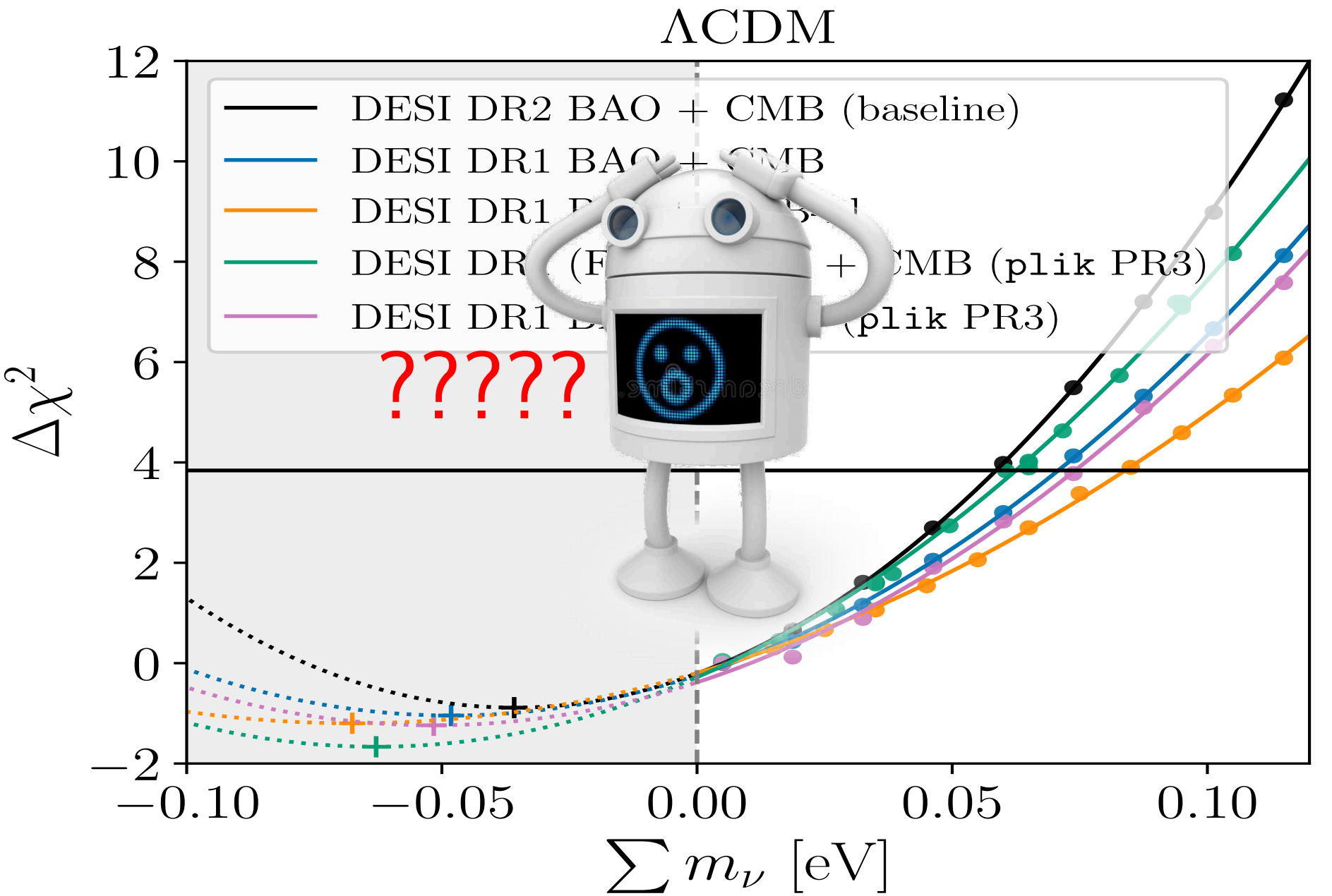
Cosmology and neutrinos

Amol Upadhye
Yunnan Univ.
July 24, 2025

Motivation: What's so hard about ν constraints?



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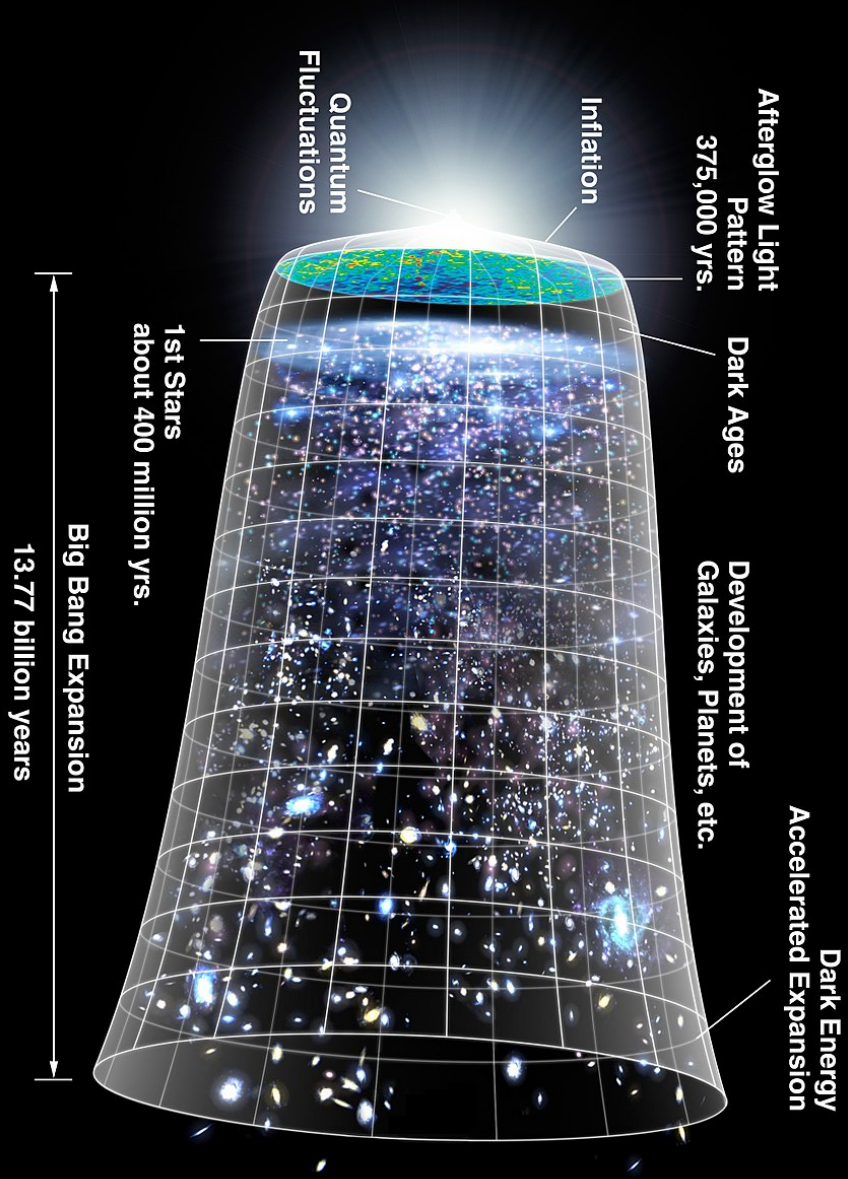

1. Neutrinos in the early universe
   - decoupling and BBN
   - $N_{\rm eff}$ and the CMB
   - neutrino masses
2. Neutrinos and geometry
   - $\Omega_{\nu 0}$, $\Omega_{\rm m0}$, H_0 degeneracy
   - DESI+CMB, $\Omega_{\rm m0}$, and τ
3. Neutrino free-streaming
   - free-streaming and large-scale structure
   - multi-fluid perturbation theory for neutrinos

Neutrino decoupling during BBN

Decoupling: $\Gamma_\nu \sim G_F^2 T^5 < H \sim T^2/M_{\rm Pl} \Rightarrow T \approx 1.5$ MeV

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T_ν, assuming full decoupling before e^+e^- annihilation

- relativistic entropy density: $s_B = \frac{2\pi^2}{45} g T^3$; $s_F = \frac{7}{8} s_B$
- e^+, e^-, γ equilibrium: $\Rightarrow$ entropy conserved
- $2 \times \frac{2\pi^2}{45} T_{\rm after}^3 = 2 \times \frac{2\pi^2}{45} T_{\rm before}^3 + 4 \times \frac{7}{8} \frac{2\pi^2}{45} T_{\rm before}^3$
  $\Rightarrow T_{\rm before}/T_{\rm after} = T_\nu/T_\gamma = (4/11)^{1/3}$

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Radiation energy density after e^+e^- annihilation:

$$\rho_r = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_\gamma$$

with Standard Model $N_{\rm eff} = 3.044$. *Bennett, Buldgen, Drewes, Wong, JCAP* **03***:003(2020)[1911.04504]*; *Bennett*, et al., *JCAP* **04***:073(2021)[2012.02726]*

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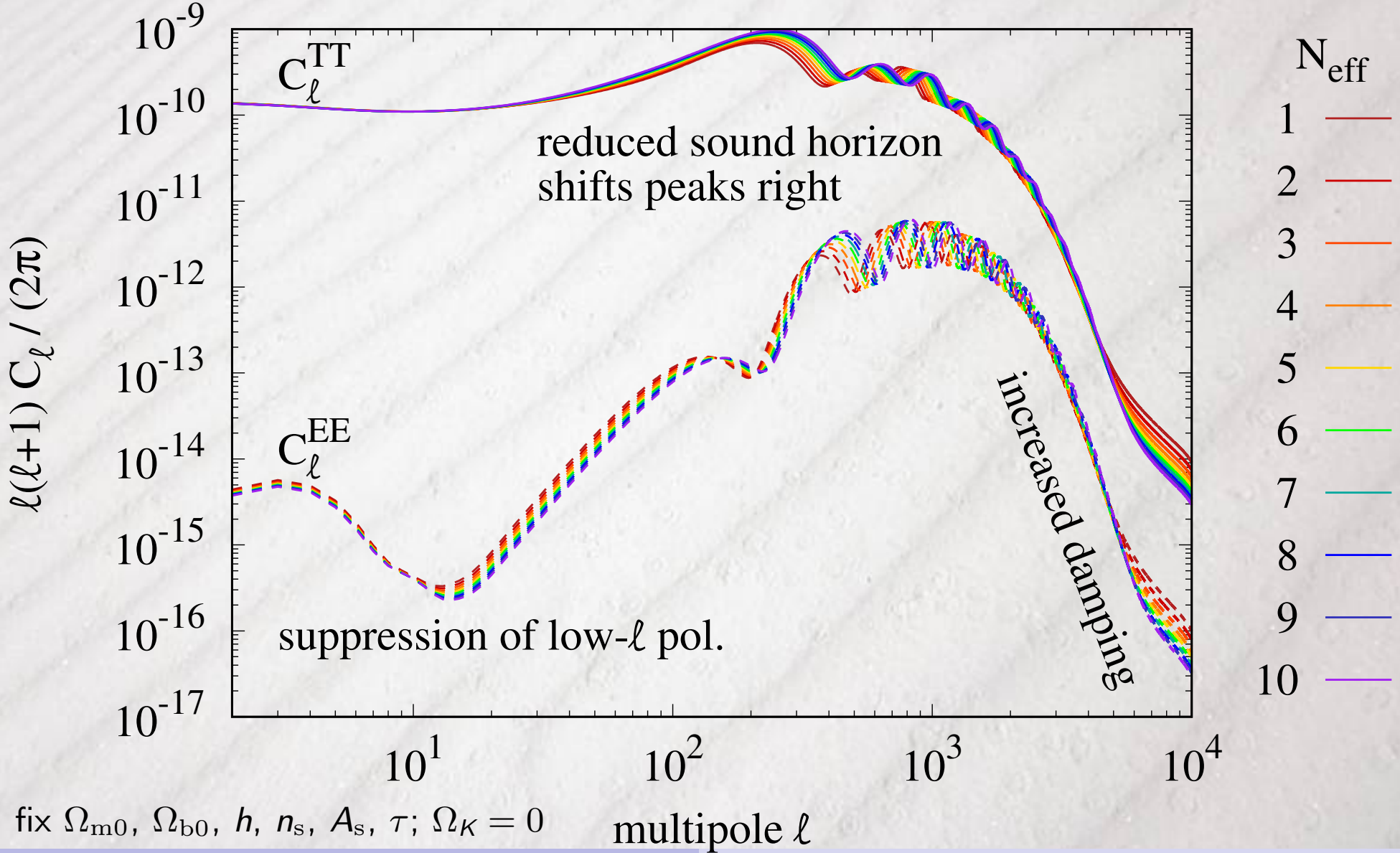
Higher $N_{\rm eff} \Rightarrow$ higher $H \Rightarrow$ earlier neutron freeze-out $\Rightarrow$ more ^{4}He

