Ultimate Precision on 2-3 Oscillation Parameters using the Synergy between DUNE and T2HK

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Neutrino Oscillation in 3ν-paradigm

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{\begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}}_{\textbf{PMNS matrix (U)}}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

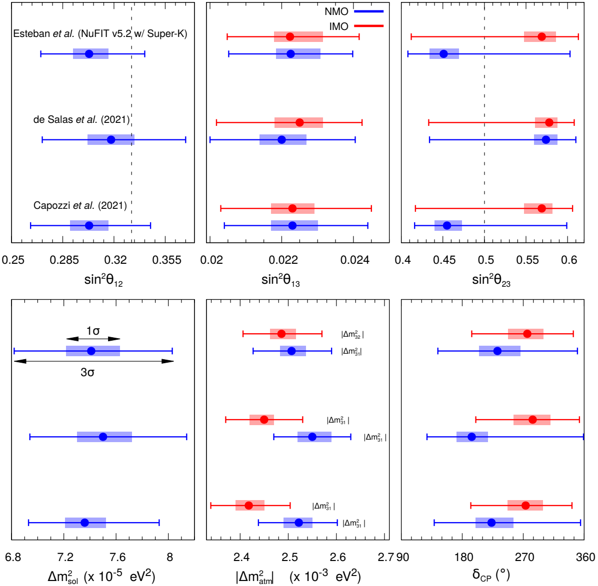
- The three-flavor neutrino oscillation probability is given by

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j<i}\mathrm{Re}(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j})\sin^2(1.27\Delta m^2_{ij}L/E)$$

$$-2\sum_{j<i}\mathrm{Im}(U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j})\sin(2.54\Delta m^2_{ij}L/E)$$

  where, $\Delta m^2_{ij} = m^2_i - m^2_j$ (in eV^2), L is the baseline (in km), and E is the energy of neutrino (in GeV).
- Neutrinos interact with matter via weak interactions by coherent forward elastic scattering.
- Charged-current interaction of ν_e with electrons creates an extra effective matter potential for ν_e, i.e, A=$2\sqrt{2}G_F N_e E = 2V_{CC}E$.
- The Hamiltonian corresponding to CC-interaction with matter is, H= U[$\frac{1}{2E}$diag(m^2_1, m^2_2, m^3_3)]$U^\dagger$ + diag(V_{CC}, 0, 0).

Present global-fit scenario in 3ν-paradigm



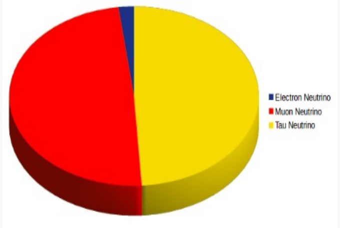
- 3σ (1σ) range of ν oscillation parameters, Esteban *et al*. www.nu-fit.org, de Salas *et al*. arXiv: 2006.11237, and Capozzi *et al*. arXiv: 2107.00532 in NMO and IMO.

Parameters	Relative 1σ error
$\sin^2\theta_{12}$	4.5%
Δm^2_{21}	2.3%
$\sin^2\theta_{23}$	**6.7%**
$\boldsymbol{\Delta m^2_{31}}$	**1.1%**
$\sin^2\theta_{13}$	3%
$\boldsymbol{\delta_{CP}}$	**16%**

- The two most uncertain parameters are θ_{23} and δ_{CP}.
- θ_{23} is in LO (HO) for NMO (IMO) in analysis of Esteban *et al*. and Capozzi *et al*.

Deviation of θ_{23} from maximal mixing

- $\mu \rightarrow \tau$ symmetry
  $|\nu_2\rangle = \cos\theta_{23}\ |\nu_\mu\rangle + \sin\theta_{23}\ |\nu_\tau\rangle$
  $|\nu_3\rangle = -\sin\theta_{23}\ |\nu_\mu\rangle + \cos\theta_{23}\ |\nu_\tau\rangle$
- If $\theta_{23} = 45^\circ$, i.e for MM, ν_2 and ν_3 have equal contributions of ν_μ and ν_τ.



