Hints from neutrinos for grand unified theories

Review of “Neutrino in GUT”-> GUT with a flavor symmetry

Nobuhiro Maekawa (KMI, Nagoya Univ.)

I. Introduction (Appeals of GUT)
II. Neutrinos suggest GUT+a flavor symmetry
III. Problems of SUSY GUT
IV. Natural GUT (GUT with a flavor symmetry)
V. Tension between neutrino mass and gauge coupling unification
VI. Several trials
VII. Summary

2025.7.25@Neutrino25 in Vietnam

Assumptions on neutrinos in this talk

Right-handed neutrinos + seasaw mechanism

Low energy neutrinos are almost left-handed Majorana neutrinos

Natural explanation of tiny neutrino masses.

Normal ordering & hierarchical structure for neutrino masses

Easier to obtain them in GUT scenario with flavor symmetry

$\Delta m_{32}^2 = m_{\nu 3}^2 - m_{\nu_2}^2 \sim 2.5 \times 10^{-3}\ \mathrm{eV}^2 \Rightarrow m_{\nu_3} \sim 0.05\ \mathrm{eV}$

$\Delta m_{21}^2 = m_{\nu 2}^2 - m_{\nu_1}^2 \sim 0.75 \times 10^{-4}\ \mathrm{eV}^2 \Rightarrow m_{\nu_2} \sim 0.009\ \mathrm{eV}$

My apologies to fans of the other possibilities.

74'Georgi-Glashow

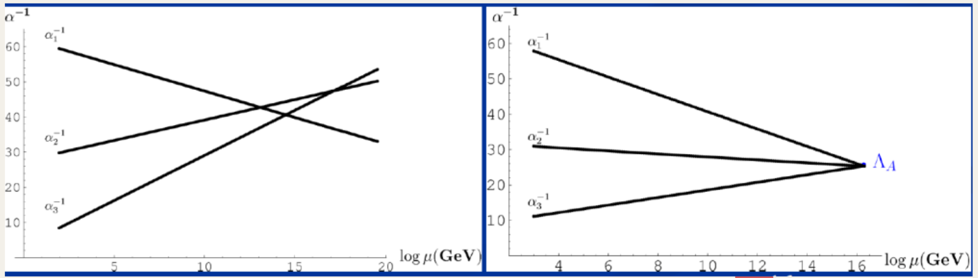
Introduction (Appeals of GUT)

Two unifications

1, Unification of forces, $SU(3)_C \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6$

2, Unification of matters, $16 = 10(\mathrm{Q} + \mathrm{U}_\mathrm{R}^\mathrm{c} + \mathrm{E}_\mathrm{R}^\mathrm{c}) + \overline{5}(D_R^c + L) + 1(N_R^c)$

SUSY



$\Rightarrow$ A, Charge quantization ($C_P = -C_e$?)

B, Anomaly cancellation in the SM.

C, Gauge coupling unification ($g_3 > g_2 > g_1$)

D, Various hierarchies in Yukawa couplings (masses & mixings for matters)

3, Experimental supports for both unifications.(C&D)

10 fields induce stronger hierarchy in Yukawa couplings than $\overline{5}$ fields

Experimental support for matter unification

Matter unification in $SU(5)$ $\quad 16 = 10(\mathrm{Q} + \mathrm{U}_\mathrm{R}^\mathrm{c} + \mathrm{E}_\mathrm{R}^\mathrm{c}) + \overline{5}(D_R^c + L) + 1(N_R^c)$

⇔ Masses and mixings of quarks and leptons

			Left	Right
up, charm, top (MeV)	2.2, 1270, 173000	Strongest	10	10
down, strange, bottom	4.7, 93, 4180	Middle	10	$\overline{5}$
e, μ, τ (MeV)	0.51, 106, 1780	Middle	$\overline{5}$	10
ν_e, ν_μ, ν_τ (eV) (NH)	<0.009, 0.009, 0.05	Weakest	$\overline{5}$	$1(\overline{5})$

10 fields induce stronger hierarchy in Yukawa couplings than $\overline{5}$ fields

⇒ Quark mixings $(\mathrm{Q} \subset 10)$ are smaller than lepton mixings $(L \subset \overline{5})$

(Because stronger hierarchy results in smaller mixings.)

