Revisit the octant ambiguity and resolvability of the θ_{23} mixing angle

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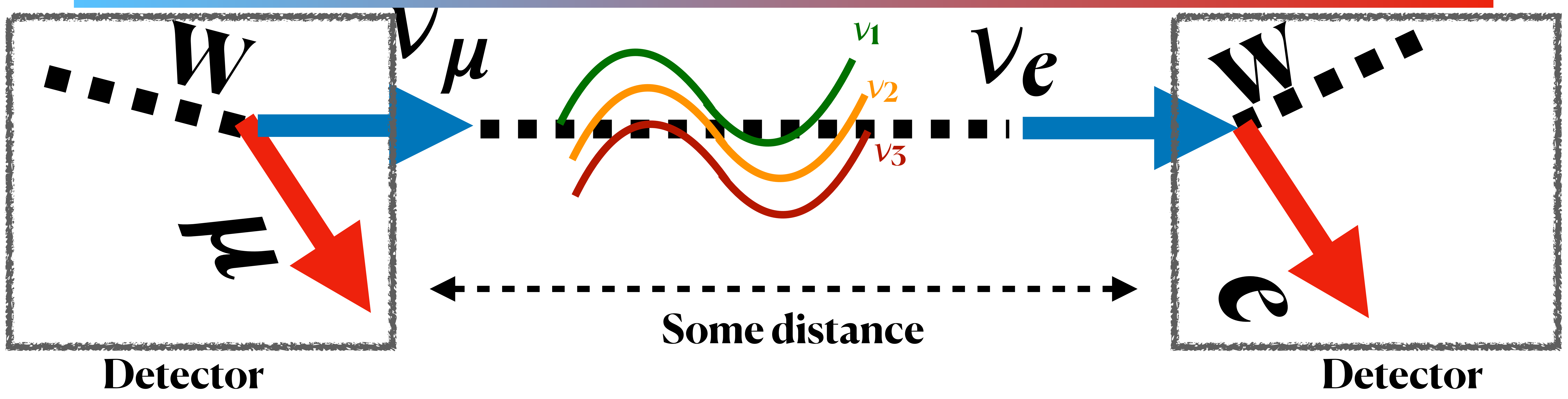
Neutrino oscillation in brief

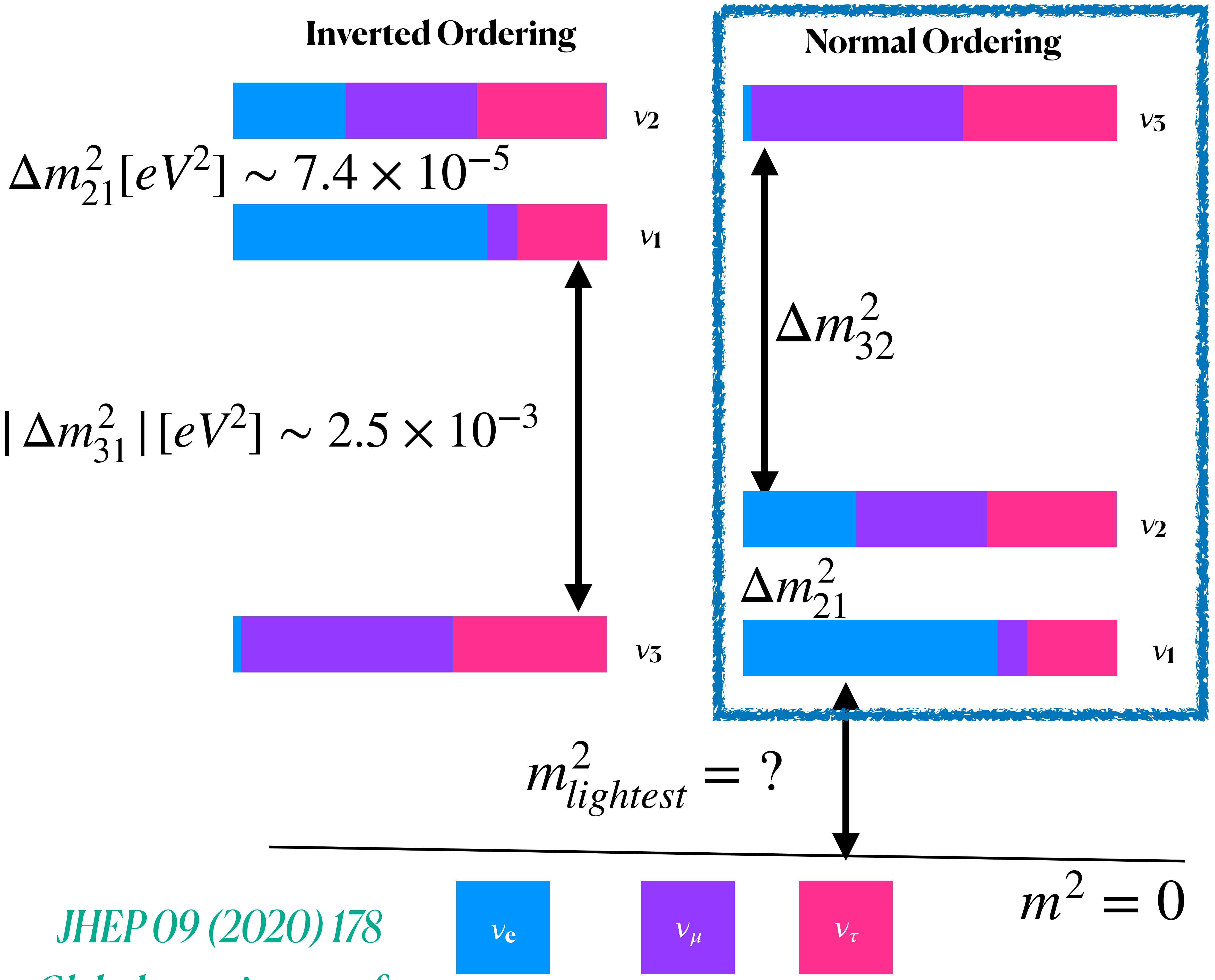
Neutrino can change its flavor when give it time to propagate



$$U_{\text{PMNS}} = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \text{Diag}(e^{i\rho_1}, e^{i\rho_2}, 0)$$

$$c_{ij} = \cos\theta_{ij}, \quad s_{ij} = \sin\theta_{ij}$$

Present landscape of the leptonic mixing



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Global neutrino exp. fit

MINOS, T2K, NOvA; Daya Bay, RENO, Double Chooz, KamLAND; SNO, Borexino; IceCube, Super-K

$$\theta_{12}[^o] = 33.41^{+0.75}_{-0.72}$$

2.2% precision

$$\theta_{13}[^o] = 8.54^{+0.11}_{-0.12}$$

1.4% precision

$$\theta_{23}[^o] = 49.1^{+1.1}_{-1.4}$$

Do not exclude $\theta_{23} = 45^o$ & lower octant yet. 1σ precision is misleading

$$\delta_{CP}[^o] = 197^{+42}_{-25}$$

Note: mild tension btw T2K vs. NOvA

Three unknowns to solve for completing neutrino oscillation picture

- CP-violation phase in the leptonic mixing matrix
- Neutrino mass ordering
- Whether the leptonic mixing angle θ_{23} maximal or not

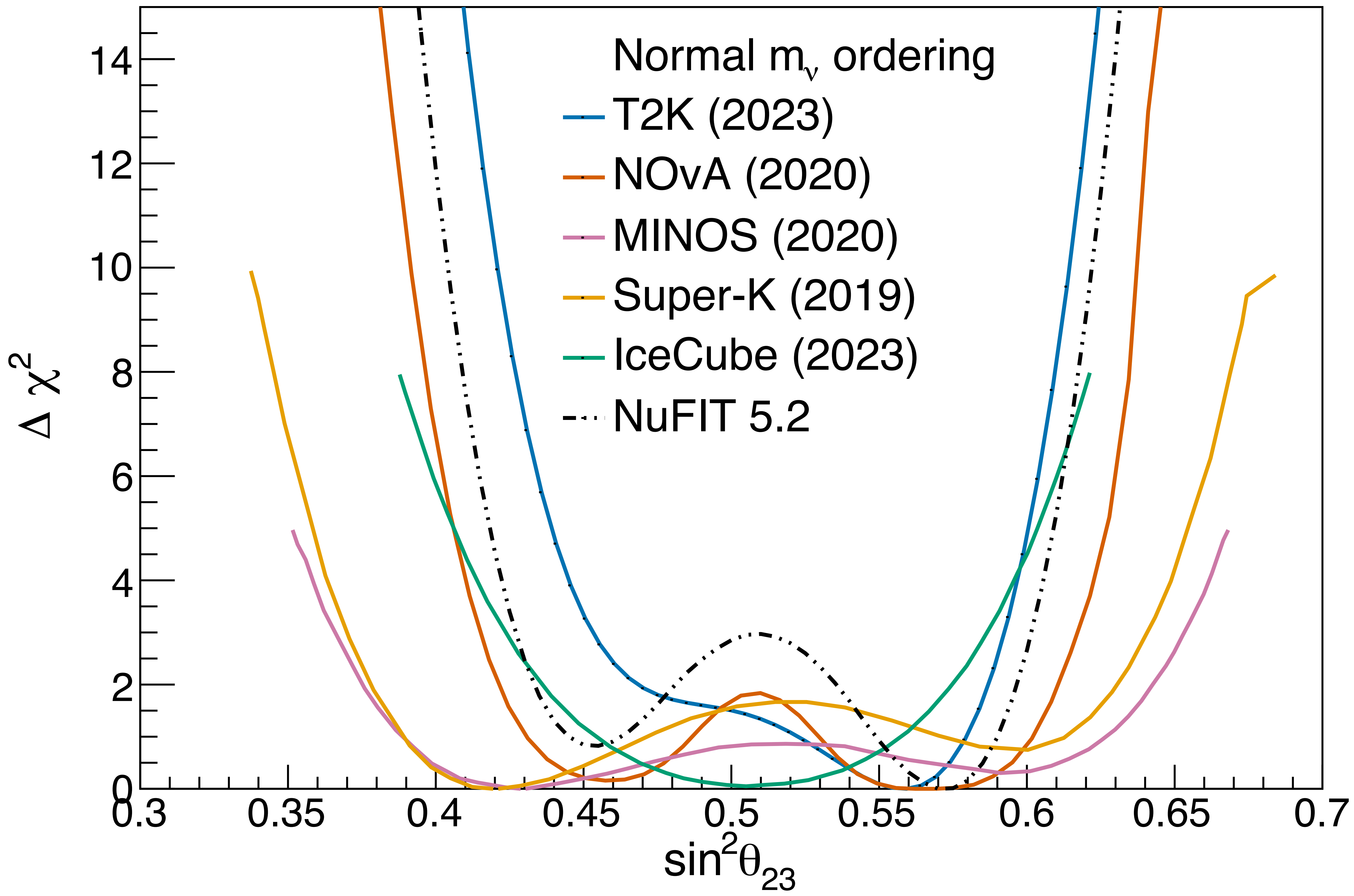
Our main objective: θ_{23} precise determination

- CP-violation phase in the leptonic mixing matrix
- Neutrino mass ordering
- Whether the leptonic mixing angle θ_{23} maximal or not

$$
\begin{array}{c} e \\ \mu \\ \tau \end{array}
\begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}
$$

If $\theta_{23} = \pi/4$, ν_3 shares the same fractions of flavor-muon and flavor-tau. Similar to ν_1 and ν_2 if $\delta_{CP} = \pm\,\pi/2$