Cyburt, Fields, Olive, Yeh, Rev. Mod. Phys. **88***:015004(2016)[1505.01076]*

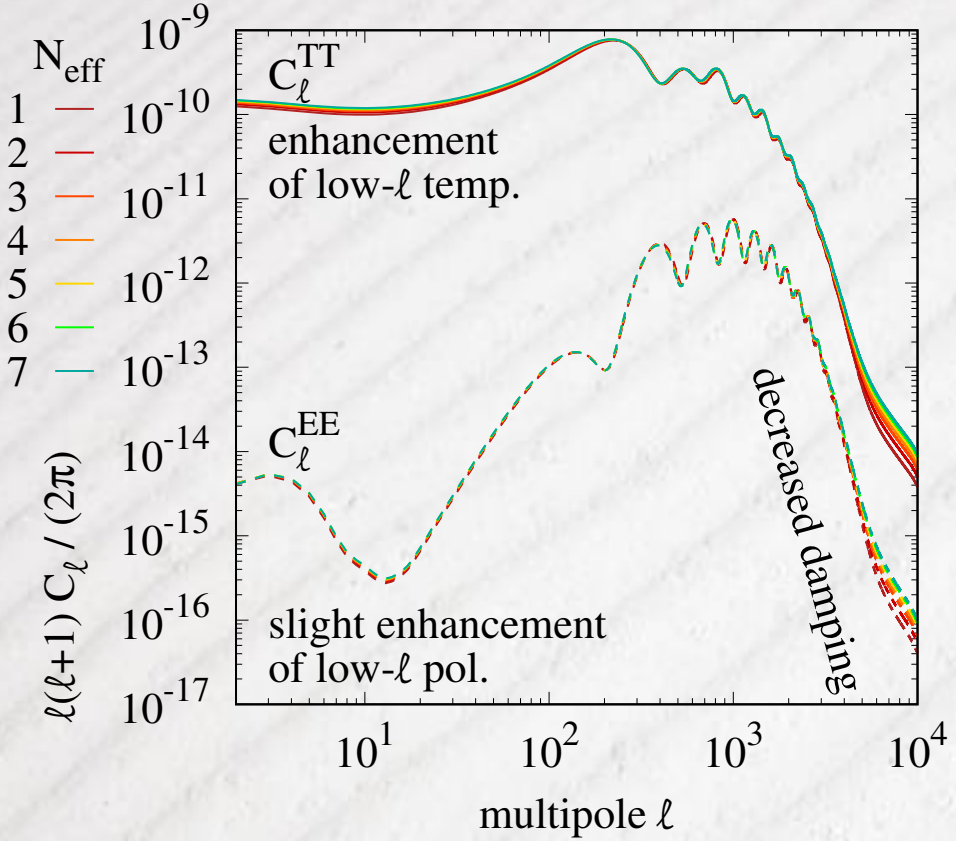
CMB power spectra and $N_{\rm eff}$

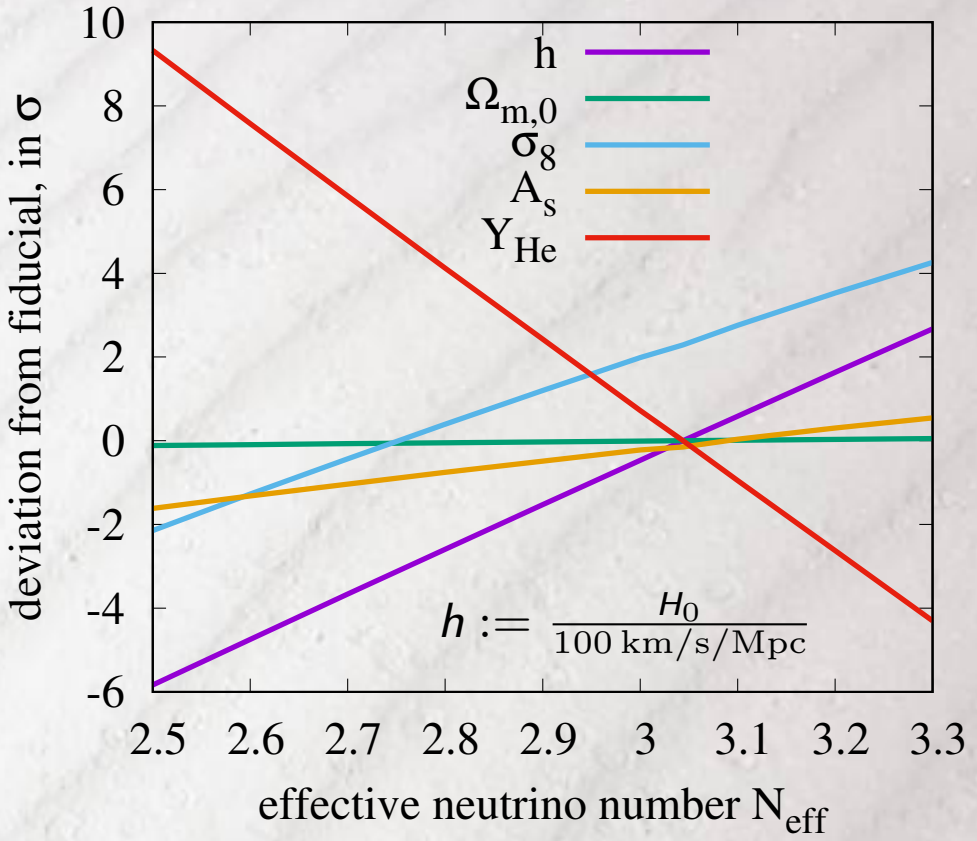
CLASS code: `class-code.net`; *Lesgourgues (2011) [1104.2932]*; *Blas, Lesgourgues, Tram, JCAP* **1107**:*034(2011)[1104.2933]*; *Lesgourgues, Tram, JCAP* **1109**:*032(2011)[1104.2935]*



fix $\Omega_{\rm m0}$, $\Omega_{\rm b0}$, h, $n_{\rm s}$, $A_{\rm s}$, τ; $\Omega_K = 0$

CMB power spectra and $N_{\rm eff}$: parameter variations





Fixing $\Omega_{\rm b0}h^2$, C_{400}^{TT}, $z_{\rm eq}$, $\theta_{\rm s}$, and $\theta_{\rm D}$ cancels most effects of varying $N_{\rm eff}$.

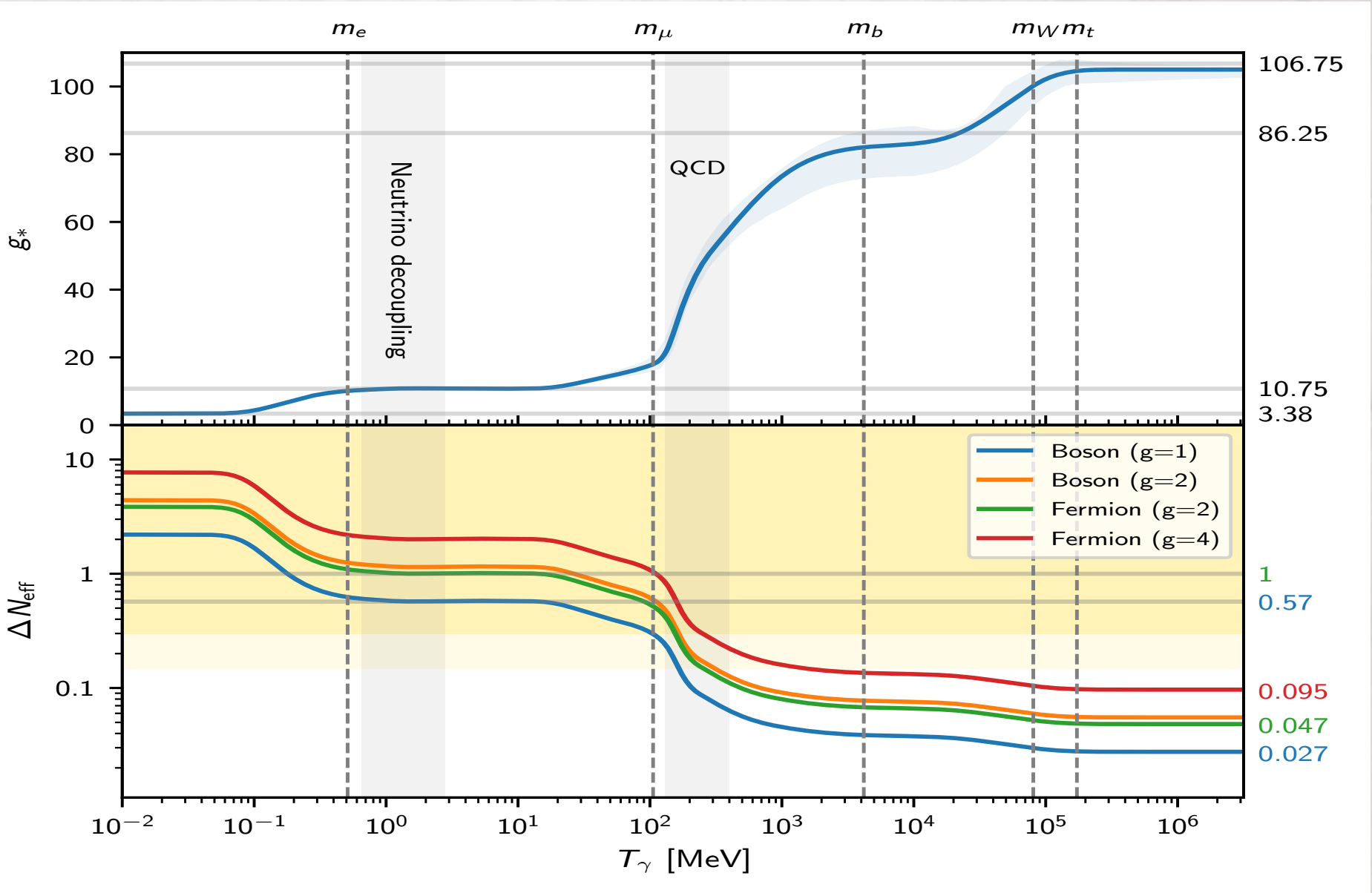
Hou, Kiesler, Knox, Millea, Reichardt,

PRD **87***:083008(2013)[1104.2333]*

The cost: required variations of other parameters $\Rightarrow$ tensions with CMB, BBN, low-z data.

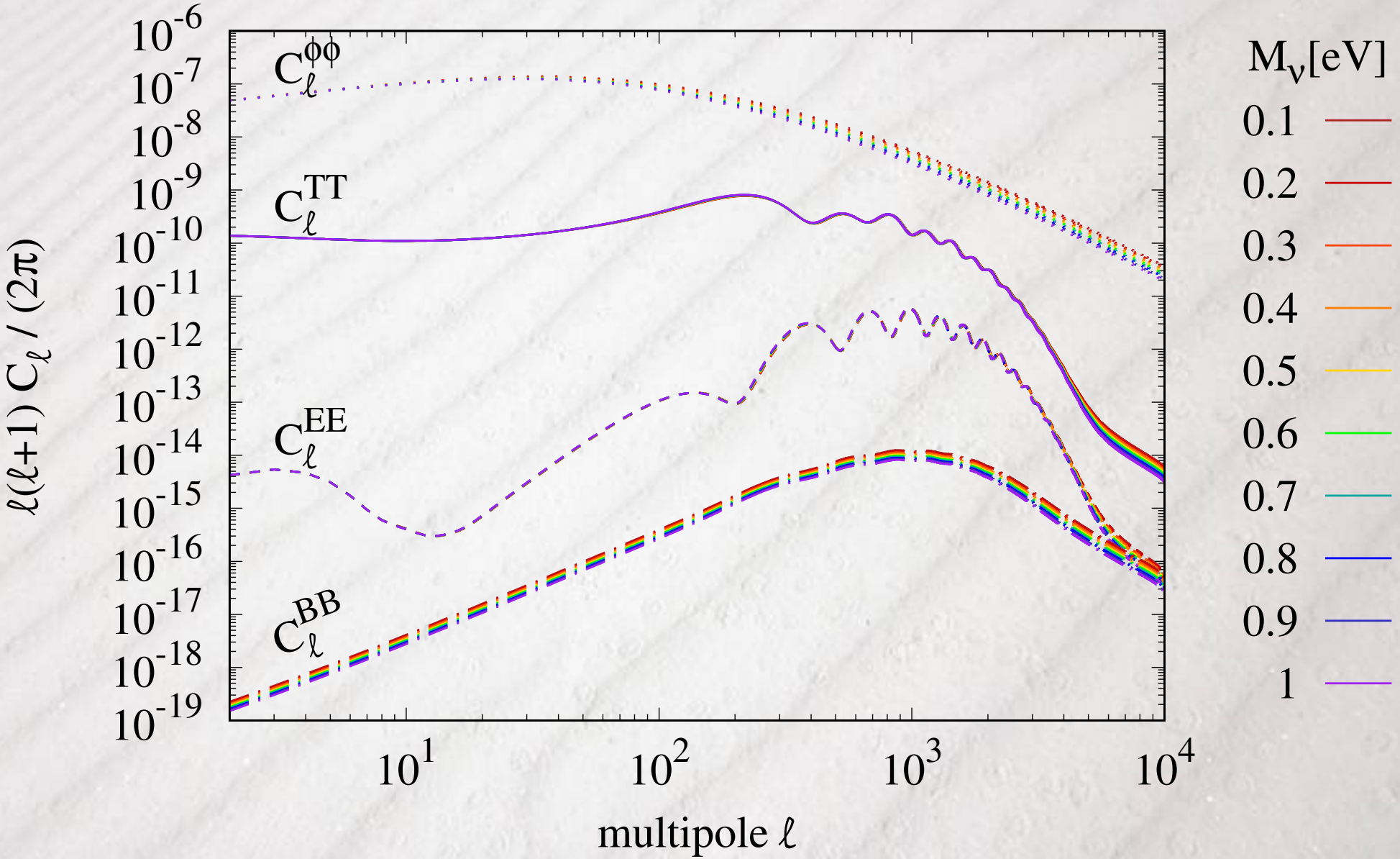
Need joint constraints.

$N_{\rm eff}$ constraints bound all relativistic species



Aghanim, et al., A&A **641**:*A6(2020)[1807.06209]* (Planck 2018)

CMB power spectra and M_ν



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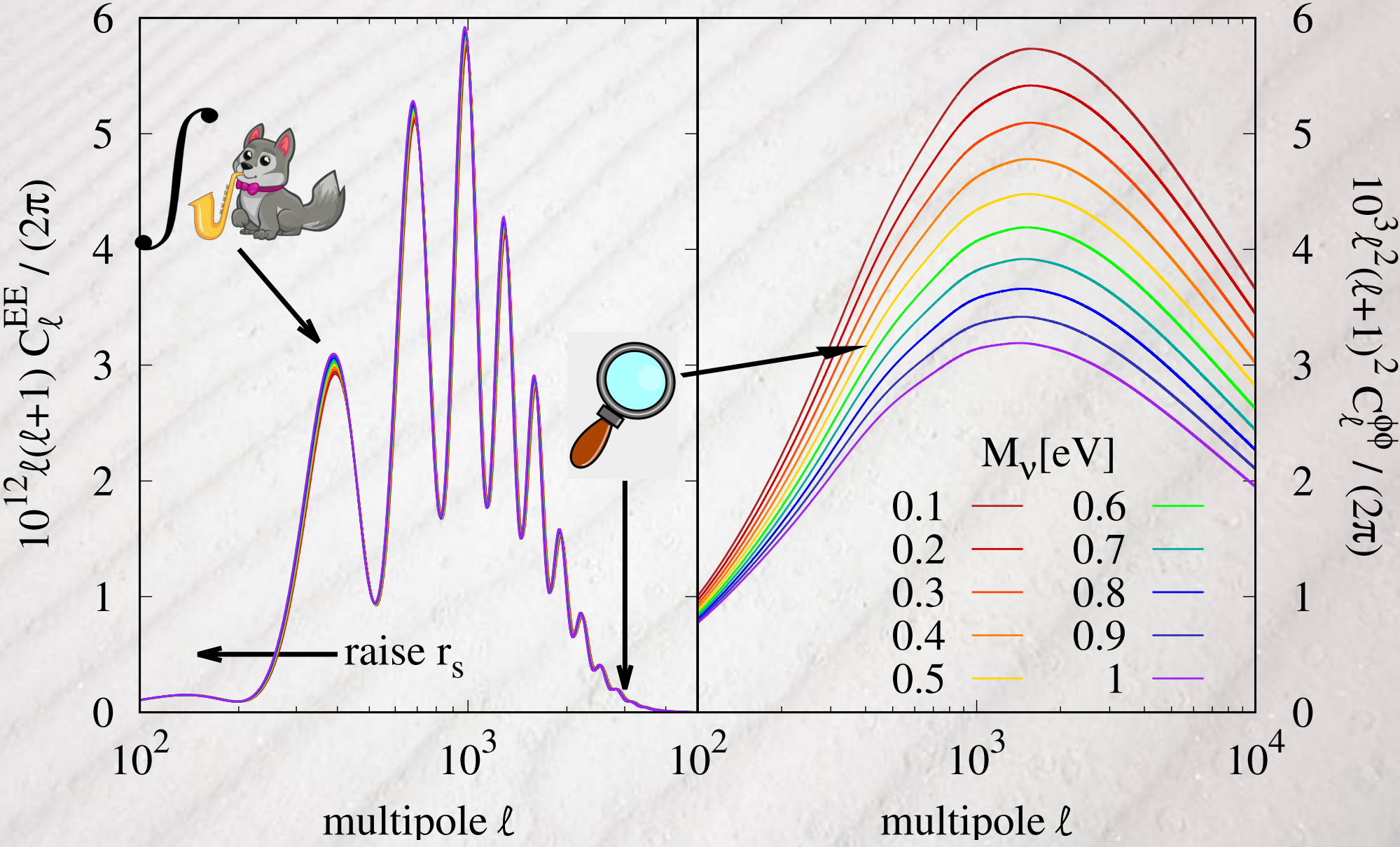

