Constraining Unitarity of Three-flavor Neutrino Mixing in Next-generation Long-baseline Experiments

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Neutrino Mixing in Three Neutrino Framework

Unitary mixing matrix (PMNS matrix)

Flavor basis *Mass basis*

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \underbrace{\begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{bmatrix}}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$\Updownarrow$$

$$\underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_{23} & \sin\theta_{23}\\ 0 & -\sin\theta_{23} & \cos\theta_{23}\end{bmatrix}}_{\text{Atmospheric}} \times \underbrace{\begin{bmatrix}\cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13}\end{bmatrix}}_{\text{Reactor}} \times \underbrace{\begin{bmatrix}\cos\theta_{12} & \sin\theta_{12} & 0\\ -\sin\theta_{12} & \cos\theta_{13} & 0\\ 0 & 0 & 1\end{bmatrix}}_{\text{Solar}}$$

Unitary condition $\Longrightarrow$ $U^\dagger U = U U^\dagger = \mathbb{I}$ $\Longrightarrow$

$$\sum_{i=1}^{3} U^*_{\alpha i} U_{\beta i} = \delta_{\alpha\beta}$$

$$\sum_{\alpha=e,\mu,\tau} U^*_{\alpha i} U_{\beta j} = \delta_{ij}$$

Neutrino Oscillation in Three-Flavor Framework

$$P(\nu_\alpha \to \nu_\beta) = |A(\nu_\alpha \to \nu_\beta)|^2 = \left|\sum_i U^*_{\alpha i} e^{-i\frac{m^2 L}{2E}} U_{\beta i}\right|^2$$

$$= |U^*_{\alpha i} U_{\beta i}|^2 - 4\sum_{i<j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\frac{\Delta_{ij}}{2}$$

$\delta_{\alpha\beta}$

$$\mp 2\sum_{i<j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\Delta_{ij}$$

$$\Delta_{ij} = \frac{\Delta m^2_{ij} L}{2E}$$

- Impact of matter in neutrino oscillation

$$V_{CC} = \sqrt{2} G_F Ne \qquad V_{NC} = \frac{1}{\sqrt{2}} G_F N_n$$

$$H_{\mathrm{mat}} = \underbrace{\frac{1}{2E_\nu} U \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix} U^\dagger}_{H_{vac}} + \begin{bmatrix} V_{CC}+V_{NC} & 0 & 0 \\ 0 & V_{NC} & 0 \\ 0 & 0 & V_{NC} \end{bmatrix}$$

$$H_{mat} = \tilde{U} Diag.[\frac{\tilde{m}_1^2}{2E}, \frac{\tilde{m}_2^2}{2E}, \frac{\tilde{m}_3^2}{2E}]\tilde{U}^\dagger = \left|\sum_i \tilde{U}^*_{\alpha i} e^{i\frac{\tilde{m}_i^2 L}{2E}} \tilde{U}_{\beta i}\right|^2$$

Non-unitarity Neutrino Mixing (NUNM)

Motivation

- Unitarity of PMNS matrix is theoritical assumption that is valid in 3ν paradigm.

  Various neutrino mass-models predict more than three neutrino states.

  JHEP 04 (2011) 123, Phys. Rev. D 92, (2015)

- In the presence of n-3 extra neutrino states, dimension of the mixing matrix is n.

Active-light mixing   Active-heavy mixing

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \\ \vdots \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \cdots \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \cdots \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{bmatrix}$$

Sterile-light mixing   Sterile-heavy mixing

N → 3×3 Active neutrino mixing matrix is not unitary anymore.

Parameterization of NUNM matrix

- General structure of N

$$N = \begin{bmatrix} \eta_{11} & 0 & 0 \\ \eta_{21} & \eta_{22} & 0 \\ \eta_{31} & \eta_{32} & \eta_{33} \end{bmatrix} \times U_{PMNS}$$

- Our convention

$$N = (\mathbb{I} + \alpha)U \qquad \alpha \to \begin{pmatrix} \alpha_{11} & 0 & 0 \\ |\alpha_{21}|e^{i\phi_{21}} & \alpha_{22} & 0 \\ \alpha_{31}e^{i\phi_{31}} & \alpha_{32}e^{i\phi_{32}} & \alpha_{33} \end{pmatrix}$$

