SELF INTRODUCTION

Nguyen Anh Ky
Mathematical-, Highe Energy- and Astro-physics Lanboratory
Institute of physics
Vietnam academy of science and technology,
Hanoi

Institute of physics (IOP)

- established in 1969,
- one of the first two institutes on basic science of the **Vietnam academy of science and technology (VAST)** and Viet Nam,
- and now, one of 34 research institutes of the VAST with about 30 other units for basic research and technology.

Our research laboratory (IOP, VAST)

- **Mathematical physics**: Theory of groups (including *supergroups* and *quantum groups*) and related topics, theories of gravitation (*GR and extensions: SUGRA, $f(R)$-gravitation, etc.*), etc.
- **Particle physics** (theory & experiment): Standard model and beyond: *Higgs physics, neutrino physics, CP violation*, etc.
- **Gravitation and cosmology**: theories of gravitation and cosmology (*BH's, DM, DE, GW's, cosmological models*, etc.).
- **Collaborations**: Belle II *and* T2K (as well as LHC_ATLAS in the past).

Vietnam school of neutrino physics

Quy Nhon, 10 - 22 July 2019

STANDARD MODEL AND NEUTRINOS

Nguyen Anh Ky

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7. Quantum field theories in brief. .

Basic courses

- Group theory (for particle physics).

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References (for this stage)

1. L. Ryder, *"Quantum field theory"*, Cambridge university press, Cambridge, 1996.
2. T.-P. Cheng & L.-F. Li, *"Gauge theory for elementary particle physics*", Oxfrod university press, Oxford, 2006.
3. Ho Kim Quang & Pham Xuan Yem, *"Elementary particles and their interactions: concepts and phenomena*", Springer-Verlag, Berlin, 1998.
4. P. Pal, *"An introduction course of particle physics*", CRC press, 2014.
5. R. Mohapatra & P. Pal, *"Massive neutrinos in physics and astrophysics*", World Sci. Lect. Notes Phys. **72**, (2004) 1.
6. S. Bilenky, *"An introduction to physics of massive and mixed neutrinos*", Springer, Berlin, 2010.
7. Nguyen Anh Ky & Nguyen Thi Hong Van, *"Was the Higgs boson discovered?*", Commun. Phys. **25**, 1 (2015) [arXiv:1503.08630 [hep-ph]].

See also *"The review of particle physics*": http://pdg.lbl.gov .

Why neutrinos?

Origin and basic nature

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Beta decays and neutrinos

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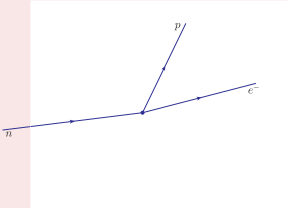

Figure: β-decays before the ν's introduction.

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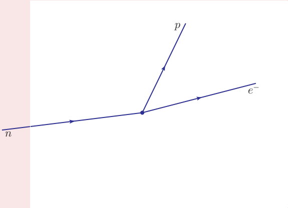

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Beta energy spectrum

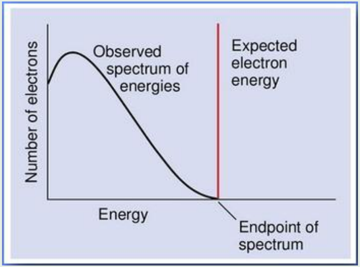

Figure: β energy spectrum [Ref. ?].

Neutrinos and Fermi's theory

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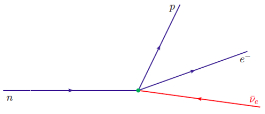

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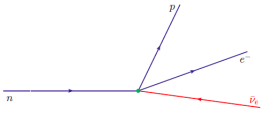

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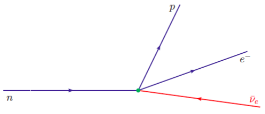

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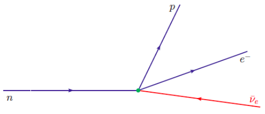

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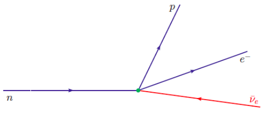

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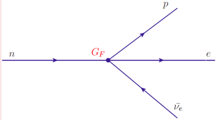
Fermi's theory



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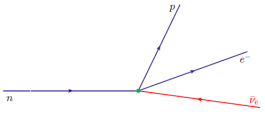

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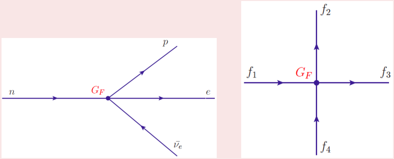

Figure: Fermi's four-fermion interactions.

