

International Center for Interdisciplinary Science and Education



**HARDWARE CAMP
FOR FAST AND LOWLIGHT DETECTION**

GROUP B

**Muon Rate and Fluorescence
Decay-time with MPPC Array**

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1 Introduction

The purpose of this scientific report is to document the experiments conducted during the Hardware Camp for Fast and Low-light Detection. Fast light detection refers to the detection of light signals that occur very quickly, typically in the range of nanoseconds to microseconds, while low light detection refers to the ability to detect and measure very low intensity of light. The experiments conducted during the camp aimed to provide participants with hands-on experience in working with MPPC to detect and measure low levels of light.

Muons are elementary particles that are similar to electrons but are much heavier. They are produced in cosmic ray showers and can be used to study various physical phenomena, including atmospheric physics and high-energy particle physics. Detecting muons requires highly sensitive detectors capable of detecting individual particles, and MPPCs have emerged as a promising technology for this application. The use of MPPCs for muon detection has the potential to enable new insights into fundamental physics and to facilitate the development of new technologies for particle detection.

Fluorescence lifetime measurement is an important technique in various scientific fields, including chemistry, biology, and physics. In the context of particle detection, fluorescence lifetime measurement can provide valuable information about the properties of scintillation materials and the behavior of high-energy particles.

Overall the result of those experiments can be used to improve the design and performance of particle detectors for a wide range of applications, from fundamental particle physics research to medical imaging.

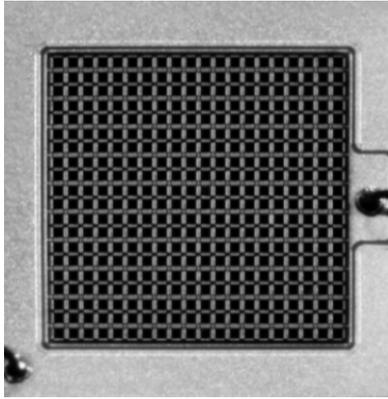
2 Materials

2.1 Multipixel Muon Counter (MPPC)

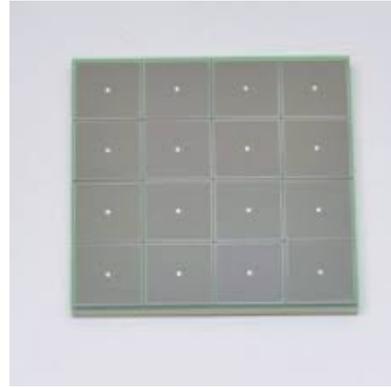
MPPC stands for Multipixel Muon Counter, also known as Silicon Multiplier (SiPM), which is a type of solid-state photodetector containing a matrix of small, independent avalanche photodiodes (pixels). MPPC are operated in Geiger mode where each pixels generate the signal of either "on" when it detects a photon or "off" when it does not. The typical application of MPPC is high-speed detecting of the low-intensity light signal.

An MPPC array is a system of photodetector consists of multiple MPPCs arranged in a grid pattern. Each MPPC in the array operates individually, so that they can detect independently the photons of light.

For our experiments, we used the MPPC array of model Hamamatsu S13361-3050AE-04[4]. Our MPPC array contains 16 pixels which connect to 16 channels, but we only use 4 pixels: 6, 7, 10 and 11.



(a) Single MPPC



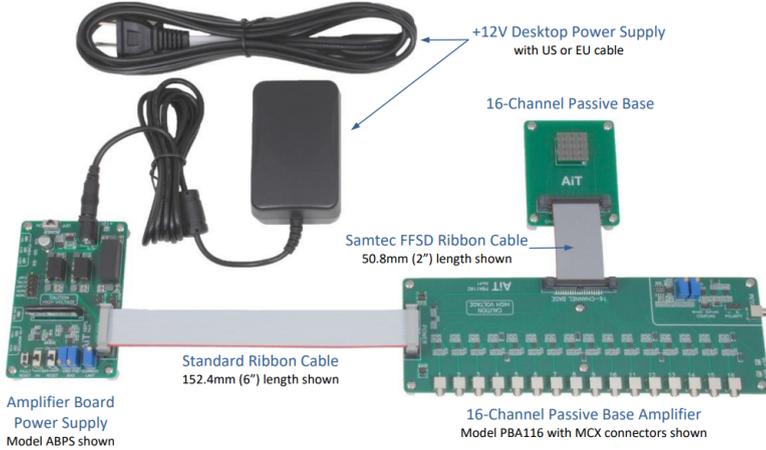
(b) Multi-channels MPPC

Parameter	Value
Number of channels	16 (4x4)
Number of pixels	3584
Operating temperature	-20 to +60($^{\circ}C$)
Spectral response range	320 to 900 (nm)
Peak sensitivity wavelength	450 (nm)
Photon detection efficiency	40%
Gain	1.7×10^6
Breakdown voltage	$53 \pm 5(V)$
Operating voltage	$V_{breakdown} + 3 (V)$

Table 1: S13361-3050AE-04 specifications

2.2 PBA116 Amplifier[1]

To support our MPPC array, we are provided with a specialized Printed Circuit Board that is designed to work in conjunction with it.



(a) 16-Channel 1-Stage Passive Base Amplifier



(b) Amplifier wrapped in tin foil

In addition, we have amplifier and power supply boards that are enclosed in a Faraday cage made of tin foil. The Faraday cage is designed to protect the sensitive electronic equipment from electromagnetic interference by canceling out the external electrical field within the cage’s interior. Essentially, the charges within the conducting material of the cage are redistributed in a way that negates the effect of the external field. This helps prevent the equipment from picking up unwanted signals and noise that could interfere with the accuracy of our measurements.

2.3 Plastic scintillators

We used 4 bars of plastic scintillators, each with the following specifications.

In our experimental set-up, we use scintillators that are coated with Titanium Dioxide on their facets. This coating serves the purpose of absorbing recoil protons. The scintillators are stacked on top of each other in a layered fashion.