In the unitary case: $\alpha_{ij} \to 0$

- Case: $\alpha_{ij} \neq 0$

$$N^\dagger N \to \begin{pmatrix} 1+\alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & 1+\alpha_{22}^2+|\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32}^* & 1+\alpha_{33}^2+|\alpha_{31}|^2+|\alpha_{32}|^2 \end{pmatrix}$$

Flavor Transition in NUNM Scenario

- Neutrino flavor transition probability

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_i N^*_{\alpha i} e^{i\frac{m_i^2 L}{2E}} N_{\beta i} \right|^2$$

$$= \left| \sum_i N^*_{\alpha i} N_{\beta i} \right|^2 - 4 \sum_{i<j} Re(N^*_{\alpha i} N_{\beta i} N_{\alpha j} N^*_{\beta j}) \sin^2 \frac{\Delta_{ij}}{2}$$

$$\mp 2 \sum_{i<j} Im(N^*_{\alpha i} N_{\beta i} N_{\alpha j} N^*_{\beta j}) \sin \Delta_{ij}$$

$\neq \delta_{\alpha\beta}$

Consequence

Zero-distance effect: $P_{\mu e}(L=0) \to \alpha_{11}^2 |\alpha_{21}|^2, \;\; P_{\mu\mu}(L=0) \to |\alpha_{21}|^2 + \alpha_{22}^2$

- Propagation Hamiltonian in matter

$$H = \frac{1}{2E_\nu} \left[\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} + N^\dagger \begin{pmatrix} a_e + a_n & 0 & 0 \\ 0 & a_n & 0 \\ 0 & 0 & a_n \end{pmatrix} N \right]$$

