Quantum Field Theory
as a COMMON LANGUAGE of Particle and Condensed Matter Physics

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IFIRSE seminar

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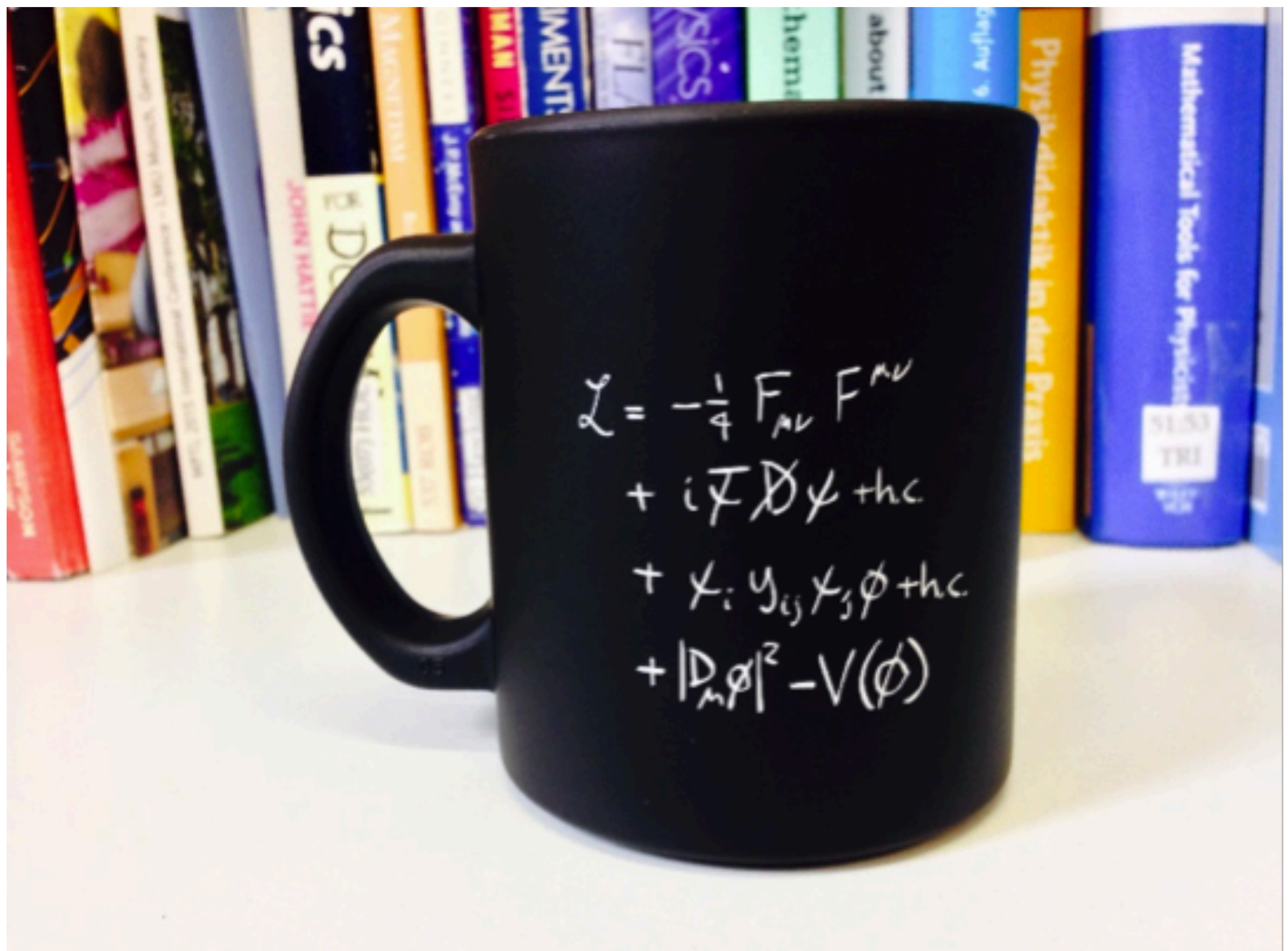
Plan

- What is quantum field theory?
- Uses of QFT in particle and condensed matter physics: commonalities and differences
- Example 1: spontaneous symmetry breaking
- Example 2: Renormalization Group
- Example 3: Duality

Quantum field theory

- “We have no better way of describing elementary particles than quantum field theory” − A. Polyakov, “Gauge fields and strings”
- Quantum field theory: unifies quantum mechanics and special relativity
- Triumph of quantum field theory: the Standard Model

The Standard Model



SM in condensed matter

Pines and Laughlin "The Theory of Everything" PNAS 1999

$$i\hbar \frac{\partial}{\partial t} |\Psi > \; = \mathcal{H} |\Psi > \qquad \textbf{[1]}$$

where

$$\mathcal{H} = -\sum_{j}^{N_e} \frac{\hbar^2}{2m} \nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha} \nabla_\alpha^2 - \sum_{j}^{N_e} \sum_{\alpha}^{N_i} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \ll k}^{N_e} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha \ll \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}. \qquad \textbf{[2]}$$