To ensure that no external light enters the set-up, we take special precautions during the light-tightening process. We wrap the two ends of the set-up with tin foil, which serves as a barrier to photons. We then reinforce the wraps with black tape. This process prevents any external

Quantity	Value (cm)
Width	2.5
Thickness	1.3
Length	25
Area	17 x 5

Table 2: Plastic scintillators specifications

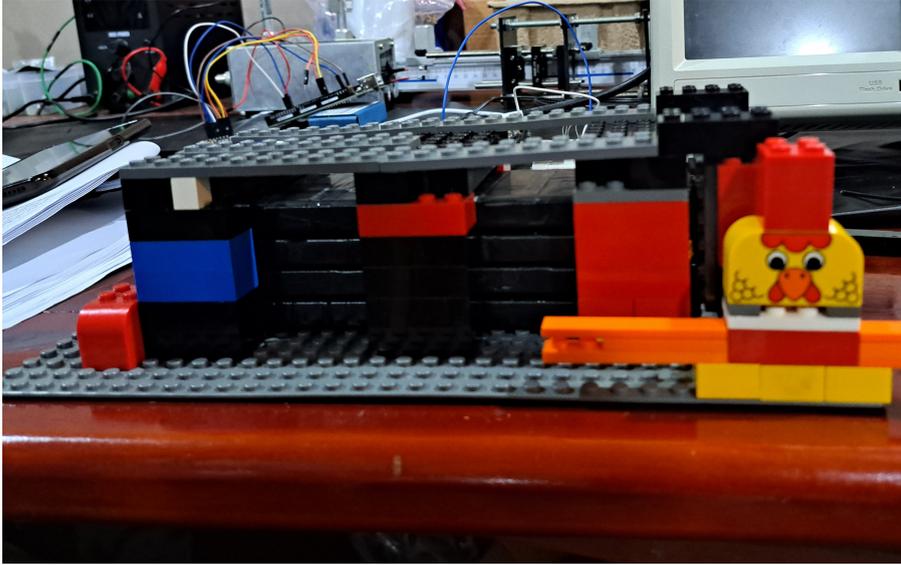


Figure 3: Preparation of scintillators

light from entering the set-up and interfering with our measurements. By taking these steps, we ensure that our measurements are reliable.

2.4 Wavelength-shifting fibers

In our experimental setup, we used wavelength-shifting fibers to collect light from the scintillation counters. To prepare the fibers, we first cut them into equal lengths, ensuring that each fiber was the same length so that the system is functional. Then, we carefully polished each fiber to optimize their light transmission efficiency.

After polishing, we light-tightened each fiber using black plastic covers to prevent any external light from entering the system. We then UV-glued the fibers to a customized plate in a specific arrangement, paying close attention to the order of the fibers and their connection to the MPPC array and plastic scintillators.

To ensure that the system worked correctly, we meticulously noted down the order of the connections and their corresponding channels on the MPPC array. This information was crucial for later channel mapping of the MPPC array and for interpreting the results of our experiments accurately.



Figure 4: Preparation of WSF

3 Theoretical Background

3.1 Muon

Cosmic rays are high-energy particles that originate from outside of our solar system and can include protons, electrons, and atomic nuclei. When cosmic rays collide with particles in the Earth's atmosphere, they create a cascade of secondary particles known as a cosmic ray shower.

One of the most abundant types of particles in a cosmic ray shower are muons, which are produced when cosmic ray particles collide with atomic nuclei in the atmosphere. Muons are able to penetrate through matter more easily than other charged particles because they are more massive and are not slowed down as much by interactions with atomic nuclei.

As the cosmic ray shower reaches the Earth's surface, most of the lower energy particles in the shower are absorbed or scattered by the atmosphere or the Earth's surface, leaving only the high-energy muons to reach the surface.

3.2 Fluorescence

Fluorescence is a phenomenon occurs where a substance absorbs light at a specific wavelength to go to a higher energy state (excited state) before reemit a photon at a longer wavelength.

Fluorescence lifetime is a characteristic property of a fluorescent molecule represent to the time it takes for a particle to come back to its ground state after being excited.

Consider a very short pulse (shorter than the lifetime of fluorophore) is applied to the sample. Consider the time $t = 0$, when the molecule reaches the excitation state and starts the emission, the fluorescence lifetime is defined as the time it takes for the intensity to decrease by an amount of $1/e$:

$$I(t) = I_0 \times e^{-\frac{t}{\tau}} \tag{1}$$

where I_0 is the peak intensity when the molecule is in the excitation state, $I(t)$ is the intensity measured at time t , τ is the fluorescence lifetime.

Fluorescence life-time can provide valuable information about the material's properties and their surroundings. The changes in the fluorescence life-time can be used to monitor the molecular interaction or the changes of the microenvironment. Overall, fluorescence lifetime has many essential applications in scientific research and medical diagnostics.

4 Data Acquisition and Scientific methods

4.1 Dark current and Muon counter set-up

For our experiments, we use 4 plastic scintillators with 4 wavelengths

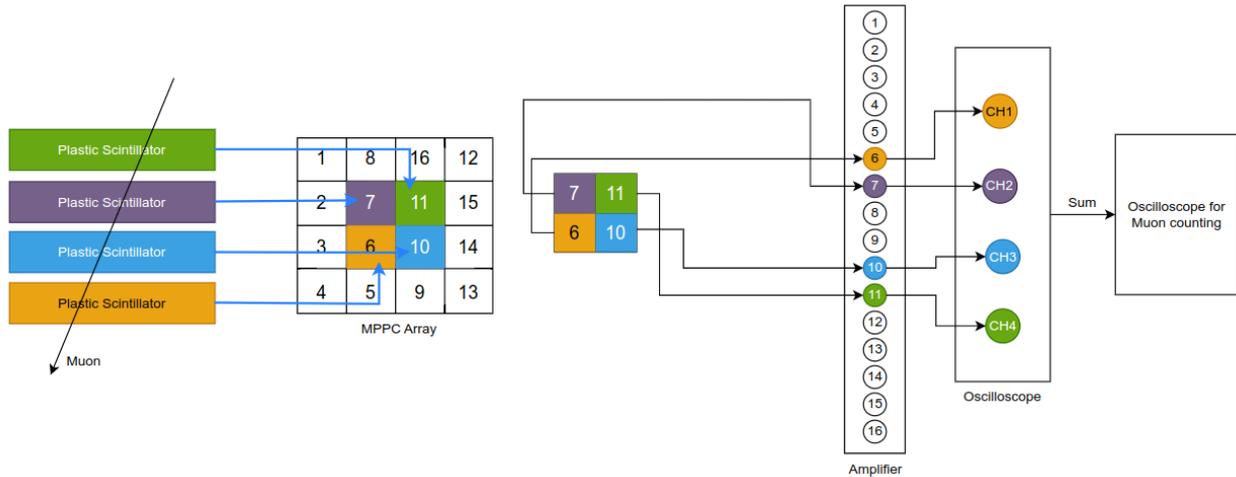


Figure 5: Muon counter set-up

In our experimental setup, we used four scintillators that were connected to an array of multi-pixel photon counters (MPPCs) through polished wavelength-shifting fibers. These fibers were carefully selected to ensure a high level of light transmission efficiency, and they were polished

to minimize light reflection at the interface between the fiber and the scintillator. The fibers were then glued to a customized plate that was specifically designed to hold the scintillators and the MPPC array in place.