Current understanding of the θ_{23} mixing angle



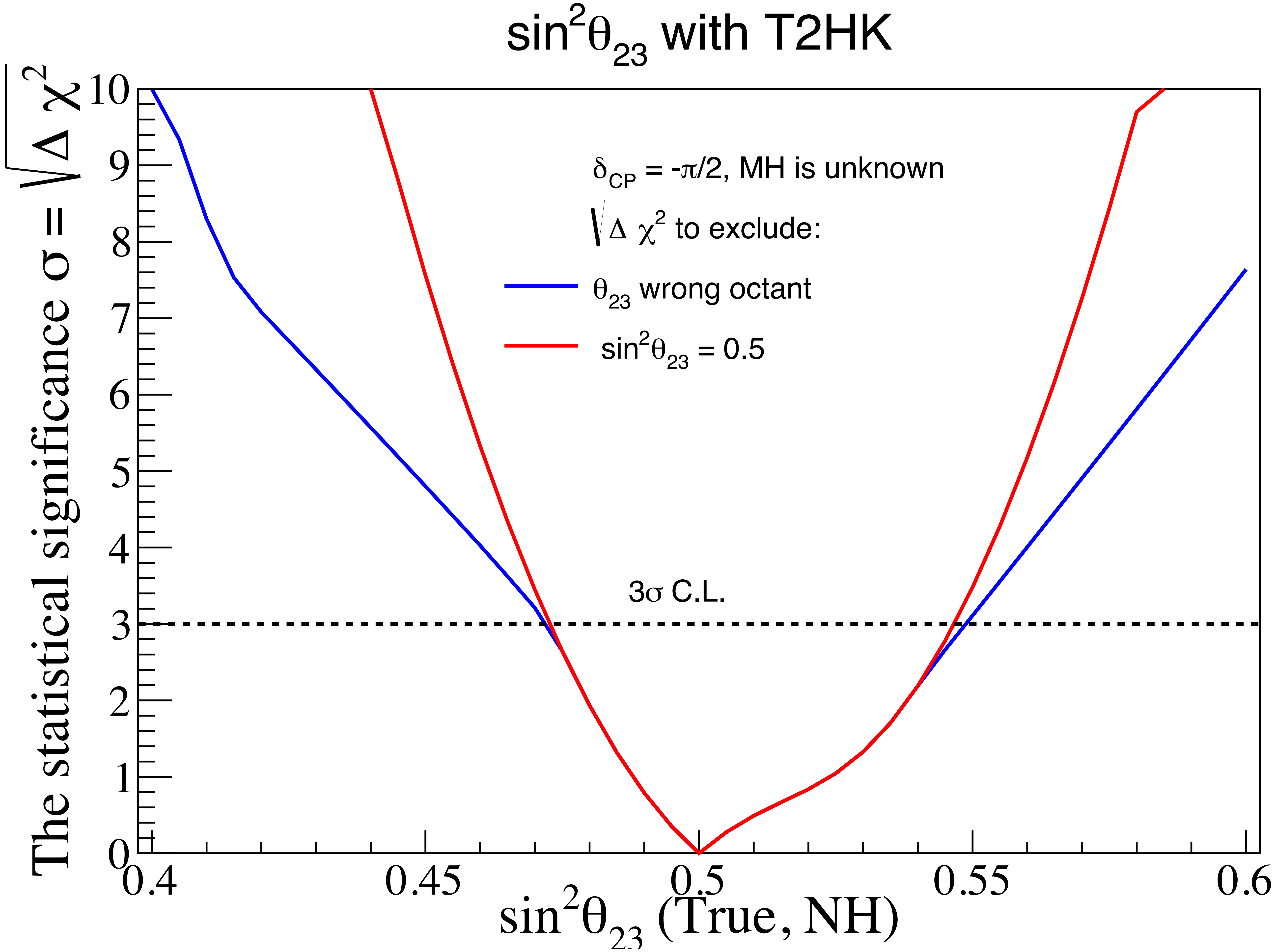
	T2K	NOνA	MINOS	Super-K	IceCube	NuFIT 5.2
Constraint $\sin^2\theta_{13}/10^{-2}$	2.18 ± 0.07	2.10 ± 0.11	2.10 ± 0.11	2.10 ± 0.11	2.224 ± 0.11	2.203 ± 0.0575
Best fit $\sin^2\theta_{23}$	$0.561^{+0.019}_{-0.038}$	$0.57^{+0.03}_{-0.04}$	$0.43^{+0.20}_{-0.04}$	$0.425^{+0.051}_{-0.034}$	0.51 ± 0.05	$0.572^{+0.018}_{-0.023}$
Octant Rej.	$\sim$1.22σ	$\sim$ 0.37σ	$\sim$ 0.53σ	$\sim$ 0.85σ	0	$\sim$ 0.89σ

Note: Most experiments use external constraint of θ_{13} from reactors but the value is not exactly the same

Two hypothesis tests for θ_{23} precise measurement

Practical approach for the statistical test:

- First, perform $\sin^2\theta_{23} = 0.5$ hypothesis test
- After we definitely exclude the maximal mixing hypothesis, we then find the "right" octant by excluding the "wrong" octant hypothesis
- The statistical significance for the former test is typically higher than the later test.

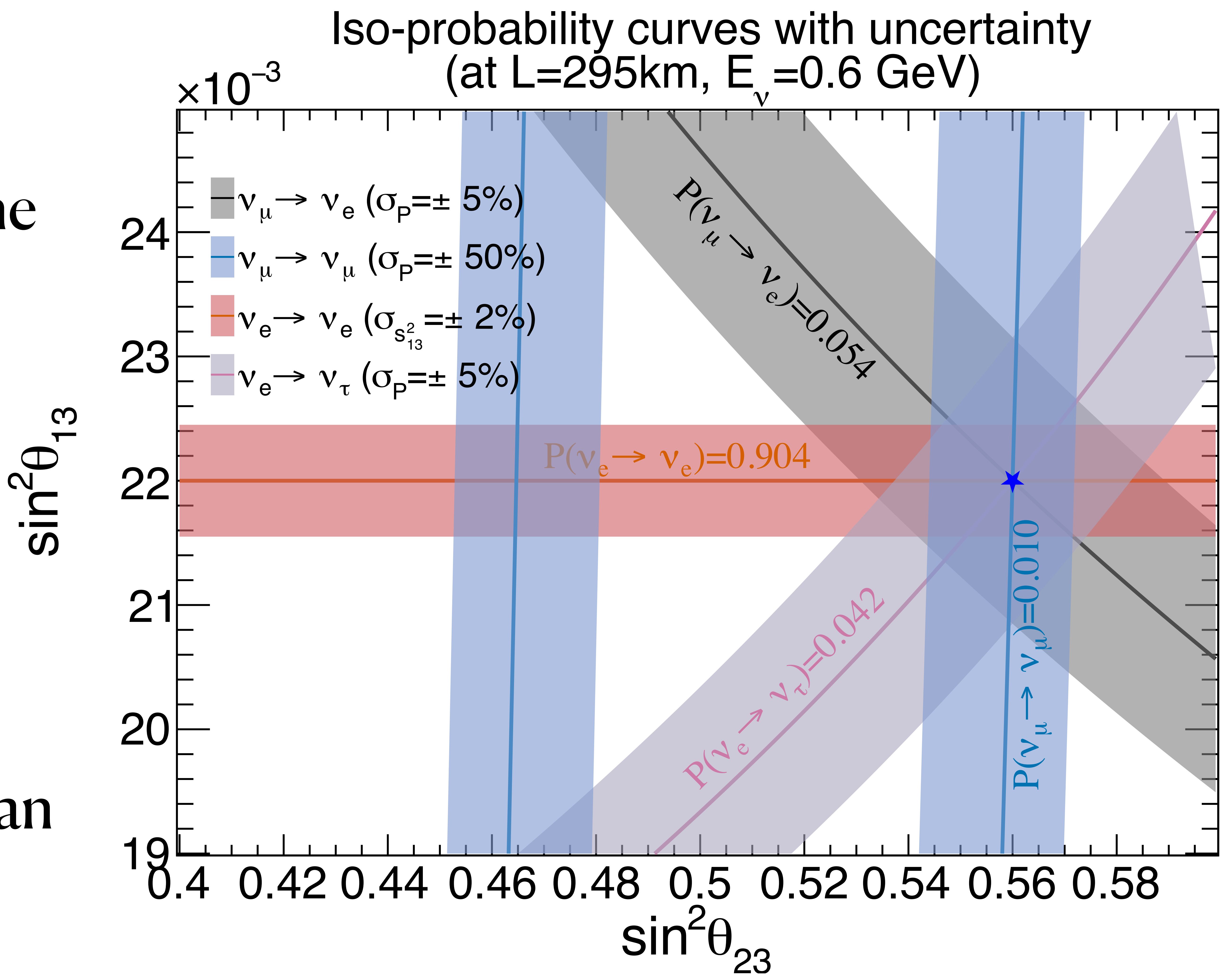


Questions we focus in this study

- Do the known unknowns (δ_{CP} *and neutrino mass ordering (MO)*) affect the θ_{23} precise determination?
- How does of the θ_{13} precision impact on the θ_{23} measurement ?
- What are contributions of neutrino and anti-neutrino appearance samples to resolve the θ_{23} ambiguity?
- Is the θ_{23} precise determination driven by statistical or systematical errors?
- Is there range of θ_{23} where its ambiguity can not be solved definitively?

Overall strategy to measure θ_{23} mixing angle

- Most effective strategy: combine disappearance — appearance samples in the accelerator/atmospheric -based exp. and constrain on θ_{13} from reactor-based exp.
  - *Both neutrinos and anti-neutrinos*
- Using other baseline/energy also can help
- Using $\nu_e \rightarrow \nu_\tau$ (*in possible neutrino factory*) can help (if the statistics is at level of $\nu_\mu \rightarrow \nu_e$)



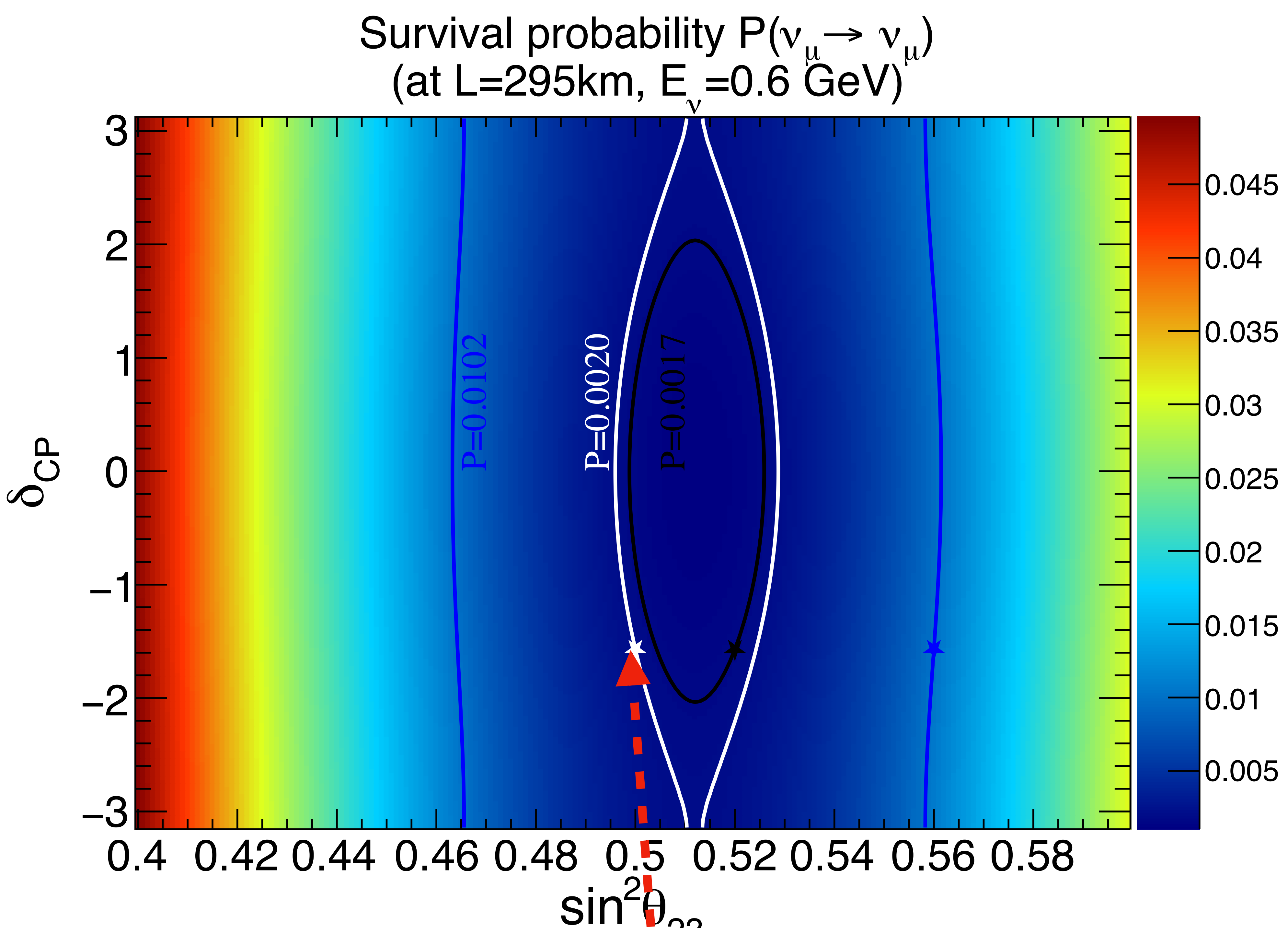
On the $\nu_\mu \to \nu_\mu$ survival (disappearance) measurement

- Discrete clone solutions of θ_{23} with marginal δ_{CP} dependence when true θ_{23} is significantly off from $\pi/4$

- When true θ_{23} is close to $\pi/4$ and in higher octant, "*continuous*" same-octant clone solution with relatively strong dependence on $\cos\delta_{CP}$ observed

- Iso-probability pattern is symmetric about

$$\sin^2\theta_{23} \sim \frac{0.5}{\cos^2\theta_{13}} \approx 0.511$$

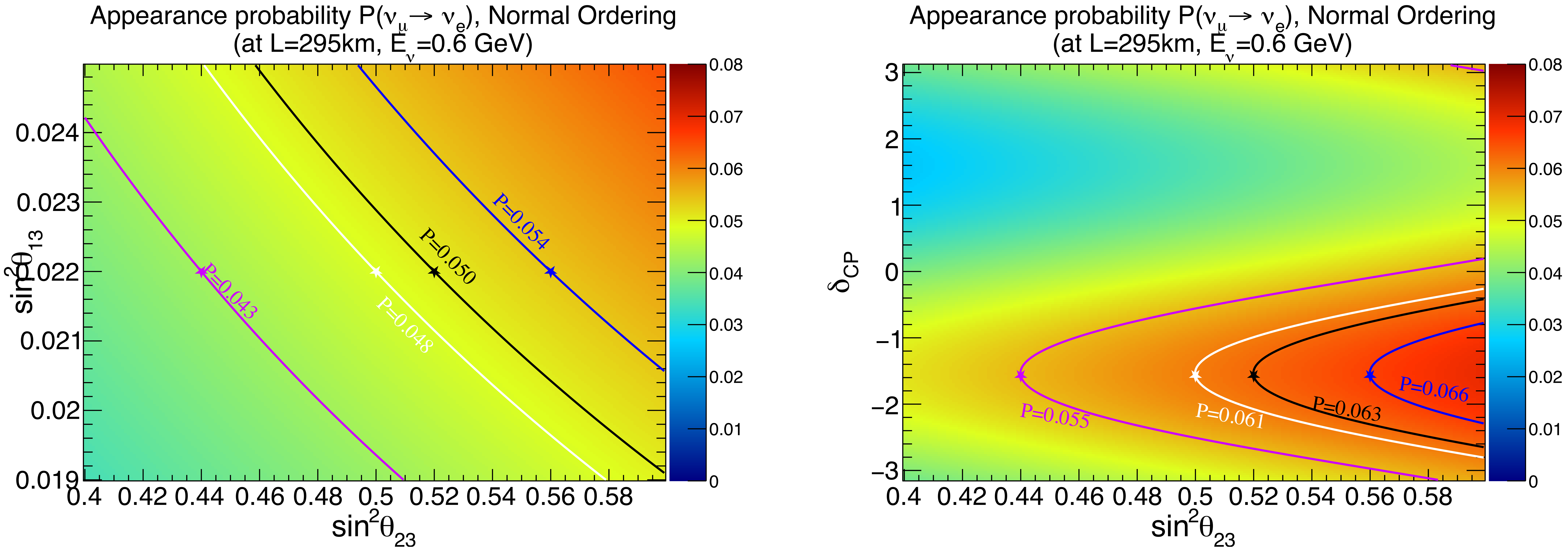


$$P_{\nu_\mu\to\nu_\mu} = 1 - \left(\cos^4\theta_{13}.\sin^2 2\theta_{23} + \sin^2 2\theta_{13}.\sin^2\theta_{23}\right)\sin^2\Phi_{31}$$
$$+ \epsilon_m \Phi_{31}\sin 2\Phi_{31}\left(\cos^2\theta_{12}\sin^2 2\theta_{23} - \sin^2\theta_{23}J_{123}\cos\delta_{CP}\right)$$

$$\Phi_{31} = \Delta m^2_{31}.\frac{L}{4E}; \quad \epsilon_m = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \approx 0.03; \quad J_{123} = \sin 2\theta_{12}.\sin 2\theta_{23}.\sin 2\theta_{13};$$

