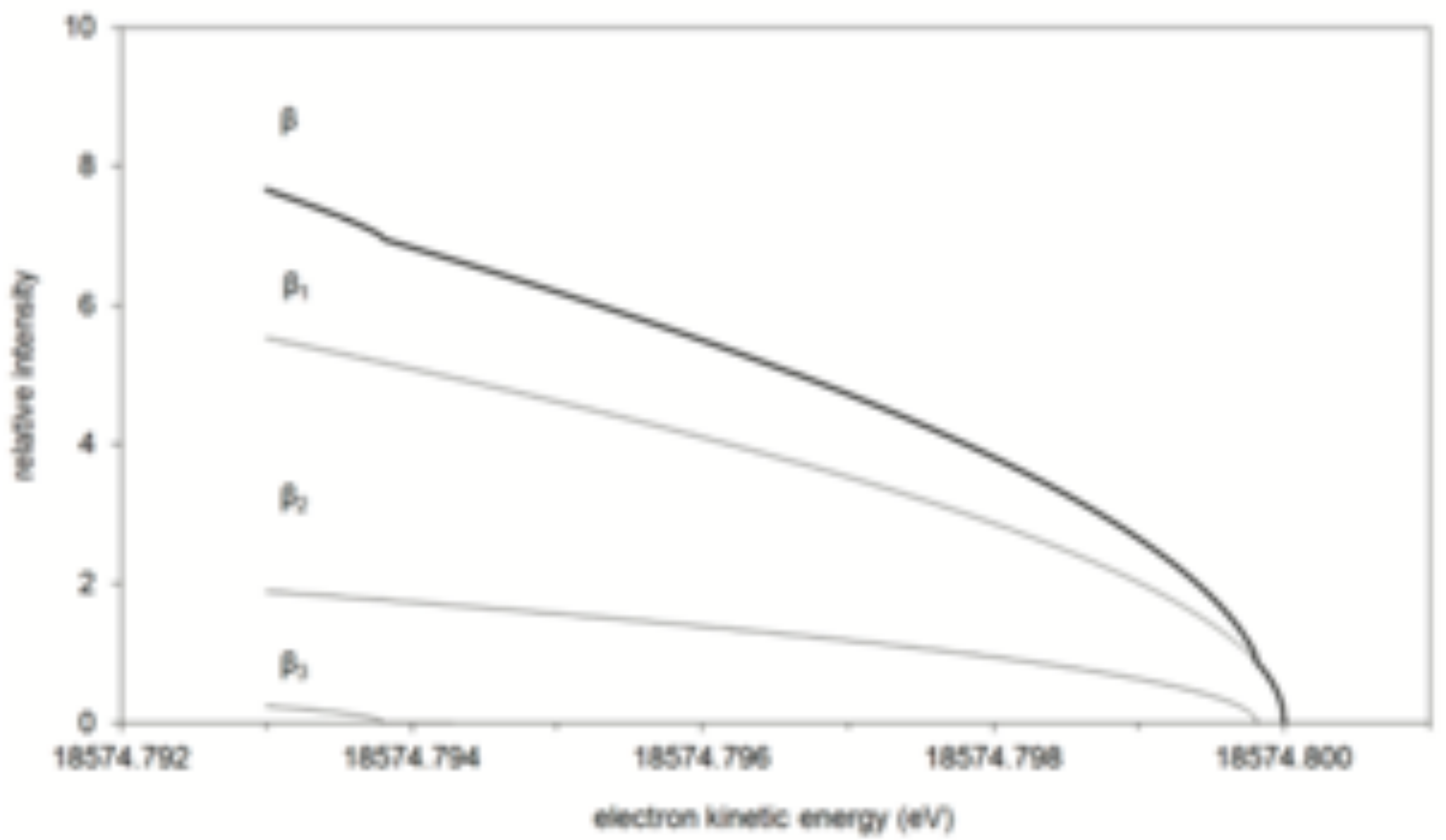

If the fine structure due to different contributions cannot be resolved, we can expand for $\varepsilon \gg m_i^2$

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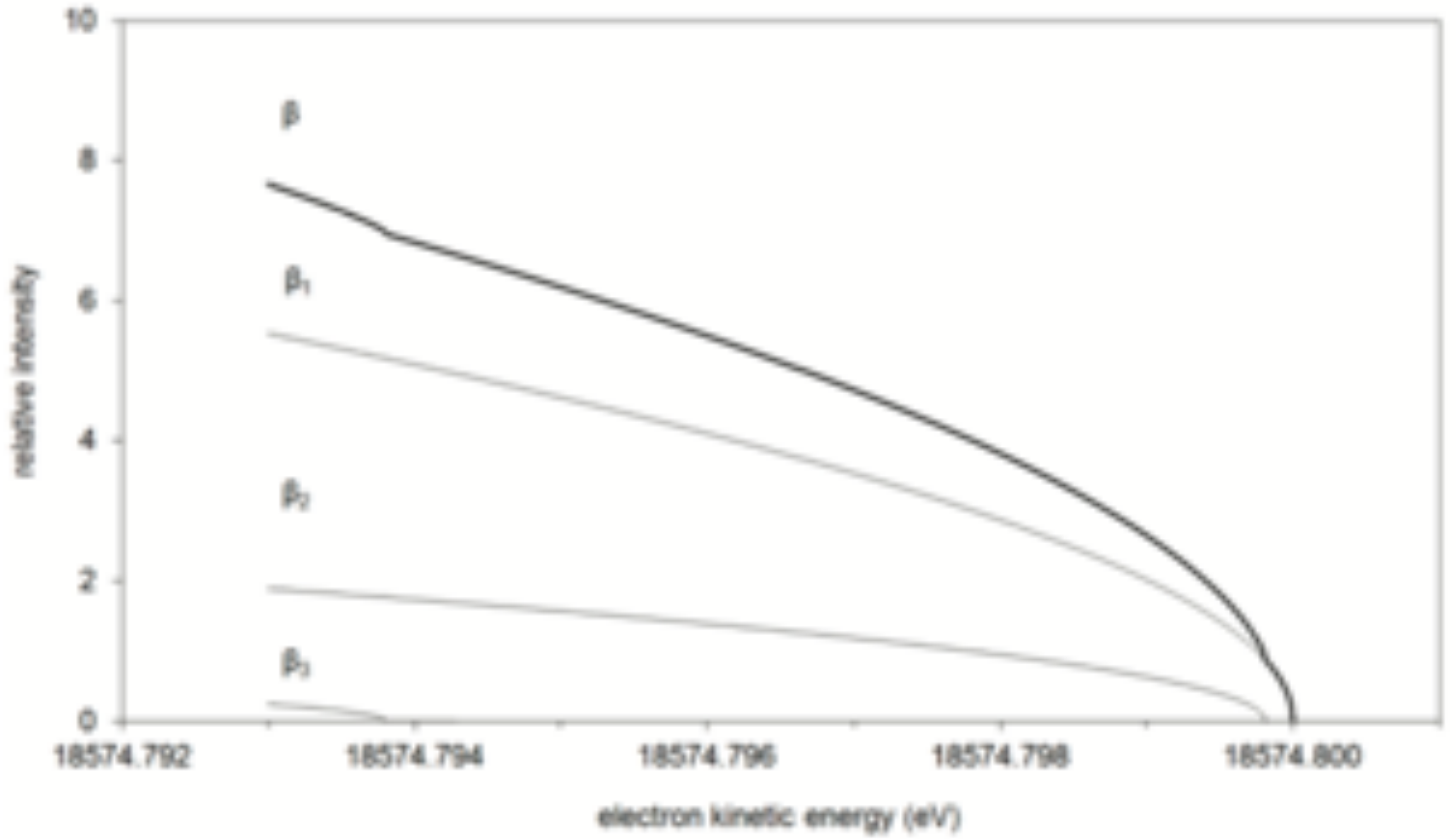

Article | Open Access | Published: 14 February 2022

Direct neutrino-mass measurement with sub-electronvolt sensitivity

The KATRIN Collaboration

Nature Physics **18**, 160–166 (2022) | Cite this article

$m_\beta < 0.8\,\mathrm{eV}$ at 90% C.L.

Are ν's their own antiparticle? The hope to figure it out with $0\nu2\beta$

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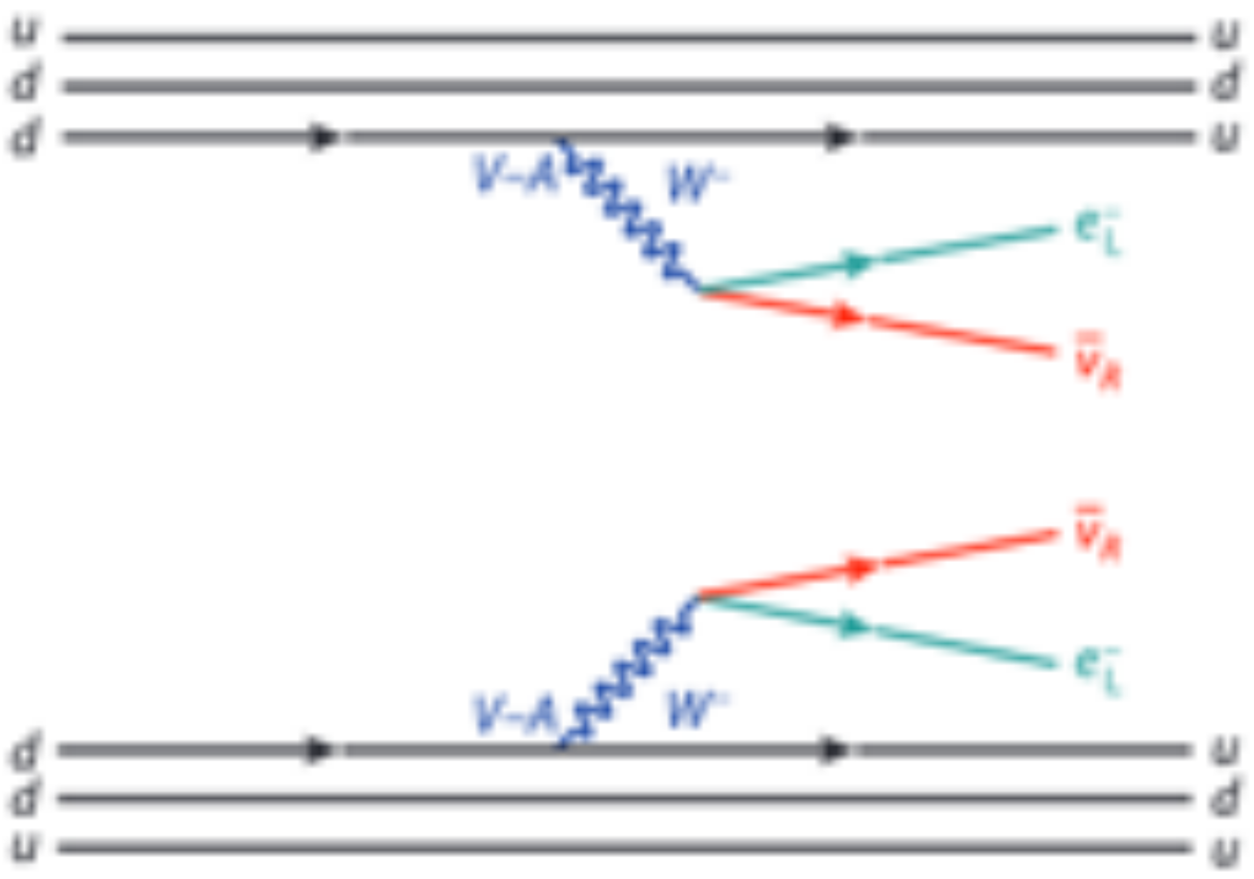
Allowed as rare weak decay in the SM, notably if the single β not energetically allowed

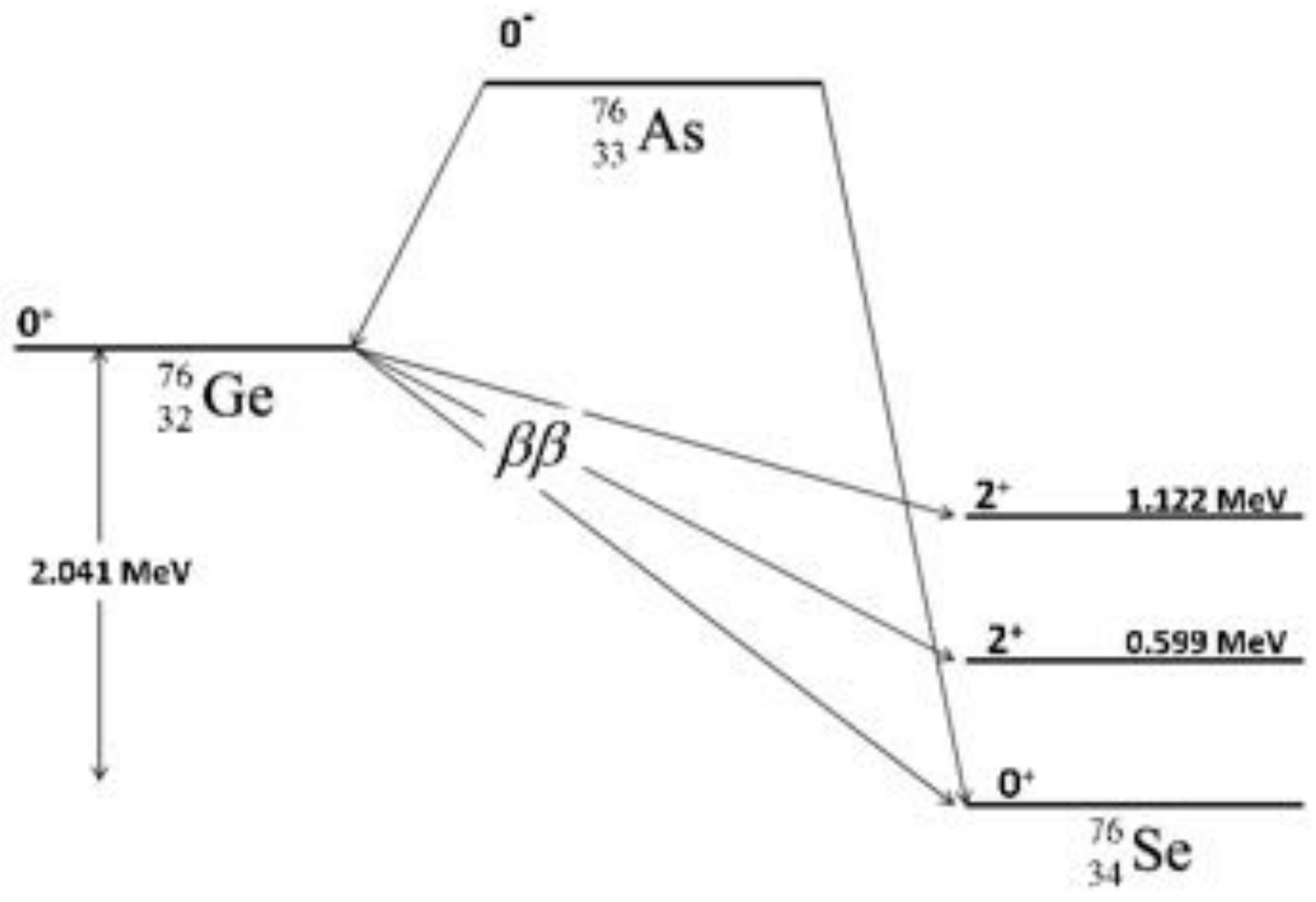
e.g. reviewed in R Saakyan 2013

Maria Goeppert Mayer

(NP 1963 for the shell model)

seminal paper on $2\nu2\beta$ in 1935





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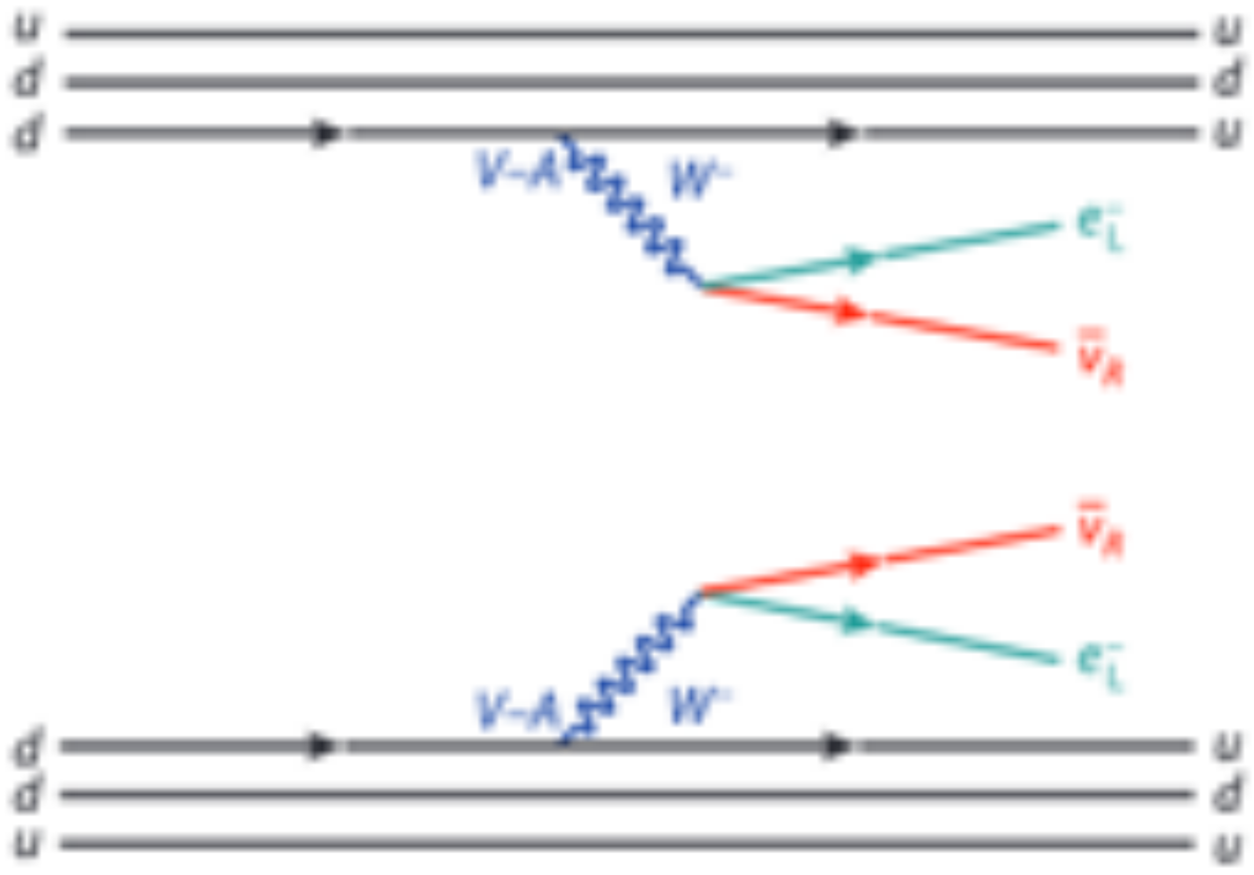