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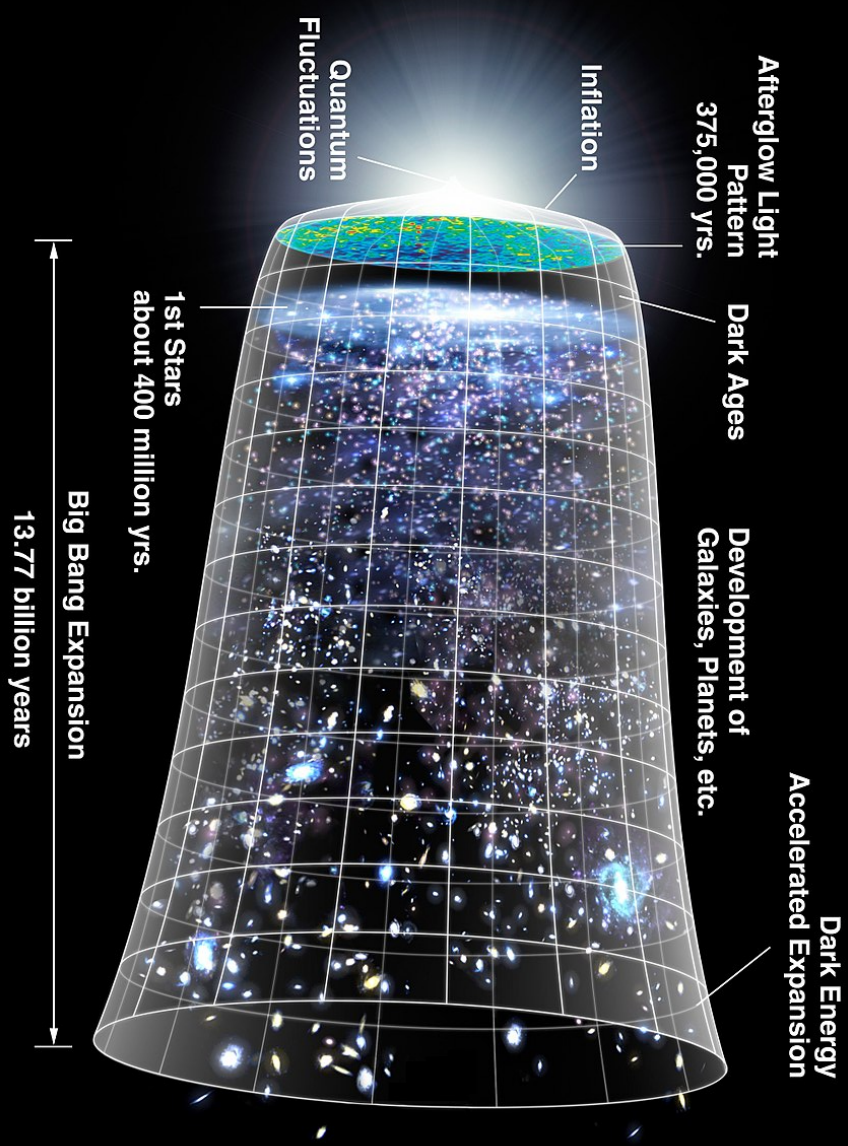

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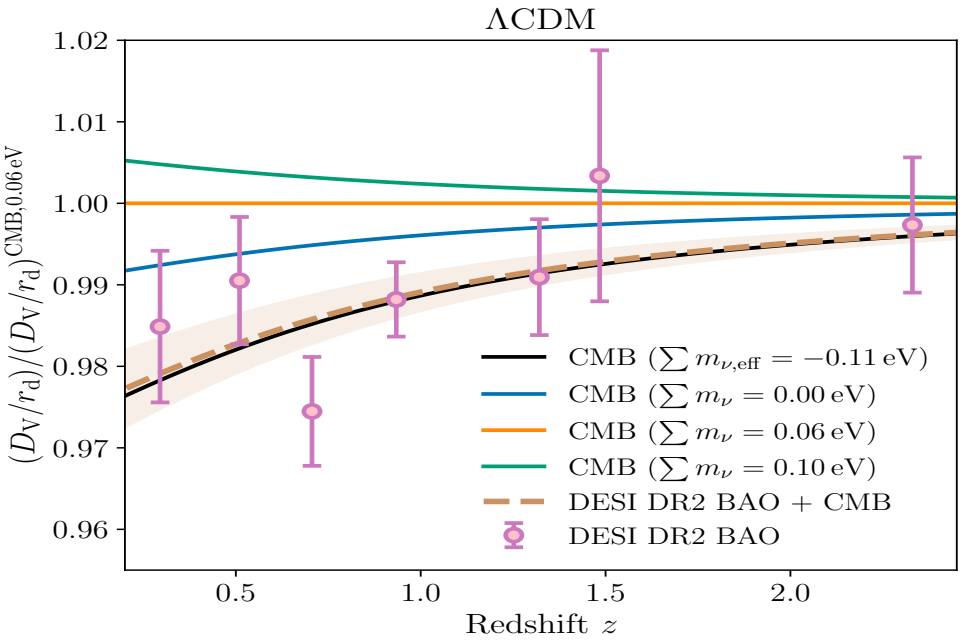
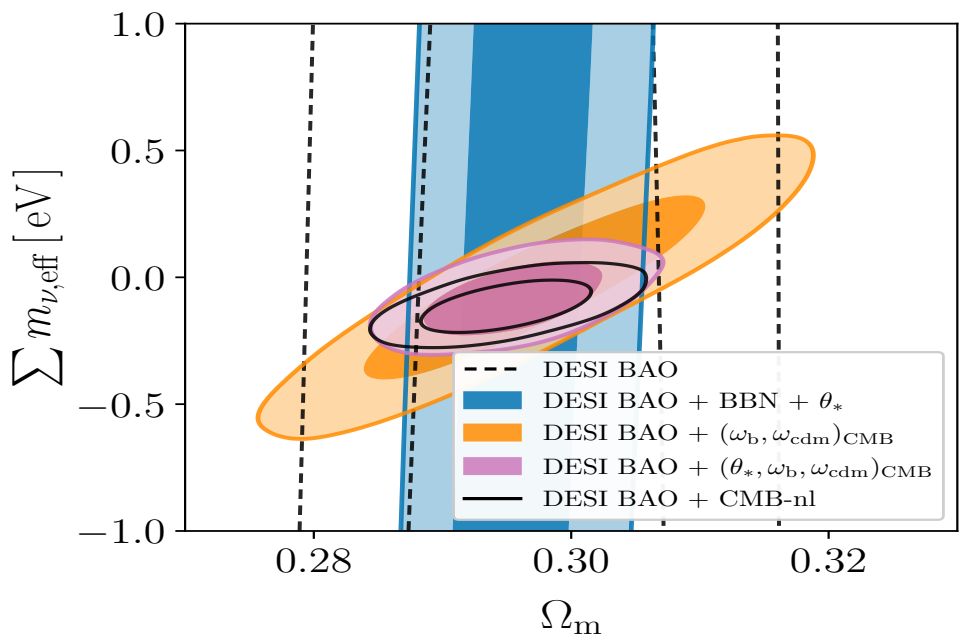
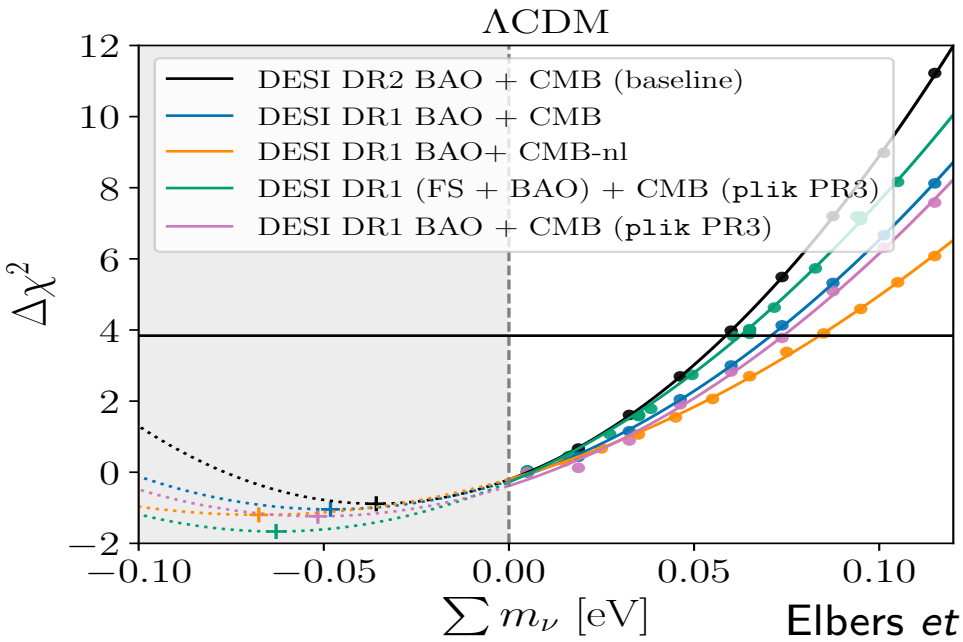
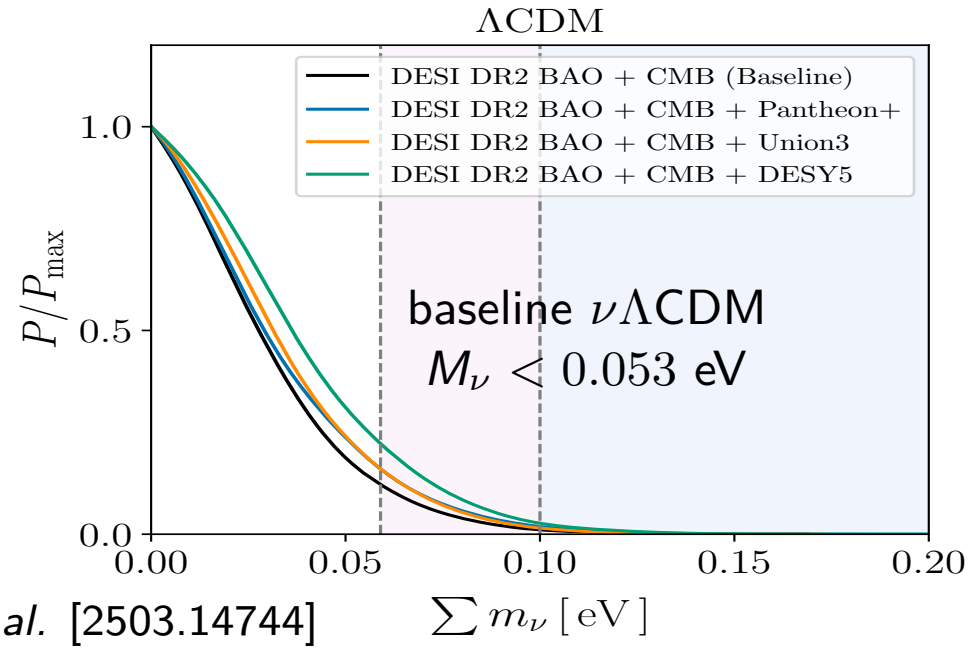
DESI hints of negative neutrino masses?









Elbers *et al.* [2503.14744]

Geometric constraints on neutrinos

Assumption: M_ν negligible before decoupling.

$\Rightarrow$ The geometric constraint is a mismatch between the physical matter density $\omega_{\rm b} + \omega_{\rm c}$ measured before decoupling and $\omega_{\rm m} = \omega_{\rm b} + \omega_{\rm c} + \omega_\nu = \Omega_{\rm m0} h^2$ measured in the late universe.

