T violation in neutrino oscillations

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1. Introduction

CP violation $\Delta P(CP) \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$

T violation $\Delta P(T) \equiv P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu)$

In the standard N_ν=3 framework

In vacuum

$$\Delta P(CP) = \Delta P(T) = 16\, J \sin\left(\frac{\Delta E_{32} L}{2}\right) \sin\left(\frac{\Delta E_{31} L}{2}\right) \sin\left(\frac{\Delta E_{21} L}{2}\right)$$

$$\Delta E_{jk} \equiv \frac{\Delta m^2_{jk}}{2E}$$

J: Jarlskog factor $J \equiv -\frac{c_{13}}{8} \sin\delta \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23}$

In matter

$$U\,\mathrm{diag}(E_1,E_2,E_3)\,U^{-1}+\mathrm{diag}(A,0,0)=\widetilde{U}\,\mathrm{diag}(\widetilde{E}_1,\widetilde{E}_2,\widetilde{E}_3)\,\widetilde{U}^{-1}$$

$$P(\nu_\alpha\to\nu_\beta)=\delta_{\alpha\beta}-4\sum_{j<k}\mathrm{Re}(\widetilde{U}_{\alpha j}\widetilde{U}^*_{\beta j}\widetilde{U}^*_{\alpha k}\widetilde{U}_{\beta k})\sin^2(\Delta\widetilde{E}_{jk}L/2)$$

$$+\boxed{2\sum_{j<k}\mathrm{Im}(\widetilde{U}_{\alpha j}\widetilde{U}^*_{\beta j}\widetilde{U}^*_{\alpha k}\widetilde{U}_{\beta k})\sin(\Delta\widetilde{E}_{jk}L),}$$

$$\boxed{\Delta\widetilde{E}_{jk}\equiv\widetilde{E}_j-\widetilde{E}_k}$$

ΔP(T) in matter is proportional to the Jarlskog factor J, so theoretically ΔP(T) seems advantageous over ΔP(CP).

$$\Delta P(CP)\neq\Delta P(T)=16\,J\,\frac{\Delta E_{21}\Delta E_{31}\Delta E_{32}}{\Delta\widetilde{E}_{21}\Delta\widetilde{E}_{31}\Delta\widetilde{E}_{32}}\sin\left(\frac{\Delta\widetilde{E}_{32}L}{2}\right)\sin\left(\frac{\Delta\widetilde{E}_{31}L}{2}\right)\sin\left(\frac{\Delta\widetilde{E}_{21}L}{2}\right)$$

$$J\equiv-\frac{c_{13}}{8}\sin\delta\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}$$

From comparison between $\Delta P(CP)$ and $\Delta P(T)$, we may be able to check consistency of the standard $N_\nu=3$ framework, e.g., confirmation of the Earth's density profile, etc.

Experimentally, measurements of T violation are challenging because they require a ν_e or $\bar{\nu}_e$ beam with energy ~O(1) GeV. Such a possibility may be realized by a neutrino factory.

ν factory: a very clear distinction between μ^- and μ^+

$\bar{\nu}_e + \nu_\mu$ (right sign muon $\nu_\mu \to \nu_\mu$; wrong sign muon $\bar{\nu}_e \to \bar{\nu}_\mu$)

or $\nu_e + \bar{\nu}_\mu$ (right sign muon $\bar{\nu}_\mu \to \bar{\nu}_\mu$; wrong sign muon $\nu_e \to \nu_\mu$)

- High energy ν factory: E_μ ~O(50) GeV
- NuSTORM (Low energy ν factory): E_μ = 1- 6 GeV → Ken Long's talk
- μTRISTAN (from muonium → μ^+ only) : E_μ >~O(1) GeV

Due to proposals for these experiments, there has been a renewed interest in T violation.

μTRISTAN

Hamada, Kitano, Matsudo, Takaura, Yoshida, PTEP 2022 (2022) 5, 053B02

Proton acceleration (Proton LINAC & RCS) ⟶ Pion production (Pion production ring)

$p(3\,\mathrm{GeV})$ $\qquad$ $p(3\,\mathrm{GeV}) + C \to \pi^+ + X$

⟶ Ultra-cold muon production ⟶ Muon acceleration (Booster ring) ⟶ Collide (Main ring)

$\pi^+ \to \mu^+ + \nu_\mu$ $\qquad$ Ultra-cold muon

Ionized by laser

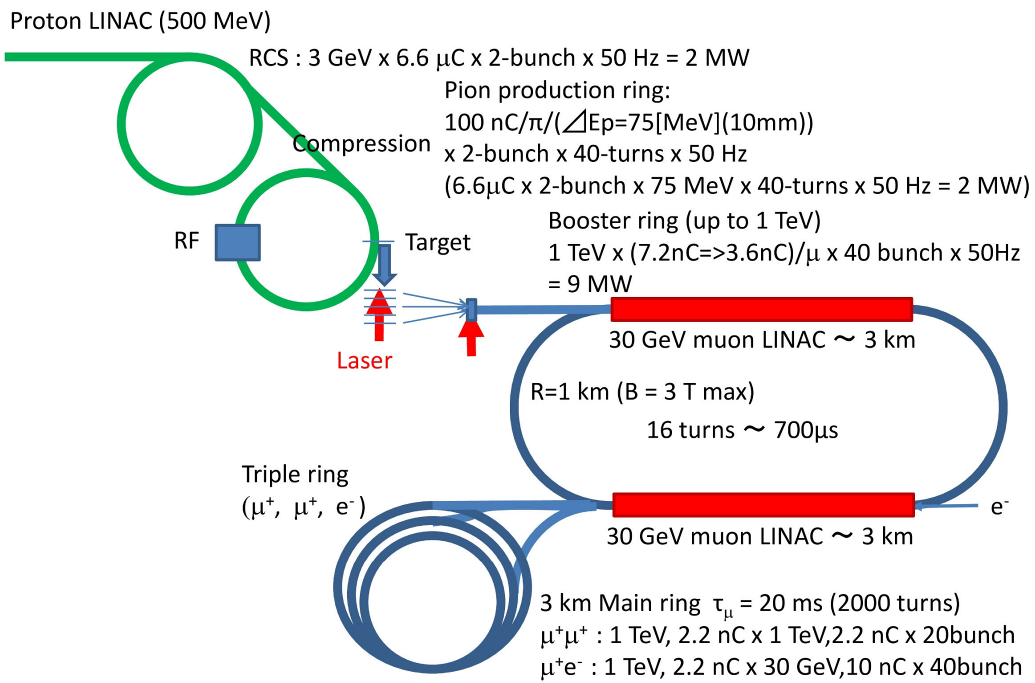
Muonium $(\mu^+ e^-)$ formation in silica aerogel

$\mu^+(1\,\mathrm{TeV})$

$[\mu^+(1\,\mathrm{TeV}), e^-(30\,\mathrm{GeV})]$

or

$[\mu^+(1\,\mathrm{TeV}), \mu^+(1\,\mathrm{TeV})]$



μTRISTAN offers only μ^+ beams, but is expected to be easier to build with current technology

μTRISTAN could be used to build a ν factory

Takaura@workshop on "Exploration of Particle Physics and Cosmology with Neutrino", 7-8 March 2022

