# Neutrinos and Proton Decay in Grand Unified Theories

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Neutrino Workshop @ IFIRSE
Quy Nhon, Vietnam
July 17, 2023

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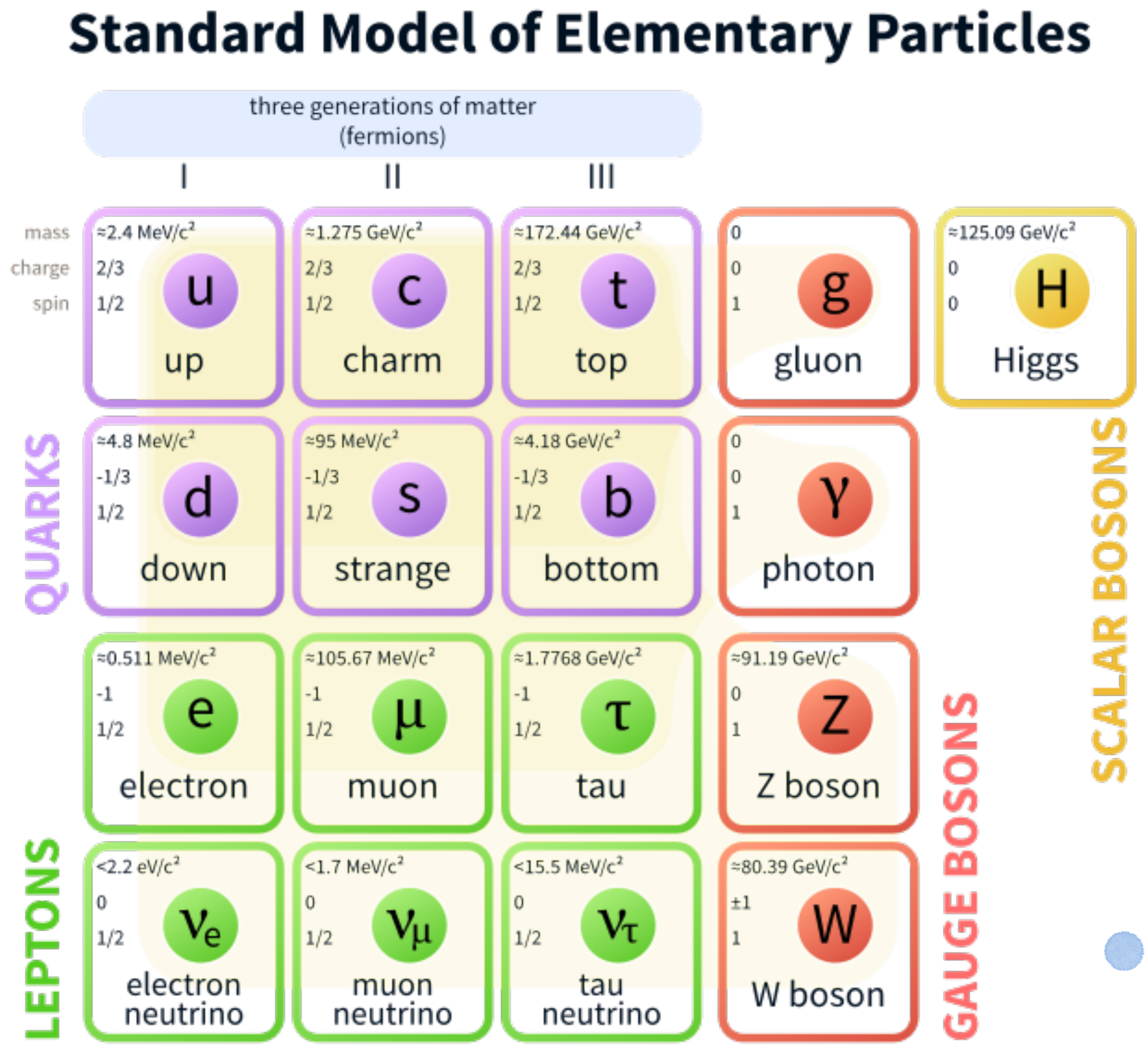
# Quarks and Leptons




[Wikipedia]

- Three generations
- Hierarchical mass

# Mixing matrices

CKM matrix

PMNS matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \qquad \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix}$$

- CKM matrix is close to unit matrix.
- PMNS matrix has large mixing.

Is it possible to understand this characteristic flavor structure of quarks and leptons in a unified way?

# Grand Unified Theories (GUTs)

Grand Unified Theories (GUTs) provide a promising framework:

- Unification of quarks and leptons
- Unification of Yukawa couplings
- Introduction of right-handed neutrinos

# Unification of quarks and leptons

## SU(5) example

$$\mathrm{SU}(5) \supset \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$$

$$\mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix}$$

$$\bar{\mathbf{5}} = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix}$$

Quarks and leptons are embedded into the same multiplet to form a representation of the GUT gauge group.

# Unification of Yukawa couplings

SU(5) example

$$\mathcal{L}_{\text{Yukawa}} \supset -\sqrt{2} f^{ij} \mathbf{10}_i \overline{\mathbf{5}}_j \bar{H}$$

$$\Rightarrow \quad -V_{ij}^* f_j Q_i \bar{D}_j H - f_i L_i \bar{E}_i H$$

$$\Rightarrow \quad f_{d_i}(M_{\text{GUT}}) = f_{e_i}(M_{\text{GUT}})$$

In some cases, threshold corrections, etc. may be needed to explain the observed values.

# Introduction of right-handed neutrinos

Some GUTs predict the presence of right-handed neutrinos:

SU(5)

$$\mathbf{1} = \bar{N} \qquad \bar{\mathbf{5}} = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix} \qquad \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix}$$

Right-handed neutrinos are introduced as a different representation.

SO(10)

$$\mathbf{16} = \mathbf{1} \oplus \bar{\mathbf{5}} \oplus \mathbf{10}$$

Right-handed neutrinos are unified with other quarks and leptons.