LoVerde, Weiner, JCAP **12***:48(2024)[2410.00090]*

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$\omega_\mathrm{b}, \omega_\mathrm{c}$ fixed by CMB $\Rightarrow \omega_\mathrm{m} = \frac{\omega_\mathrm{b}+\omega_\mathrm{c}}{1-f_\nu} \approx (\omega_\mathrm{b} + \omega_\mathrm{c})(1 + f_\nu)$

where $f_\nu = \Omega_{\nu 0}/\Omega_{\mathrm{m}0}$. Angular scale of acoustic oscillations $\theta_\mathrm{s} = r_\mathrm{s}/\chi_\star$ fixed by CMB, r_s fixed by pre-decoupling physics,

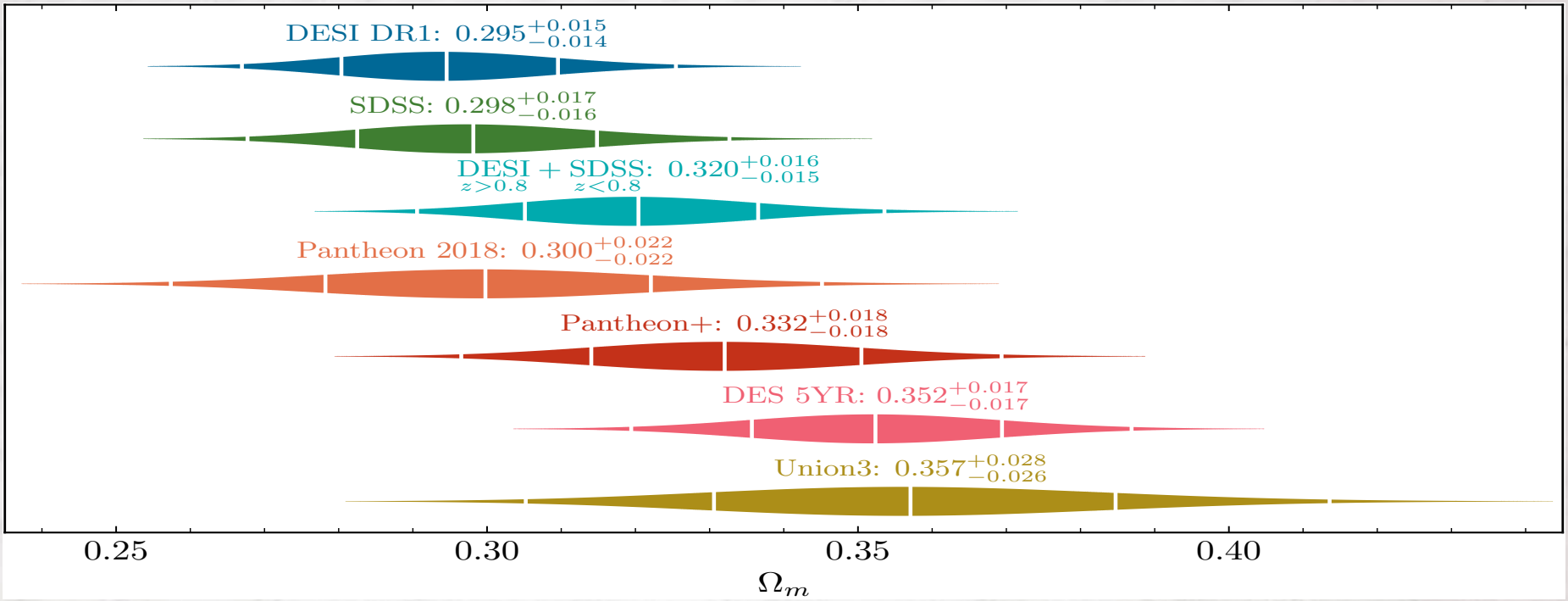
$$\chi_\star = \int_{a_\star}^{1} \frac{da}{a^2 H(a)} \approx \frac{2(\Omega_{\mathrm{m}0})^{0.1}}{H_{100}\sqrt{\omega_\mathrm{m}}} \qquad \text{where } H_{100} = \frac{100\ \mathrm{km/s}}{\mathrm{Mpc}},\ h = \frac{H_0}{H_{100}}$$

with ω_m coming from H and $\Omega_{\mathrm{m}0}$ coming from $a_{\mathrm{m}\Lambda}$.

$$\Rightarrow \Omega_{\mathrm{m}0} \propto (1 + f_\nu)^5 \approx 1 + 5 f_\nu \quad \text{and} \quad h \propto (1 + f_\nu)^{-2} \approx 1 - 2 f_\nu$$

LoVerde, Weiner, JCAP **12***:48(2024)[2410.00090]*

Geometric constraints and $\Omega_{\rm m0}$ uncertainty



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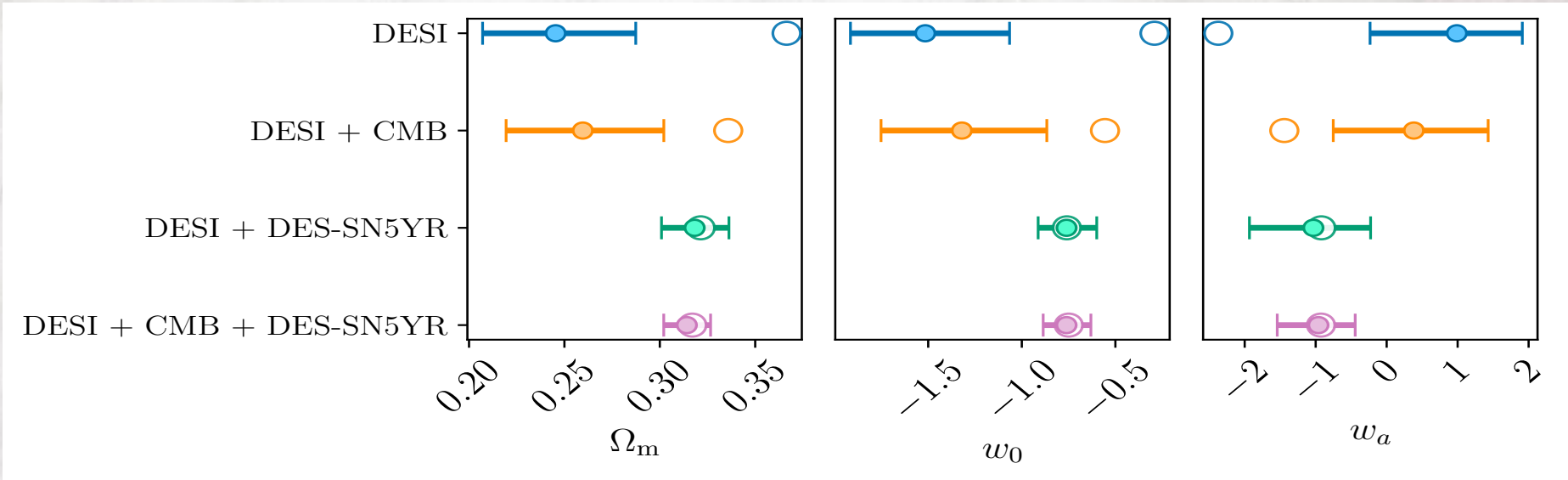
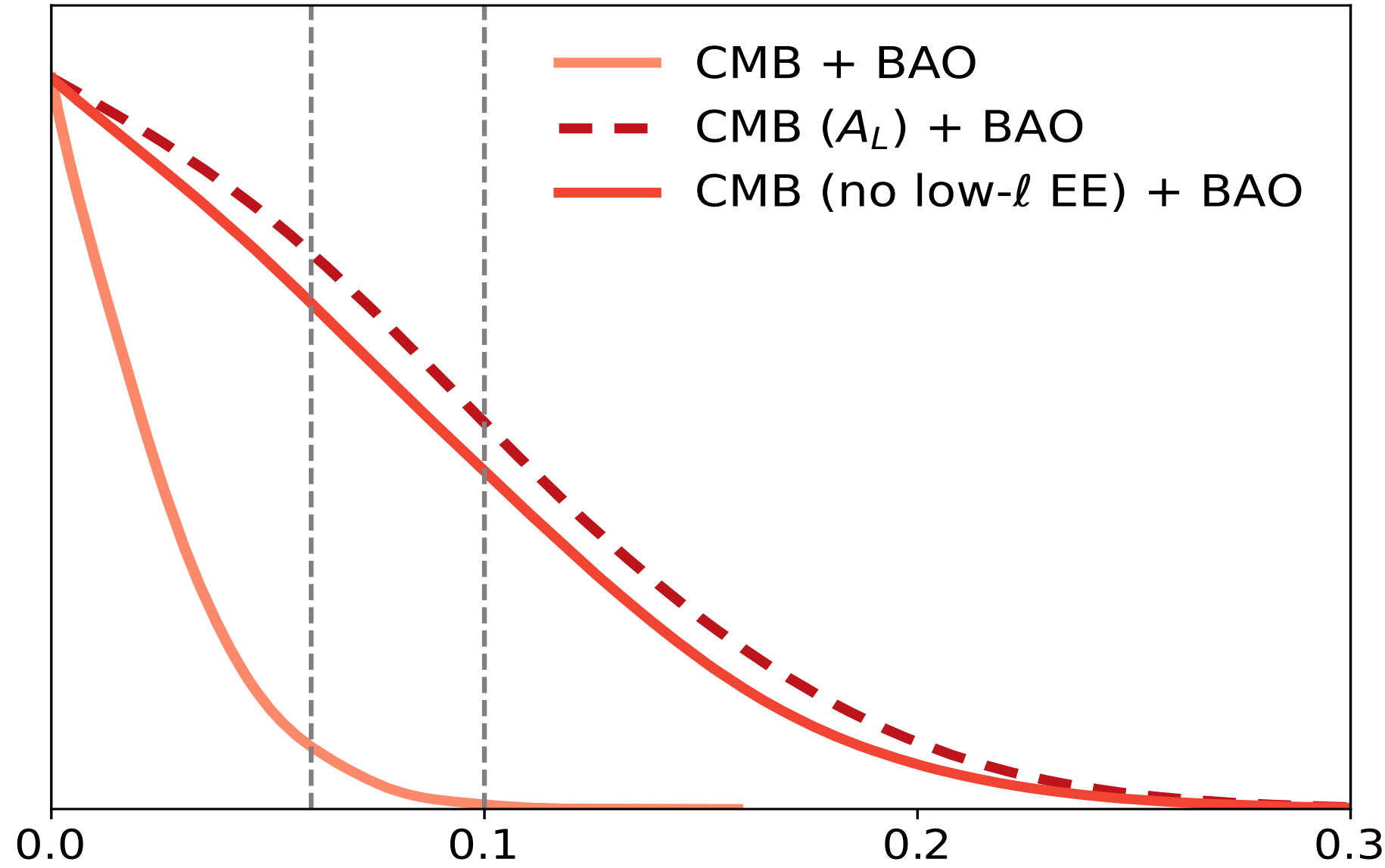


Adame *et al.* [2411.12022]

τ, $A_{\rm L}$ allow larger M_ν



Jhaveri, Karwal, Hu (2025) [2504.21813]

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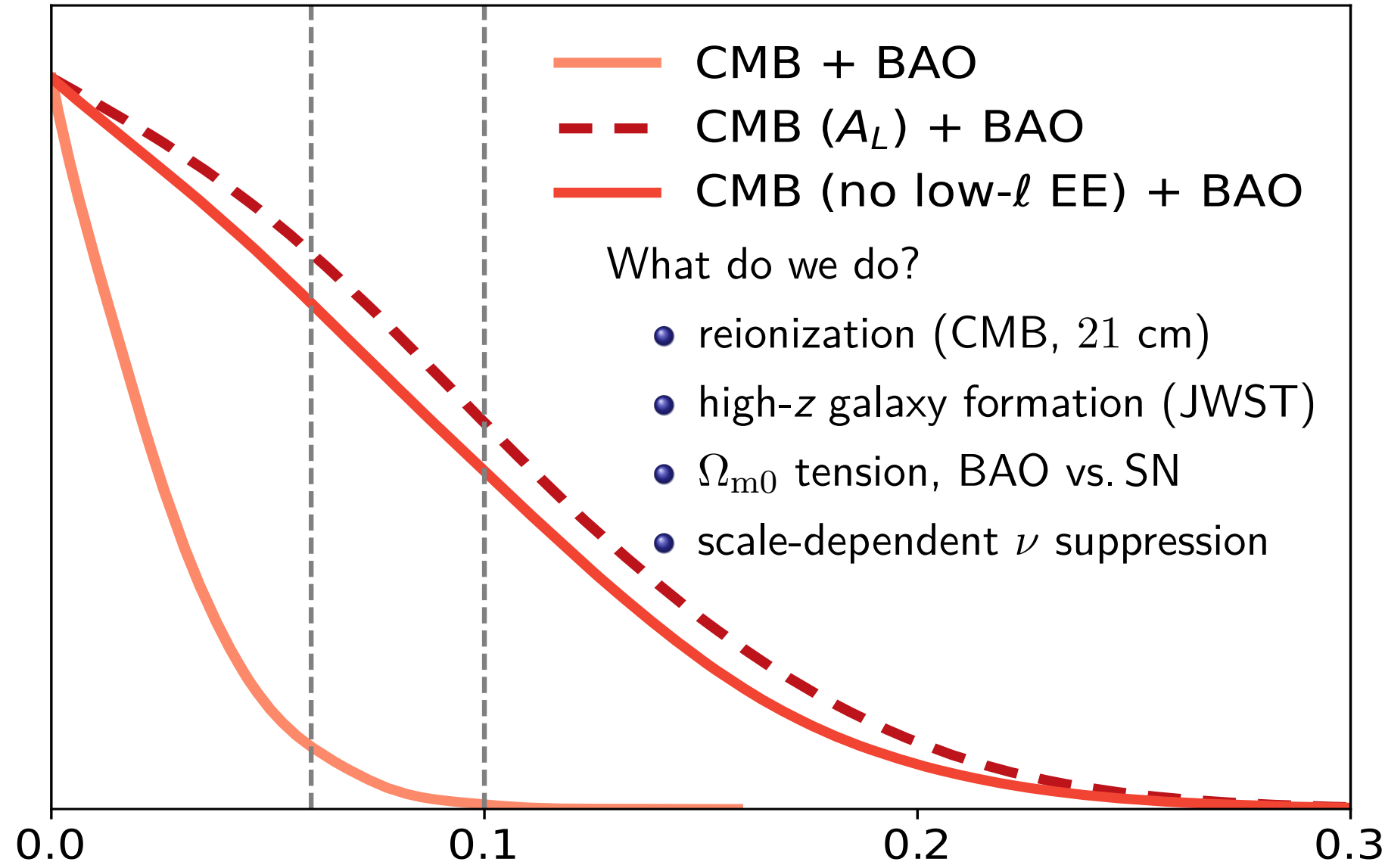

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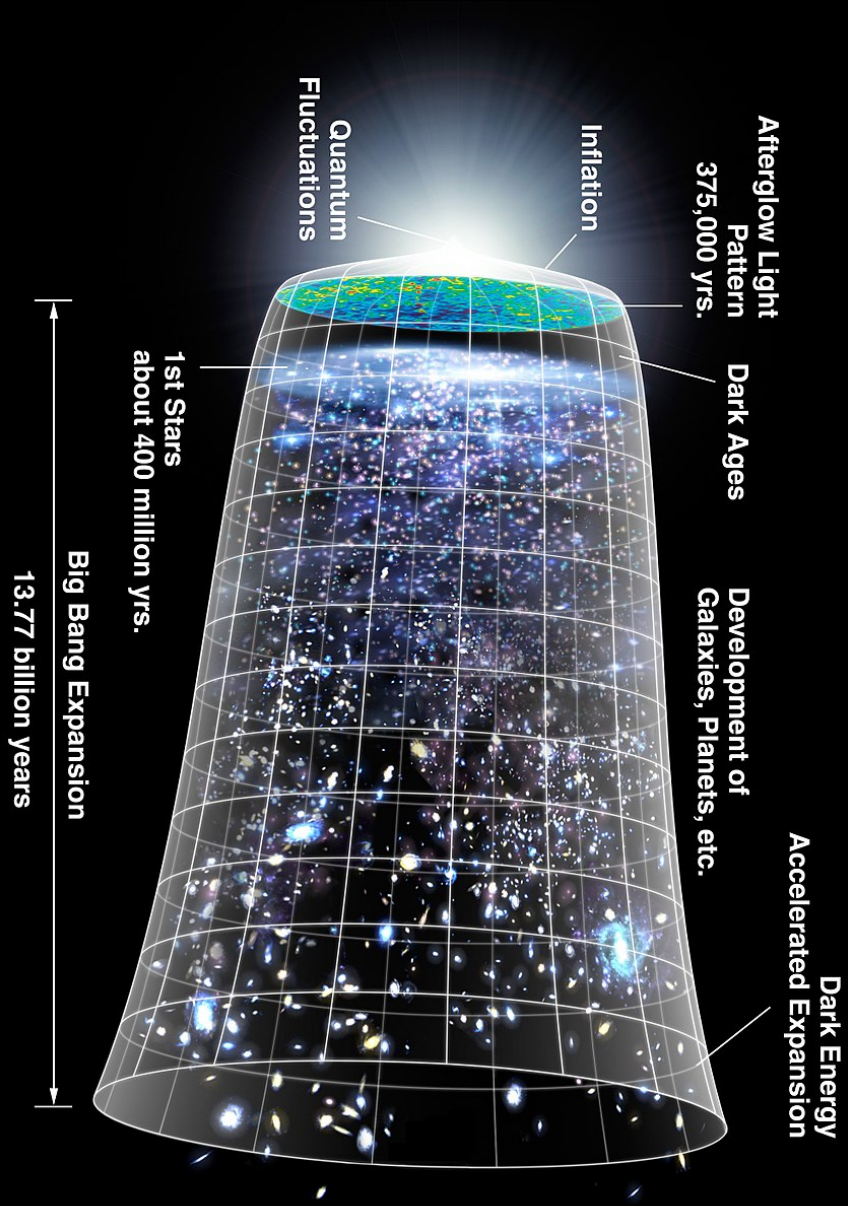