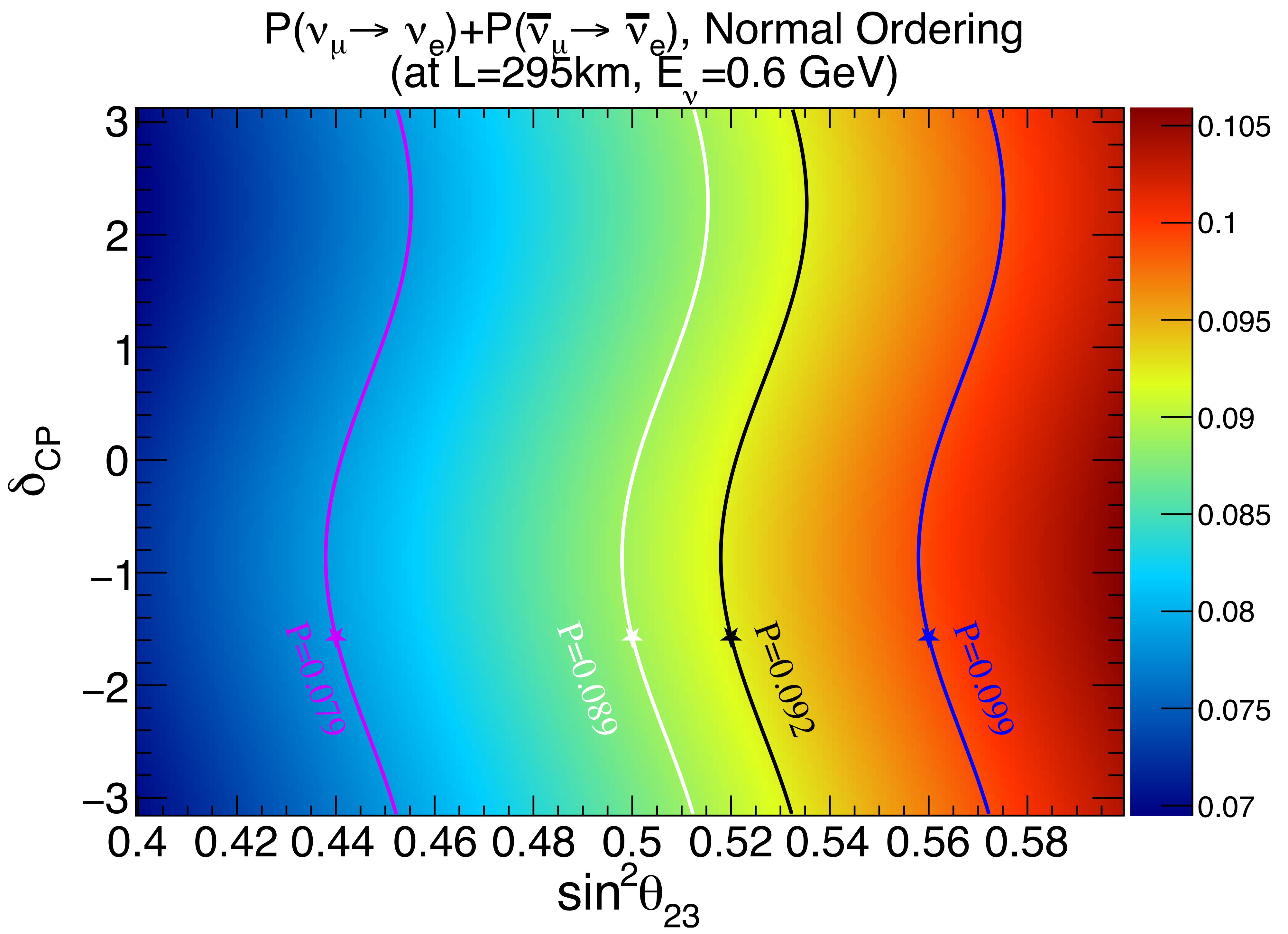
On the $\nu_\mu \to \nu_e$ appearance measurement

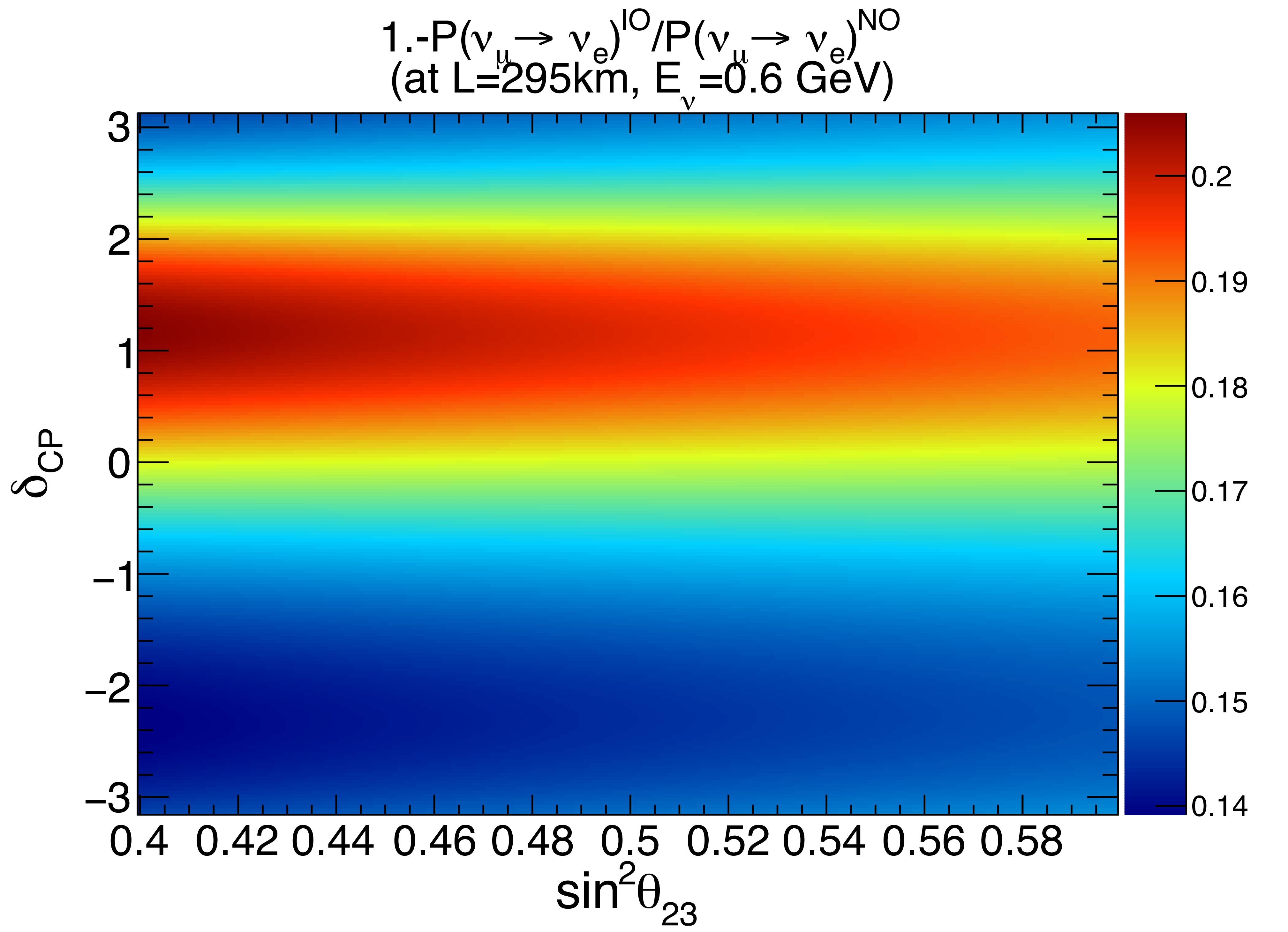
$$P(\nu_\mu \to \nu_e)(\Phi_{31} \sim \pi/2) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Phi_{31} + \epsilon_m \Phi_{31}\sin\Phi_{31} J_{123}\cos(\Phi_{31} + \delta_{CP})$$



- Strong $\theta_{23} - \theta_{13}$ *continuous* degeneracy, simply $f^{iso.prob.}(\theta_{23}, \theta_{13}) = \sin\theta_{23} \,.\, \sin 2\theta_{13}$, strong impact on the θ_{13} precise measurement
- *Energy(baseline)-dependent* $\theta_{23} - \delta_{CP}$ *continuous* degeneracy but having marginal effect on θ_{13} resolvability (*next slides*)

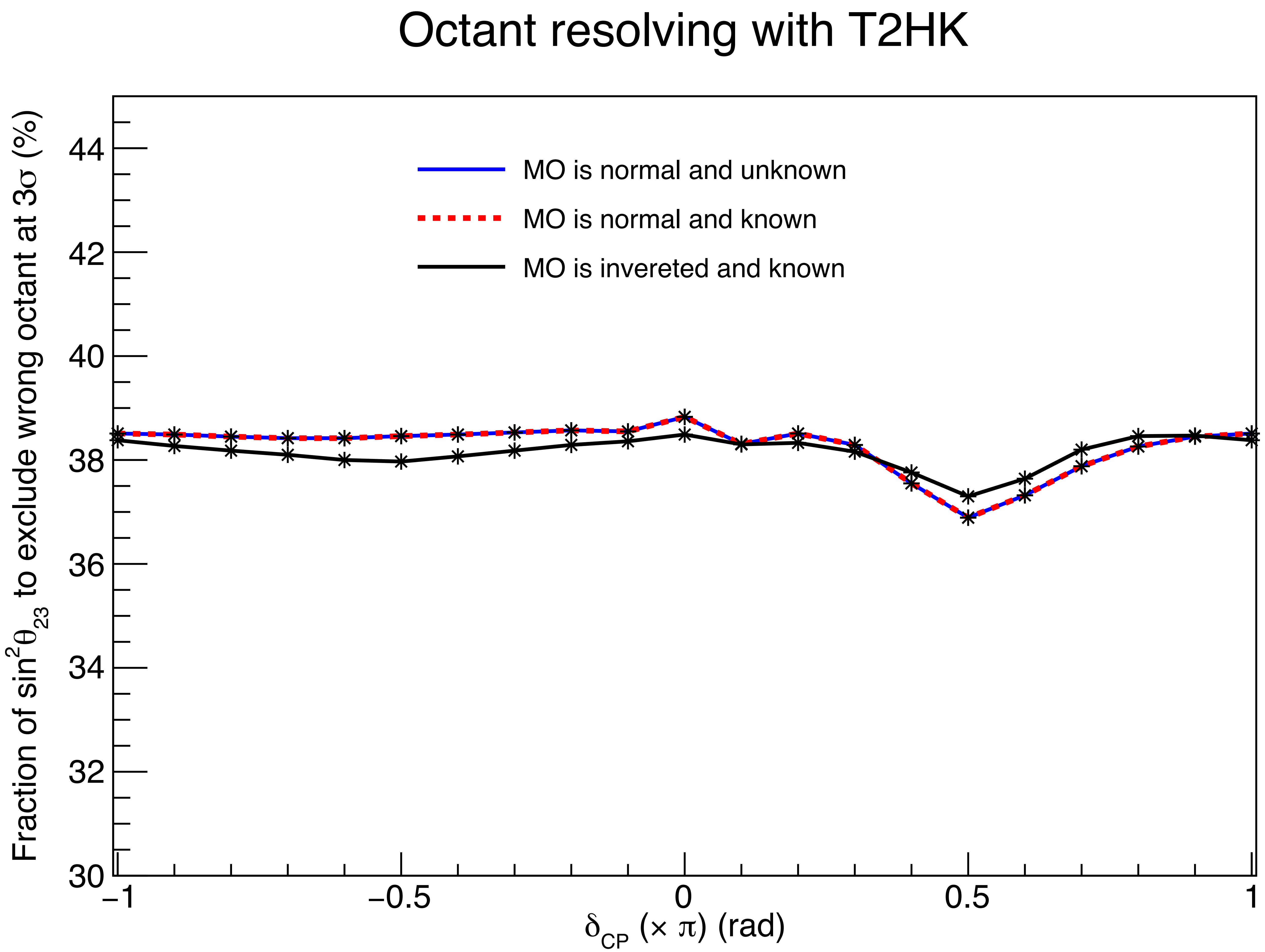
On the effect of δ_{CP} and mass ordering