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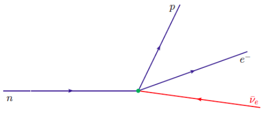

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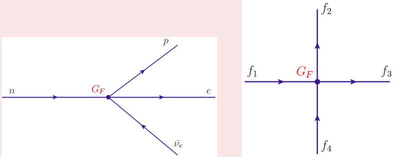

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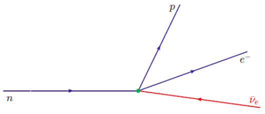

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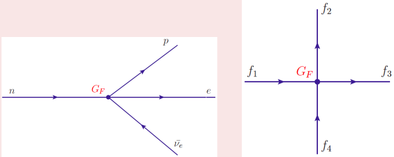

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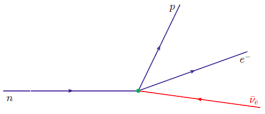

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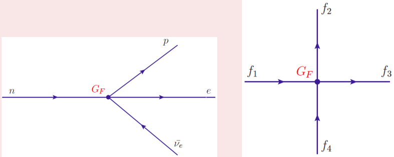

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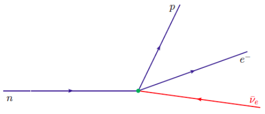

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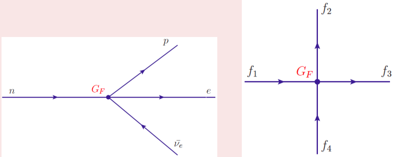

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by C. Cowan and F. Reines in 1956.

Symmetry and symmetry groups

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See also notes from the white boards!

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- All particles respect the space-time symmetry

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- Space-time symmetry groups:
  - **Lorentz group** (containing spacetime inversions, Lorentz boosts and spatial rotations): $O(1,3) \longrightarrow$ chiral (left-right spin) representation with algebra $su(2)_L \otimes su(2)_R$.
  - **Poincaré group** (containing space-time translations and Lorentz transformations): massive and massless representations $\Longrightarrow$ an arbitrary-spin massless particle has only one of two possible chiralities, either left-handed or right-handed, but not both simultaneously.
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Symmetry and symmetry groups

Symmetry could be

- External (space-time) or internal,
- Continuous or discrete,
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Symmetry groups.

- Some frequently used groups in physics:
  - **(Pseudo)-orthogonal groups**: $O(n)$, $SO(n)$, $O(p,q)$, $SO(p,q)$, etc.
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- All particles respect the space-time symmetry $\Longrightarrow$ all obey Klein-Gordon equation, and, for spin-1/2 particles, also Dirac equation.

Examples

Rotation in a 2D real or complex Euclidean space

$$\mathcal{O}_{2\times 2} = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix}.$$

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Rotation in a 3D real Euclidean space

$$\mathcal{O}_{3\times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Examples (cont.)

Transformation in an n-D real or complex space

$$x \longrightarrow x' = \mathcal{O}x$$

$\mathcal{O}$ - orthogonal matrix:

$$x^2 = x'^2 \Longleftrightarrow \mathcal{O}^T\mathcal{O} = I \Longrightarrow$$
$$\det\mathcal{O} = \pm 1.$$

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$$|z|^2 = |z'|^2 \Longleftrightarrow \mathcal{U}^\dagger\mathcal{U} = I \Longrightarrow |\det\mathcal{U}| = 1.$$

Quantum field theories in brief

Klein-Gordon equation

The Klein-Gordon equation for a (free) field of mass m

$$(\Box + m^2)\phi(x) = 0 \qquad (1)$$

can be obtained from a Lagrangian which in the case of a scalar ϕ has the form

$$\mathcal{L}_\phi = \frac{a}{2}(\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \frac{a}{2} m^2 \phi^\dagger \phi, \qquad (2)$$

where $a = 1$ for a real ϕ and $a = 2$ for a complex ϕ.

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Dirac equation

The Dirac equation for a (free) spinor of mass m

$$(i\gamma_\mu\partial^\mu - m)\psi(x) = 0 \qquad (3)$$

can be obtained from the Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi. \qquad (4)$$

Quantum field theories in brief

Space-time symmetry: some notations

Lorentz index: can take a value in [0,1,2,3]; for example, $\mu = 0, 1, 2, 3$.

Summation rule:

$$x_\mu y^\mu \equiv \Sigma_{\mu=0}^{3} \; x_\mu y^\mu .$$

Metric (for Minkowski space-time):

$$\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1), \;\; \eta^{\mu\lambda}\eta_{\lambda\nu} = \delta^\mu_\nu .$$

Lower and upper index:

$$v_\mu = \eta_{\mu\nu} v^\nu , \quad v^\mu = \eta^{\mu\nu} v_\nu .$$

Scalar product between 4-vectors, say, a_μ and b_μ:

$$a.b \equiv \eta_{\mu\nu} a^\mu b^\nu = a_\mu b^\mu = a_0 b_0 - \Sigma_{i=1}^{3} a_i b_i .$$

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$$\partial_\mu = \frac{\partial}{\partial x^\mu} , \ \Box = \partial_\mu \partial^\mu .$$

Energy-momentum vector (4-momentum):

$$p^\mu = (p^0, p^k) \equiv (\frac{E}{c}, p^k), \ \ k = 1, 2, 3,$$

$$\text{(quantization: } p_\mu \longrightarrow i\partial_\mu).$$

Pauli matrices: $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3) \equiv \sigma^k$,

Dirac matrices: $\gamma^\mu = (\gamma^0, \vec{\gamma}) \equiv (\gamma^0, \gamma^k)$,

$$\text{in the chiral basis: } \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} ,$$

$$\sigma^\mu = (1, \sigma^k), \ \ \bar{\sigma}^\mu = (1, -\sigma^k), \ \ \sigma^0 = 1.$$

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Example: Electrons are massive spinors, thus, desribed by Dirac spinors.