does not look like quantum field theory

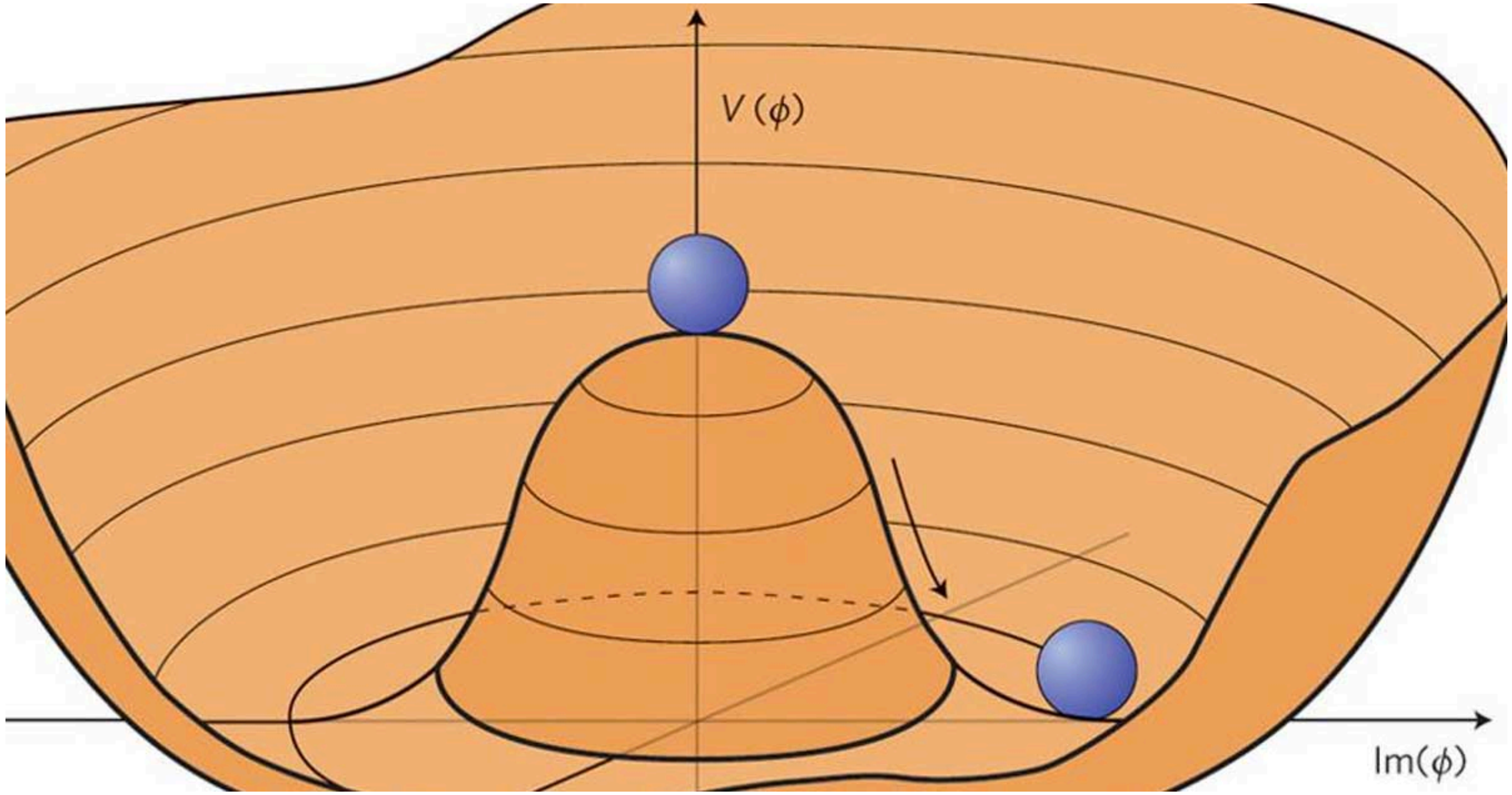
QFT in CMP

- There are many ways that QFT appear in CMP
- Through second quantization
  - $L = \psi^\dagger \partial_t \psi - \frac{1}{2m} |\nabla \psi|^2 + \cdots$
- As a long-distance description of e.g., lattice systems
- effective field theory of low-energy degrees of freedom
- We will look at a few common themes in HEP and CMP

Theme 1: Spontaneous symmetry breaking

Spontaneous symmetry breaking

- Spontaneous symmetry breaking plays a big role in particle physics
- In QCD, the SSB of the approximate chiral symmetry leads to the appearance of very light pseudoscalar mesons (pions and kaons)
- proton mass $m_p \sim 900$ MeV, pion mass $m_\pi \sim 140$ MeV
  - naive quark model: $m_\pi \approx \frac{2}{3} m_p \sim 600$ MeV



Goldstone mode: motion along the valley
Higgs mode: radial oscillation

SSB in CMP

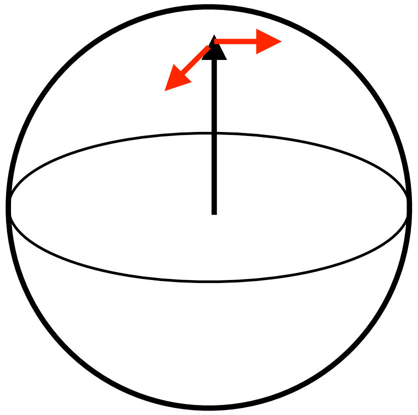
- Spontaneous symmetry breaking also occurs in CMP and underlies many physical phenomena
- Example: ferromagnetism (Heisenberg model)
- At long distances: a field theory with a real scalar
  - $L = (\nabla \phi^a)^2 + m^2 \phi^a \phi^a + \lambda(\phi^a \phi^a)^2$
- Changing T: effectively changing all parameters including m^2
  - low temperature $m^2 < 0$: ferromagnetism

$$\phi = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \qquad v = \sqrt{\frac{-m^2}{\lambda}}$$

How many Nambu-Goldstone bosons?

- Folk theorem in high-energy physics: the number of NGBs is the number of broken generators
- For the model at hand, the vacuum breaks rotations around $x, y \rightarrow$ expects 2 NGBs
- But in ferromagnet: one NGB with dispersion relation $\omega \sim q^2$
- Reason: the two broken generators form a canonically conjugate pair of operators

$$\phi = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix}$$



$$[S_x, S_y] = iS_z \neq 0$$

hep-ph/0108210

Theme 2: Renormalization

See also "How Mathematical Hocus-Pocus Saved Particle Physics", Quanta Magazine

Renormalization

- Late 1940s: Quantum electrodynamics
  - successful predictions: Lamb shift, anomalous magnetic moment of the electron
  - but requires subtraction of infinity (renormalization)
- The meaning of renormalization wasn't clear
- Nambu: *In 1947 I joined Tomonaga's group an listened to his lectures… Tomonaga called renormalization the principle of "hohki", which means either "giving up" or "broom".*

