Lectures on neutrino phenomenology - Part 1

Saturday - July 16th -VSON 2022

Pasquale Dario Serpico (LAPTh - Annecy, France)

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In physics, loosely speaking this means quantitative predictions for experimental investigation based on (sometimes simplified) theoretical models *(as opposed to theoretical models per se, or 'mere exploratory' experimental physics)*

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Lecture's Outline

- Motivation: Why focusing on ν mass?
- Some history
- ν oscillations
  - Vacuum (focus on CP violation)
  - Matter

  That's how we learned about massive ν nature & still in the process of measuring unknown/poorly known parameters
- Absolute mass scale & ν nature
  - Tritium endpoint
  - Cosmology
  - $0\nu2\beta$
- Conclusions

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1. Motivation

Will deal mostly with ν masses. Why?

- The neutrinos are spin-1/2 electrically neutral leptons.
- Besides gravity, the only known force they experience is the weak force: ν's form SU(2) doublets with charged lepton partners. Unique among SM fields → experimental challenge… & opportunity!
- Their weak interaction seems successfully described by the SM; cosmology also indicates that they gravitate as expected.

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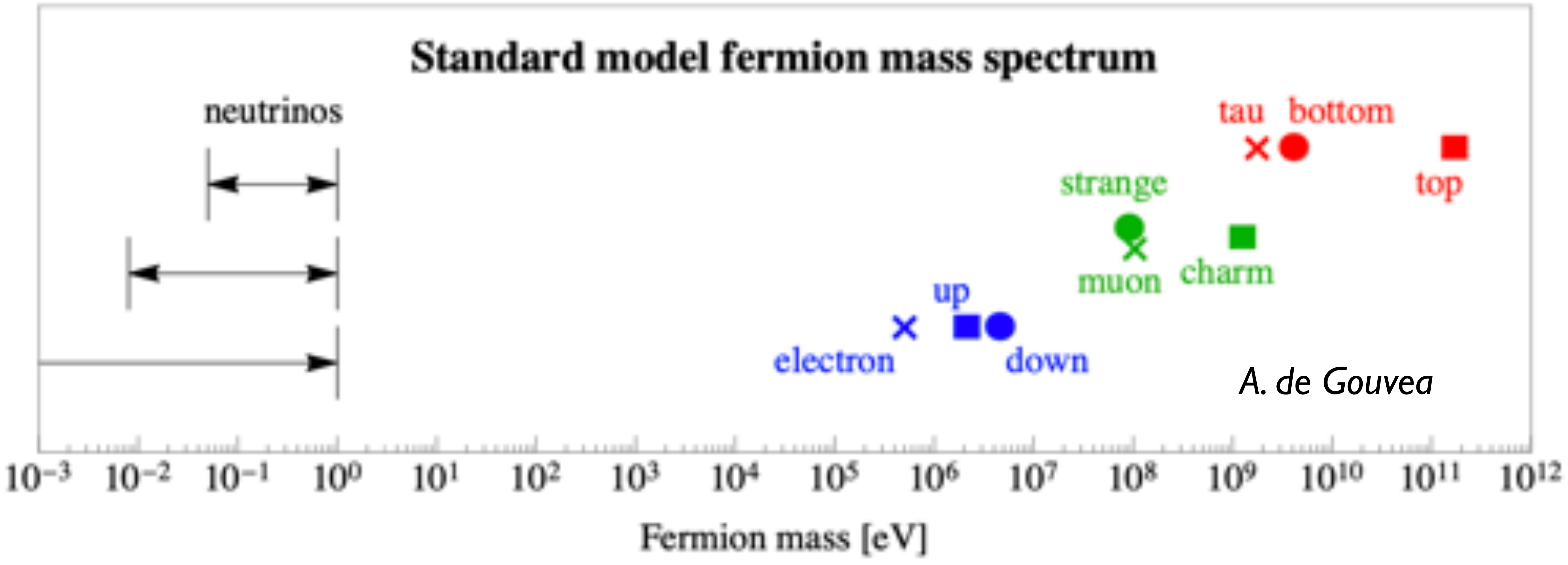
- Their weak interaction seems successfully described by the SM; cosmology also indicates that they gravitate as expected.

However

In the SM ν's are massless, while many experiments over the past decades have proven that ν's do have mass… and a very tiny one!

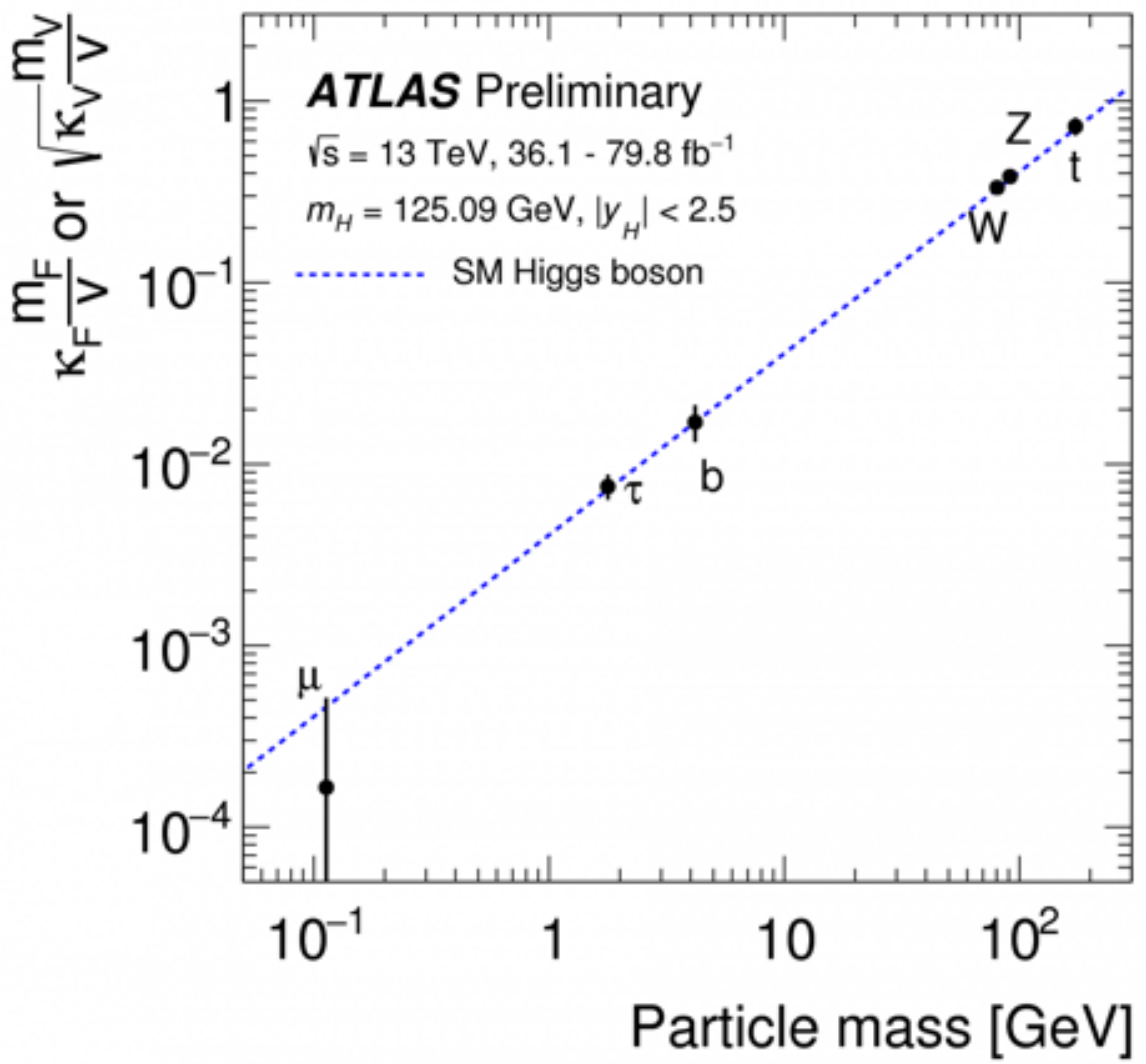


Strictly speaking, only well-established Lab deviation from the SM. But is is a 'trivial' one?

Origin of mass = a main driver of modern particle physics

Reconcile the massive nature of the weak bosons and the SM electro-weak symmetry breaking ⟺ Higgs mechanism.

Recent strong evidence that the quarks and charged leptons derive their masses from an interaction with the Higgs field



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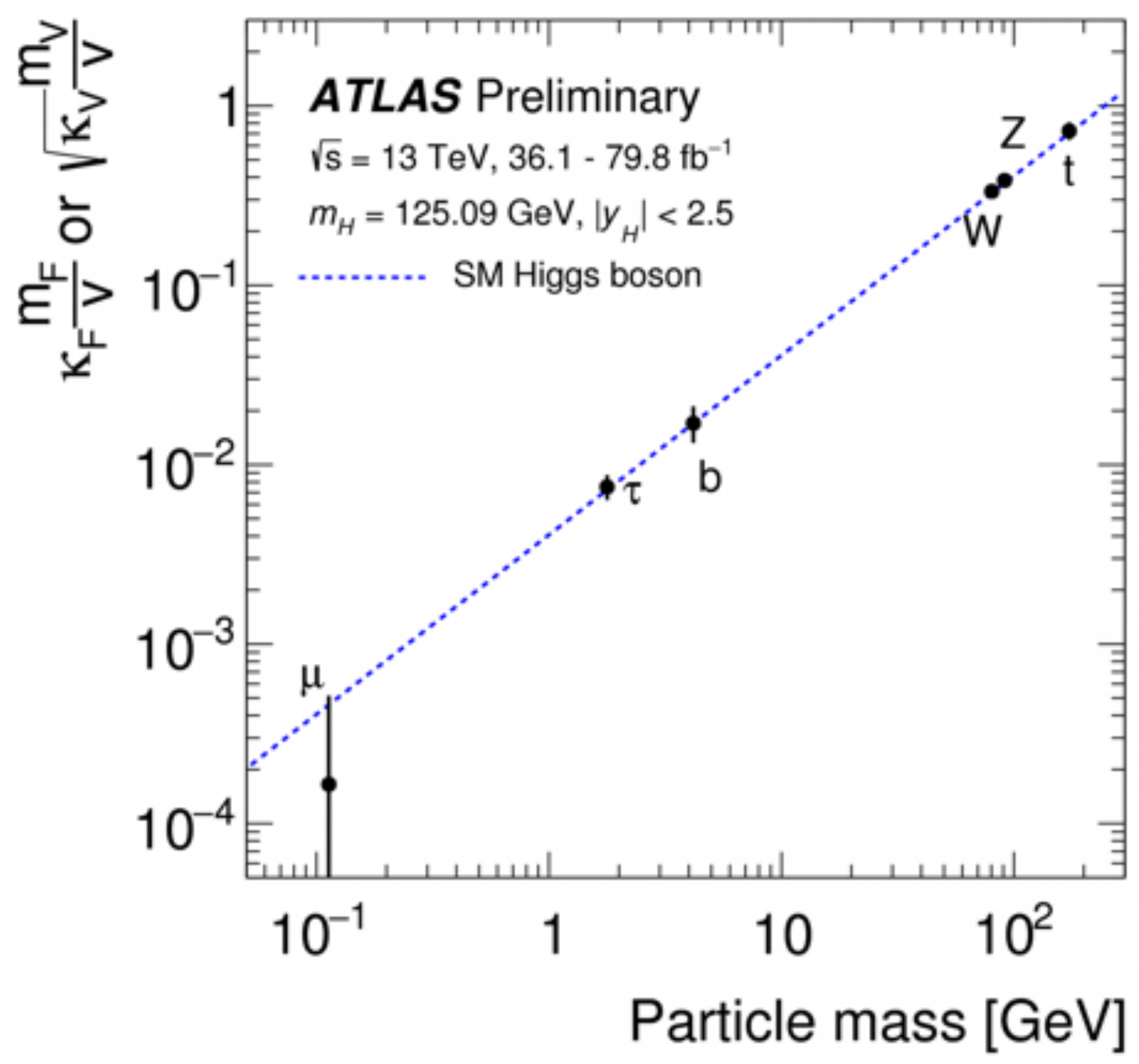

Most theorists strongly suspect that the nature and the origin of the ν masses is different

Two qualitative options:

- Keep the known field content, but drop the requirement of renormalisability of the SM → *where does it lead us to?*
- Add ν_R (although we've never 'seen' these new dof's), or more stuff → *where does it lead us to?*

Either option has deeper implications that one might naively think…

(...

Dirac mass term

$$\bar{\psi} = \psi^\dagger \gamma_0$$

$$-m(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)$$

$$\psi_L \equiv P_L \psi \equiv \frac{1-\gamma_5}{2}\psi$$

$$\psi_R \equiv P_R \psi \equiv \frac{1+\gamma_5}{2}\psi$$

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ν particle associated to *a*, $a^\dagger$

anti-***ν*** particle associated to a_c, $a_c^\dagger$

Charge conjugation C swaps a with a_c

C-operator flips the chirality of the field (does not change spin of particle excitations!)

$$(\psi_L)^c = (\psi^c)_R$$

It helps to keep the field and single-particle notions distinct…

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In the relativistic limit:

$$\begin{pmatrix}u_\ell(p\hat{\boldsymbol{z}},\frac{1}{2})\\ u_r(p\hat{\boldsymbol{z}},\frac{1}{2})\end{pmatrix} \to \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad \begin{pmatrix}u_\ell(p\hat{\boldsymbol{z}},-\frac{1}{2})\\ u_r(p\hat{\boldsymbol{z}},-\frac{1}{2})\end{pmatrix} \to \begin{pmatrix}0\\1\\0\\0\end{pmatrix},$$

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We've seen these excitations, what we call ν and anti-ν

- ψ_L ann. ferm. with hel. - & creates antifer. with hel. +
- $\psi_L^\dagger$ ann. antifer. with hel. + & creates ferm. with hel. -

For ν's we don't know if these states exist. If L is conserved, they must exist by CPT theorem, like for other fermions

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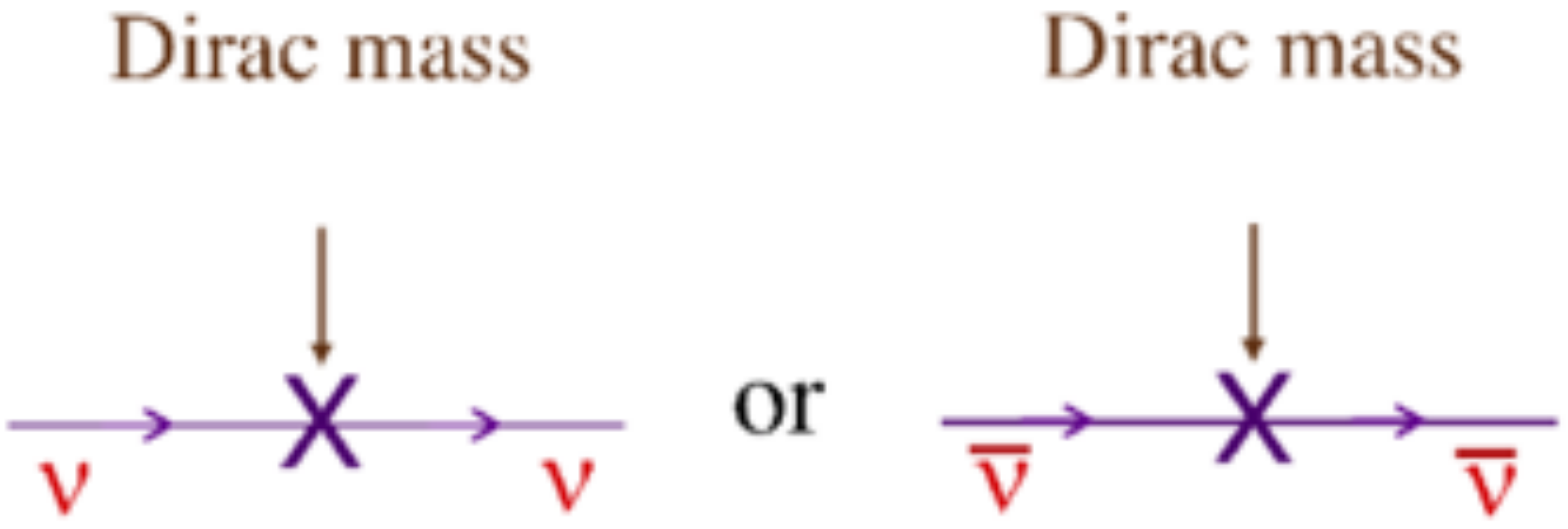
Can symbolically think of it as