A Hint from neutrinos

-Another explanation to stress neutrino contribution-

Quark and lepton masses and mixings indicate that

Unification of matters in SU(5) GUT

"A flavor symmetry"

Today's story does not respect the history so much.

($V_{e3} \sim \lambda$ is predicted in 2001, and was confirmed in 2012. But we use $V_{e3} \sim \lambda$ to build the GUT model today.)

79'Froggatt-Nielsen
94' Ibanez-Ross

A flavor symmetry for Yukawa hierarchies

-Froggatt-Nielsen mechanism-

U(1) flavor symmetry is broken by $\langle\Theta\rangle = \lambda\Lambda$, where the fravon Θ has U(1) charge $\theta = -1$. When Q_i, U^c_{Ri}, H_u have U(1) charges q_i, u^c_{Ri}, h_u, respectively, U(1) invariant interactions are written by

$$W_Y = c^u_{ij}\left(\frac{\Theta}{\Lambda}\right)^{q_i+u^c_{Rj}+h_u} Q_i U^c_{Ri} H_u \Rightarrow c^u_{ij}\lambda^{q_i+u_{Rj}c+h_u} Q_i U^c_{Rj} H_u, \qquad \Lambda\text{: cutoff}, \qquad c^u_{ij}\text{: O(1) coefficients}$$

When $\lambda < 1$, hierarchical structure of Yukawa couplings can be obtained. $\quad$ $i, j = 1,2,3$

$$(m_u, m_c, m_t) \sim (\lambda^{q_1+u_{R1}}, \lambda^{q_2+u_{R2}}, \lambda^{q_3+u_{r3}})\langle H_u\rangle$$

Any mass spectrum can be realized by choosing the U(1) charges in the standard model.

One non-trivial prediction in quark sector : $V_{13} \sim V_{12}V_{23}$ because $V_{ij} \sim \lambda^{|q_i-q_j|}$for V_{CKM}.

If we take $\lambda \sim sin\theta_C \sim 0.22$, $(q_1.q_2.q_3) = (3,2,0)$, we obtain

$$Y^u \sim \begin{pmatrix} \lambda^{q_1+u_{R1}} & \lambda^{q_1+u_{R2}} & \lambda^{q_1+u_{R3}} \\ \lambda^{q_2+u_{R1}} & \lambda^{q_2+u_{R2}} & \lambda^{q_2+u_{R3}} \\ \lambda^{q_3+u_{R1}} & \lambda^{q_3+u_{R2}} & \lambda^{q_3+u_{R3}} \end{pmatrix}\lambda^{h_u}, \qquad V_{CKM12} \sim \frac{Y^u_{12}}{Y^u_{22}} \sim \lambda^{q_1-q_2} \Rightarrow V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

A flavor symmetry for Yukawa hierarchies

-Froggatt-Nielsen mechanism-

More non-trivial predictions in lepton sector than $V_{13} \sim V_{12}V_{23}$ for $V_{MNS_{ij}} = \lambda^{|l_i - l_j|}$ because neutrino masses are determined by l_i as $m_{\nu_{ij}} \sim \lambda^{l_i + l_j + 2h_u} \langle H_u \rangle^2 / \Lambda$. For example,

$$\lambda^{(l_2 - l_3)} \sim V_{MNS23} \sim \sqrt[4]{\frac{\Delta m_{21}^2}{\Delta m_{32}^2}} \sim 0.42$$

This non-trivial relation is consistent with the observed data within the O(1) coefficients.

LMA is the best solution among four remaining solutions for solar neutrino problem (LMA, SMA, LMO, VAC) in 2001.

Note that $q_i - q_j$ and $l_i - l_j$ are basically determined by CKM and MNS matrices.

For example, the small mixings in CKM matrix lead to larger differences $(q_1, q_2, q_3) = (3,2,0)$,

while the large mixings in MNS matrix lead to smaller differences $(l_1, l_2, l_3) = (3,2.5,2)[(3,2,2)]$.

Note that large difference of charges induces stronger hierarchy in the Yukawa couplings.