Model	θ_{23}	θ_{13}	θ_{12}
Tri-bimaximal	45°	0°	35°
Bi-maximal	45°	0°	45°
Tri-maximal	δ dependent	0°	35.26°
Tri-bimaximal Cabibbo	45°	8.54°	35°
Bi-large	39°	12.12°	39°
Bi-trimaximal	36.23°	12.18°	36.23°

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https://arxiv.org/abs/hep-ph/9604415, https://arxiv.org/abs/1402.4271v1

Roles of different channels in our study

- The appearance probability for $\nu_\mu \to \nu_e$ channel

$$P(\nu_\mu \to \nu_e) \approx \sin^2\theta_{23}\sin^2(2\theta_{13})\frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

$$-\alpha\sin(2\theta_{13})\zeta\sin\delta_{CP}\sin(\Delta)\frac{\sin(\hat{A}\Delta)}{\hat{A}}\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

$$+\alpha\sin(2\theta_{13})\zeta\cos\delta_{CP}\cos(\Delta)\frac{\sin(\hat{A}\Delta)}{\hat{A}}\frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

- The disappearance probability for $\nu_\mu \to \nu_\mu$ channel

$$P(\nu_\mu \to \nu_\mu) \approx 1-\sin^2(2\theta_{23})\sin^2(\Delta)$$

$$+(\alpha\Delta)c_{12}^2\sin^2(2\theta_{23})\sin(2\Delta)$$

$$-2\alpha\zeta\cos(\delta_{CP})\cos(\Delta)\frac{\sin(\hat{A}\Delta)}{\hat{A}}\frac{\sin[(\hat{A}-1)\Delta]}{(\hat{A}-1)}$$

$$+\frac{2}{(\hat{A}-1)}\alpha\zeta\cos(2\theta_{23})\cos(\delta_{CP})\sin(\Delta)[\hat{A}\sin(\Delta)-\frac{\sin(\hat{A}\Delta)}{\hat{A}}\cos((\hat{A}-1)\Delta)]$$

$$\boxed{\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2} \approx 0.033}$$

$$\boxed{\Delta = \Delta m_{31}^2 L/4E}$$

$$\boxed{\zeta = \cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}}$$

Appearance channel helps in θ_{23} octant exclusion.

Disappearance channel helps in the precision of θ_{23}.

Salient features of DUNE and T2HK

Features	**DUNE**	**T2HK**
Baseline	1300 km (Larger matter effect)	295 km (Smaller matter effect)
Detector Mass	40 kt (Smaller statistics)	187 kt (Larger statistics)
Detection technique	LArTPC	Water Cherenkov
Beam type	Wide-band, on-axis	Narrow-band, off-axis (2.5°)
Beam Power	1.2 MW	1.3 MW
Run time	5 years ν + 5 years $\bar{\nu}$	2.5 years ν + 7.5 years $\bar{\nu}$
P.O.T/year	1.1×10^{21}	2.7×10^{21}
Systematics in App. (Disapp.) channel	2% (5%)	5% (3%)

Abi. B et al., https://arxiv.org/abs/2103.04797; K. Abe et al., https://arxiv.org/abs/1611.06118

Benchmark values of neutrino oscillation parameters in our work

Parameters	Best-fit	1σ range	3σ range	
$\Delta m^2_{21}/10^{-5}\ eV^2$	7.36	7.21-7.52	6.93-7.93	Fixed
$\sin^2\theta_{12}/10^{-1}$	3.03	2.90-3.16	2.63-3.45	Fixed
$\sin^2\theta_{13}/10^{-2}$	2.23	2.17-2.30	2.04-2.44	Fixed
$\sin^2\theta_{23}/10^{-1}$	**4.55**	4.40-4.73	4.16-5.99	Free
$\Delta m^2_{31}/10^{-3}\ eV^2$	**2.522**	2.490-2.545	2.436-2.605	Free
$\delta_{\mathrm{CP}}/^\circ$	**223**	200-256	139-355	Free