2. T violation in the framework beyond the standard N_ν=3

With 3 active and (n-3) sterile neutrinos, generic non-unitary mixing and the generic matter effect

mass eigenstate $\Psi_m = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_n \end{pmatrix}$

flavor eigenstate $\Psi_f = \begin{pmatrix} \nu_{\alpha_1} \\ \nu_{\alpha_2} \\ \vdots \\ \nu_{\alpha_n} \end{pmatrix}$

$$\Psi_f = N\,\Psi_m$$

$$N \equiv (\mathbf{1} + \eta)\,U$$

$$\mathcal{A} \equiv \begin{pmatrix} A_{\alpha_1\alpha_1} & A_{\alpha_1\alpha_2} & \cdots & A_{\alpha_1\alpha_n} \\ A_{\alpha_2\alpha_1} & A_{\alpha_2\alpha_2} & \cdots & A_{\alpha_2\alpha_n} \\ \vdots & \vdots & & \vdots \\ A_{\alpha_n\alpha_1} & A_{\alpha_n\alpha_2} & \cdots & A_{\alpha_n\alpha_n} \end{pmatrix}$$

$$i\frac{d\,\Psi_m(t)}{dt} = (\mathcal{E} + N^T \mathcal{A}\, N^*)\,\Psi_m(t)$$

$$\mathcal{E} \equiv \mathrm{diag}(E_1, E_2, \cdots, E_n)$$

Assuming constant density and assuming N can be different at production (p) and detection (d) in general, the equation of motion can be solved with a unitary matrix W.

$$\mathcal{E} + N^T \mathcal{A}\, N^* = W\,\widetilde{\mathcal{E}}\,W^{-1}$$

$$\widetilde{\mathcal{E}} \equiv \mathrm{diag}(\widetilde{E}_1, \widetilde{E}_2, \cdots, \widetilde{E}_n)$$

The modified oscillation probability and T violation

$$\hat{P}(\nu_\alpha \to \nu_\beta) \equiv P(\nu_\alpha \to \nu_\beta) \left\{ N^p (N^p)^\dagger \right\}_{\alpha\alpha} \left\{ N^d (N^d)^\dagger \right\}_{\beta\beta}$$

production detection

can be expressed as

$$\begin{aligned} \hat{P}(\nu_\alpha \to \nu_\beta) = & \left| \left\{ N^d (N^p)^\dagger \right\}_{\beta\alpha} \right|^2 - 4 \sum_{j<k} \mathrm{Re}(\widetilde{X}_j^{\alpha\beta} \widetilde{X}_k^{\alpha\beta *}) \sin^2(\Delta \widetilde{E}_{jk} L/2) \\ & + 2 \sum_{j<k} \mathrm{Im}(\widetilde{X}_j^{\alpha\beta} \widetilde{X}_k^{\alpha\beta *}) \sin(\Delta \widetilde{E}_{jk} L) \end{aligned}$$

$$\begin{aligned} & \hat{P}(\nu_\alpha \to \nu_\beta) - \hat{P}(\nu_\beta \to \nu_\alpha) \\ = & \left| \left\{ N^d (N^p)^\dagger \right\}_{\beta\alpha} \right|^2 - \left| \left\{ N^d (N^p)^\dagger \right\}_{\alpha\beta} \right|^2 + 4 \sum_{j<k} \mathrm{Im}(\widetilde{X}_j^{\alpha\beta} \widetilde{X}_k^{\alpha\beta *}) \sin(\Delta \widetilde{E}_{jk} L) \end{aligned}$$

$$\widetilde{X}_j^{\alpha\beta} \equiv (N^d W^*)^*_{\beta j} (N^p W^*)_{\alpha j} \quad (j = 1, 2, \cdots, n)$$

Assuming all the mass squared differences for sterile ν give rapid oscillations, $\boxed{|\Delta \widetilde{E}_{jk} L| \gg 1 \quad \text{for } j > 3 \text{ or } k > 3}$

$$\boxed{\sin(\Delta \widetilde{E}_{jk} L) \to 0 \quad \text{for } j > 3 \text{ or } k > 3}$$

and approximating NN† ~1, the dominant contribution to energy dependent T violation comes from the N=3 mass eigenstates:

$$
\begin{aligned}
& P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu) \simeq \hat{P}(\nu_\mu \to \nu_e) - \hat{P}(\nu_e \to \nu_\mu) \\
\simeq & \quad \underbrace{|\{N^d (N^p)^\dagger\}_{\beta\alpha}|^2 - |\{N^d (N^p)^\dagger\}_{\alpha\beta}|^2}_{} \\
& \quad = \text{constant small factor to be absorbed as systematic error of flux} \\
& + \ 4 \sum_{1 \le j < k \le 3} \mathrm{Im}(\underbrace{\widetilde{X}_j^{\mu e} \widetilde{X}_k^{\mu e *}}_{\text{small factor}}) \ \sin(\underbrace{\Delta \widetilde{E}_{jk}}_{\simeq \left[\Delta \widetilde{E}_{jk}\right]_{\text{standard } N_\nu = 3}} L)
\end{aligned}
$$

$$\boxed{\widetilde{X}_j^{\alpha\beta} \equiv (N^d W^*)^*_{\beta j} (N^p W^*)_{\alpha j} \quad (j = 1, 2, \cdots, n)}$$

Example 1: Non Standard Interaction of 3 active ν

OY, Entropy 26 (2024) 472

$$N^d = N^p = U$$

$$\mathcal{A} \equiv A \begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$

$$\Delta\widetilde{E}_{31} \simeq \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}$$

$$\left\{ \begin{array}{c} -\Delta\widetilde{E}_{21} \\ \Delta\widetilde{E}_{32} \end{array} \right\} \simeq \frac{1}{2}\left\{\Delta E_{31} + A \mp \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}\right\}$$

$$\begin{aligned} & P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu) \\ \simeq & \frac{16\,(\Delta E_{31})^2}{\Delta\widetilde{E}_{21}\Delta\widetilde{E}_{31}\Delta\widetilde{E}_{32}} \sin\left(\frac{\Delta\widetilde{E}_{32}L}{2}\right) \sin\left(\frac{\Delta\widetilde{E}_{31}L}{2}\right) \sin\left(\frac{\Delta\widetilde{E}_{21}L}{2}\right) \\ & \times \mathrm{Im}\left[U_{e3}U_{\mu3}^*\left\{\boxed{\Delta E_{21}U_{e2}U_{\mu2}^*} + \boxed{A\ \left(|U_{\tau3}|^2\epsilon_{e\mu} - U_{e3}U_{\tau3}^*\epsilon_{\tau\mu} - U_{\tau3}U_{\mu3}^*\epsilon_{e\tau}\right)}\right\}\right] \end{aligned}$$

Standard contribution ∝ $\Delta m_{21}{}^2/E$

NSI contribution ∝ const.

Precise measurements of the energy spectrum may reveal the NSI effect.

Example 2: Unitarity violation with 3 active ν

OY, Entropy 26 (2024) 472

$$N^d = N^p = (1+\eta)\, U$$

$$\eta^\dagger = \eta$$

$$\Delta\widetilde{E}_{31} \simeq \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}$$

$$\left\{ \begin{array}{c} -\Delta\widetilde{E}_{21} \\ \Delta\widetilde{E}_{32} \end{array} \right\} \simeq \frac{1}{2}\left\{\Delta E_{31} + A \mp \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}\right\}$$

This form of factorization is a consequence of 3 flavor unitarity.

$$\begin{aligned} &P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu) \\ &\simeq 16 \frac{\Delta E_{31}}{A\,\Delta\widetilde{E}_{31}\ \cos^2\theta_{13}} \boxed{\sin\left(\frac{\Delta\widetilde{E}_{31}L}{2}\right)\sin\left(\frac{\Delta\widetilde{E}_{21}L}{2}\right)\sin\left(\frac{\Delta\widetilde{E}_{32}L}{2}\right)} \\ &\quad \times \mathrm{Im}\left[U_{e3}U^*_{\mu3}\left\{\boxed{\Delta E_{21}U_{e2}U^*_{\mu2}} + \boxed{4\,A\,\eta_{\mu e} + 2\,A_n\sum_\alpha(\eta_{\mu\alpha}\,U_{\alpha3}\,U^*_{e3} + \eta_{\alpha e}\,U_{\mu3}\,U^*_{\alpha3}) - 2\,A_n\eta_{\mu e}}\right\}\right] \\ &\quad + 8\frac{\Delta E_{31}}{\Delta\widetilde{E}_{31}} \boxed{\sin\left(\Delta\widetilde{E}_{31}L\right)}\ \mathrm{Im}\left[\eta_{\mu e}U_{e3}U^*_{\mu3}\right] \end{aligned}$$

Energy dependence different from the standard one.