# Roles of neutrino experiments

Neutrino experiments, such as

- Neutrino oscillation experiments
- Cosmological determination of neutrino mass sum
- Neutrinoless double β decay

etc.

give us information about

- Neutrino masses
- Mass ordering
- Mixing
- CP phases (Dirac/Majorana)

etc.

→ Important implications for GUT models

# Implications of neutrino masses

Light neutrino masses are related to the masses of right-handed neutrinos via the seesaw formula:

$$m_\nu \simeq \frac{(y_\nu v)^2}{M_R}$$

P. Minkowski (1977), T. Yanagida (1979)
M. Gell-Mann, P. Ramond, R. Slansky (1979)
S. L. Glashow (1980)
R. N. Mohapatra and G. Senjanovic (1980)

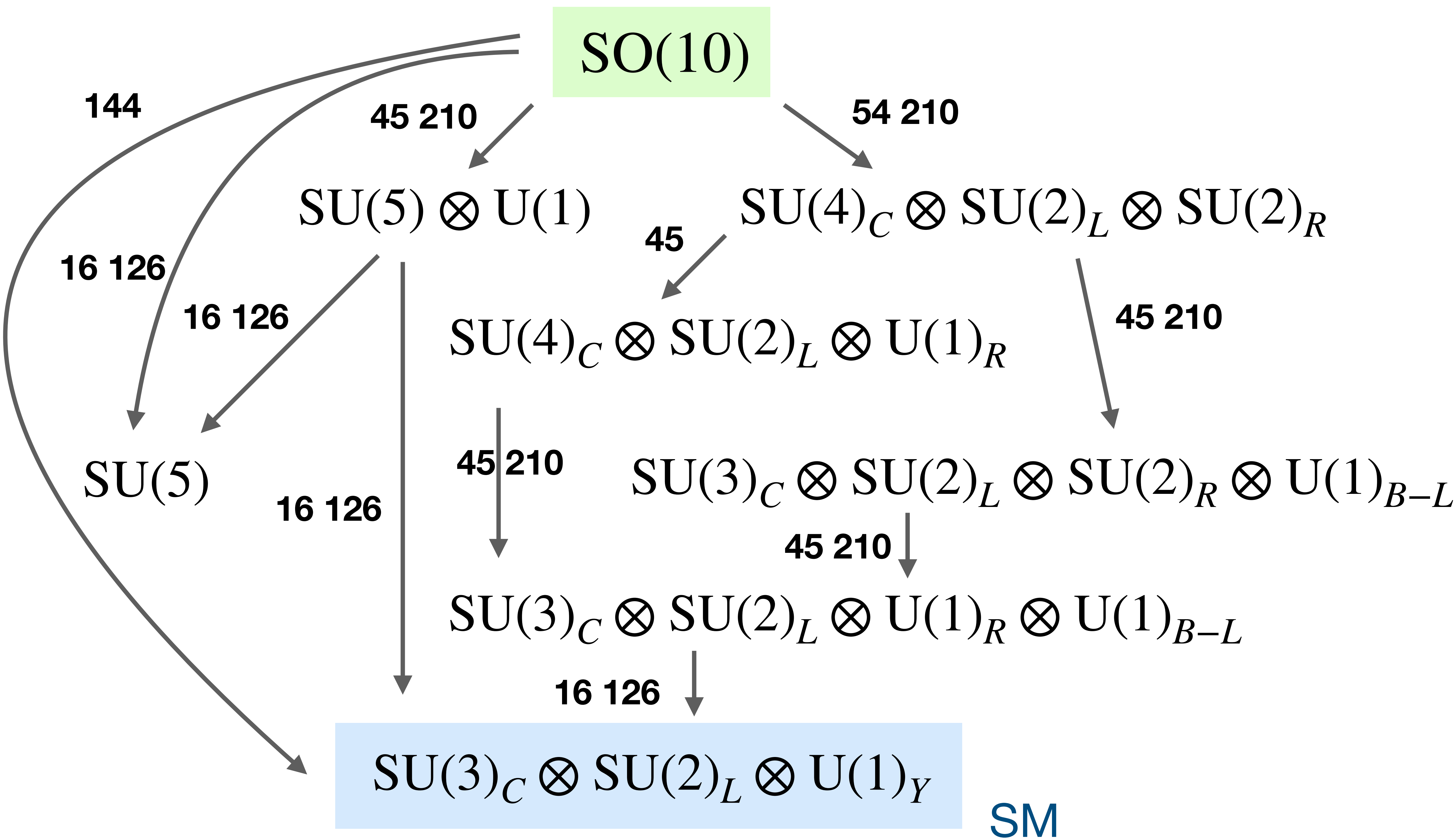
In GUTs, the neutrino Yukawa couplings can be related to other Yukawa couplings.

Determination of $m_\nu$ → Determination of $M_R$

→ Implications for the gauge symmetry breaking pattern.

# Gauge symmetry breaking in SO(10)

SO(10) can be broken to the SM gauge group in a various way:



# Intermediate gauge symmetry

Right-handed neutrinos acquire masses after the intermediate gauge symmetry is spontaneously broken.

e.g.) $$\mathrm{SO}(10) \to \mathrm{SU}(4)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \to G_{\mathrm{SM}}$$

$\mathrm{SO}(10)$

$N_R$ forms a multiplet with $Q_L, L_L, u_R, d_R, e_R$ .

⇒ $N_R$ must also be massless.