JOAN CARTIER

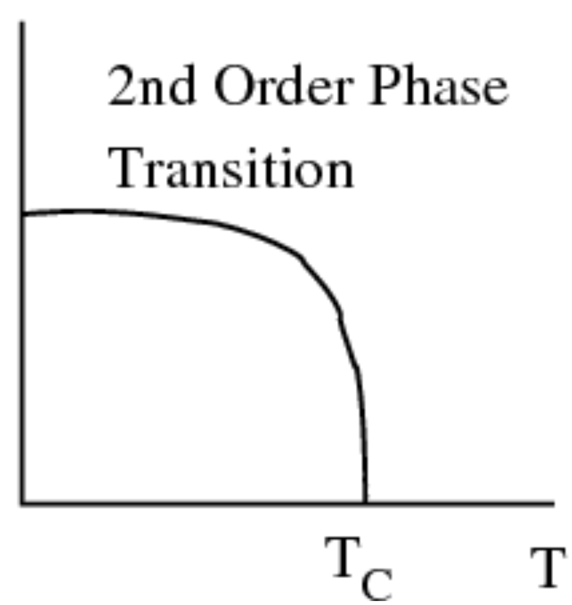
ALRIGHT RUTH, I ABOUT GOT THIS ONE RENORMALIZED.

Renormalization group

- An important step in the understanding of QFT: renormalization group
- Gell-Mann, Low, Bogoliubov, Shirkov, etc.
  - resummation of logarithmic divergences
  - coupling constants depending on renormalization scale
- The physical meaning of renormalization procedure was not clear until Wilson's work on 2nd order phase transitions

Second-order phase transition

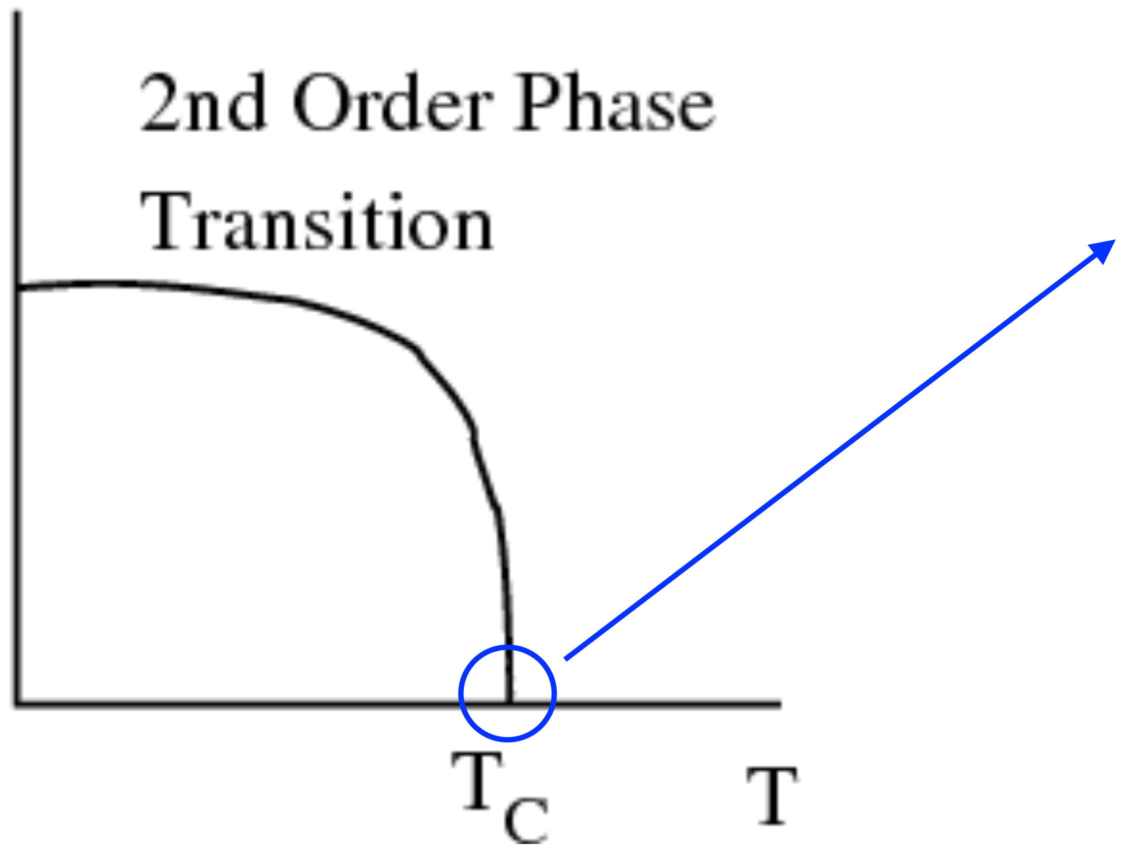
- Prototypical model of 2nd order phase transition: the Ising model
- $$H = -J \sum_{\langle ij \rangle} S_i S_j \,, \qquad S_i = \pm 1$$
- Gibbs ensemble: $\rho[S_i] \sim e^{-\beta H[S_i]}$



$$\langle S_i \rangle \neq 0 \quad T < T_c$$

$$\langle S_i \rangle = 0 \quad T > T_c$$

Critical exponents



$$\langle S \rangle \sim (T_c - T)^{\beta}$$

$$\langle S(x)S(0) \rangle \sim e^{-|x|/\xi}$$

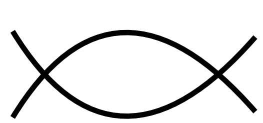
$$\xi(T) \sim \frac{1}{|T - T_c|^{\nu}}$$

Mean field theory (Landau)

- Landau's mean field theory is basically the tree-level analysis of the field theory
- $L(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi), \quad V(\phi) = m^2\phi^2 + \lambda\phi^4$
- $m^2 = \alpha(T - T_c)$
- $T < T_c\colon\ \langle\phi\rangle = \sqrt{\frac{-m^2}{\lambda}} \sim (T_c - T)^{1/2} \qquad \beta = \frac{1}{2}$
- Analogously $\nu = 1/2$