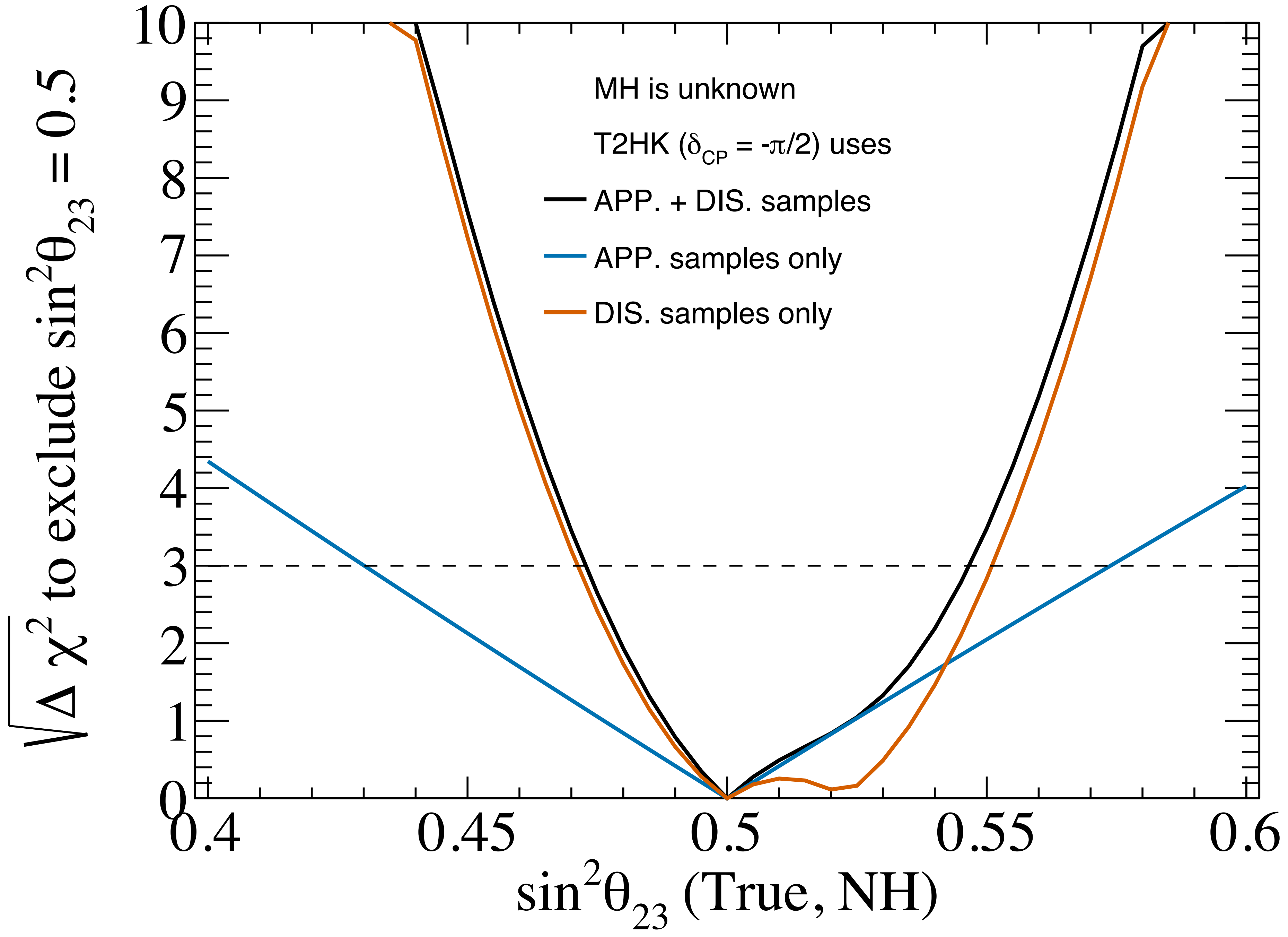
- The octant resolvability of neutrino exp. like T2HK, DUNE doesn't depend much on the truth value of δ_{CP}
  - $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ probabilities share a mirror symmetry and thus cancel out if combine neutrino and anti-neutrino (*if at the same statistical level*)
  - The iso-probability curve is energy dependence, spectral measurement helps to mitigate this degeneracy
- The relative difference btw. *normal* and *inverted* mass ordering (MO) is independent to θ_{23} → whether MO is *normal* or *inverted, known* or *unknown* has mild impact on the θ_{23} resolvability

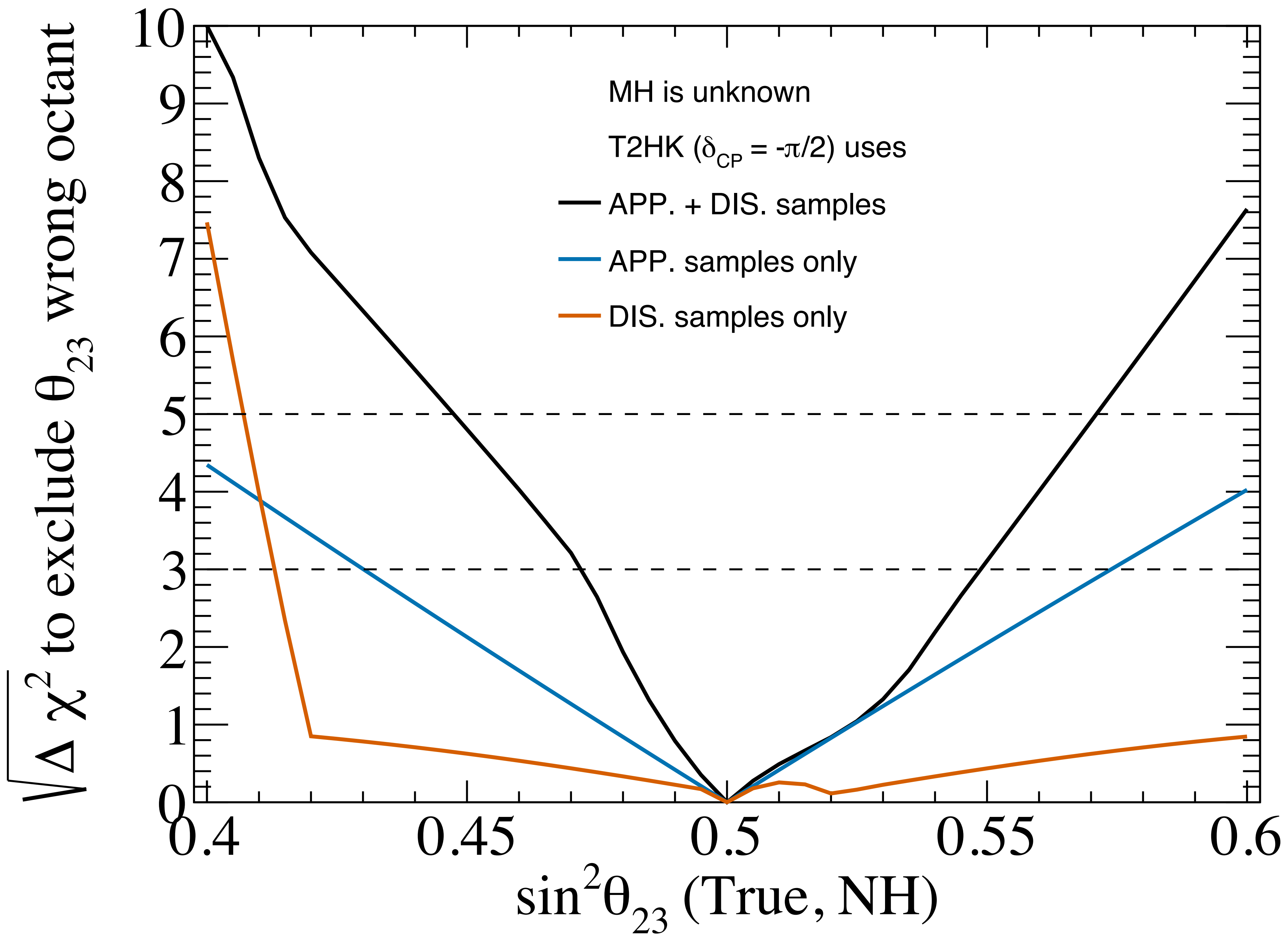
On the effect of δ_{CP} and mass ordering



The octant resolvability has marginal dependence on δ_{CP} and does not depend much on the mass ordering understanding