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Majorana mass term

Can I use *the same field* to deal with what we observe?
Yes if $a=a_c$, i.e. L is not conserved (i.e. there is no intrinsic distinction between leptons and anti leptons)

$$-\frac{1}{2}\, m(\bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c)$$

factor 1/2 since the same dof enter twice…

The Majorana (2 comp) LH field writes

$$\psi_{\ell\, M}(x) = \sum_{s=\pm 1/2} \int \frac{d^3p}{(2\pi)^{3/2}} u_\ell(\mathbf{p},s)\, e^{ip\cdot x}\, a(\mathbf{p},s) + v_\ell(\mathbf{p},s)\, e^{-ip\cdot x}\, a^\dagger(\mathbf{p},s)$$

And the RH field (same operators, not ind.!) is

$$\psi_{r\, M}(x) = \sum_{s=\pm 1/2} \int \frac{d^3p}{(2\pi)^{3/2}} u_r(\mathbf{p},s)\, e^{ip\cdot x}\, a(\mathbf{p},s) + v_r(\mathbf{p},s)\, e^{-ip\cdot x}\, a^\dagger(\mathbf{p},s)$$

Majorana mass term present

$\Longleftrightarrow$

neutrinos are their own antiparticles

Can be combined in a 4-component *Majorana* field

$$\psi_M(x) = \begin{pmatrix} \psi_{\ell M}(x) \\ \psi_{r M}(x) \end{pmatrix}$$ satisfying $$\psi_M = \psi_M^c$$

cannot exist for any fermion but ν's due to charge conservation

Majorana mass term

Cannot really talk of *ν* /anti-*ν*, just SU(2) interacting particles with opposite helicities. In that sense, the term

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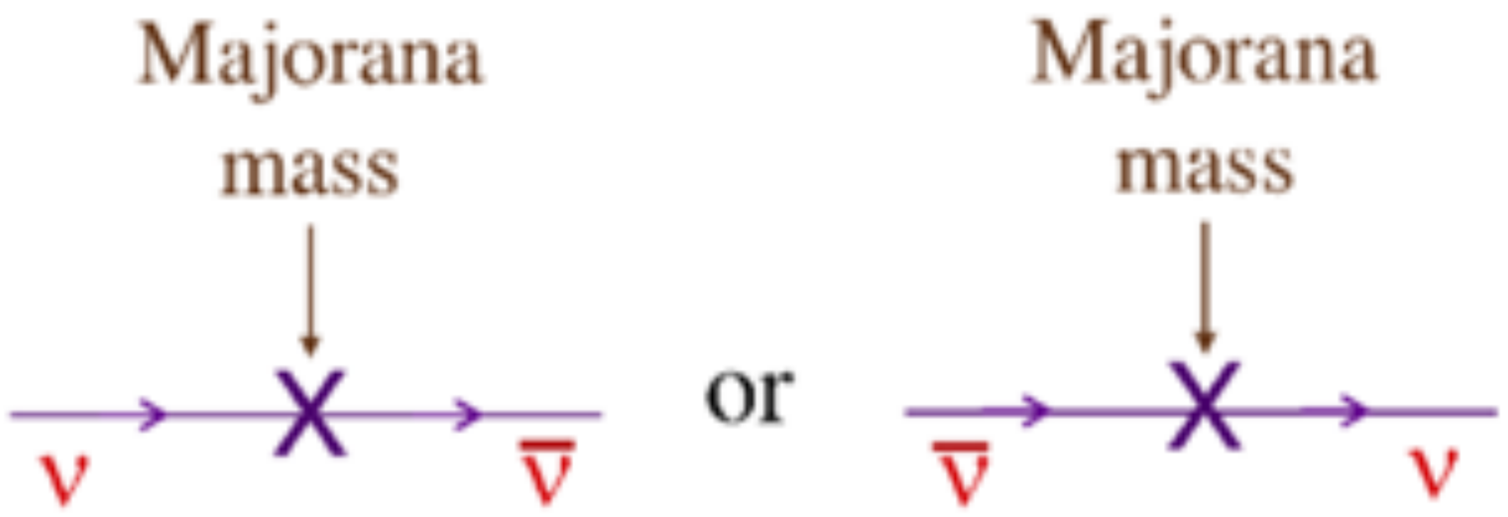
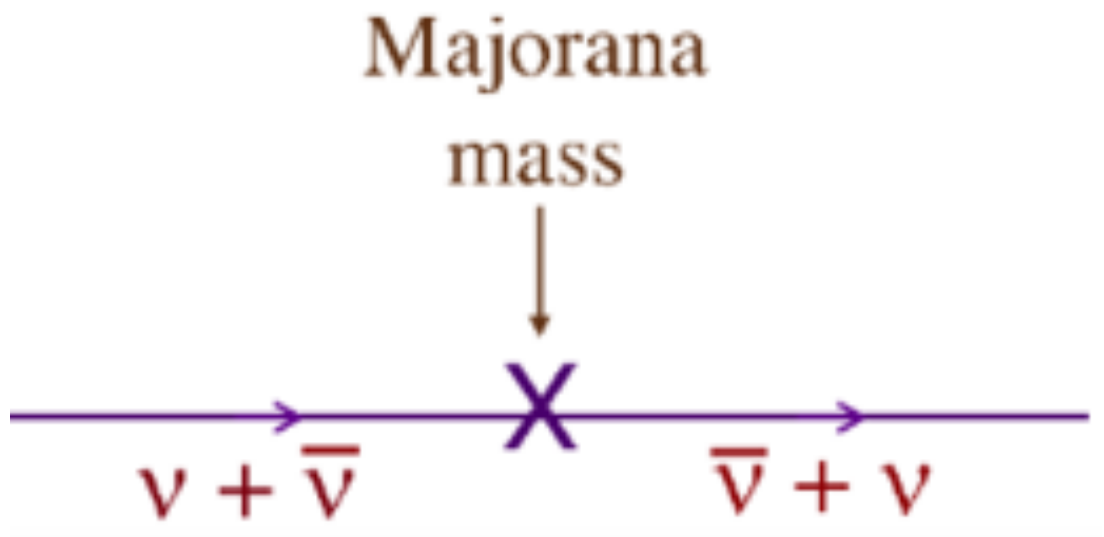
can be symbolically represented as



i.e. globally as



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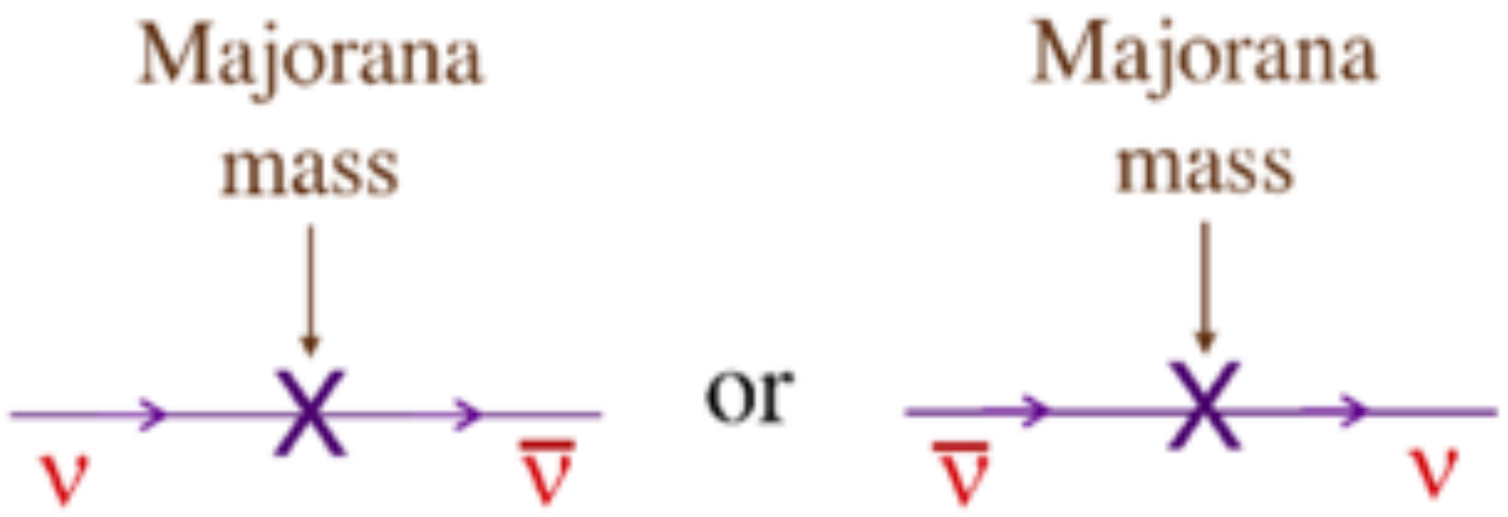

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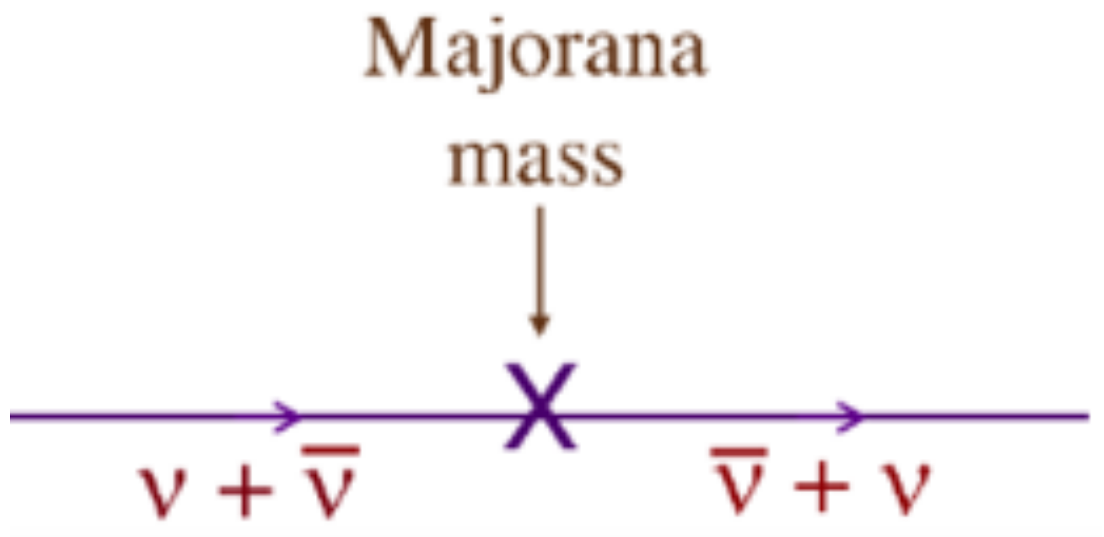

Catania 1906 - Mediterranean sea1938?
(after1959, Venezuela?)

Ettore Majorana

"There are several categories of scientists in the world; those of second or third rank do their best but never get very far. Then there is the first rank, those who make important discoveries, fundamental to scientific progress. But then there are the geniuses, like Galilei and Newton. Majorana was one of these."

Enrico Fermi

Dirac vs Majorana mass term

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As written, neither allowed in the SM, for any fermion!

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Violates SU(2) invariance

Violates both SU(2) and U(1) invariance

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Violates SU(2) invariance

Need to make *m* emerge from the Higgs SU(2) doublet coupling via Yukawa's after EWSB

e.g. $\bar{Q}_L Y_u u_R H$

The field ν_R does not exist in SM, can't be done for ν's

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Weak isospin 1, triplet-like!

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Same 'promotion' of *m* does not work here for ν at renormalisable level (dim=4) hence you did not hear about it!

The (renormalisable!) SM 'accidentally' conserves L

Result: In the SM ν are massless

...)

Option 1: Drop renormalisability, add Weinberg's operator

The trick does work at dim=5 (SU(2) triplet out of 2 Higgs doublets)

One can write a unique, dimension-5 operator that breaks L

$$\mathcal{L}_5 = \frac{1}{\Lambda}(HL)(HL)$$

S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979)

After EWSB, this yields a Majorana mass term for neutrinos

$$m\,\nu^2 \sim \frac{v^2}{\Lambda}\nu^2$$

Note the quadratic dependence of m on Higgs vev, contrary to other SM particles!

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Sensible to think that the tiny *ν* masses detected via oscillations are due to the fact that a high scale *Λ* is responsible for physics BSM breaking L

Can estimate *Λ* so that m~0.01-0.1 eV (scale bracketed by oscillations & direct searches)

It is well below the Planck scale~10^{19} GeV: **New physics scale required!**

Option II: Adding ν_R (=N)

Gauge singlets! Now we can form Yukawa mass term for *ν*, but nothing prevents Majorana mass for *N*

$$\mathcal{L} \ni -Y_{N_{ij}} \bar{N}_i L_j H - \frac{m_{N_i}}{2} \overline{N_i^c} N_i + h.c.$$

Renormalizable extension of SM

Yukawa mass term now possible

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Renormalizable extension of SM