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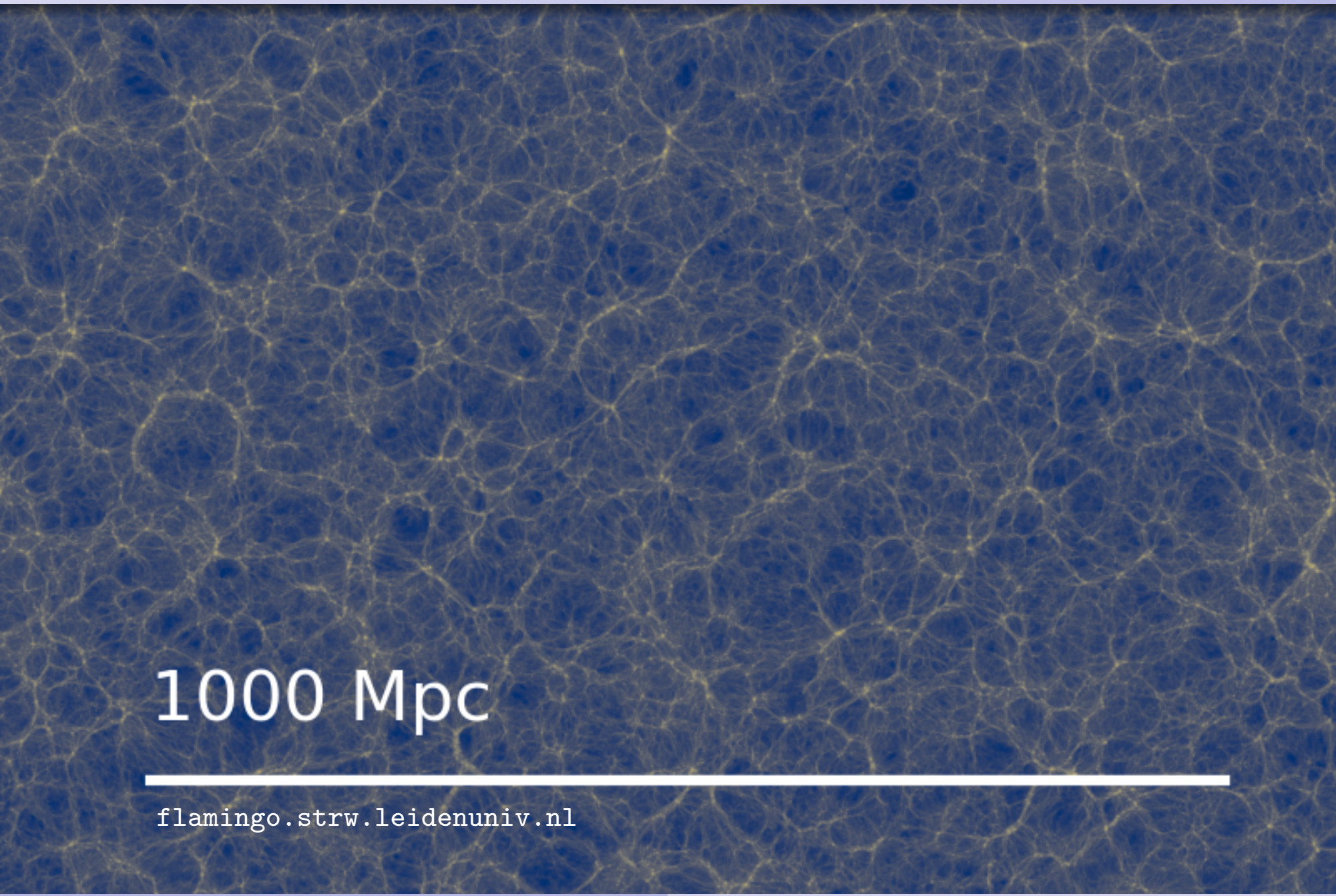
Neutrino velocity dispersion and the cosmic web



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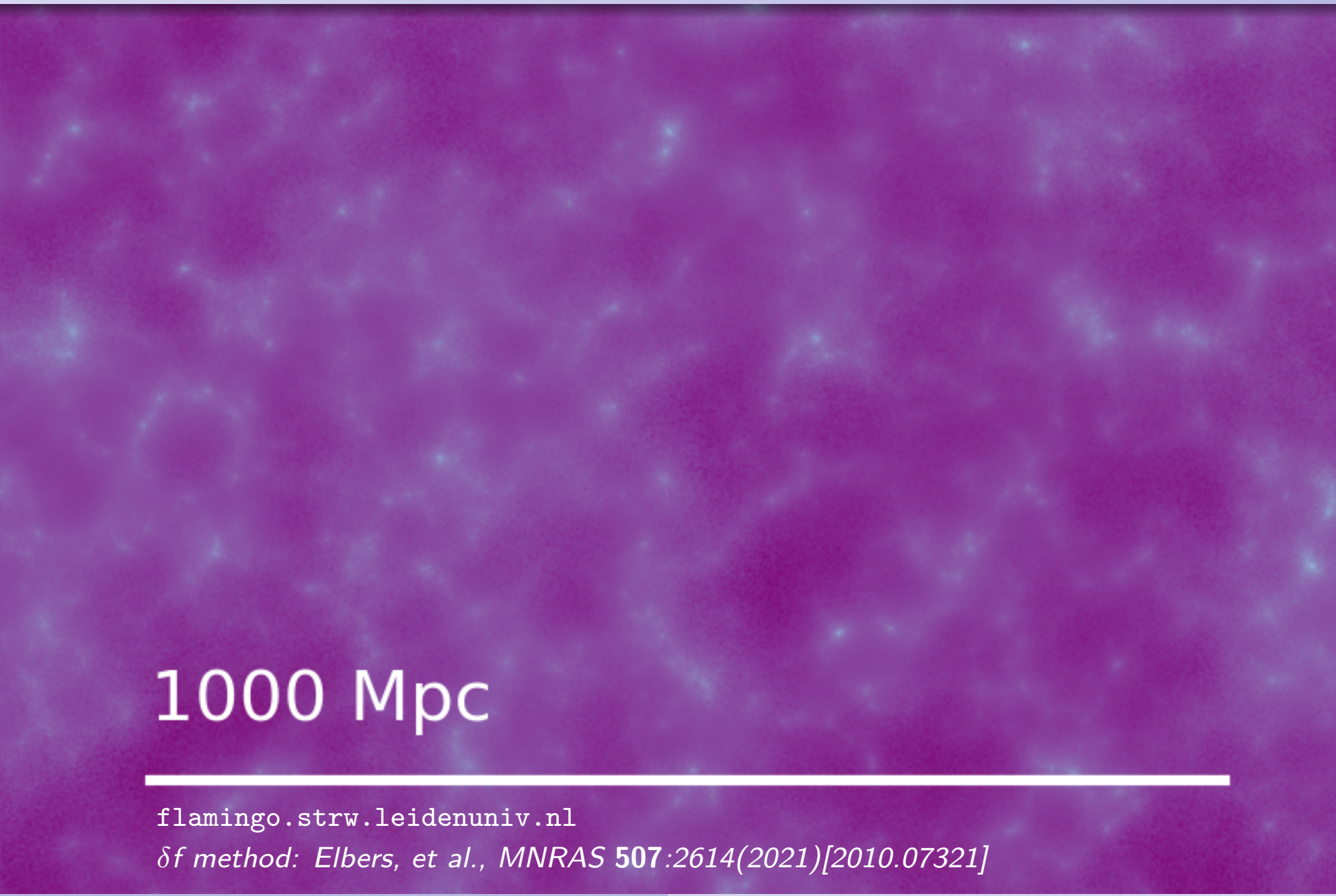

`flamingo.strw.leidenuniv.nl`

δf method: Elbers, et al., MNRAS **507***:2614(2021)[2010.07321]*

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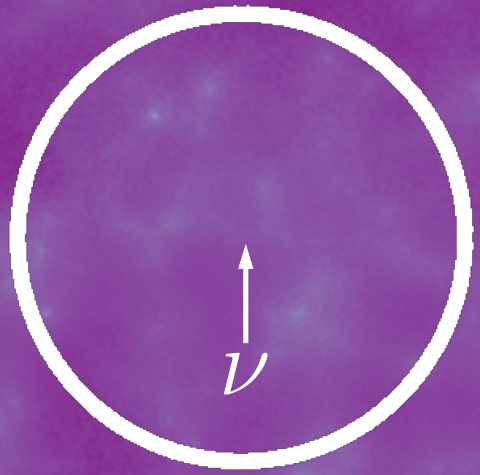
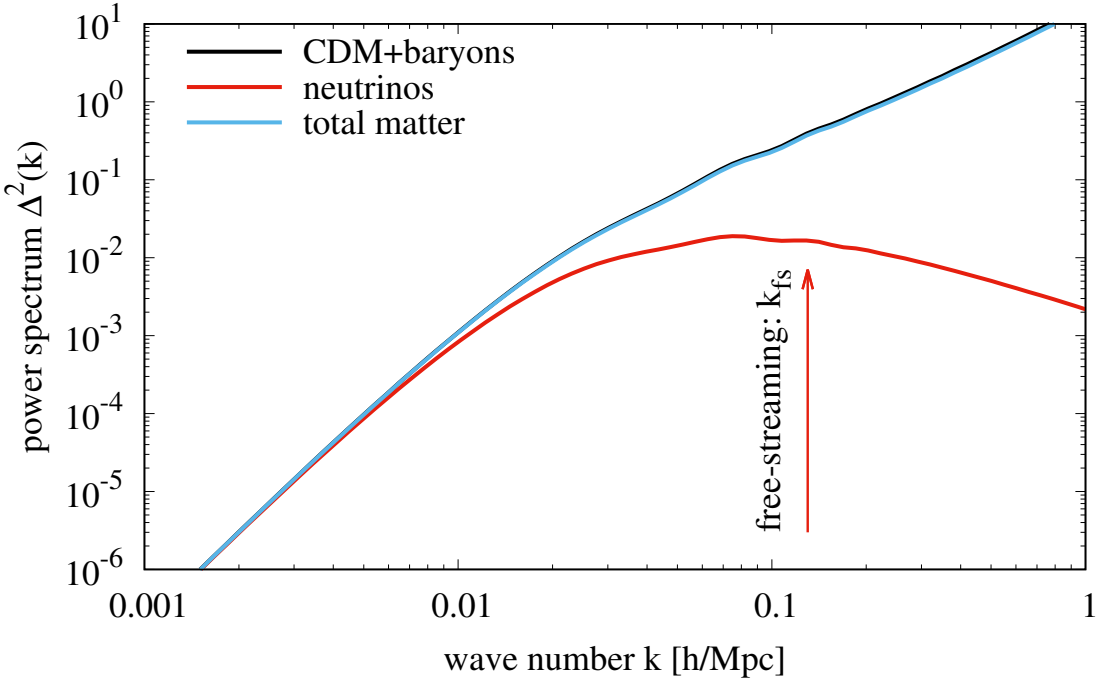


1000 Mpc

ν power suppressed on small scales (large k)



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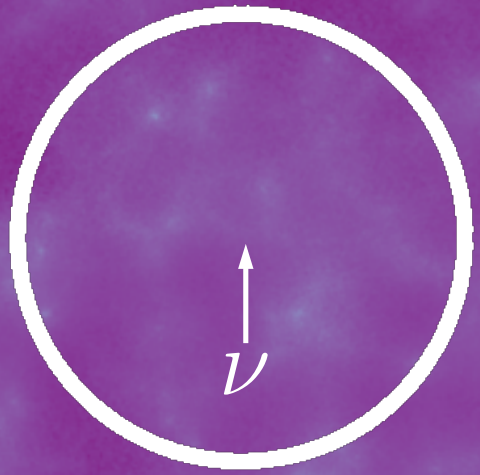

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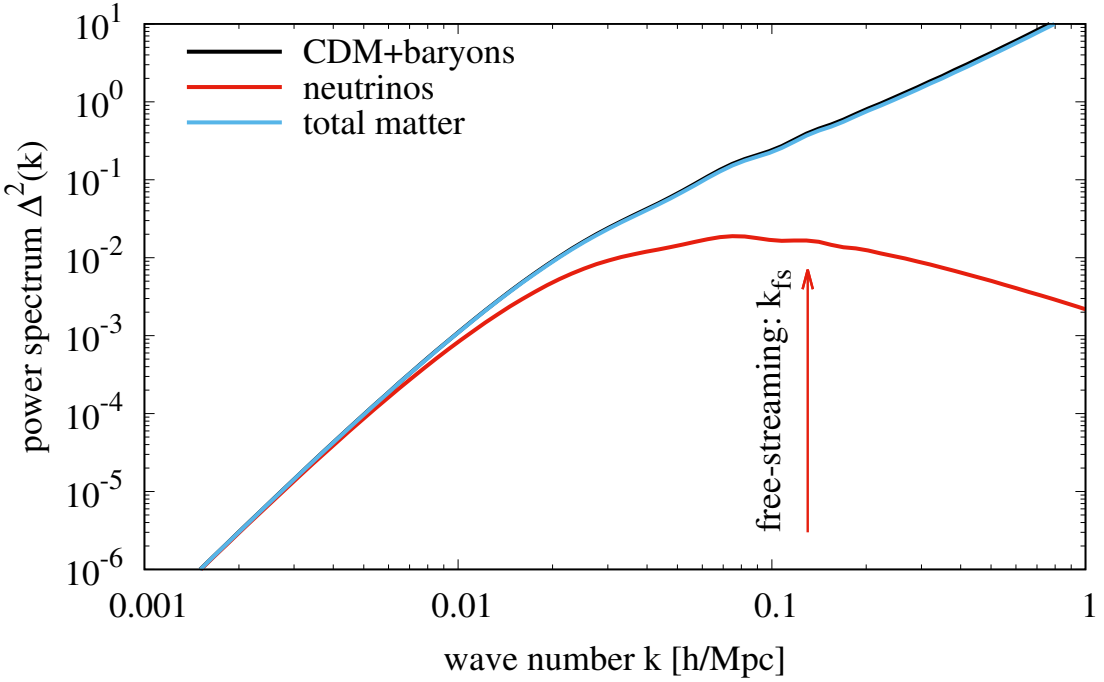