$\mathrm{SU}(4)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$

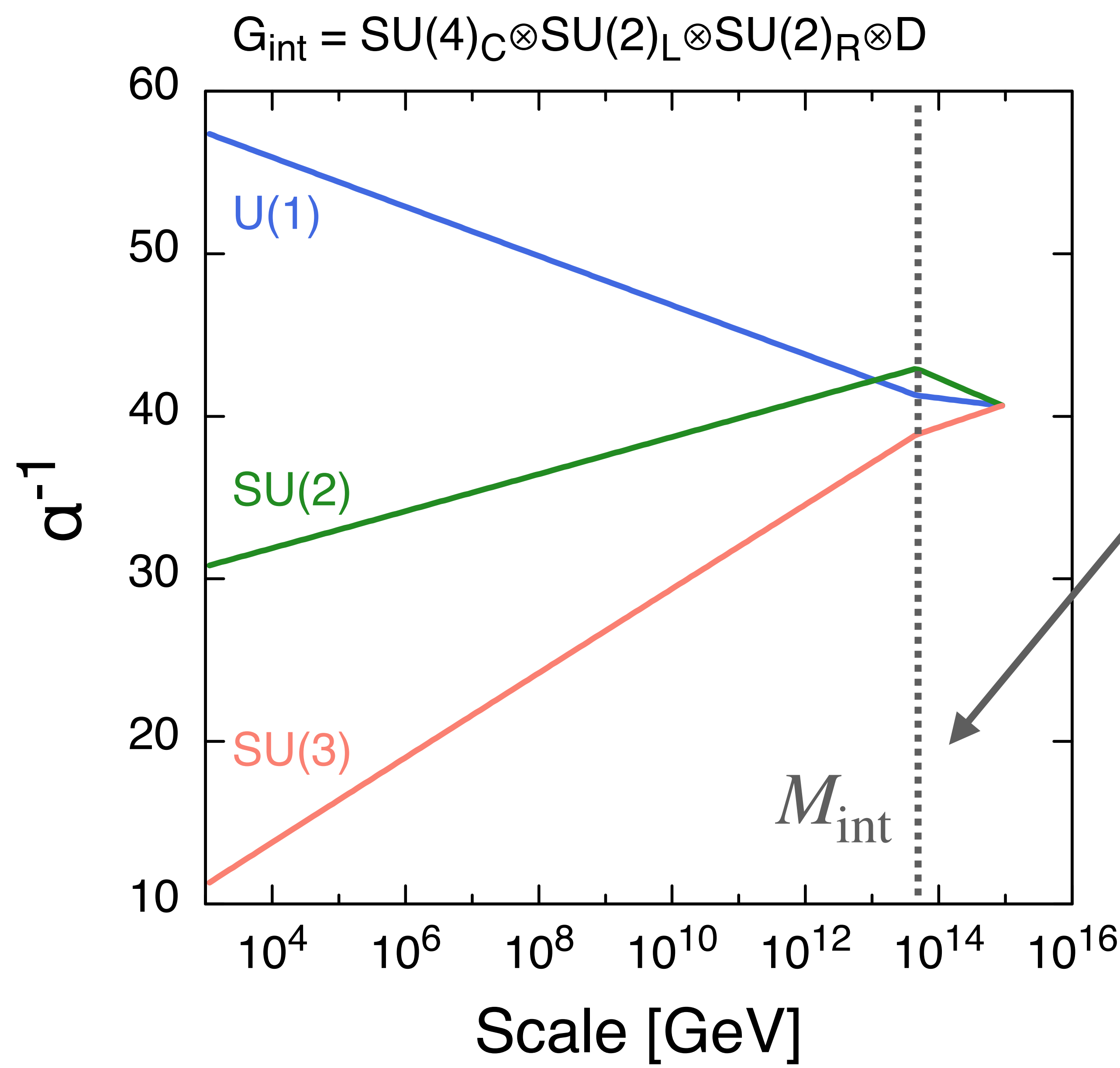
$N_R$ forms a multiplet with $e_R$

⇒ $N_R$ must also be massless.

# Intermediate scale

We thus have $M_R \simeq M_{\rm int}$ .  ($M_{\rm int}$: intermediate symmetry breaking scale)

$M_{\rm int}$ can be inferred from gauge coupling unification.




We can test if this $M_{\rm int}$ is consistent with

$$M_R \simeq \frac{(y_u v)^2}{m_\nu}$$

Up-type Yukawa coupling

# Mixing, CP phases

The determination of neutrino mixing and CP phases can also constrain GUT models.

SO(10) Yukawa structure

$$\mathcal{L} \supset h_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + f_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H$$

Dirac Yukawa couplings          Majorana mass term

Mass matrices

$$(M_u)_{ij} = h_{ij} v^u_{\mathbf{10}} + f_{ij} v^u_{\mathbf{126}} \qquad (M_D)_{ij} = h_{ij} v^u_{\mathbf{10}} - 3 f_{ij} v^u_{\mathbf{126}}$$

$$(M_d)_{ij} = h_{ij} v^d_{\mathbf{10}} + f_{ij} v^d_{\mathbf{126}} \qquad (M_l)_{ij} = h_{ij} v^d_{\mathbf{10}} - 3 f_{ij} v^d_{\mathbf{126}}$$

$$(M_R)_{ij} = f_{ij} v^R_{\mathbf{126}}$$

# Degrees of freedom

- Two complex symmetric matrices, $h_{ij}, f_{ij}$: $2 \times 12 = 24$
- Five complex VEVs, $v_{\mathbf{10}}^{u,d}, v_{\mathbf{126}}^{u,d,R}$: $5 \times 2 = 10$

Among them, 15 are unphysical, i.e., can be rotated away via the field rotation.

➡ We have 19 parameters.

Observables

- Quark, charged lepton masses: 9
- CKM: 4
- Neutrino mass difference: 2
- Neutrino mixing angles: 3

➡ 18

Already highly restrictive

# Fitting

Previous works show that the experimental data can be fitted in this framework.

e.g.) A. S. Joshipura and K. M. Patel, Phys. Rev. **D83**, 095002 (2011);
G. Altarelli and D. Meloni, JHEP **08**, 021 (2013);
A. Dueck and W. Rodejohann, JHEP **09**, 024 (2013).

+ Many efforts

Caveat

Usually, some simplification is made in the analysis.

e.g.) Universal masses for

- GUT-scale particles
- Intermediate-scale particles
- SUSY particles

Relaxing this assumption introduces extra uncertainty in the results.