To ensure accurate and reliable data collection, we paid careful attention to the arrangement and order of connection between the scintillators, MPPC array, and channels. We meticulously noted down the positions and orientations of each component, as well as the connection sequence between them. This attention to detail allowed us to optimize the sensitivity and response of the scintillation detector, which is crucial for detecting and measuring low levels of radiation.

The diagram provided above illustrates the precise arrangement and connection of the scintillators to the MPPC array. The use of a LEGO frame for the stacking of scintillators was a solution to provide a sturdy and customizable platform for the detector. Additionally, the precise connection of the MPPC array and channels further optimized the performance of the detector.

Overall, the combination of polished wavelength-shifting fibers, a customized plate, a LEGO frame, and precise connections between components resulted in an efficient and effective scintillation detector.

4.2 Width of the window measurements set-up

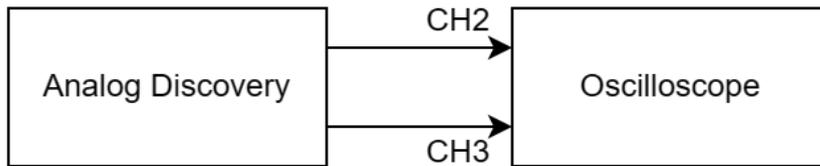


Figure 6: Width of window

To investigate the pattern of two signals generated by the Analog Discovery 2 and connected to the oscilloscope, we initially set the pattern measurement to accommodate two signals. To observe the differences in the initial phases of the signals, we gradually increased the phase angle by 5 degrees until we no longer observed any coincidence between the signals. This was done to identify a point where there is no overlap between the pulses of the two signals. We then used cursors on the oscilloscope to approximately measure the time difference between the two signals and recorded the corresponding time when no coincidence was observed. This

approach allowed us to obtain reliable measurements of the time difference between the signals and helped in understanding the pattern of the two signals generated by the Analog Discovery 2.

4.3 Fluorescence decay time set-up

The fluorescence set-up contains three main parts: the waveform generators, LED shining on fluorescence slides coupling with MPPCs and oscilloscope for read-out signals.

In order to generate the square short-duration pulse for LED, we use The Analog Discovery 2 oscilloscope which is designed to be a portable alternative to a stack of benchtop equipment. It's durable enclosure measures (3.23 inch x 3.25 inch x 7/8 inch) and fits in a pocket [3]. We connected the Analog Discovery 2 to the BNC Adapter for higher bandwidth and BNC connectors.



Figure 7: Analog Discovery 2 with BNC Adapter

We use an ultra violet LED with the wavelength of 385nm, shorter wavelength than 5 panels because higher energy photons can lead to more efficient excitation of the fluorophore. The fluorescent samples we use are Thorlabs's FSK5 fluorescent slides available in five colors: Blue, Green, Yellow, Orange, and Red with dimensions of 25.4 mm x 76.2 mm and 1.7 mm Thick.

We coupled the LED shining on each fluorescent slide to the MPPC Array, then connect to the oscilloscope to observe the average waveform. The schematic set-up are shown in the figure 9.



Figure 8: FSK5 Set of five colored fluorescence slide.

The real set-up of the experiment are shown in the figure 10.

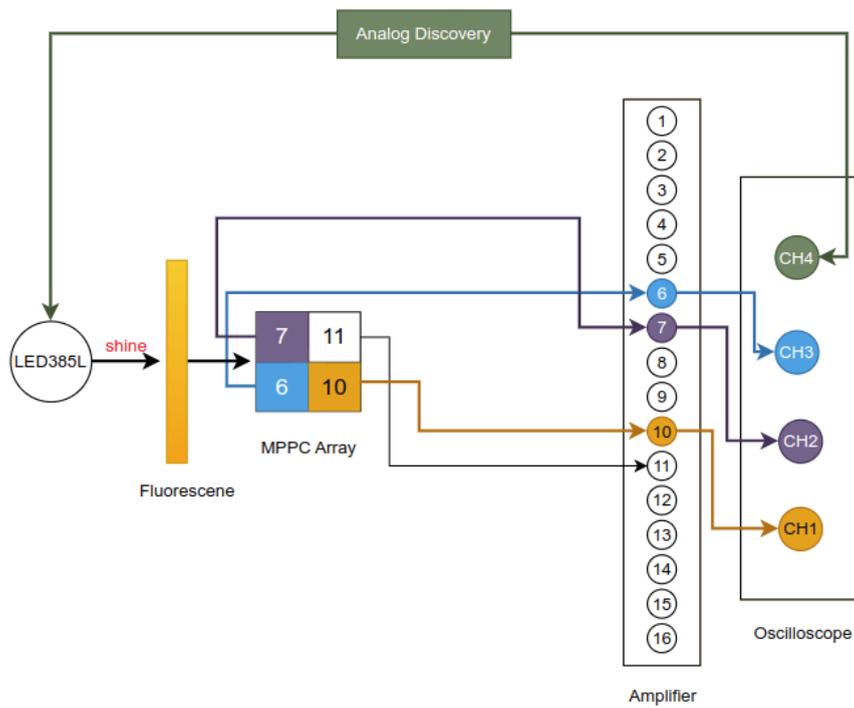
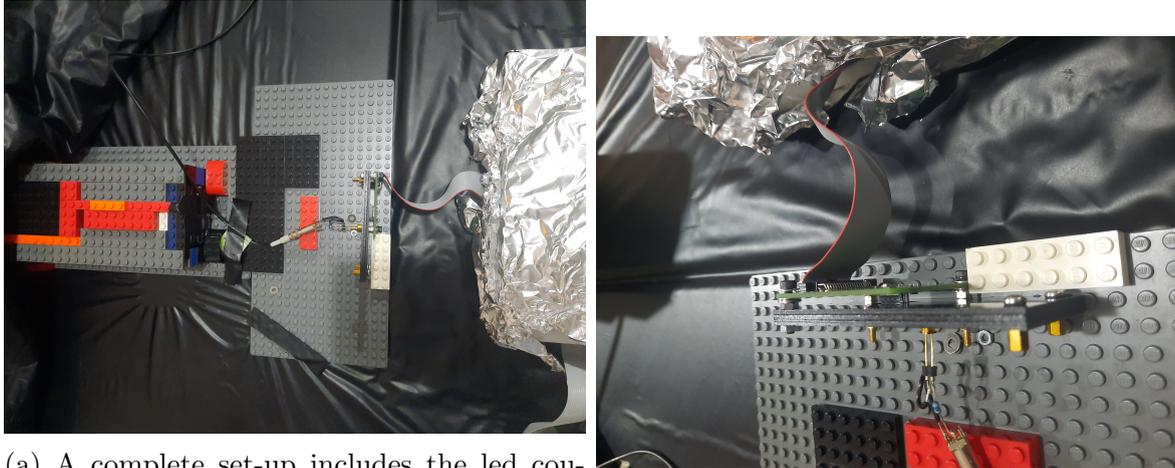


Figure 9: Fluorescence experiment schematics

We only use 3 channels 1, 2 and 3 for three corresponding pixels of MPPC Array, the fourth

channel is reserved for displaying the signal of Analog Discovery 2. The data will be collected remotely from the computer by *Root*.