Capozzi et al., https://arxiv.org/abs/2107.00532

Definition of χ^2 used in our analysis

$$\chi^2 = \min_{(\vec{\zeta}_s, \vec{\zeta}_b)} \left\{ 2\sum_{i=1}^{n} (\tilde{y}_i - x_i - x_i ln\frac{\tilde{y}_i}{x_i}) + \zeta_s^2 + \zeta_b^2 \right\},$$

where, n is the total number of bins and

$$\tilde{y}_i(\{\omega\}, \{\zeta_s, \zeta_b\}) = N_i^{th}(\{\omega\})[1 + \pi^s \zeta_s] + N_i^b(\{\omega\})[1 + \pi^b \zeta_b].$$

Here,

- $N_i^{th}(\{\omega\})$ = Predicted no. of events in i-th bin in theory for a set of osc. params ω.
- $N_i^b(\{\omega\})$ = No. of background events in the i-th bin where CC background depends on ω but NC does not.
- π^s, π^b = Systematic errors in signal and background.
- ζ_s, ζ_b ='Pulls' due to systematic errors in signal and background respectively.
- $x_i = N_i^{ex} + N_i^b$ (where, N_i^{ex} = No. of observed CC signal events in the i-th bin in data, N_i^b = No. of the background events in data at i-th bin).

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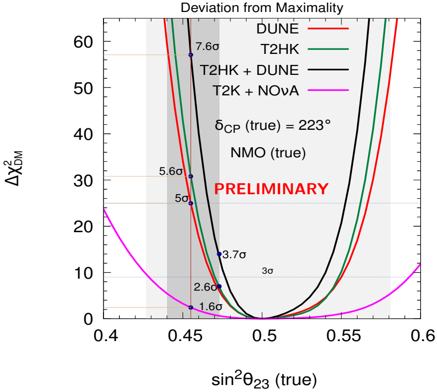
https://arxiv.org/pdf/1509.03517.pdf.

Deviation from maximal θ_{23}



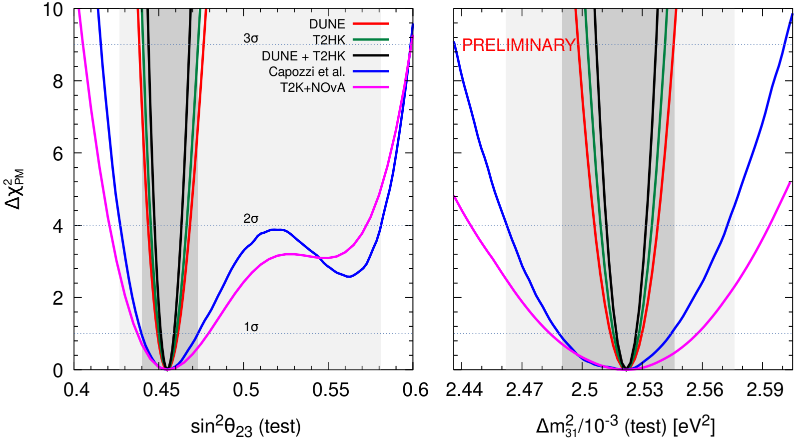
Agarwalla, Kundu, Singh.

(Work in progress)

$$\boxed{\Delta\chi^2_{\text{DM}} = \min_{\delta_{\text{CP}},\Delta m^2_{31}} \left\{\chi^2\left(\sin^2\theta_{23}^{\text{test}} = 0.5\right) - \chi^2\left(\sin^2\theta_{23}^{\text{true}} \in [0.4, 0.6]\right)\right\}}$$

- In Nature, if true $\sin^2\theta_{23}$ attains the lower value of the current 1σ uncertainty (0.473), only DUNE+T2HK can achieve 3σ sensitivity of non-maximal θ_{23} with the present benchmark values.

Precision measurement of θ_{23} and Δm^2_{31}



Agarwalla, Kundu, Singh (work in progress)

- The combination of DUNE and T2HK outperforms their individual performances to the precision measurement of θ_{23} and Δm^2_{31}.