$$a_e = 2\sqrt{2} E_\nu G_F N_e$$

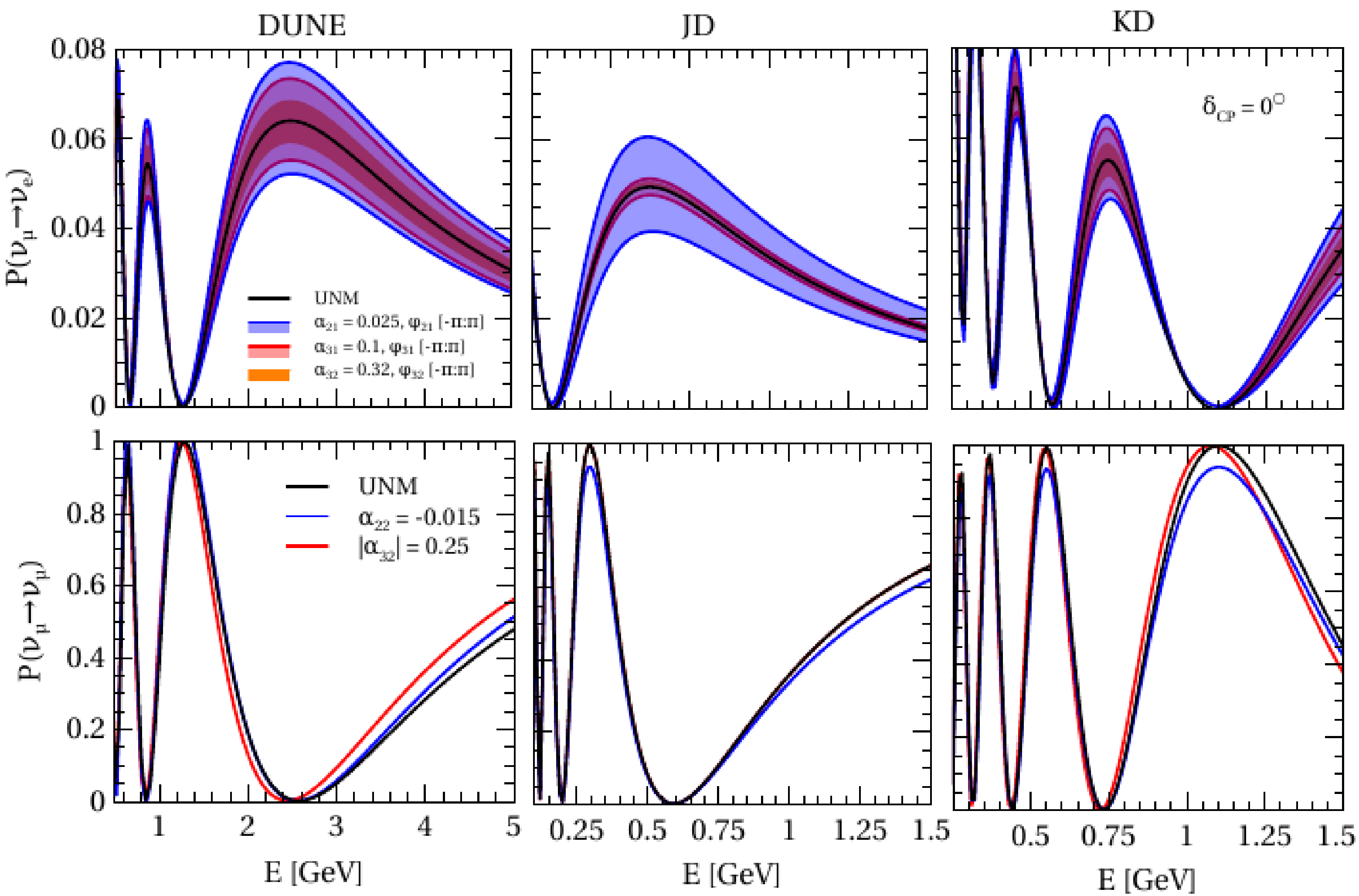
$$a_n = -\sqrt{2} E_\nu G_F N_n$$

Experimental Details

	DUNE	JD/KD*
Detector Mass	40 kt LArTPC	187 kt WC (each)
Baseline	1300 km	295/1100 km
Proton Energy	120 GeV	80 GeV
Beam Type	Wide-band,on-axis	Narrow-band,off-axis
Beam Power	1.2 MW	1.3 MW
P.O.T./year	1.1×10^{21}	2.7×10^{21}
Run time $(\nu + \bar{\nu})$	3.5 yr + 3.5 yr	2.5 yr + 7.5 yr

* JD – Japanese detector (T2HK)
KD – Korean Detector (T2HKK)

Flavor transition in NUNM scenario



- α_{21} shows largest impact in appearance channel
- α_{31} And α_{32} always coupled to matter terms
- α_{22} shows largest impact in disappearance channel

Chi-square Analysis

$$\chi^2(\vec{\omega},\, \kappa_s,\, \kappa_b) = \min_{(\vec{\lambda},\, \kappa_s,\, \kappa_b)} \left\{ 2\sum_{i=1}^{n} (\tilde{y_i} - x_i - x_i \ln \frac{\tilde{y_i}}{x_i}) + \kappa_s^2 + \kappa_b^2 \right\}$$

$$\tilde{y_i}\,(\vec{\omega}, \{\kappa_s, \kappa_b\}) = N_i^{th}(\vec{\omega})[1 + \pi^s \kappa_s] + N_i^b(\vec{\omega})[1 + \pi^b \kappa_b]$$

$$\mathrm{x}_i = N_i^{ex} + N_i^b$$

$N_i^{th}\,(\vec{\omega})$ - Predicted number of signal events in the i-th energy bin from theory