# A result

(with SUSY)

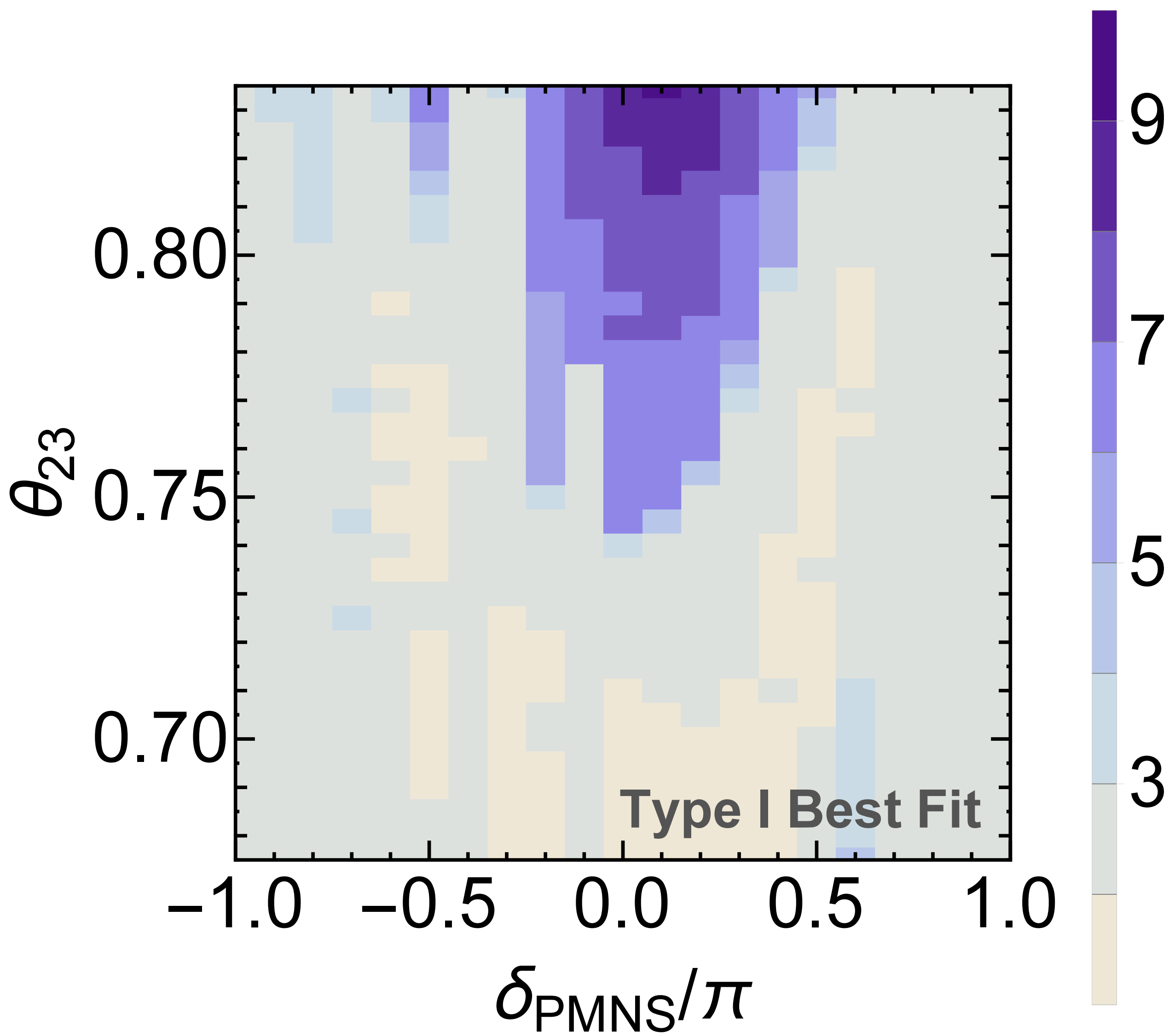
MS (with RGE)

| | $\tan\beta = 50$ | | $\tan\beta = 38$ | | $\tan\beta = 10$ | |
|---|---|---|---|---|---|---|
| Observable | best-fit | pull | best-fit | pull | best-fit | pull |
| $m_d$ | 0.00087 | -1.6714 | 0.00090 | -1.6449 | 0.00091 | -1.6381 |
| $m_s$ | 0.04512 | -0.6371 | 0.04711 | -0.5089 | 0.04870 | -0.4063 |
| $m_b$ | 2.87626 | -0.1526 | 2.88217 | -0.0870 | 2.88499 | -0.0557 |
| $m_u$ | 0.00127 | 0.0018 | 0.00127 | 0.0068 | 0.00127 | 0.0064 |
| $m_c$ | 0.62848 | 0.1129 | 0.62738 | 0.0997 | 0.62854 | 0.1135 |
| $m_t$ | 171.453 | -0.0823 | 171.522 | -0.0593 | 171.539 | -0.0537 |
| $\sin\theta_{12}^q$ | 0.22460 | -0.0040 | 0.22460 | -0.0018 | 0.22460 | -0.0009 |
| $\sin\theta_{23}^q$ | 0.04191 | -0.0675 | 0.04193 | -0.0565 | 0.04193 | -0.0543 |
| $\sin\theta_{13}^q$ | 0.00351 | 0.0241 | 0.00351 | 0.0322 | 0.00351 | 0.0314 |
| $\delta_{\text{CKM}}$ | 1.21318 | -0.0364 | 1.21398 | -0.0225 | 1.21409 | -0.0205 |
| $\Delta m_{21}^2$ | $7.50\times10^{-5}$ | 0.0021 | $7.50\times10^{-5}$ | 0.0013 | $7.50\times10^{-5}$ | 0.0009 |
| $\Delta m_{31}^2$ | $2.47\times10^{-3}$ | -0.0022 | $2.47\times10^{-3}$ | -0.0013 | $2.47\times10^{-3}$ | -0.0010 |
| $\sin^2\theta_{12}^l$ | 0.30015 | 0.0112 | 0.30007 | 0.0053 | 0.30004 | 0.0028 |
| $\sin^2\theta_{23}^l$ | 0.40960 | -0.0129 | 0.40977 | -0.0073 | 0.40987 | -0.0043 |
| $\sin^2\theta_{13}^l$ | 0.02299 | -0.0045 | 0.02297 | -0.0138 | 0.02297 | -0.0123 |
| $m_e$ | 0.00049 | 0.0702 | 0.00049 | 0.0660 | 0.00049 | 0.0605 |
| $m_\mu$ | 0.10315 | 0.0839 | 0.10306 | 0.0665 | 0.10298 | 0.0513 |
| $m_\tau$ | 1.76204 | 0.1809 | 1.75740 | 0.1278 | 1.75528 | 0.1035 |
| $\chi^2_{\text{min}}$ | | **3.294** | | **3.016** | | **2.889** |