(a) A complete set-up includes the led coupling with the MPPC, and the MPPC is connected to the amplifier circuit (b) A zoom-in to the LED coupling with the MPPC array

Figure 10: Set up of fluorescence experiment

5 Results and Discussions

5.1 Dark count

To better detect the muon’s signal, the values which are proportional to a value of one a pe, (1pe, 2pe, 3pe, etc.) are measured corresponding to each channel when the system has no light shine on it. By doing this, we can identify some threshold values to remove background noise from the dark current.

Channel	1 pe	2 pe	3 pe	4 pe	5 pe
1	5,2	10,4	15,8	21,3	26,6
2	4,2	9,9	14,8	20	25,4
3	4,2	8,7	12,3	16,5	20,5
4	5,6	11	16,6	21,5	28

Table 3: Dark current of individual channels

The default ”window” time interval is measured to verify the given equation to calculate the coincidence frequency.

degree different	0	0,5	1	1,5	2	2,5	3	3,5	4
time(ns)	0	30	45	54	67	76	85	109	DNE

Table 4: Measurement to determine the default coincidence's "window" width

The equation to calculate the coincidence frequency of two sources:

$$f_{coin} = 2 \times f_1 \times f_2 \times W$$

Parameters:

- f_{coin} : coincidence frequency of source 1 and source 2
- f_1, f_2 : frequency of source 1 and source 2
- W : time value of the interval "window"

Pe	$f_1(kHz)$	$std_1(kHz)$	$f_2(kHz)$	$std_2(kHz)$	$f_{coin}(kHz)$	$std_{coin}(Hz)$	$f_{theo}(kHz)$
1	145,98	14,36	571,41	4,97	2,38	147,84	17,93
2	80,76	2,81	77,27	2,68	1,81	150,08	1,34
3	13,16	0,36	11,35	0,37	0,15	801,89	0,03
4	2,16	0,18	2,022	0,25	0,000022	-	0,00094

Table 5: Coincidence of dark current of channel 1 and 2

For 2 channels' coincidence:

- the frequency of coincident of 4 pe. of 3 channels is approximately 0.
- For 2 pe, the frequency of coincident measured by the oscilloscope is approximately equal to the calculated one by the equation.

The equation to calculate the coincidence frequency of three sources:

$$f_{coin} = 3 \times f_1 \times f_2 \times f_3 \times W^2$$

1. f_{coin} : coincidence frequency of source 1 and source 2
2. f_1, f_2, f_3 : frequency of source 1 and source 2
3. W : time value of the interval "window"

Pe	$f_1(kHz)$	stdev $f_1(Hz)$	$f_2(kHz)$	stdev $f_2(Hz)$	$f_3(kHz)$	stdev $f_3(Hz)$
1	81,09	8,75	601,46	4,97	133	328,86
2	85,09	2,55	82,76	3,29	84,31	3,14
3	13,30	394,73	11,78	344,03	15,64	553,94

Table 6: Coincidence of dark current of channel 1, 2, and 3

Pe	$f_{coin}(Hz)$	stdev f_{coin}	$f_{coin}theoretical$
1	465,78	155,58	224,90
2	0,02	NA	20,58
3	0	NA	0,08

Table 7: Coincidence of dark current of channels 1,2, and 3 (continue)

For 3 channels' coincidence:

- The frequency of coincident of 2 and 3 pe. of 3 channels is approximately 0.
- The frequency generated by the oscilloscope is far larger than the frequency calculated by the given equation.

Pe	$f_{coin}(Hz)$	stdev f_{coin}
1	0,0167	NA
2	0	NA

Table 8: Coincidence of dark current of channels 1, 2, 3, and 4 (manually measured)

Notice here, the frequency of coincident of 2 pe of 4 channels is approximately 0. Therefore, the coincidence of 4 channels at the threshold above 2 pe is chosen to detect the appearance of muon passing the detectors.

5.2 Muon counter



Figure 11: Scintillator used in experiment

The area of the plastic scintillator used in this experiment is (17cm x 5cm). The rate of muon particles passing through the scintillator can be estimated based on previous studies and calculations. It has been found that the rate of muon particles arriving at the Earth's surface is approximately one per square centimeter per minute (here we only consider the muon rays that come in the vertical direction). Therefore, based on the size of our scintillator, we can estimate the expected muon count rate by multiplying the area of the scintillator by the estimated muon particle rate:

$$17(cm) \times 5(cm) \times 1\mu/cm^2/60s \approx 1Hz$$

This is the muon count rate that we expect to find.

However, due to our lack of experience in counting statistics and error propagation, we lack a considerable amount of necessary data and correct measurements for further processing.

Basically, what we did was count the number of hits as we increased the time intervals. We used this information to calculate the frequency by dividing the number of hits by the corresponding interval. For a single measurement, conventionally, the uncertainty or "error" is equivalent to one value of the standard deviation σ .

$$\sigma = \sqrt{x}$$

where x is the sample variance, or in our case, the number of hits.

We observed that over a long period of time, the muon arrival rate comes closer to the theoretical value, which is 1. However, this initial conclusion is incorrect and has no statistical meaning.