N_i^{ex} - Signal events at i-th bin from the prospective data

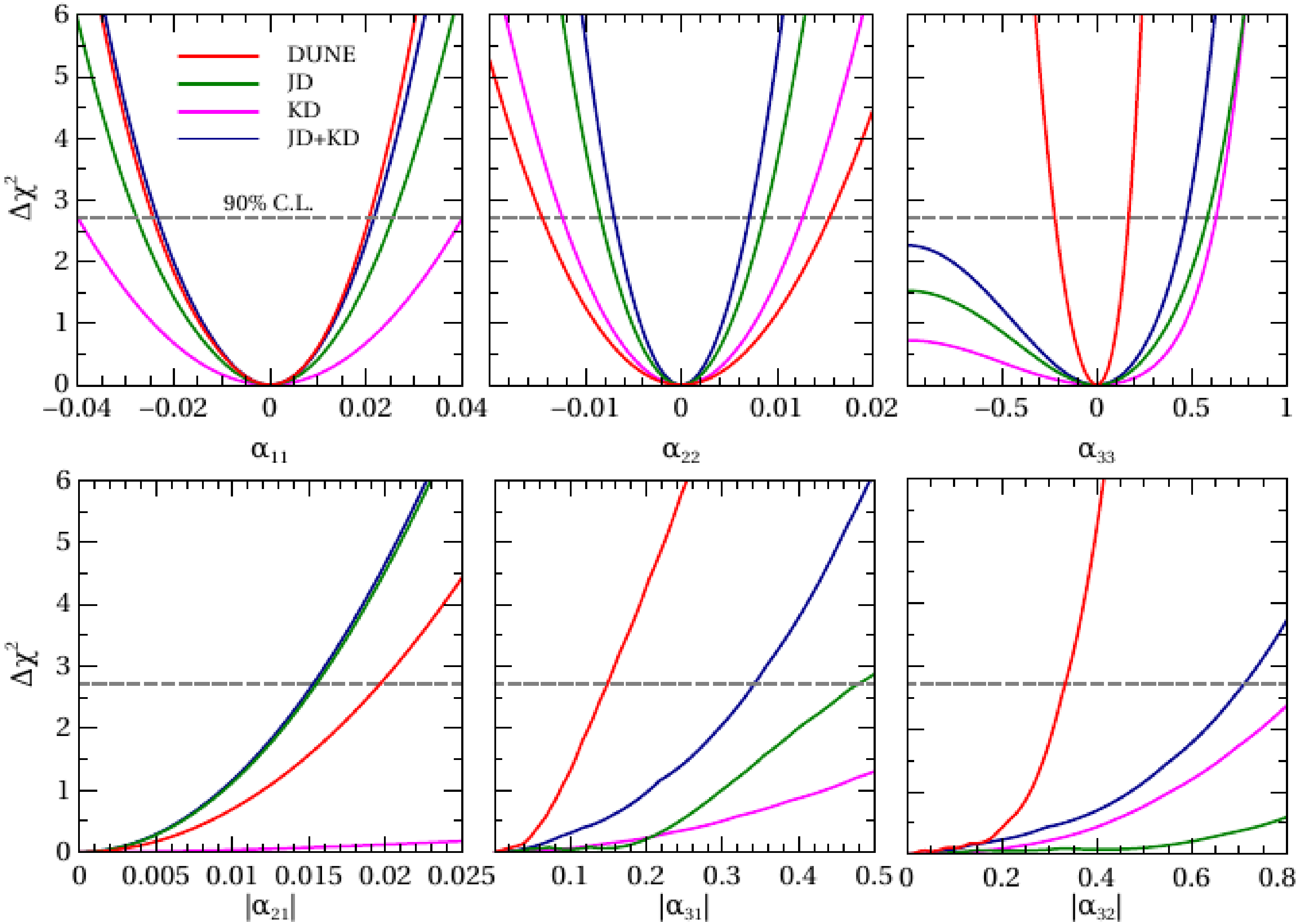
N_i^b - Background events at i-th bin from the prospective data

$$\Delta\chi^2 = \min_{(\theta_{23},\, \delta_{\mathrm{CP}},\, \phi_{ij},\, \lambda_1,\, \lambda_2)} \left[\chi^2(\alpha_{ij} \neq 0) - \chi^2(\alpha_{ij} = 0)\right] \, ,$$

$\chi^2(\alpha_{ij} \neq 0)$ - Calculated by fitting data assuming NUNM

$\chi^2(\alpha_{ij} = 0)$ - No new physics in the theory

Constraints on NUNM Parameters



- JD+KD (T2HKK) have better sensitivites to α_{21} and α_{22} than DUNE.
- DUNE shows better sensitivites to α_{31}, α_{32}, and α_{33} than JD+KD.
- Sensitivity to α_{11} is similar in both the setups.