Only normal ordering fits well.

A. Dueck and W. Rodejohann, JHEP **09**, 024 (2013).

# Prediction




$\chi^2$ map in SUSY

T. Fukuyama, K. Ichikawa, Y. Mimura, Phys. Rev. **D94**, 075018 (2016).

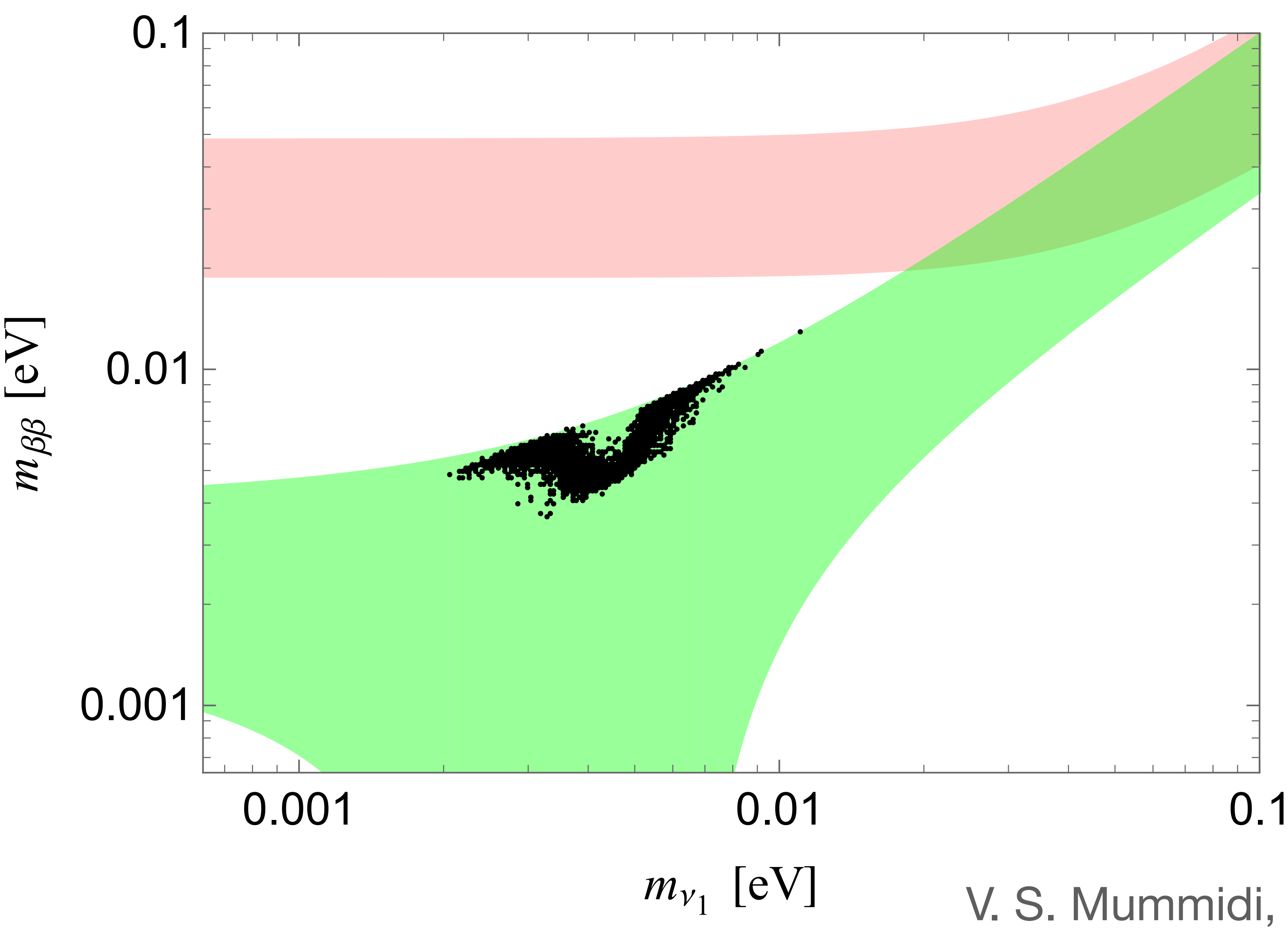
- Such predictions can be tested in future neutrino experiments.
- Depend on the assumption on mass spectrum, etc.

Case-by-case studies needed.