The term outside of the factorization is a consequence of 3 flavor unitarity violation.

Again precise measurements of the energy spectrum may reveal the NSI effect.

3. Sensitivity of future experiments to T violation

- Sensitivity of a ν factory to T violation

Ota, Sato, Kuno, PLB 520 (2001) 289

- Sensitivity of μTRISTAN to T violation

Kitano, Sato, Sugama, JHEP 12 (2024) 014

- Model-independent test of T invariance by T2HK, DUNE, T2HKK, ESSνSB

Schwetz, Segarra, PRL128 ('22) 091801

- Model-independent test of T invariance by T2HK, DUNE

Chatterjee, Patra, Schwetz, Sharma, JHEP 12 (2024) 200

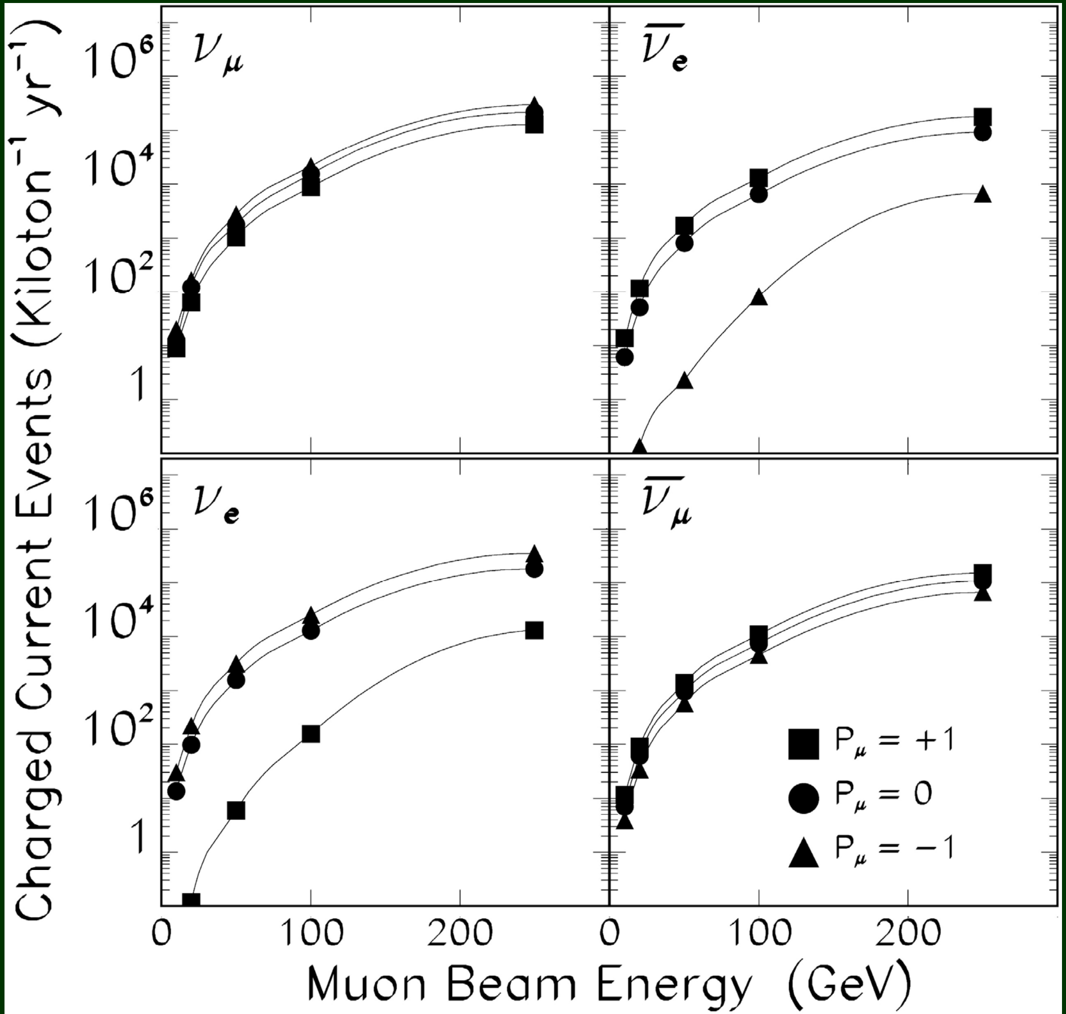
● The effect of μ polarization in a ν factory

Geer, Phys.Rev. D57 (1998) 6989

The energy spectrum varies with polarization P_μ:

$$\frac{d^2N(\nu_\mu)}{dxd\Omega} \propto \frac{2x^2}{4\pi}\left[(3-2x) \mp (1-2x)P_\mu \cos\theta\right]$$

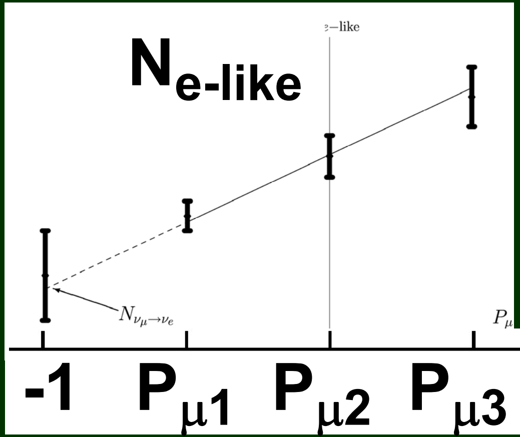
$$\frac{d^2N(\nu_e)}{dxd\Omega} \propto \frac{12x^2}{4\pi}\left[(1-x) \mp (1-x)P_\mu \cos\theta\right]$$



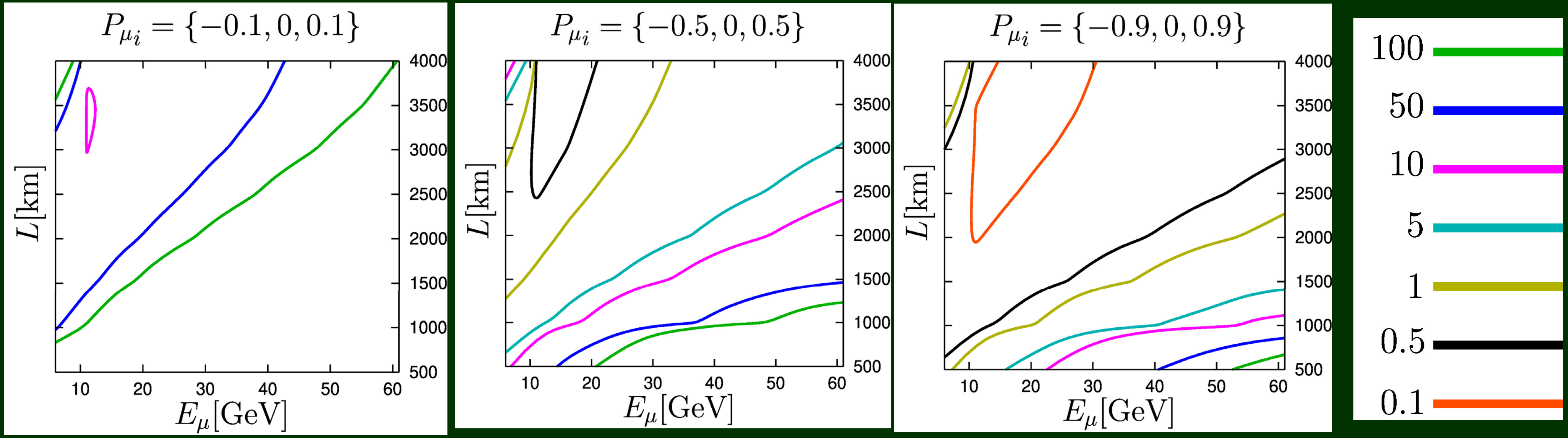
● Sensitivity of a ν factory to T violation