95%CL, DESI(FS+BAO) + BBN + n_s:

$$M_\nu < \begin{cases} 0.409 \text{ eV } (10\sigma\ n_s) \\ 0.300 \text{ eV } (1\sigma\ n_s) \end{cases}$$

Adame, et al., JCAP **07***:028(2025)[2411.12022]*

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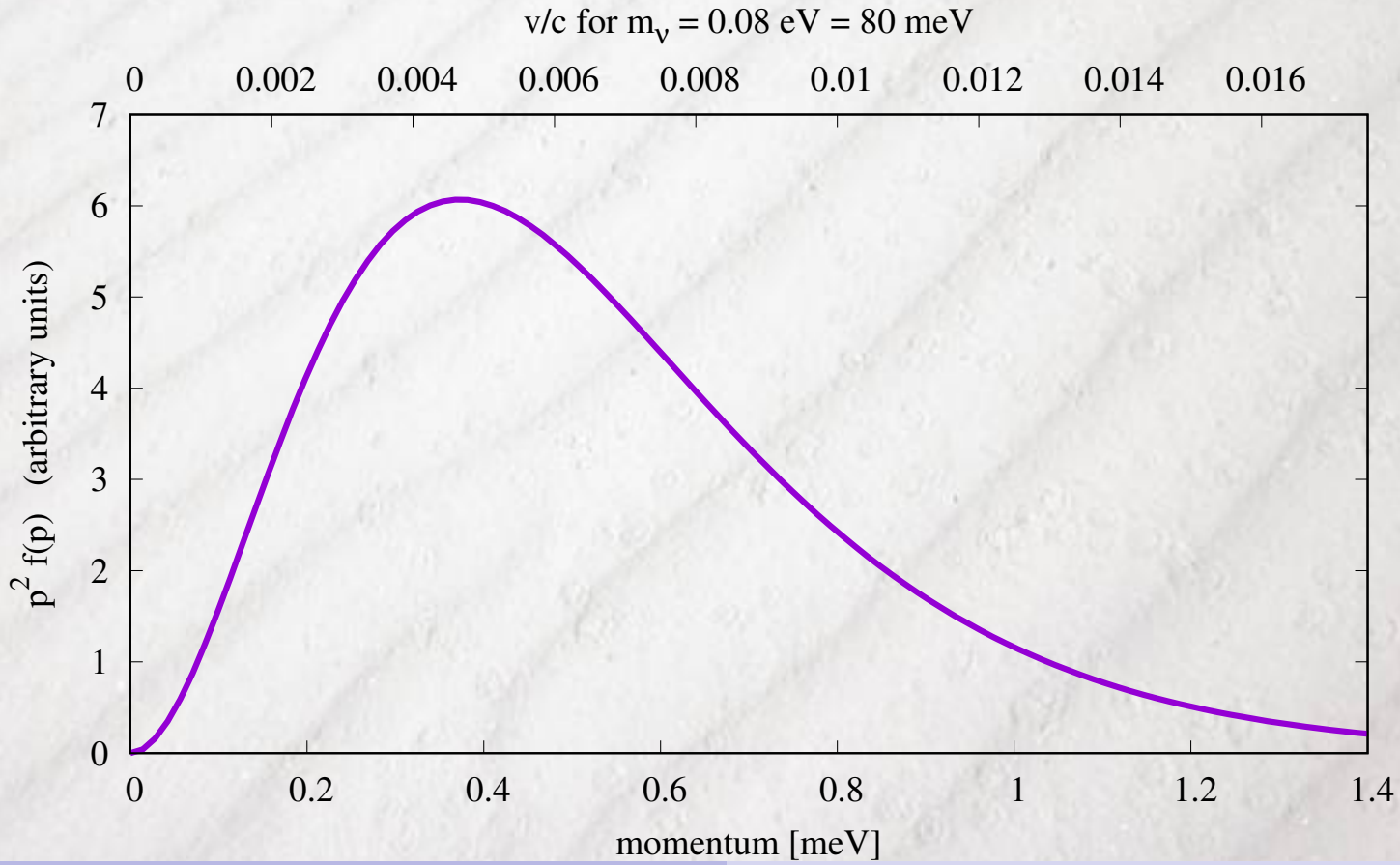
δf method: Elbers, et al., MNRAS **507***:2614(2021)[2010.07321]*

Neutrinos as multiple flows

Continuity (mass conservation): $\frac{1}{a^3}\frac{\partial(a^3\rho)}{\partial\tau} + \vec{\nabla}\cdot(\rho\vec{V}) = 0$

Euler (momentum evolution): $\frac{1}{a}\frac{\partial(a\vec{V})}{\partial\tau} + (\vec{V}\cdot\vec{\nabla})\vec{V} + \vec{\nabla}\Phi = 0$



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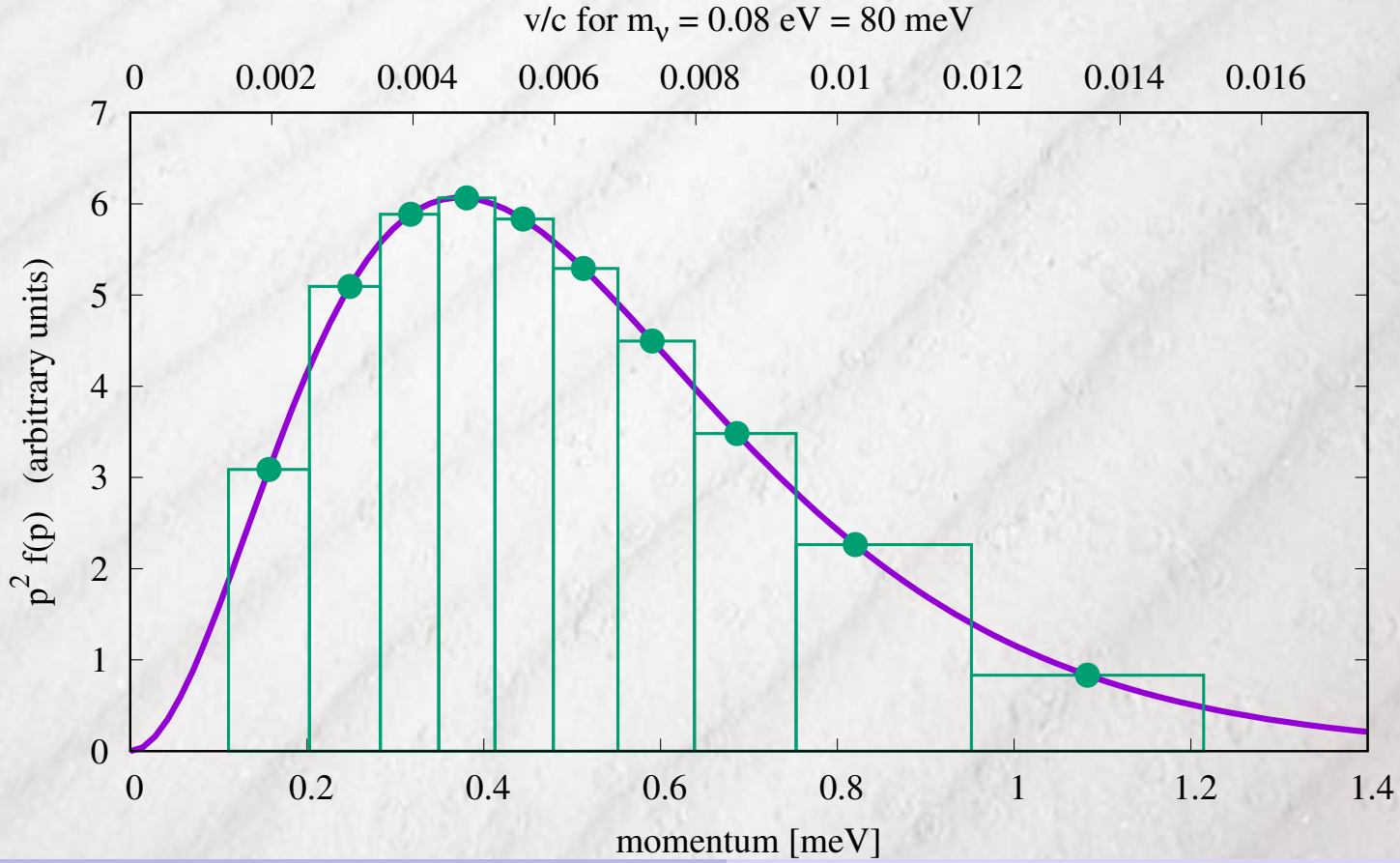
Idea: discretize $f_{\rm FD}$ into many flows, each with uniform thermal $\vec{v}_\alpha$

$\vec{V}_\alpha(\vec{x},\tau) = \vec{v}_\alpha(\tau) + \vec{\mathcal{V}}_\alpha(\vec{x},\tau)$ *Dupuy,Bernardeau, JCAP* **1401***:030(2014)[1311.5487]*



Nonlinear equations of motion for flow α

- Continuity equation: Mass is conserved. A change in density locally must be balanced by an inflow or outflow.

$$\frac{1}{a^3}\frac{\partial(a^3\rho_\alpha)}{\partial\tau} + \vec{\nabla}\cdot(\rho\vec{V}_\alpha) = 0$$

- Euler equation: Changes in the velocity of a fluid element are driven by gradients in the gravitational potential.

$$\frac{1}{a}\frac{\partial(a\vec{V}_\alpha)}{\partial\tau} + (\vec{V}_\alpha\cdot\vec{\nabla})\vec{V}_\alpha + \vec{\nabla}\Phi = 0$$

- Poisson equation: density perturbations cluster gravitationally.

$$k^2\Phi = -\tfrac{3}{2}\mathcal{H}^2\left[\Omega_{\rm cb}(\tau)\delta_{\rm cb} + \textstyle\sum_\alpha \Omega_\alpha(\tau)\int_{-1}^{1}\frac{d\mu}{2}\delta_\alpha\right]$$

where: $\mathcal{H} = aH,\ \mu = \hat{k}\cdot\hat{v}_\alpha,\quad \delta_\alpha = \frac{\rho_\alpha}{\bar{\rho}_\alpha} - 1,\quad \theta_\alpha = -\frac{\vec{\nabla}\cdot\vec{\mathcal{V}}_\alpha}{\mathcal{H}}.$

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$$\Rightarrow \frac{\partial\delta_\alpha}{\partial\tau} + (\vec{v}_\alpha\cdot\vec{\nabla})\delta_\alpha - \mathcal{H}\theta_\alpha = -\vec{\nabla}\cdot(\vec{\mathcal{V}}_\alpha\delta_\alpha)$$

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$$\Rightarrow \frac{\partial}{\partial\tau}(\mathcal{H}\theta_\alpha) + \mathcal{H}^2\theta_\alpha + (\vec{v}_\alpha\cdot\vec{\nabla})\mathcal{H}\theta_\alpha - \nabla^2\Phi = \vec{\nabla}\cdot\left[(\vec{\mathcal{V}}_\alpha\cdot\vec{\nabla})\vec{\mathcal{V}}_\alpha\right]$$