# Leptogenesis in SO(10)

Requiring to explain the baryon asymmetry of the Universe via thermal leptogenesis:

M. Fukugita and T. Yanagida (1986)

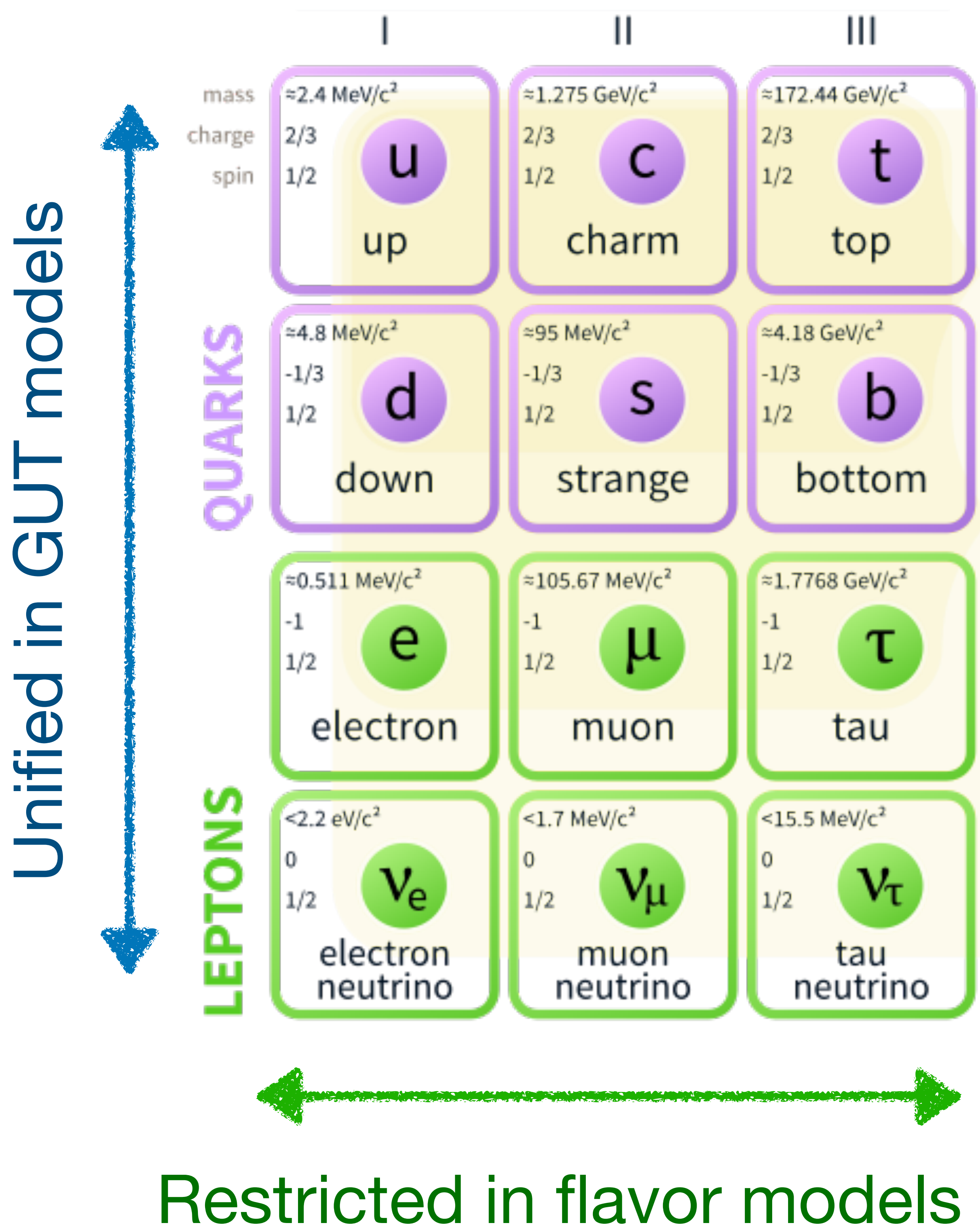



V. S. Mummidi, K. M. Patel, JHEP **12**, 042 (2021).

Could be a nice target for future 0νββ experiments.

# Flavor models

We may obtain a stronger constraint if we consider flavor models.




- The Yukawa structure is restricted in flavor models.
- Model-dependent approach is needed.

# Proton decay

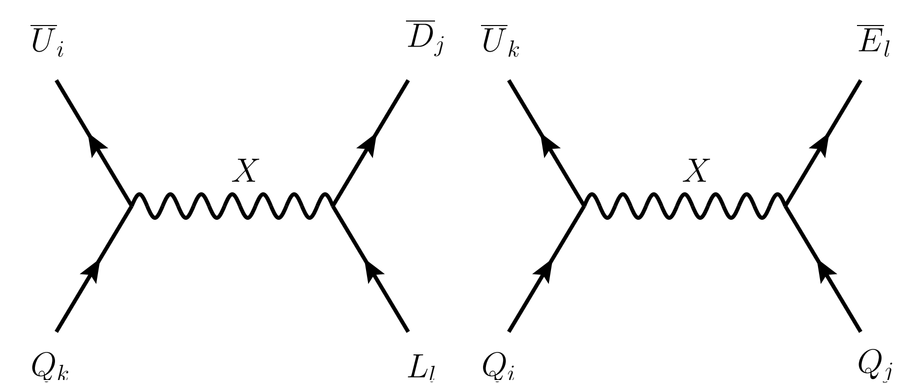
Proton decay is a universal prediction in GUTs.

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

**Unity of All Elementary-Particle Forces**

Howard Georgi* and S. L. Glashow
*Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138*
(Received 10 January 1974)

transitions $p_1 + p_2 \to W \to \bar{n}_3 + e^+$. Exchange of this vector boson contributes directly to the decay $p \to \pi^0 + e^+$. Since the proton is rather stable,[15] this vector boson must be very massive.[16] The




Proton decay branching fractions are sensitive to the GUT gauge group and flavor structure.

Let us see this by looking at the case of flipped SU(5).

# Flipped SU(5)

S. M. Barr (1982); J. P. Derendinger, J.E. Kim, D. V. Nanopoulos (1984); I. Antoniadis, J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos (1987).