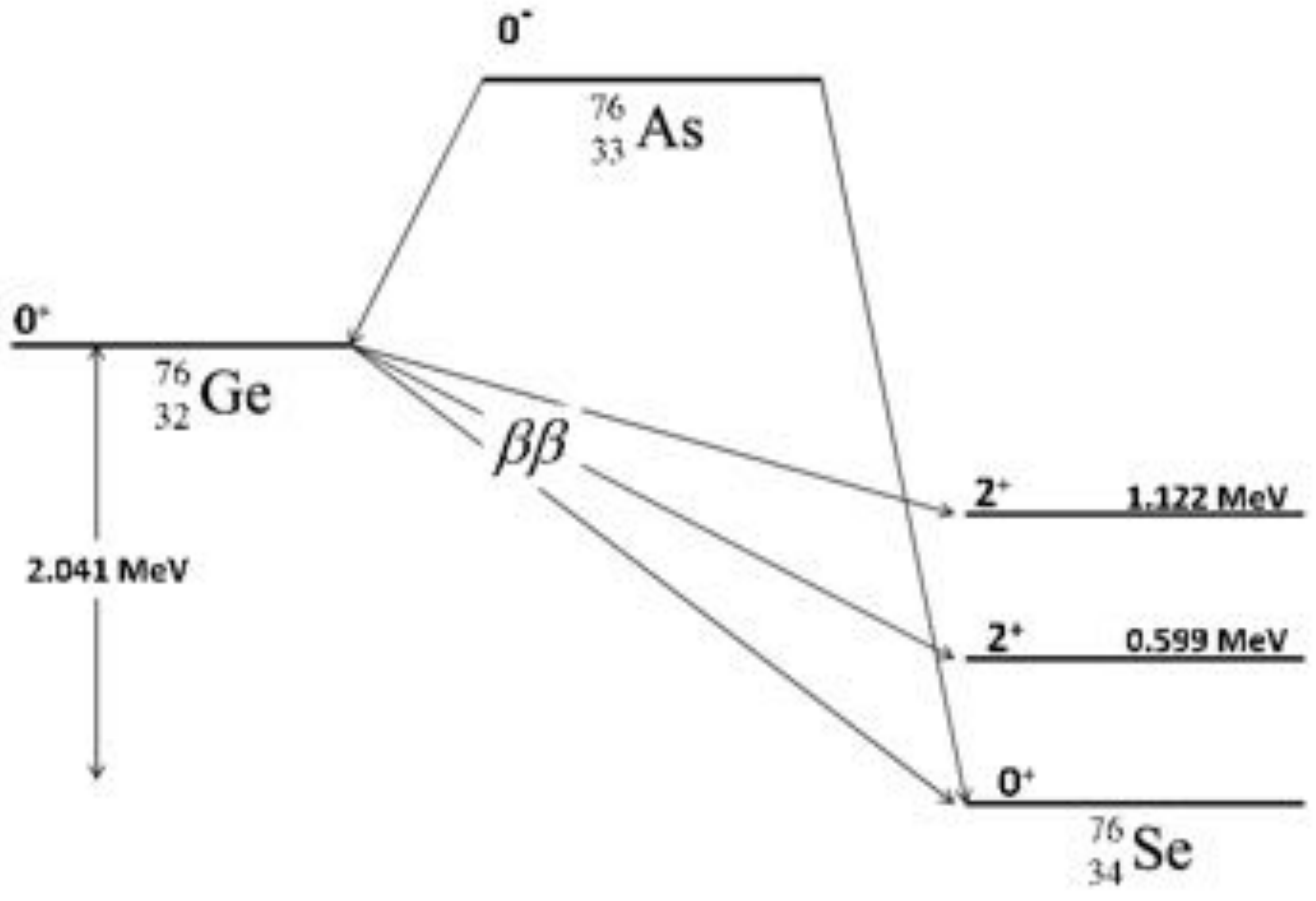

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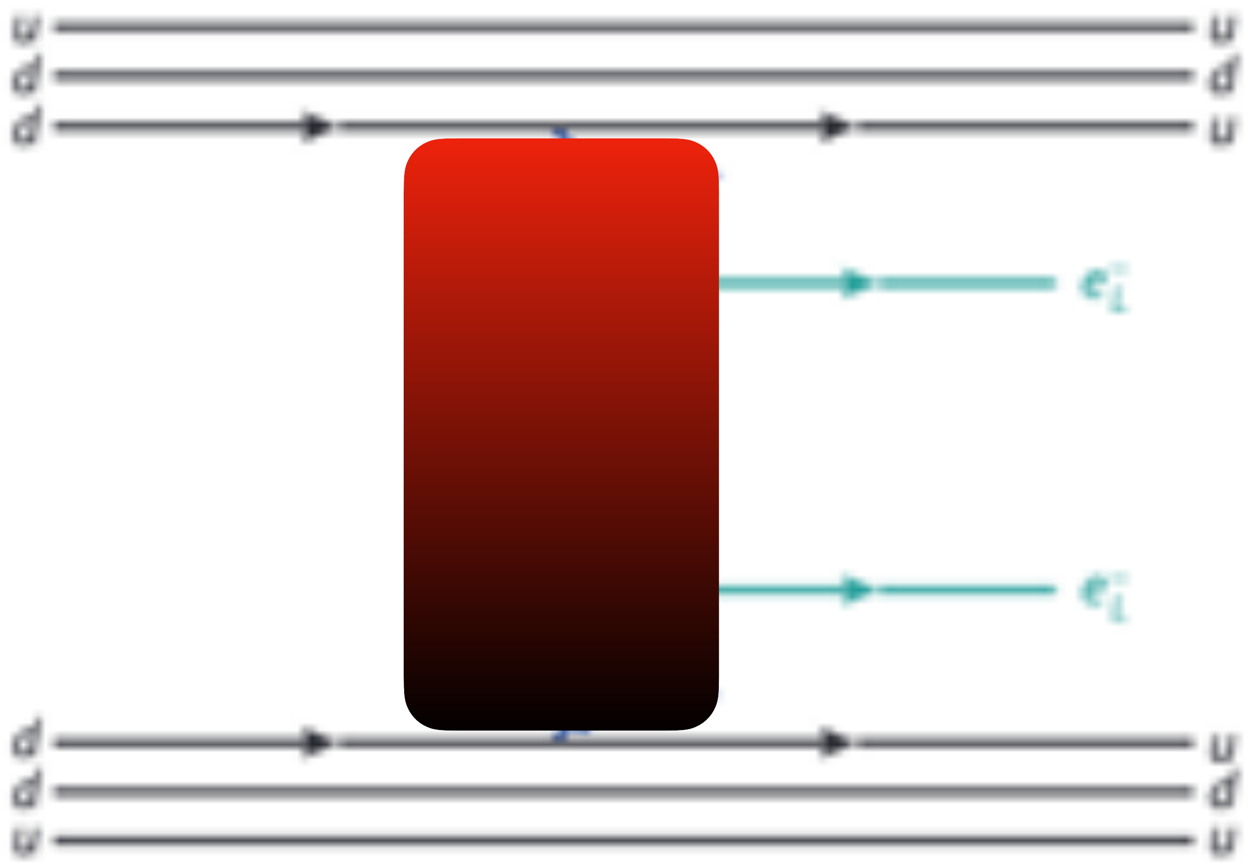

Is this mode allowed? Clearly violates L (cannot happen for Dirac ν's)

→ would imply the existence of of a Majorana mass term

Schechter and Valle PRD 25, 2951 (1982)

Neutrinos and $0\nu2\beta$

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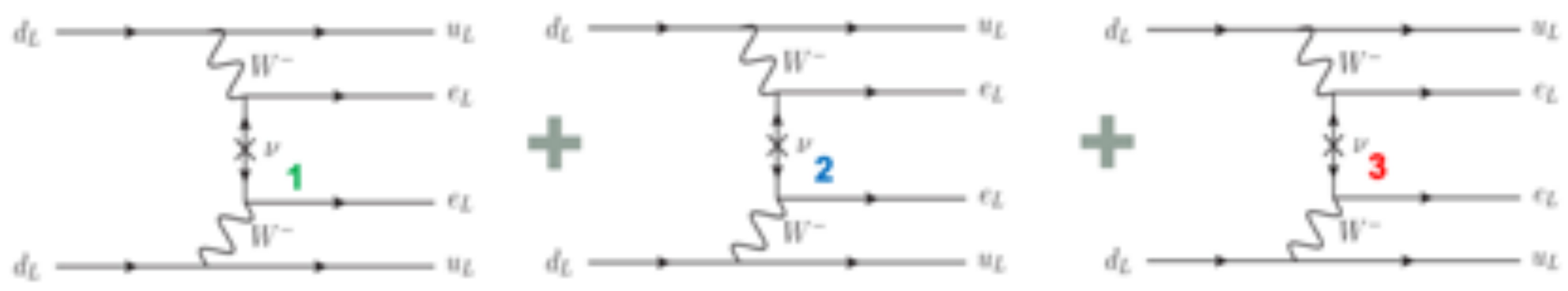
$\Gamma \propto \Big|$  $\Big|^2$

$$\propto \left| (U_{e1})^2 m_1 + (U_{e2})^2 m_2 + (U_{e3})^2 m_3 \right|^2 \equiv \left| m_{\beta\beta} \right|^2$$

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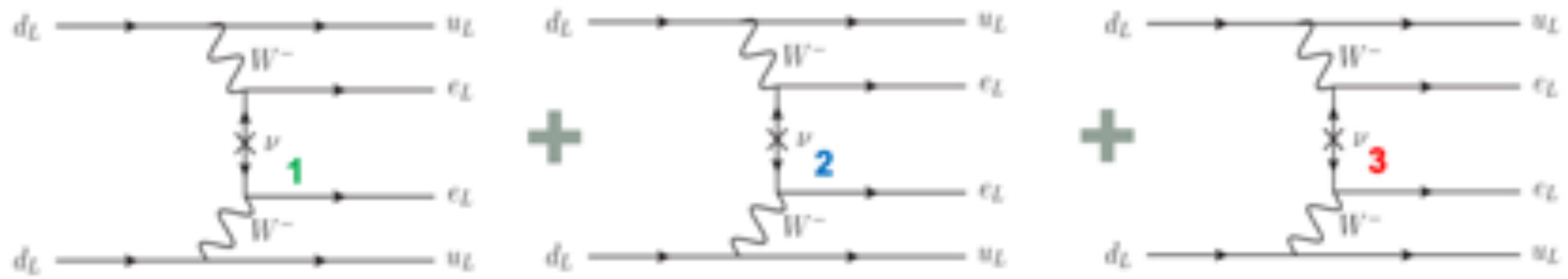
If $0\nu2\beta$ mediated by the ν mass term (there can be other contributions!), one should keep in mind that different masses enter coherently

$\Gamma \propto \Big|$



$\Big|^2$

$$\propto \left| (U_{e1})^2 m_1 + (U_{e2})^2 m_2 + (U_{e3})^2 m_3 \right|^2 \equiv |m_{\beta\beta}|^2$$

Check that Majorana phases do not disappear from this quantity!

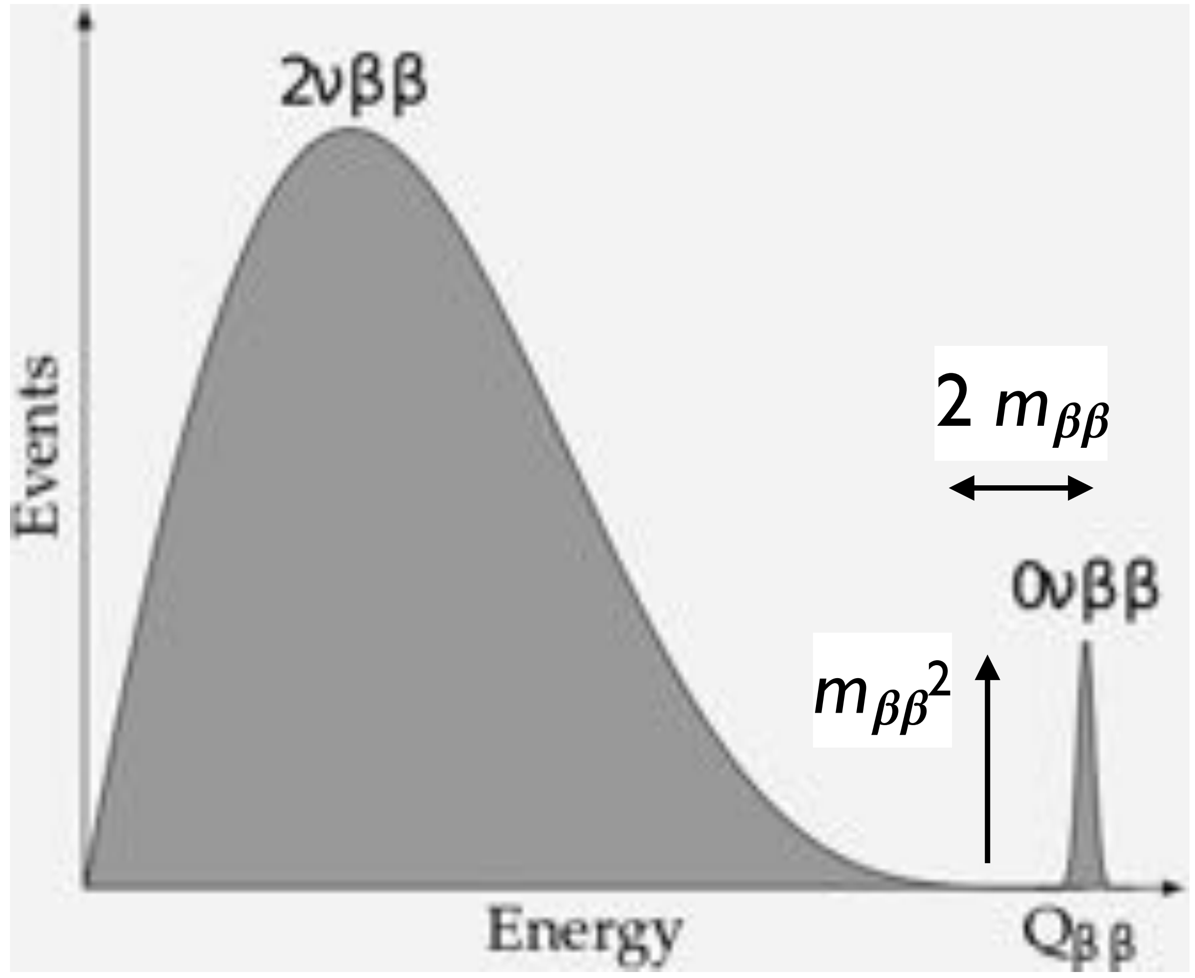
$$\mathcal{M} \propto \sum_i m_i U_{ei}^2 \equiv m_{\beta\beta}$$

$0\nu2\beta$ in the minimal scenario: Signatures and challenges

High resolution & large statistics are key!

$$\Gamma_{0\nu} = |m_{\beta\beta}|^2 |\mathcal{M}_{\text{nuc}}|^2 G_{\text{ph.sp.}}$$