Ota, Sato, Kuno,
PLB 520 (2001) 289

Extrapolating the data at three different polarizations $P_{\mu 1}$, $P_{\mu 2}$, $P_{\mu 3}$ to deduce the data at P_μ = -1



Data size in unit of 10^{21} muon decays x 100kt to exclude T invariance at 90%CL



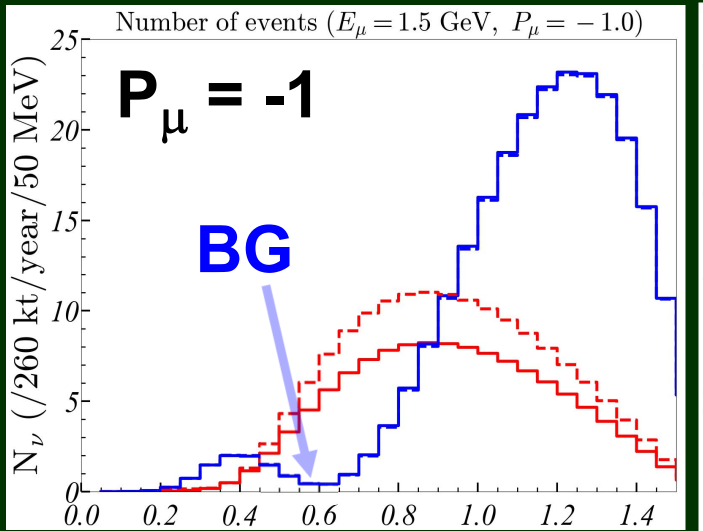
● Sensitivity of μTRISTAN to T violation

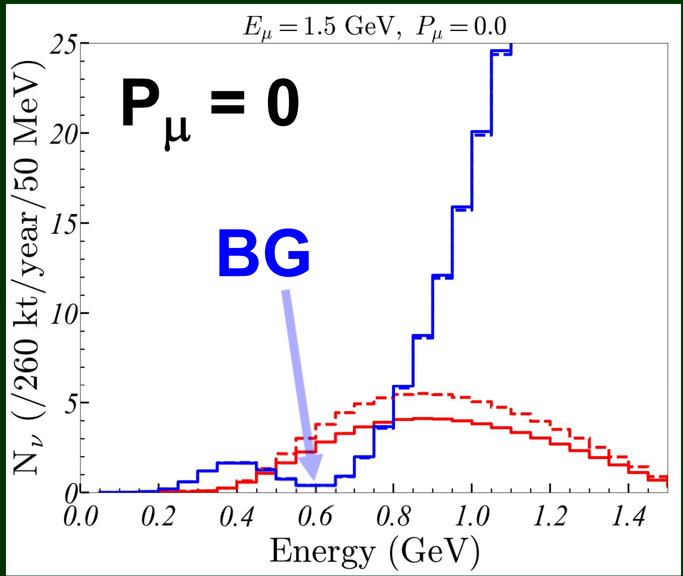
Kitano, Sato, Sugama, JHEP 12 (2024) 014

$P(\nu_e \to \nu_\mu)$ from μTRISTAN with E_μ = 1.5 GeV, L=295km and $P(\nu_\mu \to \nu_e)$ from T2HK

Efficiency of charge identification of μ^+ and μ^-: C_{id}, Polarization: P_μ

Even if C_{id} =0, there is little BG from $\bar{\nu}_\mu \to \bar{\nu}_\mu$ near E_ν= 0.6 GeV.





$N(\nu_e \to \nu_\mu)$ (δ=-π/2)

$N(\nu_e \to \nu_\mu)$ (δ=0)

$N(\bar{\nu}_\mu \to \bar{\nu}_\mu)$ (δ=-π/2)

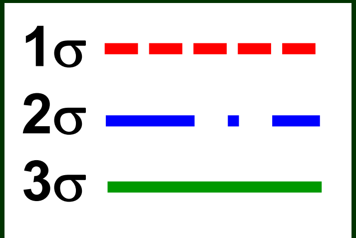
$N(\bar{\nu}_\mu \to \bar{\nu}_\mu)$ (δ=0)

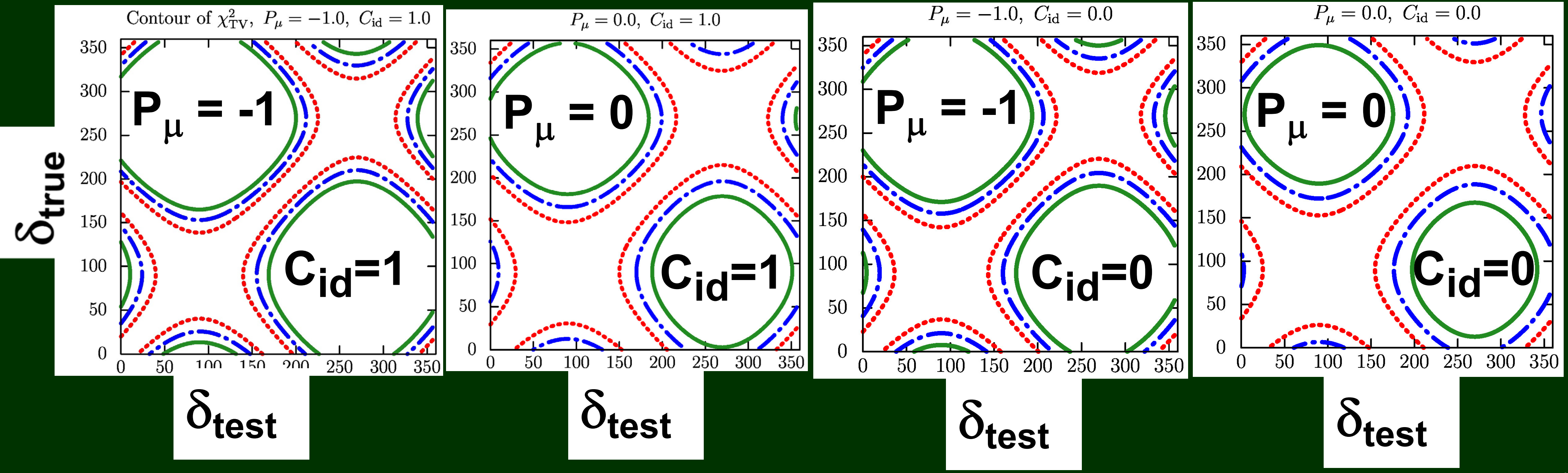
Correlation of δ_{true} and δ_{test} using the quantity

Kitano, Sato, Sugama, JHEP 12 (2024) 014

$$P_j^{\text{TV}} = P_j(\nu_e \to \nu_\mu)\big|_{\mu\text{TRISTAN}} - P_j(\nu_\mu \to \nu_e)\big|_{\text{T2HK}}$$

χ^2(TV) does not depend strongly on C_{id} and P_μ .





Model-independent test of T invariance by T2HK, DUNE, T2HKK, ESSνSB

Assuming T invariance (→ all the matrix elements become real), evaluate the significance to exclude T invariance:

Schwetz, Segarra, PRL128 ('22) 091801

$$P(\nu_\mu \to \nu_\beta; L, E) \to \left(\sum_{j=1}^{3} c_j^\beta\right)^2 - 4\sum_{j<k} c_j^\beta\, c_k^\beta\, \sin^2\left(\frac{\Delta\widetilde{E}_{jk}L}{2}\right)$$

β = (μ, e)

$$c_j^\beta \equiv \frac{\widetilde{X}_j^{\mu\beta}}{[\{N^p(N^p)^\dagger\}_{\mu\mu}]^{1/2}[\{N^d(N^d)^\dagger\}_{\beta\beta}]^{1/2}}$$

8 data at the **same energy** are required to determine 8 parameters for the **disappearance** (β = μ) and **appearance** (β = e) channels.