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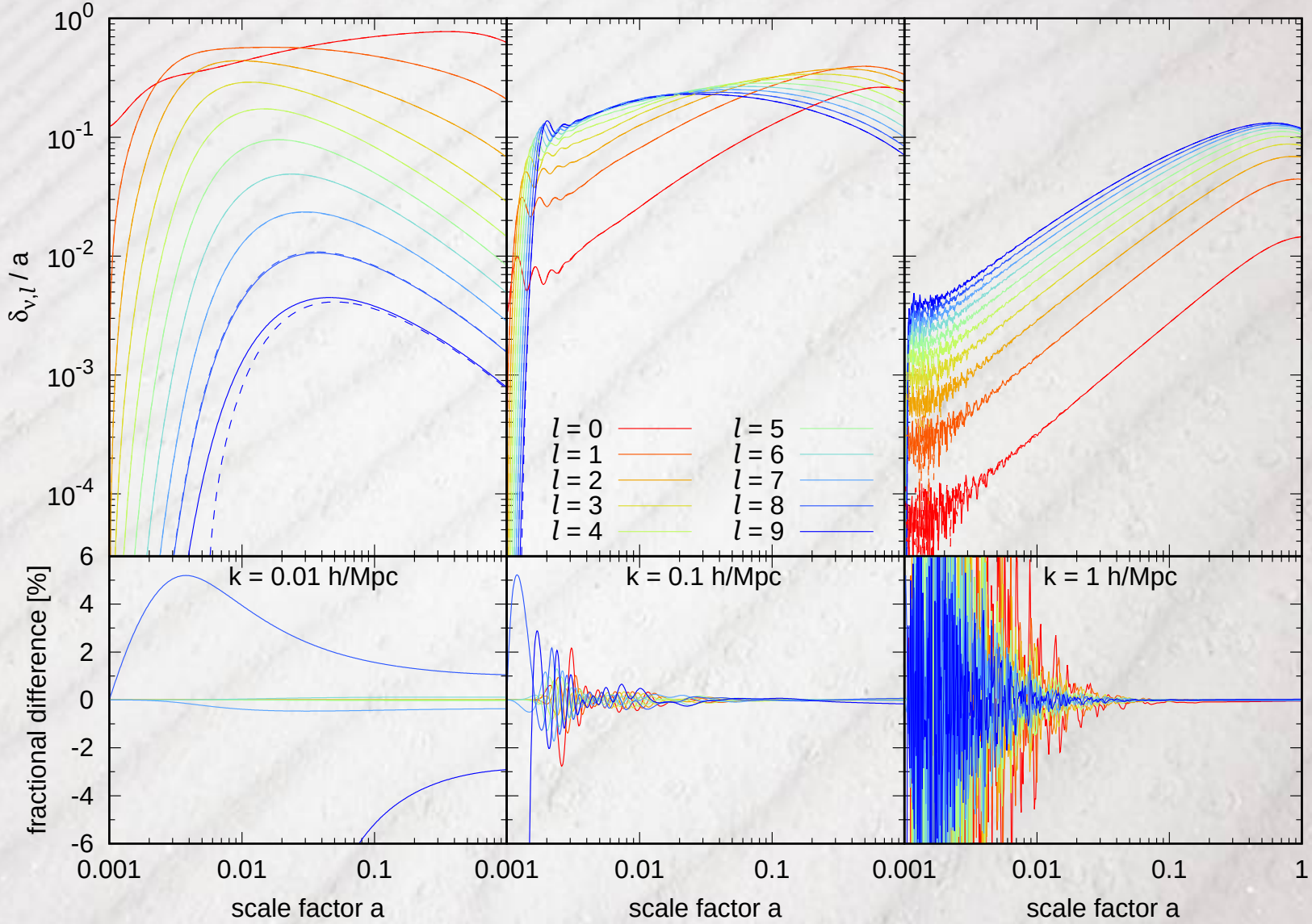
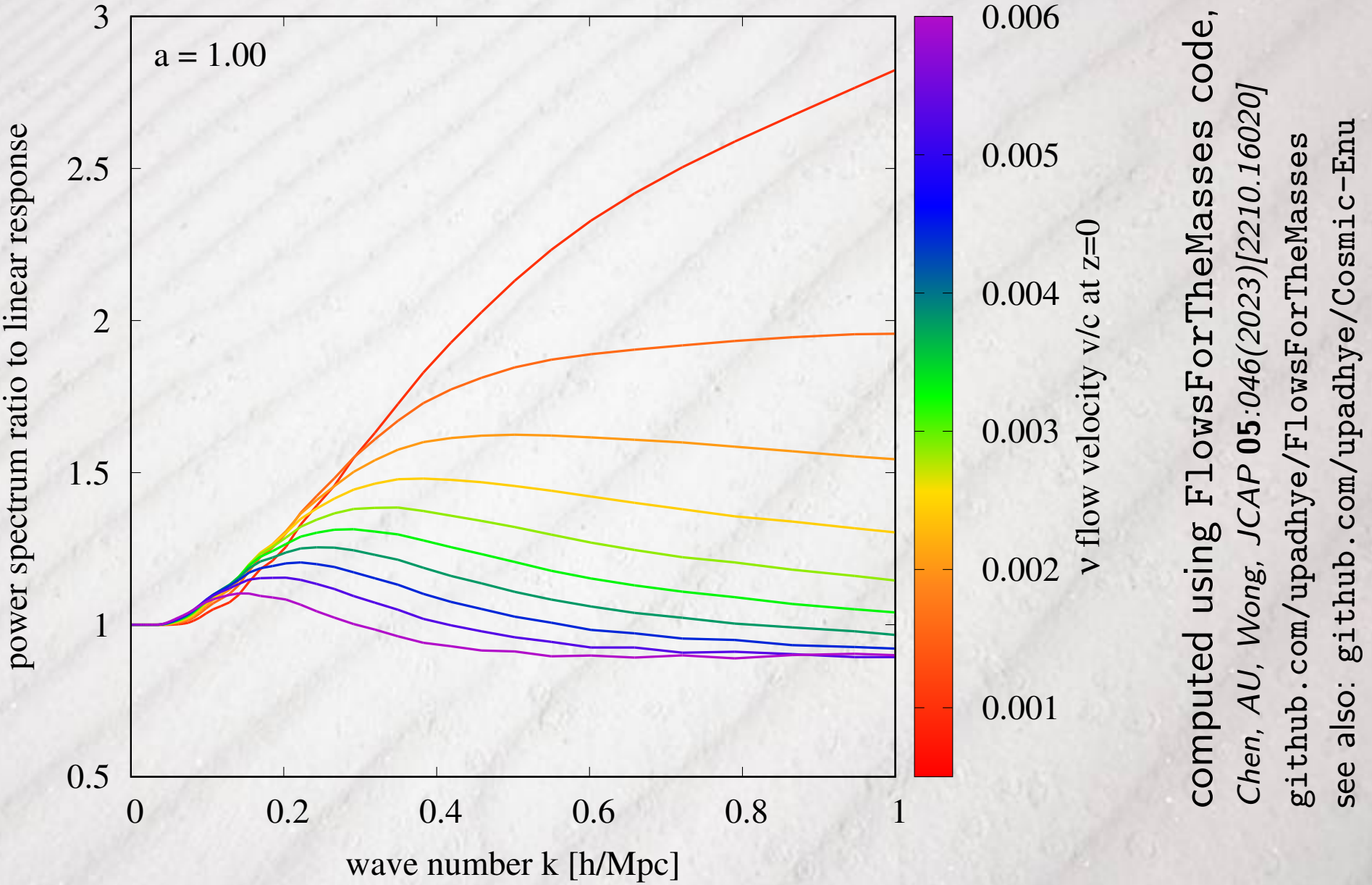
Free-streaming term and "leakage" from monopole

$$\delta_\alpha(\vec{k}) = \sum_\ell (-i)^\ell \mathcal{P}_\ell(\mu)\, \delta_{\alpha\ell}(k) \;\Rightarrow \text{"leakage"} \propto \vec{k}\cdot\vec{v}_\alpha/\mathcal{H}$$



Chen, AU, Wong, JCAP **03***:065(2021)[2011.12503]*

Nonlinear power enhancement vs. flow velocity

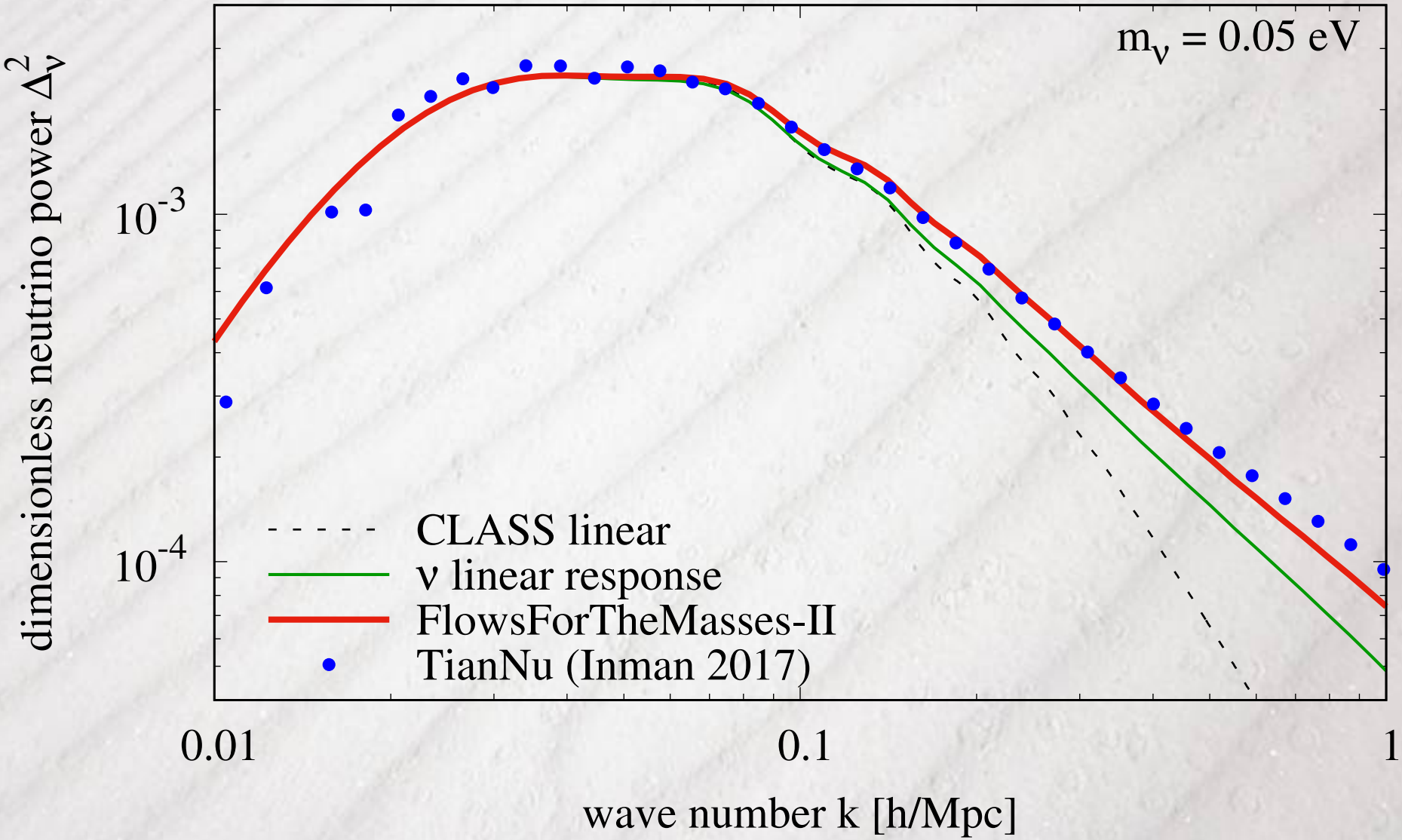


computed using `FlowsForTheMasses` code,
Chen, AU, Wong, JCAP **05**:*046(2023)[2210.16020]*
`github.com/upadhye/FlowsForTheMasses`
see also: `github.com/upadhye/Cosmic-Enu`

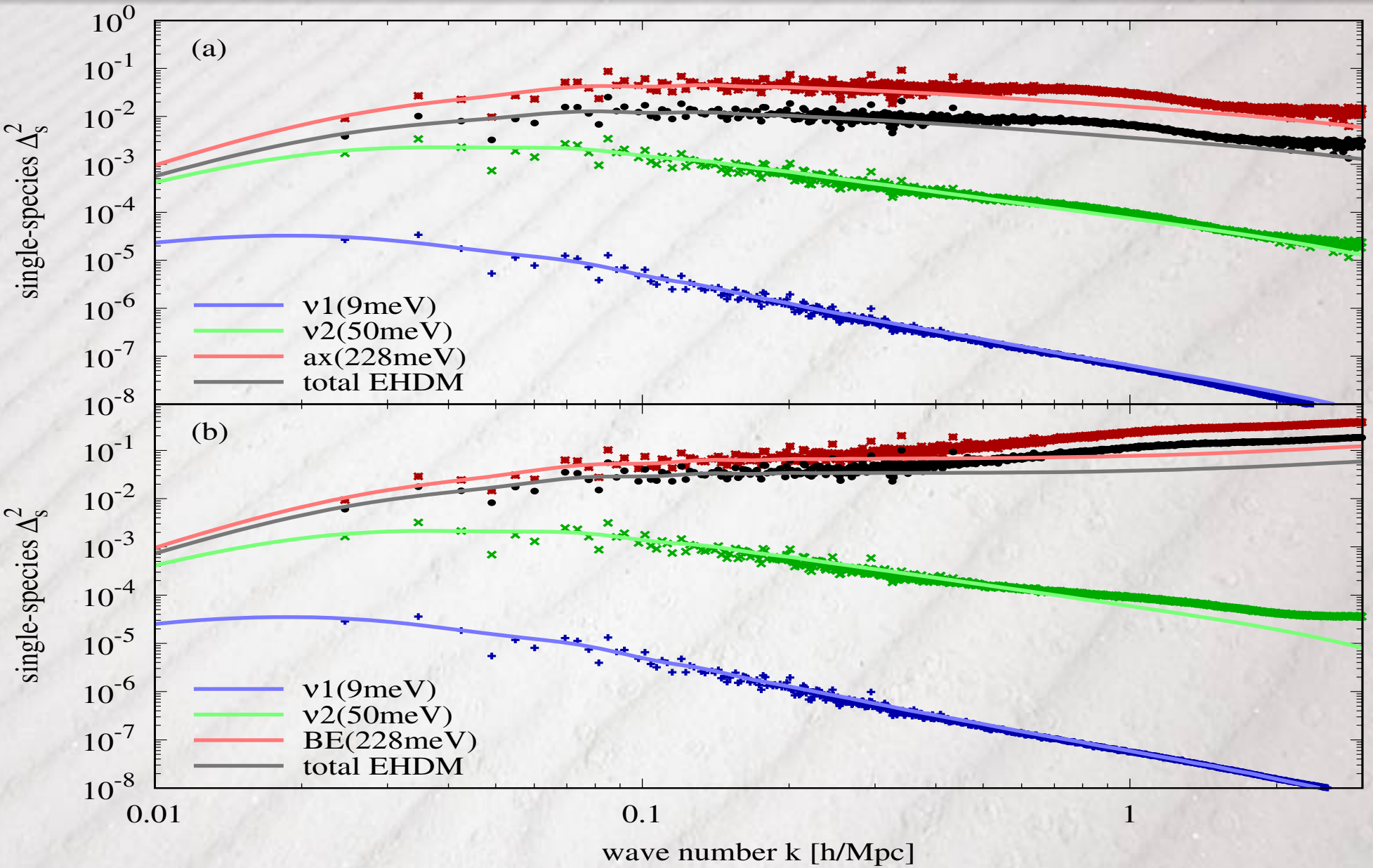
Simulations vs. perturbation theory

TianNu: *Inman, Yu, Zhu, Emberson, Pen, PRD* **95**:*083518(2017)[1610.09354]*

FlowsForTheMassesII: *AU, Mosbech, Pierobon, Wong, JCAP* **01**:*077(2025)[2410.05815]*

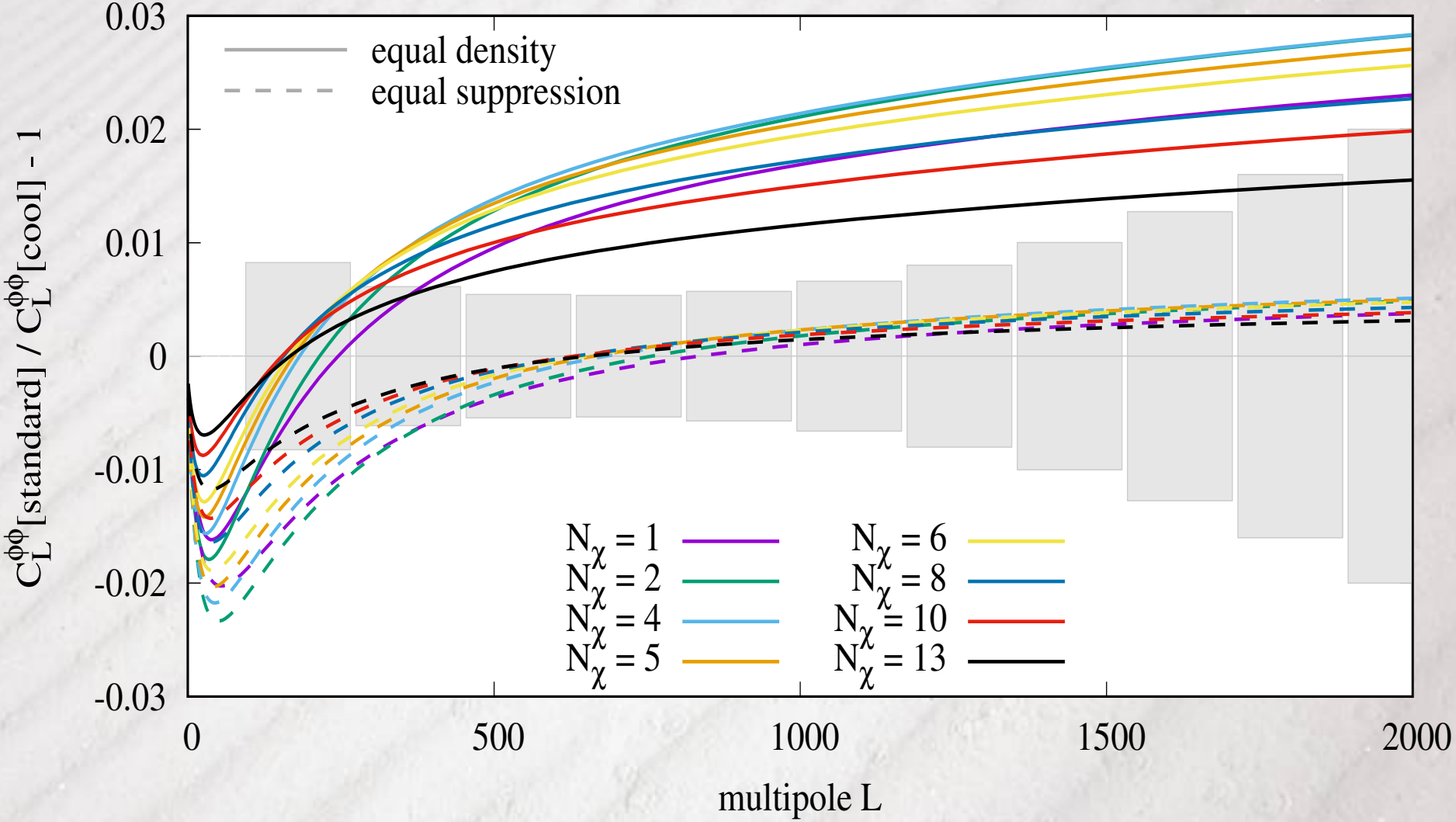


Neutrino + axion models



FFTM-II: AU, Mosbech, Pierobon, Wong [2410.05815]; axion $f(p)$: Notari++ [2211.03799]