Need also nuclear matrix elements to translate lifetime into ν mass



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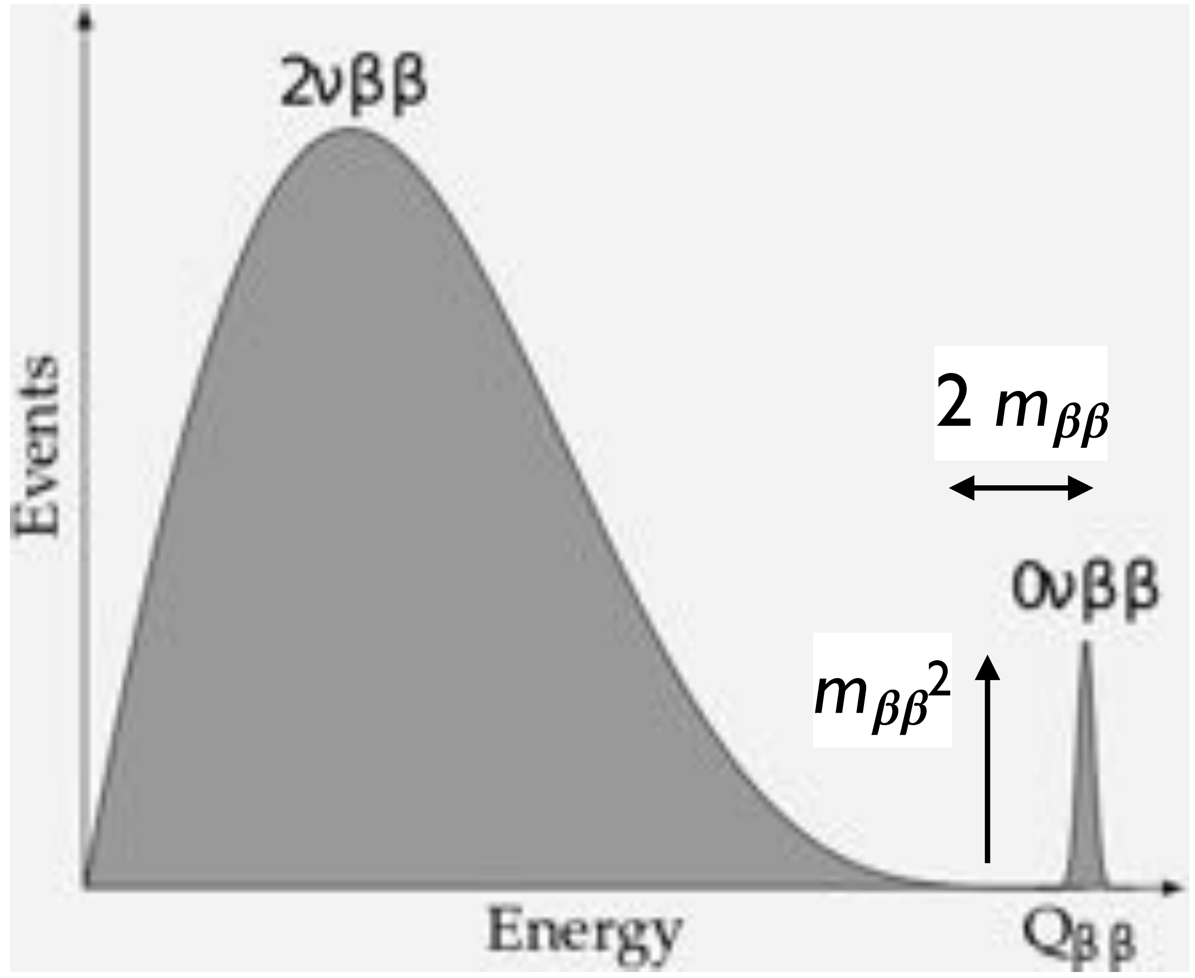

Leading limits in each isotope:

Experiment	Isotope	Exposure [kg yr]	$T^{0\nu}_{1/2}$ [10^{25} yr]	$m_{\beta\beta}$ [meV]
Gerda	^{76}Ge	127.2	18	79-180
Majorana	^{76}Ge	26	2.7	200-433
KamLAND-Zen	^{136}Xe	970	23	36-156
EXO-200	^{136}Xe	234.1	3.5	93-286
CUORE	^{130}Te	1038.4	2.2	90-305

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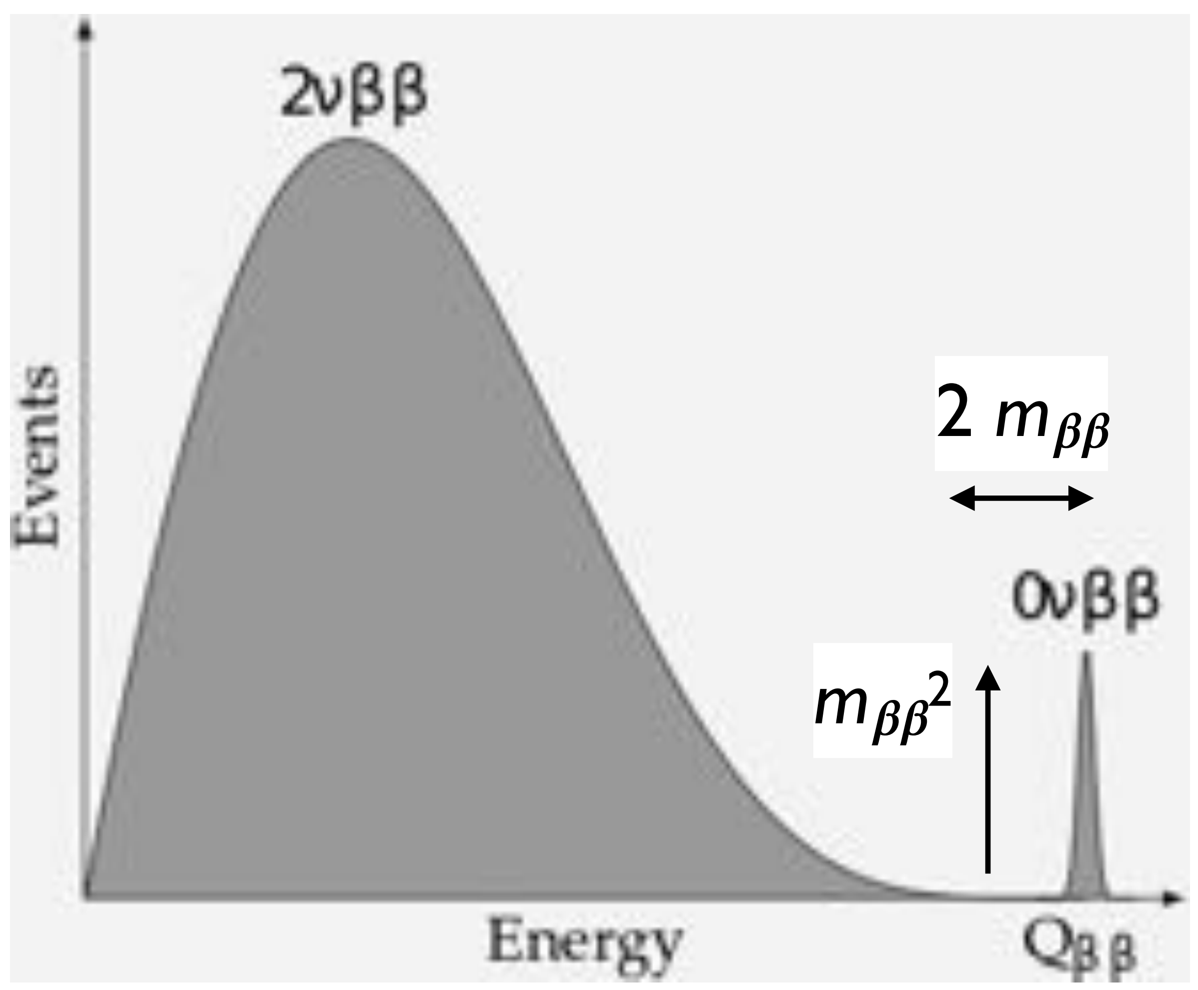

For IH, lower limit exists, currently being probed by KamLAND-Zen

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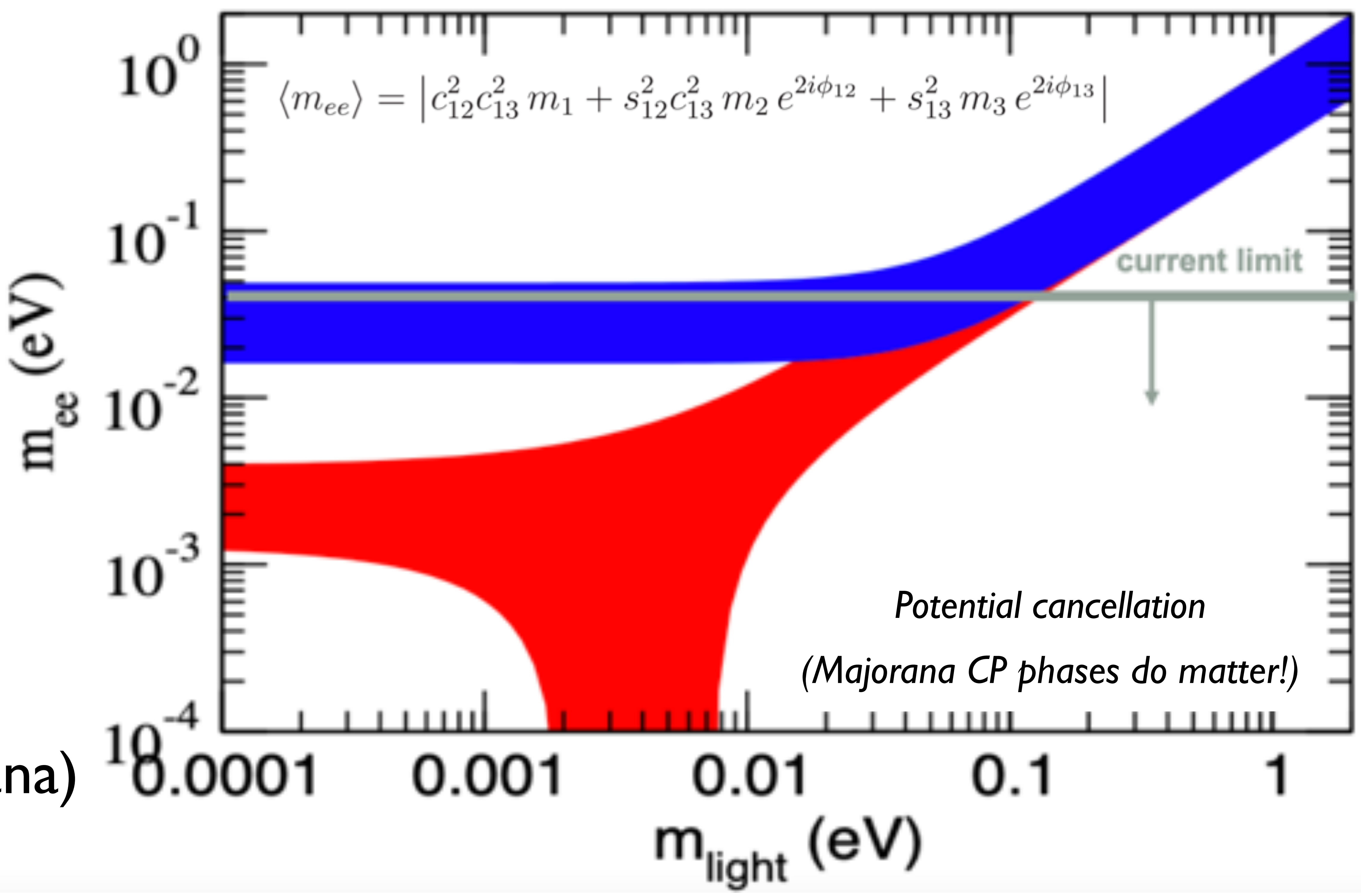
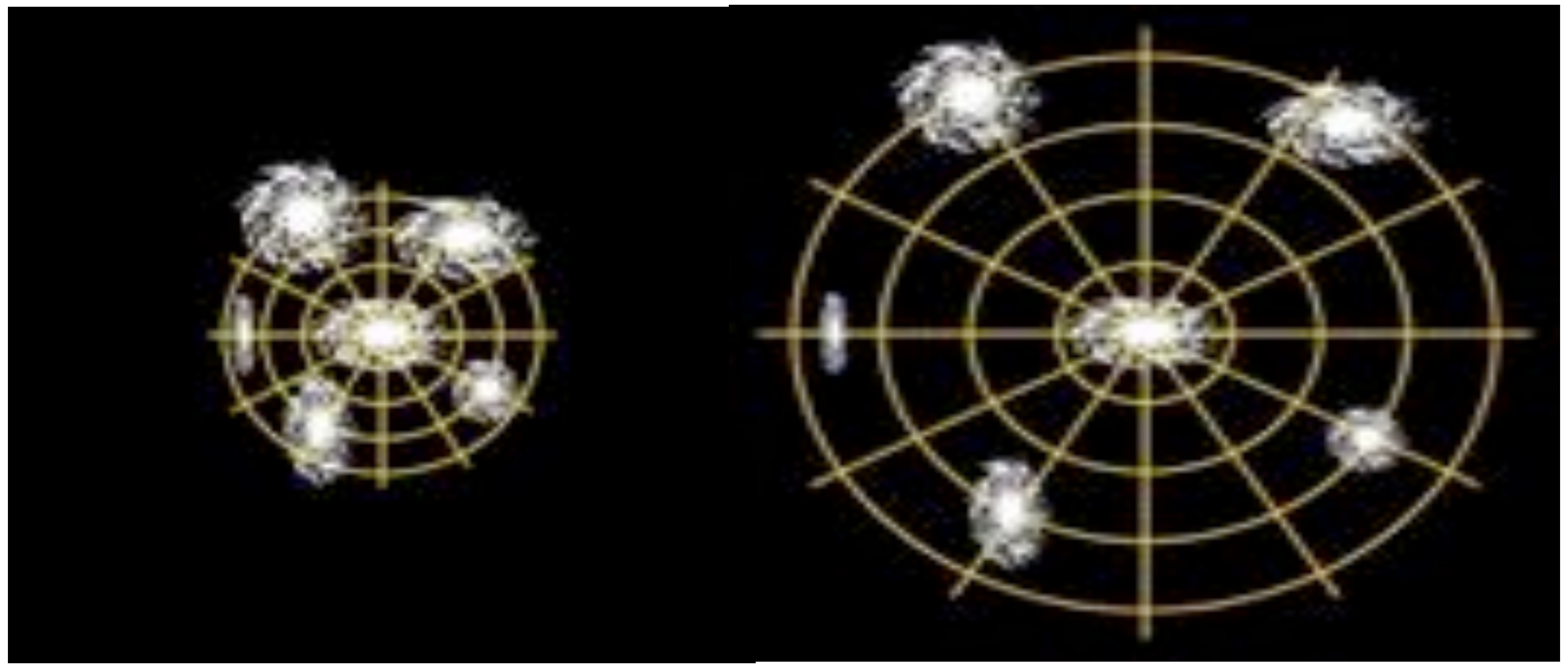


If IH, realistic path to 'guaranteed detection' (or exclude Majorana)

If NH… we need a good dose of luck!

Some notions of Cosmology (pedestrian exposition, apologies!)

- Homogeneous & isotropic solution of GR equations (used as first order proxy to describe the Universe, *Copernican principle*) leads to an ***expanding*** (or contracting) metric, with scale factor $a=a(t)$



- The expansion rate $H=a^{-1}\, da/dt$ depends on the energy content of the Universe (its acceleration further depends on the pressure, unlike in Newtonian physics)

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

In this framework, the **Hubble-Lemaître law (**Galaxies sufficiently far away from us recede with $v=H_0 d$) makes sense!