→ L = 0, $L_{b(far)}$ (b=1,…,N_L); β = (μ, e) at b-th experiments → $\boxed{N_L \geq 3}$

at least 3 baseline lengths

$\boxed{c_j^\beta \ (\beta = (\mu, e),\ j = 1, 2, 3)}$

$\boxed{\Delta\widetilde{E}_{21},\ \Delta\widetilde{E}_{31}}$

all depend on E_ν

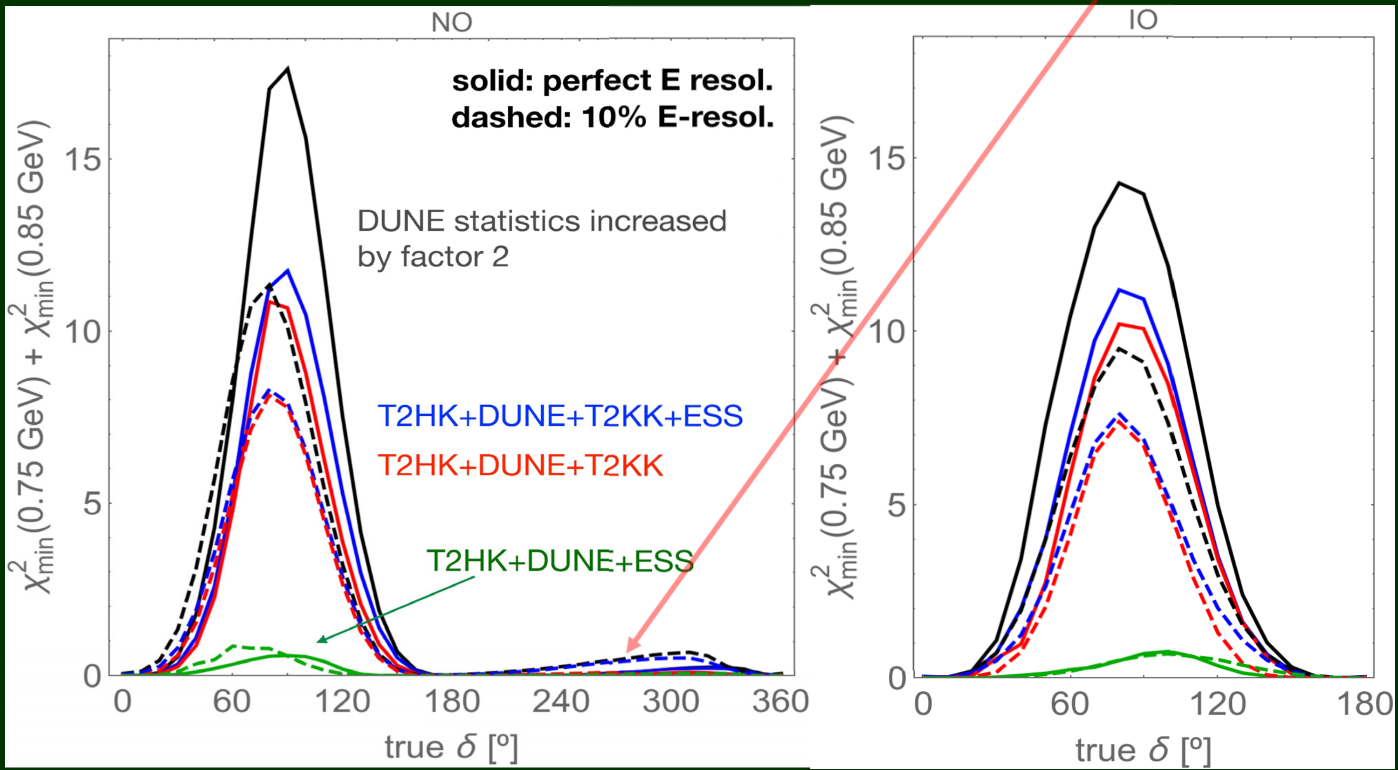
$$\chi^2_{\rm even} = \sum_{b=1}^{N_L}\left[\frac{P(\nu_\mu \to \nu_\mu; L_b, E)|_{\rm theory} - P(\nu_\mu \to \nu_\mu; L_b, E)|_{\rm exp}}{\sigma_b(\nu_\mu \to \nu_\mu)}\right]^2 + \sum_{b=1}^{N_L}\left[\frac{P(\nu_\mu \to \nu_e; L_b, E)|_{\rm theory} - P(\nu_\mu \to \nu_e; L_b, E)|_{\rm exp}}{\sigma_b(\nu_\mu \to \nu_e)}\right]^2$$

disappearance channel

appearance channel

E_ν = 0.75 GeV, 0.85 GeV and (L=0, 295km, 540km, 1100km, 1300km) and $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\mu$ are used

- **T2HKK seems to be important for better results.**
- **This method works for $\delta = \pi/2$, but not for $\delta = -\pi/2$ (inclusion of anti-ν won't help). It is the price we have to pay for discarding other data.**



Schwetz@ NuTs 2022, IFT Madrid

Chatterjee, Patra, Schwetz, Sharma, JHEP 12 (2024) 200

- **Model-independent test of T invariance by T2HK, DUNE**

$$P_{\rm even}(\nu_\mu \to \nu_e; L, E) = \left(\sum_{j=1}^{3} c_j\right)^2 - 4\sum_{j<k} c_j\, c_k\, \sin^2\left(\frac{\Delta\widetilde{E}_{jk}L}{2}\right)$$

$$c_j^e \to c_j \quad (j = 1, 2, 3)$$

The energy eigenvalues can be approximated by the standard one:

$$\Delta\widetilde{E}_{31} \simeq \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}$$

$$\left\{\begin{array}{c} -\Delta\widetilde{E}_{21} \\ \Delta\widetilde{E}_{32} \end{array}\right\} \simeq \frac{1}{2}\left\{\Delta E_{31} + A \mp \sqrt{(\Delta E_{31}\cos 2\theta_{13} - A)^2 + (\Delta E_{31}\sin 2\theta_{13})^2}\right\}$$

→ Only 3 unknown parameters are c_2, c_3, $\varepsilon = c_1 + c_2 + c_3$.

→ Only 3 data (ND + 2 baselines) are required to determine them for the appearance channel only.

ε is the oscillation probability at L=0:

$$\epsilon \equiv c_1 + c_2 + c_3 = \sqrt{P(\nu_\mu \to \nu_e; L_{ND})}$$

→ Only 2 unknown parameters are c_2, c_3.

By fitting the parameters c_2, c_3 , check if the following quantity is negative:

$$X_T \equiv P_{\text{even}}(\nu_\mu \to \nu_e; L_2) - P_{\text{even}}(\nu_\mu \to \nu_e; L_1) - \delta_0 P(\nu_\mu \to \nu_e; L_{ND})$$
$$= \delta_{12}(c_2')^2 + \delta_{13}(c_3')^2 + \delta_{23} c_2' c_3'$$
$$\geq 0 \quad (\text{iff } \delta_{12} > 0 \ \& \ \delta_{13} > 0 \ \& \ 4\delta_{12}\delta_{13} > \delta_{23}^2)$$

must be satisfied if T invariance is conserved

$$\delta_0 \equiv \frac{\delta_{12} + \delta_{13} - \delta_{23}}{\delta_{23}^2/(\delta_{12}\delta_{13}) - 4}$$