Definition of Relative 1σ precision and $\Delta\chi^2_{PM}$

The relative 1σ precision in the measurement of oscillation parameters ζ is estimated as follows:

$$p(\zeta) = \frac{\zeta^{\mathrm{max}} - \zeta^{\mathrm{min}}}{6.0 \times \zeta^{\mathrm{true}}} \times 100\%.$$

ζ^{max} and ζ^{min} are the allowed 3σ upper and lower bounds, respectively.

$$\Delta\chi^2_{\text{PM, }\sin^2\theta_{23}} = \min_{(\delta_{CP}, \Delta m^2_{31})} \left\{ \chi^2\left(\sin^2\theta_{23}^{\mathrm{test}} \in [0.4, 0.6]\right) - \chi^2\left(\sin^2\theta_{23}^{\mathrm{true}} = [0.455]\right) \right\},$$

$$\Delta\chi^2_{\text{PM, }\Delta m^2_{31}} = \min_{(\delta_{CP}, \sin^2\theta_{23})} \left\{ \chi^2\left(\sin^2\theta_{23}^{\mathrm{test}} \in [2.4, 2.6] \times 10^{-3}\right) - \chi^2\left(\sin^2\theta_{23}^{\mathrm{true}} = 2.522 \times 10^{-3}\right) \right\}$$

Parameter	Relative 1σ precision (%) **[Agarwalla, Kundu, Singh (work in progress)].**					
	T2HK	DUNE	T2HK+DUNE	T2K+NOνA	Capozzi *et al.*	JUNO
$\sin^2\theta_{23}$	1.18	1.40	0.88	7.10	6.72	—
Δm^2_{31}	0.25	0.31	0.20	0.99	1.09	0.2

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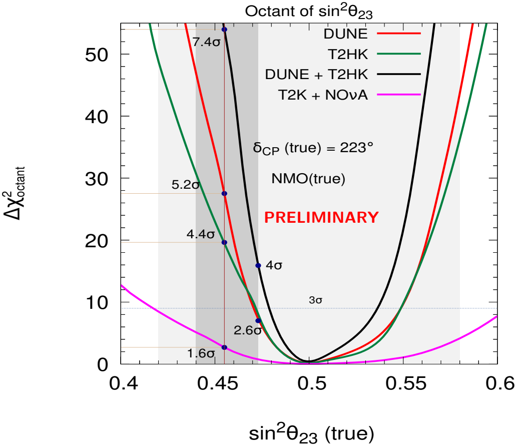
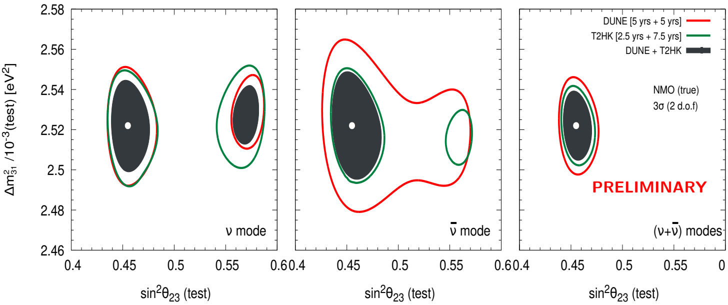
Potential of exclusion of the wrong octant of θ_{23}



Agarwalla, Kundu, Singh (work in progress).

- At lower confindence, T2HK wins due to larger statistics whereas, at higher confidence DUNE wins due to lesser systematics in appearance channel.
- The combined setup of DUNE and T2HK boosts their individual performances to exclude the wrong octant solution.