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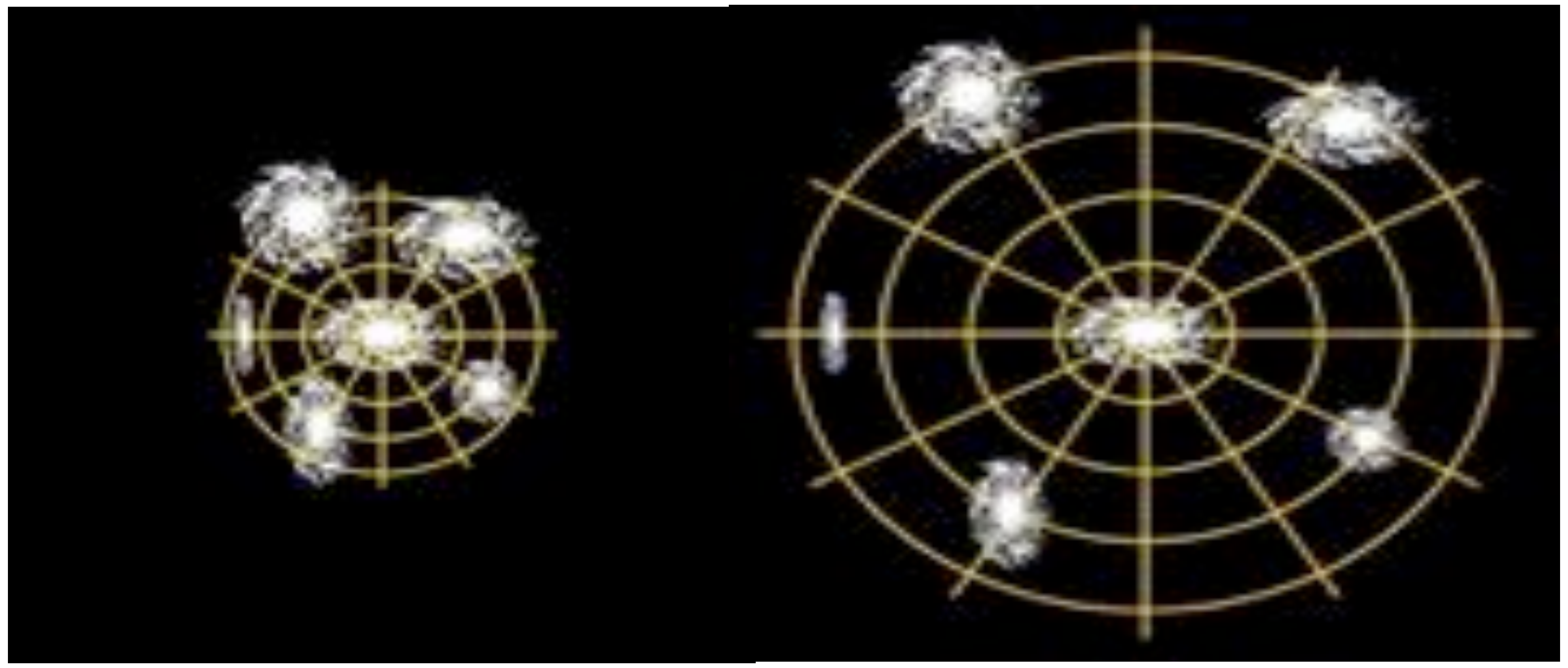

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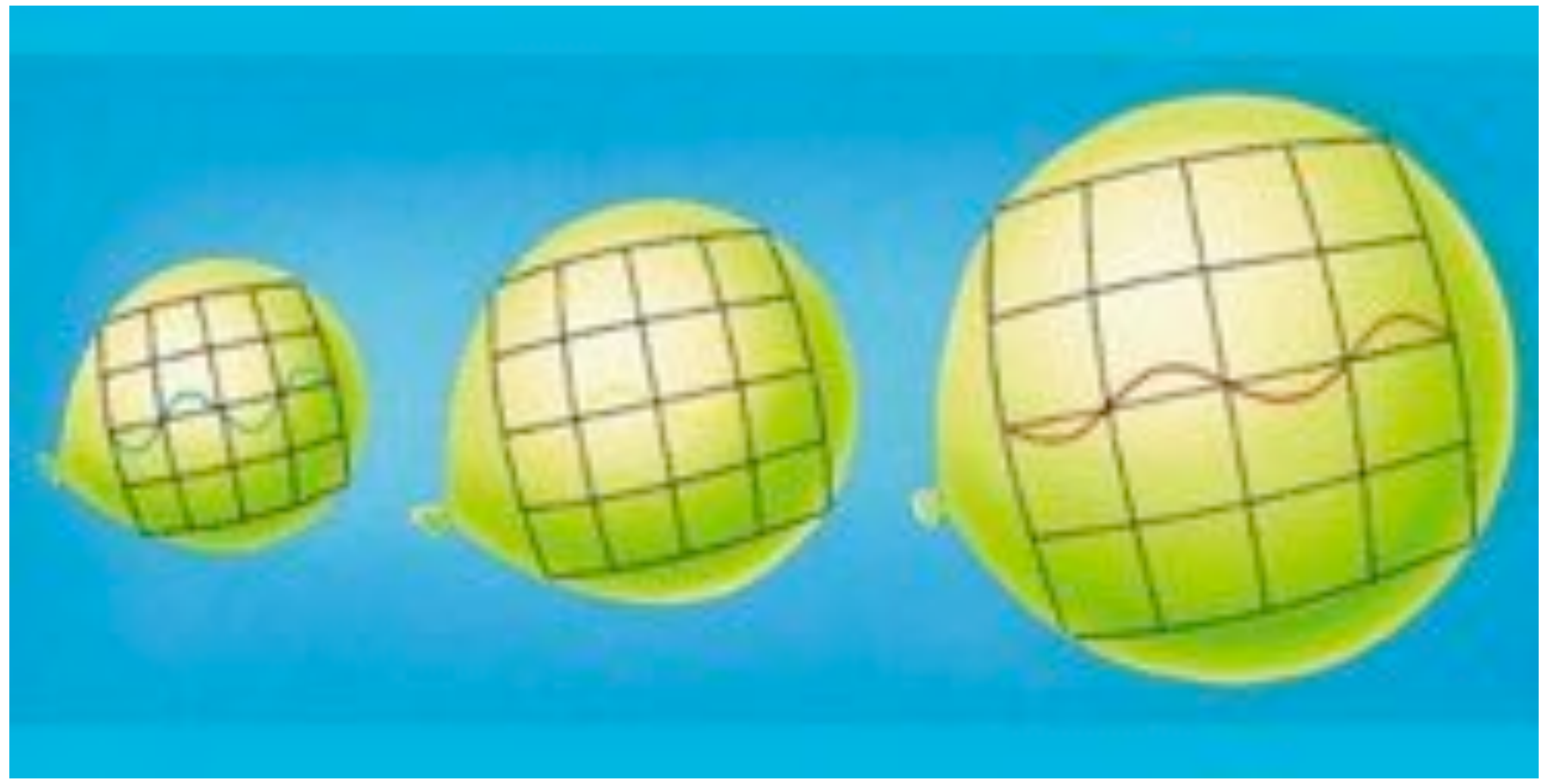
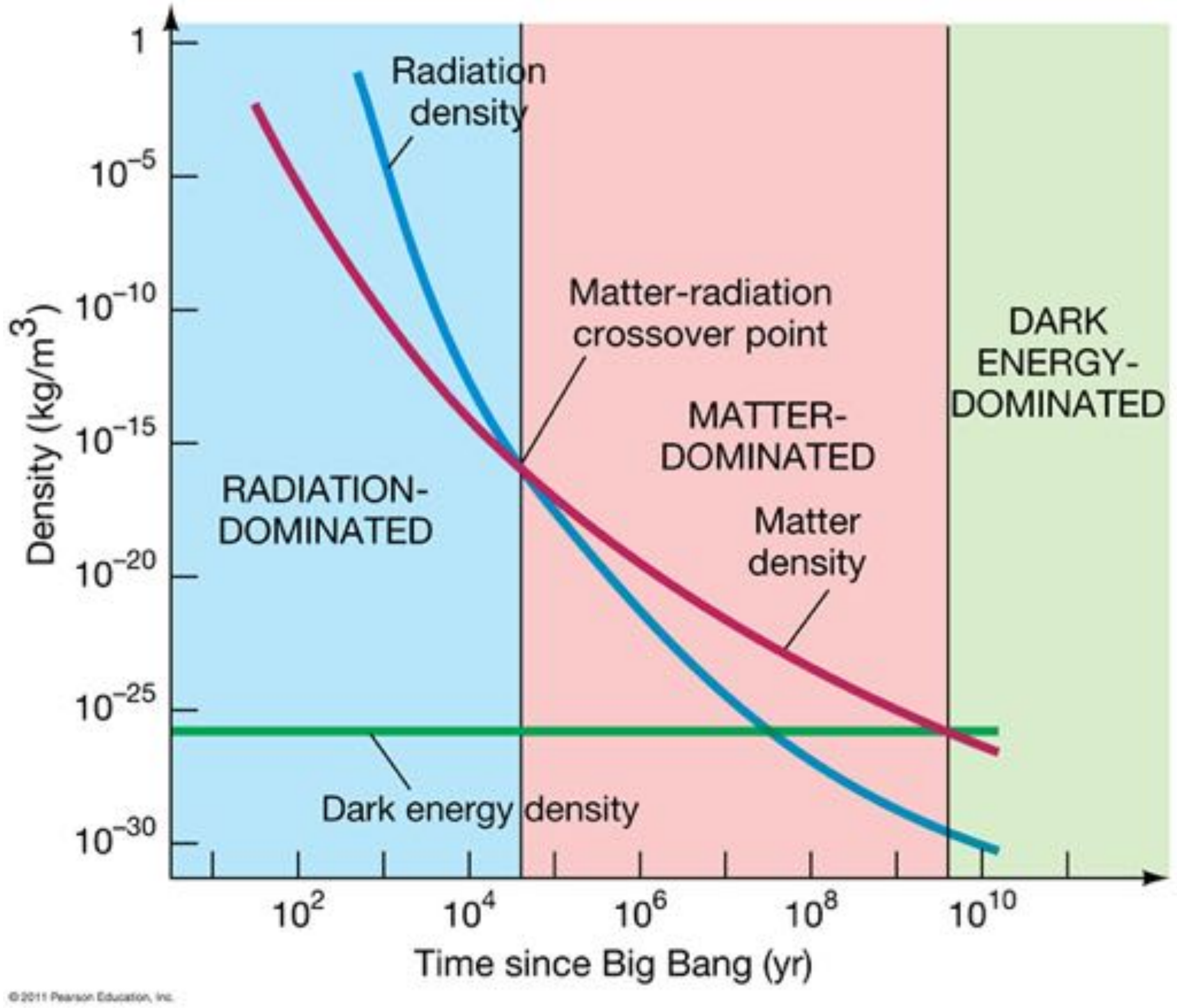
In this framework, the **Hubble-Lemaître law** (Galaxies sufficiently far away from us recede with $v=H_0 d$) makes sense!

- Not only density* higher in the early universe, but radiation wavelength contracted: More energetic! Early universe denser & hotter (eventually a plasma, $E\sim T$) & dominated by relativistic species; even weak interaction at equilibrium when T>few MeV !



**of stuff in 'free fall', going with the expansion. Not of structures decoupled from that*



“Brooklyn is not expanding!” (cit.)

From “Annie Hall”, by Woody Allen, 1977 (@ Youtube)

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Neutrinos & Cosmology

- ν's produced (# comparable with photons!) e.g. via $e^+e^- \longleftrightarrow \nu$ anti-ν & attain a FD distribution.

$$f_\nu(p) \simeq \frac{1}{e^{p/T_\nu}+1}$$

- With expansion & cooling below T ~few MeV ν decouple and 'freeze-out': number drops as a^{-3}, average momentum redshifts as a^{-1} (1 eV ~ 10^4 K)

$$H \simeq \sqrt{G_N}T^2 \qquad \Gamma_{\rm eq} = n_{\rm eq}\langle\sigma v\rangle \sim G_F^2 T^5 \quad \langle\sigma v\rangle = \sigma_{\rm weak} \sim G_F^2 E^2 \sim G_F^2 T^2 \qquad \frac{\Gamma_{\rm eq}}{H} \sim \left(\frac{T}{\rm MeV}\right)^3$$

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Key pheno consequences

- *Slightly colder than CMB: ν's decouple before $e^\pm$ annihilations*

$$\frac{T_\nu}{T_\gamma} \simeq \left(\frac{4}{11}\right)^{1/3}$$

- *Abundance*

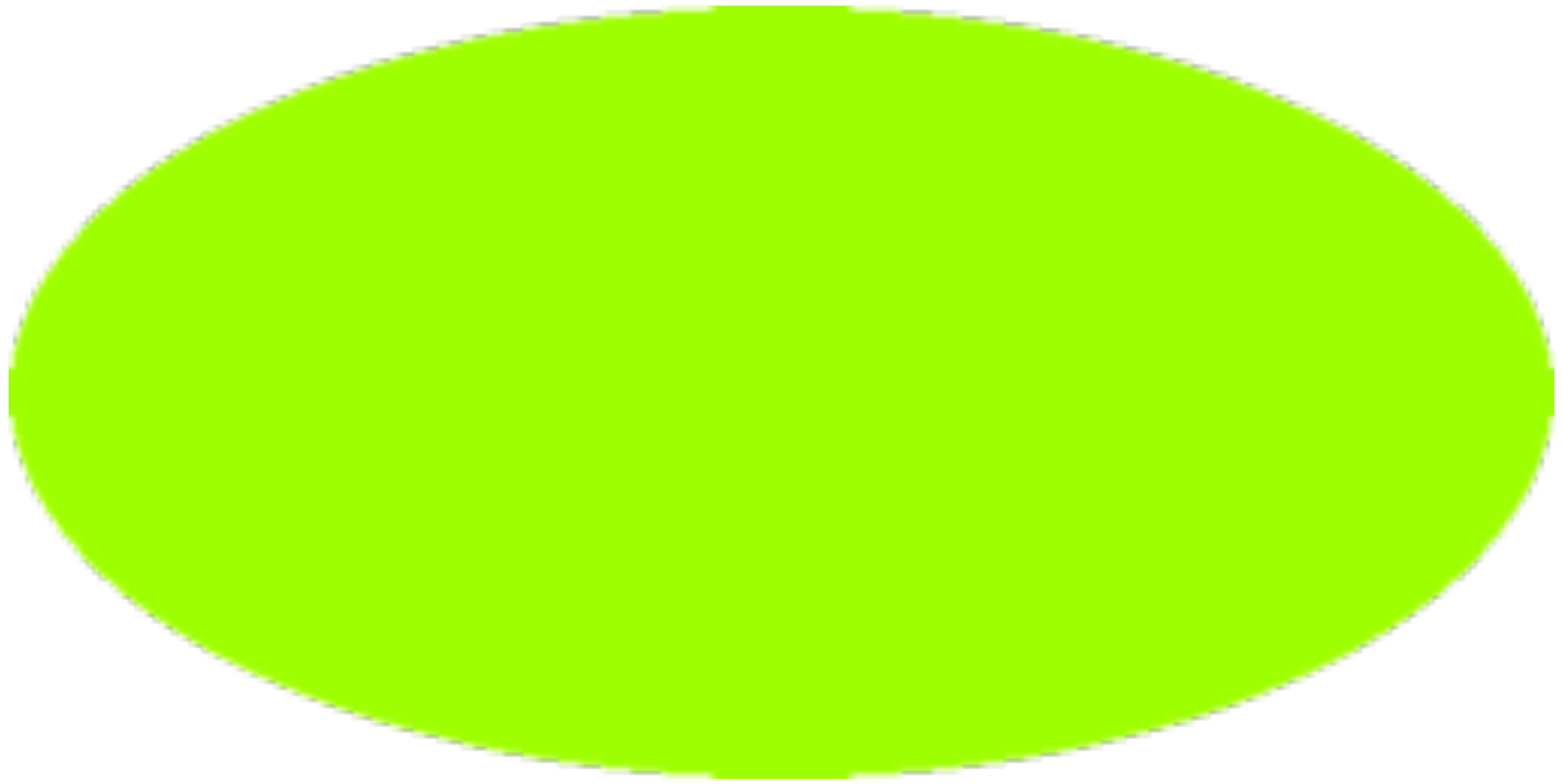
$$n = g\int f_\nu(p)\frac{d^3\mathbf{p}}{(2\pi)^3} = \frac{g}{2\pi^2}\frac{3\zeta(3)}{2}T_\nu^3 \to 110\,\mathrm{cm}^{-3}\ \text{today, per flavour}$$