The other U(1) charges are fixed to obtain observed quark and lepton masses in the SM.

A flavor symmetry for Yukawa hierarchies

01' N.M.

In SU(5) unification, U(1) charges for $10(Q + U_R^c + E_R^c)$, $\bar{5}(D_R^c + L)$ are determined by CKM and MNS matrices. For example, if we take (3,2,0) for 10_i and (3,2.5,2) for $\bar{5}_i$,

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d = Y_e^T \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}, \quad M_\nu = Y_{\nu_D} M_{\nu_R}^{-1} Y_{\nu_D}^T \langle H_u \rangle^2 \propto \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \qquad V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}, \qquad \lambda^{0.5} \sim 0.5$$

which can explain observed quark&lepton masses and mixings except m_u .

Interestingly, these explain basic hierarchical structures for quark and lepton masses:

- Up type quarks have the strongest hierarchy. $U_L, U_R^c \in 10$
- Down type quarks & charged leptons have middle hierarchy. $D_L(E_R^c) \in 10$, $E_L(D_R^c) \in \bar{5}$
- Neutrinos have the weakest hierarchy. $N_L \in \bar{5}$

This is an evidence for matter unification in SU(5) GUT & a flavor symmetry.

Brief summary up to now

Quark and lepton masses and mixings indicate SU(5) GUT and a flavor symmetry.

Flavor U(1) charges for left-handed quarks and letpons are consistently determined by CKM and MNS matrices in the SM.

In SU(5) GUT, the U(1) charges for all quarks and leptons can be fixed by the CKM & MNS matrices because of matter unification: $\mathrm{10(Q + U_R^c + E_R^c)}$, $\bar{5}(D_R^c + L)$

It is non-trivial that the predicted hierarchies of Yukawa matrices are almost consistent with observed masses in SU(5) GUT with the flavor symmetry.

Another hint from neutrino observation

The scale of the Right-handed neutrino mass is near the GUT scale.

$m_{\nu_3} \sim y_{\nu_D}^2 \langle H_u \rangle^2 / M_{\nu_R} \sim 0.05$ eV & $y_{\nu_D} \sim 1 \Rightarrow \quad M_{\nu_R} \sim 10^{14-15}$ GeV $\Leftrightarrow \Lambda_{GUT} \sim 10^{16}$ GeV

This suggests certain GUTs whose GUT symmetry forbids the right-handed neutrino masses, which are obtained after breaking the GUT symmetry.

SO(10) GUT has such a feature because

$$16 = 10(\mathrm{Q} + \mathrm{U}_\mathrm{R}^\mathrm{c} + \mathrm{E}_\mathrm{R}^\mathrm{c}) + \overline{5}(D_R^c + L) + 1(N_R^c)$$

But unrealistic GUT relations for Yukawa matrices are predicted in simple SO(10) model as

$$Y_d = Y_e^T (= Y_u = Y_{\nu_D})$$

Problems in SUSY GUT

1, The doublet-triplet(DT) splitting problem (Finetuning problem 10^{-14})

Higgs $5_H = \begin{pmatrix} 3_{H_T} \\ 2_{H_{SM}} \end{pmatrix}$ $m_{H_T} > 10^{16}$ GeV for long lifetime of proton
$m_{H_{SM}} \sim 100$ GeV to obtain the weak scale

This is possible if the GUT breaking is picked up, but finetuning is needed generically.

2, Unrealistic GUT relation for Yukawa matrices for SU(5) or SO(10)

$$16 = 10(\mathrm{Q} + \mathrm{U}_\mathrm{R}^\mathrm{c} + \mathrm{E}_\mathrm{R}^\mathrm{c}) + \overline{5}(D_R^c + L) + 1(N_R^c)$$

$$Y_d = Y_e^T (= Y_u = Y_{\nu_D})$$

RGE effects ($y_b \sim y_\tau, 3y_s \sim y_\mu, y_d \sim 3y_e$ at GUT scale.)

