VIETNAM NATIONAL UNIVERSITY, HANOI
UNIVERSITY OF SCIENCE
FACULTY OF PHYSICS

Nguyen Hoang Duy Thanh

RESOLVING DEGENERACIES FOR CP VIOLATION SEARCH IN NEUTRINO EXPERIMENTS

Submitted in partial fulfillment of the requirement for the degree of
Bachelor of Science in Physics
(Standard Program)

Hanoi - 2018

VIETNAM NATIONAL UNIVERSITY, HANOI
UNIVERSITY OF SCIENCE
FACULTY OF PHYSICS

Nguyen Hoang Duy Thanh

RESOLVING DEGENERACIES FOR CP VIOLATION SEARCH IN NEUTRINO EXPERIMENTS

Submitted in partial fulfillment of the requirement for the degree of
Bachelor of Science in Physics
(Standard Program)

Supervisor: Assoc. Prof. Nguyen Thi Hong Van

Hanoi - 2018

Acknowledgements

First and foremost, my deepest gratitude is to my supervisor, Assoc. Prof. Nguyen Thi Hong Van who gave me the guidance, helpful recommendation and leading me to complete this thesis. She helped me how to express my ideas and look for necessary information. I would like to thank Dr. Cao Van Son for his useful advices and assistance to improve my knowledge.

I also wish to give my gratefulness to every member in *Neutrino Group* at IFIRSE and *Laboratory of High Energy Physics and Cosmology* at Faculty of Physics, VNU University of Science for suggestions and always supporting me.

I would like to express my appreciation to teachers at Faculty of Physics, VNU University of Science for the encouragement and training, to my friends for their sharing and helping.

My special thankfulness gives to my family for the love and patience, especially, to my mother who takes care of and supports me to this day.

Hanoi, June 2018

Nguyen Hoang Duy Thanh

List of Figures

Table of Contents

Introduction

Nowadays, the neutrino oscillations, masses and the flavor neutrino mixing have been investigated by the experiments. The evidence for neutrino masses is now clear (Nobel Prize in Physics 2015 for the discovery of neutrino oscillations, which shows that neutrinos have mass[15]) and it turns out there are very interesting problems about masses, mixings as well as CP phases. Neutrino masses imply mixings in the lepton sector which include CP violating phases. These CP phases are not only interesting due to their appearance in mixing matrix, but they can be very important physical effects[3, 12]. CP violation in the neutrino sector is the essential ingredient in explaining the baryon asymmetry of the universe.

We measure the values of parameters in the mixing matrix through many experiments with high accuracy and it seems to exist a problem for measuring δ_{CP} and θ_{13}, the determination of the neutrino mass hierarchy and θ_{23} octant. That is the parameter degeneracies which appear when a set of values of the oscillation probabilities, $P(\nu_\alpha \to \nu_\beta)$ and $P(\overline{\nu}_\alpha \to \overline{\nu}_\beta)$ at a fixed neutrino energy and baseline, does not determine uniquely the values of δ_{CP} and θ_{13}, are intrinsic degeneracy, mass hierarchy degeneracy and θ_{23} octant degeneracy.

The goal of my thesis is understanding clearly parameter degeneracies and looking for the resolving in present and future experiments. My thesis is organized by four chapters. Chapter 1 discusses neutrino oscillations in vacuum as well as in matter and some characteristics of CP violation. Chapter 2 is about neutrino sources and types of neutrino experiment. Chapter 3 shows parameter degeneracies and chapter 4 considers the resolutions of them. Finally, Appendix A introduces briefly about Electroweak interaction and some features of neutrino in Standard Model, appendix B illustrates some calculations for CP violation asymmetry $A^{CP}_{e\mu}$.

Chapter 1

Neutrino oscillations

In the Glashow - Weinberg - Salam theory, neutrinos have the following properties:

- The masses of neutrinos are exactly zero,
- There are three flavor neutrinos ν_e, ν_μ, ν_τ,
- Neutrinos and antineutrinos are distinct,
- The left-handed neutrinos and right-handed antineutrinos are active only.

If the neutrinos are massive, the three lepton families could mix up like the three quark families and the neutrino masses could been revealed by the oscillation phenomenon like the neutral K meson oscillations. Indeed, the observation of the neutrino oscillation are evidences for the fact that neutrinos are mixed and massive. In addition, the CP violation is also observed in the neutrino oscillations. These phenomenon are, however, outside of the explaination of the Standard Model.

1.1 Neutrino oscillations in vacuum

In the theory of neutrino oscillations[6], a neutrino with flavor α and momentum $\overrightarrow{p}$, produced in a charged-current weak interaction process from a charged lepton or together with a charged antilepton, is described by the flavor state

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle \qquad (\alpha = e, \mu, \tau) \tag{1.1}$$

where U is the neutrino mixing matrix, $|\nu_j\rangle$ is the massive neutrino state, having mass $m_j \neq 0$.

The number of massive neutrinos are not limited and must be equal to or greater than three since there are three active flavor neutrinos, corresponding to ν_e, ν_μ, ν_τ.

The massive neutrino states $|\nu_j\rangle$ are eigenstates of the Hamiltonian with eigenvalues E_j

$$\mathcal{H} |\nu_j\rangle = E_j |\nu_j\rangle \tag{1.2}$$

$$E_j = \sqrt{\overrightarrow{p}^2 + m_j^2} \tag{1.3}$$

We consider the Schrödinger equation

$$i\frac{d}{dt} |\nu_j(t)\rangle = \mathcal{H} |\nu_j(t)\rangle \tag{1.4}$$

$$\Rightarrow |\nu_j(t)\rangle = e^{-iE_jt} |\nu_j\rangle \tag{1.5}$$

From (1.1) and (1.5), we have

$$|\nu_\alpha(t)\rangle = \sum_j U_{\alpha j}^* e^{-iE_jt} |\nu_j\rangle \tag{1.6}$$

such that

$$|\nu_\alpha(t=0)\rangle = |\nu_\alpha\rangle \tag{1.7}$$

The massive neutrino states can be written in terms of flavor states using the relation

$$U^\dagger U = I \Leftrightarrow \sum_\alpha U_{\alpha j}^* U_{\alpha k} = \delta_{kj} \tag{1.8}$$

Hence,

$$|\nu_j\rangle = \sum_\alpha U_{\alpha j} |\nu_\alpha\rangle \tag{1.9}$$

Substituting (1.9) into (1.6), we get

$$|\nu_\alpha(t)\rangle = \sum_{\beta=e,\mu,\tau} \left(\sum_j U_{\alpha j}^* e^{-iE_jt} U_{\beta j} \right) |\nu_\beta\rangle \tag{1.10}$$

The amplitude of $\nu_\alpha \to \nu_\beta$ transitions is

$$A_{\nu_\alpha \to \nu_\beta}(t) \equiv \langle \nu_\beta | \nu_\alpha(t) \rangle = \sum_j U_{\alpha j}^* U_{\beta j} e^{-iE_jt} \tag{1.11}$$

Then we obtain the transition probability

$$P_{\nu_\alpha\to\nu_\beta}(t) = \left|A_{\nu_\alpha\to\nu_\beta}(t)\right|^2 = \sum_{j,k} U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j}e^{-i(E_j-E_k)t} \tag{1.12}$$

For ultrarelativistic neutrino, we have

$$E_j \approx E + \frac{m_j^2}{2E} \tag{1.13}$$

$$E_j - E_k \approx \frac{\Delta m_{jk}^2}{2E} \tag{1.14}$$

where Δm_{jk}^2 is the squared-mass difference

$$\Delta m_{jk}^2 = m_j^2 - m_k^2 \tag{1.15}$$

and E is the neutrino energy

$$E \approx |\overrightarrow{p}| \tag{1.16}$$

The transition probability is approximated by

$$P_{\nu_\alpha\to\nu_\beta}(t) = \sum_{j,k} U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j}\exp\left(-i\frac{\Delta m_{jk}^2 t}{2E}\right) \tag{1.17}$$

The propagation time t can be approximated by the distance L between the source and the detector, $t \approx L$. Therefore,

$$P_{\nu_\alpha\to\nu_\beta}(L,E) = \sum_{j,k} U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j}\exp\left(-i\frac{\Delta m_{jk}^2 L}{2E}\right) \tag{1.18}$$

We have the unitarity relation

$$UU^\dagger = I \Leftrightarrow \sum_j U_{\alpha j}U^*_{\beta j} = \delta_{\alpha\beta} \tag{1.19}$$

Using (1.19), we get

$$\begin{aligned}
&\left(\sum_k U_{\alpha k}U^*_{\beta k}\right)\sum_j U^*_{\alpha j}U_{\beta j} = \delta_{\alpha\beta} \\
\Leftrightarrow &\sum_{j,k} U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j} = \delta_{\alpha\beta} \\
\Leftrightarrow &\sum_j |U_{\alpha j}|^2|U_{\alpha j}|^2 + 2\sum_{j>k}\mathfrak{Re}\left[U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j}\right] = \delta_{\alpha\beta} \\
\Leftrightarrow &\sum_j |U_{\alpha j}|^2|U_{\alpha j}|^2 = \delta_{\alpha\beta} - 2\sum_{j>k}\mathfrak{Re}\left[U_{\alpha k}U^*_{\beta k}U^*_{\alpha j}U_{\beta j}\right]
\end{aligned} \tag{1.20}$$

The probability is written in the form

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_j |U_{\alpha j}|^2 |U_{\alpha j}|^2 + 2\sum_{j>k} \mathfrak{Re}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(-i\frac{\Delta m^2_{jk} L}{2E}\right)\right] \tag{1.21}$$

Hence,

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \sum_j |U_{\alpha j}|^2 |U_{\alpha j}|^2 + 2\sum_{j>k} \mathfrak{Re}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \cos\left(\frac{\Delta m^2_{jk} L}{2E}\right) + 2\sum_{j>k} \mathfrak{Im}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.22}$$

Substituting (1.20) into (1.22), then

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 2\sum_{j>k} \mathfrak{Re}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \left[1 - \cos\left(\frac{\Delta m^2_{jk} L}{2E}\right)\right] + 2\sum_{j>k} \mathfrak{Im}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.23}$$

Therefore,

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{j>k} \mathfrak{Re}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right) + 2\sum_{j>k} \mathfrak{Im}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.24}$$

The oscillation probabilities with $\alpha \neq \beta$ are usually called *transition probabilities* and with $\alpha = \beta$ are usually called *survival probabilities.*

There are two rules of the conservation of probability[6] in which the oscillation probabilities satisfy:

- In the transitions from a flavor neutrino ν_α to all flavor neutrinos ν_β (including $\alpha = \beta$), the sum of the probabilities is

$$\sum_\beta P_{\nu_\alpha \to \nu_\beta}(L,E) = 1 \tag{1.25}$$

- In the transitions from any flavor neutrino ν_α to a flavor neutrino ν_β (including $\alpha = \beta$), the sum of the probabilities is

$$\sum_\alpha P_{\nu_\alpha \to \nu_\beta}(L,E) = 1 \tag{1.26}$$

For example, we introduce two massive neutrino states ν_1 and ν_2 such that

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{1.27}$$

The states ν_1 and ν_2 at the time $t > 0$ is given by

$$\nu_1(t) = e^{-iE_1 t}\nu_1(0), \nu_2(t) = e^{-iE_2 t}\nu_2(0) \tag{1.28}$$

where $E_i^2 = \overrightarrow{p}^2 + m_i^2, i = 1, 2$ and since $m_i \ll |p_i|$ then $|p_i| \approx E$, $E_i \approx E + m_i^2/2E$
Substituting (1.28) into (1.27) we get

$$\nu_e(t) = \left(e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta\right)\nu_e(0) + \cos\theta\sin\theta\left(e^{-iE_2 t} - e^{-iE_1 t}\right)\nu_\mu(0) \tag{1.29}$$
$$\nu_\mu(t) = \left(e^{-iE_1 t}\sin^2\theta + e^{-iE_2 t}\cos^2\theta\right)\nu_\mu(0) + \sin\theta\cos\theta\left(e^{-iE_2 t} - e^{-iE_1 t}\right)\nu_e(0) \tag{1.30}$$

The probability for ν_μ at $t = 0$ remains ν_μ[9] at $t > 0$ is

$$P_1 = |\langle\nu_\mu(t)|\nu_\mu(0)\rangle|^2 = 1 - \frac{1}{2}\sin^2 2\theta + \frac{1}{2}\sin^2 2\theta\cos\left(\frac{\Delta m_{21}^2}{2E}t\right) \tag{1.31}$$

where $\Delta m_{21}^2 = m_2^2 - m_1^2$.
The probability for ν_μ at $t = 0$ to be converted into ν_e at $t > 0$ is

$$P_2 = |\langle\nu_e(t)|\nu_\mu(0)\rangle|^2 = \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\sin^2 2\theta\cos\left(\frac{\Delta m_{21}^2}{2E}t\right) \tag{1.32}$$

We have the approximation $t \approx L$. Hence,

$$P_1(L, E) = 1 - \frac{1}{2}\sin^2 2\theta + \frac{1}{2}\sin^2 2\theta\cos\left(\frac{\Delta m_{21}^2}{2E}L\right) \tag{1.33}$$
$$P_2(L, E) = \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\sin^2 2\theta\cos\left(\frac{\Delta m_{21}^2}{2E}L\right) \tag{1.34}$$

The conditions for oscillations[9] are θ and Δm_{21}^2 have nonzero values, the traveling distance L of the Neutrinos must not be too different from the oscillation length L_0

$$L_0 = \frac{4\pi E}{\left|\Delta m_{21}^2\right|} \tag{1.35}$$

Let us consider the antineutrinos. In this case, we also have

$$|\overline{\nu}_\alpha\rangle = \sum_j U_{\alpha j}|\overline{\nu}_j\rangle \qquad (\alpha = e, \mu, \tau) \tag{1.36}$$

With the same way in the neutrino oscillation probability, we obtain the antineutrino oscillation, $\overline{\nu}_\alpha \to \overline{\nu}_\beta$, probability

$$P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(L,E) = \sum_{j,k} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \exp\left(-i\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.37}$$

Then,

$$P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(L,E) = \sum_{j,k} U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \exp\left(i\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.38}$$

Finally, we have

$$P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(L,E) = \delta_{\alpha\beta} - 4\sum_{j>k} \mathfrak{Re}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right)$$
$$- 2\sum_{j>k} \mathfrak{Im}\left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}\right] \sin\left(\frac{\Delta m^2_{jk} L}{2E}\right) \tag{1.39}$$

1.2 Neutrino oscillations in matter

In the experiments, when neutrinos propagate, they interact with the particles forming the matter[6, 13]. The Hamiltonian of the neutrino system now is

$$\mathcal{H}_m = \mathcal{H}_0 + \mathcal{H}_{int} \tag{1.40}$$

where $\mathcal{H}_0$ is the Hamiltonian in vacuum and $\mathcal{H}_{int}$ is the Hamiltonian describes the interaction between the neutrinos and the particles of matter. Let us consider an ultrarelativistic neutrino with flavor α ($\alpha = e, \mu, \tau$) described by the flavor state

$$|\nu_\alpha\rangle = \sum_j U^*_{\alpha j} |\nu_j\rangle \tag{1.41}$$

The massive neutrino state $|\nu_j\rangle$, with momentum $\overrightarrow{p}$, is an eigenstate of the vacuum Hamiltonian $\mathcal{H}_0$

$$\mathcal{H}_0 |\nu_j\rangle = E_j |\nu_j\rangle \tag{1.42}$$

where $E_j = \sqrt{\overrightarrow{p}^2 + m_j^2}$

We have

$$\mathcal{H}_{int} |\nu_\alpha\rangle = V_\alpha |\nu_\alpha\rangle \tag{1.43}$$

V_α is the effective potential of left-handed neutrino with flavor α propagating through the medium.

The Schrödinger equation

$$i\frac{d}{dt}|\nu_\alpha(t)\rangle = \mathcal{H}_m|\nu_\alpha(t)\rangle \quad \text{with} \quad |\nu_\alpha(0)\rangle = |\nu_\alpha\rangle \tag{1.44}$$

The amplitude of $\nu_\alpha \to \nu_\beta$ transitions is

$$\mathcal{A}_{\alpha\beta}(t) = \left\langle \nu_\beta | \nu_\alpha(t) \right\rangle \tag{1.45}$$

Hence, the transition probability is

$$P_{\nu_\alpha \to \nu_\beta} = \left|\mathcal{A}_{\alpha\beta}(t)\right|^2 \tag{1.46}$$

We have from (1.44)

$$\begin{aligned} i\frac{d}{dt}\left\langle \nu_\beta | \nu_\alpha(t) \right\rangle &= \left\langle \nu_\beta | \mathcal{H}_m | \nu_\alpha(t) \right\rangle \\ \Rightarrow i\frac{d}{dt}\mathcal{A}_{\alpha\beta}(t) &= \left\langle \nu_\beta | \mathcal{H}_0 | \nu_\alpha(t) \right\rangle + \left\langle \nu_\beta | \mathcal{H}_{int} | \nu_\alpha(t) \right\rangle \end{aligned}$$

Note that

$$\left\langle \nu_\beta \right| \mathcal{H}_0 = \sum_j U_{\beta j} \left\langle \nu_j \right| \mathcal{H}_0 \tag{1.47}$$

$$\left\langle \nu_j \right| \mathcal{H}_0 = E_j \left\langle \nu_j \right| \tag{1.48}$$

$$\left\langle \nu_j \right| = \sum_\zeta U^*_{\zeta j} \left\langle \nu_\zeta \right| \tag{1.49}$$

$$\left\langle \nu_\beta \right| \mathcal{H}_{int} = V_\beta \left\langle \nu_\beta \right| = \sum_\zeta \delta_{\beta\zeta} V_\beta \left\langle \nu_\zeta \right| \tag{1.50}$$

Then,

$$\begin{aligned} i\frac{d}{dt}\mathcal{A}_{\alpha\beta}(t) &= \sum_j U_{\beta j} E_j \sum_\zeta U^*_{\zeta j} \left\langle \nu_\zeta | \nu_\alpha(t) \right\rangle + \sum_\zeta \delta_{\beta\zeta} V_\beta \left\langle \nu_\zeta | \nu_\alpha(t) \right\rangle \\ &= \sum_\zeta \left(\sum_j U_{\beta j} E_j U^*_{\zeta j} + \delta_{\beta\zeta} V_\beta \right) \left\langle \nu_\zeta | \nu_\alpha(t) \right\rangle \end{aligned} \tag{1.51}$$

Therefore,

$$i\frac{d}{dt}\mathcal{A}_{\alpha\beta}(t) = \sum_\zeta \left(\sum_j U_{\beta j} E_j U^*_{\zeta j} + \delta_{\beta\zeta} V_\beta \right) \mathcal{A}_{\alpha\zeta}(t) \tag{1.52}$$

For ultrarelativistic neutrinos[6],

$$E_j \approx E + \frac{m_j^2}{2E}, \; p \approx E, \; t \approx x \tag{1.53}$$

where x is the distance from the source.

Then (1.52) can be written in the form as

$$i\frac{d}{dx}\mathbf{A}_\alpha = \mathcal{H}_F\mathbf{A}_\alpha \tag{1.54}$$

For instance, we consider the two neutrino mixing between ν_e, ν_μ and ν_1, ν_2. The initial neutrino is an electron neutrino ($\alpha = e$). The system of evolution equations describing the $\nu_\alpha \leftrightarrow \nu_\beta$ oscillation in matter:

$$i\frac{d}{dx}\begin{pmatrix}\mathcal{A}_{ee}\\ \mathcal{A}_{e\mu}\end{pmatrix} = \frac{1}{4E}\begin{pmatrix}-\Delta m^2\cos 2\theta + A_{CC} & \Delta m^2 \sin 2\theta\\ \Delta m^2\sin 2\theta & \Delta m^2\cos 2\theta - A_{CC}\end{pmatrix}\begin{pmatrix}\mathcal{A}_{ee}\\ \mathcal{A}_{e\mu}\end{pmatrix} \tag{1.55}$$

where $\Delta m^2 = m_2^2 - m_1^2$ and θ is the mixing angle

$A_{CC} = 2\sqrt{2}EG_FN_e(t) \approx 2\sqrt{2}EG_FN_e(x)$, G_F is the Fermi constant, N_e is the electron density in matter.