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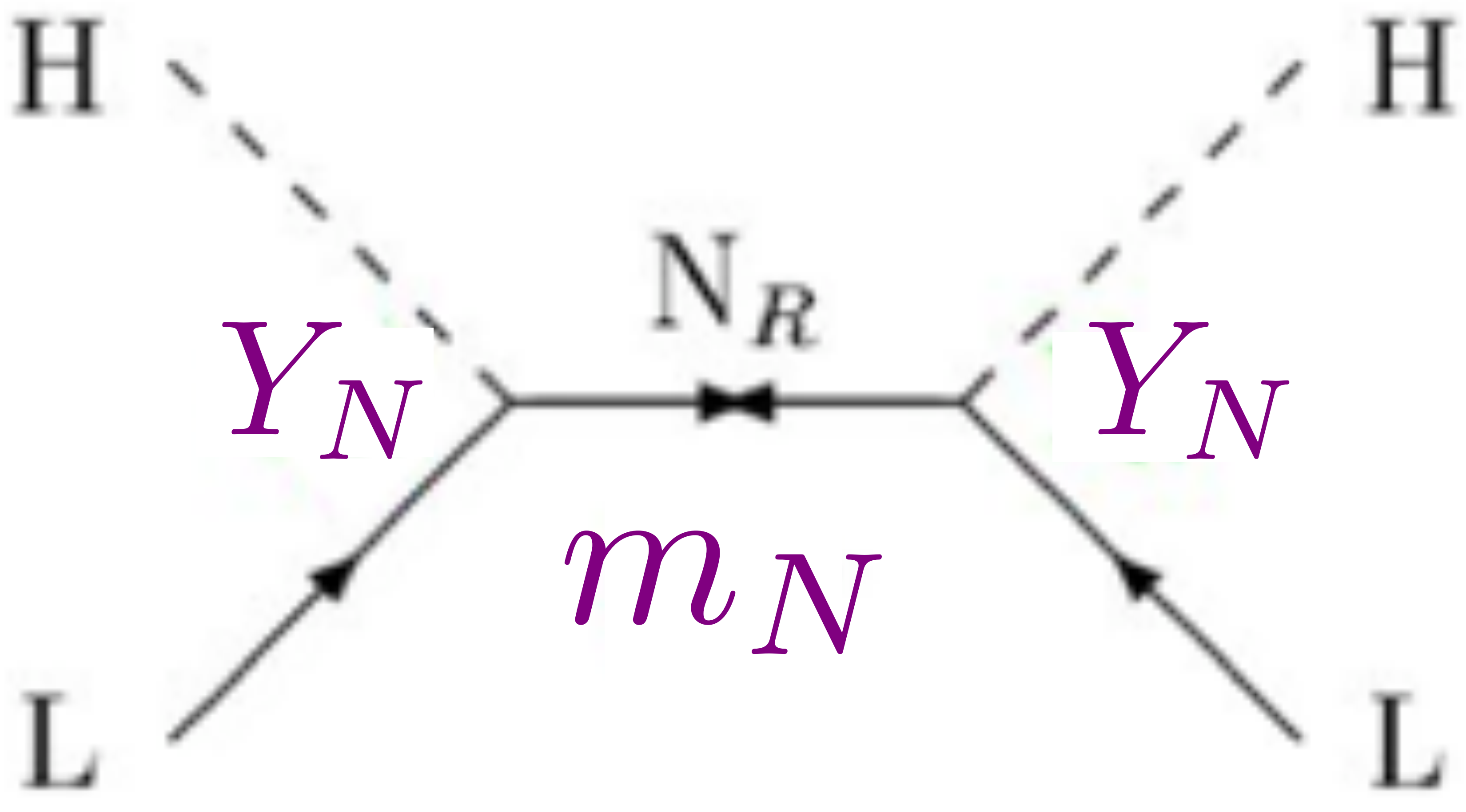
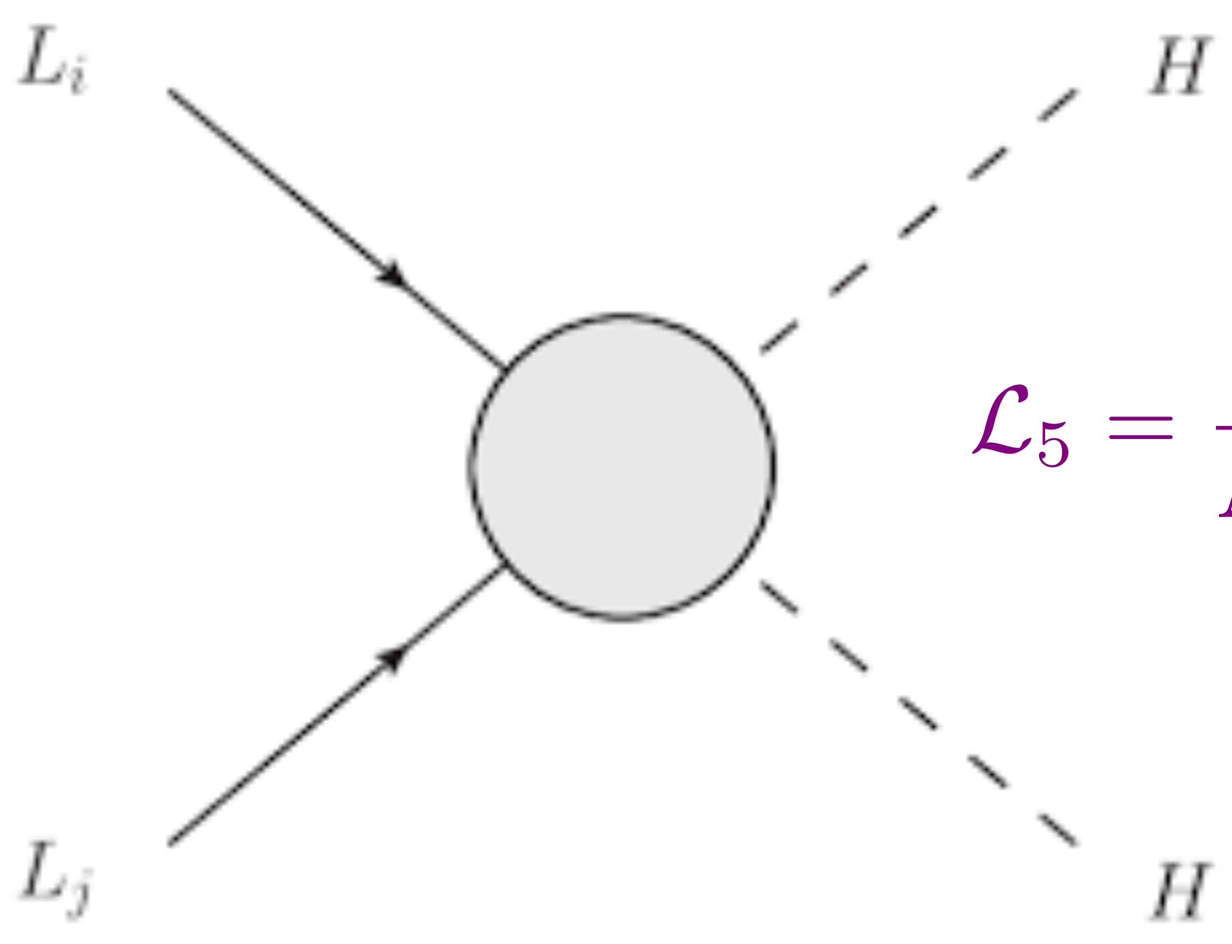
Following diagram possible



At $E << m_N$



$$\mathcal{L}_5 = \frac{1}{\Lambda}(HL)(HL)$$

Back to **effective Majorana mass term for ν's** (Seesaw, type I)

$$m_\nu = Y_N^T \frac{1}{m_N} Y_N v^2$$

Tiny Yukawa's is a possible alternative, but in general $m_N \neq 0$ unless explicitly **enlarging symmetry group** of the SM (*why?*)

So, why focusing on ν masses?

They do achieve what people have been trying to do since the 70's, to 'break' the SM!

New fields/energy scale/'meaningful' symmetry (or breaking thereof) out there, below Planck scale!

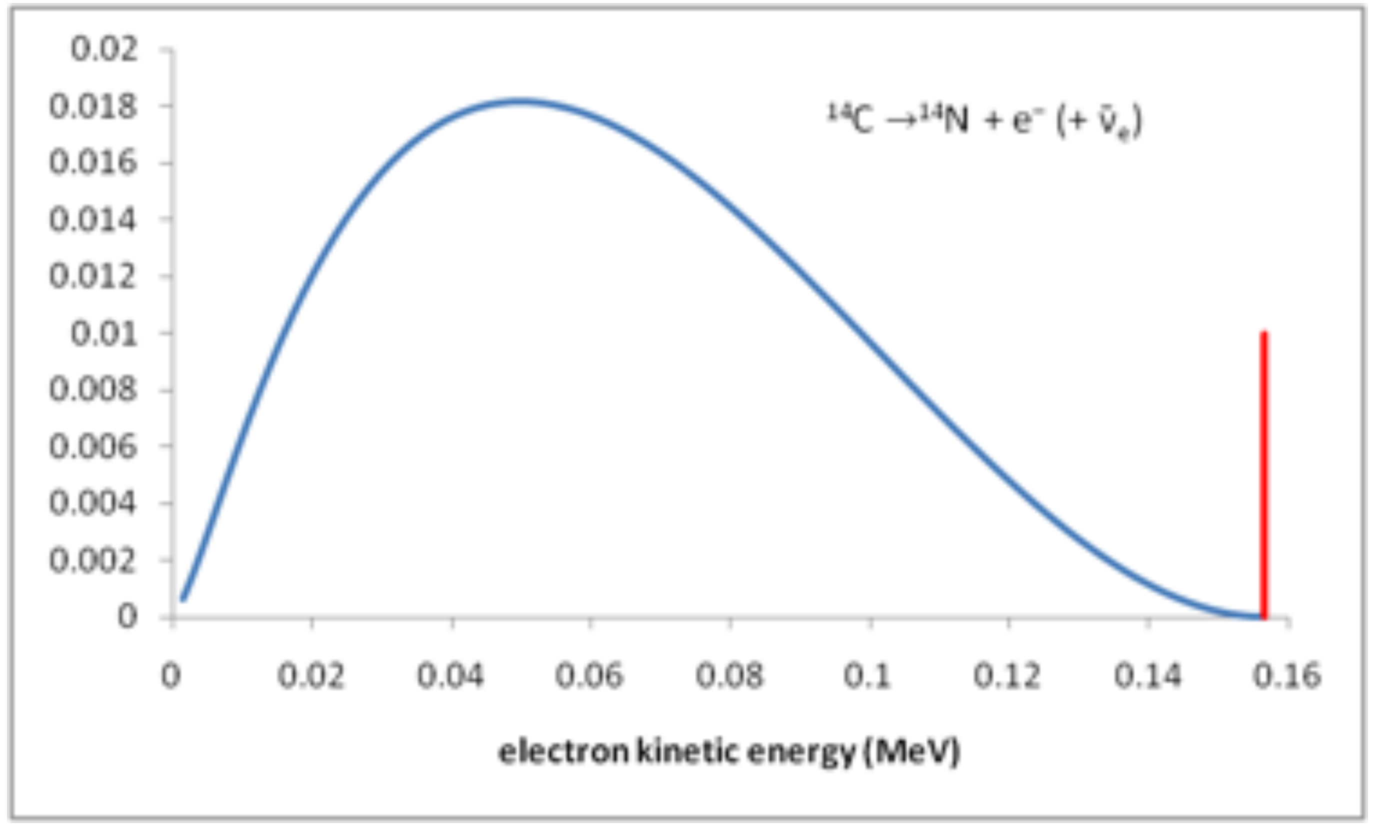
Yet, unlikely that we will understand deeper structure *only* with low-E experiments.
But our duty to collect as much info as we can…

Now time to review how this achievement was attained

II. Historical notes

Partial and incomplete, just to give you a sense of the main events and dates! Apologies to dozens of collaborations and thousands of colleagues!

A (brief) ν history, I



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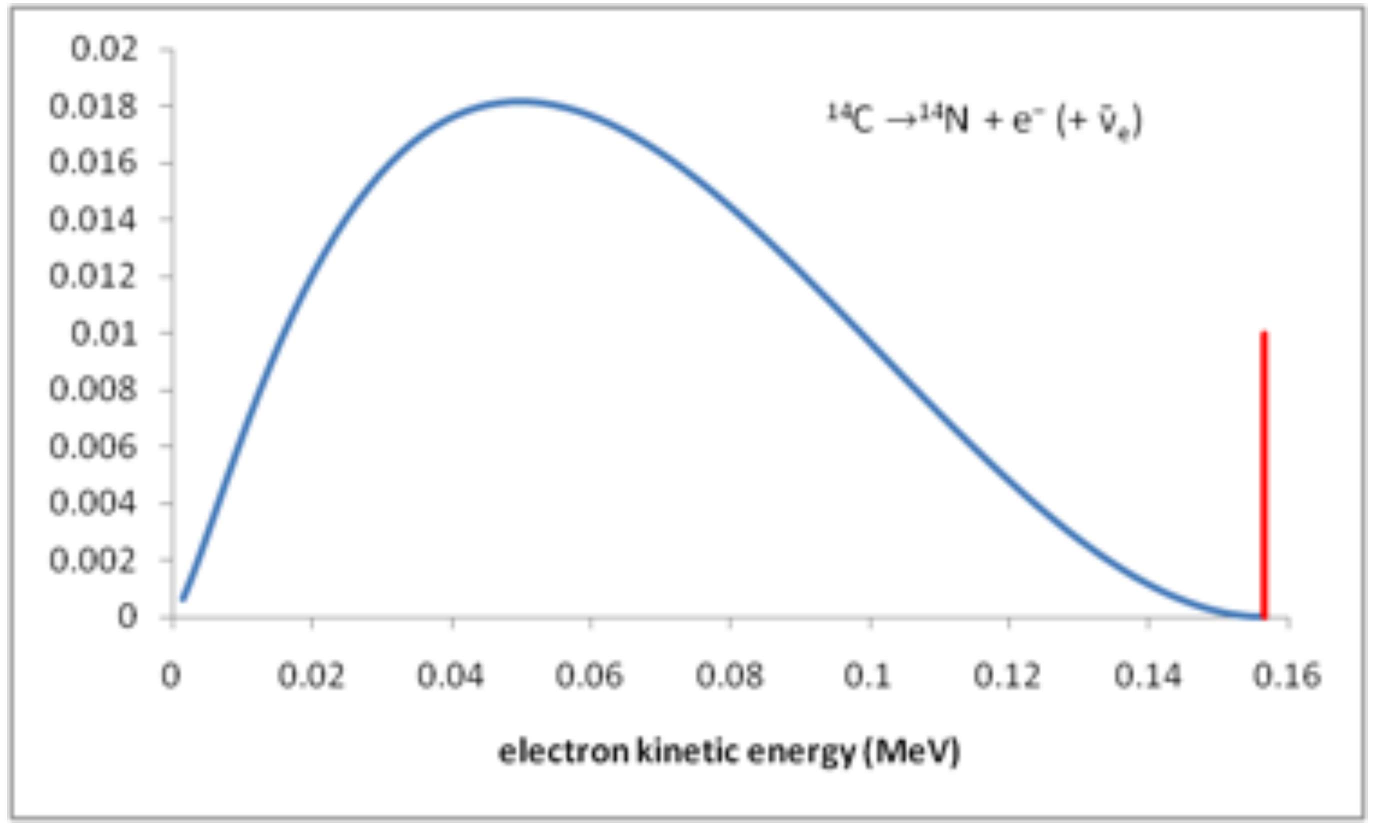
  Dear radioactive ladies and gentlemen, […] I have hit upon a desperate remedy to save the […]energy theorem. Namely the possibility that there could exist in the nuclei electrically neutral particles that I wish to call neutrons, which have spin 1/2 […] The mass of the neutron must be […] not larger than 0.01 proton mass. […] in β decay a neutron is emitted together with the electron, in such a way that the sum of the energies of neutron and electron is constant.



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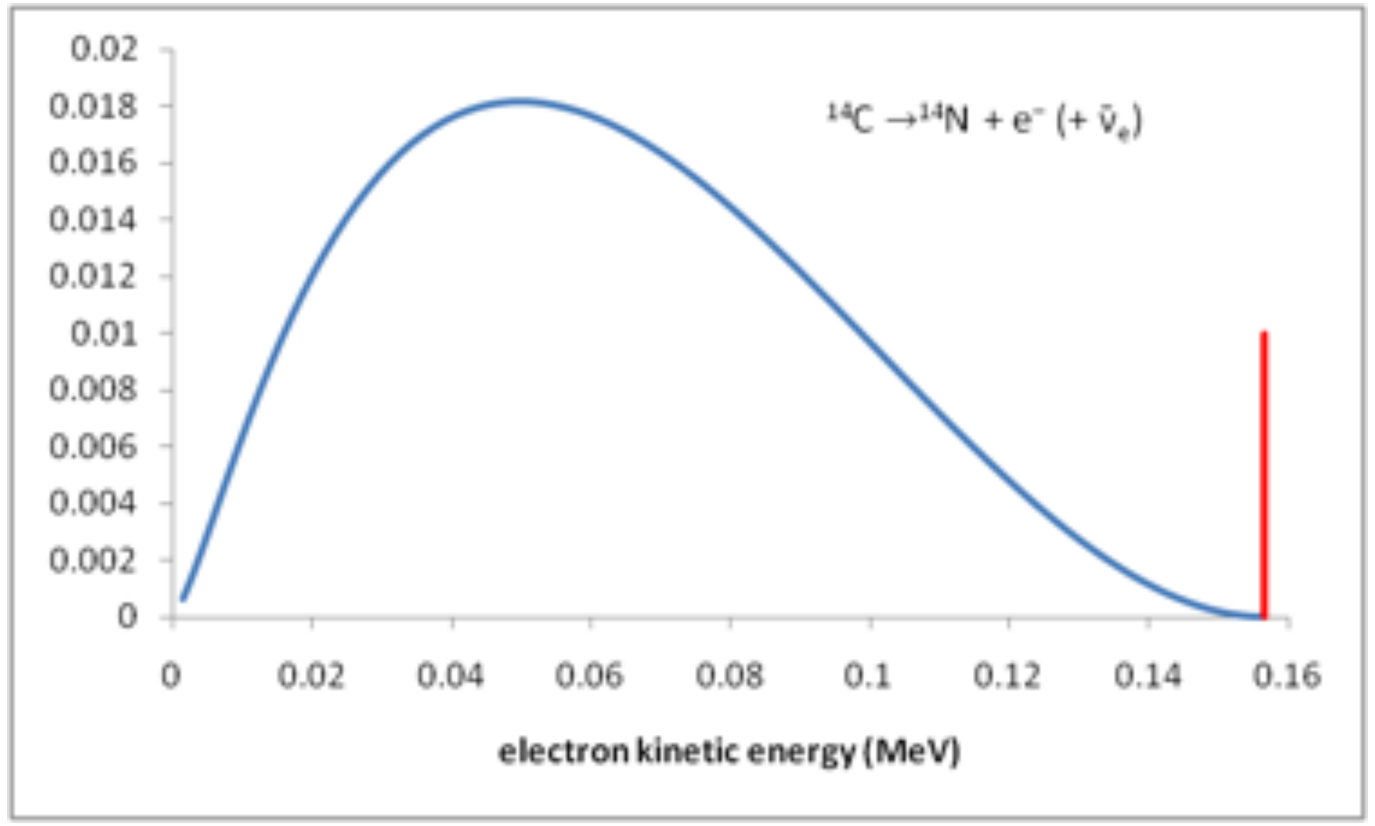