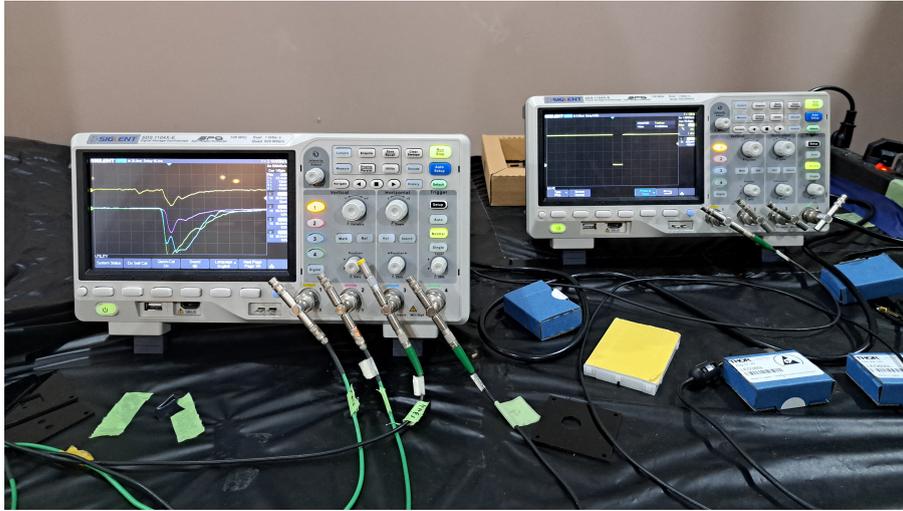


Figure 12: Muon count rate practical set-up

Time (s)	Number of hits	Frequency (Hz)	Uncertainty
3840	1796	0.5	42.37
9132	8326	0.91	91.25

Table 9: Muon count rate measurements

The issue at hand is that raw data, which includes the count of events over a specific time period, is often not the ultimate point of interest. Typically, this data is processed by performing various mathematical operations such as multiplication or addition to obtain a derived quantity of more immediate relevance.[5] We could enhance our experiment by taking a series of N repeated counts from the same source with equivalent counting times. Let Σ be the sum of the results of these multiple counts.

$$\Sigma = x_1 + x_2 + .. + x_N$$

then,

$$\sigma_{\Sigma} = \sqrt{\Sigma}$$

and with \bar{x} as the mean value from these N independent measurements,

$$\bar{x} = \frac{\Sigma}{N}$$

and,

$$\sigma_{\bar{x}} = \sqrt{\frac{\bar{x}}{N}}$$

With N independent counts, the mean value is expected to have a smaller error by a factor \sqrt{N} , in comparison to any single measurement on which the mean is established.

5.3 Muon decay

We employed the oscilloscope to perform a measurement of coincidence events in our experiment. Specifically, we set the thresholds at 2.5 photoelectrons for channels 1, 2, and 3, and we recorded a total of 8000 events. With the assistance of Dr. Son Cao's C code, we were able to analyze the data and identify 65 candidate events that satisfied the selection criteria. The experiment was conducted for roughly 14 hours, commencing at 6 PM and concluding at 8 AM the next day. The frequency of candidates is approximately $1.3e - 3Hz$.

