

# Search for phenomena beyond the Standard Model in events with large $b$ -jet multiplicity using the ATLAS detector at the LHC

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# Contents

- 1 Theoretical framework
  - Standard Model
  - Supersymmetry Model
- 2 Experimental setup
  - Large Hadron Collider
  - ATLAS detector
- 3 Search for new physics in fully hadronic final states
  - Analysis strategy
  - Multijet background estimation:  $\text{TRF}_{\text{MJ}}$  method
  - Results
- 4 Conclusions



## Standard Model (SM) of Particle Physics

- Quantum field theory under gauge symmetry group:

- $SU(3)_C \times SU(2)_L \times U(1)_Y$

- Describes elementary particles and their interactions (**strong** and **electroweak**)

- Fundamental particles:

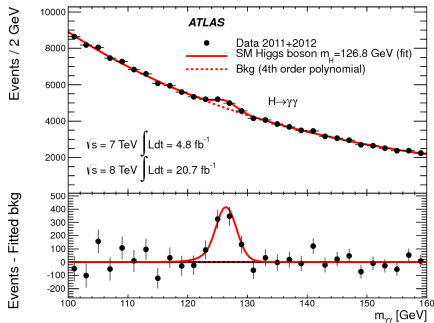
- Fermions: **quarks** and **leptons**

- Vector boson: **gluons**, **Z** and  **$W^\pm$**  bosons, **photon**

- Scalar boson: **Higgs** boson

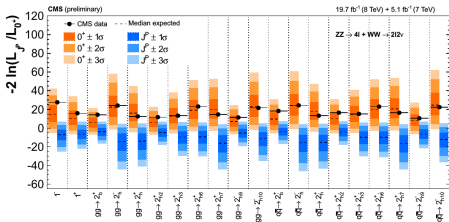
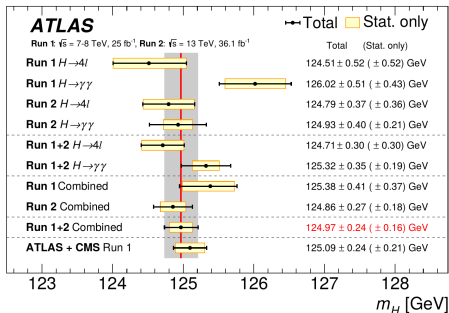
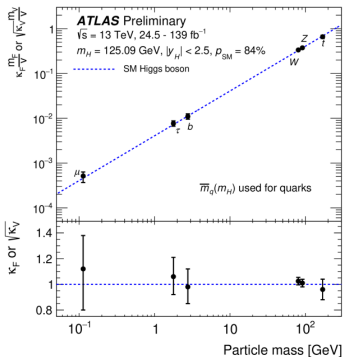
- generate **mass** for fundamental particles

- In 2012, the last piece of the particles in SM picture, the **Higgs** boson, has been discovered by the LHC



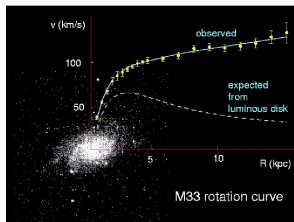
[[arXiv:1207.7214](https://arxiv.org/abs/1207.7214)]

# The Higgs boson measurements



- Higgs coupling to 3<sup>rd</sup> generation of fermions and vector bosons are measured
  - $y_f = \sqrt{2} \frac{m_f}{v}$  and  $g_V \sim 2 \frac{m_V^2}{v}$
- Mass measurement Run 1+2:
  - $m_H = 124.97 \pm 0.24 \text{ GeV}$
- Compatible with SM spin-0 and even parity prediction

## Standard Model shortcomings



[[arXiv:astro-ph/9909252](https://arxiv.org/abs/astro-ph/9909252)]

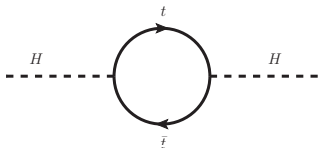


[[Matter-antimatter asymmetry](#)]