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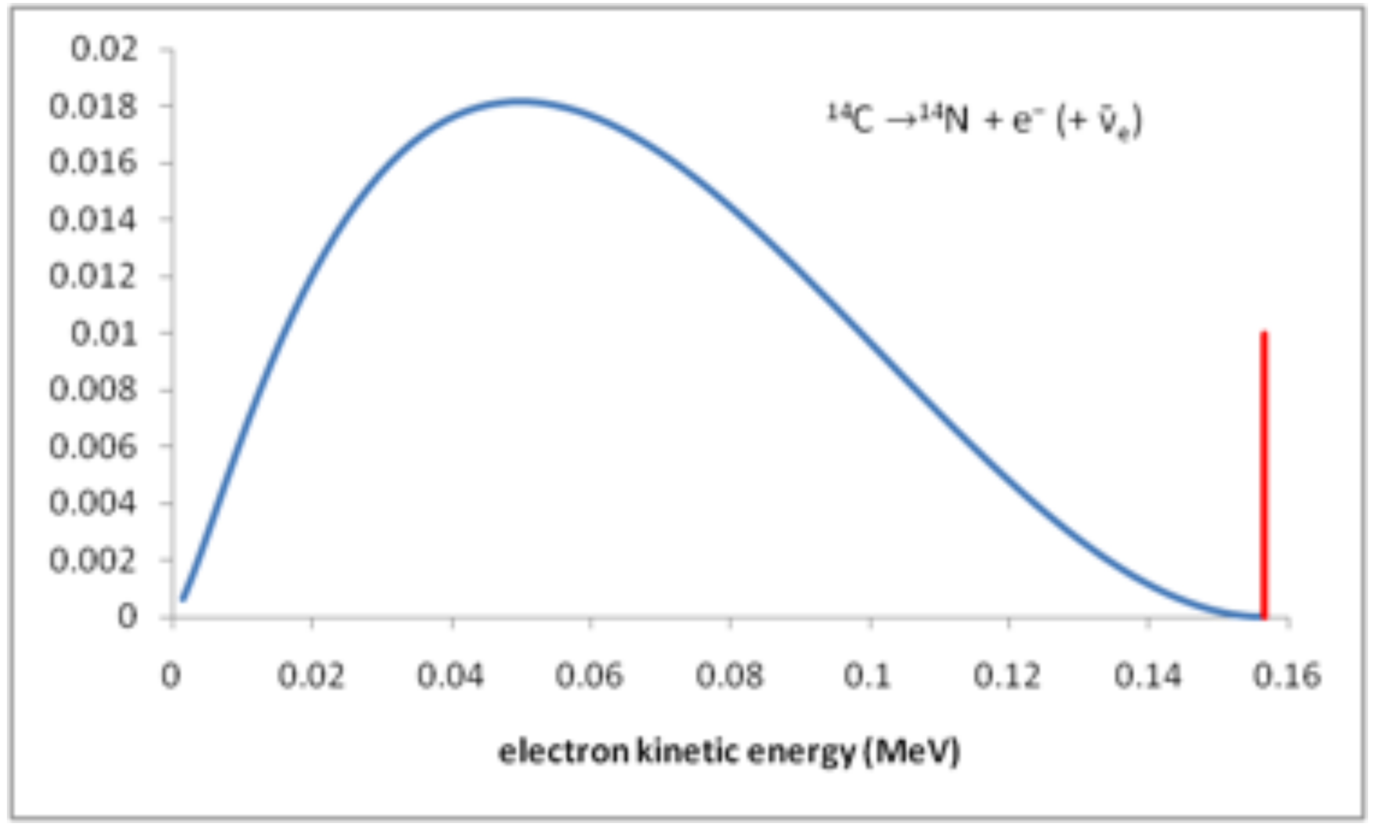

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"I have done a terrible thing today, something which no theoretical physicist should ever do. I have suggested something that can never be verified experimentally." (Pauli to Baade)

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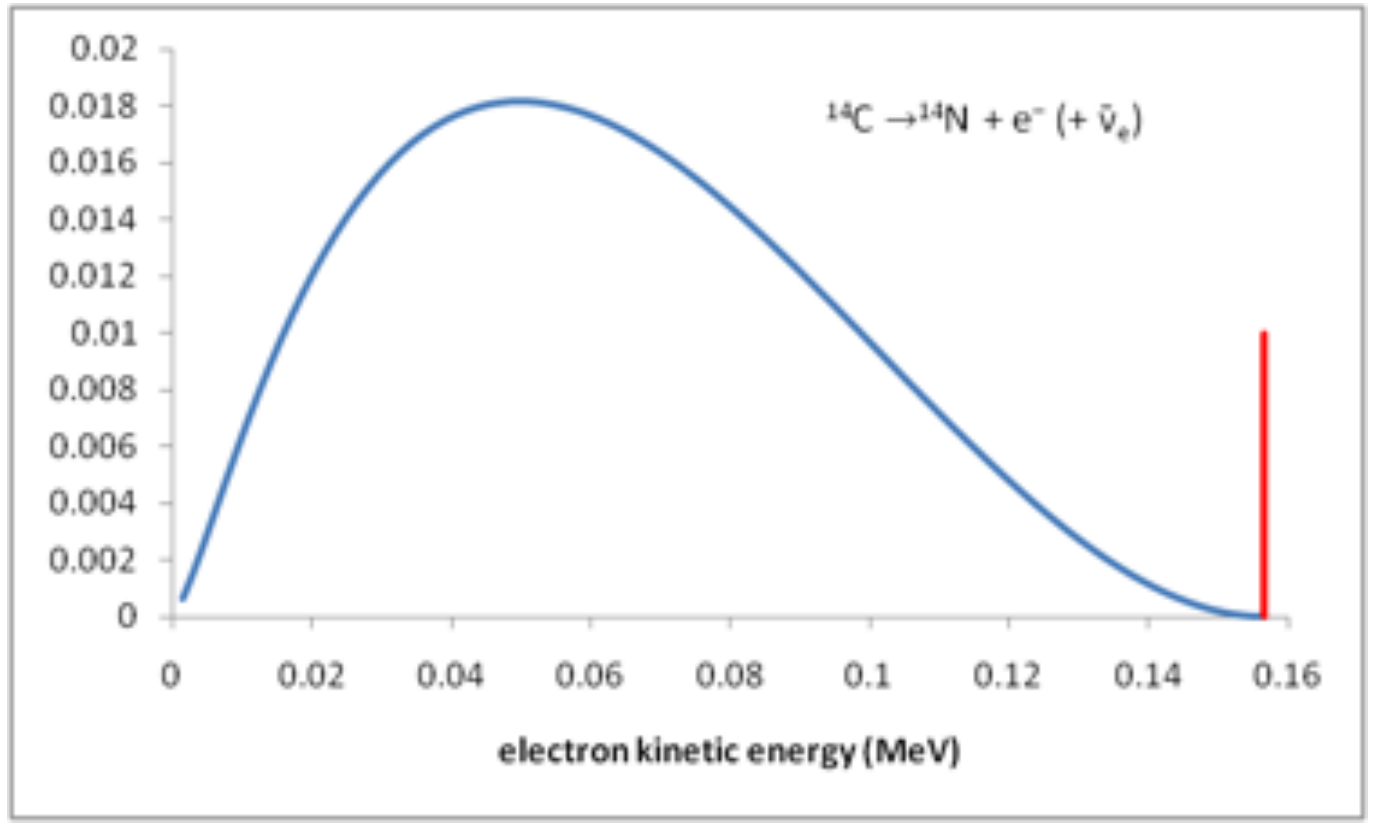

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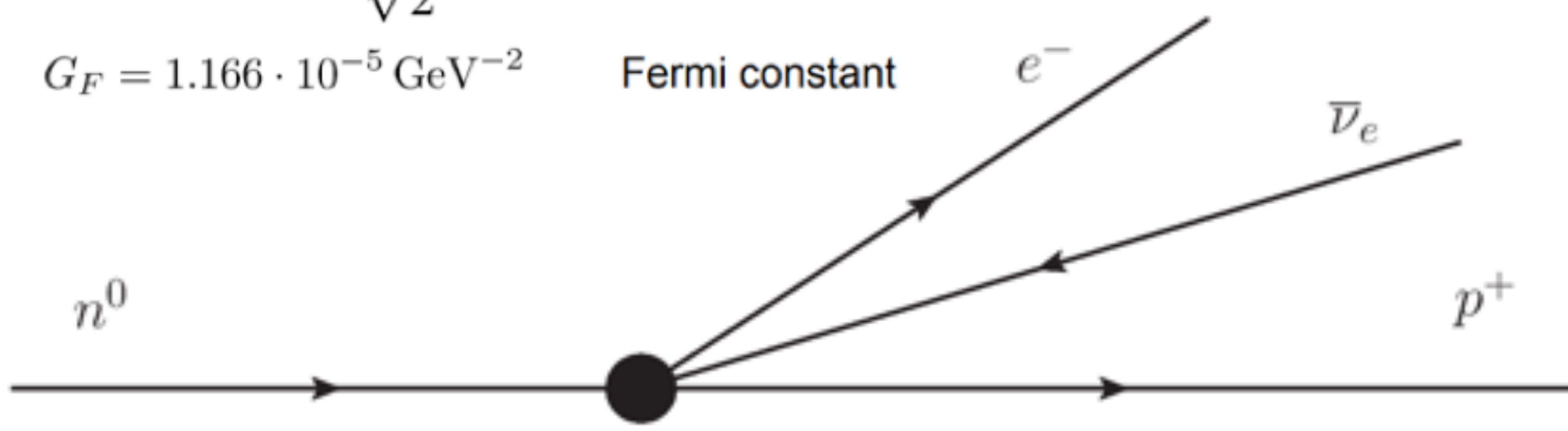
- 1932-34: Fermi (NP 1938) names it "neutrino" (little neutral one) to distinguish it from the neutron recently discovered by J. Chadwick (NP 1935), and later proposes the 'Fermi' theory of beta decay.

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\bar{p}\gamma_\mu n\,\bar{e}\gamma^\mu\nu + \text{h.c.}$$

$G_F = 1.166 \cdot 10^{-5}\,\text{GeV}^{-2}$ Fermi constant



A (brief) ν history, II

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- 1956-57: Lee & Yang (NP 1957) propose and Madame Wu (no NP?!?) proves that P is violated in weak interactions. Sudarshan, Marshak, Gell-Mann & Feynman propose the V-A current structure.

Tsung-Dao Lee (李政道) advisor: Fermi

Chen-Ning Yang (杨振宁); advisors: Teller, Fermi

Mme (Chieng-Shiung) Wu (吴健雄)

A (brief) ν history, III

- 1957: First idea of ν (anti-ν) oscillation, by B. Pontecorvo

- Early '60: Leptonic mixing introduced
  Maki (牧二郎) Nakagawa (中川昌美) Sakata (坂田昌)

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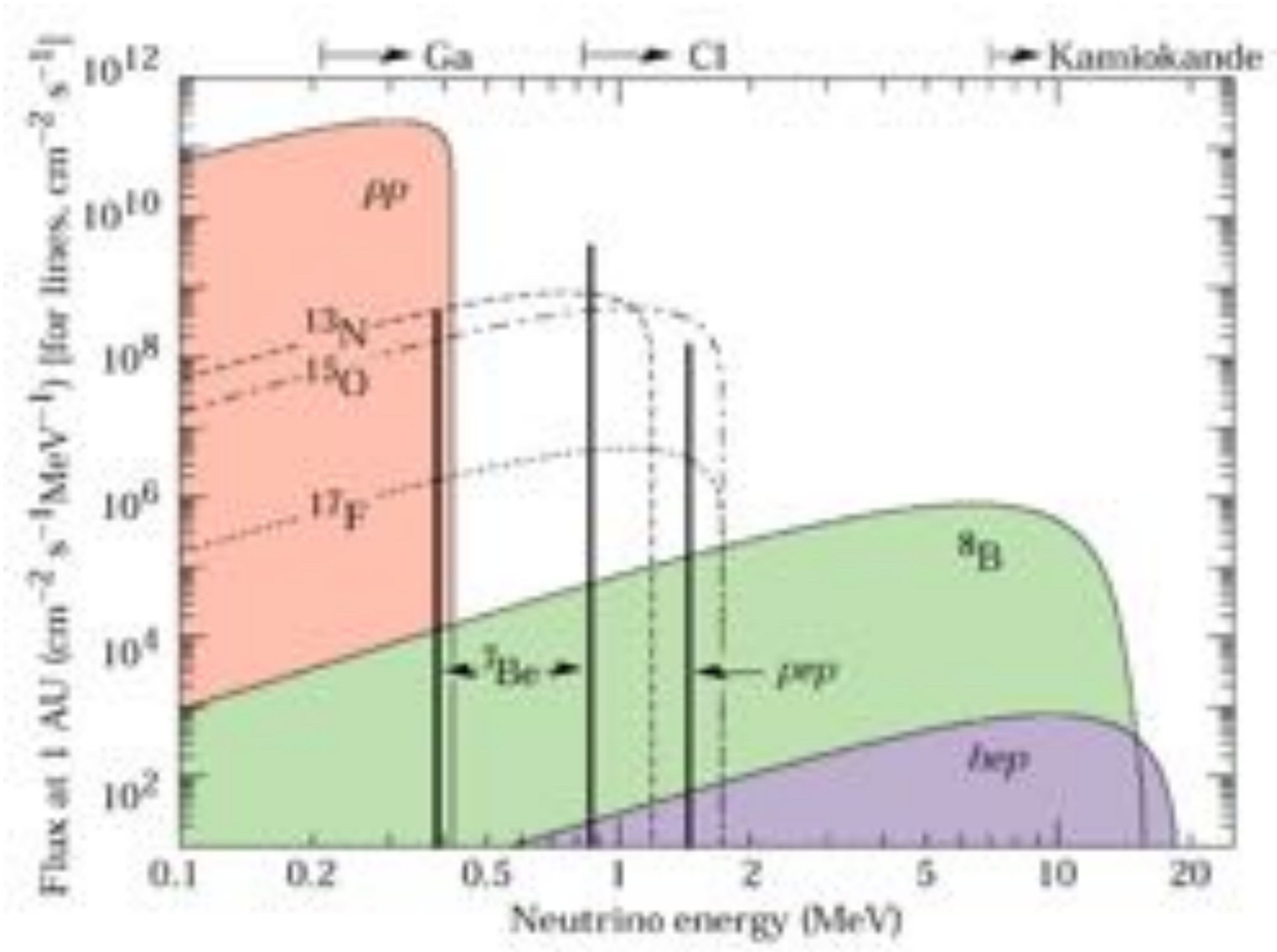
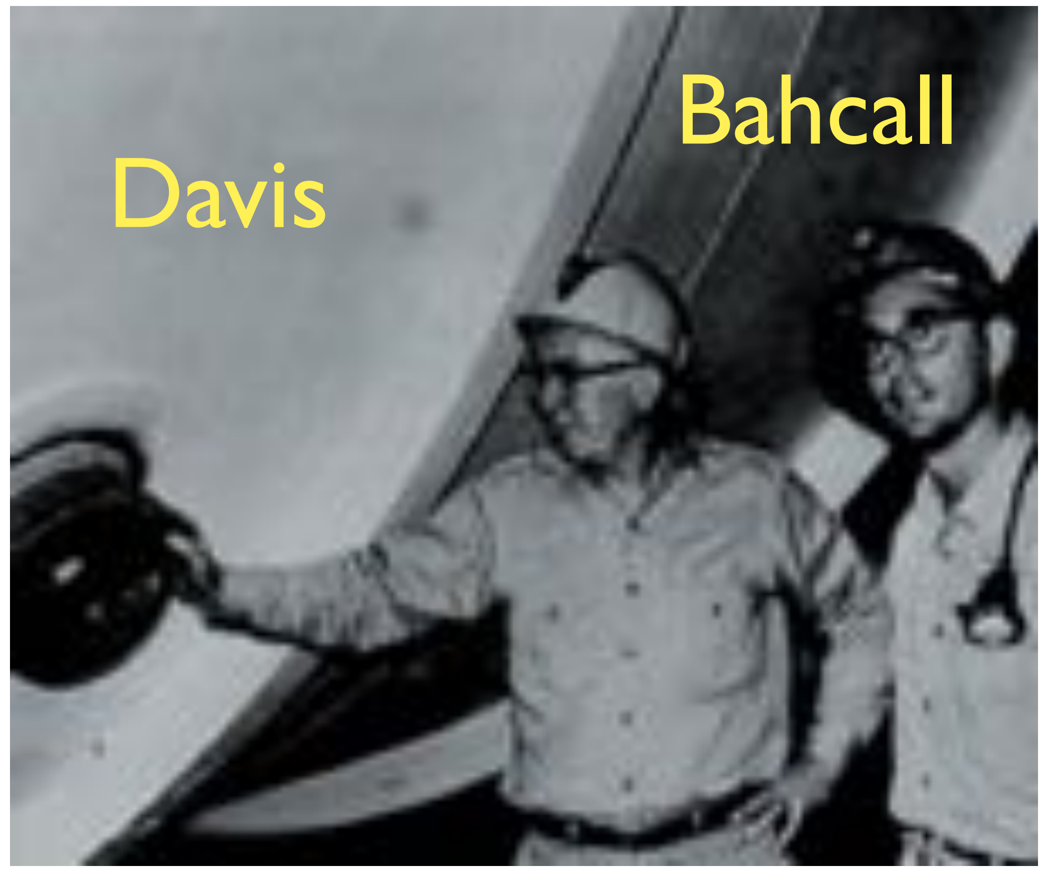
- 1962: ν_μ discovered by L. Lederman, M. Schwartz and J. Steinberger
  (NP 1988…awarded before the NP for the (anti)-ν_e !)