Breakdown of mean-field theory

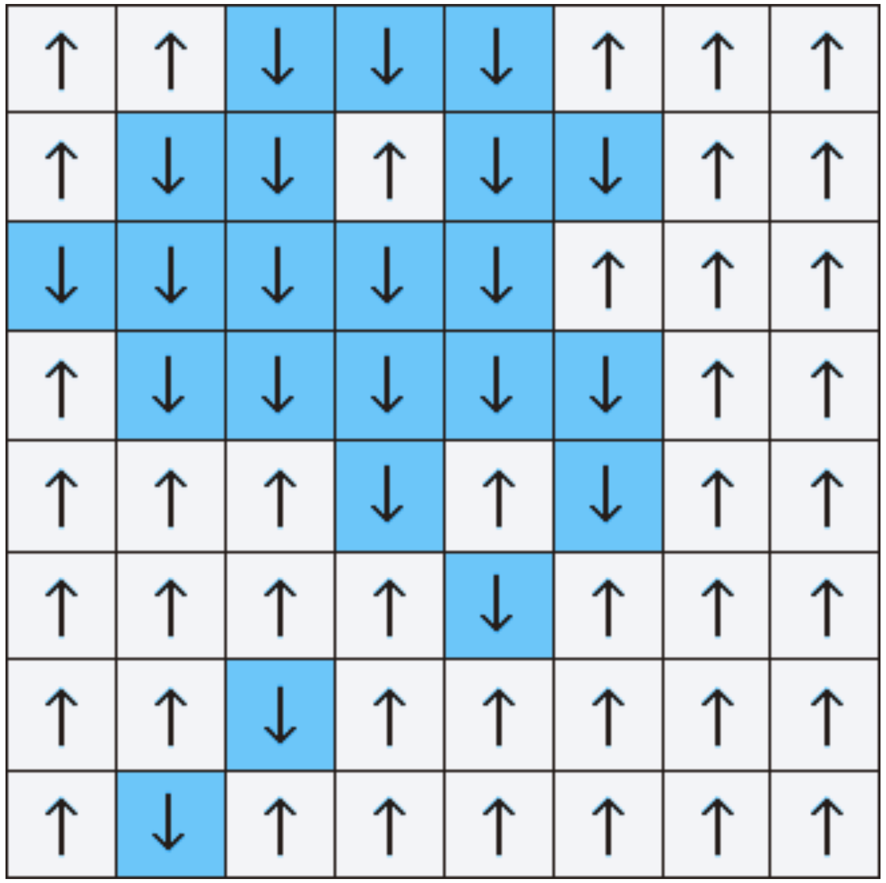
- We have to go beyond the tree-level analysis
- Loop correction: shifts the location of the phase transition: $m^2_{\text{phys}} = m^2_{\text{bare}} + \delta m^2$
- But when $m^2_{\text{phys}} \to 0$: infrared divergences