Contribute to octant resolving: disappearance and appearance

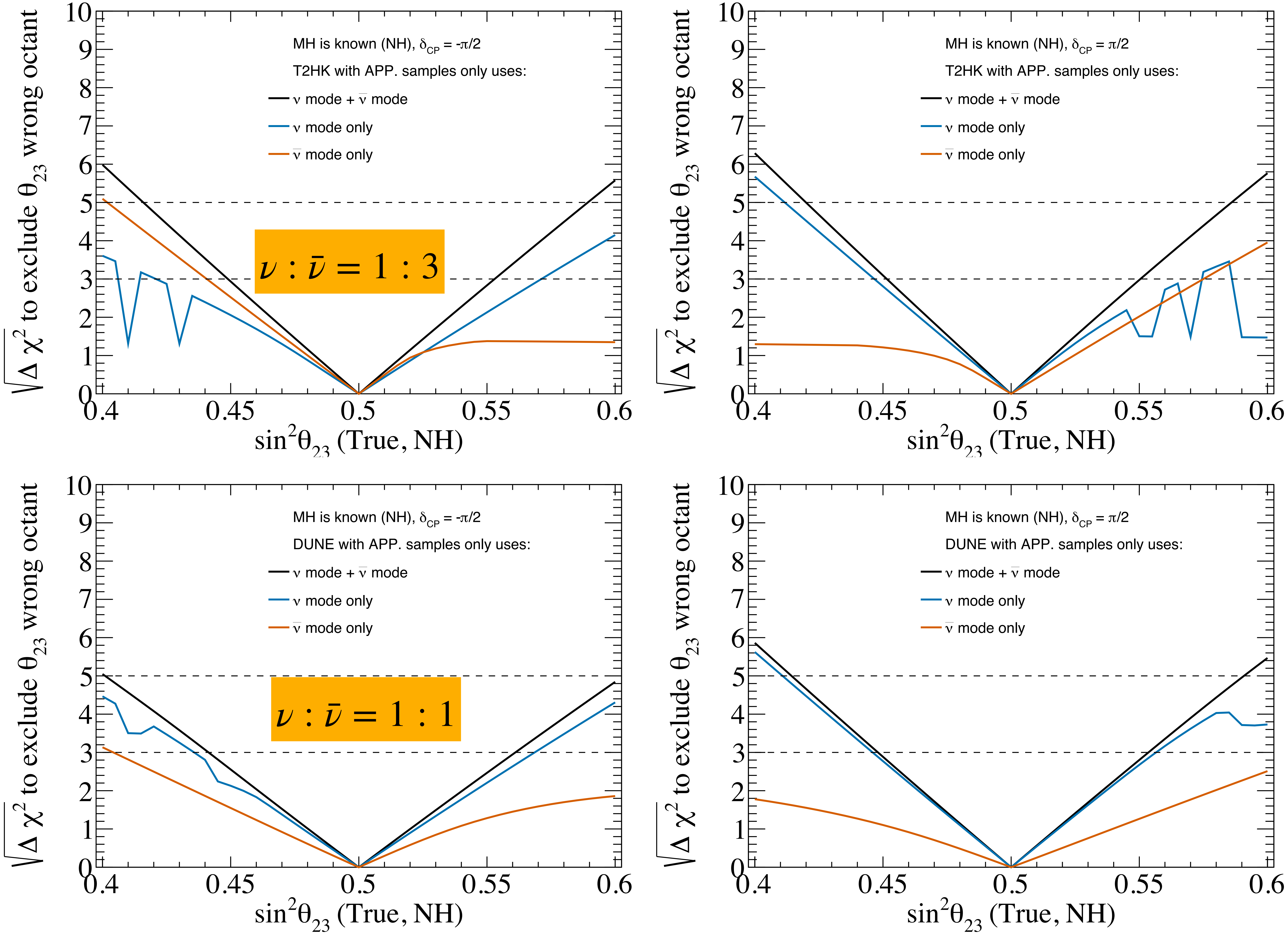




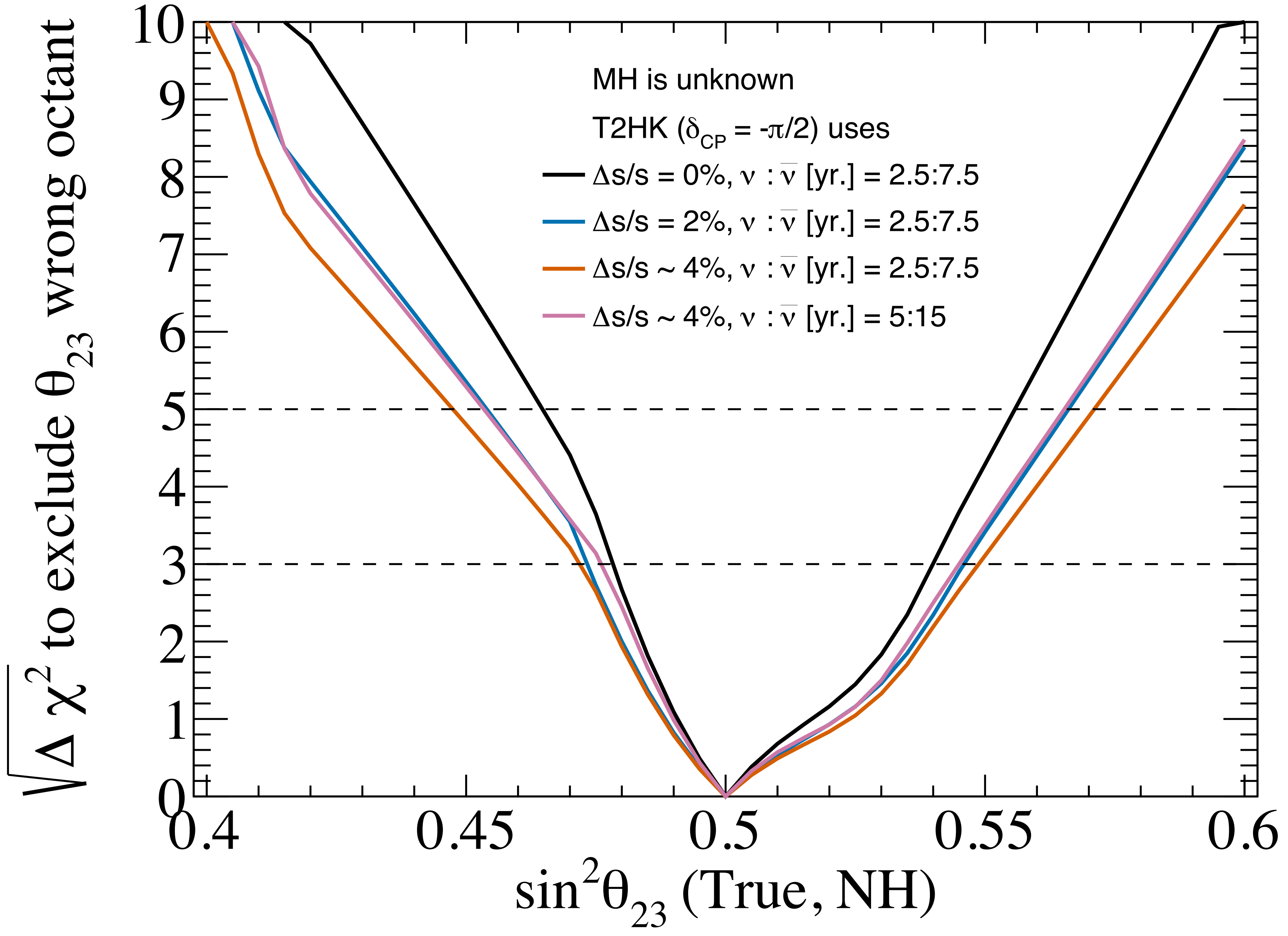
- For excluding $\sin^2\theta_{23} = 0.5$, main contribution is from disappearance measurement unless $0.5 < \sin^2\theta_{23} \lesssim 0.54$
- For octant resolving, mostly driven by the appearance unless $\sin^2\theta_{23} \lesssim 0.4$
- Typically, octant resolvability is better if θ_{23} lies in the lower octant.

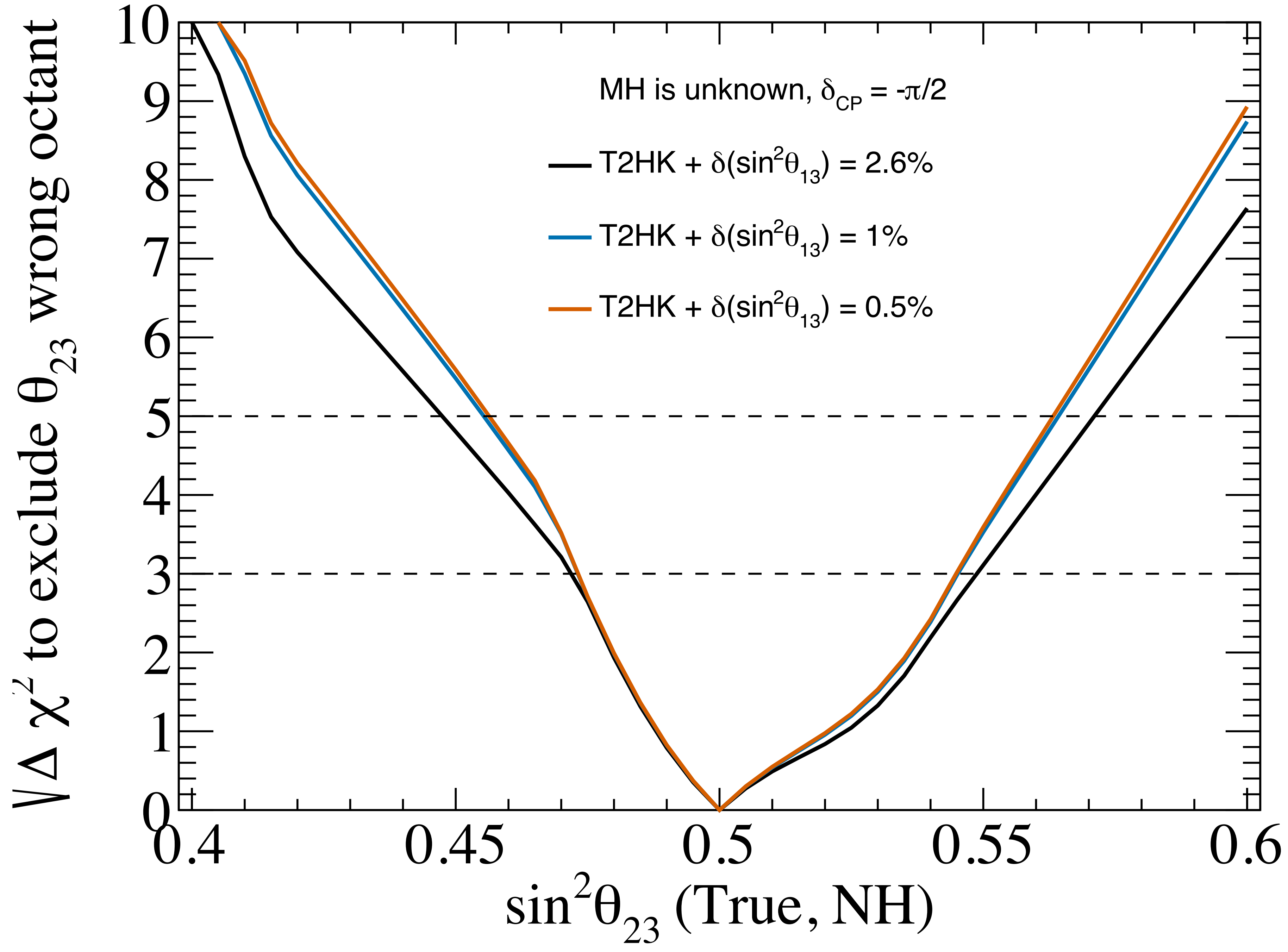
Contribution of neutrino appearance and anti-neutrino sample

- If $\nu_e, \bar{\nu}_e$ appearance samples are the same statistical level, their contribution to the θ_{23} octant resolving are not the same
- In DUNE, with $\nu : \bar{\nu} = 1 : 1$, statistics of neutrino is more dominant, thus a driving factor for octant resolving



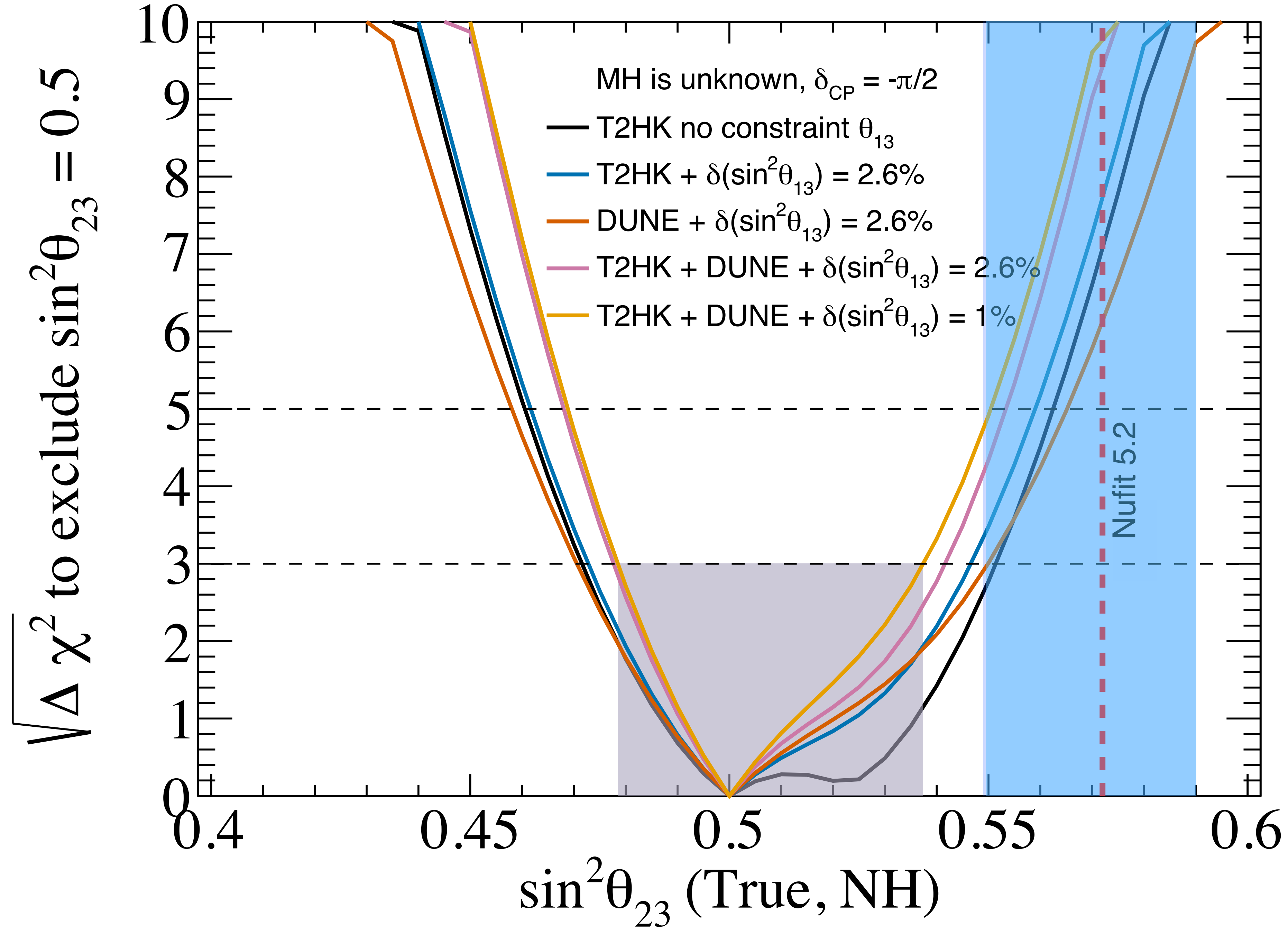
Systematic effects and θ_{13} constraint

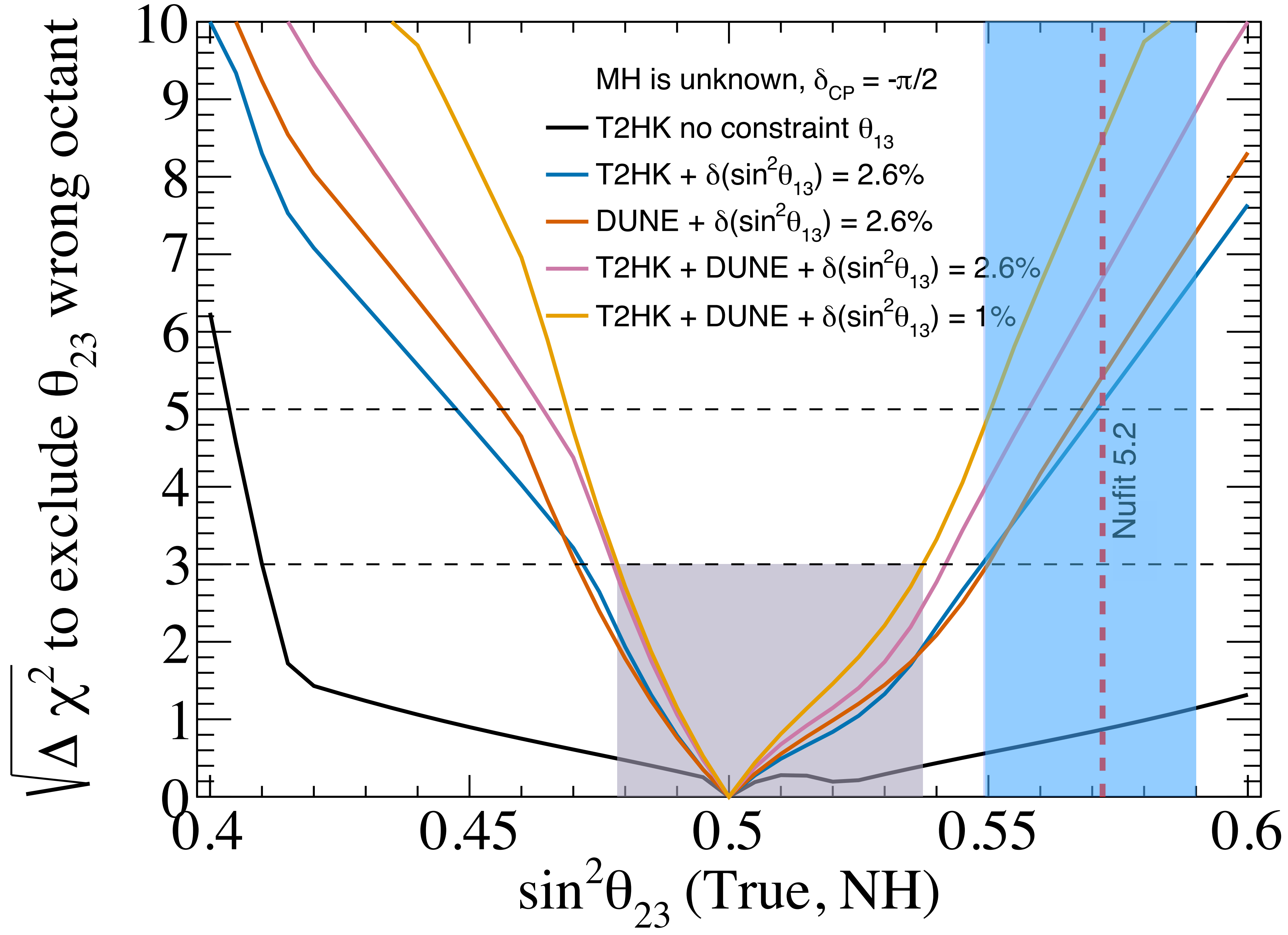




- Systematic improvement (if possible) impact on the octant resolvability: 4% → 2% systematic error is equivalent to a double of statistics
- Further improvement of θ_{13} (2.6% → 1.0%) give some extension of 5σ reach but not 3σ

Ultimate reach for θ_{23} precision measurement?