SU(5) GUT relation for 3rd generation is good and the others are not so bad if $Y_d \sim Y_e^T$

79'Froggatt-Nielsen
94' Ibanez-Ross

An assumption with the flavor symmetry

U(1) gauge symmetry is broken by $\langle\Theta\rangle = \lambda\Lambda$, where the fravon Θ has U(1) charge $\theta = -1$. When Q_i, U^c_{Ri}, H have U(1) charges q_i, u^c_{Ri}, h, respectively, U(1) invariant interactions are written by

$$W_Y = c^u_{ij}\left(\frac{\Theta}{\Lambda}\right)^{q_i+u^c_{Rj}+h} Q_i U^c_{Ri} H \Rightarrow c^u_{ij}\lambda^{q_i+u_{Rj}c+h} Q_i U^c_{Rj} H,$$

Λ: cutoff , c^u_{ij}: O(1) coefficients

When $\lambda < 1$, hierarchical structure of Yukawa couplings can be obtained. $i, j = 1,2,3$

If we take $\lambda \sim sin\theta_C \sim 0.22$, $(q_1.q_2.q_3) = (u^c_{R1}.u^c_{R2}.u^c_{R3}) = (3,2,0)$ and $h = 0$,

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \qquad U_L \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Assumption: All interactions allowed by symmetry are introduced with O(1) coefficients.

SUSY zero mechanism: If $q_i + u^c_{Rj} + h < 0$, the Yukawa int. is forbidden.

Natural GUT in a nutshell

Principle: All interactions allowed by symmetry are introduced with O(1) coefficients.

This principle is applied to the Higgs sector in which GUT group is spontaneously broken.

Reckless?

Infinite number of terms can be controlled by the SUSY zero mechanism.

All int. are fixed by the symmetry except O(1) coefficients. $\langle Z \rangle \sim \begin{cases} 0 & (z > 0) \\ \lambda^{-z} & (z \leq 0) \end{cases}$ with $\Lambda = 1$ unit

So, VEVs and mass spectrum of superheavy fields are fixed except O(1) coefficients.

Under this natural assumption, the various problems of SUSY GUT can be solved.

Realistic quark and lepton masses and mixings are obtained.

New explanation for the gauge coupling unification is given.

SO(10) natural GUT ($\boldsymbol{SO(10) \times U(1)_A \times Z_2}$)

$SO(10)$	negative charge	positive charge	matter fields
45	$A(a=-1,-)$	$A'(a'=3,-)$	
16	$C(c=-4,+)$	$C'(c'=4,-)$	$\Psi_i\left((\psi_1,\psi_2,\psi_3)=(\frac{9}{2},\frac{7}{2},\frac{3}{2}),+\right)$
$\overline{16}$	$\bar{C}(\bar{c}=-1,+)$	$\bar{C}'(\bar{c}'=6,+)$	
10	$H(h=-3,+)$	$H'(h'=4,+)$	$T\left(t=\frac{5}{2},+\right)$
1	$\Theta(\theta=-1,+)$	$Z'(z'=5,+)$	
	$Z_i(z_1=z_2=-2,-)$		

$\langle A\rangle \sim \lambda$ $\qquad\qquad$ $\langle C\rangle = \langle\bar{C}\rangle \sim \lambda^{2.5}$

$$SO(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to G_{SM}$$

01'N.M.

How to obtain the assumption for Yukawa hierarchies?

10 fields induce stronger hierarchy in Yukawa couplings than $\bar{5}$ fields ($SO(10) \supset SU(5)$)

matters $\quad 16_{\Psi_i} = 10_{\Psi_i} + \bar{5}_{\Psi_i} + 1_{\Psi_i}, \quad 10_T = 5_T + \bar{5}_T$

Higgs $\quad \langle 16_C \rangle = 10_C + \bar{5}_C + \langle 1_C \rangle \quad 10_H = 5_H + \bar{5}_H$

$$W = \lambda^{\psi_i+\psi_j+h} 16_{\Psi_i} 16_{\Psi_j} 10_H + \lambda^{\psi_i+t+c} 16_{\Psi_i} 10_T \langle 16_C \rangle + \lambda^{2t} 10_T 10_T$$

$$\Rightarrow \lambda^{\psi_3+t+c} \langle C \rangle \bar{5}_{\Psi 3} 5_T + \lambda^{2t} \bar{5}_T 5_T$$