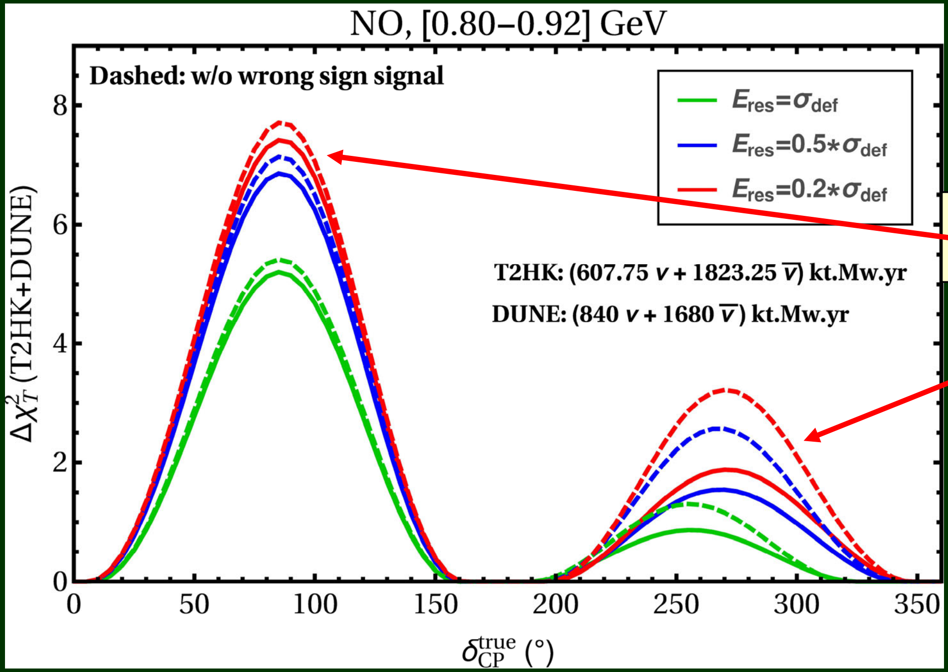
$$c_2{}' \equiv c_2 + \epsilon \frac{\delta_{13}\delta_{23} - 2\delta_{12}\delta_{13}}{\delta_{23}^2 - 4\delta_{12}\delta_{13}}$$

$$c_3{}' \equiv c_3 + \epsilon \frac{\delta_{12}\delta_{23} - 2\delta_{12}\delta_{13}}{\delta_{23}^2 - 4\delta_{12}\delta_{13}}$$

$$\delta_{jk} = \sin^2\left(\frac{\Delta \widetilde{E}_{jk} L_2}{2}\right) - \sin^2\left(\frac{\Delta \widetilde{E}_{jk} L_1}{2}\right)$$

Best choice for this test is L_1=295 km, L_2=1300km, E_ν=0.86GeV

Chatterjee, Patra, Schwetz, Sharma, JHEP 12 (2024) 200



Significance is at most less than 3σ.

ν mode only has no sensitivity for δ = -π/2.
➞ anti-ν mode must be added to improve sensitivity for δ = -π/2.

4. Conclusions

- Conceptually, T violation in matter seems attractive because it is proportional to the Jarlskog factor as in vacuum.
- If we can measure T violation precisely, then we gain information on physics beyond the standard $N_\nu=3$ framework from the energy spectrum.
- Experimentally, analysis with only T-odd or T-even part may lose statistics and it will not yield significant conclusions. T violation plays a role only in the analysis with full data.