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Dirac eq. for massless particles

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Dirac eq. for massless particles

Eq. (5) at $m = 0$ is broken into two Weyl equations:

$$\bar{\sigma}_\mu \partial^\mu \xi_L(x) = 0, \tag{7}$$

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$$\psi^c \equiv C\psi^*, \tag{9}$$

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Quantum field theories in brief

Dirac Lagrangian

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$$\mathcal{L}_{D_0} = \overline{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x), \tag{11}$$

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$\Longrightarrow$ **to be considered in the next section!**.

GAUGE THEORY

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1. Abelian gauge theory

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2. Non-Abelian gauge theory

Abelian gauge theory

$U(1)$-gauge transformations

As stated above, the Lagrangian

$$\mathcal{L}_{D_0} = \overline{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x), \tag{4}$$

is not invarianr under the local $U(1)$-transformations

$$\psi \to \psi' = U(x)\psi(x), \quad U(x) = e^{-i\alpha(x)}, \tag{12}$$

$$\mathcal{L}_{D_0} \to \mathcal{L}'_{D_0} = \mathcal{L}_{D_0} + \Delta\mathcal{L}_{D_0}, \tag{13}$$

$$\partial_{\mu}\psi \to (\partial_{\mu}\psi)' \neq U(x)\partial_{\mu}\psi \Longrightarrow \Delta\mathcal{L}_{D_0} \neq 0 :$$

$$\Delta\mathcal{L}_{D_0} = ... \tag{14}$$

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$$\Delta\mathcal{L}_{D_0} = ... \tag{14}$$

We need to add something to compensate $\Delta\mathcal{L}_{D_0}$ in order to recover the symmetry !

Abelian gauge theory

$U(1)$-gauge symmetry

Abelian gauge theory

$U(1)$-gauge symmetry

Replace the derivative ∂_μ in (11) by the so-called **covariant derivative** D_μ:

$$D_\mu = \partial_\mu + ieA_\mu(x)$$

with $A_\mu(x)$ a new field transforming under (12) as

$$A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x).$$

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Thus, the resulting Lagrangian

$$\mathcal{L}_D = \overline{\psi}(x)\left(i\gamma^\mu D_\mu - m\right)\psi(x), \qquad (15)$$

is invariant under the gauge transformations (12)

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The $U(1)$-symmetry is recovered !

Abelian gauge theory

$U(1)$-gauge theory

To give the field $A_\mu(x)$ an own life $\Longrightarrow$ add its kinetic term $\mathcal{L}_{gauge}$ to (15):

Abelian gauge theory

$U(1)$-gauge theory

To give the field $A_\mu(x)$ an own life $\Longrightarrow$ add its kinetic term $\mathcal{L}_{gauge}$ to (15):

$$\mathcal{L}_{tot} = \mathcal{L}_D + \mathcal{L}_{gauge} = \overline{\psi}(x)\left(i\gamma^\mu D_\mu - m\right)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{16}$$

$$\mathcal{L}_{gauge} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{17}$$

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It is an example of an $U(1)$-gauge theory with A_μ being a gauge field.

Note: there is no mass term for A_μ in (16) as either it is not needed (for a massless gauge field, e.g., the electromagnetic (EM) field, see below) or it violates the gauge symmetry.

Abelian gauge theory

$U(1)$-gauge theory

Note: $\mathcal{L}_{tot}$ is exactly the Lagrangian of an EM interaction,

$$\begin{aligned}\mathcal{L}_{tot} &= \mathcal{L}_{free} + \mathcal{L}_{interac} \\ &= \overline{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - e\overline{\psi}(x)\gamma^{\mu}\psi(x).A_{\mu}(x), \qquad (18)\end{aligned}$$

with the interaction term

$$\mathcal{L}_{interac} = -e\overline{\psi}(x)\gamma^{\mu}\psi(x).A_{\mu}(x) = -eJ^{\mu}(x).A_{\mu}(x), \qquad (19)$$

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It is a current×field interaction familiar in electrodynamics with the EM field being a gauge field.

Non-Abelian gauge theory

$SU(2)$ gauge symmetry

Start again with a Lagrangian of type (11)

$$\mathcal{L}_{\mathcal{D}_0} = \overline{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x), \tag{21}$$

Non-Abelian gauge theory

$SU(2)$ gauge symmetry

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The Lagrangian (21) is invariant under a global- but not local $SU(2)$ gauge transformation,

$$\psi^{'}(x) = U(x)\psi(x), \quad U(x) = e^{-\frac{i}{2}\ \vec{\theta}(x).\vec{\sigma}}, \tag{23}$$

where, $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3) \equiv \sigma_i$ are Pauli matrices, and where, $\theta_i(x),\ \ i = 1, 2, 3,$ are arbitrary functions of x.