If $\lambda^{\psi_3+t+c}\langle C \rangle > \lambda^{2t}$, $\bar{5}_{\Psi 3}$ becomes superheavy and 3 massless $\bar{5}$ become $(\bar{5}_{\Psi 1}, \bar{5}_T, \bar{5}_{\Psi 2})$.

$$Y_u \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad Y_d \sim Y_e^T \sim Y_{\nu_D}^T \sim \begin{pmatrix} \lambda^6 & \lambda^{5.5} & \lambda^5 \\ \lambda^5 & \lambda^{4.5} & \lambda^4 \\ \lambda^3 & \lambda^{2.5} & \lambda^2 \end{pmatrix}, \quad M_\nu \propto \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}, \quad V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

$(10_{\Psi 1}, 10_{\Psi 2}, 10_{\Psi 3}) \Leftrightarrow (\bar{5}_{\Psi 1}, \bar{5}_T, \bar{5}_{\Psi 2}) \Rightarrow \sin\theta_c \sim V_{e3} \sim \lambda$

(Right-handed) neutrino masses

$$16_{\Psi_i} = 10_{\Psi_i} + \overline{5}_{\Psi_i} + 1_{\Psi_i}, \qquad \overline{16}_{\bar{C}} = \overline{10}_{\bar{C}} + 5_{\bar{C}} + \langle 1_{\bar{C}} \rangle$$

$$W = \lambda^{\psi_i+\psi_j+2\bar{c}} 16_{\Psi_i} 16_{\Psi_j} \langle \overline{16}_{\bar{C}} \rangle \langle \overline{16}_{\bar{C}} \rangle$$

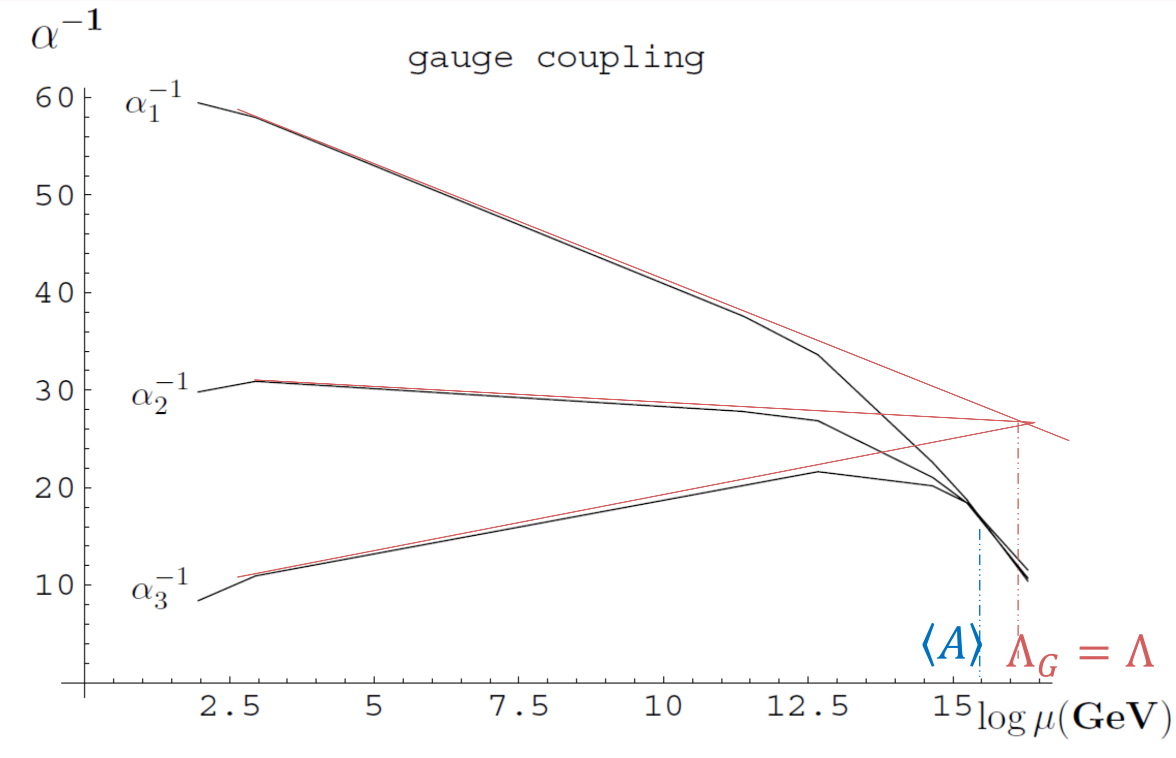
$$\Rightarrow \left(M_{\nu_R}\right)_{ij} \sim \lambda^{\psi_i+\psi_j+2\bar{c}} \langle C \rangle^2$$