- 1967-69: First pheno elaboration of flavour oscillations and thoughts of connection to the solar problem
  Pontecorvo, Gribov (Влади́мир Нау́мович Гри́бов)

A (brief) ν history, IV

- 1964-1968: Deficit in solar ν flux measured by R. Davis (NP 2002) at Homestake if compared with the predicted solar ν fluxes (Bahcall et al.)

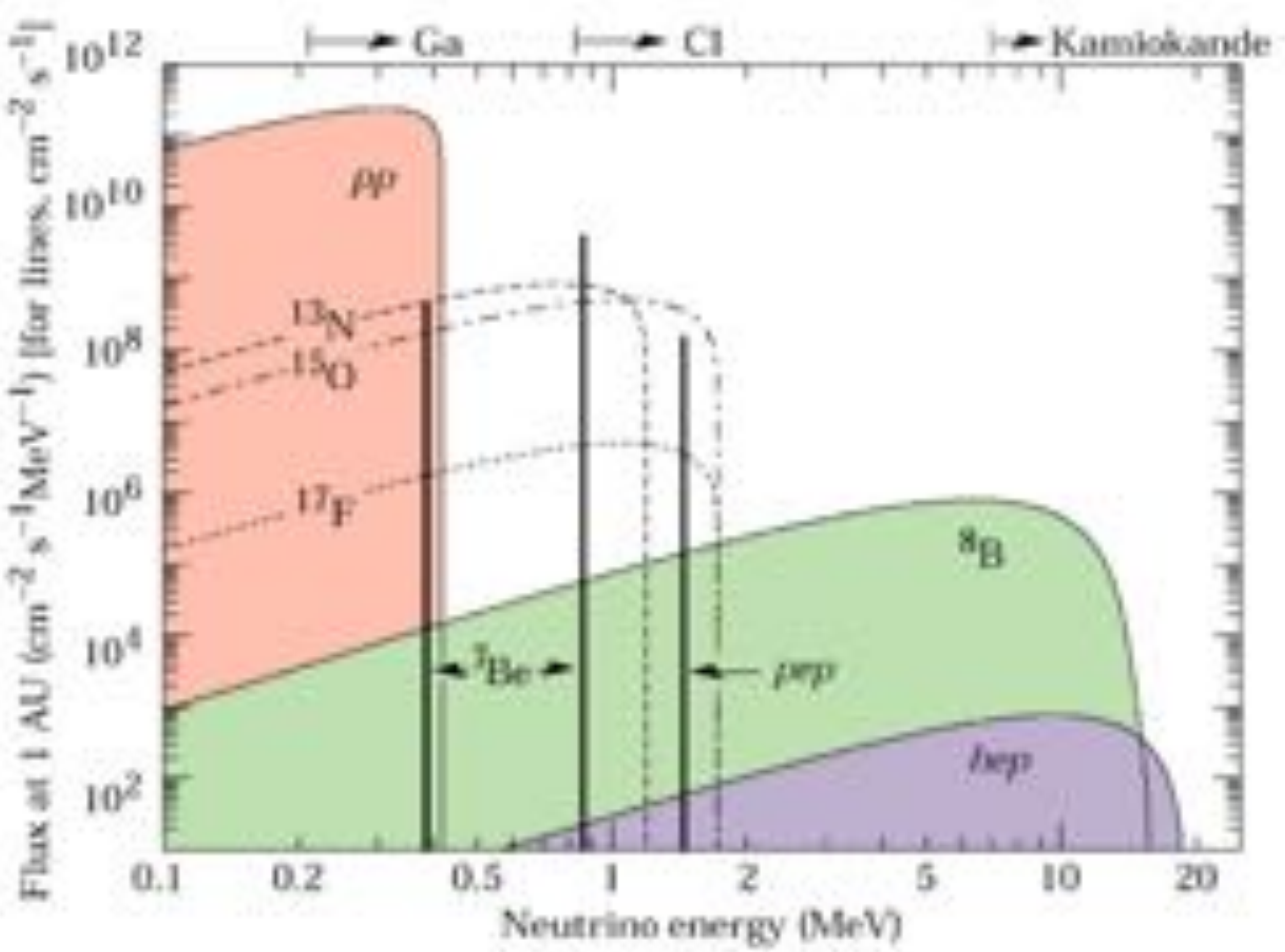


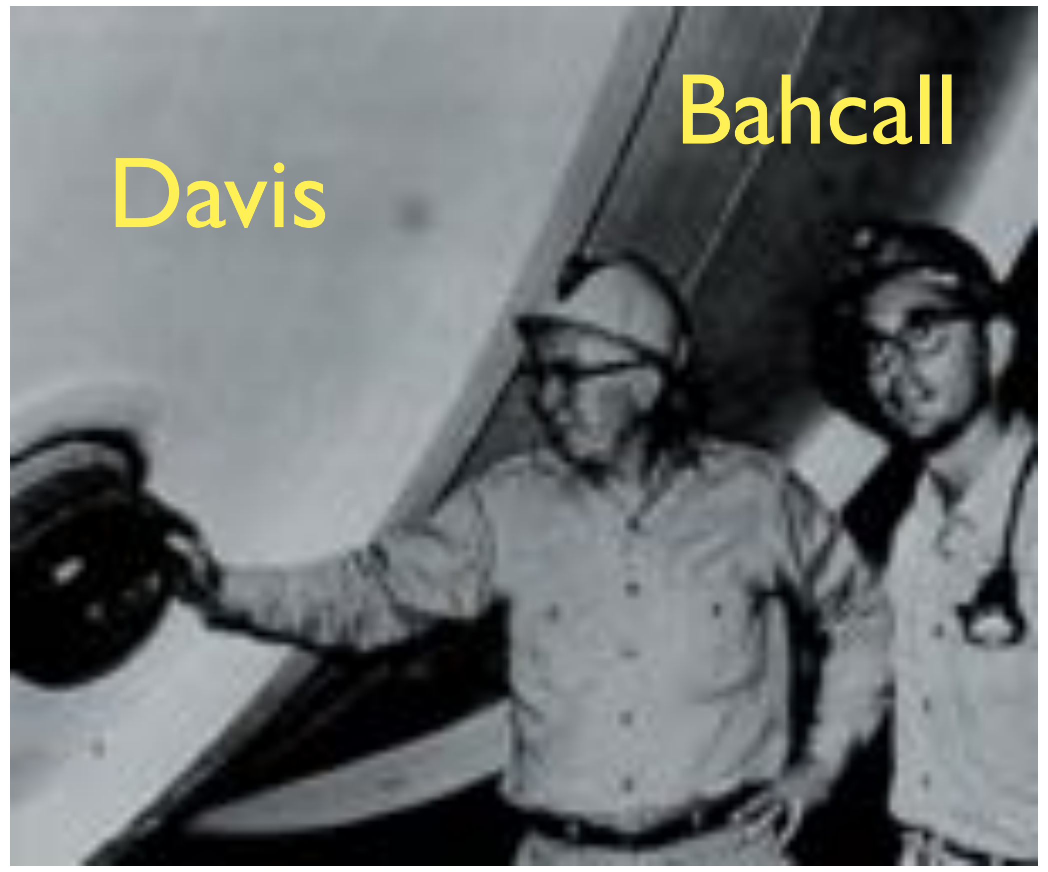


A (brief) ν history, IV

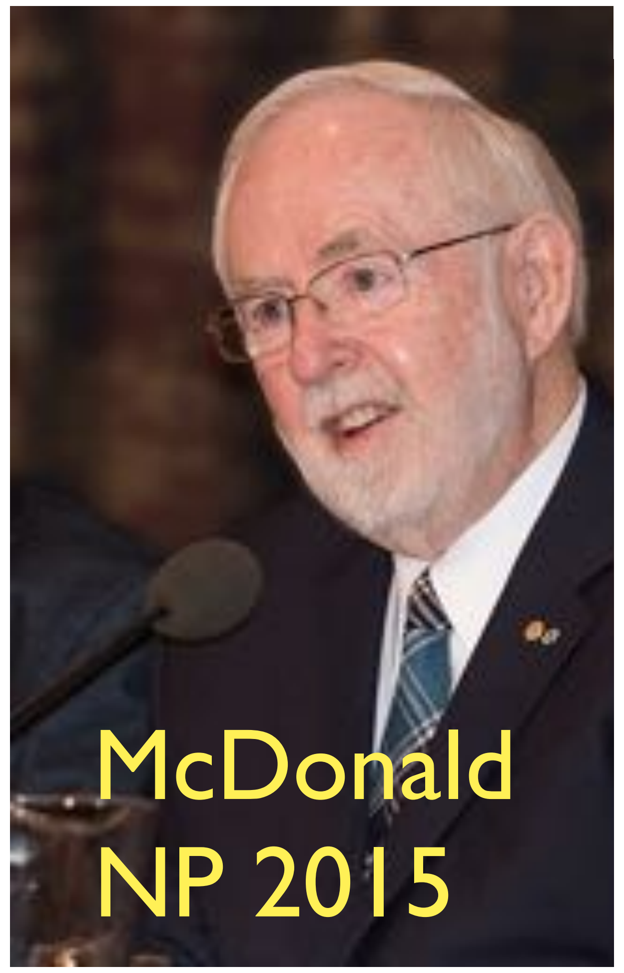
- 1964-1968: Deficit in solar ν flux measured by R. Davis (NP 2002) at Homestake if compared with the predicted solar ν fluxes (Bahcall et al.)

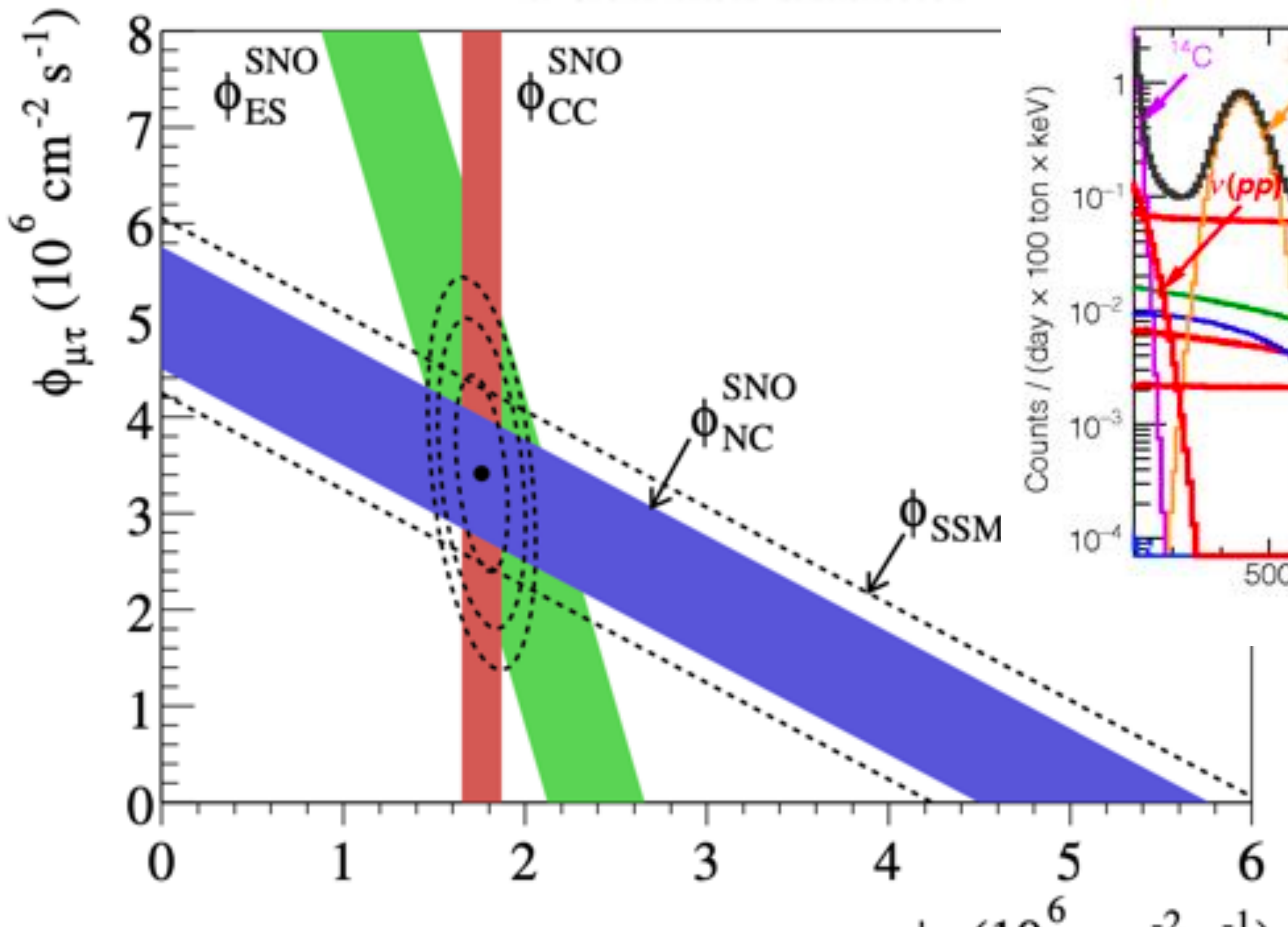
- Anomaly received further confirmation (SAGE, GALLEX, KamiokaNDE…) eventually interpretation due to mixing, sealed by SNO