Constraints on the NUNM Parameters

90% C.L. limits

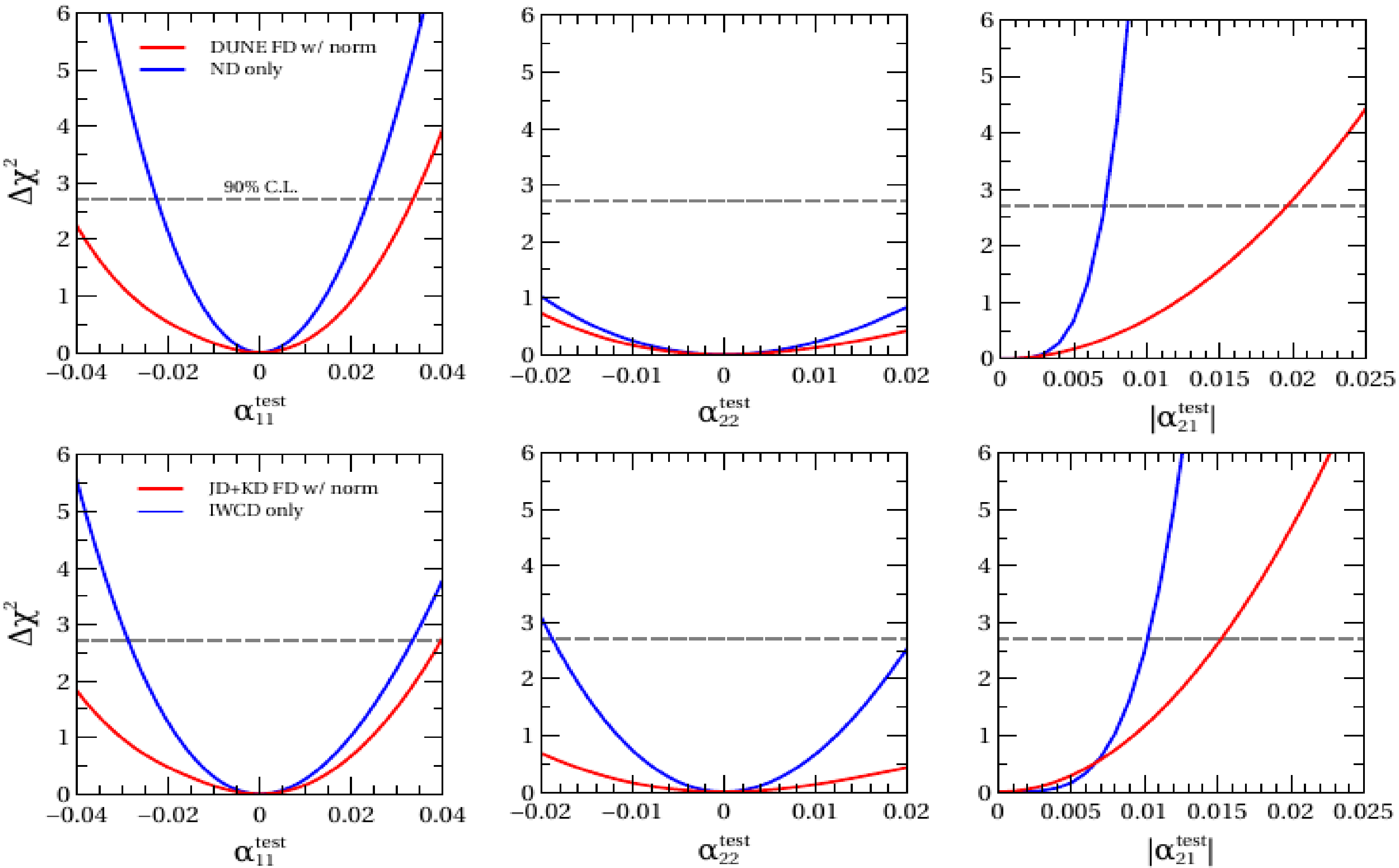
Individual

	DUNE	JD
α_{11}	[-0.02,0.02]	[-0.025,0.025]
α_{22}	[-0.014,0.014]	[-0.009,0.009]
α_{33}	[-0.2,0.17]	< 0.6
$\|\alpha_{21}\|$	< 0.022	< 0.015
$\|\alpha_{31}\|$	< 0.15	< 0.48
$\|\alpha_{32}\|$	< 0.33	< 1.2

Combinations

	JD+KD	JD+KD+DUNE
α_{11}	[-0.022,0.022]	[-0.017,0.017]
α_{22}	[-0.007,0.004]	[-0.006,0.006]
α_{33}	< 0.476	[-0.17,0.17]
$\|\alpha_{21}\|$	< 0.016	< 0.012
$\|\alpha_{31}\|$	< 0.34	< 0.11
$\|\alpha_{32}\|$	< 0.71	< 0.27

Benefits of Near Detectors and Impact of Normalization



$$P_{\alpha\beta}^{eff} = \frac{P_{\alpha\beta}}{|(NN^\dagger)_{\alpha\alpha}|^2}$$

- DUNE ND
  67 tons LarTPC detector, 574 m from the source.
- T2HK ND (IWCD)
  1 kt water Cerenkov detector, 1 km from the source.
- Data from ND alone have better sensititvities.