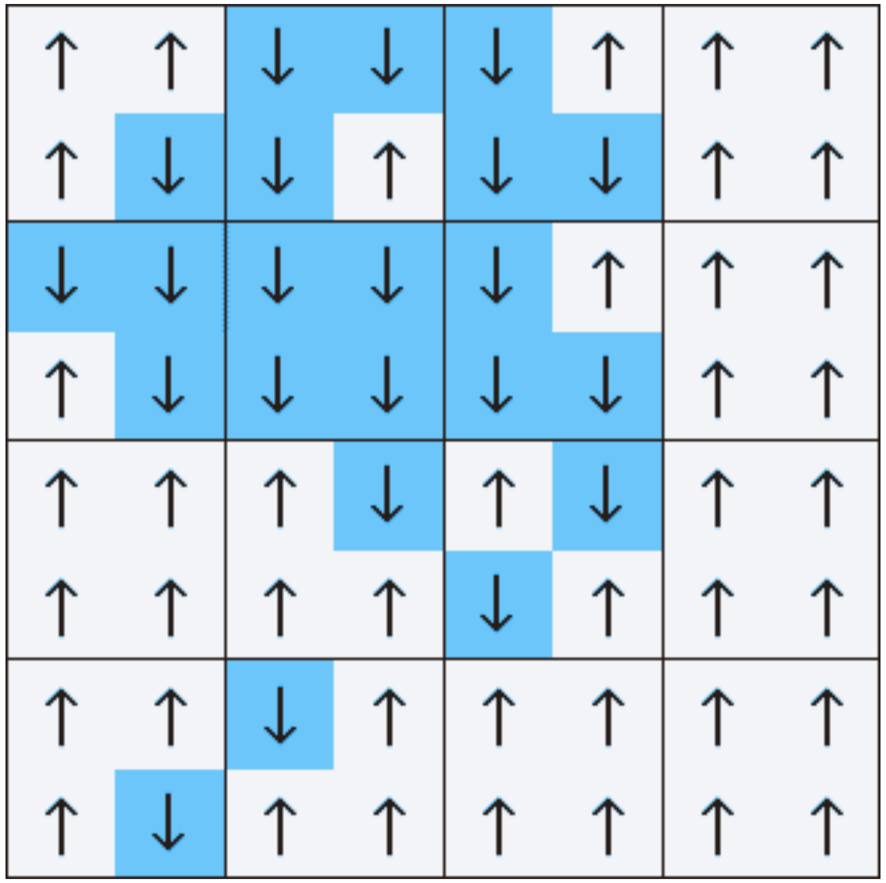
$$\int \frac{d^3q}{q^4}$$

Kadanoff block spin

Kadanoff block spin



Kadanoff block spin



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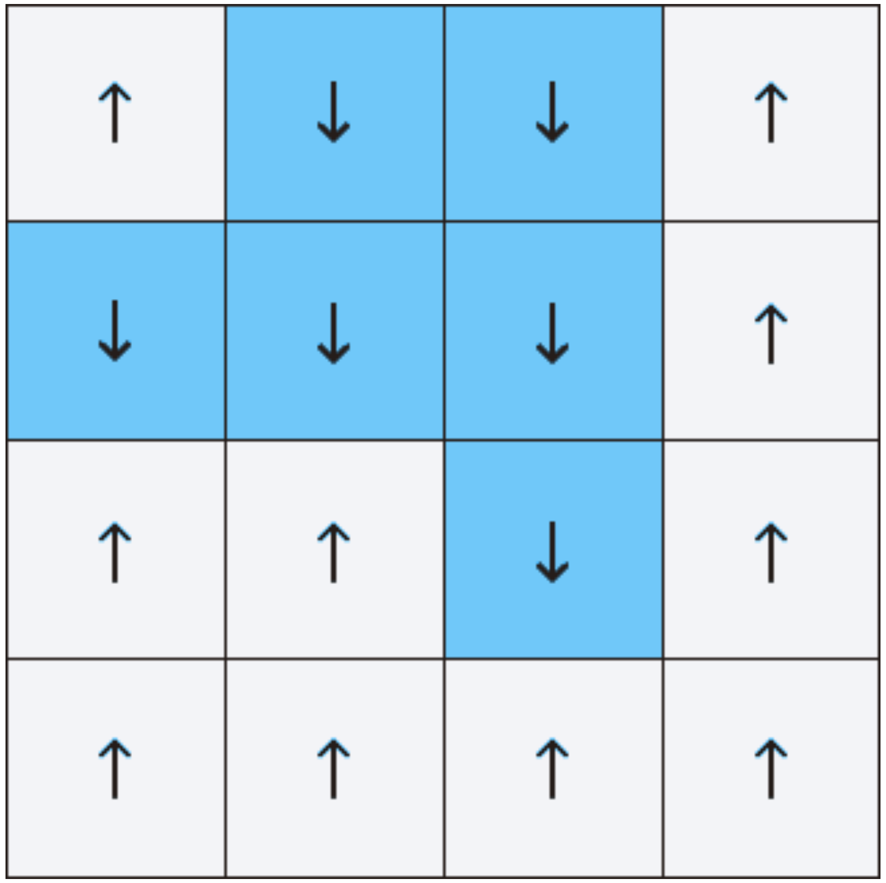

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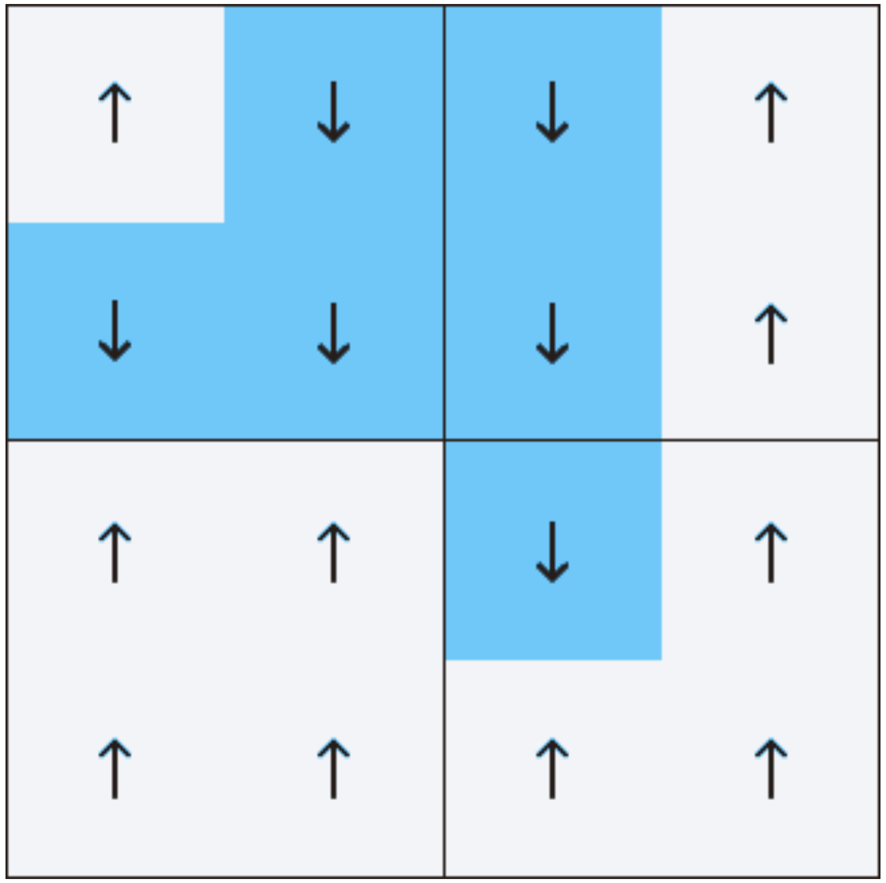

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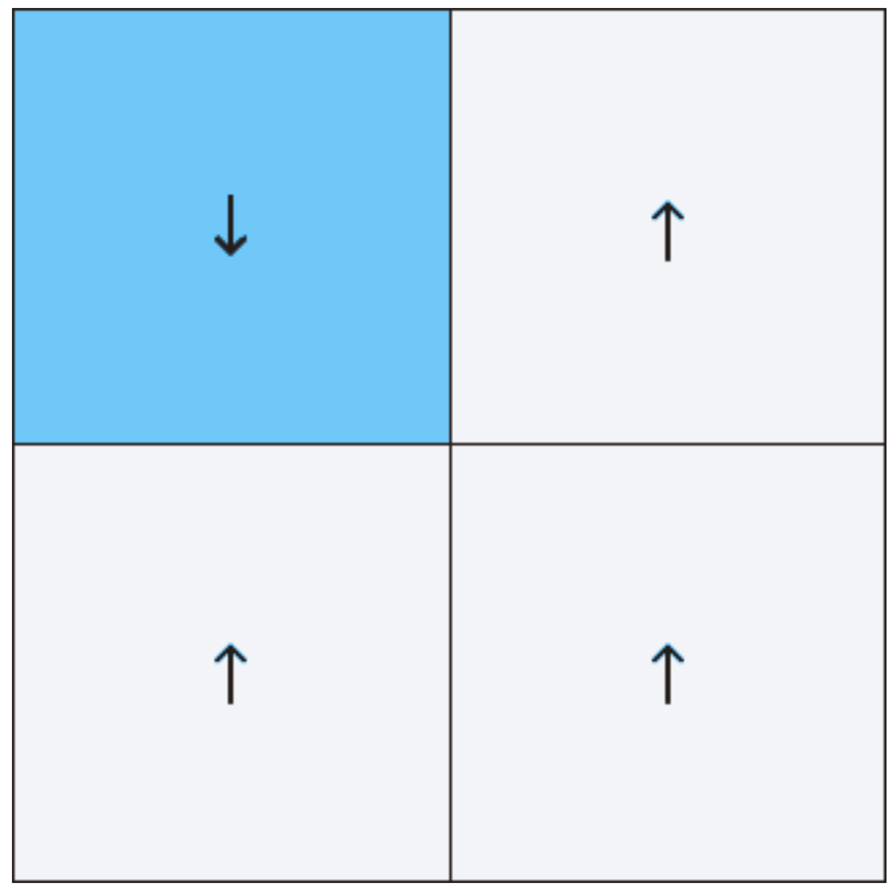