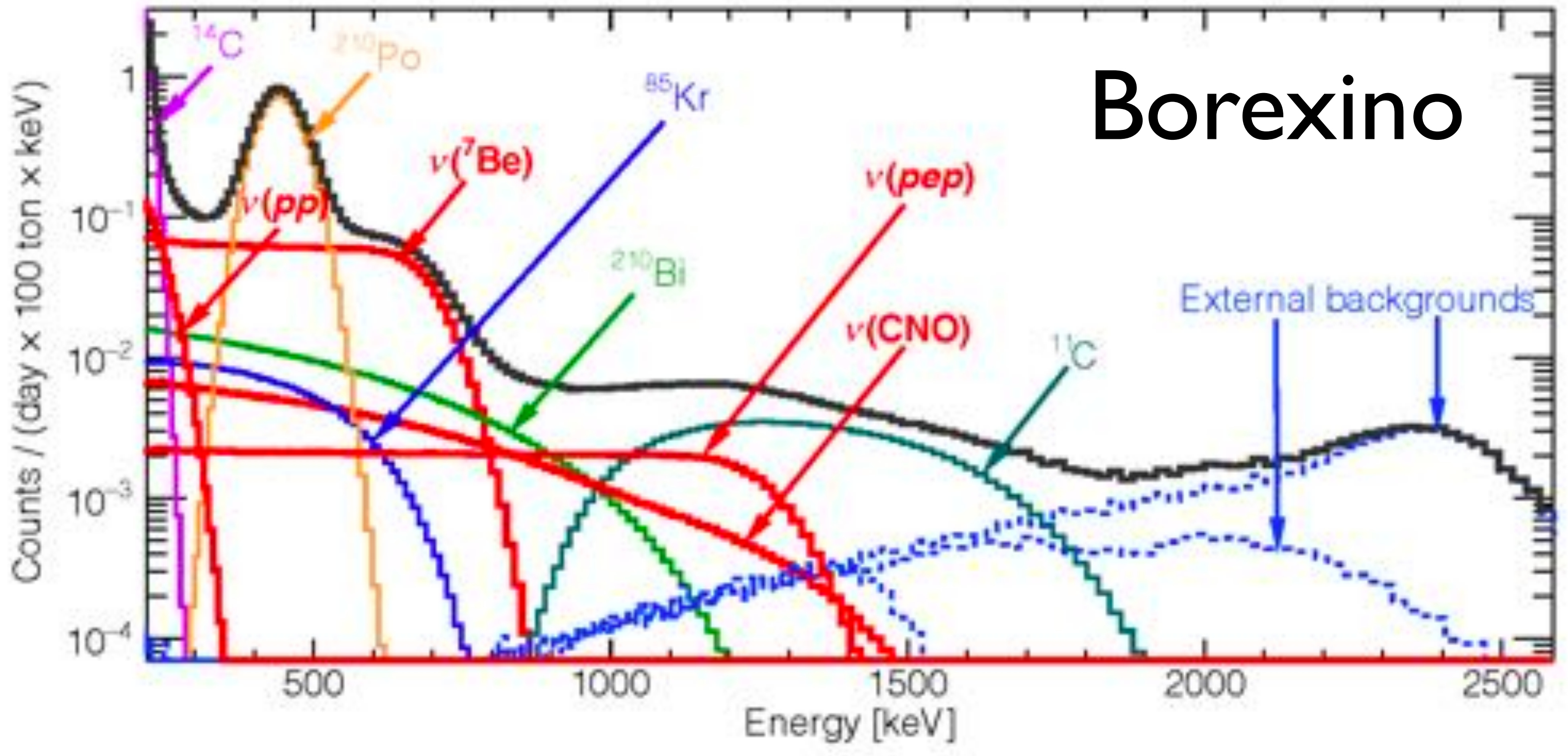


No p-decay, but solar ν & SN1987A





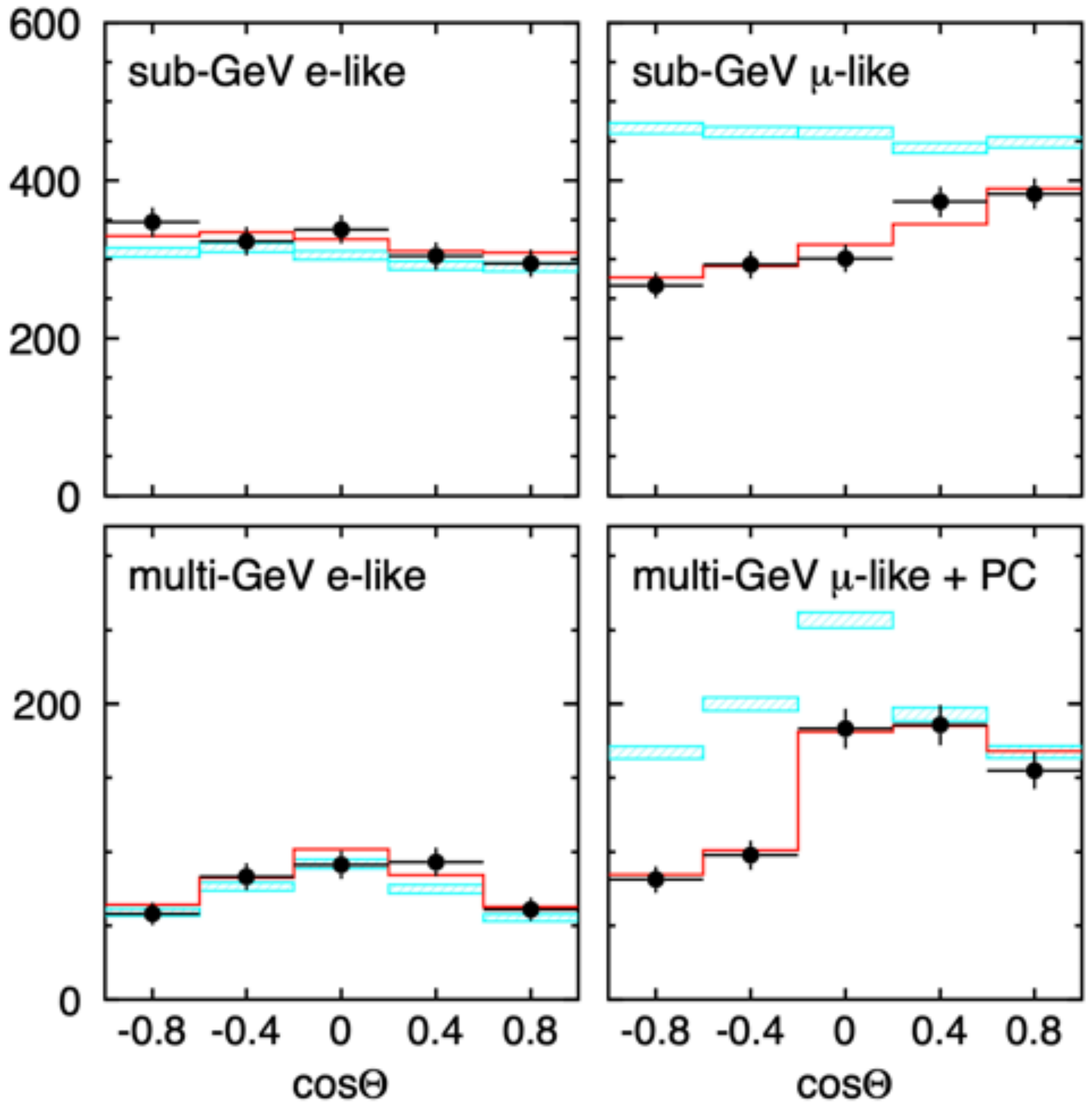
SNO, PRL 89 2002

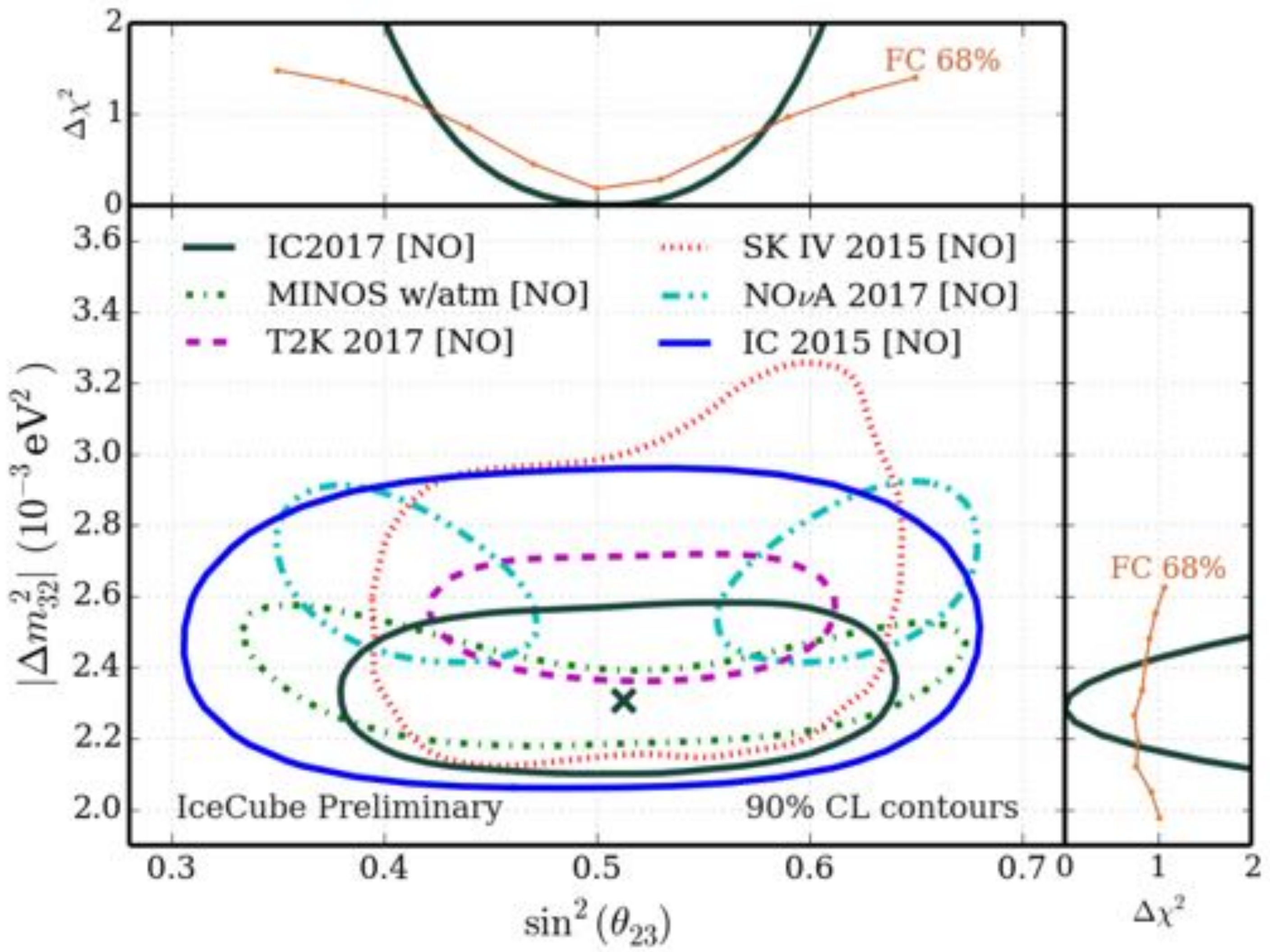


2020: even the CNO solar ν measured!

…In parallel, starting point of “modern ν physics”

- 1988: First convincing evidence for atmospheric neutrino anomaly [Kamiokande], confirmed e.g. by MACRO
- 1998: Strong evidence by SuperKamiokande, confirmed by Soudan2 & MACRO



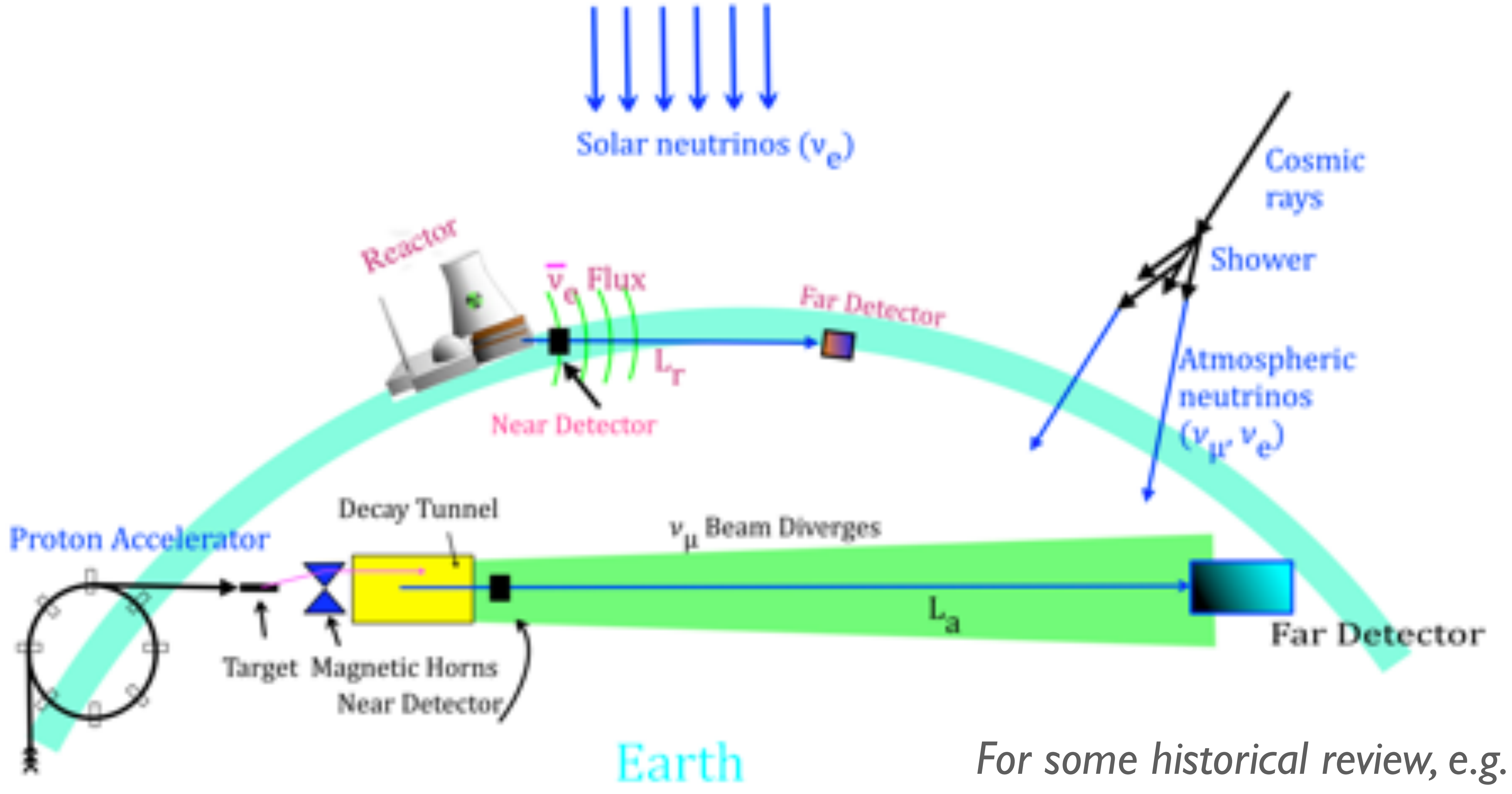


- Over the past decade, also HE telescopes (mostly astro!) joined these studies (Antares, 1206.0645, IceCube…)

… till long baseline & modern reactor ν projects

- K2K (1999-2004), T2K (2010-2021), MINOS (2005-2016), OPERA (2008-2012), NOνA (>2014) : Long baseline confirmation and refinement of the picture

- "Solar" parameters further explored by KamLAND (>2002) via 'long distance' studies of reactor fluxes.



For some historical review, e.g. Diwan et al. 1608.06237

- Greatly improved reactor experiments (…CHOOZ, Palo Verde…) eventually lead to the generation capable of measuring third mixing angle (From 2012: Daya Bay, Double Chooz, RENO…)

Some references

- G. Altarelli and K. Winter, *Neutrino Mass,* Springer, Berlin/Heidelberg, 2003, in particular B. Kayser's chapter on mixing and flavour change, hep-ph/0211134
- M. Fukugita, T. Yanagida, *Physics of Neutrinos and applications to astrophysics*, Springer 2003
- A. De Gouvea, *TASI lectures on neutrino physics*, hep-ph/0411274
- A. Strumia and F. Vissani, *neutrino masses and mixing and…* hep-ph/0606054.
- C. Giunti, C.W. Kim, *Fundamentals of Neutrino Physics and Astrophysics*, Oxford University Press 2007
- Z.-Z. Xing, S. Zhou, *Neutrinos in Particle Physics, Astronomy and Cosmology*, Springer 2011
- V. Barger, D. Marfatia, and K. Whisnant, *The Physics of Neutrinos*, Princeton University Press 2012

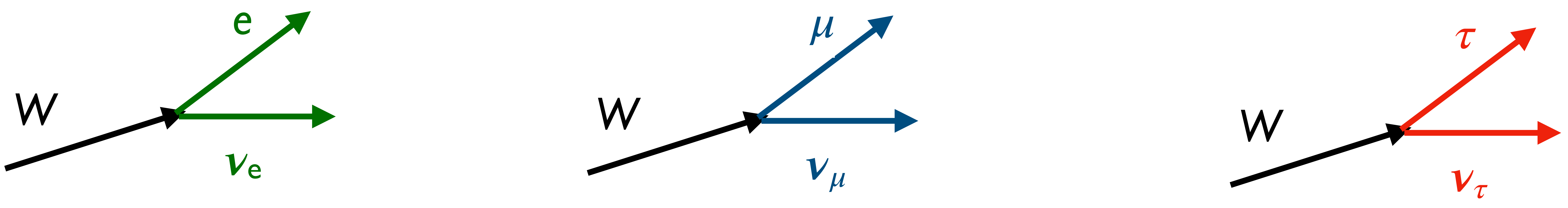
III. Neutrino oscillations (in vacuum)

The discovery that ν's have masses comes from the observation that ν's oscillate (due to lepton mixing), a concept which we now introduce

Oscillations experiments are also the primary tool to measure the parameters governing this new mass sector, hence we'll focus on some of their key aspects

(Cartoon) meaning of flavour

ν flavour **defined** via the charged current weak interaction vertex involved in its production/detection



The weak interaction couples the ν of a given flavour only to the charged lepton ℓ of the same flavour.

Note

The 'flavour' of charged leptons (typically studied/measured via their e.m. interactions) is 'defined' by their mass, which determines their properties, like their decays.

Cartoon translates into equations in the SM, of course!