Note: the vector notation here should not be confused with that for an ordinary 3D-space vector.

Non-Abelian gauge theory

$SU(2)$ gauge symmetry

Following the way for the Abelian case, introduce the covariant derivative

$$\mathcal{D}_\mu = \partial_\mu + \frac{1}{2} ig\vec{\sigma}.\vec{A}_\mu, \tag{24}$$

with $\vec{A}_\mu$ transforming under (23) as

$$\vec{A}_\mu(x) \to \vec{A}_\mu^{\,\prime}(x) = \vec{A}_\mu(x) - \frac{1}{g}\partial_\mu \vec{\theta}(x) - \vec{\theta}(x) \times \vec{A}_\mu(x). \tag{25}$$

Replacing ∂_μ in (11) by $\mathcal{D}_\mu$, the resulting Lagrangian

$$\mathcal{L}_{\mathcal{D}} = \overline{\psi}(x) \left(i\gamma^\mu \mathcal{D}_\mu - m \right) \psi(x), \tag{26}$$

is invariant under the local $SU(2)$-transformation (23). Thus,

the $SU(2)$-symmetry is recovered.

Non-Abelian gauge theory

$SU(2)$-gauge theory

Adding a free Lagrangian of $\vec{A}_\mu$ to (26) we get the total Lagrangian

$$\mathcal{L}_{Tot} = \overline{\psi}(x)\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi(x) - \frac{1}{4}\vec{F}_{\mu\nu}(x).\vec{F}^{\mu\nu}(x), \tag{27}$$

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NOTE: there is no mass term for $\vec{A}$, as it violates the $SU(2)$-gauge invariance.
There is also a term of current×field interaction (as in the Abelian case),

$$\mathcal{L}_{Int} = -g\vec{J}_\mu(x).\vec{A}^\mu(x) = -g\sum_{i=1}^{3} J^i_\mu A^\mu_i(x), \tag{29}$$

$$J^i_\mu(x) = \overline{\psi}(x)\gamma_\mu \frac{1}{2}\tau^i\psi(x), \tag{30}$$

but there is a self-interaction term $\mathcal{L}_{s.i.}$ which is absent in the Abelian case

$$\mathcal{L}_{s.i.} \propto -2g\left[\partial_\mu \vec{A}_\nu(x) - \partial_\nu \vec{A}_\mu(x)\right].\left[\vec{A}^\mu \times \vec{A}^\nu\right] + g^2\left[\vec{A}_\mu \times \vec{A}_\nu\right].\left[\vec{A}^\mu \times \vec{A}^\nu\right]. \tag{31}$$

Non-Abelian gauge theory

Electroweak theory as a non-Abelian gauge theory

An example of a gauge theory is the electroweak (EW) model with the gauge group being a direct product of an Abelian subgroup and a non-Abelian subgroup:

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Standard model

The **standard model** (SM) based on the gauge symmetry

$$SU(3)_c \times SU(2)_L \times U(1)_Y. \qquad (33)$$

combines the EW theory and QCD based on the non-Abelian gauge symmetry $SU(3)_c$.

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The symmetry $SU(3)_c$ remaining always unbroken, unlike the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ which will be broken at the end to a stable $U(1)$ symmetry, leading to a massless gauge field - the EM field.

SPONTANEOUS SYMMETRY BREAKING

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1. Spontaneous breaking of discrete symmetry

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Consider a Lagrgangian of a scalar field $\phi(x)$,

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi), \tag{34}$$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4. \tag{35}$$

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Abbreviation used below: VEV = vacuum expectation value.

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Z_2 symmetry breaking

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Observation from (37):

- H is a massive field with mass M_H,

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Spontaneous breaking of Abelian symmetry

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$U(1)$ symmetry

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Consider now a theory of a complex scalar field $\phi(x)$,

$$\mathcal{L}_\phi = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - V(\phi), \tag{39}$$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \tag{40}$$

where, λ and μ^2 are real, but $\lambda > 0$.

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$\Longrightarrow$ **fixing the phase of $\langle \phi \rangle$ breaks the $U(1)$ symmetry spontaneously!**.