The initial condition of the equations (1.55) is

$$\begin{pmatrix}\mathcal{A}_{ee}(0)\\ \mathcal{A}_{e\mu}(0)\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} \tag{1.56}$$

The system of equations (1.55) can always be solved in numerical methods. The transition and survival probabilities are

$$P_{\nu_e\to\nu_\mu}(x) = |\mathcal{A}_{e\mu}(x)|^2, P_{\nu_e\to\nu_e}(x) = |\mathcal{A}_{ee}(x)|^2 \tag{1.57}$$

In the case of the oscillations in matter with constant density: $N_e(x) = const$, we have

$$\mathcal{A}_{ee}(x) = \cos\left(\frac{\Delta m_M^2}{4E}x\right) + i\sin\left(\frac{\Delta m_M^2}{4E}x\right)\cos 2\theta_M \tag{1.58}$$

$$\mathcal{A}_{e\mu}(x) = -i\sin\left(\frac{\Delta m_M^2}{4E}x\right)\sin 2\theta_M \tag{1.59}$$

where

$$\Delta m_M^2 = \sqrt{(\Delta m^2\cos 2\theta - A_{CC})^2 + (\Delta m^2 \sin 2\theta)^2} \tag{1.60}$$

is the squared-mass difference in matter. The mixing angle in matter θ_M is given by

$$\sin 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m_M^2}, \cos 2\theta_M = \frac{\Delta m^2\cos 2\theta - A_{CC}}{\Delta m_M^2} \tag{1.61}$$

Hence, the transition probability is

$$P_{\nu_e \to \nu_\mu}(x) = \sin^2\left(\frac{\Delta m_M^2}{4E}x\right)\sin^2 2\theta_M \tag{1.62}$$

and the survival probability is

$$P_{\nu_e \to \nu_e}(x) = \cos^2\left(\frac{\Delta m_M^2}{4E}x\right) + \sin^2\left(\frac{\Delta m_M^2}{4E}x\right)\cos^2 2\theta_M \tag{1.63}$$

In order to determine the parameters, especially CP phases, in the mixing matrix from the data with high precision, we need the analytic expressions for the oscillation probabilities in matter which should reveal the dependence of the probabilities on the parameters as well as on the experiment characteristics and then the experimental setup[1]. The probabilities are expanded in $\alpha = \Delta m_{21}^2/\Delta m_{31}^2, \sin\theta_{13}$ or in both of them based upon the approximations $\alpha, \sin\theta_{13} \ll 1$.
In my thesis, we use the expansion formulas for three-flavor neutrino oscillation probabilities in constant matter density up to second order in both α and $\sin\theta_{13}$[1].

$$P(\nu_e \to \nu_e) = 1 - \alpha^2 \sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} - 4\sin^2\theta_{13}\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \tag{1.64}$$

$$\begin{aligned} P(\nu_\mu \to \nu_e) = {} & 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + \\ & + 2\alpha\sin\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(\delta+\Delta)\frac{\sin(A\Delta)\sin[(A-1)\Delta]}{A(A-1)} \\ & + \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} \end{aligned} \tag{1.65}$$

where

$$\alpha = \frac{\Delta m_{21}^2}{\Delta m_{31}^2}, \Delta = \frac{\Delta m_{31}^2 L}{4E}, A = 2\sqrt{2}G_F N_e^{man}\frac{E}{\Delta m_{31}^2} \tag{1.66}$$

The probability of antineutrino oscillation can be obtained from the probability of neutrino oscillation by changing the sign of the A and δ.

1.3 CP violation

Understanding the origin of CP violation is one of the challenges of elementary particle physics. There are Dirac CP phases and Majorana CP phases in the neutrino mixing matrix. We discuss CP phases through neutrino oscillations in vacuum and matter.

Under the CP transformation, neutrinos become antineutrinos:

$$\nu_\alpha \overset{CP}{\leftrightarrows} \overline{\nu}_\alpha \tag{1.67}$$

and the $\nu_\alpha \to \nu_\beta$ channel becomes the $\overline{\nu}_\alpha \to \overline{\nu}_\beta$ channel. The condition of CP invariance[6] is:

$$P_{\nu_\alpha \to \nu_\beta} = P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta} \tag{1.68}$$

Then the CP violation in neutrino oscillation experiments can be measured in transition between different flavors and could reveal by measuring the asymmetry:

$$A^{CP}_{\alpha\beta} = P_{\nu_\alpha \to \nu_\beta} - P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta} \tag{1.69}$$

In vacuum, from (1.24) and (1.39), we obtain

$$A^{CP}_{\alpha\beta} = 4 \sum_{j>k} \mathfrak{Im} \left[U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j} \right] \sin \left(\frac{\Delta m^2_{jk} L}{2E} \right) \tag{1.70}$$

It can be seen that the quartic products $U_{\alpha k} U^*_{\beta k} U^*_{\alpha j} U_{\beta j}$ depend only on the Dirac phases in the mixing matrix, then CP violation depends only on the Dirac phases in the mixing matrix.

For instance, in the case of three neutrinos mixing, the unitary mixing matrix:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = U^D D^M \tag{1.71}$$

$$U^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{1.72}$$

$$D^M = diag(1, e^{i\lambda_2}, e^{i\lambda_3}) \tag{1.73}$$

where $c_{mn} = \cos\theta_{mn}$, $s_{mn} = \sin\theta_{mn}$, θ_{mn} are the three mixing angles ($0 \leqslant \theta_{mn} \leqslant \pi/2$) ($m, n = 1, 2, 3; m \neq n$), δ is the Dirac CP violation phase ($0 \leqslant \delta \leqslant 2\pi$) and λ_2, λ_3 are the two physical Majorana CP violation phases.
Substituting the mixing matrix into (1.70), we have (see Appendix B)

$$A^{CP}_{e\mu} = 2 \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} \tag{1.74}$$

where

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E} L \quad i = 2,3, j = 1,2 \tag{1.75}$$

In matter, the oscillation probability expressions appear matter effect terms so CP violation depends not only on the Dirac phases but also the matter effect terms. For example, the formula (1.65) is the $\nu_\mu \to \nu_e$ transition probability

$$\begin{aligned} P(\nu_\mu \to \nu_e) = 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + \\ + 2\alpha\sin\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(\delta+\Delta)\frac{\sin(A\Delta)\sin[(A-1)\Delta]}{A(A-1)} \\ + \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} \end{aligned} \tag{1.76}$$

and the $\overline{\nu}_\mu \to \overline{\nu}_e$ transition probability

$$\begin{aligned} P(\overline{\nu}_\mu \to \overline{\nu}_e) = 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2[(A+1)\Delta]}{(A+1)^2} + \\ + 2\alpha\sin\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(-\delta+\Delta)\frac{\sin(A\Delta)\sin[(A+1)\Delta]}{A(A+1)} \\ + \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} \end{aligned} \tag{1.77}$$

We derive

$$\begin{aligned} A_{\mu e}^{CP} =& P(\nu_\mu \to \nu_e) - P(\overline{\nu}_\mu \to \overline{\nu}_e) \\ =& 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + \\ &+ 2\alpha\sin\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(\delta+\Delta)\frac{\sin(A\Delta)\sin[(A-1)\Delta]}{A(A-1)} \\ &- 4\sin^2\theta_{13}\sin^2\theta_{23}\frac{\sin^2[(A+1)\Delta]}{(A+1)^2} \\ &- 2\alpha\sin\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\cos(-\delta+\Delta)\frac{\sin(A\Delta)\sin[(A+1)\Delta]}{A(A+1)} \end{aligned} \tag{1.78}$$

Chapter 2

Neutrino experiments

2.1 Neutrino sources

2.1.1 Astrophysical neutrinos

Solar neutrinos

The solar energy is generated by the process of fusing hydrogen to helium through some of the fusion reactions in the pp chain or CNO cycle and then the solar neutrinos are produced as pure electron neutrinos. The solar neutrino flux is about 60 billion per square centimeter per second. However, the neutrinos have very low energy[16] (< 0.42 MeV) so the neutrino detectors have to be sensitive to such low energy neutrinos. Solar neutrinos have been instrumental in studying neutrino oscillations and from solar neutrino experiments, the data we get is still a key input into calculations of neutrino mixing angles.

Supernova neutrinos

Two main types of supernovae are *Type Ia supernovae*, not expected to produce many neutrinos, and *Core-collapse supernovae*[16]. We know 99% of the energy released by a core-collapse supernova comes in the form of neutrinos.neutrino production in core-collapse supernovae is an essential ingredient in understanding the chemical evolution of the Galaxy. Furthermore, the neutrinos come from much deeper in the stellar collapse than the light, and therefore have the potential to provide information from much closer to the initial neutron star formation.

Atmospheric neutrinos

The cosmic ray particles, mainly protons, from space hit the atmosphere, high energy protons interact with air molecules to produce pions and also kaons[16]. After that, pions decay to muons and muon neutrinos, muons decay to muon neutrinos and electron neutrinos, kaons decay to muons or muon neutrinos or electron neutrinos. Atmospheric neutrinos have a wide range of energies, so it is good to provide a means of studying neutrino oscillations.

2.1.2 Artificial neutrinos

Accelerator neutrinos

At a high energy proton accelerator, to produce neutrino beams, a proton beam is extracted from the accelerator and directed onto a nuclear target. From the collisions, mesons are produced then decay to produce neutrinos (antineutrinos), with high energy collisions, pions (kaons) are dominantly produced and also decay to produce neutrinos (antineutrinos). The neutrino energy spectrum can be calculated from the beam parameters and (or) derived from the measured muon spectrum[16].

Reactor neutrinos

There are mainly nuclear fission of four heavy isotopes, ^{235}U, ^{238}U, ^{239}Pu and ^{241}Pu [16]. Mostly pure electron antineutrinos are produced from β decays of fission products. Modern reactor experiments use a near detector and a far detector to get more neutrino information.

2.2 Detecting neutrinos

Neutrinos can be detected via their interations with detectors. There are two ways that neutrinos interact: *charged current interactions*, the neutrinos convert into the corresponding charged leptons through the exchange of $W^{\pm}$, and *neutral current interactions*, the neutrino remains a neutrino through the exchange of Z^0.

Because electrons and muons are easy to identify, charged current interactions are easier to detect, also if we have the signal of an electron then it came from an electron neutrino, but it must be enough energy to allow the lepton to be created. Depending on the requirements of the particular study, the features of a neutrino experiment are follow: low energy threshold, good angular resolution, good particle identification, good energy measurement, good time resolution, charge identification[16].

2.3 Neutrino oscillation experiments

As we know, there are two types of neutrino oscillation experiments[6]:

- **Appearance experiments** Measuring transitions between different neutrino flavors. The final flavor is not present in the initial beam, if so, the background can be very small. The experiment can be sensitive to small values of mixing angles.

- **Disappearance experiments** Measuring the transitions between same neutrino flavors. Since the number of events fluctuate statistically, it is very difficult to reveal a small disappearance. Hence, this type is hard to detect small values of mixing angles.

Furthermore, since the value of Δm^2 is fixed by nature[6], experiments can be construct to be sensitive to different values of Δm^2 by choosing appropriate values of L/E. In an experiment, the value of Δm^2 such that

$$\frac{\Delta m^2 L}{2E} \sim 1 \tag{2.1}$$

is so-called sensitivity to Δm^2.

Depending on the average value of the ratio L/E for an experiment, related to the sensitivity, there are three types of the experiments: Short Baseline experiments, Long Baseline experiments, Very Long-Baseline experiments.

Chapter 3

Parameter degeneracies

In this chapter, we assume that the input true solution is (s_1, δ_1). The sign of Δm^2_{31}, we consider $\Delta m^2_{31} > 0$ known as normal hierarchy (NH) and $\Delta m^2_{31} < 0$ known as inverted hierarchy (IH), $\theta_{23} > 45^o$ is called higher octant (HO) and $\theta_{23} < 45^o$ is called lower octant (LO) $(\theta_{23} \neq \pi/4)$[2, 5, 10, 11]. We also discuss the probability of $\nu_\mu \to \nu_e$ transition and $\overline{\nu}_\mu \to \overline{\nu}_e$ transition to look for the resolution of the degeneracies.