## Minimal SU(5)

- Gauge group is SU(5)
- $\bar{\mathbf{5}} \ni \{\overline{D}, L\}\,, \quad \mathbf{10} \ni \{Q, \overline{U}, \overline{E}\}\,, \quad \mathbf{1} = \overline{N}$
- Hypercharge is proportional to the SU(5) generator

## Flipped SU(5)

- Gauge group is SU(5) x U(1)
- $\bar{\mathbf{5}} \ni \{\overline{U}, L\}\,, \quad \mathbf{10} \ni \{Q, \overline{D}, \overline{N}\}\,, \quad \mathbf{1} = \overline{E}$
- Hypercharge is a linear combination of the SU(5) generator and the U(1).

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S. M. Barr (1982); J. P. Derendinger, J.E. Kim, D. V. Nanopoulos (1984); I. Antoniadis, J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos (1987).

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- Gauge group is SU(5) x U(1)
- $\bar{\mathbf{5}} \ni \{\overline{U}, L\}\,, \quad \mathbf{10} \ni \{Q, \overline{D}, \overline{N}\}\,, \quad \mathbf{1} = \overline{E}$
- Hypercharge is a linear combination of the SU(5) generator and the U(1).

# Flipped vs unflipped SU(5)

SU(5) and flipped SU(5) predict different branching ratios for the SU(5) gauge boson exchange processes.

Minimal SU(5)

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{Minimal SU(5)}}}{\Gamma(p \to \pi^0 e^+)_{\text{Minimal SU(5)}}} \simeq \frac{|V_{ud} V_{us}^*|^2}{1 + (1 + |V_{ud}|^2)^2}$$

Depends on CKM.

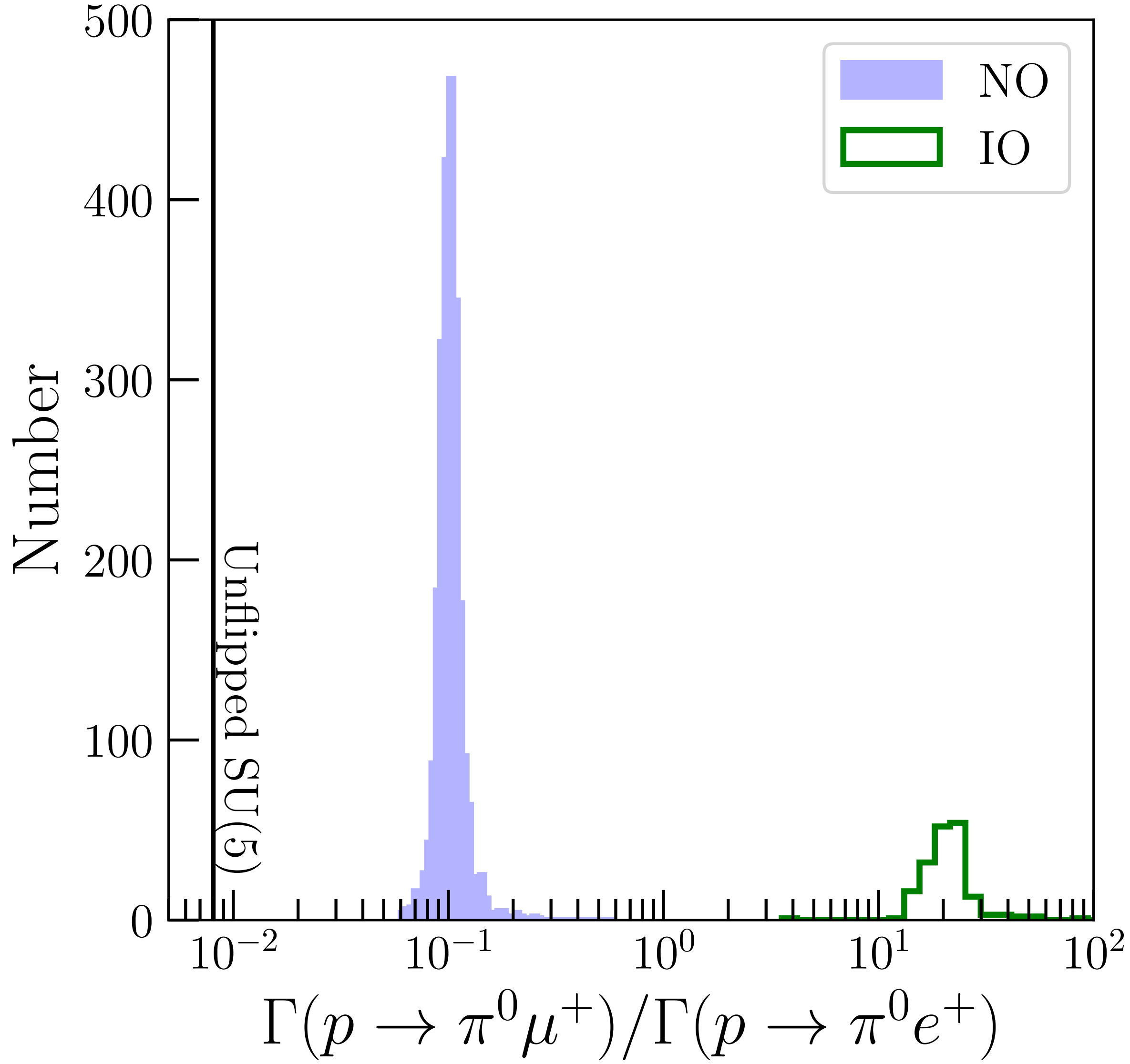
Flipped SU(5)

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} \simeq \frac{|(U_{\text{PMNS}})_{21}|^2}{|(U_{\text{PMNS}})_{11}|^2} \quad \text{(NO)}$$

$$\frac{\Gamma(p \to \pi^0 \mu^+)_{\text{flipped}}}{\Gamma(p \to \pi^0 e^+)_{\text{flipped}}} \simeq \frac{|(U_{\text{PMNS}})_{23}|^2}{|(U_{\text{PMNS}})_{13}|^2} \quad \text{(IO)}$$

Depends on PMNS.