From SM Weak interaction to Effective Fermi Theory

$$\frac{g}{\sqrt{2}}\left(J_W^\mu W_\mu^+ + J_W^{\mu\dagger} W_\mu^-\right) + \frac{g}{\cos\vartheta_W} J_Z^\mu Z_\mu$$

$$J_W^\mu \equiv \sum_{\text{gen.}} \bar{u}\gamma^\mu P_L d + \bar{\nu}\gamma^\mu P_L \ell, \qquad J_Z^\mu \equiv \sum_f \bar{f}\gamma^\mu \left(I_3^f P_L - \sin^2\vartheta_W Q_f\right) f, \quad \supset \quad \frac{1}{4}\sum_\alpha \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5)\nu_\alpha$$

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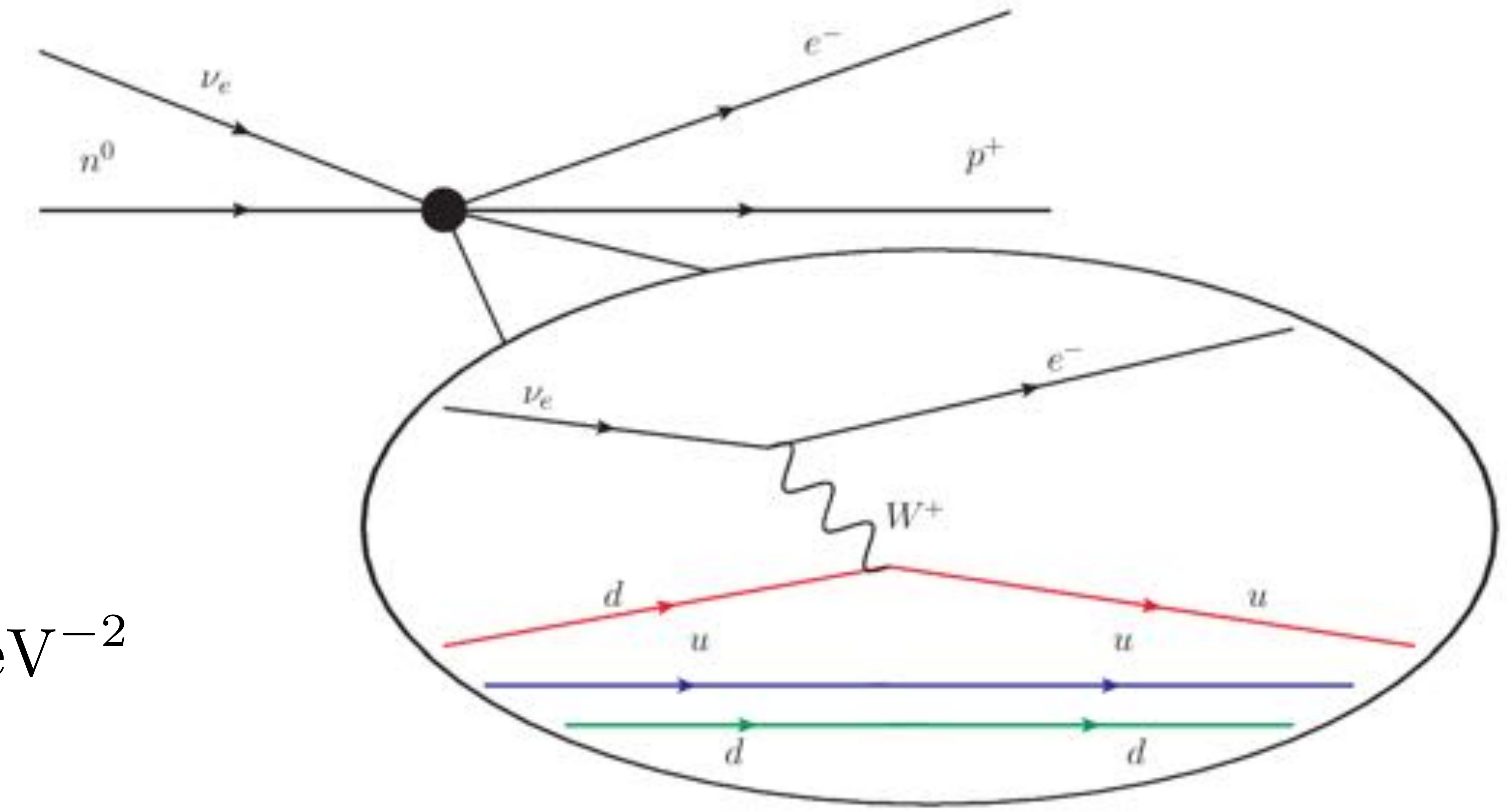
For phenomenology at $E \ll M_W, M_Z$, useful to 'integrate out' the gauge bosons

(set their kinetic term to zero, neglect all terms that involve more than two heavy particle like triple and quartic gauge couplings, gauge-Higgs interactions, as well as currents with the top quark)

$$\frac{\partial \mathcal{L}}{\partial W_\mu^+} = \frac{g}{\sqrt{2}} J_W^\mu + M_W^2 W^{-\mu} = 0, \qquad \frac{\partial \mathcal{L}}{\partial Z_\mu} = \frac{g}{\cos\vartheta_W} J_Z^\mu + M_Z^2 Z^\mu = 0.$$

$$\mathcal{L}_{\text{weak}}^{\text{eff}} = -2\sqrt{2} G_F (J_W^\mu J_{W\mu}^\dagger + J_Z^\mu J_{Z\mu}); \qquad G_F \equiv \frac{\sqrt{2}g^2}{8M_W^2} \simeq 1.166 \times 10^{-5}\,\text{GeV}^{-2}$$



Lepton number conservation

The SM Lagrangian is invariant under a global U(1) transformation for each generation (each α)

$$\ell_\alpha \to e^{i\phi}\ell_\alpha \qquad \nu_\alpha \to e^{i\phi}\nu_\alpha$$

The associated conserved quantum number (via Noether's theorem) is the generation Lepton number L_α, whose sum is the (global) lepton number L

Number operators, counting # leptons - antileptons

$$L = \sum_{\text{gen}} L_\alpha = \sum_{\text{gen}} \int \mathrm{d}x^3 \left[\nu_\alpha^\dagger(x)\nu_\alpha(x) + \ell_\alpha^\dagger(x)\ell_\alpha(x)\right]$$

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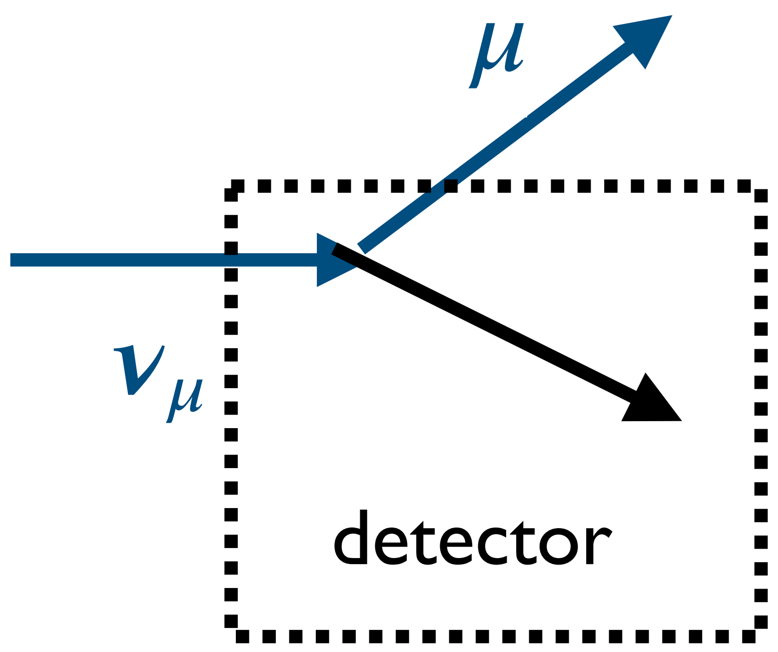
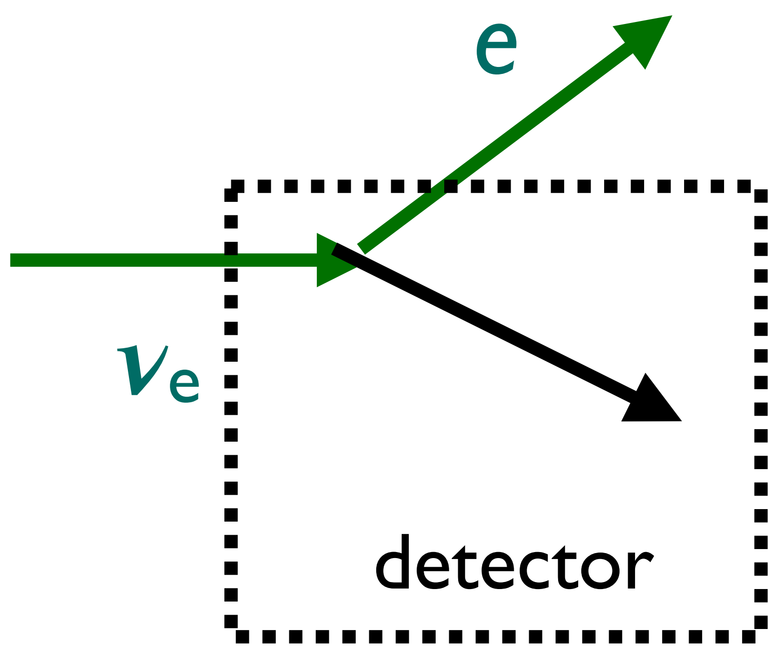
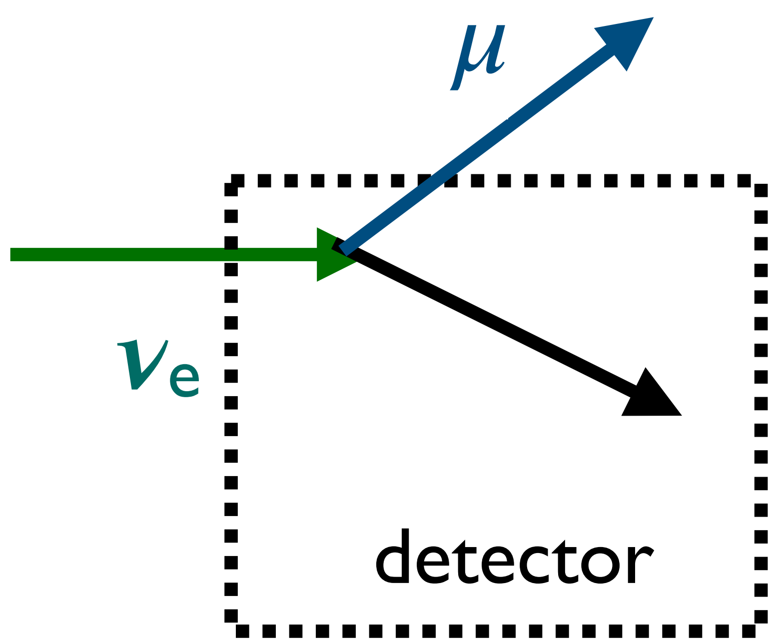
This formalism translates the experimental evidences (over several decades!) that the ν flavour at detection is the same as it was at production

E.g. we see





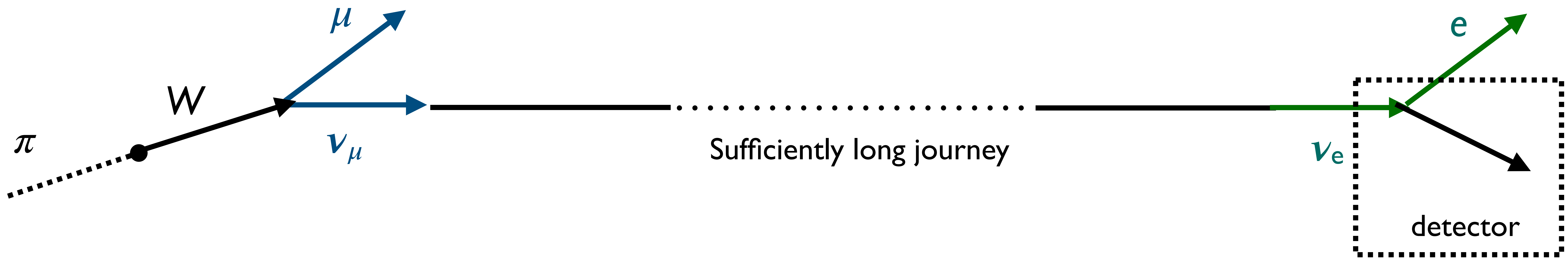
we **do not** see



…**do we?**

Violation of L_α's conservation in ν experiments!

…until evidence collected that, if you make ν's propagate long enough, this **may not** be true! E.g. can have:



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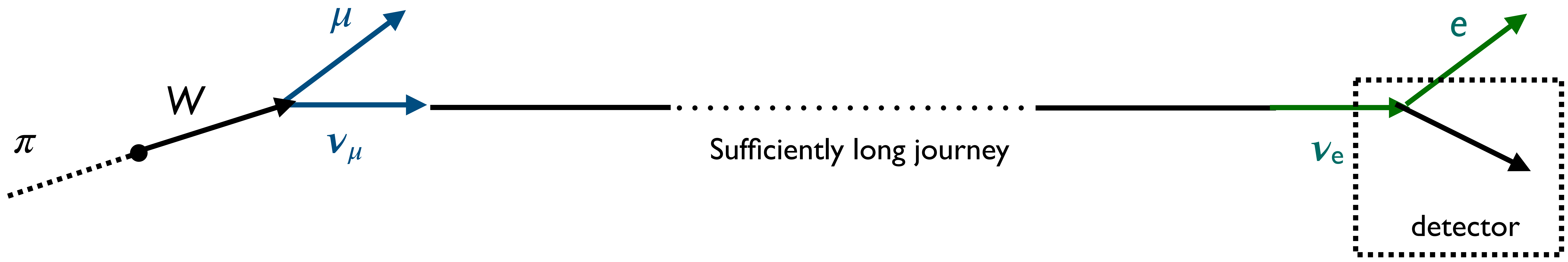

Will see that this requires *leptonic mixing and that (some) ν mass $\neq 0$*

Something non-trivial in flavour space must happen in the propagation of the (free) ν's.

We know how to describe the propagation of mass eigenstates m_i (eigenstates of the Hamiltonian) which we denote ν_i

Free ν propagation & Leptonic mixing

Free ν_i obey Dirac eq.

$$(i\partial\!\!\!/ - m)\psi(x) = 0$$

Solved in terms of plane waves

$$\psi(x) = u(p)e^{-i\,p\cdot x}$$

ct=x

with dispersion relation

$$E^2 = p^2 + m^2 \Rightarrow E \simeq p + \frac{m^2}{2p}$$

ultra-relativistic limit

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Note: vacuum evolution equivalent to ν states evolving as $$i\frac{\partial}{\partial t}\psi = E\psi \simeq \left(p + \frac{m^2}{2p}\right)\psi$$

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Complete bases in flavour and mass space are related by a unitary matrix *U* (PMNS for *N*=3)

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In terms of ν fields

$$\psi_\alpha = \sum_{i=1,2,3} U_{\alpha i}\psi_i$$

In terms of single-ν state

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U^*_{\alpha i}|\nu_i\rangle$$

Since $|\nu\rangle = \psi^\dagger|0\rangle$

Anti-ν are instead created as

$$|\bar{\nu}\rangle = \psi|0\rangle$$

Hence

$$|\bar{\nu}_\alpha\rangle = \sum_{i=1,2,3} U_{\alpha i}|\bar{\nu}_i\rangle$$

For anti-ν, $U \to U^*$

ν_α's of definite flavour (i.e. associated to a given charged lepton mass) are not mass eigenstates.

The mixing matrix and its meaning

We can thus rewrite the CC weak interaction in the massive ν basis as (now these indicates ν fields!)

$$\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

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(

Theorist's rant

Technically, once taking into account the fact that one may also rotate the charged lepton basis, the *U* entering the W ν ℓ coupling is given by

$$U = U_{\nu}\, U_{\ell}^{\dagger}$$

Hence, properly one should talk of leptonic (rather than ν) mixing

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Meaning of U

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

e-row: *linear combination of mass states that couple to the electron.*

Second column: *linear combination of charged leptons that couples to* ν_2

… and so on, you get the idea!