3.1 Intrinsic degeneracy

To determine δ_{CP} and θ_{13}, we measure the oscillation probabilities $P(\nu_\alpha \to \nu_\beta)$ $(\alpha, \beta = e, \mu, \tau)$, for example $P(\nu_\mu \to \nu_e)$, in the neutrino experiments with fixed baseline (L) and energy (E). If we get the value of one oscillation probability from the data, we would obtain the equation which has a continuous number $(\delta_{CP}, \theta_{13})$ solution[14]. Figure 3.1 is an equiprobability curve of $P(\nu_\mu \to \nu_e)$, assuming from the data $P(\nu_\mu \to \nu_e) = 0.05$, shows the correlation of δ_{CP} and θ_{13}.

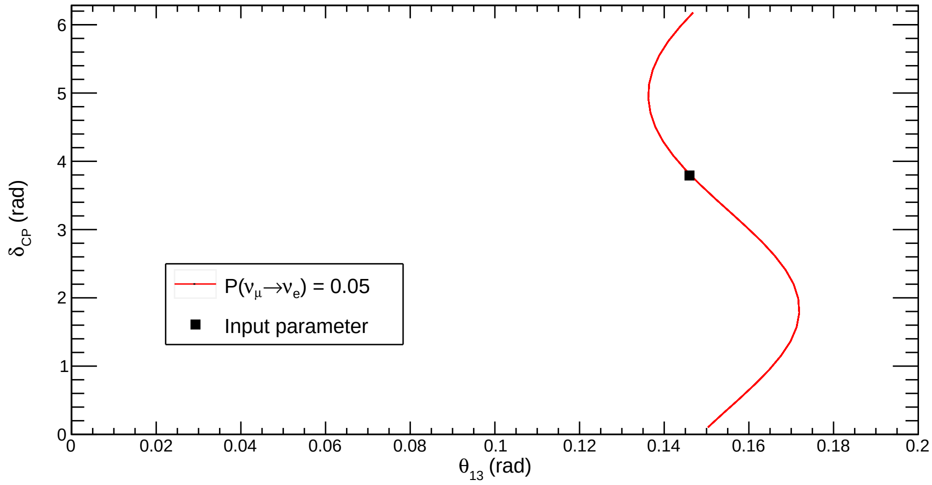


Figure 3.1: Equiprobability curve for intrinsic degeneracy

Input parameter point: δ_{CP} = 3.79, $\sin^2\theta_{13}$ = 0.0212 and other parameters[13] in the above figure: L = 810 km, E_ν = 2 GeV, $\sin^2\theta_{12}$ = 0.297, $\sin^2\theta_{23}$ = 0.425, Δm^2_{21} = 7.37×10^{-5}eV^2, Δm^2_{31} = 2.56×10^{-3}eV^2

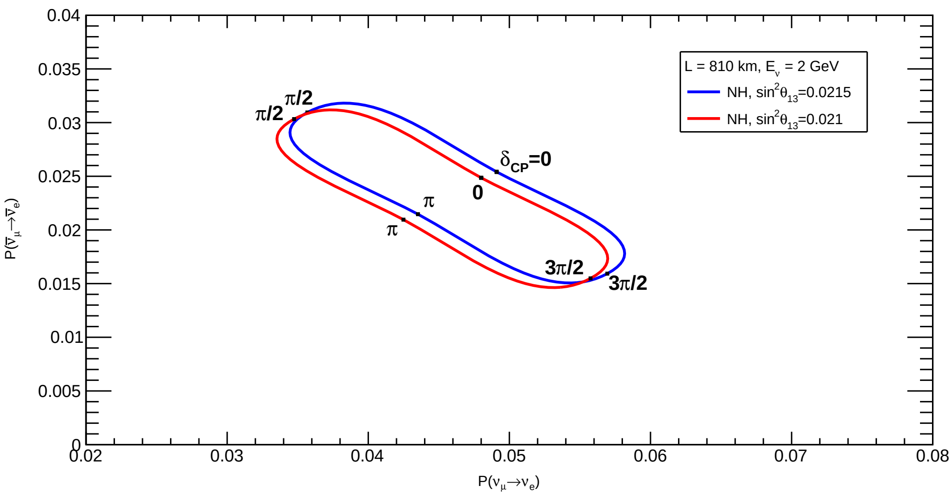


Figure 3.2: (δ, θ_{13}) ambiguity with L = 810 km, E_ν = 2 GeV, $\sin^2\theta_{12}$ = 0.297, $\sin^2\theta_{23}$ = 0.425, Δm^2_{21} = 7.37×10^{-5}eV^2, Δm^2_{31} = 2.56×10^{-3}eV^2

In many cases, there are two or more value of δ_{CP} and θ_{13} can give the same values of the probabilities for fixed values of the other parameters, it is known as the intrinsic degeneracy or $(\delta_{CP}, \theta_{13})$ degeneracy. The figure 3.2 is an example of intrinsic degeneracy. It is two trajectories on the $P - \overline{P}$ plane which drawn by varying the CP violating phase, δ, from 0 to 2π and there are two crossing points of the two trajectories corresponding with $\sin^2\theta_{13} = 0.0215$ and $\sin^2\theta_{13} = 0.021$ as in the figure.

From formula (1.65), we can write $P \equiv P(\nu_\mu \to \nu_e)$ as a simple form[11]

$$P = Xs^2 + Ys\cos(\delta + \Delta) + Z \tag{3.1}$$

where $s = \sin\theta_{13}$. The functions X, Y, Z are

$$X = 4\sin^2\theta_{23}\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \tag{3.2}$$

$$Y = 2\alpha\sin 2\theta_{12}\sin 2\theta_{23}\frac{\sin(A\Delta)\sin[(A-1)\Delta]}{A(A-1)} \tag{3.3}$$

$$Z = \alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} \tag{3.4}$$

For the normal hierarchy and lower octant, in the $\nu_\mu \to \nu_e$ channel, from (3.1), the intrinsic degeneracy solutions (δ_j, s_j) $(j = 1, 2)$ are defined by

$$P = Xs_1^2 + Ys_1\cos(\delta_1 + \Delta) + Z \tag{3.5}$$

$$P = Xs_2^2 + Ys_2\cos(\delta_2 + \Delta) + Z \tag{3.6}$$

In the $\overline{\nu}_\mu \to \overline{\nu}_e$ channel, we also have

$$\overline{P} = X^+s_1^2 + Y^+s_1\cos(-\delta_1 + \Delta) + Z \tag{3.7}$$

$$\overline{P} = X^+s_2^2 + Y^+s_2\cos(-\delta_2 + \Delta) + Z \tag{3.8}$$

where $\overline{P} \equiv P(\overline{\nu}_\mu \to \overline{\nu}_e)$

$$X^+ = 4\sin^2\theta_{23}\frac{\sin^2[(A+1)\Delta]}{(A+1)^2} \tag{3.9}$$

$$Y^+ = 2\alpha\sin 2\theta_{12}\sin 2\theta_{23}\frac{\sin(A\Delta)\sin[(1+A)\Delta]}{A(1+A)} \tag{3.10}$$

From equations (3.5) to (3.8), we obtain

$$\frac{X}{Y}(s_1^2 - s_2^2) + s_1\cos(\delta_1 + \Delta) - s_2\cos(\delta_2 + \Delta) = 0 \tag{3.11}$$

$$\frac{X^+}{Y^+}(s_1^2 - s_2^2) + s_1\cos(-\delta_1 + \Delta) - s_2\cos(-\delta_2 + \Delta) = 0 \tag{3.12}$$

Hence, the expressions of $\cos\delta_2$ and $\sin\delta_2$ are

$$s_2\cos\delta_2 = \frac{C^+}{2\cos\Delta}(s_1^2 - s_2^2) + s_1\cos\delta_1 \tag{3.13}$$

$$s_2\sin\delta_2 = \frac{C^-}{2\sin\Delta}(s_1^2 - s_2^2) + s_1\sin\delta_1 \tag{3.14}$$

where

$$C^+ = \frac{X^+}{Y^+} + \frac{X}{Y}, C^- = \frac{X^+}{Y^+} - \frac{X}{Y} \tag{3.15}$$

Therefore, the clone solution ($s_2 \neq s_1$) is

$$s_2^2 = s_1^2 + \left[1 + s_1\left(\frac{\cos\delta_1}{\cos\Delta}C^+ + \frac{\sin\delta_1}{\sin\Delta}C^-\right)\right]\frac{\sin^2 2\Delta}{(C^+\sin\Delta)^2 + (C^-\cos\Delta)^2} \tag{3.16}$$

Considering equations (3.16) and (3.14), when $\Delta \neq n\pi/2 \ \ (n \in \mathbb{Z})$, if $\sin\delta_1 = 0$ then $\sin\delta_2 \neq 0$ if $C_- \neq 0$. It means that we can get (δ, θ_{13}) degeneracy with the confusion between CP violating and CP conserving solutions.

3.2 Mass hierarchy degeneracy

We still have problem of degeneracy, the sign of Δm^2_{31}, beside the intrinsic degeneracy. That is some values of (δ, θ_{13}) with $\Delta m^2_{31} > 0$ and other values of (δ, θ_{13}) with $\Delta m^2_{31} > 0$ can get equal probabilities, it is called mass hierarchy degeneracy[2, 11]. The two figures below give an example of the degeneracy, there are two crossing points of two lines and each point indicates the value of (δ, θ_{13}) that give the same value of the probabilities. In the figure 3.3, the red curve is drawn by $P_{\nu_\mu\to\nu_e}(\theta_{13}, \delta, \Delta m^2_{31})$ equals to 0.001 and the blue one is drawn by equation $P_{\nu_\mu\to\nu_e}(\theta_{13}, \delta, -\Delta m^2_{31}) = 0.001$.