- A combination of T2HK, DUNE with further constraint on θ_{13} is supposed to provide an ultimate reach for θ_{23}
  - Can rule out whole present 1σ region (NuFIT 5.2) with more than 3σ
- If θ_{23} lies between [0.48,0.54], it is almost impossible to resolve the θ_{13} ambiguity with high statistic significance

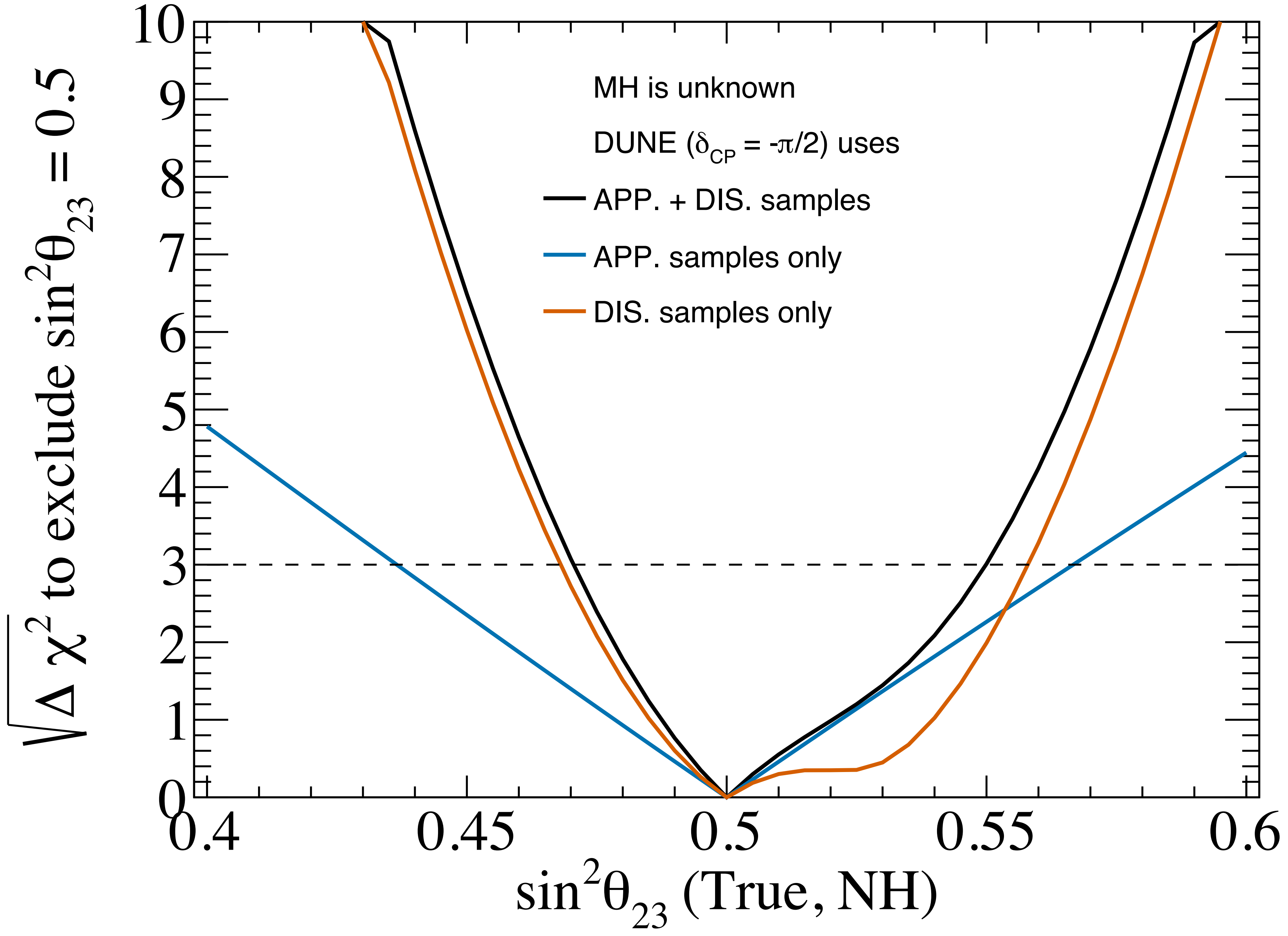
Summary

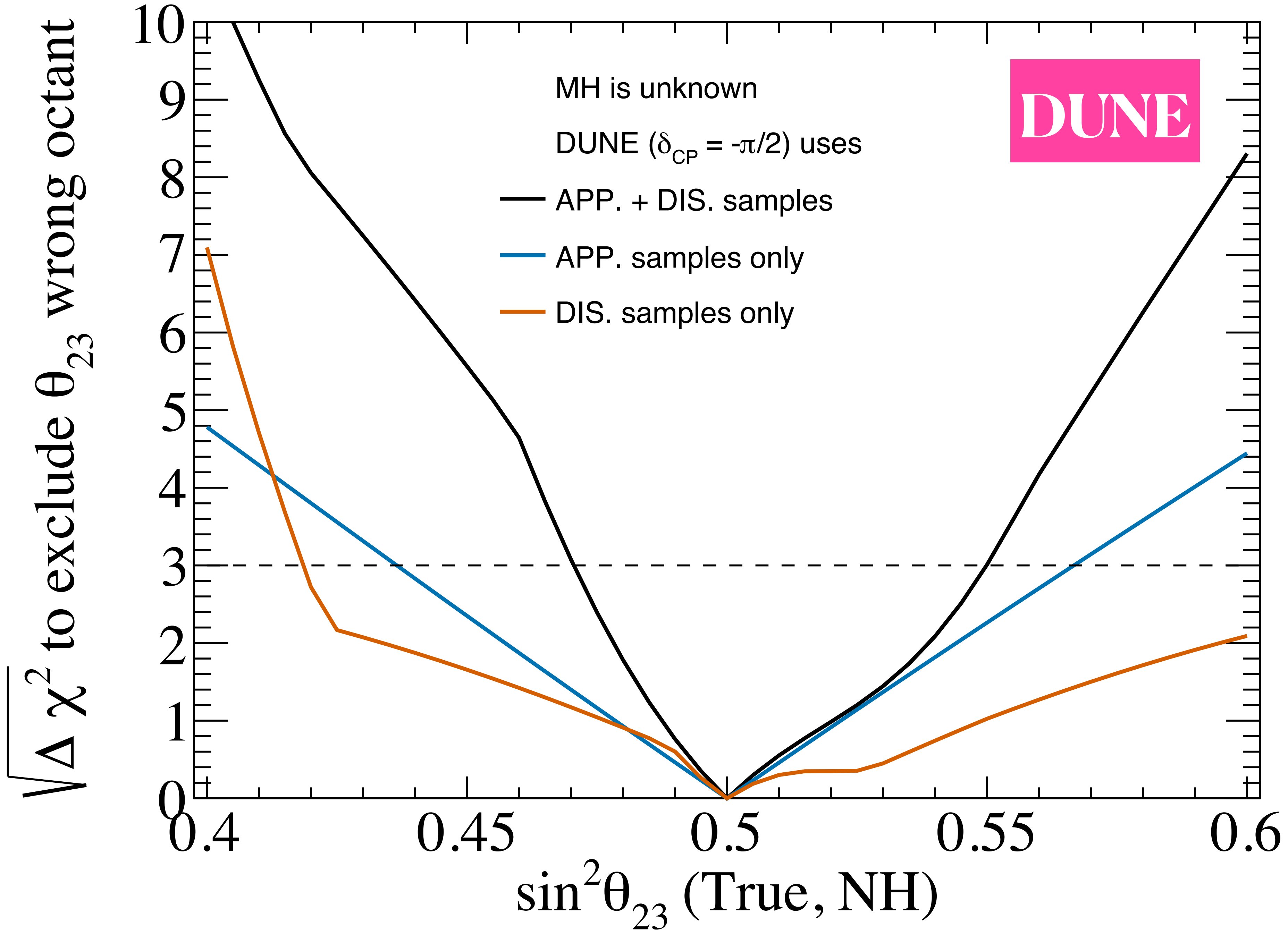
- Do the known unknowns (δ_{CP} *and neutrino mass ordering*) affect the θ_{23} precise determination? → **Marginal effect**
- How does of the θ_{13} precision impact on the θ_{23} measurement ? → **Not much when enhancing θ_{13} precision 2.6% -> 1%**
- What are contributions of neutrino and anti-neutrino appearance samples to resolve the θ_{23} ambiguity? → **Interesting to see their difference. Maybe good practice for joint analysis to consider their individual contribution**
- Is the θ_{23} precise determination driven by statistical or systematical errors? → **Both, but 4% to 2% sys. improvement is equivalent to statistic double**
- Is there range of θ_{23} where its ambiguity can not be solved definitively? → **Yes, if θ_{23} lies [0.48-0.54]**