Backup slides

ν factory

Geer, Phys.Rev.D 57 (1998) 6989

$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

$\nu_\mu \to \nu_\mu \to \mu^-$

Right sign muon

$\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$

$\bar{\nu}_\mu \to \bar{\nu}_\mu \to \mu^+$

Right sign muon

golden channel

$\bar{\nu}_e \overset{\text{osc}}{\to} \bar{\nu}_\mu \to \mu^+$

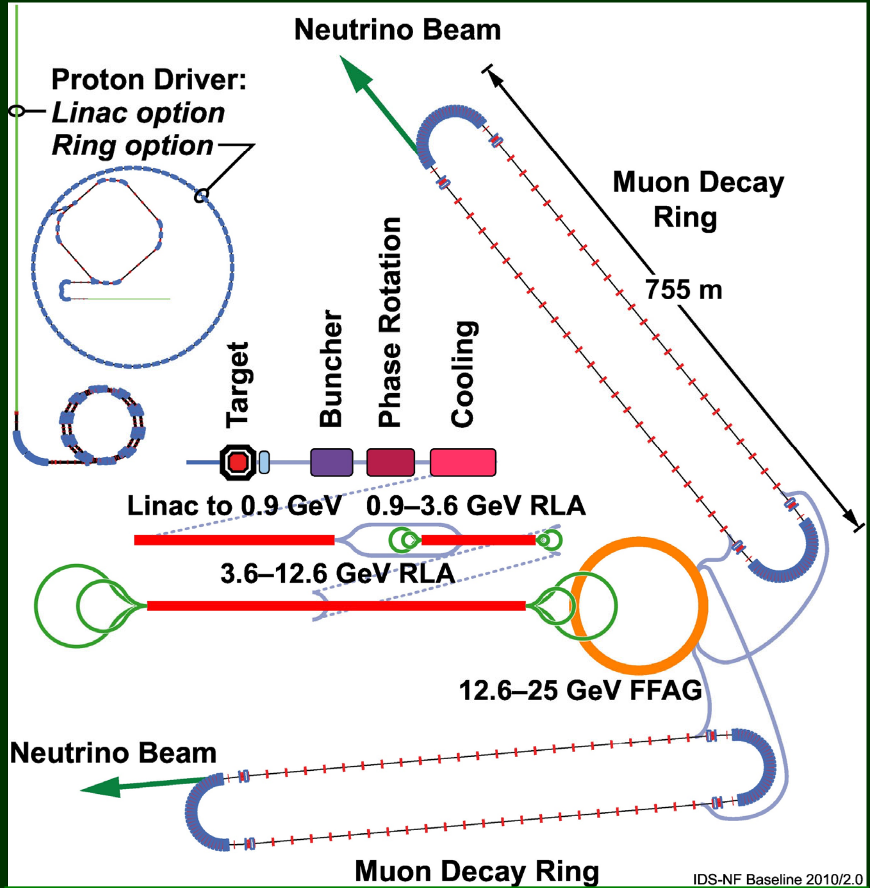
Wrong sign muon

$\nu_e \overset{\text{osc}}{\to} \nu_\mu \to \mu^-$

Wrong sign muon

- Large number of events
- A very clear distinction between μ^- and μ^+

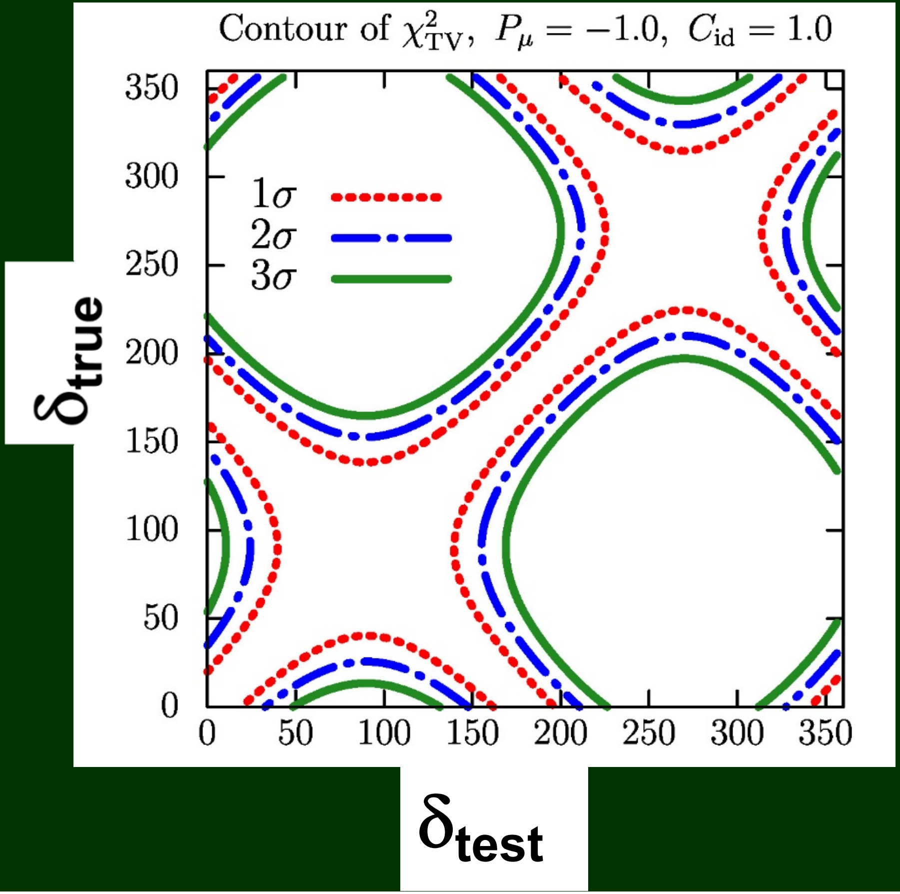
→ Low backgrounds

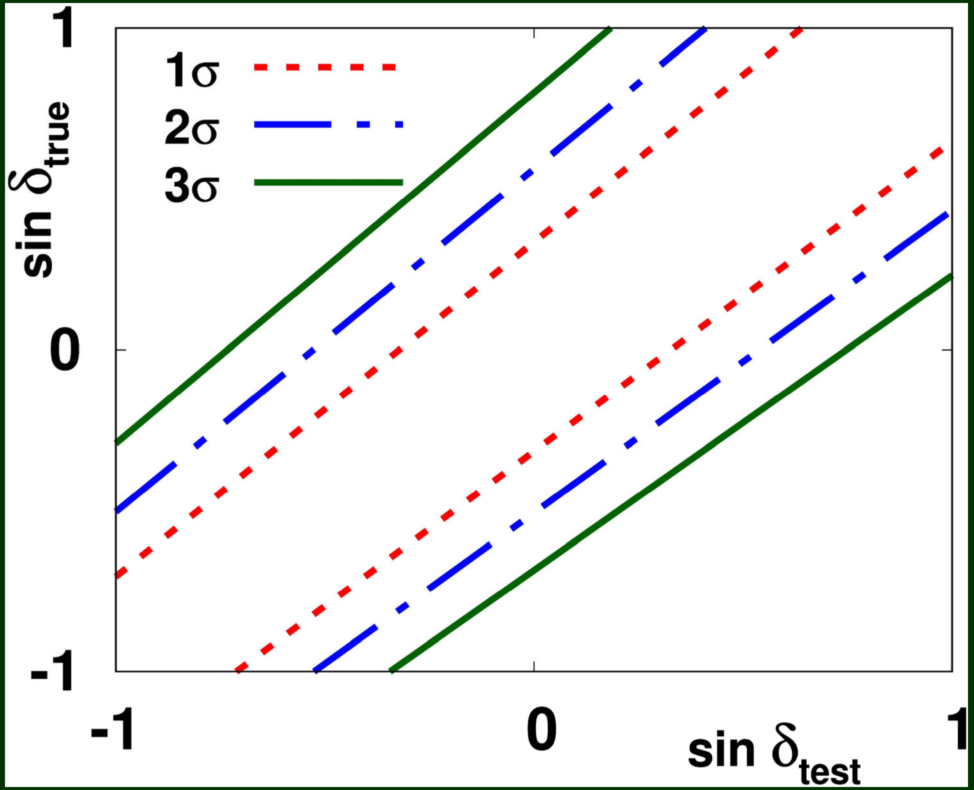


IDS-NF Collaboration, 1112.2853 [hep-ex]

There seems to be degeneracy in δ, but in terms of sinδ, there is no degeneracy. This is because ΔP(T) in matter is proportional to sinδ.

Kitano, Sato, Sugama, JHEP 12 (2024) 014





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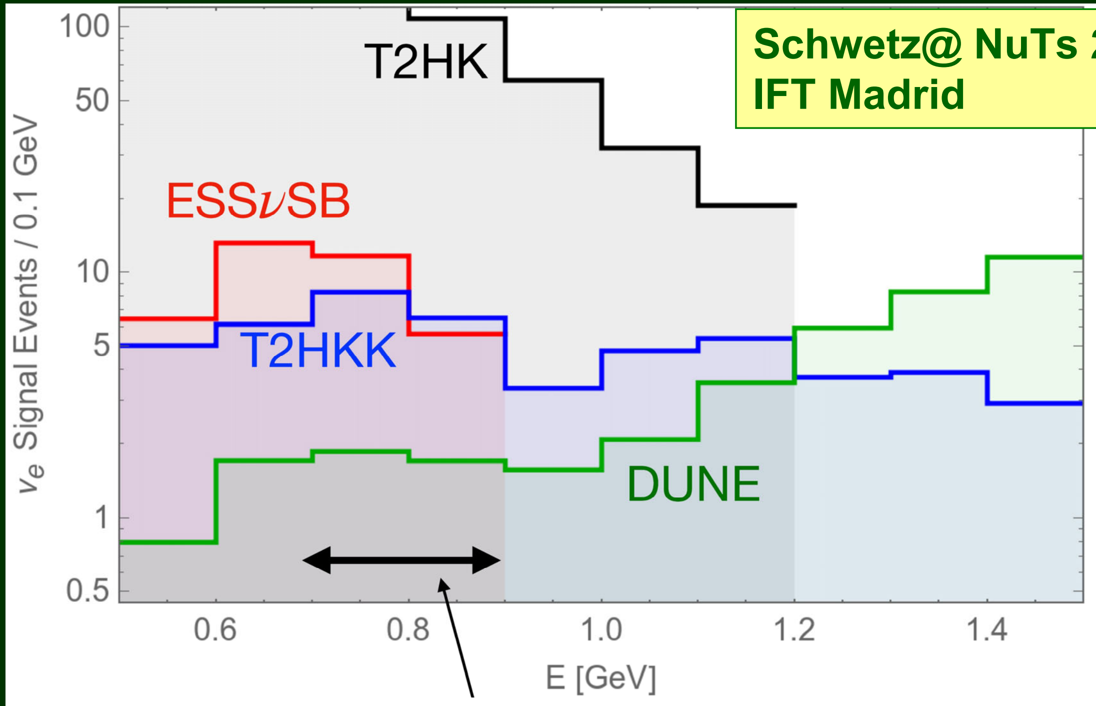
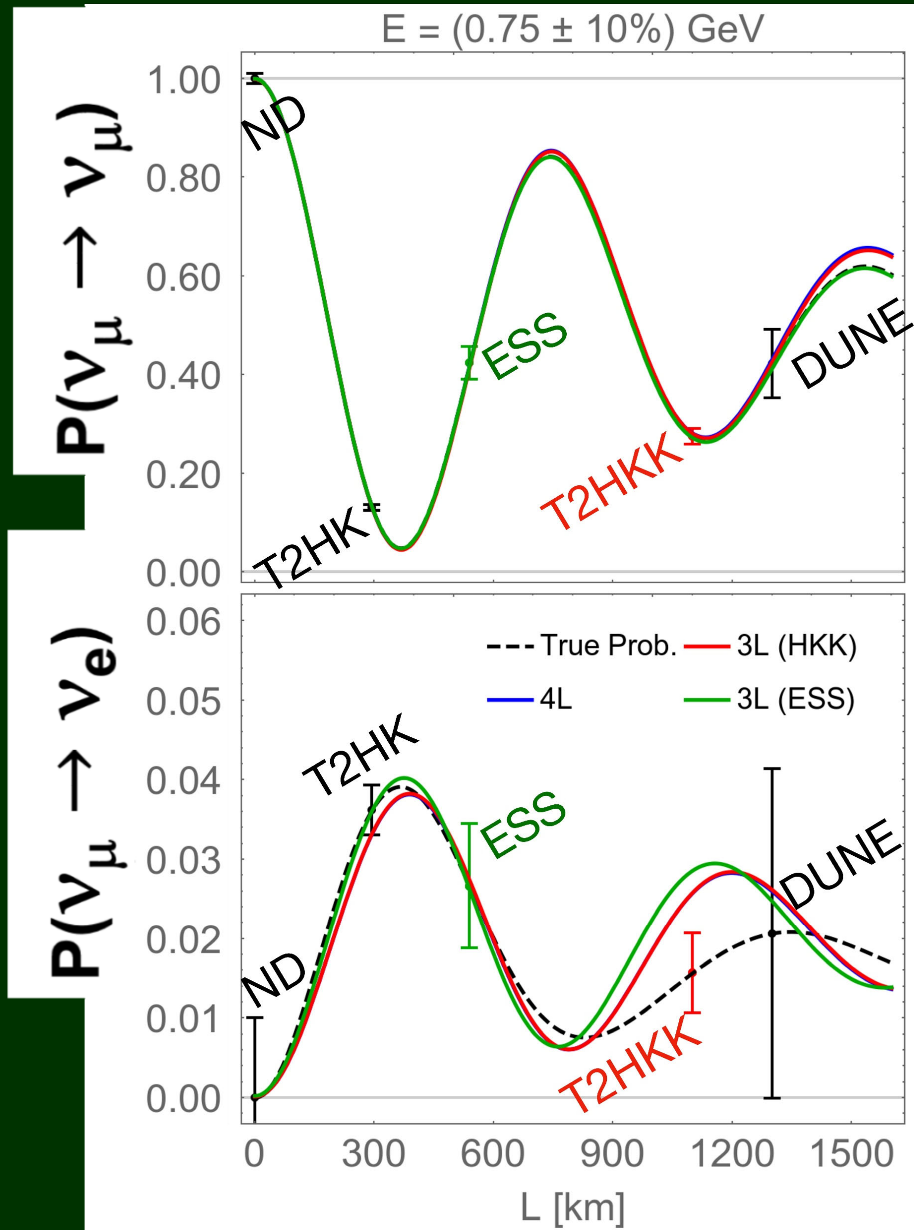