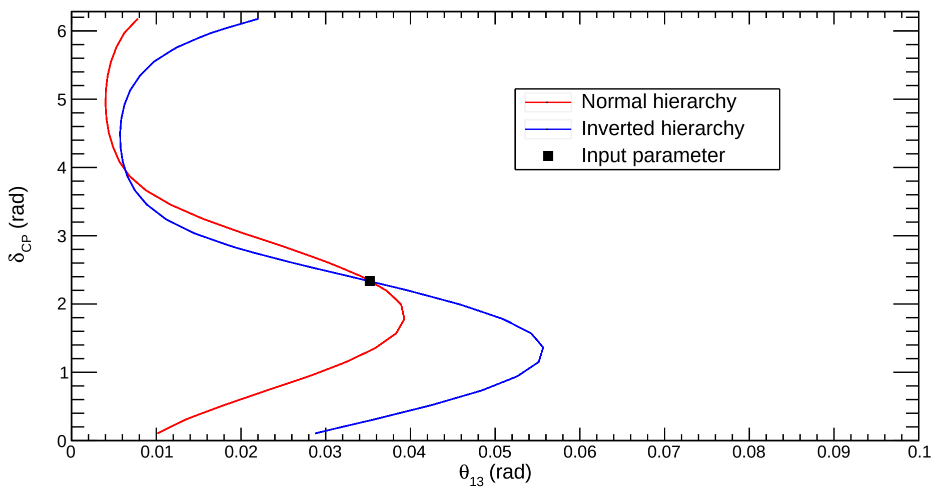


Figure 3.3: Two equiprobability curves for mass hierarchy degeneracy

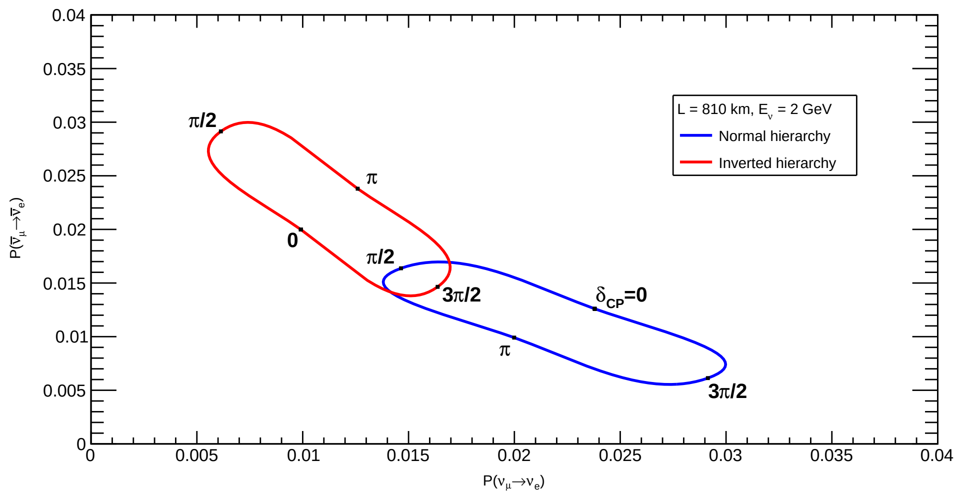


Figure 3.4: Sign of Δm^2_{31} ambiguity with $\sin^2\theta_{13} = 0.01$

Input parameters[13] in the above figures: L = 810 km, E_ν = 2 GeV, $\sin^2\theta_{23} = 0.425$, $\sin^2\theta_{12} = 0.297$, $\Delta m^2_{21} = 7.37\times10^{-5}\text{eV}^2$, $\left|\Delta m^2_{31}\right| = 2.56\times10^{-3}\text{eV}^2$

For lower octant, we assume that the true solution (s_1, δ_1) corresponds with normal hierarchy $(\Delta m^2_{31} > 0)$ and the solution (s_3, δ_3) corresponds with inverted hierarchy $(\Delta m^2_{31} < 0)$ satisfy the equations

$$P = Xs_1^2 + Ys_1\cos(\delta_1 + \Delta) + Z \tag{3.17}$$
$$\overline{P} = X^+s_1^2 + Y^+s_1\cos(-\delta_1 + \Delta) + Z \tag{3.18}$$

$$P = X^+s_3^2 - Y^+s_3\cos(\delta_3 - \Delta) + Z \tag{3.19}$$
$$\overline{P} = Xs_3^2 - Ys_3\cos(\delta_3 + \Delta) + Z \tag{3.20}$$

From the above equations, we have

$$\frac{X}{Y^+}s_1^2 + \frac{Y}{Y^+}s_1\cos(\delta_1 + \Delta) - \frac{X^+}{Y^+}s_3^2 + s_3\cos(\delta_3 - \Delta) = 0 \tag{3.21}$$
$$\frac{X^+}{Y}s_1^2 + \frac{Y^+}{Y}s_1\cos(-\delta_1 + \Delta) - \frac{X}{Y}s_3^2 + s_3\cos(\delta_3 + \Delta) = 0 \tag{3.22}$$

We obtain the expression of $\cos\delta_3$ and $\sin\delta_3$

$$s_3\cos\delta_3 = \frac{1}{2\cos\Delta}\left(C^+s_3^2 - D^+\right) \tag{3.23}$$
$$s_3\sin\delta_3 = \frac{1}{2\sin\Delta}\left(C^-s_3^2 + D^-\right) \tag{3.24}$$

where

$$D^+ = \left(\frac{X}{Y^+} + \frac{X^+}{Y}\right) s_1^2 + \left[\frac{Y}{Y^+}\cos(\delta_1 + \Delta) + \frac{Y^+}{Y}\cos(-\delta_1 + \Delta)\right] s_1 \tag{3.25}$$

$$D^- = \left(\frac{X^+}{Y} - \frac{X}{Y^+}\right) s_1^2 + \left[\frac{Y^+}{Y}\cos(-\delta_1 + \Delta) - \frac{Y}{Y^+}\cos(\delta_1 + \Delta)\right] s_1 \tag{3.26}$$

Hence, we have the equation for s_3

$$Hs_3^4 + Ks_3^2 + N = 0 \tag{3.27}$$

where

$$H = \frac{1}{4\cos^2\Delta}(C^+)^2 + \frac{1}{4\sin^2\Delta}(C^-)^2 \tag{3.28}$$

$$K = \frac{1}{2\sin^2\Delta}C^- D^- - \frac{1}{2\cos^2\Delta}C^+ D^+ - 1 \tag{3.29}$$

$$N = \frac{1}{4\cos^2\Delta}(D^+)^2 + \frac{1}{4\sin^2\Delta}(D^-)^2 \tag{3.30}$$

Equation (3.27) is in the form of biquadratic equation and there would exist real solutions if $K^2 - 4HN > 0$. It means the degeneracy appears.

3.3 θ_{23} octant degeneracy

As mass hierarchy degeneracy, if we have

$$P_{\nu_\alpha \to \nu_\beta}(\theta_{13}, \delta, \theta_{23}) = P_{\nu_\alpha \to \nu_\beta}(\theta'_{13}, \delta', \theta'_{23}) \tag{3.31}$$

and (or)

$$P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(\theta_{13}, \delta, \theta_{23}) = P_{\overline{\nu}_\alpha \to \overline{\nu}_\beta}(\theta'_{13}, \delta', \theta'_{23}) \tag{3.32}$$

where θ_{23} is lower octant and θ'_{23} is higher octant, then it turns out that θ_{23} octant degeneracy would show up.

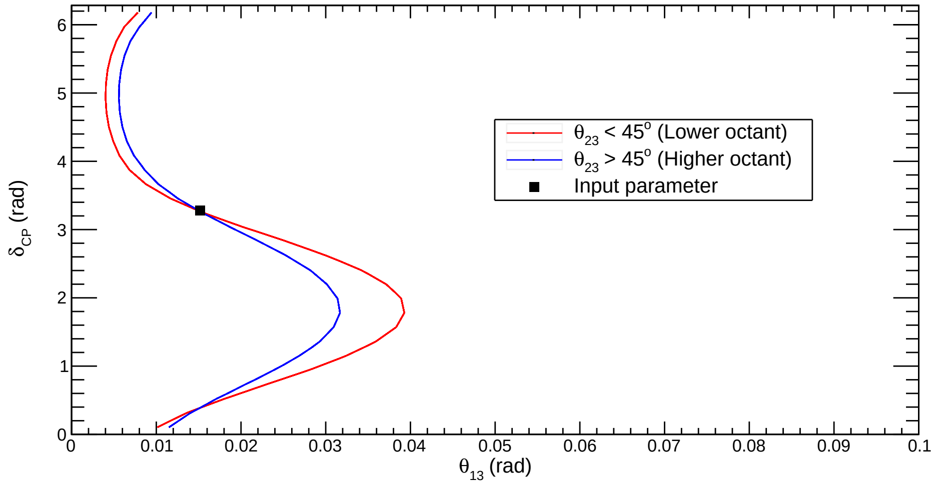


Figure 3.5: Equiprobability curves for θ_{23} octant degeneracy

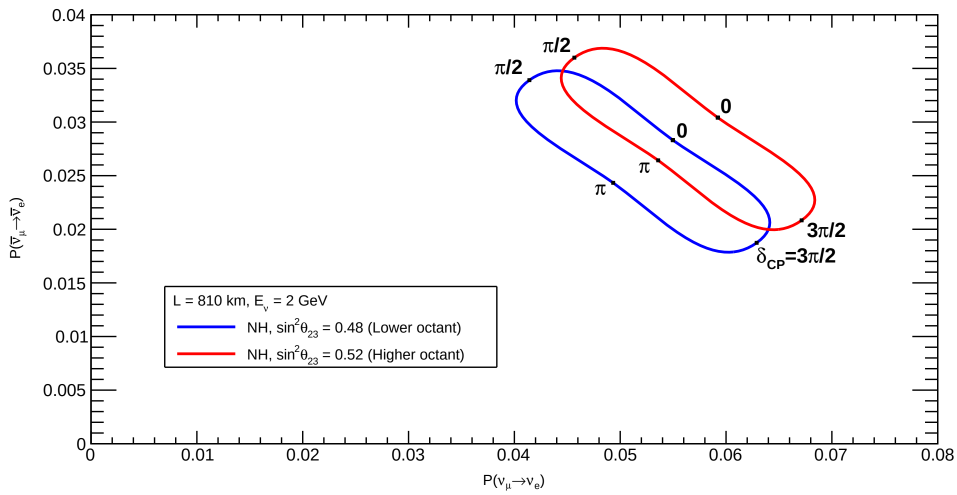


Figure 3.6: θ_{23} octant ambiguity with $\sin^2\theta_{13} = 0.0215$

Input parameters[13] in the above figures: L = 810 km, E_ν = 2 GeV, $\sin^2\theta_{12} = 0.297$, $\Delta m^2_{21} = 7.37{\times}10^{-5}\text{eV}^2$, $\Delta m^2_{31} = 2.56{\times}10^{-3}\text{eV}^2$

For normal hierarchy, we suppose the clone solution in this case is (s_4, δ_4) corresponds with the higher octant ($\theta_{23} > 45^o$), and (s_1, δ_1) with the lower octant ($\theta_{23} < 45^o$)

satisfy the following equations

$$P = Xs_1^2 + Ys_1\cos(\delta_1 + \Delta) + Z \tag{3.33}$$

$$P = X_f s_4^2 + Ys_4\cos(\delta_4 + \Delta) + Z_f \tag{3.34}$$

$$\overline{P} = X^+ s_1^2 + Y^+ s_1\cos(-\delta_1 + \Delta) + Z \tag{3.35}$$

$$\overline{P} = X_f^+ s_4^2 + Y^+ s_4\cos(-\delta_4 + \Delta) + Z_f \tag{3.36}$$

where

$$X_f = 4\sin^2\left(\frac{\pi}{2} - \theta_{23}\right)\frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \tag{3.37}$$

$$Z_f = \alpha^2\cos^2\left(\frac{\pi}{2} - \theta_{23}\right)\sin^2 2\theta_{12}\frac{\sin^2(A\Delta)}{A^2} \tag{3.38}$$

$$X_f^+ = 4\sin^2\left(\frac{\pi}{2} - \theta_{23}\right)\frac{\sin^2[(A+1)\Delta]}{(A+1)^2} \tag{3.39}$$