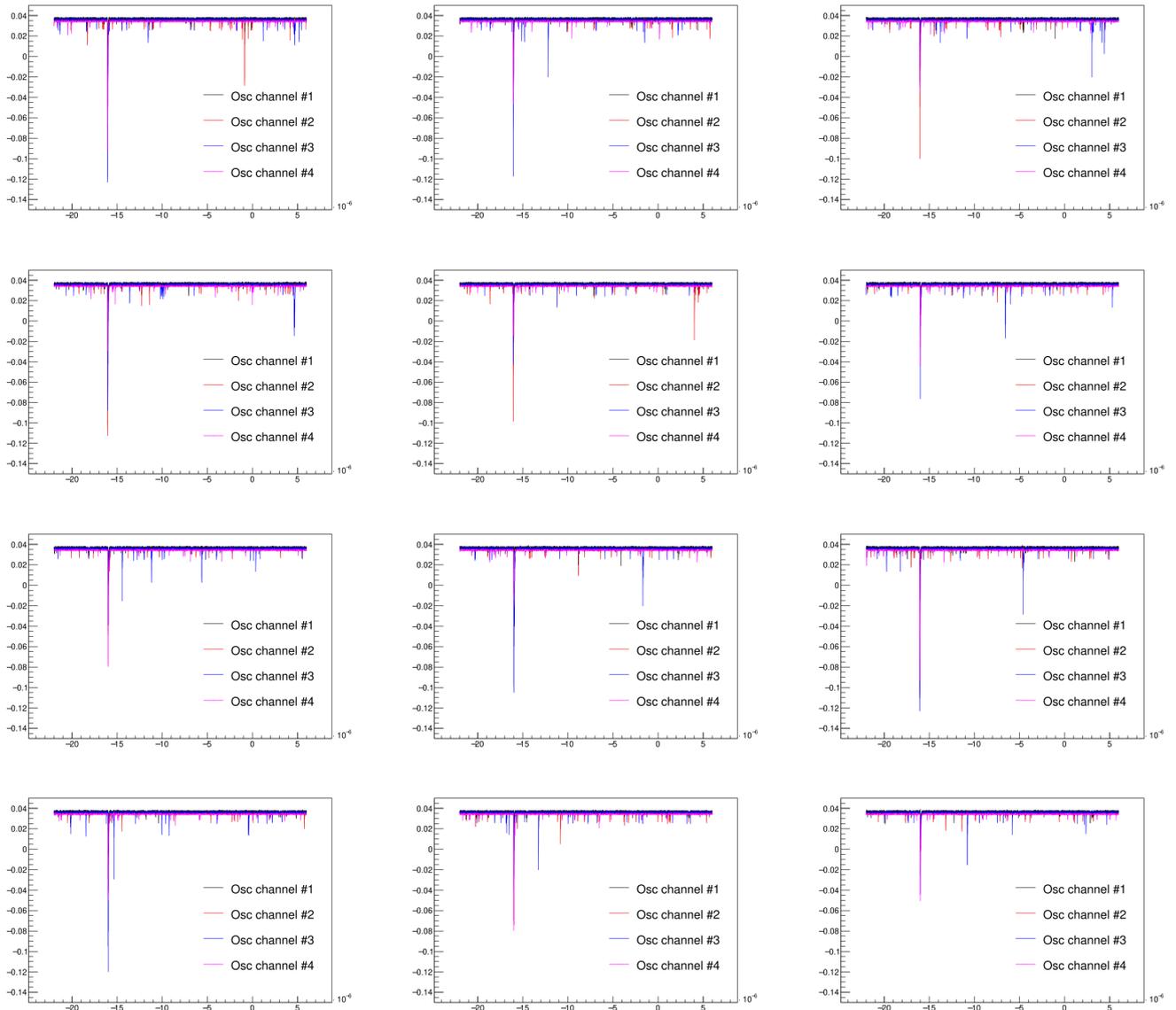


Figure 13: Data retrieved on remote PC

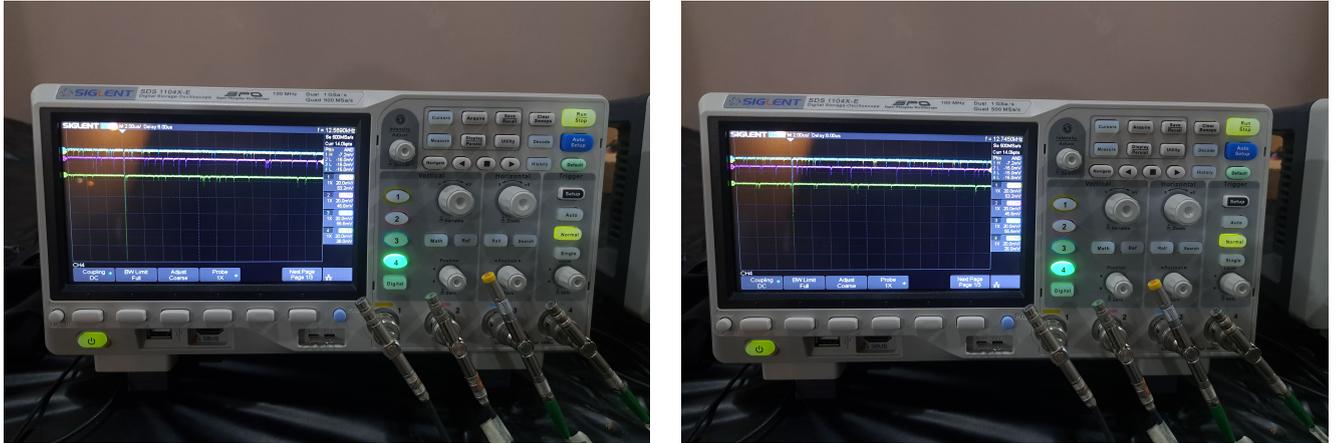


Figure 14: Events on oscilloscope

Although this should allow us to collect a sufficient amount of data to perform a detailed analysis of the events recorded during the experiment, we were only able to retrieve the full data on the last day of the program and thus could not make further progress.

There are still, nonetheless, some interesting observations and discussions to make. Essentially, the mean lifetime of a muon at rest is known to be $(2.1969811 \pm 0.22e - 5\mu s)$, but in our experiment, we measured muons that were traveling near the speed of light. This means we need to take into account the effects of special relativity, which predicts that the muon lifetime while in motion will be longer than when at rest. This phenomenon is known as time dilation, and it arises due to the fact that moving objects experience time differently than stationary objects.[2]

To calculate the actual muon lifetime in our experiment, we need to take into account the speed of the muons and apply the appropriate time dilation factor. This requires a more complex analysis that involves the Lorentz factor and the muon's velocity relative to our detector.

$$t = \gamma\tau$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where t is the muon lifetime while in motion, τ is the muon lifetime at rest and γ is the relativistic factor (Lorentz factor).

The decay of muons is characterized by the decay constant λ :

$$N = N_0e^{-\lambda t}$$

To find the "lifetime" of a muon, one can use the reciprocal of λ . The relationship between the lifetime and λ is straightforward, as it follows the same pattern observed in radioactive decay, characterized by an exponential function.

To measure the surviving number of muons, it is not possible to obtain a single clump of muons. Instead, we detect muon decays as they randomly enter the detector, usually one at a time. However, the exponential nature of the decay remains consistent, as described above. The decay time distribution, represented by $f(t)$, refers to the time-dependent probability of a muon decaying between time intervals t and $t + dt$. By considering a single muon, the probability density function of the decay time can be calculated using the following function:

$$f(t) = \lambda e^{-\lambda t}$$

The idea is that to estimate the value of muon decay constant λ , one can model the Probability Density Function (PDF) of the muon decay time.

5.4 Fluorescence



Figure 15: The signal for two channels at the amplitude of 4.6V

In this experiment, the 5 fluorophore panels of Thorlab were excited by an short pulse at of UV LED (the wavelength of 385nm) with the square waveform generated by Analog Discovery 2, the ideal pulse duration that we desired was 1nm (approximate the fluorescence lifetime provided by the manufacturer). We tried to create a square pulse on the software Digilent WaveForms designed for Analog Discovery 2 by examining the voltage of the Analog Discovery from 4V to 5V, the voltages from 4 to 4.4 are weak, therefore we choose the amplitude of 4.6V, also the symmetry under 5% does not generate the signal and over 7% cause the shutdown on the system. Due to the technical limit of Analog Discovery 2, the analog discovery can only produce the shortest pulse of 50nm.

Pulse shape	Square
Amplitude	4.6 V
Frequency	1MHz
Symmetry	5%

If the pulse of LED used for fluorescence excitation is not short enough, it can result in the convolution of the fluorescence decay curve and the MPPC response function. The MPPC response function refers to the temporal response of the detector to a single photon of light. This convolution effect can make it more difficult to accurately determine the fluorescence lifetime.

Following the set up we described in section ..., we use *Root* to retrieve the data remotely from the oscilloscope, the Code for this is provided by Dr. Son Cao, then we convert the data from *T_Graph* format to *.csv* format.

Following the method we described in section 3.2, after having the *.csv* data file, we make some adjustments to convert them into easy handling DataFrame format, then we utilize the function *curve_fit* which uses non-linear least squares in the package *scipy* on *Python* to fit the defined equation 1 for constraint of the best fit parameter of fluorescence decay-time. The code and result of fitting function for each fluorophore are shown in Appendix.

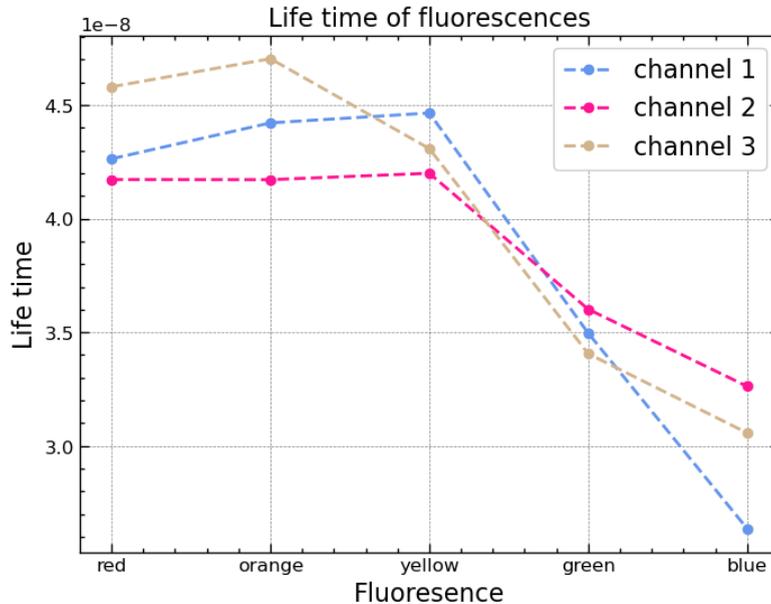


Figure 16: Fluorescence decaytime in compare of three channel

From the result, we can see that the fluorescence decay-times measured from this experiment are longer than the the value in theory (around 1nm) as we expected, the reason is that the decay time is not only contributed from fluorescence but also from MPPC response function.