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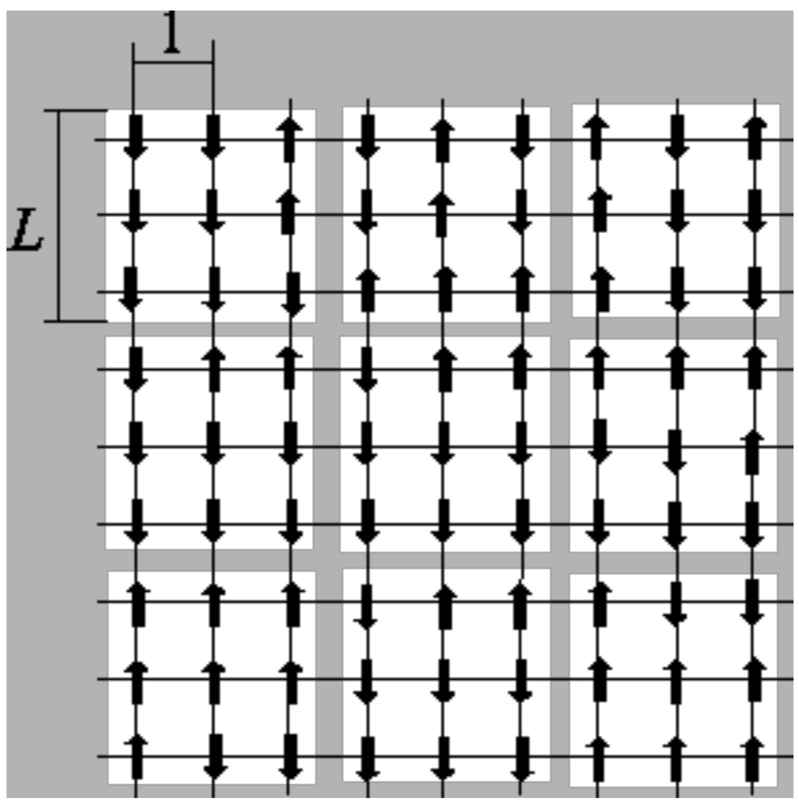
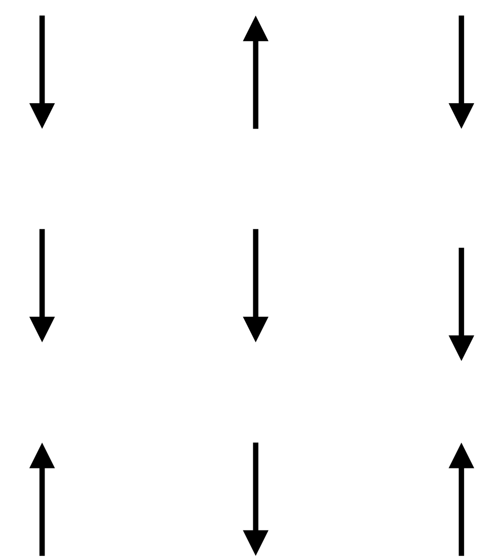




$$H = -J \sum_{\langle ij \rangle} S_i S_j$$

$$H' = -J' \sum_{\langle ij \rangle} S_i S_j + \cdots$$

Ken Wilson's insight

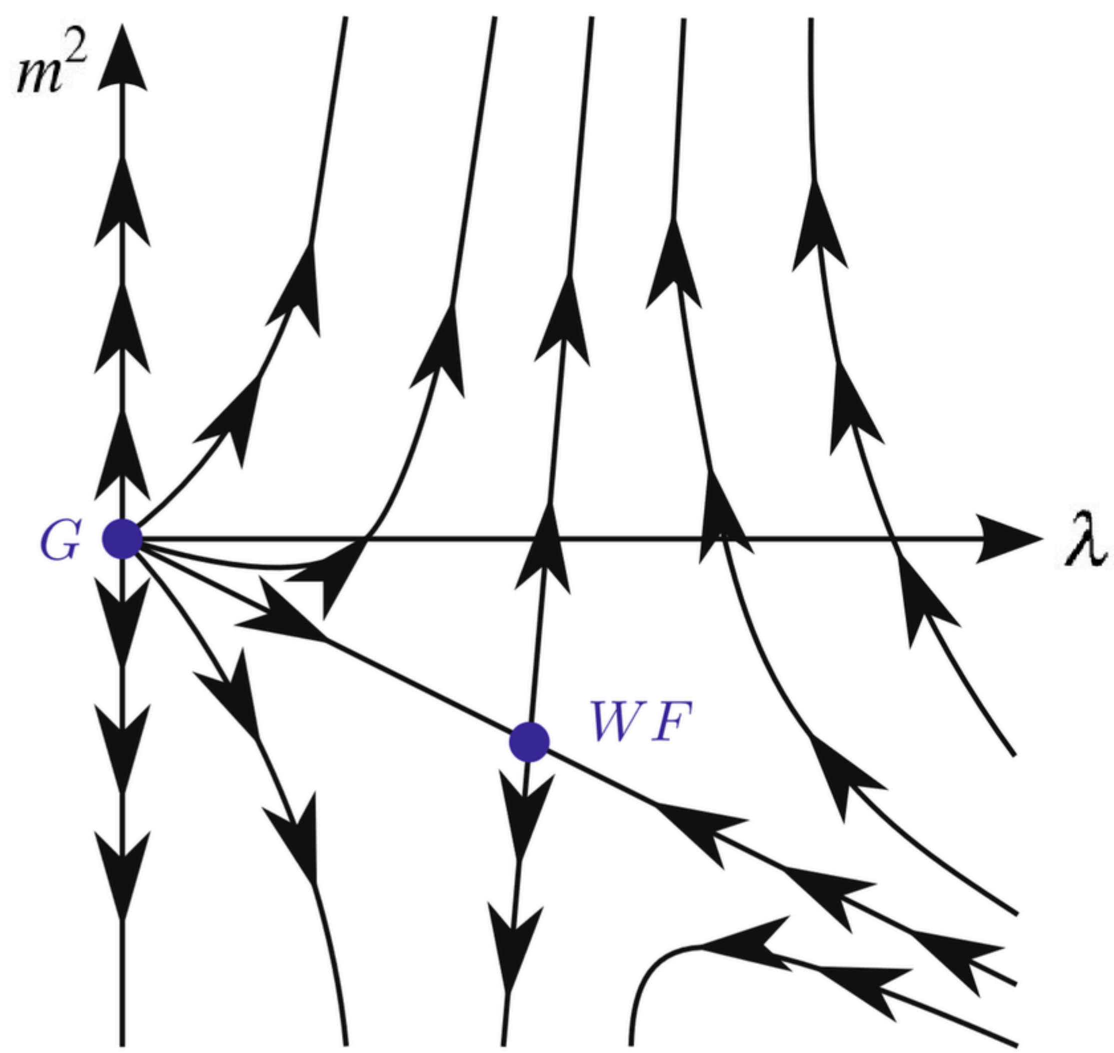
- K. Wilson: Renormalization Group is a rigorous way to perform the Kadanoff block spin transformation