From the equations (3.33)-(3.36), we use the same way as the previous section to get the following equations

$$s_4\cos\delta_4 = \frac{1}{2\cos\Delta}\left(R^+ - U^+ s_4^2\right) \tag{3.40}$$

$$s_4\sin\delta_4 = \frac{1}{2\sin\Delta}\left(R^- + U^- s_4^2\right) \tag{3.41}$$

where

$$R^+ = C^+ s_1^2 + 2\cos\Delta\cos\delta_1 + \left(Z - Z_f\right)\left(\frac{1}{Y} + \frac{1}{Y^+}\right) \tag{3.42}$$

$$R^- = C^- s_1^2 + 2\sin\Delta\sin\delta_1 + \left(Z - Z_f\right)\left(\frac{1}{Y^+} - \frac{1}{Y}\right) \tag{3.43}$$

$$U^+ = \frac{X_f}{Y} + \frac{X_f^+}{Y^+} \tag{3.44}$$

$$U^- = \frac{X_f}{Y} - \frac{X_f^+}{Y^+} \tag{3.45}$$

Therefore, we have the equation for s_4

$$Ts_4^4 + Vs_4^2 + W = 0 \tag{3.46}$$

where

$$T = \frac{1}{4\cos^2\Delta}(U^+)^2 + \frac{1}{4\sin^2\Delta}(U^-)^2 \tag{3.47}$$

$$V = \frac{1}{2\sin^2\Delta}R^-U^- - \frac{1}{2\cos^2\Delta}R^+U^+ - 1 \tag{3.48}$$

$$W = \frac{1}{4\cos^2\Delta}(R^+)^2 + \frac{1}{4\sin^2\Delta}(R^-)^2 \tag{3.49}$$

Equation (3.46) has real solutions when $V^2 - 4TW > 0$ which means there exists the degeneracy.

Chapter 4

Resolution of parameter degeneracies

4.1 Resolution of intrinsic degeneracy

If we measure the oscillation probabilities $P(\nu_\mu \to \nu_e)$ and $P(\overline{\nu}_\mu \to \overline{\nu}_e)$ with the same L, E and other parameters, the continuous solution would change into two solutions. For example, comparing figure 3.1 with figure 4.1, figure 4.1 shows two equiprobability curves and it turns out that we have two solutions. Besides, there exists the clone solutions that we do not identify exactly which solution is true[14].

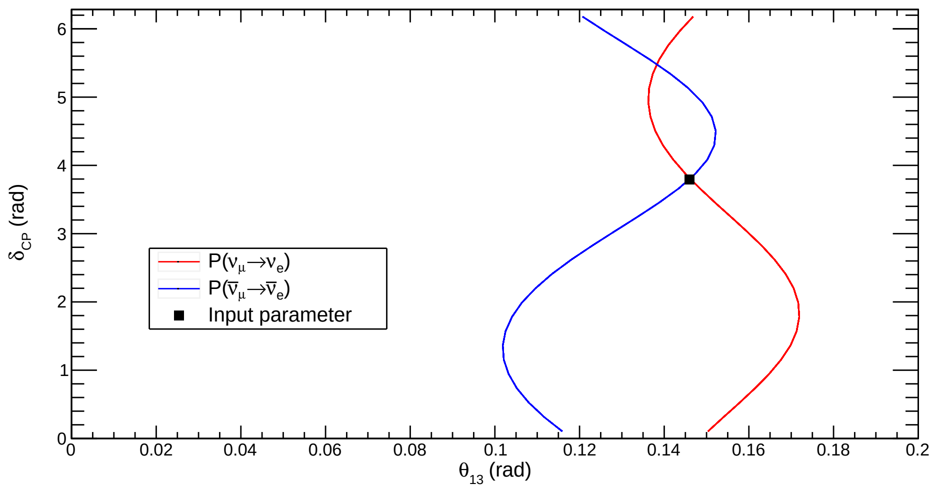


Figure 4.1: Two equiprobability curves for intrinsic degeneracy

Input parameter point: $\delta_{CP} = 3.79$, $\sin^2\theta_{13} = 0.0212$ and other parameters[13] in the above figures: L = 810 km, E_ν = 2 GeV, $\sin^2\theta_{12} = 0.297$, $\sin^2\theta_{23} = 0.425$, $\Delta m^2_{21} = 7.37\times10^{-5}\text{eV}^2$, $\Delta m^2_{31} = 2.56\times10^{-3}\text{eV}^2$

Moreover, from expression (3.16)

$$s_2^2 = s_1^2 + \left[1 + s_1\left(\frac{\cos\delta_1}{\cos\Delta}C^+ + \frac{\sin\delta_1}{\sin\Delta}C^-\right)\right]\frac{\sin^2 2\Delta}{(C^+\sin\Delta)^2 + (C^-\cos\Delta)^2} \tag{4.1}$$

We have $s_2 = s_1$ when $\sin^2 2\Delta = 0$ then $\Delta = n\pi/2$ ($n \in \mathbb{Z}$). Thus θ_{13} is no longer degeneracy. For instance, figure 4.2 indicates the degeneracy resolution using baseline L = 295 km, energy E_ν = 0.6 GeV ($\Delta \approx \pi/2$) (blue curve) and baseline L = 810 km, energy E_ν = 2 GeV (red curve). Comparing figure 4.2 with figure 4.1, we can see the degeneracy is almost eliminated.

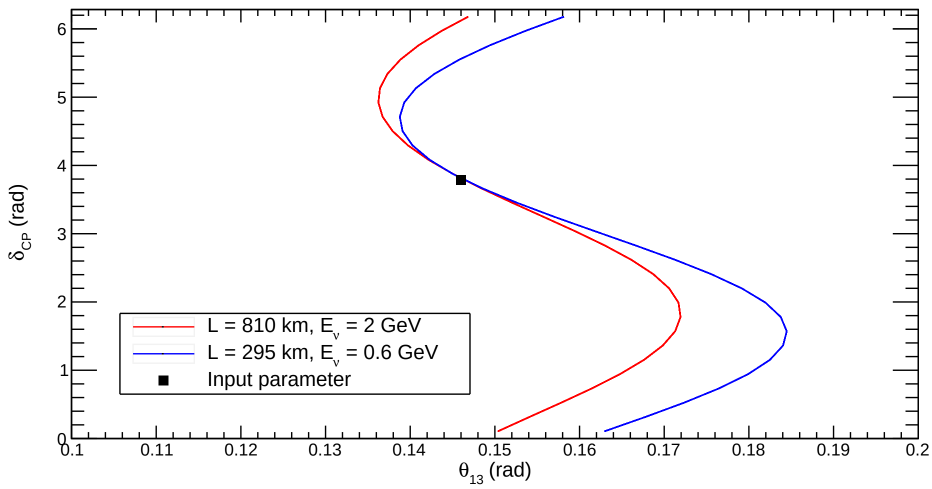


Figure 4.2: Equiprobability curves for resolving intrinsic degeneracy; $\sin^2\theta_{12} = 0.297$, $\sin^2\theta_{23} = 0.425$, $\Delta m^2_{21} = 7.37\times10^{-5}\text{eV}^2$, $\Delta m^2_{31} = 2.56\times10^{-3}\text{eV}^2$, input parameter point: $\delta_{CP} = 3.79$, $\sin^2\theta_{13} = 0.0212$

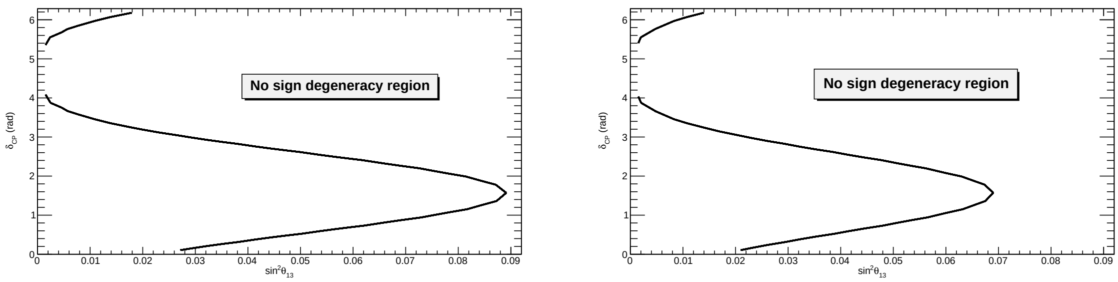
4.2 Resolution of mass hierarchy degeneracy

In this case, we consider equation (3.27) of the clone solution s_3

$$Hs_3^4 + Ks_3^2 + N = 0 \tag{4.2}$$

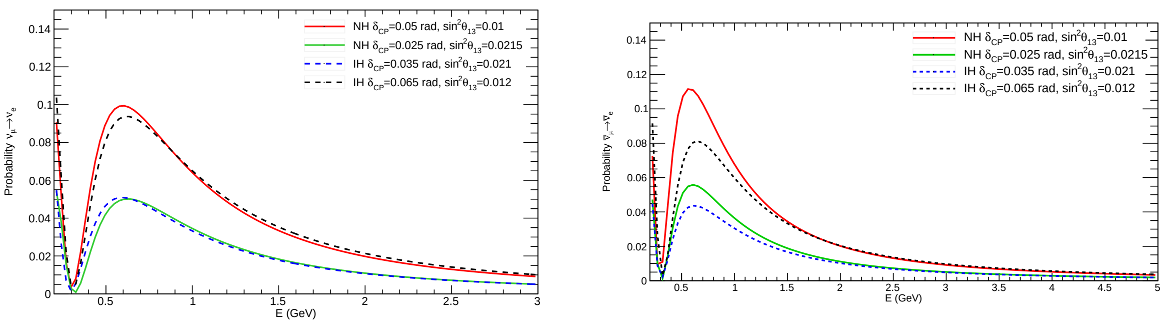
The equation has no real solution if $K^2 - 4HN < 0$. Moreover, mass hierarchy degeneracy would be eliminated if there are not any clone solutions so it can be revealed the region of no sign Δm^2_{31} degeneracy on the $(\sin^2\theta_{13}, \delta)$ plane through the condition $K^2 - 4HN < 0$.

Furthermore, the mass hierarchy degeneracy resolution also depends on the matter effect as shown in the figure 4.3 and 4.4. It can be eliminated the degeneracy by setting up the experiments with the baseline and neutrino energy that should be included a large matter effect.