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Choosing, without loss of generality, $\langle\phi\rangle = v/\sqrt{2}$, and re-expressing $\phi(x)$ via two QF's $H(x)$ and $\eta(x)$ as follows,

$$\phi(x) = \frac{1}{\sqrt{2}} \left[v + H(x) + i\eta(x) \right], \tag{43}$$

we get a $U(1)$-symm.-breaking theory of a massive field $H(x)$ with mass $m_H = 2\lambda v^2$, and a massless field $\eta(x)$:

$$\begin{aligned} \mathcal{L}_\phi = & \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) \\ & - \lambda v H^2 - \lambda v H (H^2 + \eta^2) \\ & - \frac{\lambda}{4}(H^2 + \eta^2)^2. \end{aligned} \tag{44}$$

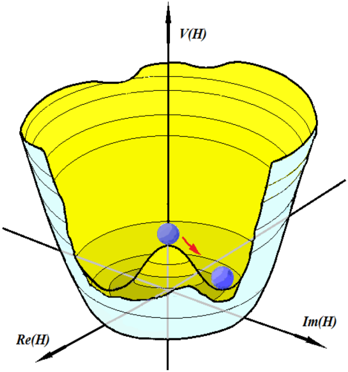

Figure: Spontaneous symmetry breaking

Spontaneous breaking of non-Abelian symmetry

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$SU(2)$ symmetry

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Consider an $SU(2)$-doublet complex scalar

$$\phi = \left(\begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right)$$

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$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \equiv \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \quad (45)$$

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with an $SU(2)$-symmetric Lagrangian

$$\mathcal{L}_S = \partial_\lambda \phi^\dagger \partial^\lambda \phi - V(\phi^\dagger \phi), \quad (46)$$

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STANDARD MODEL

Outline

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1. The matter world and elementary particles.

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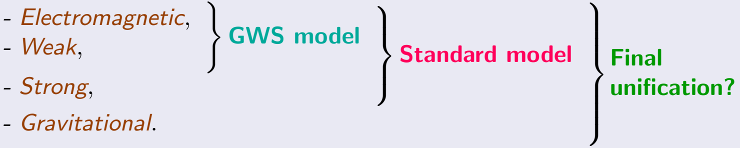

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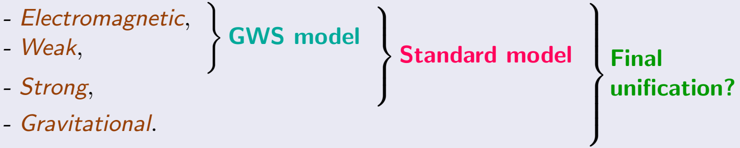

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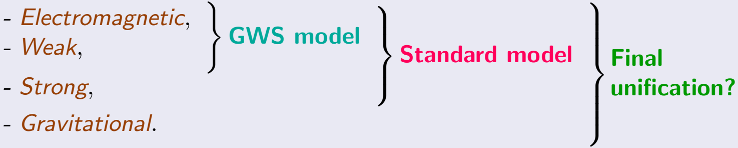

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See, the *"History of the Universe"*.

Neutrinos and the standard model

β-decays

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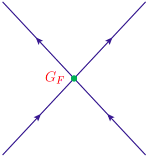

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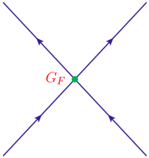

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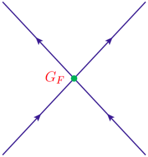

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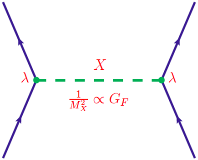

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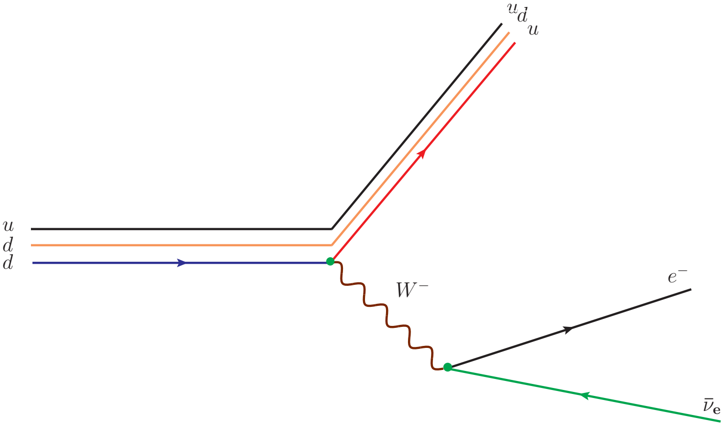

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Symmetry and structure of the standard model

Gauge group

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$SU(3)_C$: unbroken and describing the strong interaction.

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Gell-Mann-Nishijima formula

$$Q = I_3 + \frac{Y}{2}. \qquad (57)$$

Symmetry and structure of the standard model

Particle-field structure

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Particle-field structure

Matter sector:

$$\text{Generation 1:} \quad \begin{pmatrix} u^i \\ d^i \end{pmatrix}_L, \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad u_R, d_R, e_R; \tag{58}$$

$$\text{Generation 2:} \quad \begin{pmatrix} c^i \\ s^i \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad c_R, s_R, \mu_R; \tag{59}$$

$$\text{Generation 3:} \quad \begin{pmatrix} t^i \\ b^i \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \quad t_R, b_R, \tau_R, \tag{60}$$

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Scalar sector:

$$\phi(x) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}. \tag{62}$$

Brout-Englert-Higgs mechanism

Recall: the Glashow-Weiberg-Salam electroweak model is based on the gauge group $SU(2)_L \times U(1)_Y$.