Benefits of Near Detectors and Impact of Normalization

90% C.L. limits

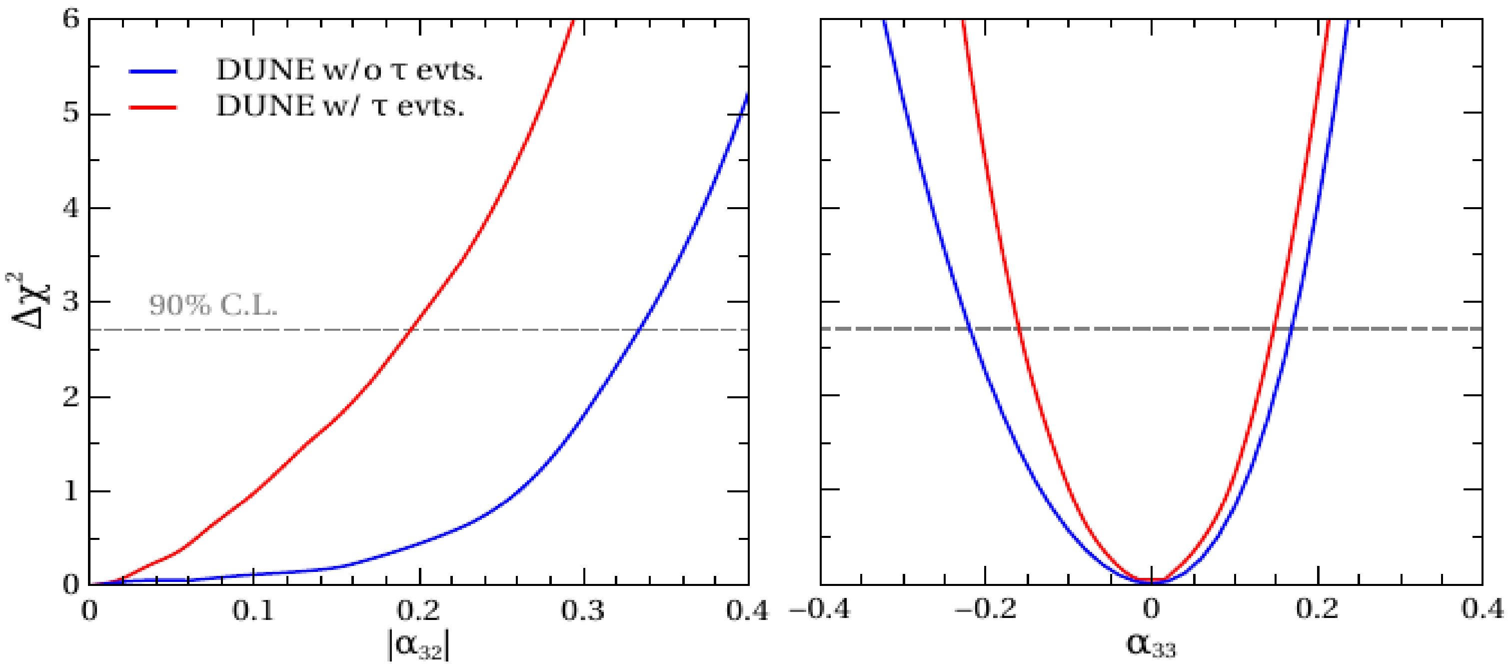
DUNE

	DUNE ND	DUNE (FD w/ norm)
α_{11}	[-0.02, 0.024]	[-0.043, 0.034]
α_{22}	[-0.033, 0.037]	[0.036, 0.048]
$\vert\alpha_{21}\vert$	$<$ 0.007	$<$ 0.022

T2HKK

	JD+KD (IWCD)	JD+KD (FD w/ norm)
α_{11}	[-0.029, 0.033]	[-0.048, 0.04]
α_{22}	[-0.019, 0.02]	[0.038, 0.05]
$\vert\alpha_{21}\vert$	$<$ 0.01	$<$ 0.015

Improvement Due to τ Neutrino Sample in DUNE



90% C.L. limits

	w/o ν_τ appearanece	w/ ν_τ appearanece
α_{33}	[-0.2, 0.17]	[-0.16, 0.15]
$\lvert\alpha_{32}\rvert$	$<$ 0.33	$<$ 0.19

Conclusion

1. T2HKK constrains α_{21} and α_{22} better compared to DUNE as it has larger statistics in appearance channel and less systematics in disappearance channel.

2. DUNE has better sensitivities for α_{31}, α_{32}, and α_{33} as their sensitivities depend on matter effect.

3. Limits on the NUNM parameters can be improved further by combining the data from DUNE and T2HKK.

4. Possible FD and ND correlation has significant impact on the sensitivities for α_{11} α_{21}, and α_{22} due to the zero-distance effect.

5. ν_τ appearance events in DUNE will improve the limits on α_{32} and α_{33}.

Thank You