Allowed regions in $\sin^2\theta_{23} - \Delta m^2_{31}$ plane

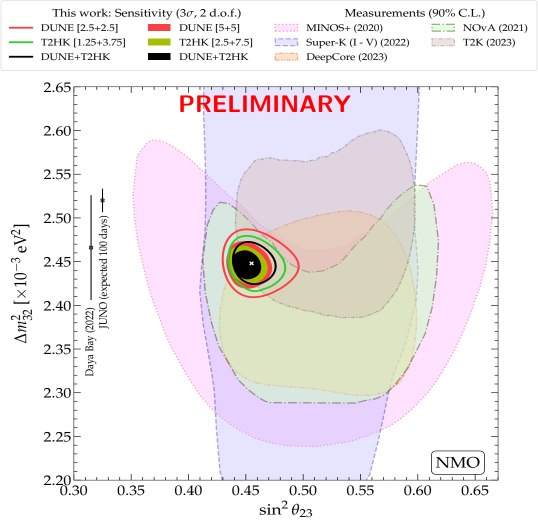


Agarwalla, Kundu, Singh (work in progress)

- The combination of DUNE and T2HK can exclude the higher octant (HO) only in antineutrino mode at 3σ C.L. breaking $\sin^2\theta_{23} - \delta_{\rm CP}$ degeneracy due to higher $\bar{\nu}$ statistics in T2HK. So, majority of the appearance events are free from fake (matter-induced) CP-phase.

- HO can be ruled out when both ν and $\bar{\nu}$ modes are considered together.

Comparison of the precision measurements of $\sin^2\theta_{23}$ & Δm^2_{32} by the other experiments



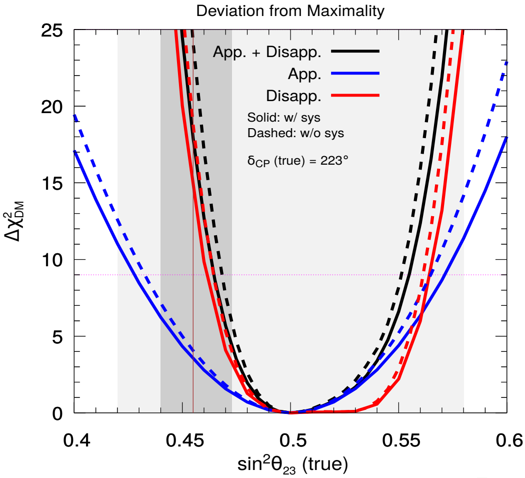
Agarwalla, Kundu, Singh (work in progress)

Takehome messages

- Ongoing long-baseline and atmospheric experiments (e.g.- T2K, NOνA, MINOS+, Super-K etc.) strongly suggest deviation from maximal mixing of θ_{23}.
- DUNE has large matter effect so is expected to measure Δm^2_{31} precisely. But the larger matter effect induces fake CP-asymmetry which is negligible in T2HK.
- The disappearance statistics of T2HK for the antineutrinos is larger whereas, DUNE has larger matter effect. So, the combined setup improves the present achievable precision of $\sin^2\theta_{23}$ and Δm^2_{31} by a factor of 7 and 5, respectively.
- Synergy between DUNE and T2HK helps to boost the performance of measuring non-maximal θ_{23}, precision of $\sin^2\theta_{23}$ and Δm^2_{31}, and exclusion of the wrong octant solution than their standalone performances with a high confidence.

Thank You !!!

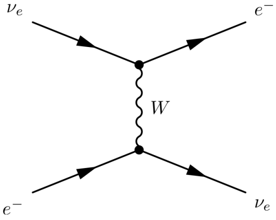
Backup 01 - Effect of Systematics of DUNE in probing non-maximal θ_{23}

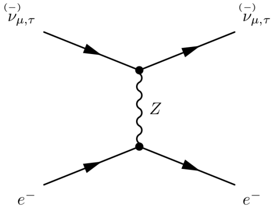


Backup 02 - Table of systematics in DUNE

True $\sin^2\theta_{23}$	Channels	2%, 5%	0%, 0%	5%, 5%	5%, 10%	10%, 10%
0.455 (Best-fit)	App.+Disapp.	**17.64**	**24.13**	**16.88**	**16.74**	**15.42**
	App.	**3.52**	**4.05**	**2.33**	**2.33**	**1.05**
	Disapp.	**14.31**	**18.79**	**14.31**	**14.16**	**14.16**
0.473 (1σ upper bound)	App.+Disapp.	**4.28**	**5.72**	**3.88**	**3.84**	**3.42**
	App.	**1.27**	**1.47**	**0.84**	**0.84**	**0.38**
	Disapp.	**2.99**	**3.88**	**2.99**	**2.97**	**2.97**