However, we can still deduce the relationship between the decay-time and wavelength. In the figure, we can observe that data on three different channels suggest the same trend that the decay-time of fluorescence decreases proportional to the wavelength.

6 Scientific conclusion

The experimentally measured frequency of the dark count coincidence was found to deviate from the values predicted by the formula. Despite setting thresholds for the coincidence from three or four channels, the frequency of the dark count coincidence remained inconsistent with the expected results. However, it was observed that these thresholds helped in reducing the impact of dark current on the measurements.

On the other hand, the measured muon rate in the experiment was found to be consistent with the expected value. However, this does not necessarily imply that the method used to measure the muon decay time was accurate since it did not account for counting statistics and error propagation. Nevertheless, the experimental setup was able to successfully detect muons, indicating that the detection system was functioning properly. These findings suggest that further investigation is needed to determine the factors that affect the dark count coincidence frequency measurements, and more comprehensive statistical analyses should be performed in future experiments.

The decay time of fluorescence decrease proportionally with the wavelengths. The reason is because as the wavelength of the excitation light becomes shorter, its energy increases. This can result in a greater likelihood that the excited fluorescent molecule will lose energy through non-radiative processes, such as collisional deactivation or energy transfer, before it has a chance to emit a photon and return to its ground state.

Looking ahead to future work, there are a few potential avenues for improvement that should be considered. One option is to modify the fluorescence setup, possibly by incorporating better waveform generators to enable faster pulses. This could result in more precise measurements of the muon's lifetime.

Furthermore, in order to achieve more accurate measurements of the muon decay time and count rate, it is recommended that careful consideration be given to the use of statistics. Specifically, evaluating the Probability Density Function (PDF) of muon decay time could lead to a better understanding of the underlying processes involved. Additionally, more accurate interpretation of the muon rate measurements can be obtained through statistical methods.

Finally, it may be worthwhile to investigate the possibility of finding a more accurate equation for the coincidence frequency. This could potentially provide valuable insights into the mechanisms underlying the decay of muons, and could help to improve the accuracy of future experiments in this area.

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7 Appendix

7.1 Appendix A: Python code for fluorescence fitting and plotting

```

1 # Import the necessary Package and set up.
2 import numpy as np
3 import matplotlib
4 import matplotlib.pyplot as plt
5 import pandas as pd
6 import scienceplots
7 from scipy.optimize import curve_fit
8 plt.style.use(['science', 'notebook', 'grid'])
9 matplotlib.rc('xtick', labels=12)
10 matplotlib.rc('ytick', labels=12)
11
12 import warnings
13 warnings.filterwarnings("ignore")

```

Listing 1: Code for importing the packages

```

14 def read_fluore(color):
15     all_channel = []
16     for i in range(4):
17         # Read CSV file for the given color and channel
18         df = pd.read_csv('~Documents/hardware_camp/fluorescence_data/{}/'.
19 format(color) + 'ch{}_ave_{}.csv'.format(i+1, color),
20 sep='delimiter', header=None, skiprows=5)
21
22         # Extract time and value data from the CSV file
23         time = np.array(df[0].iloc[:174].map(lambda x: str(x).rstrip(','))).
24 astype(float))
25         time_2 = np.array(float(df[0].iloc[174].rstrip(";")))
26         np.append(time, time_2)
27         value = np.array(df[0].iloc[176:350].map(lambda x: str(x).rstrip(','))).
28 astype(float))
29         value_2 = np.array(float(df[0].iloc[350].rstrip(";")))
30         np.append(value, value_2)
31
32         # Combine time and value data into a pandas DataFrame
33         df = pd.DataFrame(np.vstack((time,value)).T, columns = ['Second', '
34 Value'])
35         all_channel.append(df)
36
37     # Return a list of DataFrames for each channel
38     return all_channel

```

Listing 2: Code for defining the function for reading and transform the data in .csv file into a more readable DataFrame.

```

35 # Define the list of colors to read fluorescence data for
36 color = ['red', 'orange', 'yellow', 'green', 'blue']
37
38 # Read fluorescence data for each color and channel
39 # The resulting variables red, orange, yellow, green, blue will each be a
    list of DataFrames
40 red, orange, yellow, green, blue = [read_fluore(c) for c in color]
41
42 # Combine the fluorescence data for each color into a single list
43 data = [red, orange, yellow, green, blue]

```

Listing 3: Code for reading data for each fluorescence according to their colors

```

44 # Define the decay function with parameters I (initial intensity), c (decay
    constant), and d (background signal)
45 def decay_func(t, I, c, d):
46     return I*np.exp(-t/c) + d

```

Listing 4: Code for defining the exponential function of decay time

```

47 def decay_time_func(channel):
48     # Find the range of x and y data for fitting
49     xdata = channel["Second"][channel["Value"].idxmin():]*1e7
50     ydata = channel["Value"][channel["Value"].idxmin():]
51
52     # Perform curve fitting using the decay function
53     # maxfev is the maximum number of function evaluations allowed
54     popt, potv = curve_fit(decay_func, xdata, ydata, maxfev = 10000)
55
56     # Calculate the decay time from the fitted parameters
57     decay_time = popt[1]*1e-7
58
59     # Return the fitted parameters, xdata, and decay time
60     return popt, xdata, decay_time
61
62 def decay_time_plot(channel, popt, xdata, decay_time, c, channel_index):
63     # Create a figure and axis object for plotting
64     fig, ax = plt.subplots(figsize = (10,8))
65
66     # Plot the original data
67     ax.plot(channel['Second'], channel['Value'], color = c)
68
69     # Plot the fitted decay function with a black dashed line
70     ax.plot(channel["Second"][channel["Value"].idxmin():], decay_func(xdata,
    popt[0], popt[1], popt[2]),
71             color = 'black', linestyle='--', linewidth = 4, label = f"${popt
    [0]:.2f}\\cdot e^{-t/{decay_time:.2e}} + {popt[2]:.2f}$")
72