The work is in preparation for arXiv

Backup

Contribute to octant resolving: disappearance and appearance





- For excluding $\sin^2\theta_{23} = 0.5$, main contribution is from disappearance measurement unless $0.5 < \sin^2\theta_{23} \lesssim 0.54$
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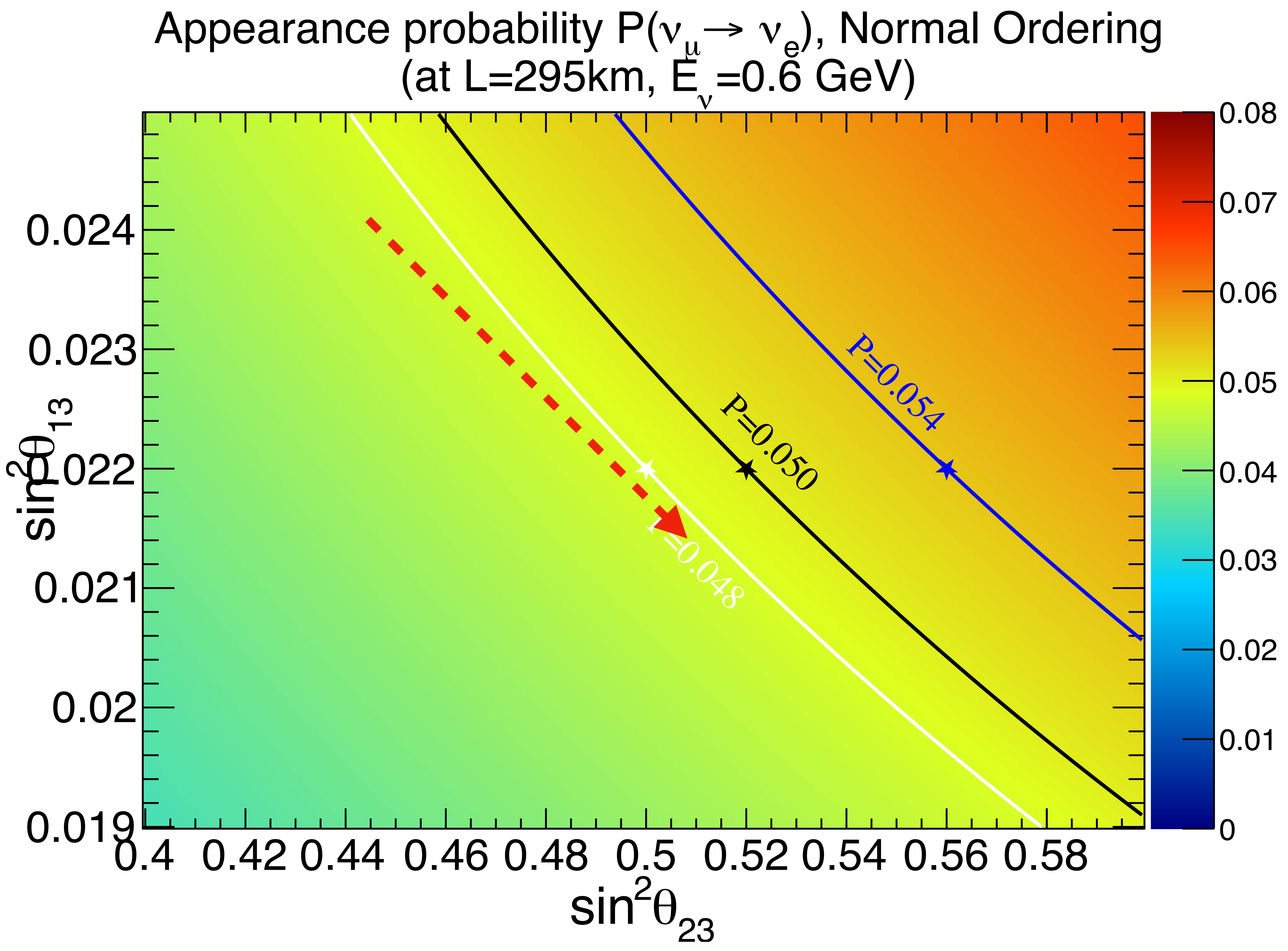
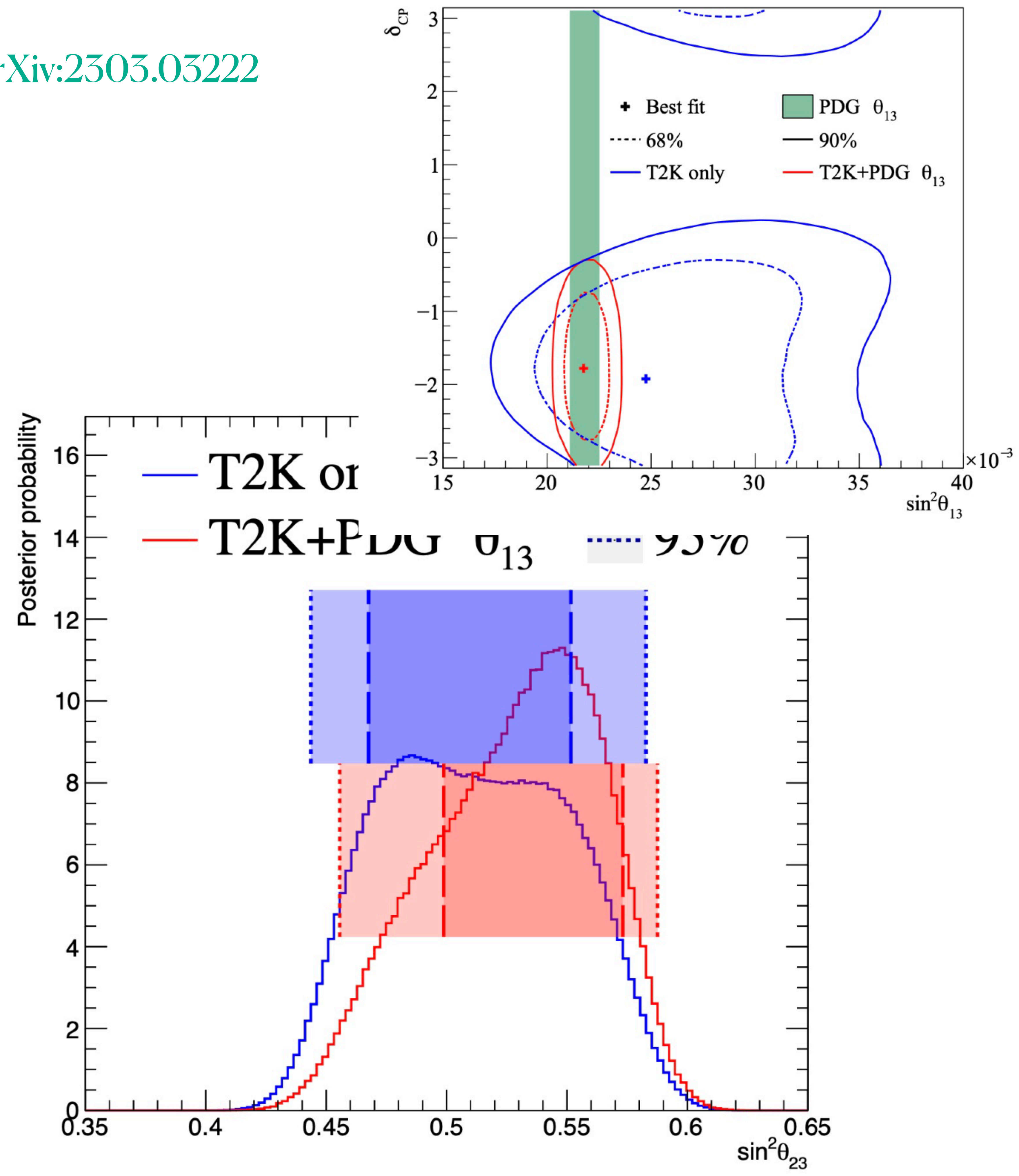
- Typically, octant resolvability is better if θ_{23} lies in the lower octant.

Not depend on the Mass ordering / CP violation

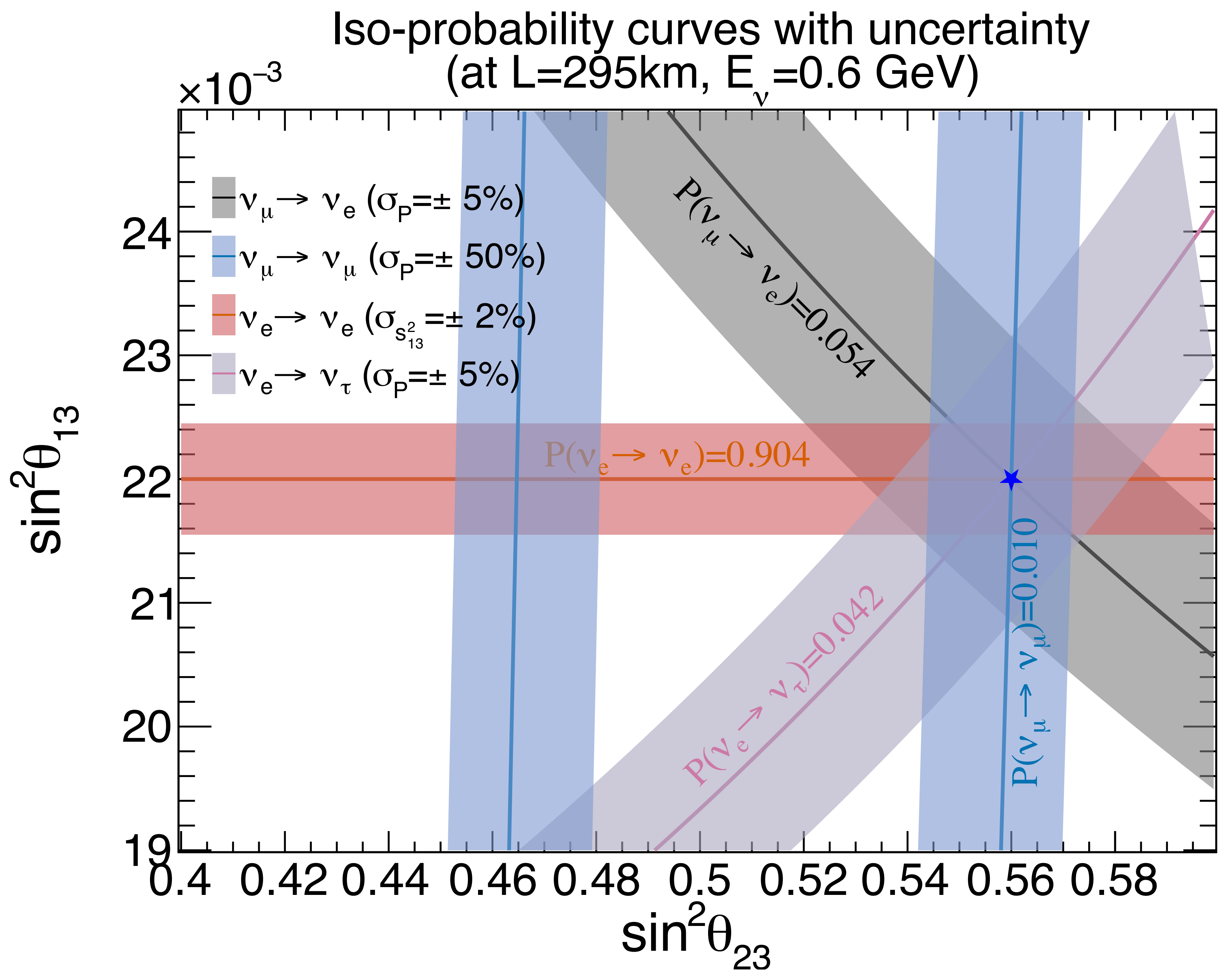


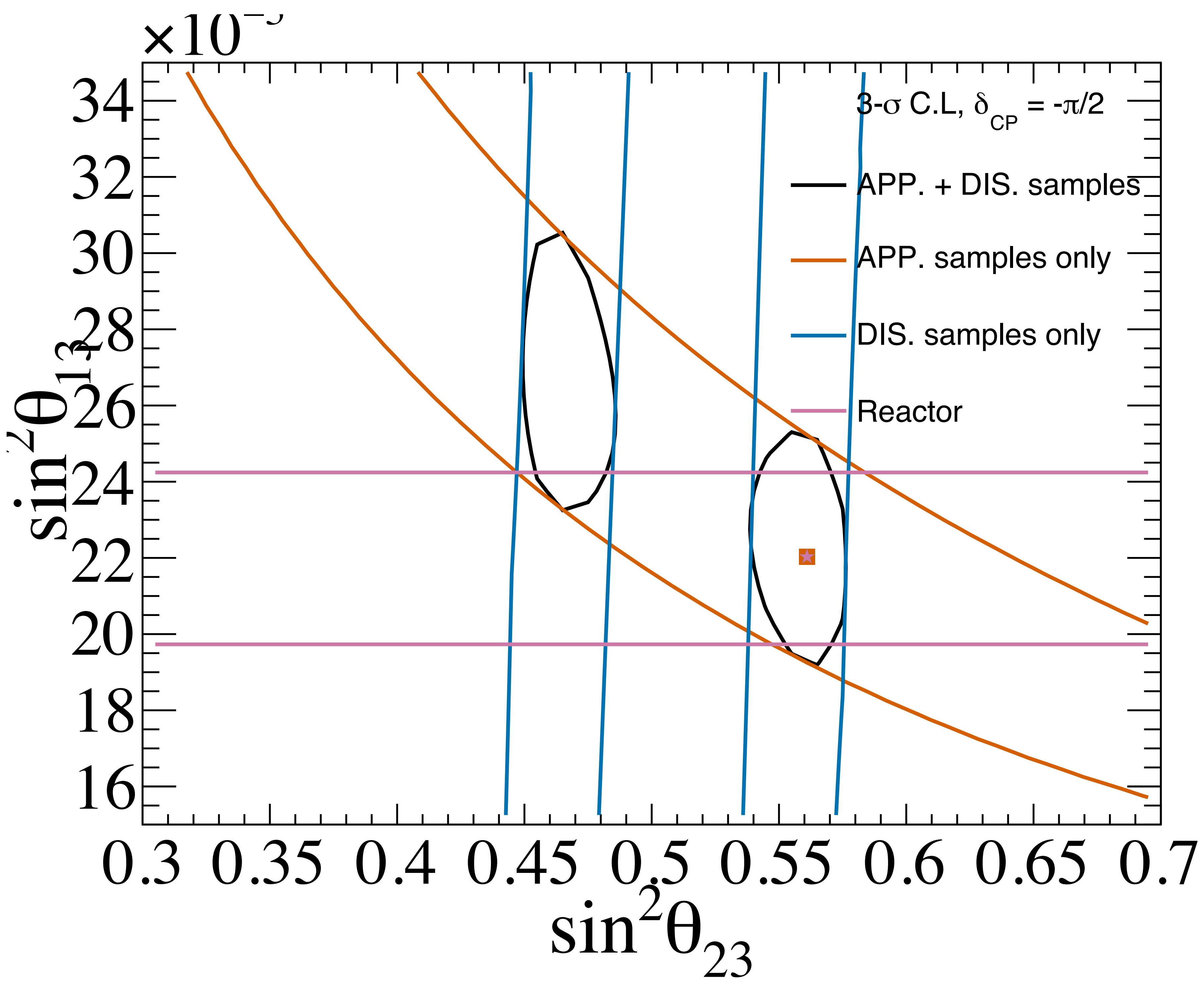
arXiv:2303.03222



Note: use reactor and they are not the same external constraint

Correlation with dcp in appearance

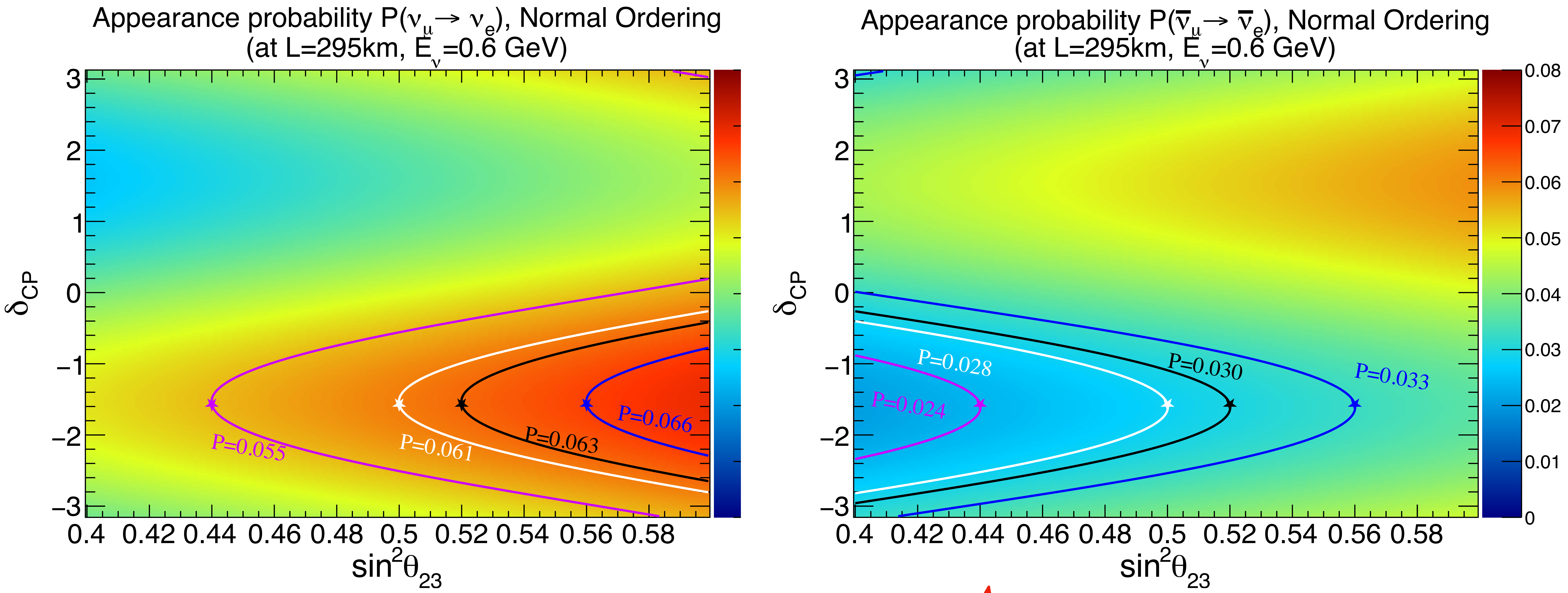




- Mirror symmetric
- Sum of neutrino and anti-neutrino probability is breaking the degeneracy between dcp and sinsq23
  - theta_23 resolving is marginally depending on the center value of dcp and its uncertainty