Backup 03 - Matter Effect





- Neutrinos interact with matter by coherent forward elastic scattering.
- Charge current interaction of ν_e with electrons creates an extra effective matter term for ν_e, i.e,
  A=$2\sqrt{2}G_F N_e E$.
- Matter term changes sign when we switch from neutrino to anti-neutrino mode.
- Matter term modifies oscillation probability differently depending on the sign of Δm^2.
- The Hamiltonian corresponding to interaction with matter via CC-interaction is,
  H= U$[\frac{1}{2E}\text{diag}(m_1^2,\ m_2^2,\ m_3^2)]U^\dagger + \text{diag}(V_{CC},\ 0,\ 0)$

Backup 04 - Total number of events in DUNE and T2HK

Channel		LO ($\sin^2\theta_{23} = 0.455$)		MM ($\sin^2\theta_{23} = 0.5$)		HO ($\sin^2\theta_{23} = 0.599$)	
		DUNE	T2HK	DUNE	T2HK	DUNE	T2HK
App.	ν	**1601** [1586]	**1598** [1588]	**1729** [1712]	**1725** [1713]	**2004** [1983]	**1996** [1981]
	$\bar{\nu}$	**297** [187]	**919** [755]	**328** [209]	**1021** [844]	**399** [260]	**1251** [1044]
Disapp.	ν	**15529** [14286]	**10064** [9487]	**15209** [13974]	**9628** [9057]	**15857** [14597]	**10661** [10074]
	$\bar{\nu}$	**9008** [4433]	**13949** [8985]	**8884** [4333]	**13541** [8643]	**9252** [4648]	**14613** [9553]

Table: Total (Signal + Background) appearance and disappearance event rates in DUNE and T2HK assuming 480 kt·MW·years and 2431 kt·MW·years of exposure, respectively. Events in parenthesis does not include the effect of wrong-sign contamination. The events are simulated by General Long Baseline Experiment Simulator (GLoBES).

- Contribution of wrong sign events is more in $\bar{\nu}$ mode than ν due to the cross-section suppression.
- Initially pions or kaons are produced due to pp or pn collision. Positive chaged mesons are abundant than the negative one. Hence, contamination of ν in $\bar{\nu}$ beam is more.

Backup 05 - Definition of $\Delta\chi^2_{\mathsf{DM}}$

$$\Delta\chi^2_{\mathsf{DM}} = \min_{(\vec{\lambda})} \left\{ \chi^2 \left(\sin^2\theta_{23}^{\mathrm{true}} \in [0.4, 0.6] \right) - \chi^2 \left(\sin^2\theta_{23}^{\mathrm{test}} = 0.5 \right) \right\},$$

where, $\lambda = \delta_{\mathrm{CP}}$, Δm^2_{31} are the marginalised parameters.

Backup 06 - Definition of $\Delta\chi^2_{\text{octant}}$

- For $\sin^2\theta_{23}$ (true)$<$0.5 (LO),

$$\Delta\chi^2_{\text{octant}} = \min_{(\vec{\lambda})}\left\{\chi^2\left(\sin^2\theta_{23}^{\text{true}} = [0.4, 0.5)\right) - \chi^2\left(\sin^2\theta_{23}^{\text{test}} = (0.5, 0.6]\right)\right\}$$

- For $\sin^2\theta_{23}$ (true)$>$0.5 (HO),

$$\Delta\chi^2_{\text{octant}} = \min_{(\vec{\lambda})}\left\{\chi^2\left(\sin^2\theta_{23}^{\text{true}} = (0.5, 0.6]\right) - \chi^2\left(\sin^2\theta_{23}^{\text{test}} = [0.4, 0.5)\right)\right\}$$

where, $\lambda = \delta_{\text{CP}}$, Δm^2_{31} is the marginalized parameters.

Total event rates in DUNE and T2HK