```

```

73     # Add a vertical line at the minimum value of the data
74     ax.axvline(channel["Second"][channel["Value"].idxmin()], linestyle = ":"
, linewidth = 2, color = 'black')
75
76     # Set the x-axis limits
77     ax.set_xlim(-1e-7,3e-7)
78
79     # Set the axis labels and title
80     ax.set_xlabel("Time (s)")
81     ax.set_ylabel("Amplitude (V)")
82     ax.set_title("Fluorescence: {} on channel {}".format(c, channel_index+1)
)
83
84     # Add a legend to the plot
85     plt.legend()
86
87     # Show the plot
88     plt.show()
89
90     # Save the plot to a file (if desired)
91     plt.savefig("output/_channel_{}_{}".format(channel_index, c))

```

Listing 5: Code for defining the function for fitting and retrieving the decay time (*decay_time_func*) and function for plotting the fitting curve on the signal (*decay_time_plot*)

```

92 # This code processes each channel of the data and plots the fitted decay
   curve for each color,
93 # then calculates the decay time for each channel and saves it as an array.
94 # Finally, it assigns the decay times to separate variables for each channel
   .
95 def process_channel(channel_index):
96     decay_times = []
97     for i, color_data in enumerate(data):
98         popt, xdata, decay_time = decay_time_func(color_data[channel_index])
99         decay_time_plot(color_data[channel_index], popt, xdata, decay_time,
   color[i], channel_index)
100         decay_times.append(decay_time)
101     return np.asarray(decay_times)
102
103 decay_time_ch1, decay_time_ch2, decay_time_ch3 = [process_channel(i) for i
   in range(3)]

```

Listing 6: Code for running the whole process of fitting and return the decay time of fluorescence on each channel.

```

104 # plot the lifetime of each fluorescence in each channel
105 plt.plot(color, decay_time_ch1, 'o--', label = 'channel 1', color = '
   cornflowerblue')

```

```
106 plt.plot(color, decay_time_ch2, 'o--', label = 'channel 2', color = '  
    deeppink')  
107 plt.plot(color, decay_time_ch3, 'o--', label = 'channel 3', color = 'tan')  
108  
109 # add legend, axis labels, and title to the plot  
110 plt.legend()  
111 plt.xlabel("Fluorescence")  
112 plt.ylabel("Life time")  
113 plt.title("Life time of fluorescences")  
114  
115 # save the plot as an image  
116 plt.savefig("output/Lifetime")
```

Listing 7: Code for plotting of fluorescence decaytime

7.2 Appendix B: Plotting of exponential function fitting

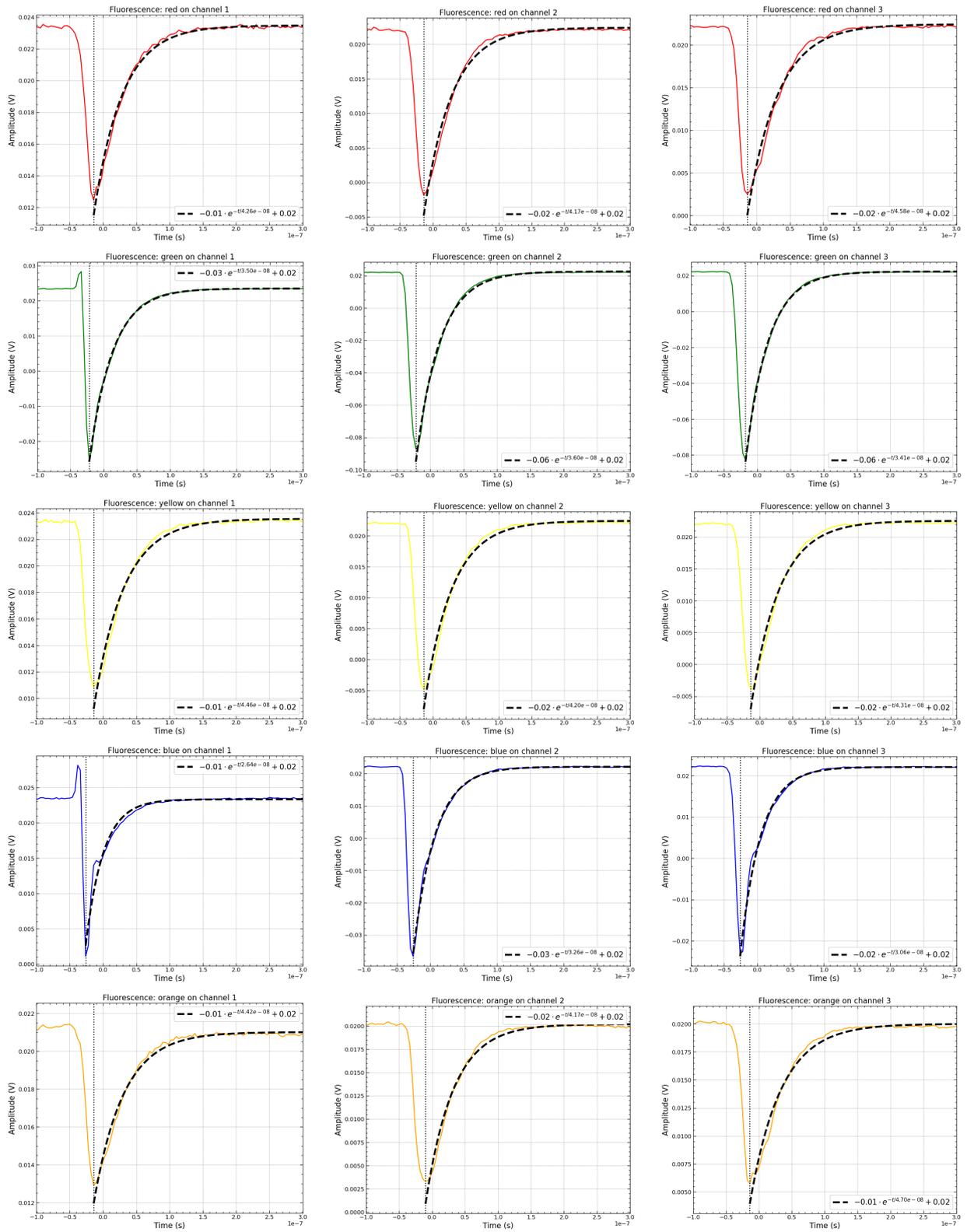


Figure 17: Fluorescence fitting with exponential function