(a) L=295 km, E=0.6 GeV, N_e=2.2$N_A$cm^{-3} (b) L=295 km, E=0.6 GeV, N_e=2.5$N_A$cm^{-3}

Figure 4.3: The regions of no sign Δm^2_{31} degeneracy in the $\sin^2\theta_{13} - \delta$ space



(a) L = 295 km, N_e = 2.2$N_A$cm^{-3} (b) L = 295 km, N_e = 4$N_A$cm^{-3}

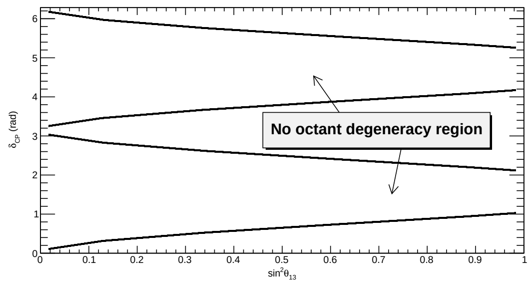
Figure 4.4: The matter effect for sign Δm^2_{31} degeneracy

4.3 Resolution of θ_{23} octant degeneracy

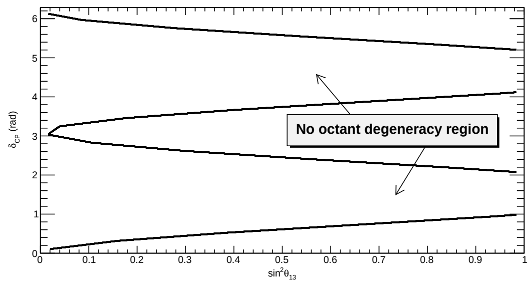
Similar to mass hierarchy degeneracy, we can also obtain no octant degeneracy solution regions. From equation (3.46) of the clone solution s_4

$$Ts_4^4 + Vs_4^2 + W = 0 \tag{4.3}$$

We derive the condition of no real solution is $V^2 - 4TW < 0$. Hence, the regions of no octant degeneracy satisfy $V^2 - 4TW < 0$. Figure 4.5a indicates the resolving degeneracy regions for baseline and energy in T2K experiment and figure 4.5b for baseline and energy in NOνA experiment, they show that δ_{CP} in no degeneracy regions is around $\pi/2$ and $3\pi/2$, octant degeneracy region intersperses with no octant degeneracy region. In the experiments, we measure two channel such as ν_μ disappearance and ν_μ appearance in the same L, E to resolve the degeneracy.



(a) L=295 km, E=0.6 GeV, N_e=2.2$N_A$cm^{-3}



(b) L=810 km, E=2 GeV, N_e=2.2$N_A$cm^{-3}

Figure 4.5: The regions of no θ_{23} octant degeneracy in the $\sin^2\theta_{13} - \delta$ space

Conclusion

In my thesis, I have analyzed mathematically three parameter degeneracies (intrinsic degeneracy, mass hierarchy degeneracy and θ_{23} octant degeneracy) for understanding the features of them to resolve through the oscillation probabilities and the combination of experiments with different baseline and neutrino energy. In fact, the resolution of degeneracies is complicated when they combine together (eight fold degeneracies) and we have to set up proper baseline and neutrino energy for resolving as well as measuring precisely parameters (δ_{CP} and θ_{13}, the neutrino mass hierarchy and θ_{23} octant).

At present, we have some long baseline experiments which are running have the result with high accuracy such as T2K experiment, NOνA experiment. In the future, Hyper - Kamiokande and DUNE are two neutrino experiments with more sensitive on CP violation. We hope there will be more neutrino experiments which are higher accuracy can resolve completely the degeneracies.

References

[1] Akhmedov E. K., Johansson R., Lindner M., Ohlsson T. and Schwetz T. (2004), "Series expansions for three flavor neutrino oscillation probabilities in matter", *JHEP*, 04, 078

[2] Barger V., Marfatia D. and Whisnant K. (2002), "Breaking eight fold degeneracies in neutrino CP violation, mixing, and mass hierarchy", *Phys. Rev.*, D65, 073023

[3] Dick K., Freund M., Lindner M. and Romanino A. (1999), "CP violation in neutrino oscillations", *Nucl. Phys. B*, **562** 29

[4] Freund M. (2001), "Analytic approximations for three neutrino oscillation parameters and probabilities in matter", *Phys. Rev. D*, **64** 053003

[5] Ghosh M., Ghoshal P., Goswami S.,Nath N. and Raut S. K. (2016), "New look at the degeneracies in the neutrino oscillation parameters, and their resolution by T2K, NOνA and ICAL", *Phys. Rev.*, D93

[6] Giunti C. and Kim C. W. (2007), *Fundamentals of Neutrino Physics and Astrophysics*, Oxford Univ. Pr., UK

[7] Grinstein B. (2015), "Lectures on Flavor Physics and CP Violation", CERN–Latin-American School of High-Energy Physics, Ibarra, Ecuador

[8] Halzen F. and Martin A. D. (1984), *QUARKS AND LEPTONS: An Introductory Course in Modern Particle Physics*, John Wiley & Sons, New York, USA

[9] Ho-Kim Q. and Pham X.Y. (2013), *Elementary Particles and Their Interactions: Concepts and Phenomena*, Springer, Berlin, Heidelberg

[10] Minakata H. and Nunokawa H. (2001), "Exploring neutrino mixing with low-energy superbeams", *JHEP*, **0110** 001

[11] Minakata H. and Uchinami S. (2010), "Parameter Degeneracy in Neutrino Oscillation – Solution Network and Structural Overview –", *JHEP*, **1004** 111

[12] Nunokawa H., Parke S. J. and Valle J. W. F. (2008), "CP Violation and Neutrino Oscillations", *Prog. Part. Nucl. Phys.* , **60** 338

[13] Patrignani C. *et al.* (2016), "Review of Particle Physics" [Particle Data Group], *Chin. Phys.*, C40

[14] Rigolin S. (2006), "Physics reach of β-beams and ν-factories: The Problem of degeneracies", *Nucl. Phys. Proc. Suppl.*, **155**, 33

[15] Royal Swedish Acad. Sci. (2015), "Scientific Background on the Nobel Prize in Physics 2015: Neutrino oscillations," *The Universe*, **3**, no.4, 38.

[16] *t2k-experiment.org/neutrinos/sources-and-experiments/*

Appendix A

An overview of Electroweak interaction

The Electromagnetic interaction in QED [8] is

$$-iej_{\mu}^{em}A^{\mu} = -ie(\overline{\psi}\gamma_{\mu}Q\psi)A^{\mu} \tag{A.1}$$

where Q is the charge operator or the generator of the $U(1)$ group
The Lagrangian of QED:

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - e\overline{\psi}\gamma^{\mu}Q\psi A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{A.2}$$

We have two basic Weak interactions [8]

$$-ig\mathbf{J}_{\mu}\mathbf{W}^{\mu} = -ig\overline{\varphi}_{L}\gamma_{\mu}\mathbf{T}.\mathbf{W}^{\mu}\varphi_{L} \tag{A.3}$$

$$-i\frac{g'}{2}j_{\mu}^{Y}B^{\mu} = -ig'\overline{\psi}\gamma_{\mu}\frac{Y}{2}\psi B^{\mu} \tag{A.4}$$

where $\mathbf{J}_{\mu}$ is the isotriplet of Weak currents, three vector bosons $\mathbf{W}^{\mu}$, Weak hypercharge current j_{μ}^{Y} and a fourth vector boson B^{μ}. $\mathbf{T}$ and Y are the generators of the $SU(2)_L$ and $U(1)_Y$ groups. We note that

$$W_{\mu}^{\pm} = \sqrt{\frac{1}{2}}\left(W_{\mu}^{1} \mp iW_{\mu}^{2}\right) \tag{A.5}$$

$W_{\mu}^{\pm}$ are the fields describe massive charged bosons $W^{\pm}$ whereas W_{μ}^{3}, B^{μ} are neutral fields.

The transformations under $SU(2)_L \times U(1)_Y$ are

$$\varphi_L \to \varphi'_L = e^{i\alpha(x)\mathbf{T}+i\beta(x)Y}\varphi_L$$
$$\psi_R \to \psi'_R = e^{i\beta(x)Y}\psi_R \tag{A.6}$$

where φ_L are the isospin doublets of the left-handed fermions and ψ_R are isosinglets of the right-handed fermions.

The Electromagnetic current is related to the two neutral currents J^3_μ and j^Y_μ by

$$j^{em}_\mu = J^3_\mu + \frac{1}{2}j^Y_\mu \tag{A.7}$$

We take together two interactions (A.3) and (A.4). Thus, the Electromagnetic interaction should be revealed. Indeed, we introduce A_μ and Z_μ are the neutral gauge fields having the form

$$A_\mu = B_\mu \cos\theta_W + W^3_\mu \sin\theta_W \tag{A.8}$$

$$Z_\mu = -B_\mu \sin\theta_W + W^3_\mu \cos\theta_W \tag{A.9}$$

where θ_W is called the Weinberg angle and $\tan\theta_W = g'/g$.

Then, the Electroweak interaction [8] is

$$-ig\mathbf{J}_\mu\mathbf{W}^\mu - i\frac{g'}{2}j^Y_\mu B^\mu \tag{A.10}$$

We have the Electroweak neutral current interaction

$$\begin{aligned}
&-igJ^3_\mu(W^3)^\mu - i\frac{g'}{2}j^Y_\mu B^\mu\\
&= -igJ^3_\mu(A^\mu\sin\theta_W + Z^\mu\cos\theta_W) - i\frac{g'}{2}j^Y_\mu(A^\mu\cos\theta_W + Z^\mu\sin\theta_W)\\
&= -i\left(gJ^3_\mu\sin\theta_W + \frac{g'}{2}j^Y_\mu\cos\theta_W\right)A^\mu - i\left(gJ^3_\mu\cos\theta_W + \frac{g'}{2}j^Y_\mu\sin\theta_W\right)Z^\mu\\
&= -iej^{em}_\mu A^\mu - \frac{ie}{\sin\theta_W\cos\theta_W}\left(J^3_\mu - \sin^2\theta_W j^{em}_\mu\right)Z^\mu
\end{aligned} \tag{A.11}$$

where $e = g\sin\theta_W = g'\cos\theta_W$

Note that from (A.7) we get

$$ej^{em}_\mu = eJ^3_\mu + \frac{1}{2}ej^Y_\mu = gJ^3_\mu\sin\theta_W + \frac{g'}{2}j^Y_\mu\cos\theta_W \tag{A.12}$$

The Electroweak Lagrangian is required to be invariant under $SU(2)\times U(1)$ transformation. For instance, the Lagrangian of the electron-neutrino pair:

$$\begin{aligned}
\mathcal{L} = &\overline{\varphi}_L\gamma_\mu\left[i\partial^\mu - g\frac{1}{2}\tau\mathbf{W}^\mu - g'(-\frac{1}{2})B^\mu\right]\varphi_L\\
&+\overline{e}_R\gamma_\mu\left[i\partial^\mu - g'(-1)B^\mu\right]e_R - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}
\end{aligned} \tag{A.13}$$

where

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu \tag{A.14}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{A.15}$$

$$\varphi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \text{ with } T = \frac{1}{2}, Y = -1$$

$$e_R^- \text{ with } T = 0, Y = -2$$

However, there are not mass terms of the gauge bosons and fermions in the Lagrangian (A.13). We have to find those mass terms which are gauge invariant and the theory remains renormalizable. Hence, we use the Higgs mechanism and spontaneously break the gauge symmetry.