- Regardless of the SM successes, some **unsolved questions** still remain:
  - Why is Higgs boson so light (so-called "naturalness" problem)?
  - What is the dark matter ( $\sim 25\%$  total mass-energy of the universe)?
  - What is the origin of neutrino masses?
  - What is the origin of matter-antimatter asymmetry in our universe?
  - Is there unification of the forces?
- Many extensions of the SM have been developed to answer those questions

## Top quark as a probe for new physics

- Top quark play an important role in any BSM theory



$$\Delta m_H^2 = -\frac{|y_t|^2}{8\pi^2} \Lambda^2 + \dots$$

- Destabilizes the Higgs mass (so-called "naturalness" problem)
- Solved by BSM models which have **top-partner: Supersymmetry**, Composite Higgs models.

- Supersymmetry (SUSY):**

Standard Model particles	Supersymmetric partners
u, c, t, g	$\tilde{u}$ , $\tilde{c}$ , $\tilde{t}$ , $\tilde{g}$ gluino
d, s, b, $\gamma$	$\tilde{d}$ , $\tilde{s}$ , $\tilde{b}$ , $\tilde{\gamma}$ photino
$\nu_e, \nu_\mu, \nu_\tau$ , Z	$\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$ , $\tilde{Z}$ zino
e, $\mu$ , $\tau$ , W	$\tilde{e}, \tilde{\mu}, \tilde{\tau}$ , $\tilde{W}$ wino
H	$\tilde{H}$ higgsino
● quarks	● squarks
● leptons	● sleptons & sneutrinos
● force particles	● neutralinos $\tilde{\chi}^0$ & charginos $\tilde{\chi}^\pm$



$$\Delta m_H^2 = \frac{y_t^2}{8\pi^2} \Lambda^2 + \dots$$

- Naturally stabilizes Higgs mass
- SUSY is a broken symmetry  
→ natural SUSY  $m_{\tilde{t}} \leq O(1) \text{ TeV}$

## Superpotential of Minimal Supersymmetric Standard Model (MSSM)

- The superpotential of MSSM can be separated into two parts:

$$W_{MSSM} = \tilde{u} \mathbf{y}_u \tilde{Q} H_u - \tilde{d} \mathbf{y}_d \tilde{Q} H_d - \tilde{e} \mathbf{y}_e \tilde{L} H_d + \mu H_u H_d,$$

$$W_{R_p} = \frac{1}{2} \lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu'^i L_i H_u + \frac{1}{2} \lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

- New symmetry R-parity:

$$R = (-1)^{3(B-L)+2S}$$

B: Baryonic number  
L: Leptonic number  
S: Spin number





## Superpotential of MSSM

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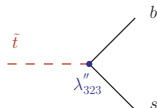
B: Baryonic number  
L: Leptonic number  
S: Spin number

### R-parity violating (RPV) model

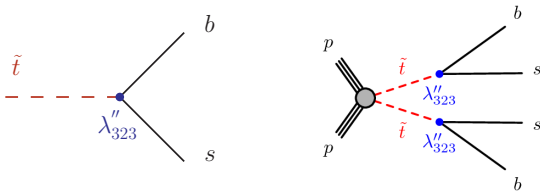
- Violate B or L
- Single sparticle production is possible
- LSP not necessary stable  $\rightarrow$  possibility for new signals
- MET not necessarily large



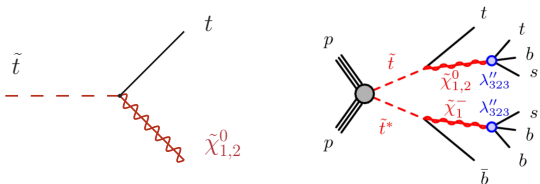
- Many constraints on SUSY searches disappear
- Top squark decays via  $\lambda''_{323}$  favored by MFV