most sensitive energy interval



E_ν = 0.75 GeV, 0.85 GeV and (ND + 4 baselines) and $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\mu$ are used

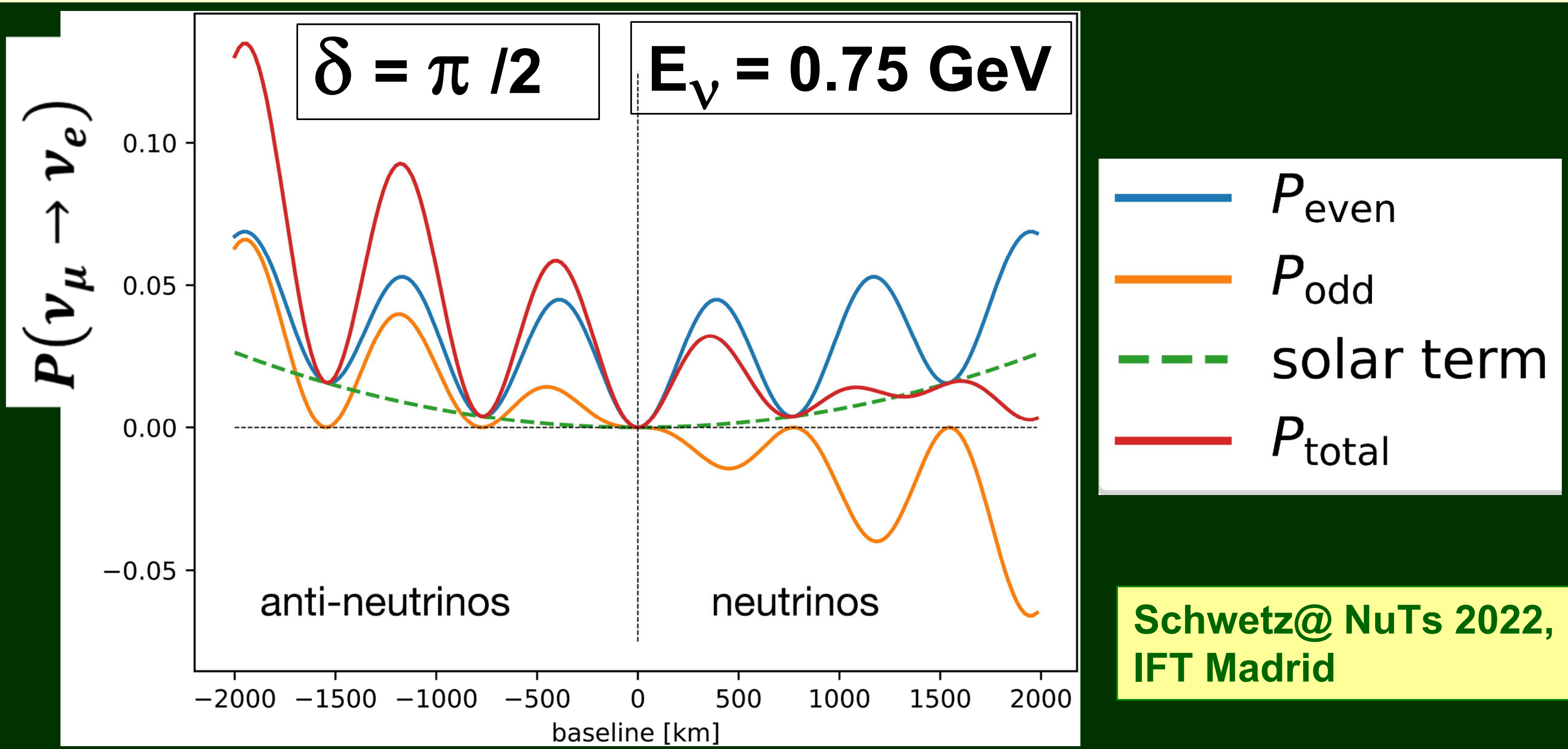
In general $P_{even}(\nu_\mu \rightarrow \nu_e;\text{2nd max}) \geq P_{even}(\nu_\mu \rightarrow \nu_e;\text{ 1st max})$

This test works only if

$P_{total}(\nu_\mu \rightarrow \nu_e;\text{2nd max}) < P_{total}(\nu_\mu \rightarrow \nu_e;\text{ 1st max})$

This happens for ν at $\delta \sim \pi/2$, and for anti-ν at $\delta \sim -\pi/2$.

Unfortunately, DUNE at L=1300km has a large error in $P_{total}(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.



Schwetz@ NuTs 2022, IFT Madrid

Chatterjee, Patra, Schwetz, Sharma, JHEP 12 (2024) 200

$$P_{\text{even}}(\nu_\mu \to \nu_e; L_2) - P_{\text{even}}(\nu_\mu \to \nu_e; L_1)$$
$$= \delta_{12}\, c_2(c_2 - \epsilon) + \delta_{13}\, c_3(c_3 - \epsilon) + \delta_{23}\, c_2 c_3$$
$$= \underbrace{\delta_{12}\,({c_2}')^2 + \delta_{13}\,({c_3}')^2 + \delta_{23}\, {c_2}'{c_3}'}_{\geq 0 \ \text{ iff } \ \delta_{12} > 0 \ \ \& \ \ \delta_{13} > 0 \ \ \& \ \ 4\delta_{12}\delta_{13} > \delta_{23}^2} + \delta_0\, \epsilon^2$$

$$\delta_{jk} = \sin^2\left(\frac{\Delta\widetilde{E}_{jk}L_2}{2}\right) - \sin^2\left(\frac{\Delta\widetilde{E}_{jk}L_1}{2}\right) \qquad (j,k = 1,2,3)$$

$${c_2}' \equiv c_2 + \epsilon\frac{\delta_{13}\delta_{23} - 2\delta_{12}\delta_{13}}{\delta_{23}^2 - 4\delta_{12}\delta_{13}} \qquad {c_3}' \equiv c_3 + \epsilon\frac{\delta_{12}\delta_{23} - 2\delta_{12}\delta_{13}}{\delta_{23}^2 - 4\delta_{12}\delta_{13}}$$

$$\delta_0 \equiv \frac{\delta_{12} + \delta_{13} - \delta_{23}}{\delta_{23}^2/(\delta_{12}\delta_{13}) - 4}$$