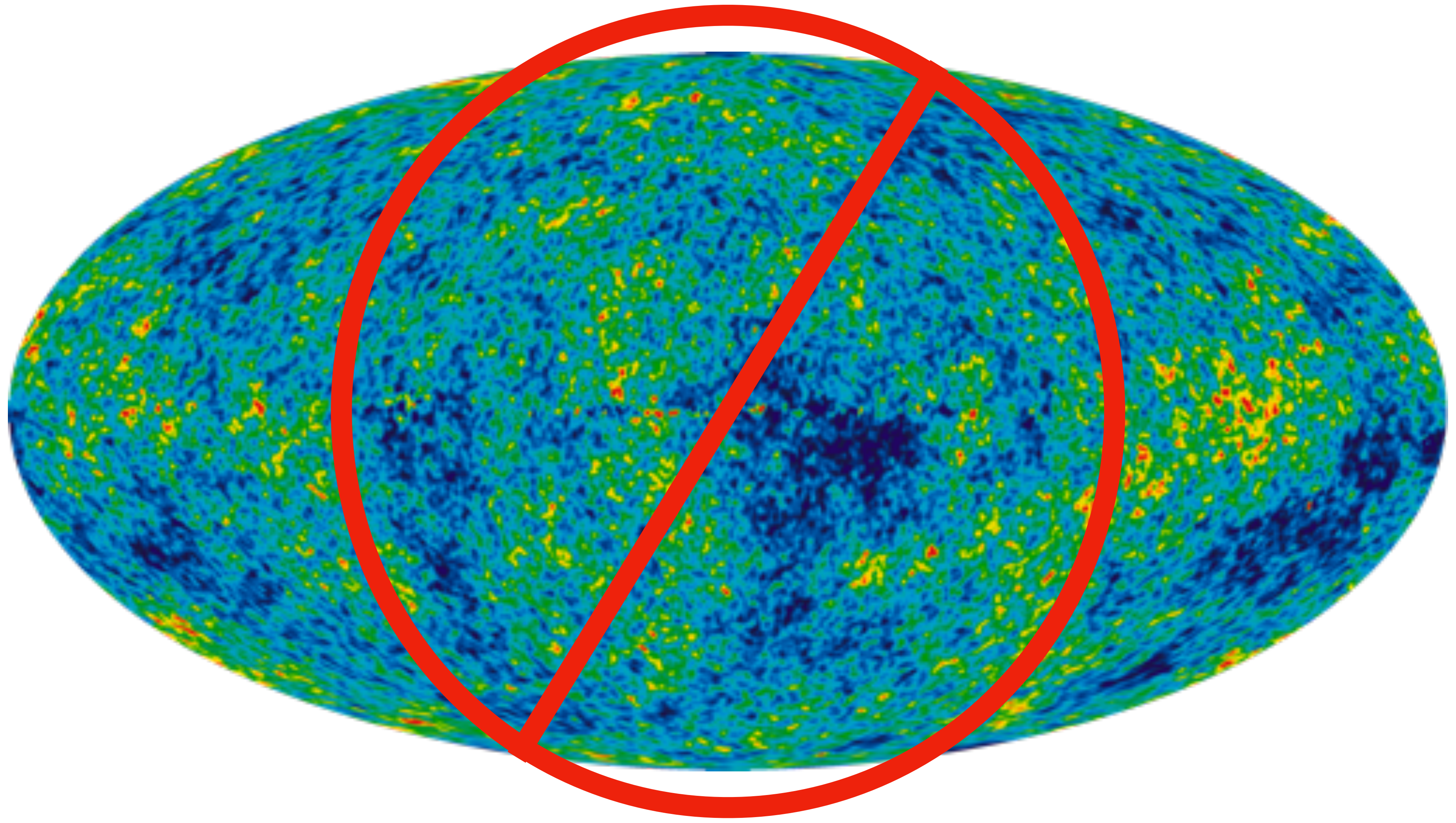
- *Energy density*

$$\rho = \sum_{i=1}^{3}\int [f_{\nu_i}(p)+f_{\bar\nu_i}(p)]\sqrt{m_i^2+p^2}\frac{d^3\mathbf{p}}{(2\pi)^3} \begin{cases} \equiv \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3}\frac{T^4}{15}N_{\rm eff} \ (T_\nu \gg m_i) \\ \simeq \sum m_i n_\nu \ (T_\nu \ll m_i) \end{cases}$$

Neutrinos in the early universe

- Very close to isotropic and homogeneous (think of the **tiny** anisotropies in the CMB!); relativistic ν *E*-density contributes to the ***expansion of the Universe via H***. Parameterised via N_{eff}~3.





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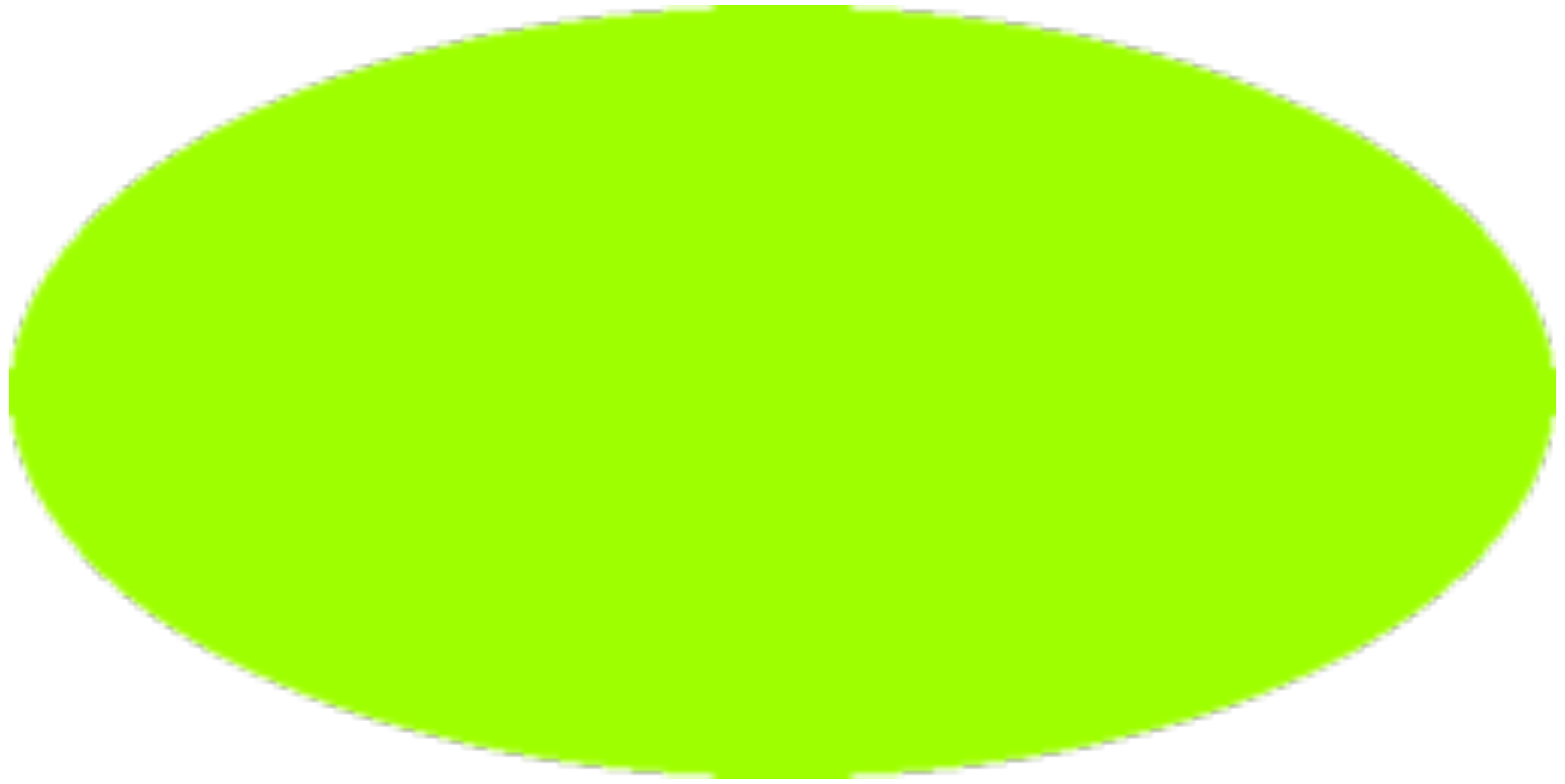
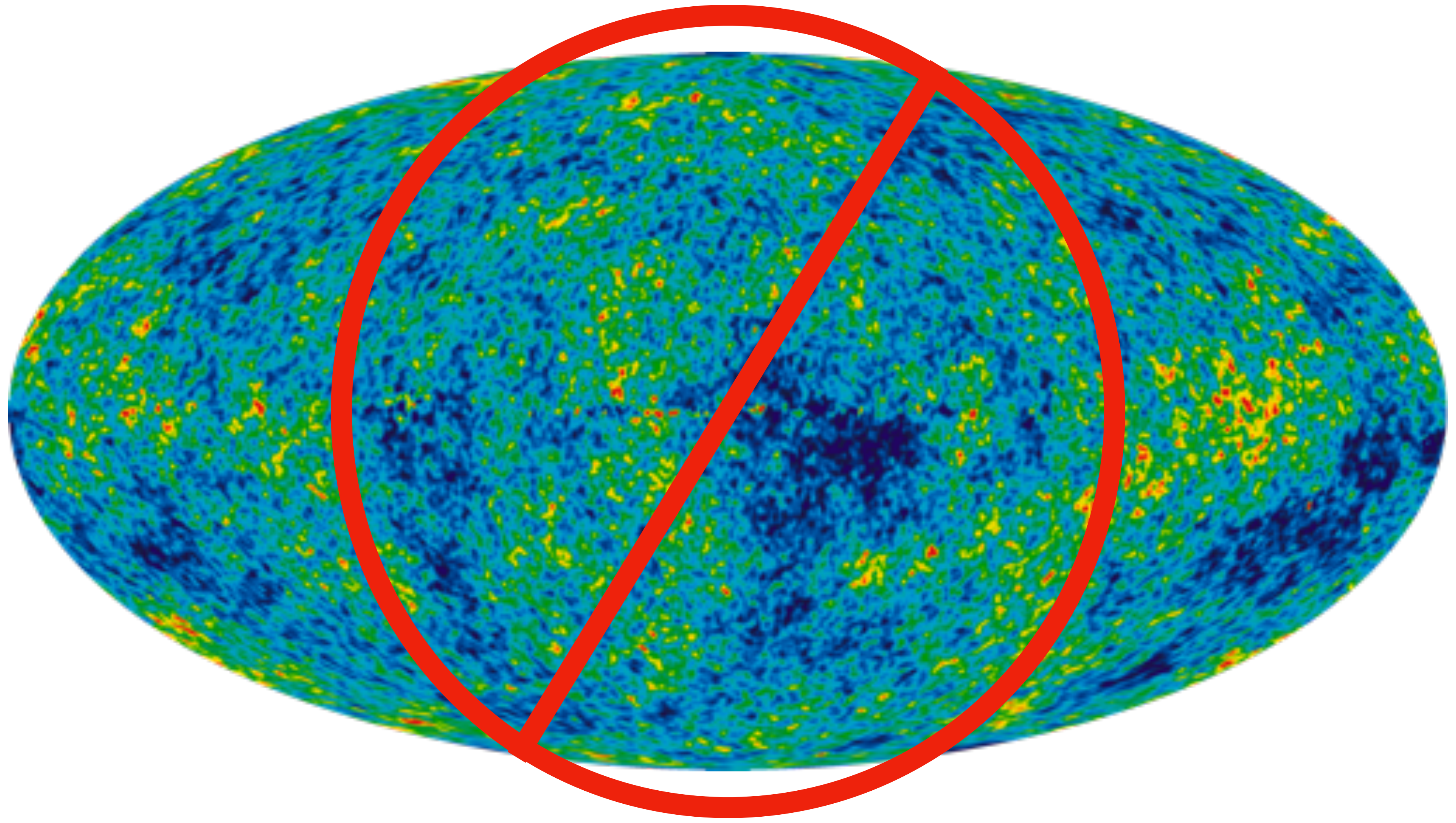




Gravitational effect, but…*

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

Statistics Flavour, NP…

$$\rho_R = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]$$

$0.1\ m_e << T_{dec} << 0.1\ m_\mu$

N_{eff} also sensitive to new dof's e.g. coupled to ν's

N_{eff} = 2.99 ± 0.34 (95% C.L.) Planck 2018 + BAO

N_{eff} = 2.88 ± 0.54 (95% C.L.) BBN; Pitrou et al. 1801.08023

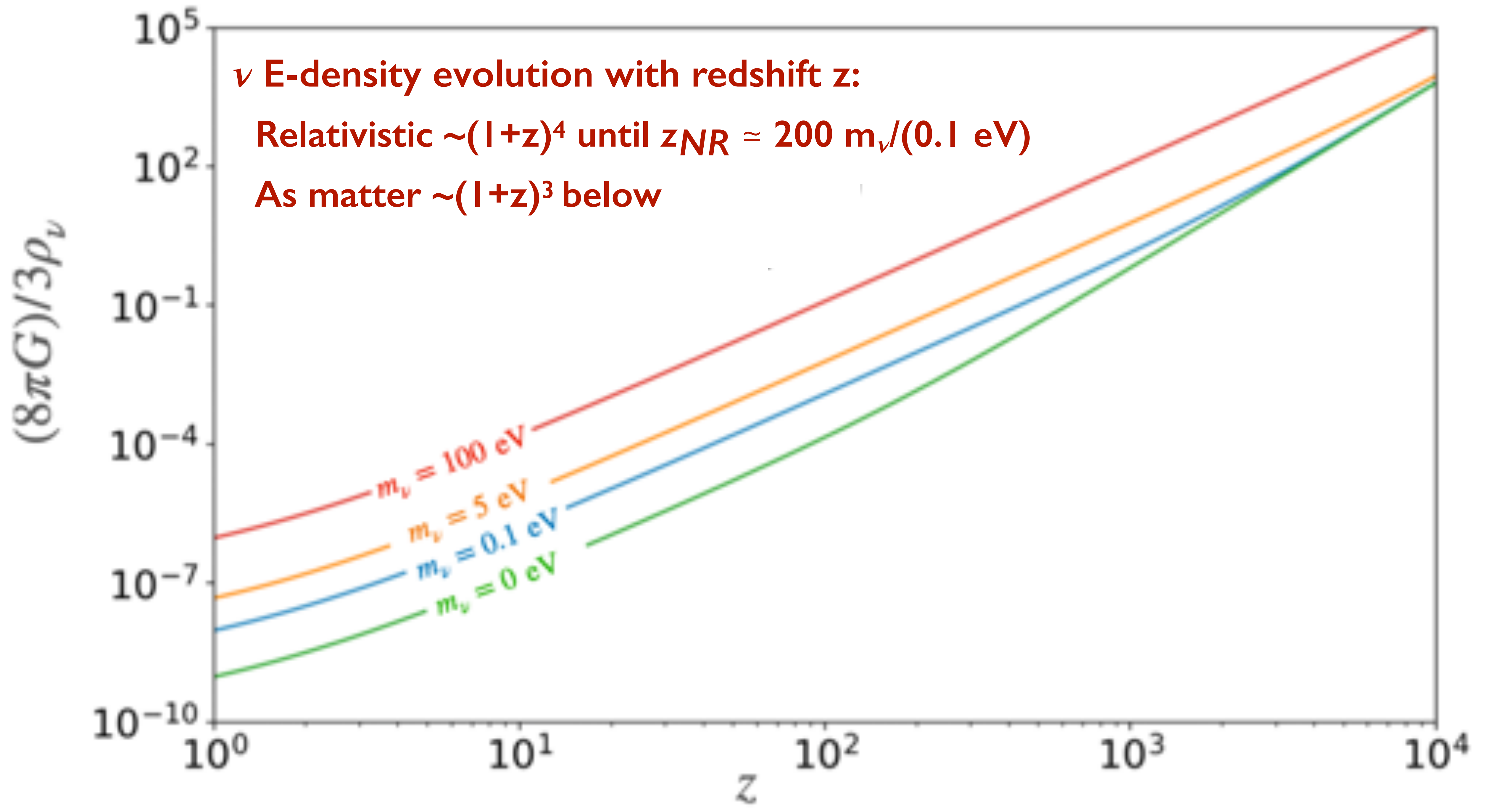
**BBN is also affected by (anti-)ν_e distributions via p-n (departure from) equilibrium*

Neutrinos in the 'late' universe

- In the late universe:

a) ν *E*-density influenced by their mass

$$\rho \simeq \sum m_i n_\nu \; (T_\nu \ll m_i)$$



Neutrinos in the 'late' universe

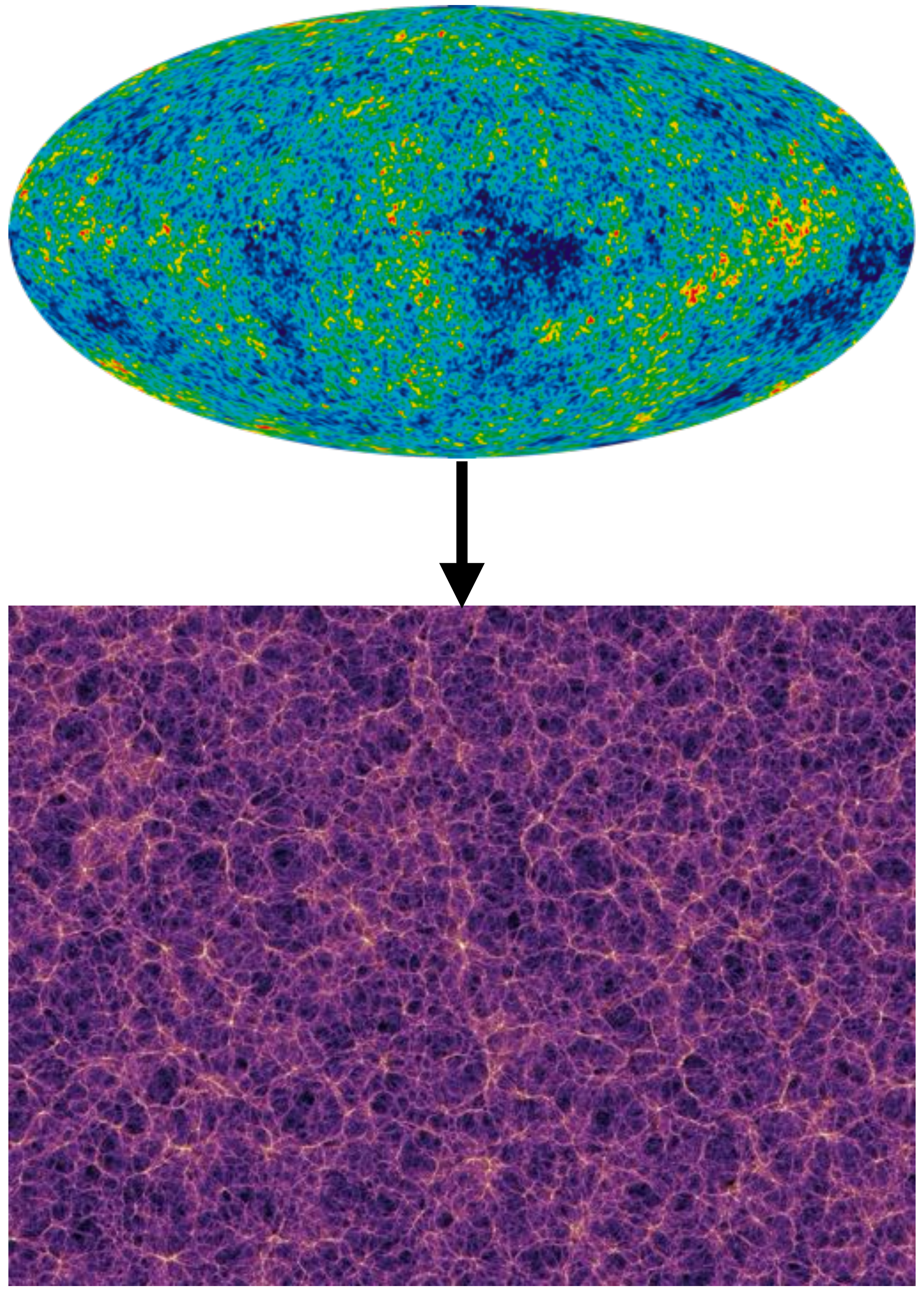
- In the late universe:

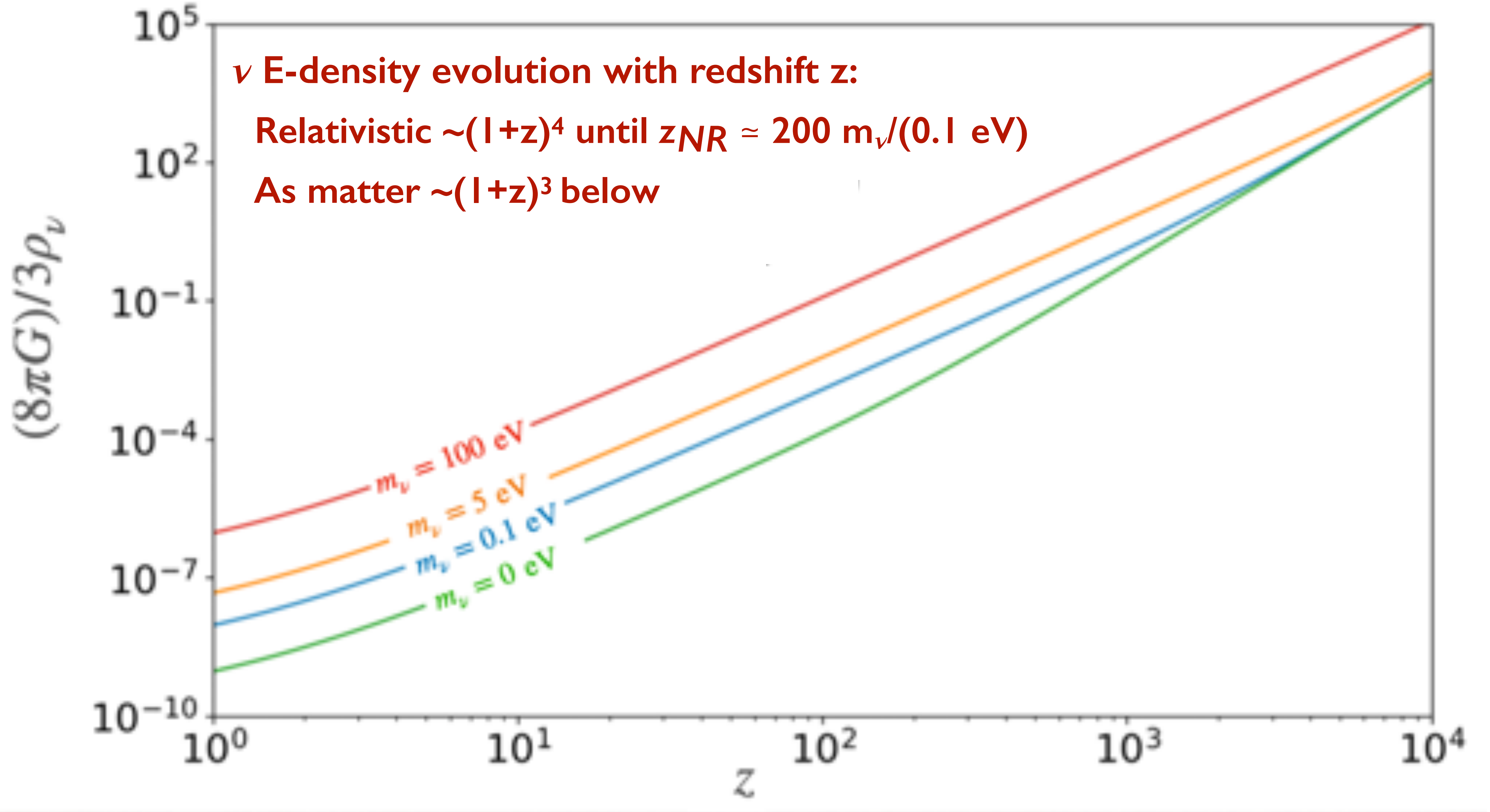
a) ν *E*-density influenced by their mass $\qquad \rho \simeq \sum m_i n_\nu \; (T_\nu \ll m_i)$

b) Formation of structures; $\qquad \delta G_{\mu\nu} = 8\pi G_N \delta T_{\mu\nu}$

since ν's have large velocities than typical (both baryonic & dark) matter, they oppose small structure forming.

From the pattern and growth of perturbations, we can constrain (the total) ν mass+exotic interactions (drag, decay…)





Neutrinos & structure growth, some key formulae

For non-relativistic pressureless particles: 2 degrees of freedom describe perturbations $\delta \equiv \delta\rho/\rho, \quad \phi$

Continuity eq. $$\delta'' + \frac{a'}{a}\delta' = -k^2\phi$$

Poisson eq. $$k^2\phi = -4\pi G_N a^2 \rho \sum \delta_i$$

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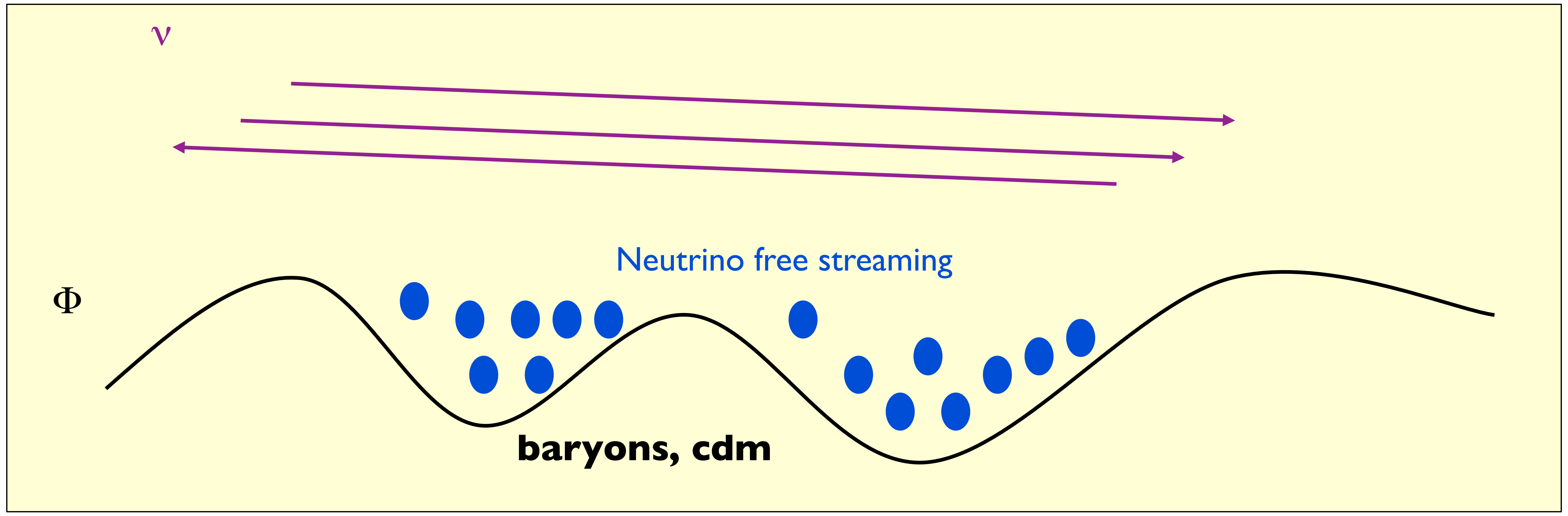
Continuity eq. $\delta'' + \frac{a'}{a}\delta' = -k^2\phi$ *Poisson eq.* $k^2\phi = -4\pi G_N a^2 \rho \sum \delta_i$

ν's 'free stream' (decoupled with *large* velocity dispersion)

$$\delta'' + \frac{a'}{a}\delta' + (k^2 - k_J^2)c_s^2\delta = -k^2\phi \qquad c_s \simeq \frac{3.15T_\nu}{m_\nu} \qquad k_J^2 = \frac{3a'}{a\,c_s^2}$$

ν's "do not settle" in potential wells that they can overcome by their typical velocity: compared with CDM, they *suppress power at small-scales (perturbations oscillate, do not grow exponentially)*



Can erase the 'free-streaming' feature with (very!) large secret self-coupling~10^{10} G_F: strongly disfavoured even for a single species.

e.g. *Schöneberg et al. 2107.10291*

Neutrinos & large scale structures in simulation

ΛCDM with massless vs. massive neutrinos (total mass of 6.9 eV), with same total matter

Power spectrum of large scale structures

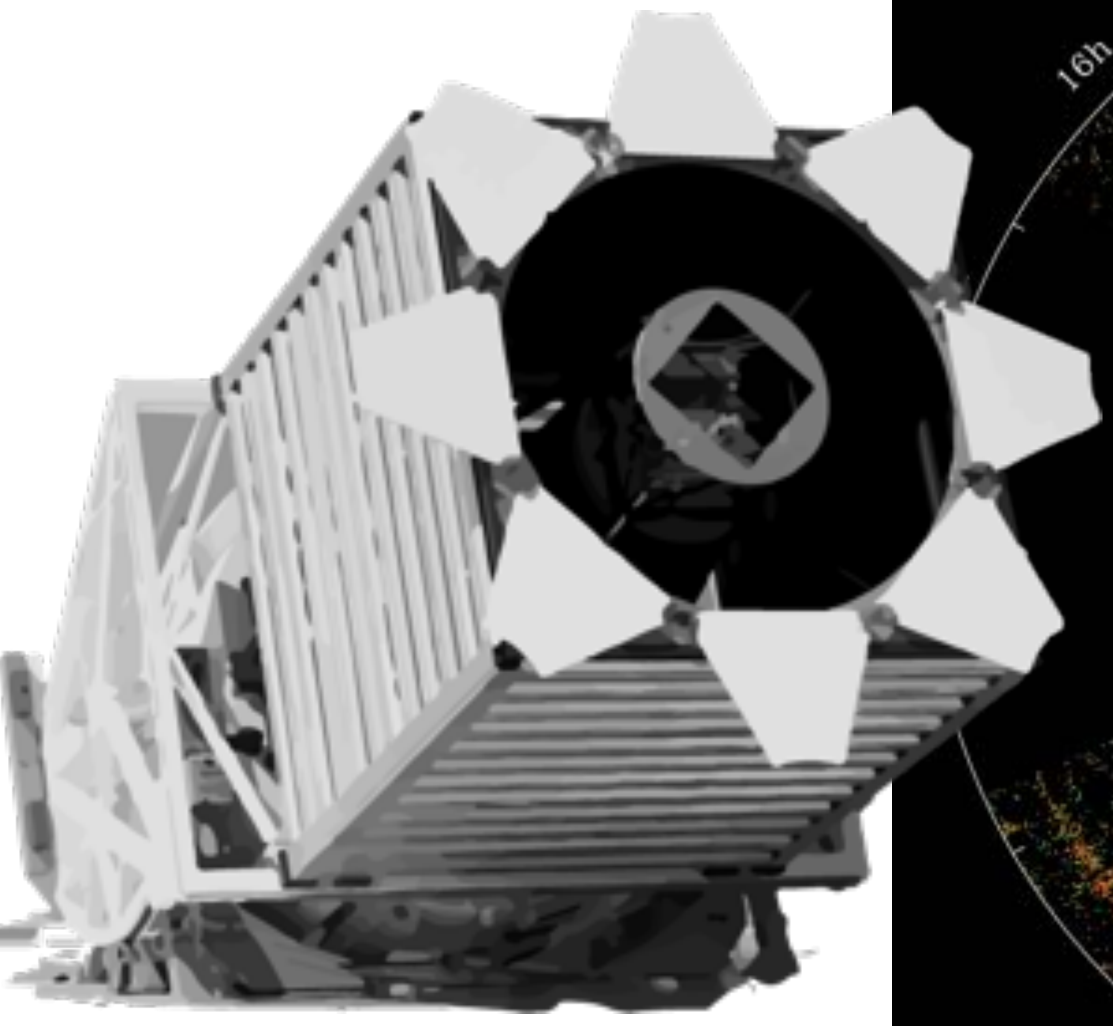
$$\delta = \frac{\rho}{\langle \rho \rangle} - 1$$

Density contrast

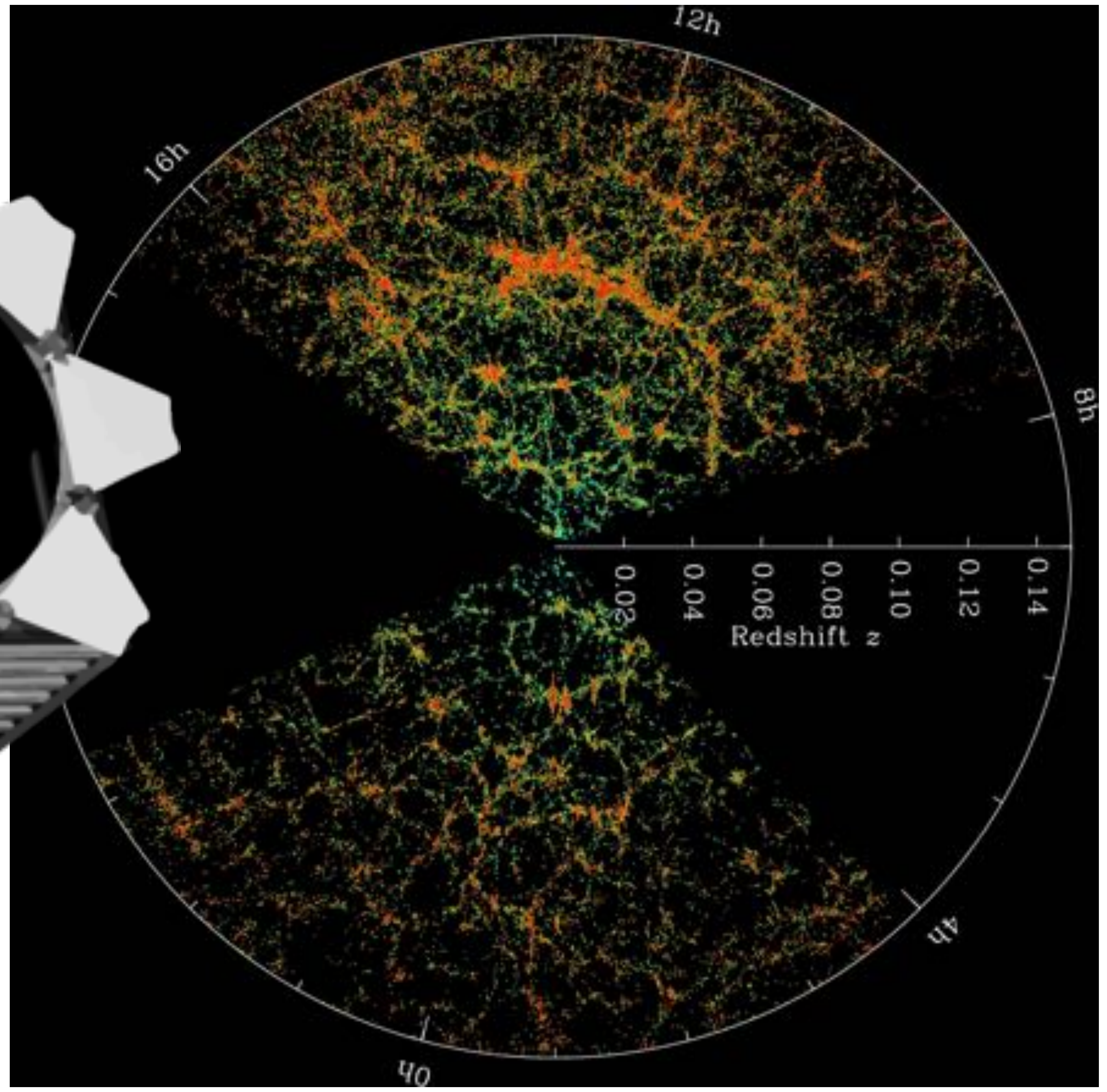
$$= \sum_{\mathbf{k}} \tilde{\delta}_{(\mathbf{k})} \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}$$