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Covariant derivative

$$D_\lambda \phi = \left(\partial_\lambda + ig\frac{\vec{\tau}}{2}.\vec{A}_\lambda + \frac{i}{2}g^{'} B_\lambda\right)\phi, \tag{63}$$

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Again with the Lagrangian of type (46)

$$\mathcal{L}_S = (D_\lambda \phi)^\dagger \left(D^\lambda \phi \right) - V(\phi^\dagger \phi). \tag{64}$$

At the minimum of $V(\phi^\dagger \phi)$ as illustrated in Fig. 5, we choose VEV

$$\langle \phi \rangle_0 = \left(\begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} \end{array} \right) \tag{65}$$

Brout-Englert-Higgs mechanism

Use the unitary parametrization

$$\phi(x) = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix} = \frac{1}{\sqrt{2}} e^{i\tau^i \theta^i(x)} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{66}$$

where $\theta^i(x)$ are NG bosons.

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where $\theta^i(x)$ are NG bosons. They can be rotated out. The corresponding degrees of freedom are transferred to those gauge fields becoming massive due to the SSB !

$$\phi' = U(x)\phi = \begin{pmatrix} 0 \\ (v + H(x))/\sqrt{2} \end{pmatrix}, \quad U(x) = e^{-i\tau^i\theta^i(x)}, \qquad (67)$$

$$\frac{\vec{\tau}}{2}.\vec{A}'_\mu = U(x)\frac{\vec{\tau}}{2}.\vec{A}_\mu U(x)^{-1} - \frac{i}{g}(\partial_\mu U(x))U^\dagger(x) \qquad (68)$$

Brout-Englert-Higgs mechanism

From (64) we get

$$\mathcal{L}_S = \frac{1}{2}\partial_\lambda H \partial^\lambda H + \frac{g^2}{4}(v+H)^2 W^\dagger_\lambda W^\lambda + \frac{g^2+g^{'2}}{8}(v+H)^2 Z_\lambda Z^\lambda - \frac{\lambda}{4}(2vH+H^2)^2, \tag{69}$$

where,

$$W^\lambda = \frac{A_1^\lambda - iA_2^\lambda}{\sqrt{2}}, \tag{70}$$

$$Z_\lambda = \cos\theta_W A_\lambda^{'3} - \sin\theta_W B_\lambda,\ A_\lambda = \sin\theta_W A_\lambda^{'3} + \cos\theta_W B_\lambda, \tag{71}$$

$$\sin\theta_W = \frac{g^{'}}{\sqrt{g^2+g^{'2}}},\quad \cos\theta_W = \frac{g}{\sqrt{g^2+g^{'2}}}. \tag{72}$$

Brout-Englert-Higgs mechanism

Boson masses

Brout-Englert-Higgs mechanism

Boson masses

Boson mass terms

$$\mathcal{L}_{B.M.} = m_W^2 W_\lambda^\dagger W^\lambda + \frac{1}{2} m_Z^2 Z_\lambda Z^\lambda - \frac{1}{2} m_H^2 H^2, \tag{73}$$

$$\begin{aligned} m_W^2 = \frac{1}{4} g^2 v^2, \quad m_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2, \\ m_H^2 = 2\lambda v^2 = 2\mu^2, \quad m_\gamma = 0. \end{aligned} \tag{74}$$

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Experimental data

$$\begin{aligned} m_W &= 80.385 \pm 0.015 \ GeV, \quad m_Z = 91.1876 \pm 0.0021 \ GeV, \\ m_H &= 125.15 \pm 0.24 \ GeV, \quad m_\gamma < 10^{-18} \ eV. \end{aligned} \tag{75}$$

Brout-Englert-Higgs-Weinberg mechanism

Fermion masses

Fermion Lagrangian

$$\begin{aligned}\mathcal{L}_F = &\sum_{k=1}^{3} \overline{Q}_{kL} i\gamma^{\mu} D_{\mu}^{(q)} Q_{kL} + \sum_{k=1}^{3} \overline{\mathcal{U}}_{kR} i\gamma^{\mu} D_{\mu}^{(q,l)} \mathcal{U}_{kR} + \sum_{k=1}^{3} \overline{\mathcal{D}}_{kR} i\gamma^{\mu} D_{\mu}^{(q,l)} \mathcal{D}_{kR} \\ &+ \sum_{\ell=e,\mu,\tau} \overline{\psi}_{\ell L} i\gamma^{\mu} D_{\mu}^{(lep)} \psi_{\ell L} + \sum_{\ell=e,\mu,\tau} \overline{\ell}_R D_{\mu}^{(q,l)} \ell_R + \text{Yukawa terms},\end{aligned} \tag{76}$$

$$D_{\mu}^{(q)} = \left(\partial_{\mu} + ig\frac{\vec{\tau}}{2}.\vec{A}_{\mu} + i\frac{1}{2}g^{'}Y_L^{q}B_{\mu}\right), \tag{77}$$

$$D_{\mu}^{(lep)} = \left(\partial_{\mu} + ig\frac{\vec{\tau}}{2}.\vec{A}_{\mu} + i\frac{1}{2}g^{'}Y_L^{lep}B_{\mu}\right), \tag{78}$$

$$D_{\mu}^{(q,l)} = \left(\partial_{\mu} + i\frac{1}{2}g^{'}Y_R^{q,l}B_{\mu}\right). \tag{79}$$