But the octant resolvability doesn't depend on the truth value of dCP and its precision

Correlation with dcp in appearance

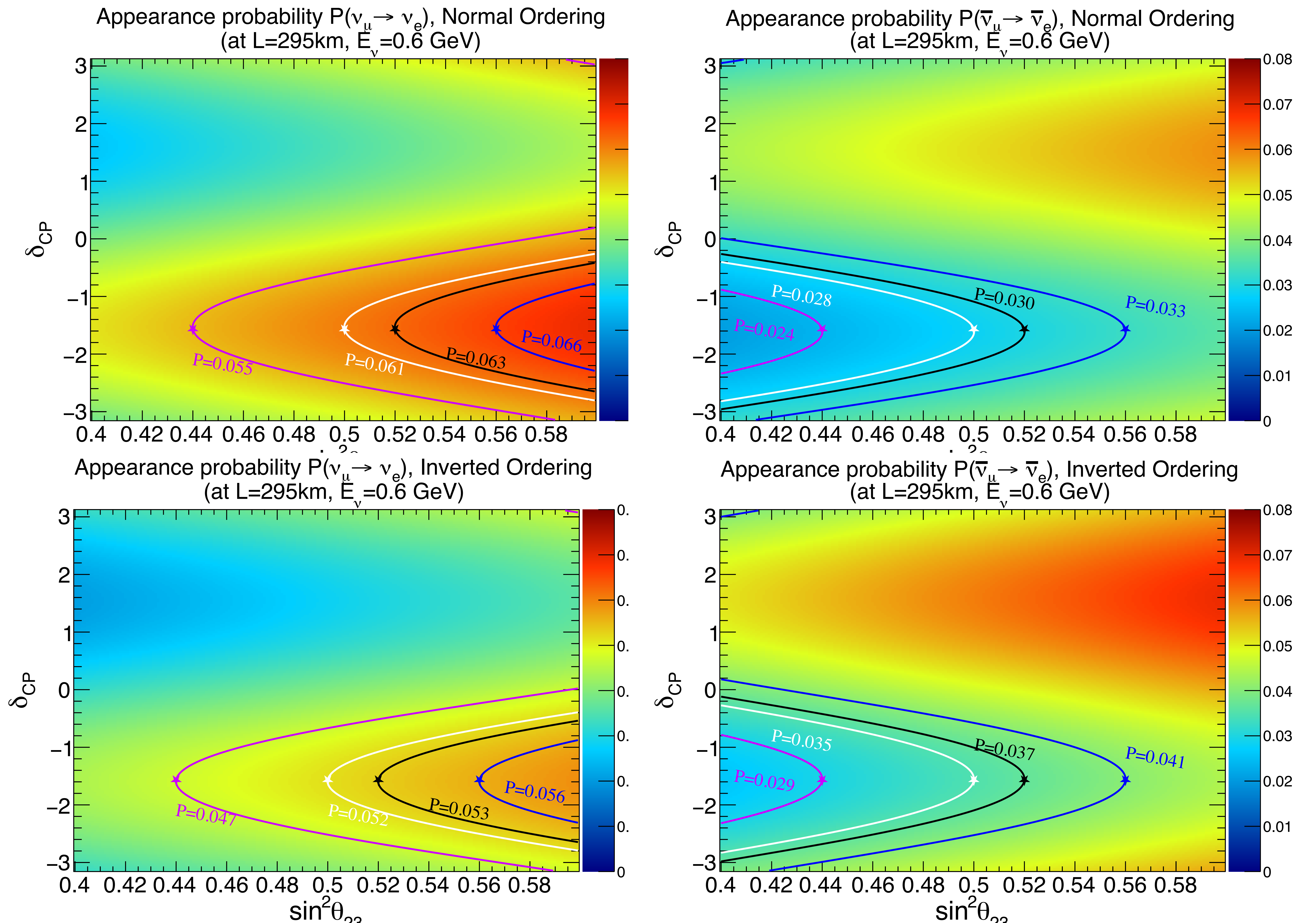


$$P^{iso.}(\theta_{23}, \delta_{CP}) = \sin^2\theta_{23} \mp \frac{A_m}{2\theta_{13}^0} \sin 2\theta_{23} \sin\delta_{CP}$$

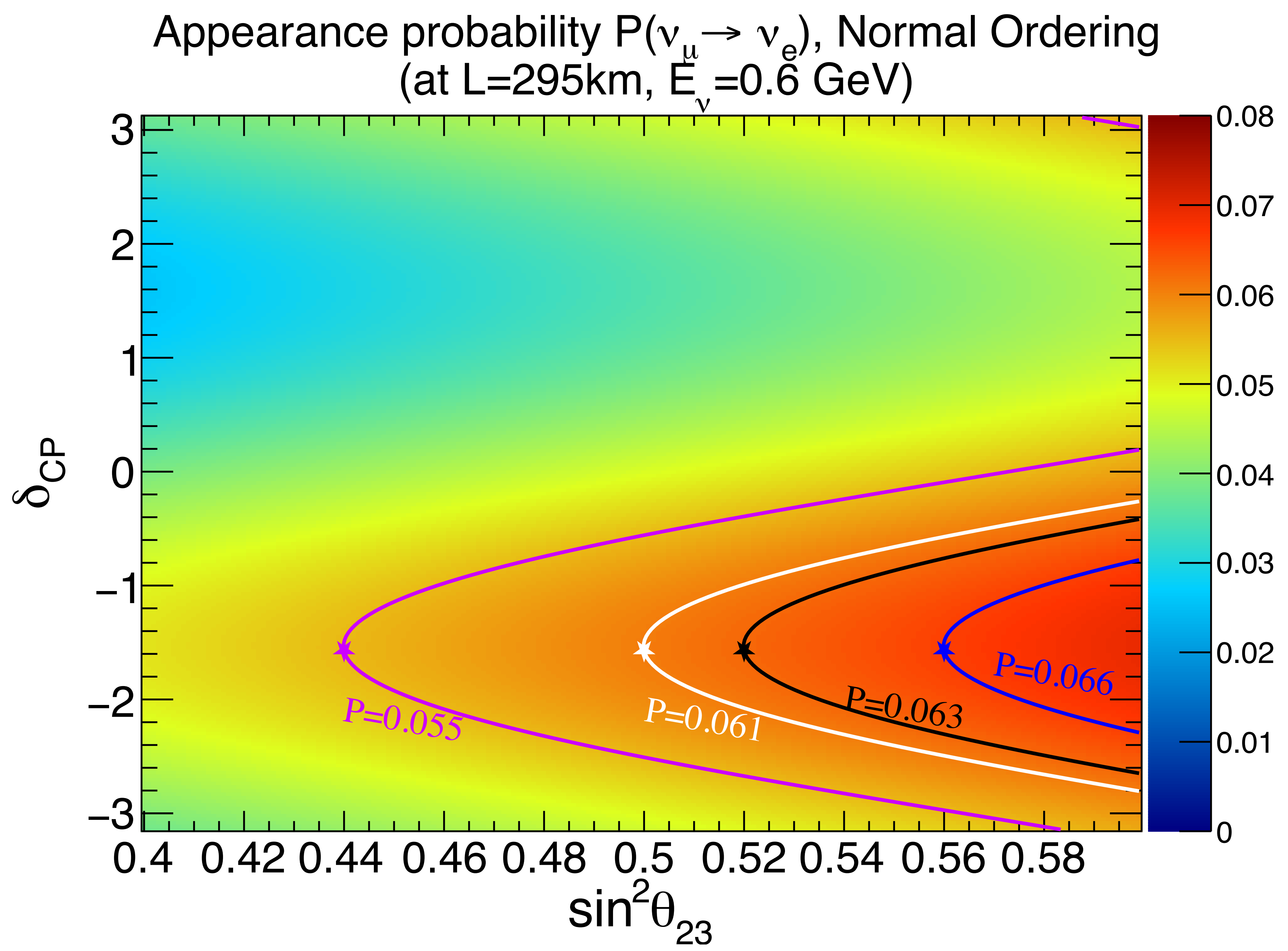
$$A_m = \frac{\pi}{2} \frac{\Delta m_{21}^2}{|\Delta m_{31}^2|} . \sin 2\theta_{12}$$

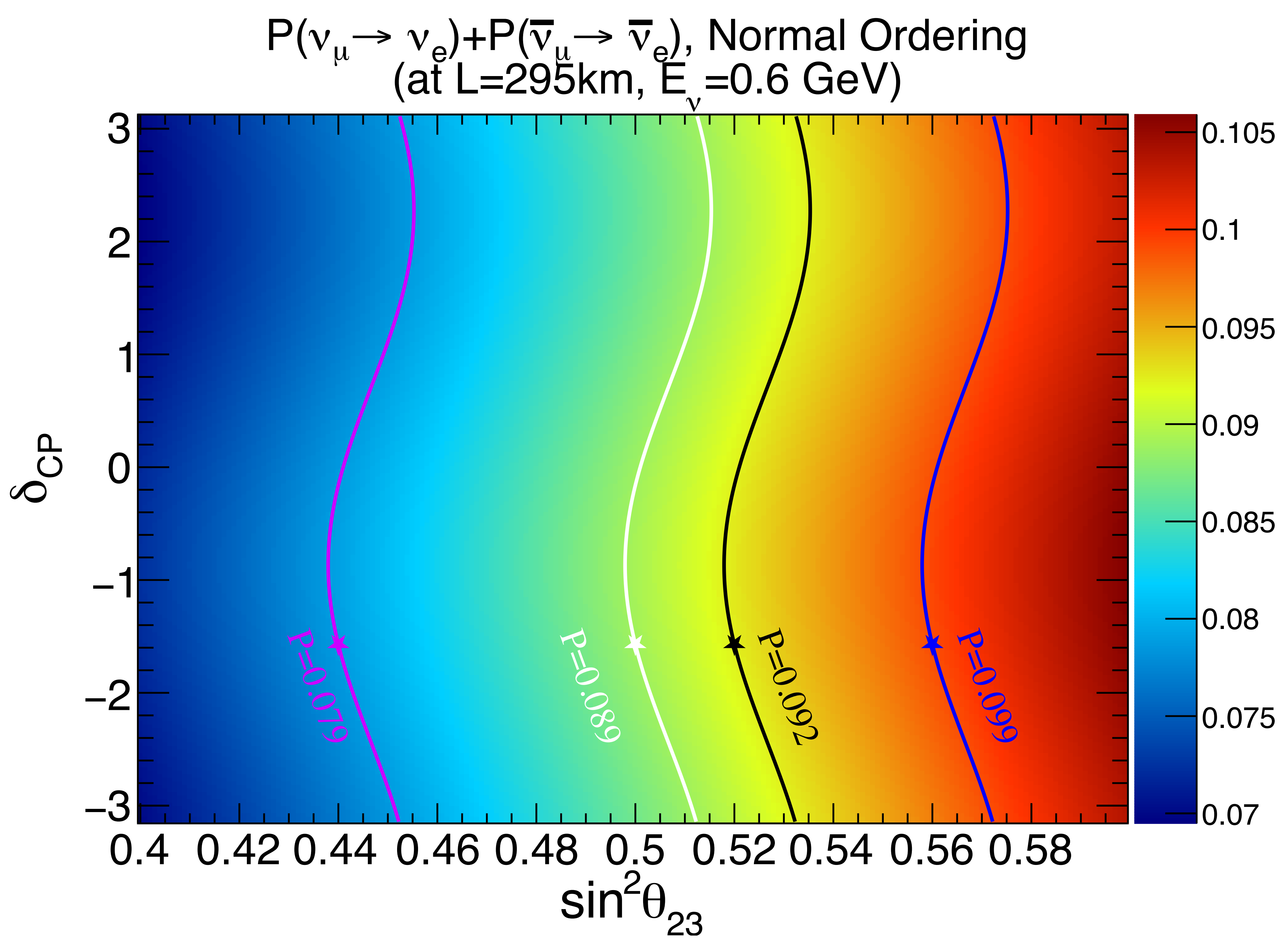
But the octant resolvability doesn't depend on the truth value of dCP and its precision

Inverted almost the same pattern, but with



On correlation with δ_{CP} in $\nu_\mu \rightarrow \nu_e$ appearance





- Strong correlation between θ_{23} and δ_{CP} presented in the appearance probability
- But the octant resolvability of neutrino experiment like T2HK, DUNE doesn't depend much on the truth value of δ_{CP}
  - Neutrino and anti-neutrino share a mirror symmetry and thus cancel out if combine neutrino and anti-neutrino (if at the same statistical level)
  - The iso-probability curve is energy dependence, spectral measurement helps to mitigate the degeneracy
- Contribution of neutrino and anti-neutrino sample for octant resolving are not the same