- Field theory: integrate out high energy degrees of freedom, reducing the UV cutoff
  - how coupling constants change under this procedure is determined by the RG equation

$$\Lambda(s) = e^{-s}\Lambda_0 \qquad\qquad \frac{\partial g(s)}{\partial s} = -\,\beta(g)$$

2nd order pt as RG fixed point

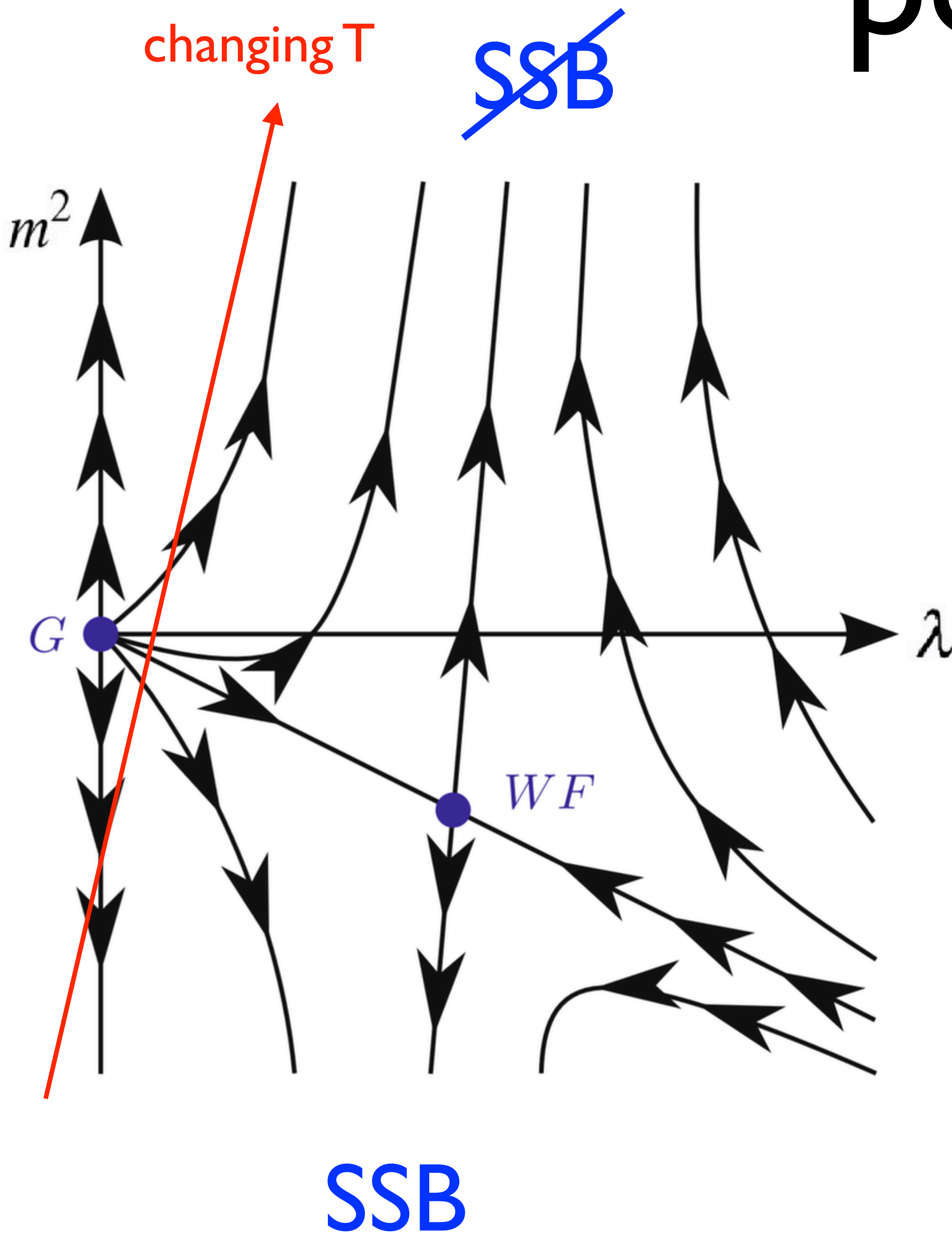
~~SSB~~



SSB

$$S = \int d^3x \, [(\partial_\mu \phi)^2 + m^2\phi^2 + \lambda\phi^4]$$

2nd order pt as RG fixed point



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Critical exponents

- At the IR fixed point the theory is interacting
  - non-mean-field critical exponents
- Advanced theoretical methods allow very precise determination of the critical exponents
  - $\nu = 0.629971(4)$
  - $\beta = 0.326419(3)$

Wilsonian view on QFT

- Wilson's work completely changed our views of QFT
- The central object in QFT is the renormalization group flow. Some would say that QFT *is* RG flow.
- Organization by energy scales. Previously, organization in number of loops.
- Effective Field Theory (Weinberg, 1970s). Almost all field theories are EFTs, with cutoff.
- Renormalizable field theories are field theories where the cutoff can be pushed to be exponentially large without increasing the number of coupling constants.