Brout-Englert-Higgs-Weinberg mechanism

Fermion masses

Yukawa couplings

$$\mathcal{L}_Y = -\sum_{k,j}\left(\Gamma^{(D)}_{kj}\overline{Q}_{kL}\phi\mathcal{D}_{jR} + \Gamma^{(U)}_{kj}\overline{Q}_{kL}\widetilde{\phi}\,\mathcal{U}_{jR}\right) - \sum_{\ell,\ell'=e,\mu,\tau}\Gamma^{(lep)}_{\ell\ell'}\overline{\psi}_{\ell L}\phi\ell'_R + H.c. \qquad (80)$$

$$\widetilde{\phi} = i\tau_2\phi^* = \begin{pmatrix} \varphi^{0*} \\ -\varphi^- \end{pmatrix} \qquad (81)$$

Brout-Englert-Higgs-Weinberg mechanism

Fermion masses

Mass terms

$$\mathcal{L}^{(ql)}_{mass} = -\sum_{k,j} \overline{\mathcal{U}}_{iL} M^{(U)}_{kj} U_{jR} - \sum_{k,j} \overline{\mathcal{D}}_{iL} M^{(D)}_{k,j} D_{jR} - \sum_{\ell,\ell'=e,\mu,\tau} \overline{\ell}_L M^{(lep)}_{\ell\ell'} \ell'_R + h.c. \qquad (82)$$

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$$M^{(U)}_{kj} = \Gamma^{(U)}_{kj}\frac{v}{\sqrt{2}}, \quad M^{(D)}_{kj} = \Gamma^{(D)}_{kj}\frac{v}{\sqrt{2}}, \quad M^{(lep)}_{\ell\ell'} = \Gamma^{(lep)}_{\ell\ell'}\frac{v}{\sqrt{2}} \tag{83}$$

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$$\begin{aligned} M^{(U)} &= V_U \ m^{(U)} \ W^{\dagger}_U, \\ M^{(D)} &= V_D \ m^{(D)} \ W^{\dagger}_D, \\ M^{(lep)} &= V_\ell \ m^{(charged \ \ lep.)} \ W^{\dagger}_\ell, \end{aligned} \qquad (84)$$

Brout-Englert-Higgs-Weinberg mechanism

Fermion masses

Quark masses

$$m^{(U)} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \; m^{(D)} = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}. \qquad (85)$$

Fermion masses

Lepton masses

$$m^{(charged\;\; lep.)} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \; \mathrm{m}^{(\nu)} = 0. \qquad (86)$$

Charged and neutral currents

Interaction Lagrangian

$$\mathcal{L}_I = \mathcal{L}_I^{CC} + \mathcal{L}_I^{NC}, \tag{87}$$

where

$$\mathcal{L}_I^{CC} = \left(-\frac{g}{2\sqrt{2}} J_\mu^{CC} W^\mu + H.c. \right), \tag{88}$$

with J_μ^{CC} being a charged current, and

$$\mathcal{L}_I^{NC} = -\frac{g}{2\cos\theta_W} J_\mu^{NC} Z^\mu - eJ_\mu^{EM} A^\mu, \tag{89}$$

with

$$J_\mu^{NC} = 2J_\mu^3 - 2\sin^2\theta_W J_\mu^{EM} \tag{90}$$

being a neutral current.

Charged and neutral currents (cont.)

Interaction Lagrangian (full)

$$\mathcal{L}_I = \left(-\frac{g}{2\sqrt{2}} J_\mu^{CC} W^\mu + h.c.\right) - \frac{g}{2\cos\theta_W} J_\mu^{NC} Z^\mu - e J_\mu^{EM} A^\mu. \qquad (91)$$

Charged and neutral currents (cont.)

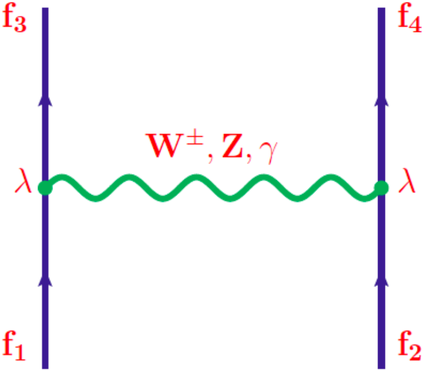

Figure: Electro-weak currents.

Neutrinos beyond the standard model

Neutrino masses and mixing

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- **Experimental observations of neutrino oscillations** [Super-Kamiokande, SNO, 1998 – 2002].