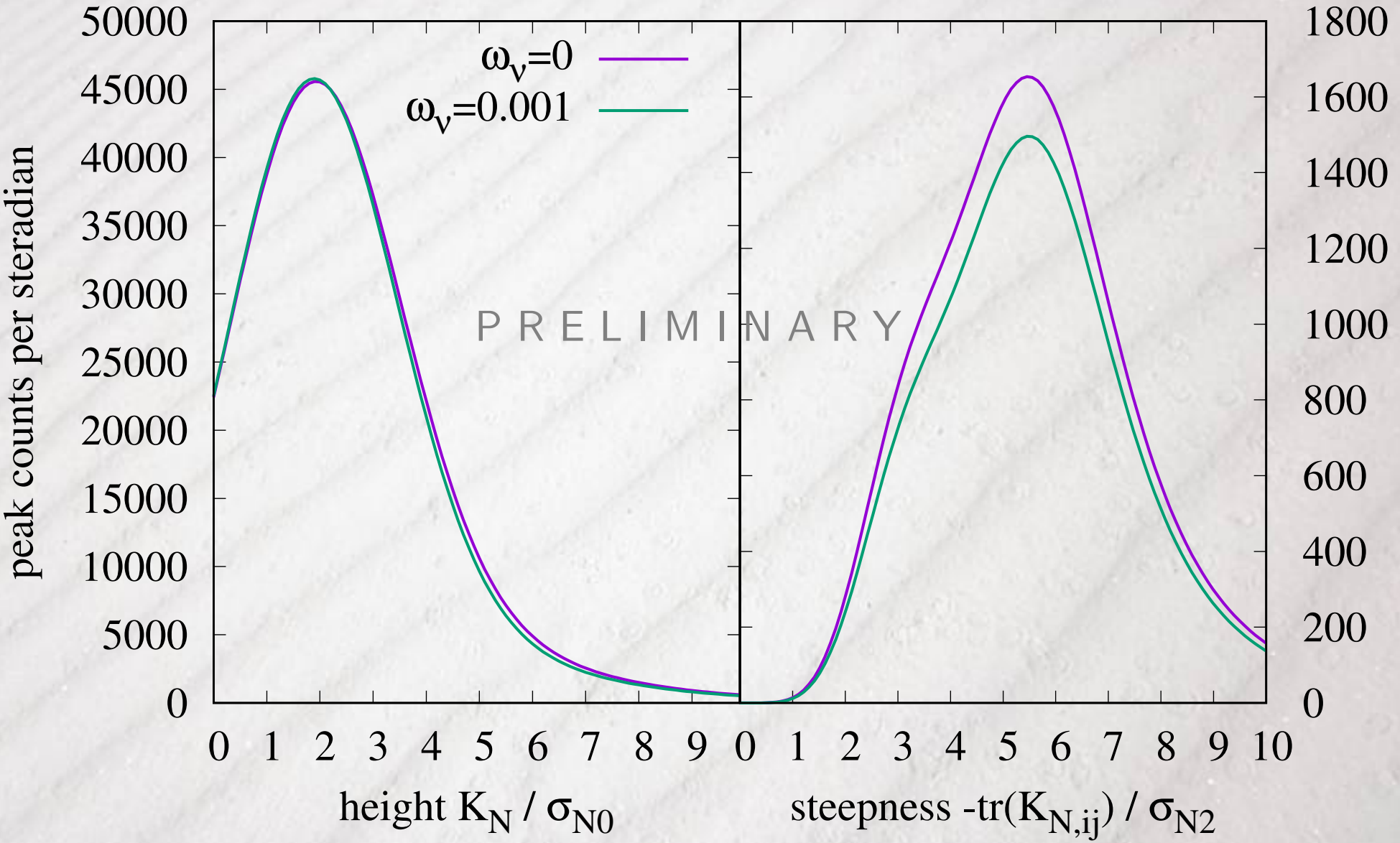
Cool ν and weak lensing of the CMB



FFTM-II: AU, Mosbech, Pierobon, Wong 2410.05815; CMB-S4: Abazajian++ 1610.02743; cool ν: Escudero++ JHEP **02**:142(2023); Farzan and Hannestad JCAP **02**:058(2016); CMB lensing: *AU* et al., *MNRAS* **529***:1862(2024)[2308.09755]*

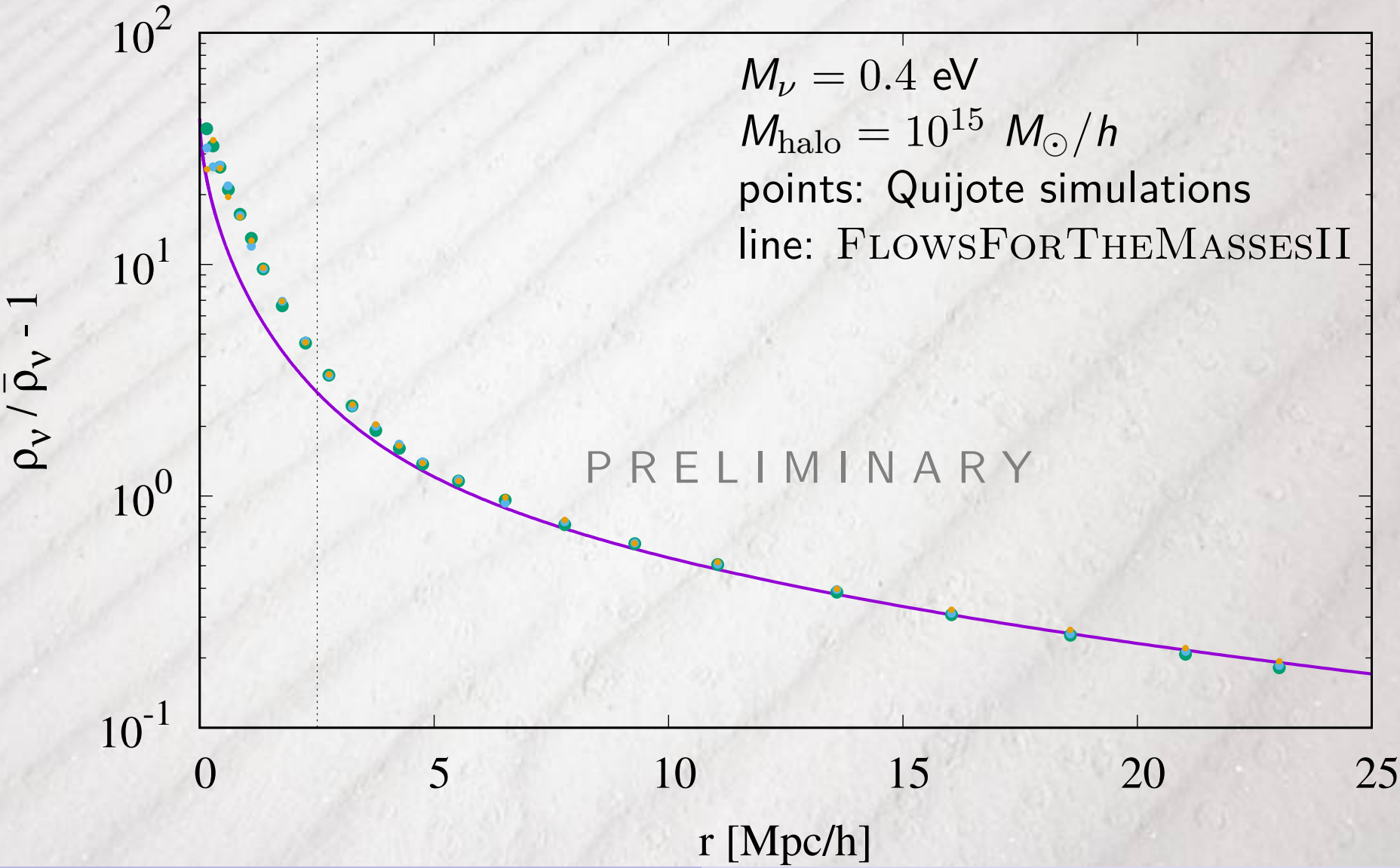
In progress: Weak lensing peak statistics and halo model

Halo model of *Li, Liu, Fan, MNRAS* **520**:*6382(2023)[2302.08255]*

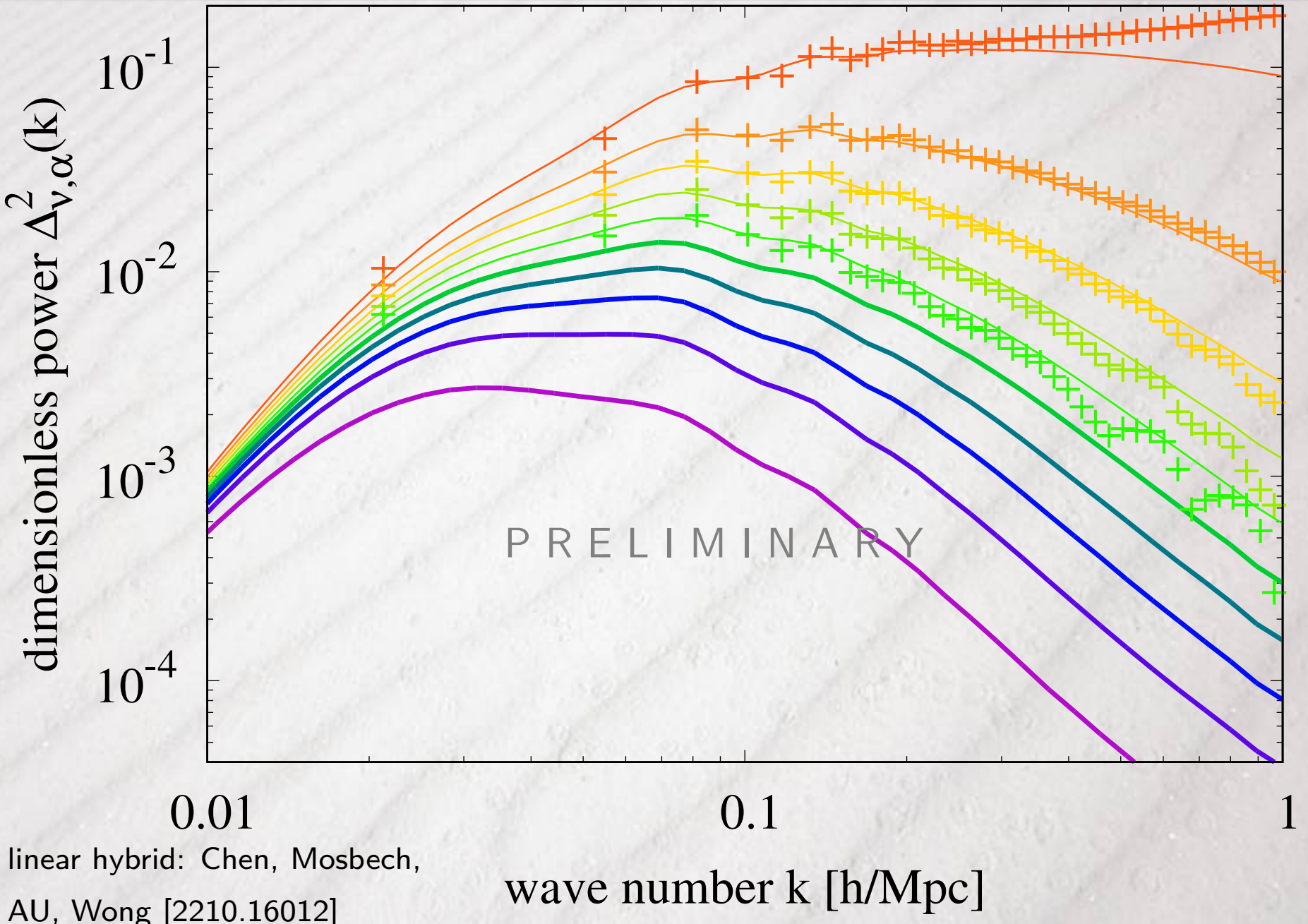


In progress: Neutrino clustering around halos

Quijote simulations: *Villaescusa-Navarro, et al., Ap.J.S.* **250**:*2(2020)[1909.05273]*



In progress: N-body NL response/hybrid simulations



linear hybrid: Chen, Mosbech, AU, Wong [2210.16012]

Conclusions

1. Early-universe cosmology can measure $N_{\rm eff}$ and M_ν.
   - parameter degeneracies $\Rightarrow$ joint constraints
   - other hot dark matter contributes to $N_{\rm eff}$
   - M_ν constraints from lensing, Φ evolution
2. Geometric constraints on M_ν are the strongest today
   - tension between DESI+CMB and oscillation experiments
   - cosmological tensions: $\Omega_{\rm m0}$, H_0, σ_8, τ
3. ν free-streaming and scale-dependent clustering suppression
   - velocity dispersion $\Rightarrow$ less small-scale clustering
   - DESI free-streaming upper bounds on M_ν: 0.3 eV – 0.4 eV
   - simulations and perturbation theory estimate suppression
   - new observables beyond the power spectrum