We introduce the Lagrangian for the scalar fields (Higgs fields) ϕ that is $SU(2) \times U(1)$ invariant[8] to add to the Lagrangian (A.13)

$$\mathcal{L}' = \left| \left(i\partial_\mu - g\mathbf{T}.\mathbf{W}_\mu - g' \frac{Y}{2} B_\mu \right) \phi \right|^2 - V(\phi) \tag{A.16}$$

where

$$V(\phi) = \alpha^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi \right)^2 \tag{A.17}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \text{ with } \phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \text{ and } \phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \tag{A.18}$$

In the case $\alpha^2 < 0$ and $\lambda > 0$, the minimum of the potential $V(\phi)$ is at

$$|\phi| = \frac{1}{2} \left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \right) = -\frac{\alpha^2}{2\lambda} \tag{A.19}$$

We choose

$$\phi_1 = \phi_2 = \phi_4 = 0 \text{ and } \phi_3^2 = -\frac{\alpha^2}{\lambda} = \rho^2 \tag{A.20}$$

Thus, the appropriate choice of a vacuum expectation value is

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho \end{pmatrix} \tag{A.21}$$

with

$$T = \frac{1}{2}, T^3 = -\frac{1}{2}, Y = 1$$

Substituting (A.21) into the Lagrangian (A.16) we have the term

$$\left|\left(i\partial_\mu - \frac{g}{2}\tau.\mathbf{W}_\mu - \frac{g'}{2}B_\mu\right)\phi\right|^2 = \frac{1}{8}\left|\begin{pmatrix} gW_\mu^3 + g'B_\mu & g\left(W_\mu^1 - iW_\mu^2\right) \\ g\left(W_\mu^1 + iW_\mu^2\right) & -gW_\mu^3 + g'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ \rho \end{pmatrix}\right|^2$$
$$= \frac{1}{8}(g\rho)^2\left[\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2\right] + \frac{1}{8}\rho^2\left(-gW_\mu^3 + g'B_\mu\right)\left(-gW^{3\mu} + g'B^\mu\right)$$
$$= \frac{1}{4}(g\rho)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}\rho^2\left[g^2\left(W_\mu^3\right)^2 - 2gg'W_\mu^3 B^\mu + g'^2 B_\mu^2\right] \tag{A.22}$$

We expect the Dirac mass term for a charged boson has the form $M_W^2 W^+ W^-$ and compare with (A.22). We have

$$M_W = \frac{1}{2}g\rho \tag{A.23}$$

Consider the term

$$N = \frac{1}{8}\rho^2\left[g^2\left(W_\mu^3\right)^2 - 2gg'W_\mu^3 B^\mu + g'^2 B_\mu^2\right] = \frac{1}{8}\rho^2\left(gW_\mu^3 - g'B_\mu\right)^2 + 0\left(gW_\mu^3 + g'B_\mu\right)^2$$

From (A.8)(A.9) and $\tan\theta_W = g'/g$, we have

$$N = \frac{1}{8}\rho^2\left(\frac{g}{\cos\theta_W}\right)^2 Z_\mu^2 + 0A_\mu^2 \tag{A.24}$$

The Dirac mass terms of the neutral bosons[8] have the form:

$$\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2$$

So

$$M_A = 0, M_Z = \frac{1}{2}\frac{g\rho}{\cos\theta_W}$$

The mass terms of the bosons have been identified.

To generate the mass of electron, we add the $SU(2)\times U(1)$ gauge invariant term:

$$\mathcal{L}_1 = -G_e\left[(\overline{\nu}_e, \overline{e})_L\begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}e_R + \overline{e}_R\left(\phi^-, \overline{\phi^0}\right)\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L\right] \tag{A.25}$$

After spontaneously breaking the symmetry, we choose

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \rho + h(x)\end{pmatrix}$$

We substitute into (A.25), so

$$\mathcal{L}_1 = -\frac{G_e}{\sqrt{2}}\rho\left(\overline{e}_L e_R + \overline{e}_R e_L\right) - \frac{G_e}{\sqrt{2}}\left(\overline{e}_L e_R h + \overline{e}_R e_L h\right) \tag{A.26}$$

Then,

$$M_e = \frac{G_e \rho}{\sqrt{2}}$$

Hence,

$$\mathcal{L}_1 = -M_e \overline{e} e - \frac{M_e}{\rho} \overline{e} e h \tag{A.27}$$

For the quarks, we also use the same way to find the mass terms. However, to generate the mass terms of the upper members of the quark doublets, we introduce the new Higgs doublet[8]:

$$\phi_c = \begin{pmatrix} -\overline{\phi^0} \\ \phi^- \end{pmatrix} \tag{A.28}$$

Spontaneously breaking the symmetry, we have

$$\phi_c = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho + h(x) \\ 0 \end{pmatrix} \tag{A.29}$$

We include the quark Lagrangian

$$\mathcal{L}_2 = -G_d^{ij} \left(\overline{u}_i, \overline{d'}_i\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_{jR} - G_u^{ij} \left(\overline{u}_i, \overline{d'}_i\right)_L \begin{pmatrix} -\overline{\phi^0} \\ \phi^- \end{pmatrix} u_{jR} + \text{hermitian conjugate} \tag{A.30}$$

where $i, j = 1, 2, ..., n$ and n is the number of quark doublets

$$d'_i = \sum_j U_{ij} d_j \tag{A.31}$$

with U is the CKM matrix.

Hence, we get the quark Lagrangian in the form

$$\mathcal{L}_2 = -M_d^i \overline{d}_i d_i \left(1 + \frac{h}{\rho}\right) - M_u^i \overline{u}_i u_i \left(1 + \frac{h}{\rho}\right) \tag{A.32}$$

The masses depend on the couplings G_d and G_u.

The complete Lagrangian in the Glashow - Weinberg - Salam theory:

$$\begin{aligned} \mathcal{L} = &- \frac{1}{4} \mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} + \overline{L} \gamma_\mu \left[i\partial^\mu - g\frac{1}{2}\tau \mathbf{W}^\mu - g'\frac{Y}{2} B^\mu \right] L \\ &+ \overline{R} \gamma_\mu \left[i\partial^\mu - g'\frac{Y}{2} B^\mu \right] R + \left| \left(i\partial_\mu - g\frac{1}{2}\tau \mathbf{W}_\mu - g'\frac{Y}{2} B_\mu \right) \phi \right|^2 - V(\phi) \\ &- G_1 \overline{L} \phi R + G_2 \overline{L} \phi_c R + \text{hermitian conjugate} \end{aligned} \tag{A.33}$$

where L denotes a left-handed fermion doublet and R denotes a right-handed fermion singlet.

Appendix B

Compute the asymmetry $A_{e\mu}^{CP}$

The three flavor neutrinos mixing matrix in vacuum:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = U^D D^M \tag{B.1}$$

$$U^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{B.2}$$

$$D^M = diag(1, e^{i\lambda_2}, e^{i\lambda_3}) \tag{B.3}$$

where $c_{mn} = \cos\theta_{mn}$, $s_{mn} = \sin\theta_{mn}$, θ_{mn} are the three mixing angles $(0 \leqslant \theta_{mn} \leqslant \pi/2)$ $(m, n = 1, 2, 3; m \neq n)$, δ is the Dirac CP violation phase $(0 \leqslant \delta \leqslant 2\pi)$ and λ_2, λ_3 are the two physical Majorana CP violation phases.

We have general formula:

$$A_{\alpha\beta}^{CP} = 4\sum_{j>k} \mathfrak{Im}\left[U_{\alpha k}U_{\beta k}^{*}U_{\alpha j}^{*}U_{\beta j}\right]\sin\left(\frac{\Delta m_{jk}^2 L}{2E}\right) \tag{B.4}$$

Thus,

$$A_{e\mu}^{CP} = \mathfrak{Im}\left[U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}\right]\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) + \mathfrak{Im}\left[U_{e1}U_{\mu 1}^{*}U_{e3}^{*}U_{\mu 3}\right]\sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) + \\ + \mathfrak{Im}\left[U_{e2}U_{\mu 2}^{*}U_{e3}^{*}U_{\mu 3}\right]\sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) \tag{B.5}$$

From the mixing matrix, we derive

$$\mathfrak{Im}\left[U_{e1}U_{\mu1}^*U_{e2}^*U_{\mu2}\right] = \mathfrak{Im}\left[U_{e2}U_{\mu2}^*U_{e3}^*U_{\mu3}\right] = \frac{1}{8}\sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta \tag{B.6}$$

$$\mathfrak{Im}\left[U_{e1}U_{\mu1}^*U_{e3}^*U_{\mu3}\right] = -\mathfrak{Im}\left[U_{e1}U_{\mu1}^*U_{e2}^*U_{\mu2}\right] = -\frac{1}{8}\sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta \tag{B.7}$$

Hence,

$$A_{e\mu}^{CP} = \frac{1}{2}J\left[\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) - \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right)\right] \tag{B.8}$$

where

$$J = \sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta \tag{B.9}$$

$$\begin{aligned}
&\sin\left(\frac{\Delta m_{21}^2 L}{2E}\right) - \sin\left(\frac{\Delta m_{31}^2 L}{2E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) \\
&= -2\cos\left(\frac{\Delta m_{21}^2 + \Delta m_{31}^2}{4E}L\right)\sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) + \sin\left(\frac{\Delta m_{32}^2 L}{2E}\right) \\
&= -2\cos\left(\frac{\Delta m_{21}^2 + \Delta m_{31}^2}{4E}L\right)\sin\left(\frac{\Delta m_{32}^2 L}{4E}\right) + 2\sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)\cos\left(\frac{\Delta m_{32}^2 L}{4E}\right) \\
&= 4\sin\left(\frac{\Delta m_{21}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{31}^2 L}{4E}\right)\sin\left(\frac{\Delta m_{32}^2 L}{4E}\right)
\end{aligned} \tag{B.10}$$

Therefore, we obtain the asymmetry $A_{e\mu}^{CP}$

$$A_{e\mu}^{CP} = 2\sin 2\theta_{12}\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin\delta\sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32} \tag{B.11}$$

where

$$\Delta_{ij} = \frac{\Delta m_{ij}^2}{4E}L \quad i = 2,3, j = 1,2 \tag{B.12}$$