Can develop in Fourier modes, evolve independently in linear theory

Measured via surveys



e.g. SDSS



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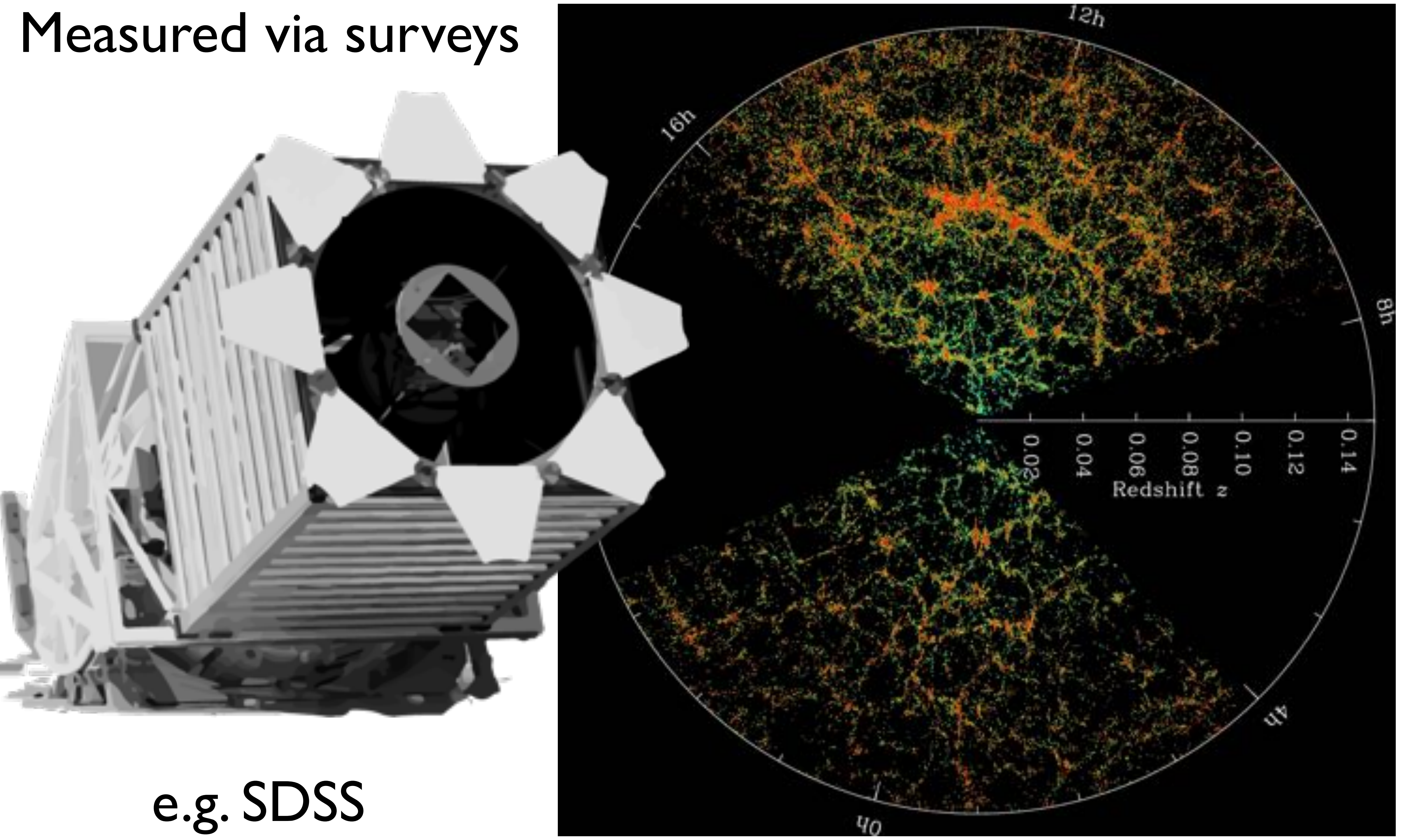
Ensemble variance is the power spectrum *P(k)*

$$\langle \tilde{\delta}(\mathbf{k})\tilde{\delta}^*(\mathbf{k}')\rangle = (2\pi)^3 P(k)\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

(Simplification for homogeneity and isotropy)

Measured via surveys



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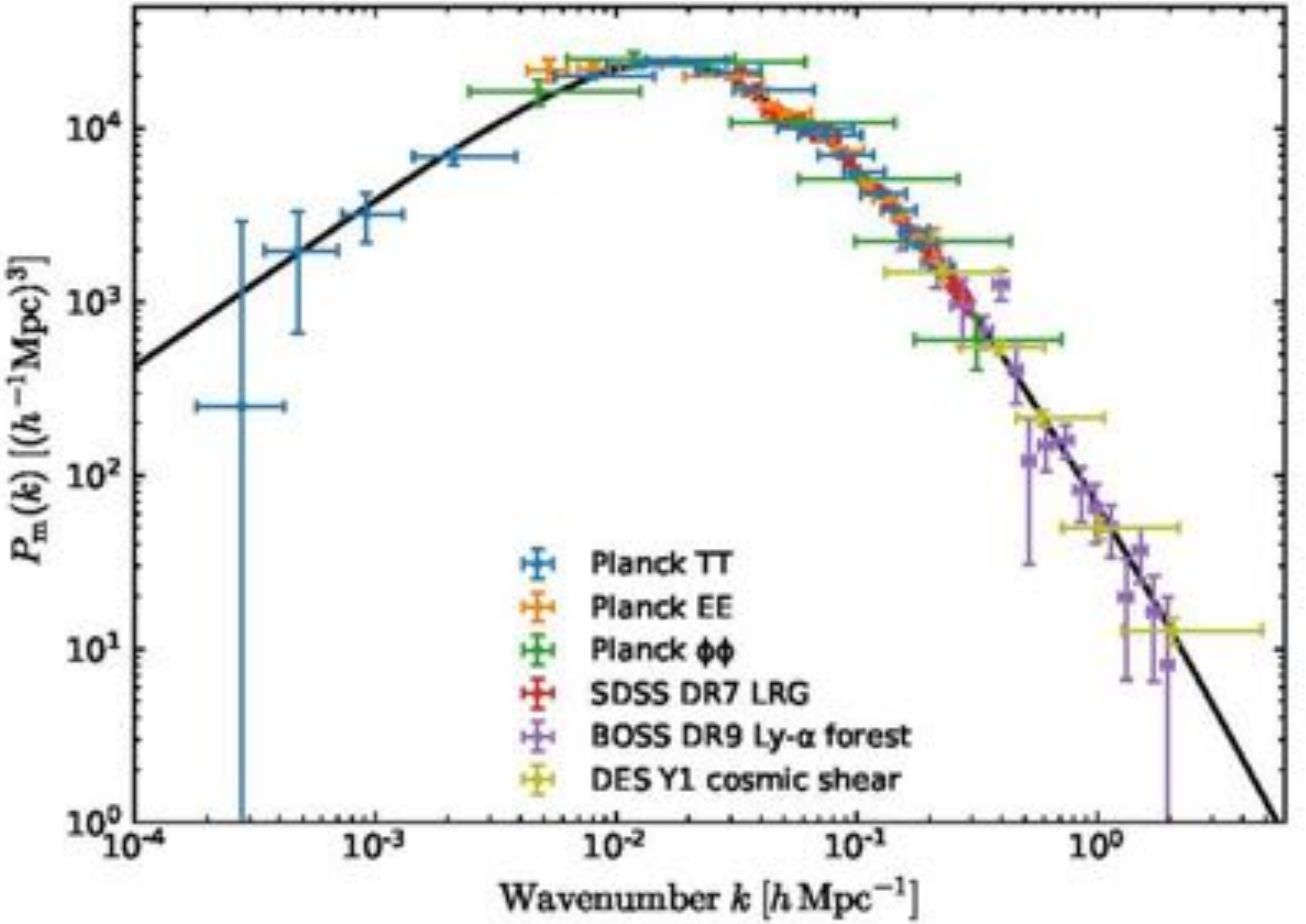


Neutrinos & large scale structures, more quantitative

Cosmologies with same total matter Ω_m but massive ν's lead to a *P(k)* suppression at small scales

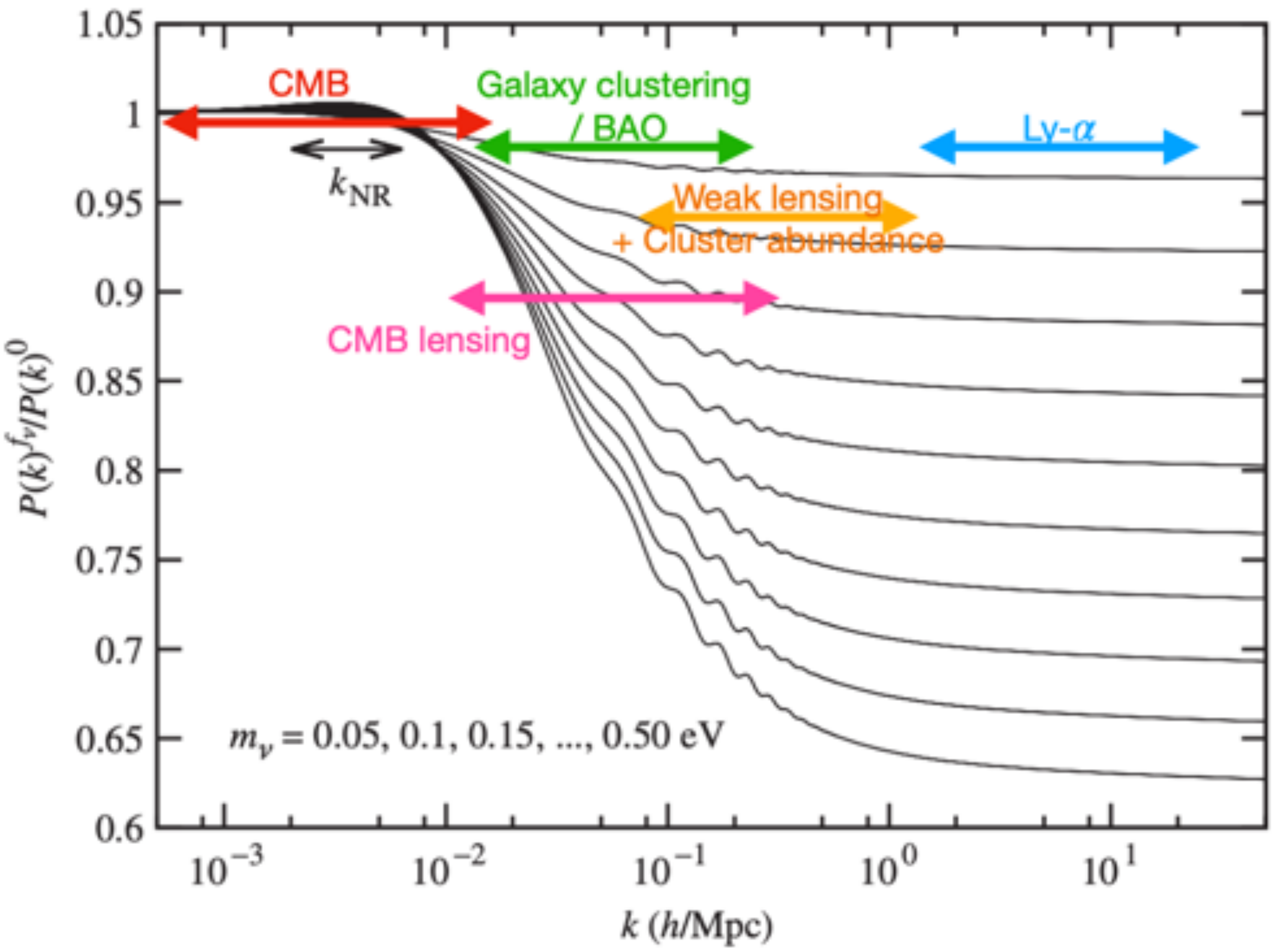
$$k > k_{\rm NR} = 0.01\sqrt{\frac{\sum m_\nu}{\rm eV}}\sqrt{\frac{\Omega_m}{0.3}}\,h\,{\rm Mpc}^{-1}$$

In linear perturbation theory,

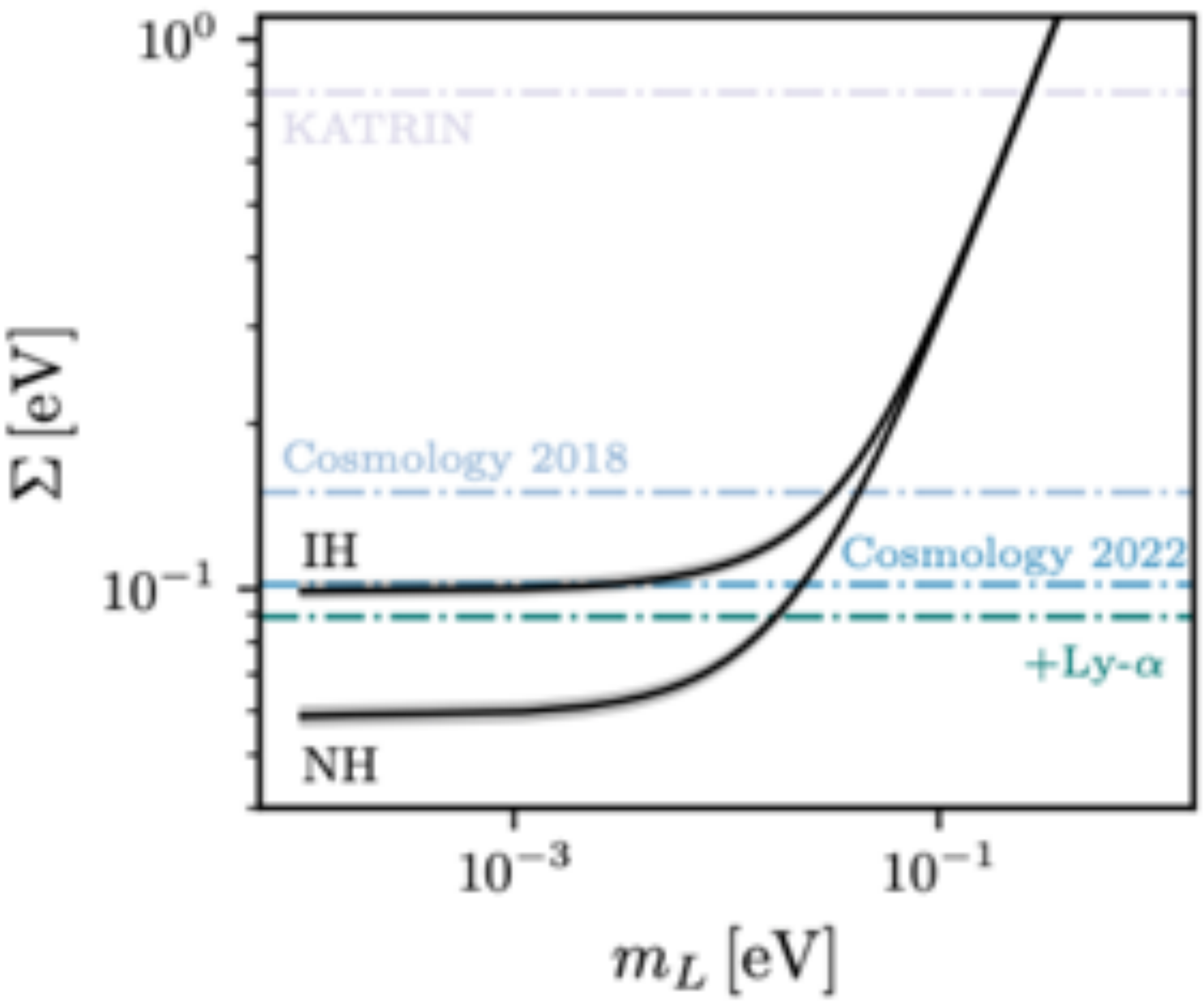
$$\frac{\Delta P}{P} \simeq -\frac{\sum m_\nu}{1.7\,{\rm eV}}\frac{0.3}{\Omega_m}$$

(Improvements exist both via analytical and numerical approaches)

Partial degeneracies exist with other parameters. Actual bound (mildly) depends on reference model and also on how many (consistent!) datasets are used.