$$\Rightarrow M_\nu = Y_{\nu_D} M_{\nu_R}^{-1} Y_{\nu_D}^T \langle H \rangle^2 \sim \lambda^{4+h+c-\bar{c}} \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \langle H \rangle^2$$

To obtain sufficiently large neutrino masses, $h \leq -3$ is required for $m_{\nu 3} \sim 0.05$ eV.

01'N.M.
02'N.M.-T.Yamashita

Gauge coupling unification



All VEVs and mass spectrum of superheavy particles are fixed by their $U(1)_A$ charges.

⇒ The running gauge couplings can be calculated.

$$\alpha_1(\langle A \rangle) = \alpha_2(\langle A \rangle) = \alpha_3(\langle A \rangle)$$

$$\Rightarrow \quad \Lambda = \Lambda_G \sim 2 \times 10^{16}\mathrm{GeV}, \quad h = 0$$

All $U(1)_A$ charges except h is cancelled in the conditions.

The real GUT scale $\langle A \rangle \sim \lambda^{-a}\Lambda$ is smaller than the usual GUT scale Λ_G.

⇒ Proton lifetime via superheavy gauge boson exchange becomes shorter than the usual GUT.

New explanation for the success of the gauge coupling unification in MSSM!

Predictions on proton decay in natural GUT

Proton decay via dim. 6 ($P \to e\pi$ mediated by gauge fields) is enhanced by smaller unification scale.

$$\Lambda_u \sim \lambda^{-a}\Lambda \sim \lambda\Lambda_G \sim 5 \times 10^{15}\mathrm{GeV}. \quad (a = -1, \Lambda \sim \Lambda_G \sim 2 \times 10^{16}\mathrm{GeV})$$

It can be found in near future experiment

Proton decay via dim. 5 ($P \to K\nu$ mediated by triplet Higgs) is suppressed.

$$m_{H_T}^{eff} \sim \lambda^{2h}\Lambda \sim 2 \times 10^{20}\mathrm{GeV}. \ (h = -3.)$$

If $h = 0$, it becomes also important.

In original SUSY SU(5) GUT, the proton decay via dim. 5 is important.

Tension between neutrino masses ($h = -3$) & gauge coupling unification condition $h = 0$

Neutrino masses require $h \leq -3$.

The DT splitting needs $h < 0$ to forbid H^2 term by SUSY zero mechanism.

Gauge coupling unification can be recovered by choosing a lot of O(1) coefficients from 0.5 to 2 instead of 1 even if $h = -3$. (It is like 100 times 50% tuning.)

The model with $h = -3$ is not killed in that sense but unnatural. (The most unsatisfied issue.)

Unknown structure may be hiding behind this problem.

The unknown structure may lead to smaller coefficients.

Which terms have smaller coefficients to solve this tension?

Are the important predictions on proton decay changed? ($\Lambda_u \sim \lambda^{-a}\Lambda, m_{H_T}^{eff} \sim \lambda^{2h}\Lambda$)

N.M.-T. Tanii 2312.14214.

Several trials

Model 1, The easiest way to solve this problem is to enhance the neutrino masses by smaller right-handed neutrino masses as

$$W = \epsilon\lambda^{\psi_i+\psi_j+2\bar{c}} 16_{\Psi_i} 16_{\Psi_j} \langle\overline{16}_{\bar{C}}\rangle\langle\overline{16}_{\bar{C}}\rangle \qquad \epsilon \ll 1$$

$$\Rightarrow \left(M_{\nu_R}\right)_{ij} \sim \epsilon\lambda^{\psi_i+\psi_j+2\bar{c}}\langle C\rangle^2$$

The natural GUT with $h = 0$ is not good because the mass term H^2 is not forbidden by the SUSY zero mechanism.