Theme 3: Duality

Duality

Two quantum field theories that look different are in fact equivalent

$$\mathcal{L}_1 = |\partial_\mu \phi|^2 - m^2|\phi|^2 - \lambda|\phi|^4$$

two phases: $\langle\phi\rangle \neq 0$ and $\langle\phi\rangle = 0$
The two are separated by a second order phase transition
(in the mean field theory: $m^2 = 0$)

$$\mathcal{L}_2 = |(\partial_\mu - iA_\mu)\Phi|^2 + m^2|\Phi|^2 - \lambda|\Phi|^4 - \frac{1}{4e^2}F_{\mu\nu}^2$$

two phases: $\langle\Phi\rangle = 0$: massless photon
$\langle\Phi\rangle \neq 0$: massive photon

Bosonic particle-vortex duality

Peskin; Dasgupta, Halperin 1978-81: exact equivalence of two theories in 2+1 D

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Theory 1 Theory 2

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	Theory 1	Theory 2
$m^2 < 0$	Goldstone boson	photon

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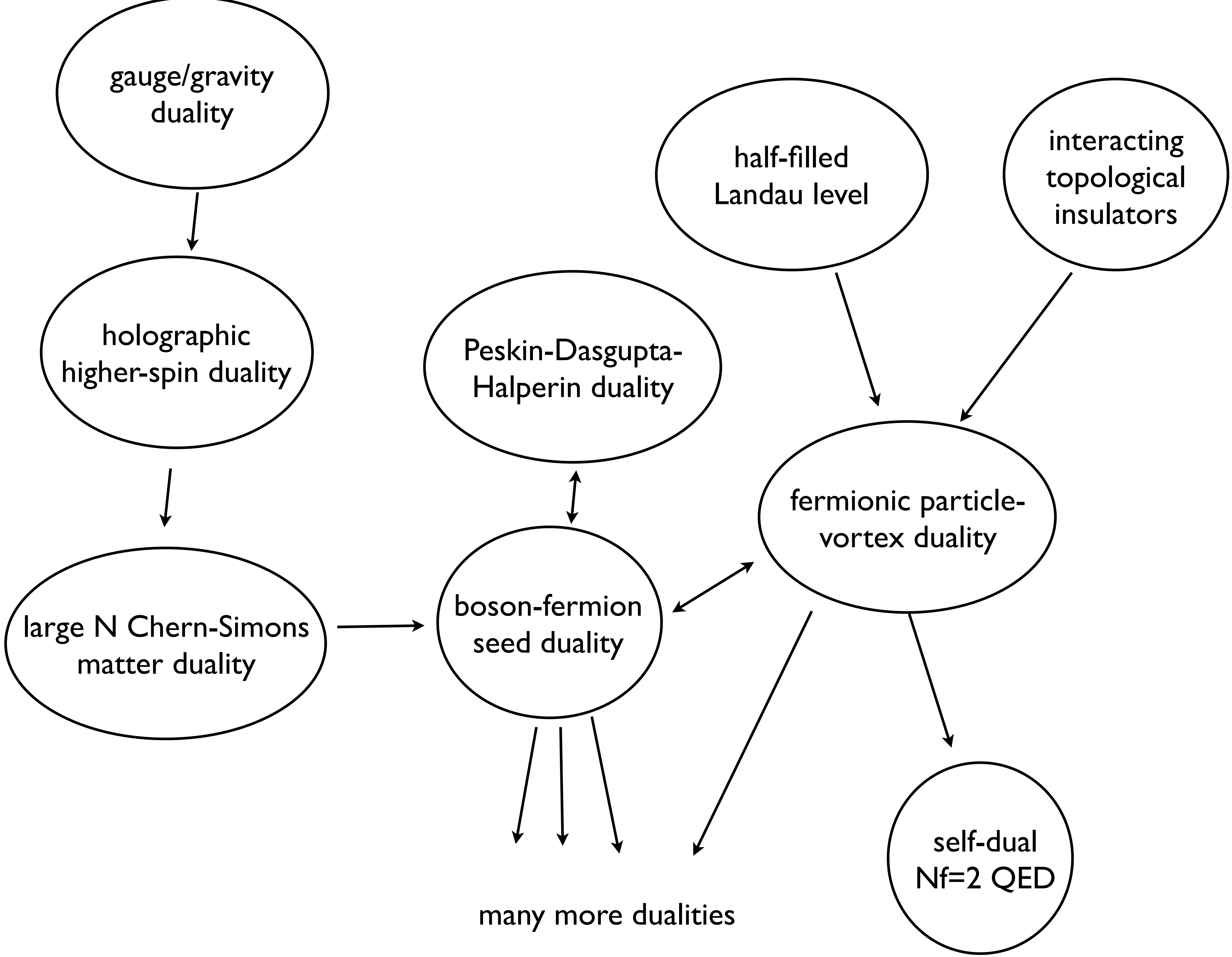
NUMERICAL EVIDENCE: the same critical exponent

Modern developments in duality

- A fermionic version of the duality may be underlying some phenomena in fractional quantum Hall effect arxiv:1805.04472
  - explains the emergence of a quasiparticle called "composite fermion"
- Many more example of dual pairs of QFTs have been discovered in (2+1) dimensions


gauge/gravity duality
holographic higher-spin duality
large N Chern-Simons matter duality
Peskin-Dasgupta-Halperin duality
boson-fermion seed duality
half-filled Landau level
interacting topological insulators
fermionic particle-vortex duality
self-dual Nf=2 QED
many more dualities

Epilogue

- What is Quantum Field Theory?
- Most field theories are effective field theories
- New types of field theories needed in CMP?
  - Landau's Fermi liquid theory as an effective field theory
  - "Non-Fermi liquids", possibly underlying high-Tc superconductivity
  - "Fractons"

Thank you