Cosmological neutrino mass bounds (95% CL)



Jimenez et al. 2203.14247

Impact of dataset combination

Planck 2018 (all T and pol, plus lensing) < 0.24 eV

Planck 2018 + BAO < 0.12 eV

(A&A 1807.06209)

Planck 2018 + BAO + Ly−α < 0.089 eV

(Palanque-Delabrouille et al. 1911.09073)

Planck 2018 + BOSS + eBOSS < 0.082 eV

(Brieden et al. 2204.11868)

Impact of cosmological model

ΛCDM: Planck 2018 + BAO < 0.12 eV

ΛCDM+w+running+N_{eff} : Planck 2018 + BAO < 0.167 eV

(Di Valentino et al. 1908.01391)

In the coming decade, expected to reach sensitivity to measure the minimum NH mass at 3-4 sigma
e.g. *T. Brinckmann et al.1808.05955*

(Very?) Long term: ν's & cosmology for Dirac vs. Majorana

- We need non-relativistic ν's to distinguish Dirac vs. Majorana
- CνB provides (lots of) them for free! If we could detect them via weak interaction, since the flux is known, we can exploit the fact that the interaction rate is twice as large in the Majorana vs. Dirac case.

Long et al 1405.7654 … Hernandez-Molinero et al. 2205.00808…

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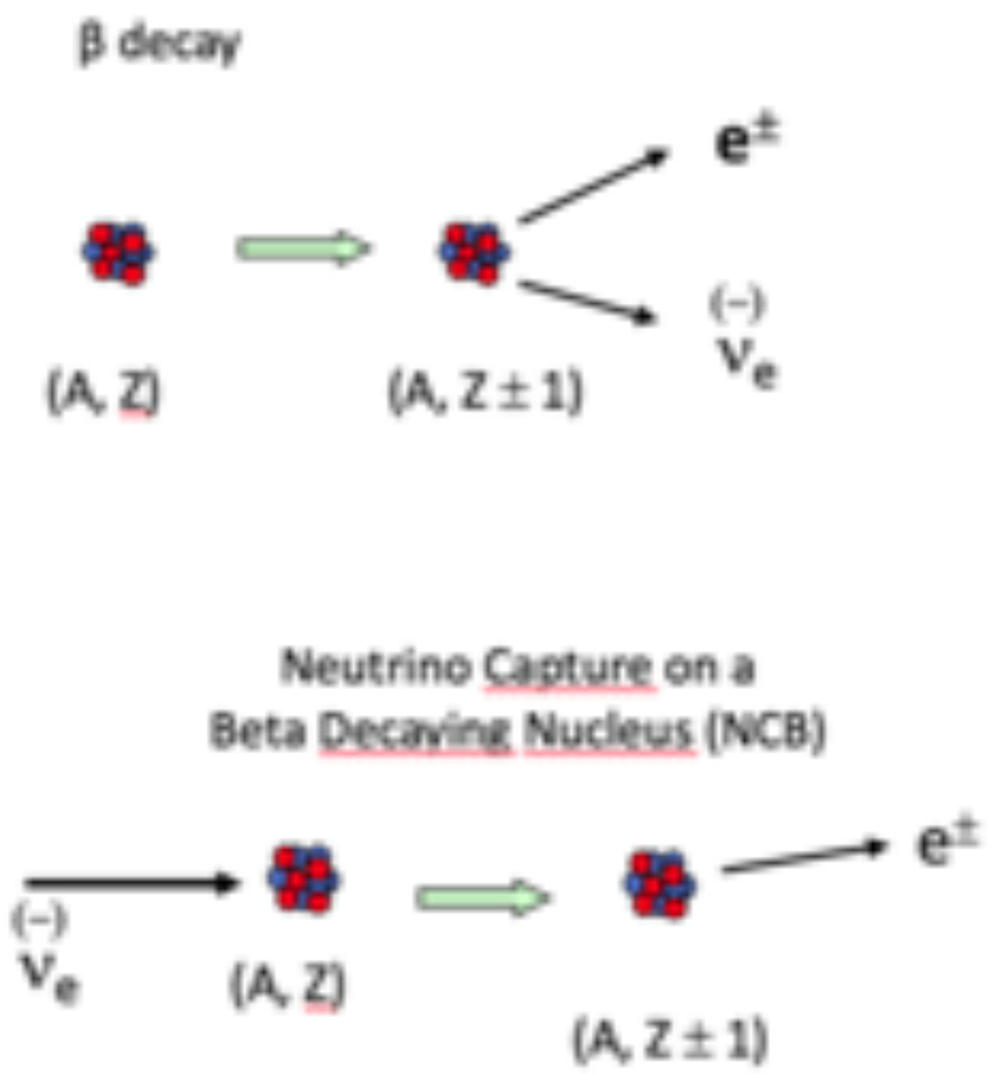
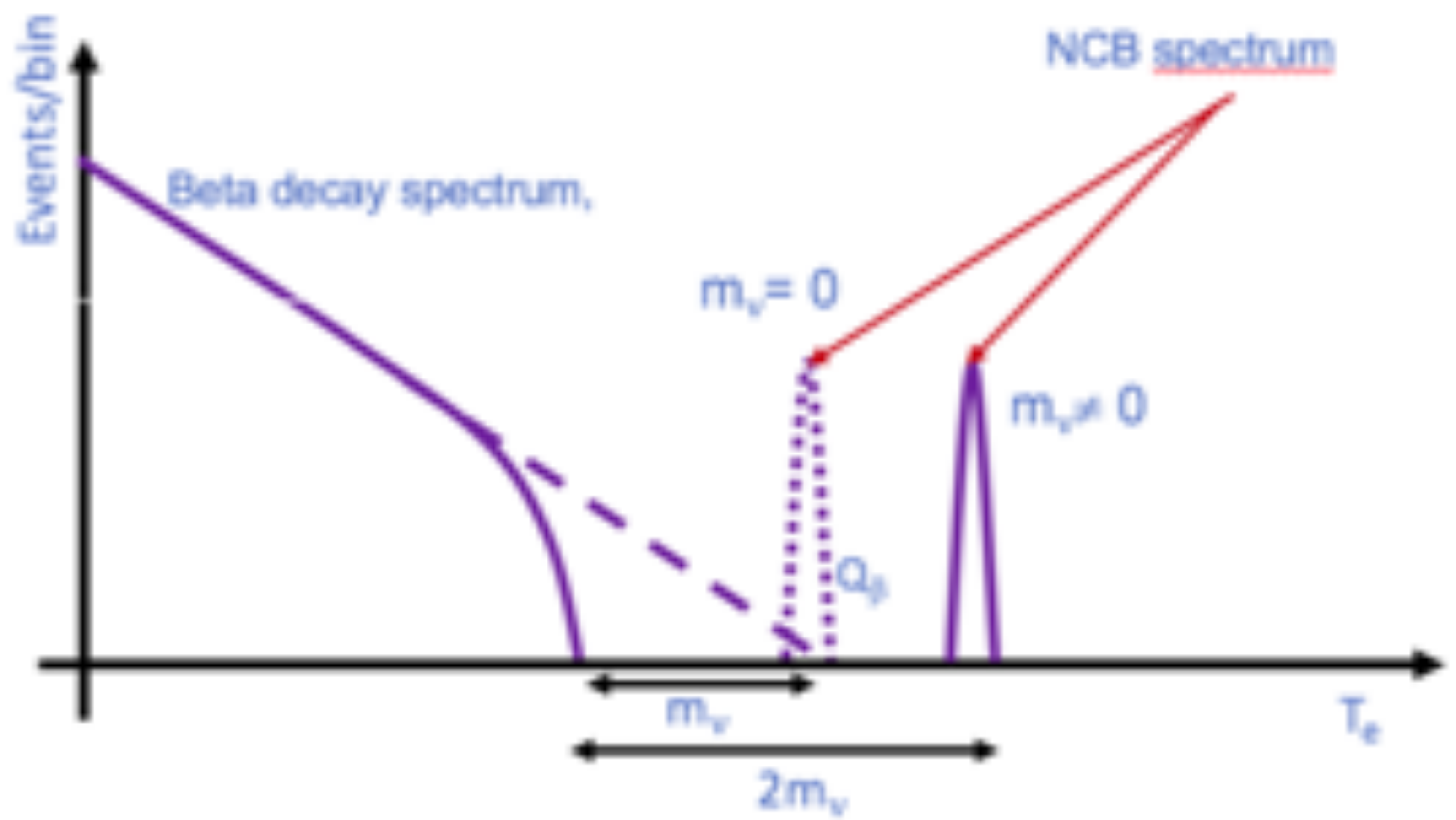
The PTOLEMY collaboration has the long-term goal to detect the CνB, via ν-induced β-decay with significant quantities of tritium atoms bound to graphene sheets.

PTOLEMY

https://ptolemy.lngs.infn.it





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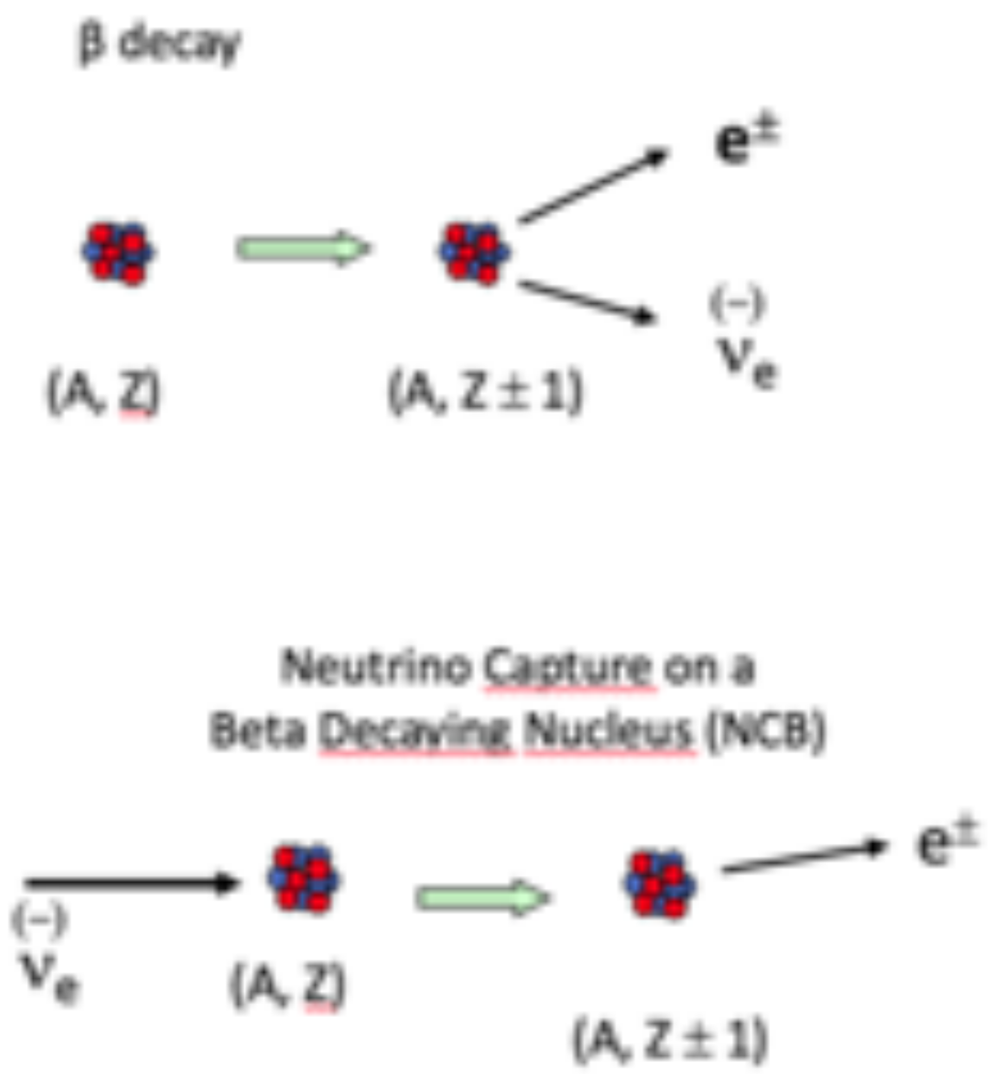

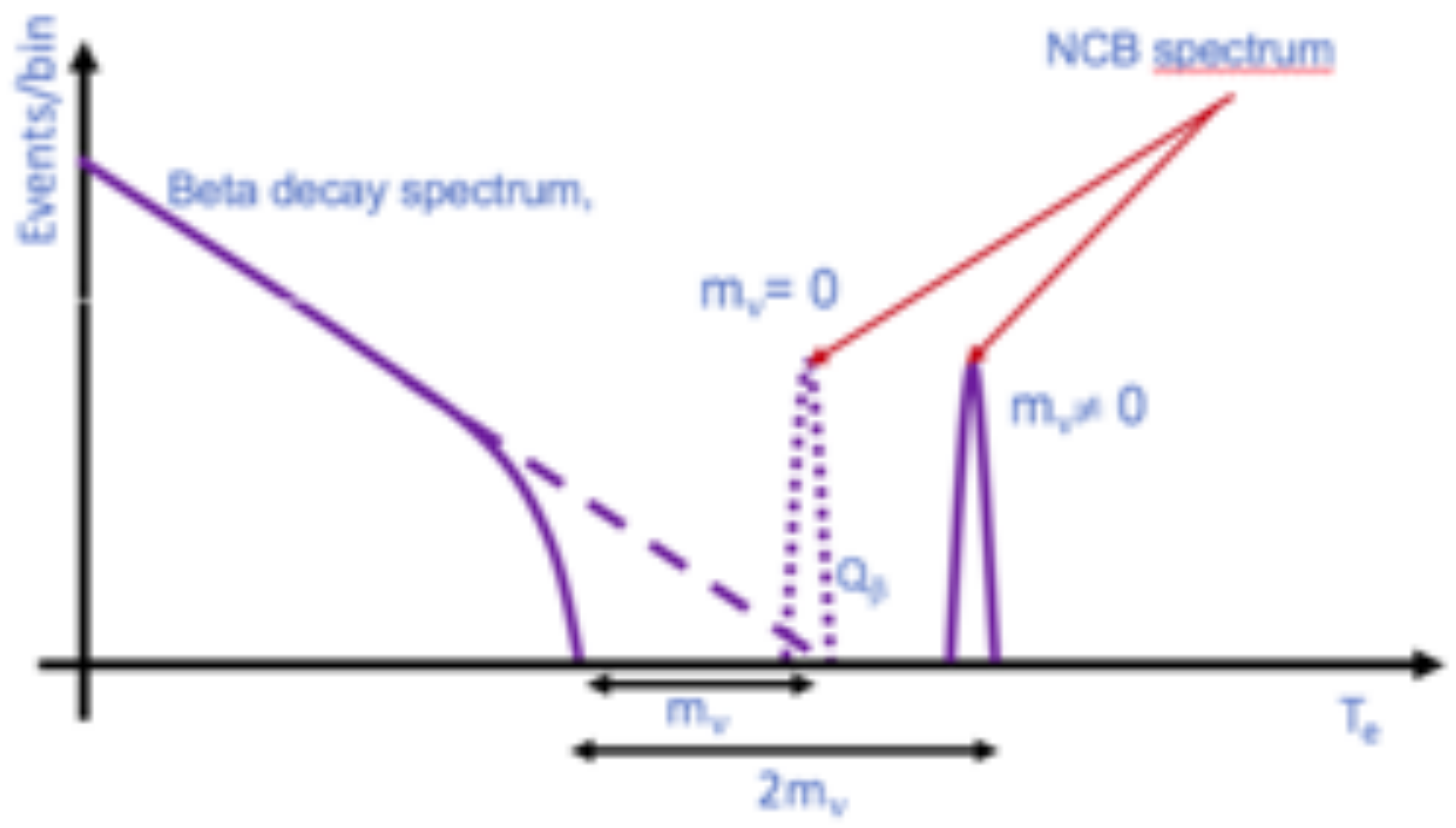

Lots of technical challenges, also conceptual ones: The localisation in the graphene induces a quantum spread on momentum

Cheipesh et al. 2101.10069, Apponi et al 2203.11228

VII. Conclusions: Exotica? Surprises?

Some conclusions and perspectives

- We care *particularly* about ν mass since motivated suspect that it is a messenger of BSM physics
- Proving that ν's are Majorana particles (best shot: $0\nu2\beta$) is the single best way to experimental prove the above…
- Absolute mass measurements also serves to pin down the scale of NP (and has cosmo consequences, for instance!)
- Establishing if (and the extent to which) CP is violated in the leptonic sector is another key objective (may be linked to cosmological ν / anti-ν asymmetry, unfortunately model-dependent)
- We have also to complete our knowledge of the mass-mixing measurement (NO vs IO, maximality of θ_{23} …essentially oscillation experiments), hopefully achieving enough precision to attempt e.g. meaningful unitarity checks; associated advances in nuclear and particle physics often required (reactor flux models, x-sec…)

Room for surprises?

- Theoretically, if RH ν's exist (N), no reasons for them to be 3! Search for sterile ν's…

- … On the other hand, no reason why they should be (only at) eV scale!
  - Maybe keV-MeV mass range (link to dark matter? Impact on astrophysics & cosmology?)
  - Maybe MeV-GeV (e.g. for ARS leptogenesis?)
  - In general, possibility of ν portal $N(LH)$ to new physics

- Neutrinos are feebly interacting, maybe a reason to:
  - (More easily) see new interactions
  - (More easily) expect visible if tiny violations of known physics (e.g. CPT invariance, Lorentz Invariance…)

Cảm ơn