The model with $h = -1$ is more reasonable, that requires $\epsilon \sim 0.001$.

Unfortunately, we have not found any approximate symmetry to realize this suppression.

Several trials(many possibilities)

N.M.-T. Tanii 2312.14214

2, Changing RGEs. We assume that the suppression factors depends only on the positively charged fields so that the VEVs of fields $\langle Z \rangle \sim \lambda^{-z}$ do not change.

$$W = \epsilon_{A'}\left(\lambda^{a'+a}A'A + \lambda^{a'+3a}A'A^3\right) + \epsilon_{2A'}\lambda^{2a'}A'^2 + \epsilon_{C'}\lambda^{c'+\bar{c}}C'(\lambda^a A + \lambda^{z_1}Z_1)\bar{C}$$
$$+\epsilon_{\bar{C}'}\lambda^{\bar{c}'+c}\bar{C}'(\lambda^a A + \lambda^{z_2}Z_2)C + \epsilon_{\bar{C}'C'}\bar{C}'C' + \epsilon_{H'}\lambda^{h'+a+h}H'AH + \epsilon_{2H'}H'^2$$

+Correct neutrino masses and gauge coupling unification $(\Lambda_u \sim \lambda\Lambda(a=-1))$

⇒ Model 2: $\epsilon_{H'} \sim 10^{-2}$ and the other $\epsilon \sim 1 \Rightarrow m_{H_T}^{\rm eff} \sim \Lambda \sim 2\times 10^{16}$GeV.

Model 3: $\epsilon_{A'} \sim 10^{-3}, \epsilon_{H'} \sim 2\times 10^{-4} \Rightarrow \Lambda \sim 2\times 10^{18}$GeV. $m_{H_T}^{\rm eff} \sim 2\times 10^{16}$GeV.

Model 4: $\epsilon_{H'} \sim 10^{-4},\ \epsilon_{2H'} < \epsilon_{H'} \Rightarrow \Lambda \sim 2\times 10^{16}$GeV. $m_{H_T}^{\rm eff} \sim \epsilon_{H'}/\epsilon_{2H'} > 2\times 10^{16}$GeV.

Predictions on proton decay can change by these suppression factors. Generically $m_{H_T}^{\rm eff} \sim \Lambda_G$.

Approximate Z_2 symmetry(odd fields A' and/or H') makes solutions 2 and 3 natural.

Spontaneously broken Z_2 does not work in natural GUT. (Breaking is maximal as $\lambda^z\langle Z \rangle \sim 1$)

⇒It may suggest hiding structure (for example, extra dimension.)

Summary (Neutrinos suggest GUT with flavor symmetry)

CKM matrix and MNS matrix essentially determine the U(1) flavor symmetry for matters in SU(5) GUT, which explain the various hierarchies of quark and lepton masses at the same time. (Non-trivial)

Natural assumption that all terms including higher dimensional terms are introduced with O(1) coefficients, which is an essential assumption of the flavor symmetry, can be applied to GUT Higgs sector, and the almost all problems of SUSY GUT can be solved under this natural assumption. P->e pi, which is enhanced in the natural GUT, becomes the main decay mode.

Tension between neutrino mass and gauge coupling unification in natural GUT may provide a mystery to neutrino physics. Although it may be avoided by introducing smaller coefficients, we may not understand something in neutrino. Solving this tension may change the prediction on proton decay mode.

Thermal leptogenesis reproduces the observed Baryon asymmetry. N.M.-Shibata-Yamanaka, in preparation

When a singlet is removed from a natural GUT, SUSY is spontaneously broken.

Split SUSY, D term dominance, and long-lived charged lepton are predicted.

Although SUSY flavor and CP problems can be avoided by split SUSY, the weak scale is destabilized.

It is a big challenge to realize the stability in natural GUT with sp. SUSY breaking.