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$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times P$$

$$= \begin{pmatrix} c_{13}s_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \times P.$$

Neutrinos beyond the standard model

Current experimental data

- $\sum m_i < 0.12\ eV$.
- $\Delta m^2_{12} = 7.54 \times 10^{-5}\ eV^2, \ \ |\Delta m^2_{31}| = 2.43 \times 10^{-3}\ eV^2$.
- $\theta_{23} \approx 41.4^\circ$, $\theta_{12} \approx 33.7^\circ$,
- $\theta_{13} \approx 8.8^\circ$ [Daya Bay, SNO, RENO, T2K, 2010 – 2011], $\theta_{13} \approx 8.47^\circ$ [T2K, 2016].

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$\theta_{13} = 0$ (by 2010), thus, U has a tribimaximal form

$$U_{tbm} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

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$\theta_{13} \neq 0$

There is no unique theory or formalism to describe this case, $\theta_{13} \neq 0$.

Neutrinos beyond the standard model

Experimental Pontecorvo-Maki-Nakagawa-Sakata matrix

$$U^{exp}_{PMNS} = \begin{pmatrix} 0.82 \pm 0.01 & 0.54 \pm 0.02 & -0.15 \pm 0.03 \\ -0.35 \pm 0.06 & 0.70 \pm 0.06 & 0.62 \pm 0.06 \\ 0.44 \pm 0.06 & -0.45 \pm 0.06 & 0.77 \pm 0.06 \end{pmatrix} \times P,$$

with the Dirac phase δ omitted.

Neutrinos BSM: neutrino mass generation

Neutrinos have masses but very tiny. How to generate them?

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Seesaw mechanism

https://www.overleaf.com/project/5fd81dbbdf15

Figure: Illustration (by an unknown author).

Neutrinos BSM: Relation to physics at colliders

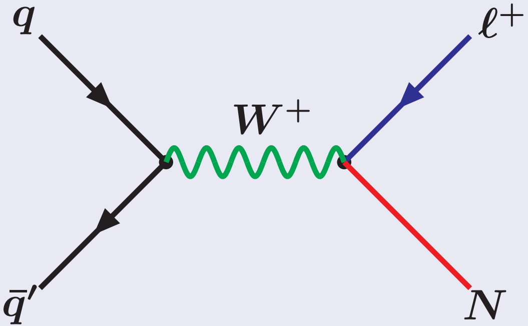
General neutrino mixing

$$\nu_{L\alpha} = U_{\alpha i}\nu_i + \Theta_{\alpha k} N_k \,,$$

$$\begin{aligned} &\alpha = e, \mu, \tau, \\ &i = 1, 2, 3, \\ &k = 1, 2, ..., n. \end{aligned}$$

Neutrinos BSM: Relation to physics at colliders

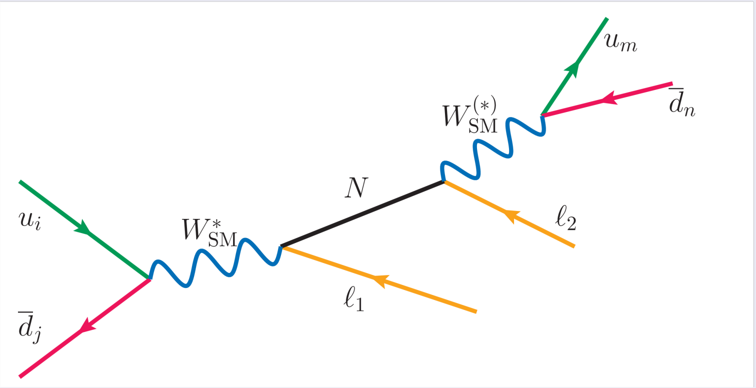
Heavy neutrino production at colliders



A scheme of a heavy neutrino production [NJP **17** (2015) 075019].

Neutrinos BSM: Relation to physics at colliders

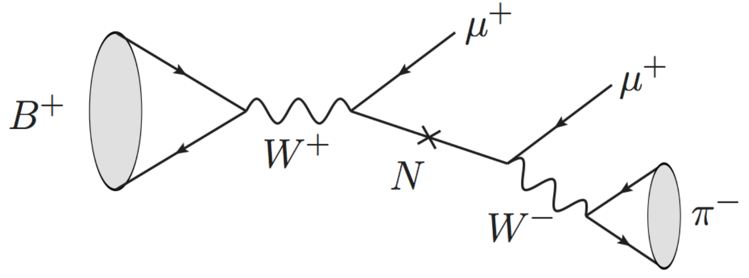
Heavy neutrino production and decay



A scheme of a heavy neutrino production and decay [arXiv: 1703.04669 [hep-ph]].

Neutrinos BSM: Relation to physics at colliders

Lepton-number-violation processes



A lepton-number-violation B-decay [Phys. Lett. B **763** (2016) 393].

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- Determination of ν's absolute masses (*very tiny but only upper limits known*).
- Sterile neutrinos (*their existence and masses*).

THANK YOU FOR YOUR ATTENTION!

GOOD LUCK WITH NEUTRINO PHYSICS!