Reminder: Quark mixing matrix

Flavour basis defined by the weak interactions. $\left(\frac{v+h}{\sqrt{2}}\right)\bar{\mathbf{u}}_L\mathbf{Y}_u\mathbf{u}_R$ After EWSB,

Mass basis defined by the Yukawa term $\left(\frac{v+h}{\sqrt{2}}\right)\bar{\mathbf{d}}_L\mathbf{Y}_d\mathbf{d}_R$ **bold** = matrices in **flavour space**

Mass matrices $\mathbf{M}_u = \mathbf{Y}_u v/\sqrt{2}$, $\mathbf{M}_d = \mathbf{Y}_d v/\sqrt{2}$ can be diagonalized by *biunitary transformations*

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Mass basis defined by the Yukawa term

$$\left(\frac{v+h}{\sqrt{2}}\right)\bar{\mathbf{u}}_L\mathbf{Y}_u\mathbf{u}_R$$

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Mass matrices $\mathbf{M}_u = \mathbf{Y}_u v/\sqrt{2}$, $\mathbf{M}_d = \mathbf{Y}_d v/\sqrt{2}$ can be diagonalized by *biunitary transformations*

Denote with primes the fields in terms of which the mass is diagonal

$$\mathbf{u}'_L = V_L^u \mathbf{u}_L$$

$$\mathbf{u}'_R = V_R^u \mathbf{u}_R$$

$$\mathbf{M}_u^{\text{diag}} = V_L^{u\dagger}\mathbf{M}_u V_R^u$$

Now, how does the weak current rewrite?

$$J_W^\mu \equiv \sum_{\text{gen.}} \bar{u}\gamma^\mu P_L d + \bar{\nu}\gamma^\mu P_L \ell,$$

$$J_W^\mu \supset \sum_{\text{flavors}} \bar{u}'_L \left(V_L^{u\dagger}V_L^d\right)\gamma^\mu P_L d'_L$$

Convention here:

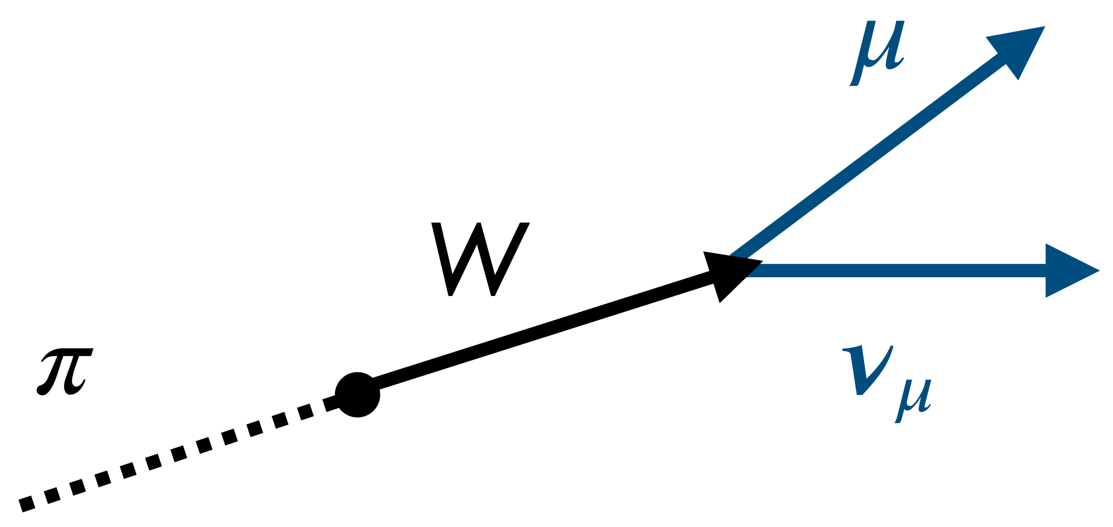
$$U_{\text{CKM}} \equiv V_L^{u\dagger}V_L^d$$

The product of unitary matrix affecting **up and down left quark fields** now enters (CKM matrix)

Note: Rotations of the right-handed fields have no physical consequence in the SM!

Towards ν oscillations (in vacuum)

Let's make sense of our previous scheme:



Production:
Flavour state, i.e.
coherent combination of mass states

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} U^*_{\alpha i}|\nu_i\rangle$$

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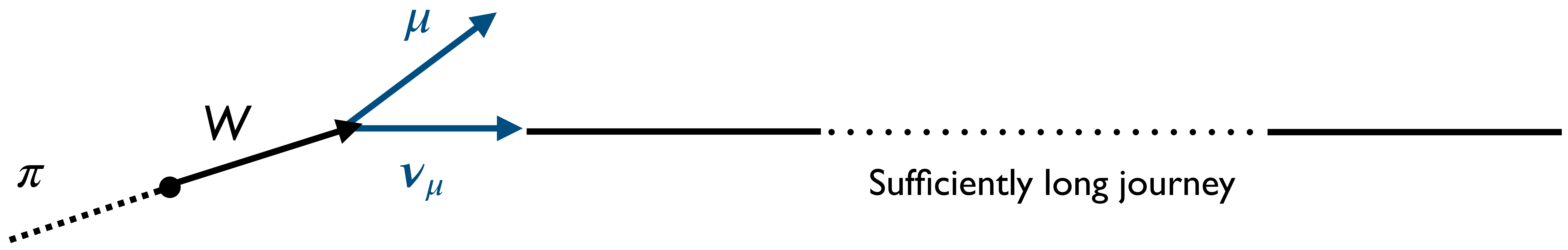

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Propagation:
Each mass state propagates independently, relative phases build-up

$$|\nu_k(t)\rangle = e^{-iE_k t}|\nu_k\rangle$$

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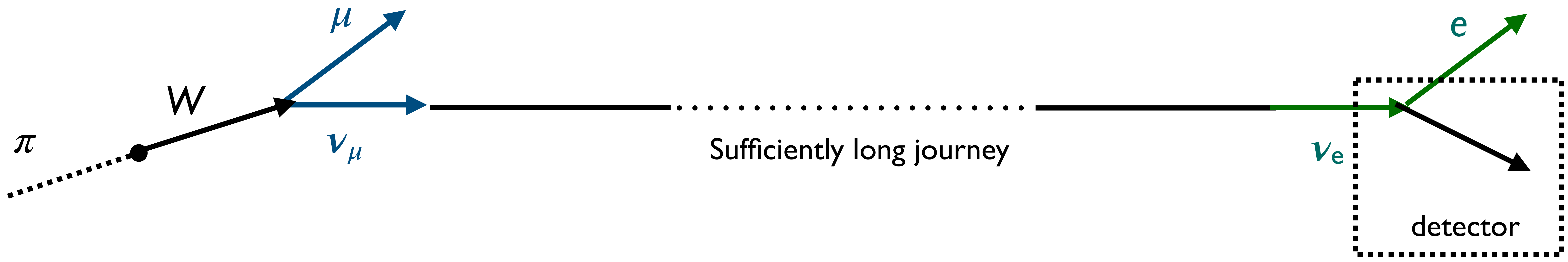

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Measure flavour, which ones depends on the combination of mass states here

Project $\langle\nu_\beta|$

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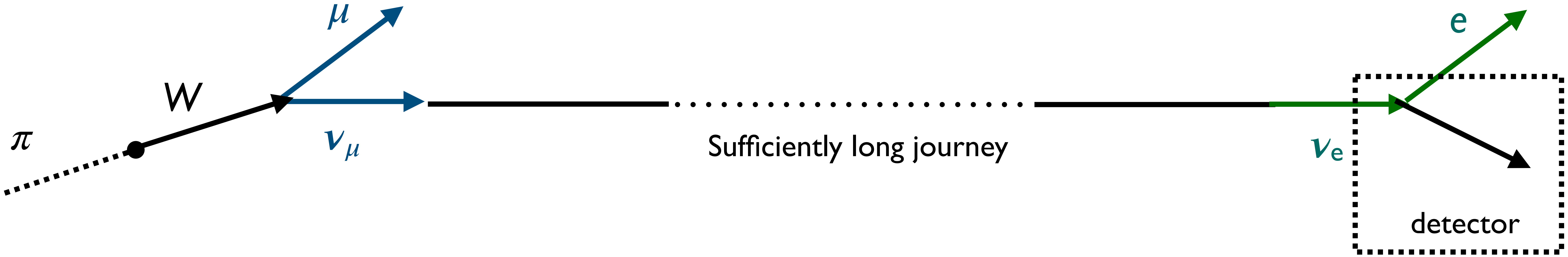

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Key quantity, the transition amplitude $A_{\nu_\alpha\to\nu_\beta}(t) \equiv A_{\alpha\beta} = \langle\nu_\beta|\nu_\alpha(t)\rangle = \sum_k U^*_{\alpha k}U_{\beta k}e^{-iE_k t}$

ν oscillation (in vacuum): The basic math

The transition probability is the modulus square of the amplitude:

$$P_{\alpha\to\beta}(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = A^*_{\alpha\beta}A_{\alpha\beta} = \sum_{j,k} J^{\alpha\beta}_{kj}\, e^{-i(E_k-E_j)t}$$

where we introduced the *quartic rephasing invariant* $J^{\alpha\beta}_{kj} \equiv U^*_{\alpha\,k}U_{\beta\,k}U_{\alpha\,j}U^*_{\beta\,j}$ *e.g. E.E. Jenkins and A.V. Manohar, Nucl. Phys. B792 (2008) 187, 0706.4313.*

encoding the information of the mixing matrix independent of phase redefinitions of the lepton fields

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encoding the information of the mixing matrix independent of phase redefinitions of the lepton fields

The double sum can be split in $\sum_{j=k}+\sum_{j>k}+\sum_{k>j}$ the latter two terms are the sum of two complex conjugate expressions, hence

$$P_{\alpha\to\beta}(t) = \sum_j |U_{\alpha\, j}|^2|U_{\beta\, j}|^2 + 2\Re\left[\sum_{k>j} J^{\alpha\beta}_{kj}\, e^{-i(E_k-E_j)t}\right] .$$

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The transition probability is the modulus square of the amplitude:

$$P_{\alpha\to\beta}(t) = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = A^*_{\alpha\beta}A_{\alpha\beta} = \sum_{j,k} J^{\alpha\beta}_{kj}\, e^{-i(E_k-E_j)t}$$

where we introduced the *quartic rephasing invariant* $J^{\alpha\beta}_{kj} \equiv U^*_{\alpha\,k}U_{\beta\,k}U_{\alpha\,j}U^*_{\beta\,j}$

e.g. E.E. Jenkins and A.V. Manohar, Nucl. Phys. B792 (2008) 187, 0706.4313.

encoding the information of the mixing matrix independent of phase redefinitions of the lepton fields

The double sum can be split in $\sum_{j=k} + \sum_{j>k} + \sum_{k>j}$ the latter two terms are the sum of two complex conjugate expressions, hence

$$P_{\alpha\to\beta}(t) = \sum_j |U_{\alpha\,j}|^2|U_{\beta\,j}|^2 + 2\Re\left[\sum_{k>j} J^{\alpha\beta}_{kj}\, e^{-i(E_k-E_j)t}\right].$$

The standard derivation of the oscillation formula relies on the approximations for ultra-relativistic ν's

$$E^2 = p^2 + m^2 \Rightarrow E \simeq p + \frac{m^2}{2p}$$

And the expression is further simplified using the unitarity of *U* and the identities

$$\Re[(a+i\alpha)(b+i\beta)] = a\,b - \alpha\,\beta \qquad e^{ix} = \cos x + i\sin x \qquad \cos x = 1 - 2\sin^2(x/2)$$

ν oscillation (in vacuum): General formula

$$P_{\alpha\to\beta}(L) = \delta_{\alpha\beta} - 4\sum_{k>j} \Re J_{kj}^{\alpha\beta}\ \sin^2\left(\frac{\Delta m^2_{kj}\,L}{4\,E}\right) + 2\sum_{k>j} \Im J_{kj}^{\alpha\beta}\ \sin\left(\frac{\Delta m^2_{kj}\,L}{2\,E}\right)$$

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- Valid for arbitrary number of generations/mass states, provided that the bases are complete (unitarity used!)

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ν oscillations & CP violation

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- Measuring CP-violation requires observing the spectral dependence. If oscillations are averaged out (e.g. due to a poor energy resolution of the detector)

CP-conserving factor $\left\langle \sin^2\left(\frac{\Delta m_{kj}^2\, L}{4\,E}\right)\right\rangle = \frac{1}{2}$ *CP-violating factor* $\left\langle \sin\left(\frac{\Delta m_{kj}^2 L}{2E}\right)\right\rangle = 0$

Analogues of ν oscillations

Aanalogous to other quantum systems where the initial state is a **coherent superposition of eigenstates of the Hamiltonian:**

Spins: for example a state with spin up in the *z*-direction in a B-field aligned in the *x*-direction. This gives raise to *spin-precession*, i.e. the state changes the spin orientation with a typical oscillatory behaviour.

K /anti-K: difference between the mass/strong interaction eigenstates (ruling production) and the weak interactions eigenstates K_S, K_L, controlling the decay.

Photon polarization state can be written as a superposition of states with H and V linear polarisations, or as a superposition of states with R and L circular polarizations. Think of ν of a given flavour as being linearly polarised, while propagating ν as circularly polarized states (those have well defined propagation characteristics such as velocity). Allows for analogical realization of the "flavour oscillation phenomenon" with lasers, e.g. *arXiv:1001.2749*

Actually, I cheated!

Can derive the formulae e.g. assuming that ν can be described by plane-waves, with definite momentum (which implies spatially infinite sources!) or assuming that the interference of different *E*-states vanishes unless they have the same *E* (implying sources constant in time, since ever and forever)

production and detection are always localised and not eternal.

e.g. production in decay: the relevant timescale is the pion lifetime (or the time travelled in the decay pipe)

$$\Delta E \gtrsim \frac{1}{2\tau_\pi}$$

For details see, Akhmedov, Smirnov, 1008.2077; or textbooks, like Giunti and Kim's

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There is always an energy-momentum spread which can be correctly described by a wave-packet formalism
For oscillations to be measurable, we need to make sure that *coherence is preserved,* hence:

Error on *p*, *E* not small enough to measure the mass, $\sigma_p \gg \frac{\Delta m^2}{\langle p \rangle}$

packets at detection not spatially separated more than size $\sigma_x \gg \frac{\Delta m^2 L}{2\langle p \rangle}$ equivalent to *E*-spread condition $\frac{\Delta E}{E} \ll \frac{\ell_{\rm osc}}{L}$

As long as those hold (often if not always!) the previous formalism yields the correct results

For details see, Akhmedov, Smirnov, 1008.2077; or textbooks, like Giunti and Kim's

IV. Parameters of the mixing matrix

How many *physical* free parameters for *N* families?

Generic *NxN* complex matrix has 2 N^2 real parameters

Unitarity implies:

- Each row vector of unit length: *N* constraints
- Each couple of rows are orthogonal: *N(N-1)* constraints

Hence a Unitary *NxN* complex matrix has N^2 real parameters, but not all are physical!

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Rephasing a charged lepton field $\ell \rightarrow e^{i\theta} \ell$ leaves the mass term invariant, but changes by a phase $e^{-i\theta}$ a row of the matrix. Phases in *N* 'up' fields and *N* 'down' … means that 2*N-1* dofs* in *U are unphysical*

Final counting $N^2-(2N-1)= N^2-2N+1=(N-1)^2$ (for N=2 → 1 dof; for N=3 → 4 dofs)

*Naively, you may think it's 2*N*; but a common phase of all the up and down fields won't affect at all the mixing matrix, hence only 2*N* -1 dofs are unphysical.

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For Majorana ν's, instead of $2N-1$ *independent rephasings,* you can rephase only *N* (charged leptons), since if the active ν are Majorana particles, then no rephasing at all is allowed*.

$N^2-N=N(N-1)$ (for $N=2 \rightarrow$ 2 dof; for $N=3 \rightarrow$ 6 dofs)

*Naively, you may think it's 2*N*; but a common phase of all the up and down fields won't affect at all the mixing matrix, hence only 2*N* -1 dofs are unphysical.

*Easiest way to see this: Transformation $\nu \rightarrow e^{i\theta_q}\,\nu$ associated to Lepton number conservation, which is violated since a conserved charge would require $\nu^c \rightarrow e^{-i\theta_q}\,\nu^c$ inconsistent with the Majorana condition $\nu = \nu^c$

Mixing angles and phases

CP-even and CP-odd parameters are called angles and phases, respectively.

Equivalently, angles are the parameters in U when it is real, which means it's an orthogonal matrix: $U^T U=I$, a set of $N(N+1)/2$ conditions since (both sides are) symmetric.

The group O(N) has dimension N^2 -$(N(N+1)/2)$= $N (N - 1)/2$, hence this is the number of independent angles (for N=2 → 1 angle; for N=3 → 3 angles)

Hence, we have

$(N\text{-}1)^2$ -$N (N - 1)/2$=$(N - 1)(N - 2)/2$ phases for Dirac ν's (for N=2 → no phase; for N=3 → 1 phase)

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Leptonic CP violation requires phases. So, it can certainly take place in 3 generations, as for quarks, but may happen in 2 generations if ν's are Majorana.

A different question is: Can it arise in 2 generations in oscillation experiments?