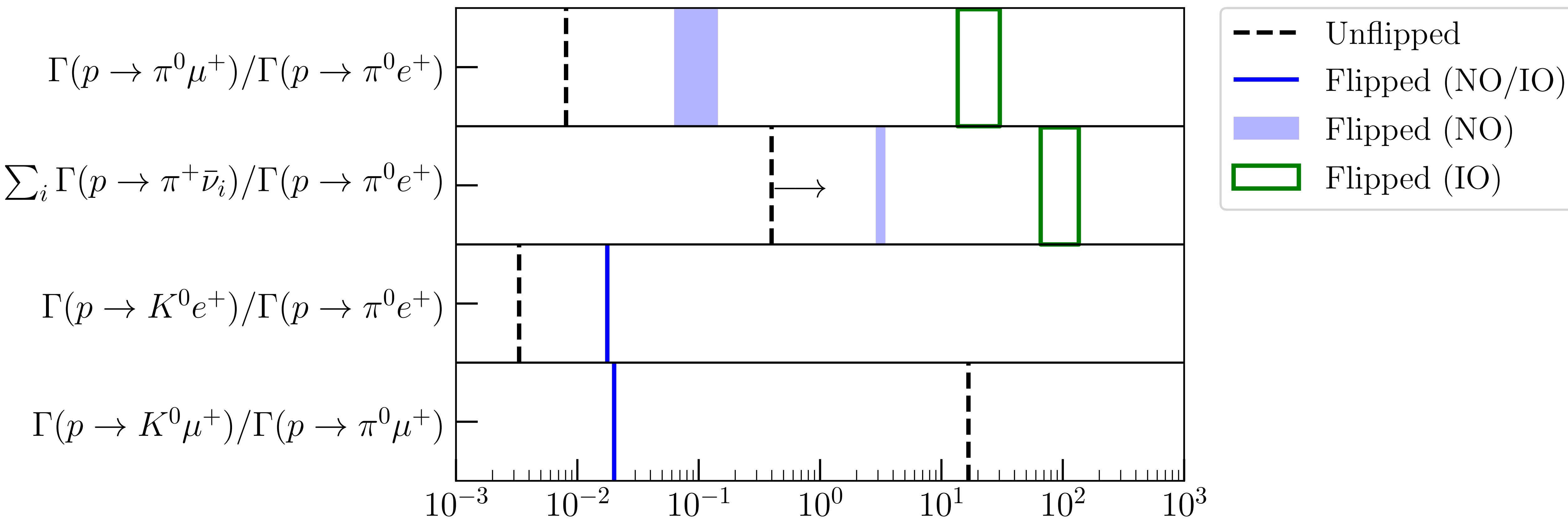
# Flipped vs unflipped SU(5)




- NO and IO give different results.
- Branching fraction to the muon mode is larger in flipped SU(5).

J. Ellis, M. A. G. Garcia, D. Nanopoulos, N. Nagata, K. A. Olive, JHEP **2005**, 021 (2020).

# Flipped vs unflipped SU(5)




- In addition, $\Gamma(p \to K^+\bar{\nu}) = 0$ in flipped SU(5).

Follows from the unitarity of CKM.

- We may distinguish GUT groups by measuring the ratio of branching fractions.

J. Ellis, M. A. G. Garcia, D. Nanopoulos, N. Nagata, K. A. Olive, JHEP **2005**, 021 (2020).

# Proton decay in SO(10)

Different pattern of proton decay branching ratios is found in SO(10) models.

An example in non-SUSY SO(10)

| Process | Branching ratio |
|---|---|
| $p \to \pi^0 e^+$ | $\approx 47\%$ |
| $p \to \pi^0 \mu^+$ | $\approx 1.00\%$ |
| $p \to \eta^0 e^+$ | $\approx 0.20\%$ |
| $p \to \eta^0 \mu^+$ | $\approx 0.004\%$ |
| $p \to K^0 e^+$ | $\approx 0.16\%$ |
| $p \to K^0 \mu^+$ | $\approx 3.62\%$ |
| $p \to \pi^+ \overline{\nu}$ | $\approx 48\%$ |
| $p \to K^+ \overline{\nu}$ | $\approx 0.22\%$ |

K. Babu and S. Khan, Phys. Rev. **D92**, 075018 (2015).

Again, this highly depends on scenarios.

Case-by-case studies needed.

# Summary

- Neutrino mass is a probe of the gauge symmetry breaking pattern.
- Observation of neutrino mixing and CP phases can constrain GUT models.
- Proton decay searches can play a complementary role in examining the flavor structure of GUTs.

# Backup

# Neutrino sector in flipped SU(5)

We introduce singlet superfields $\phi_i$ .

$$W \supset \lambda_2^i \Psi_{\mathbf{10},i} \Psi_{\bar{\mathbf{5}},i} h + \lambda_6^{ij} \Psi_{\mathbf{10},i} \phi_j \Phi + \frac{1}{2} \mu_i \phi_i^2 + \lambda_8^{ijk} \phi_i \phi_j \phi_k$$

$\Psi_{\mathbf{10},i} \Supset \bar{N}$, $\Psi_{\bar{\mathbf{5}},i} \Supset L$, $h$: Higgs field, $\Phi$: GUT Higgs

Neutrino mass matrix

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} \begin{pmatrix} \nu_i & \nu_i^c & \tilde{\phi}_j \end{pmatrix} \begin{pmatrix} 0 & \lambda_2^i \langle h \rangle & 0 \\ \lambda_2^i \langle h \rangle & 0 & \lambda_6^{ij} \langle \Phi \rangle \\ 0 & \lambda_6^{ij} \langle \Phi \rangle & \mu_j \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_i^c \\ \tilde{\phi}_j \end{pmatrix} + \text{h.c.}$$

$$\lambda_2 \langle h \rangle \simeq (m_u, m_c, m_t)$$

The smallness of neutrino masses can be explained in a similar manner as in the seesaw mechanism.