# Top-squark RPV phenomenology and searches in ATLAS

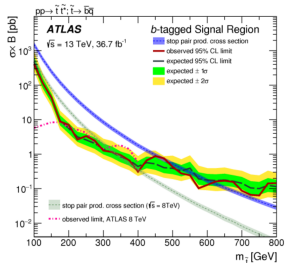


- Top-squark is LSP

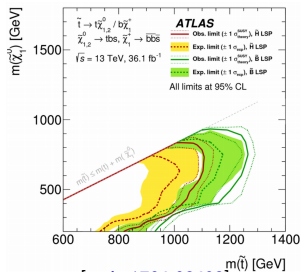


- Higgsino ( $\tilde{\chi}_2^0, \tilde{\chi}_1^\pm, \tilde{\chi}_1^0$ ) is LSP

- $m_{\tilde{t}} - m_{\tilde{\chi}_1^\pm, \tilde{\chi}_1^0} > m_t$



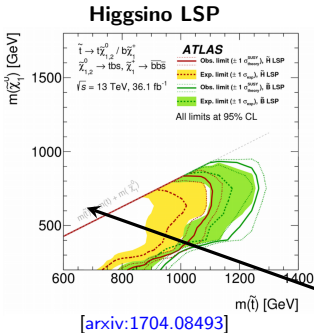
[arxiv:1710.07171]



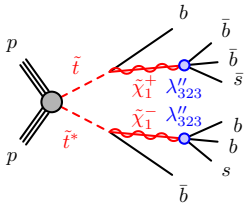
[arxiv:1704.08493]

Search for RPV decay of top squarks pair in events with multi- $b$ -jets

For natural SUSY, a triplet of higgsino-like states are LSP ( $\tilde{\chi}_2^0, \tilde{\chi}_1^\pm, \tilde{\chi}_1^0$ )



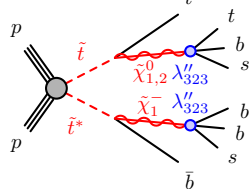
$$\tilde{t} \rightarrow b\chi_1^+ (\chi_1^+ \rightarrow \bar{b}\bar{s})$$



Saturate BR in  $m_{\tilde{t}} - m_{\tilde{\chi}_1^+} < m_t$

$$\tilde{t} \rightarrow b\chi_1^+ (\chi_1^+ \rightarrow \bar{b}\bar{s}) \text{ BR} = 0.5$$

$$\tilde{t} \rightarrow t\chi_{1,2}^0 (\chi_{1,2}^0 \rightarrow tbs) \text{ BR} = 0.5$$

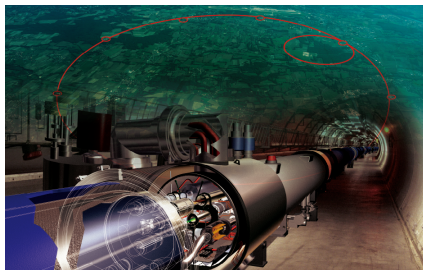
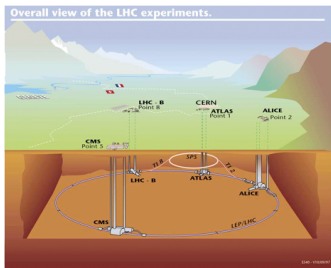


Dominant in  $m_{\tilde{t}} - m_{\tilde{\chi}_1^+} > m_t$

No leptonic final states can be used for this scenario  
 Search for signal in events with high  $b$ -jet multiplicity

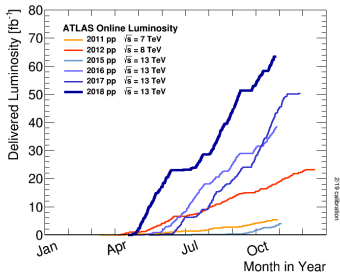
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- 2 Experimental setup**
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  - ATLAS detector
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# The Large Hadron Collider (LHC) at CERN



- Circular collider of protons/ions
- Proton-proton ( $pp$ ) collisions:
  - 2010-2012:  $\sqrt{s} = 7, 8$  TeV (Run 1)
  - 2015-2018:  $\sqrt{s} = 13$  TeV (Run 2)
- Long shutdown in 2019 – 2020
  - Prepare for high luminosity increase
  - 2021-2023:  $\sqrt{s} = 13/14$  TeV (Run 3)

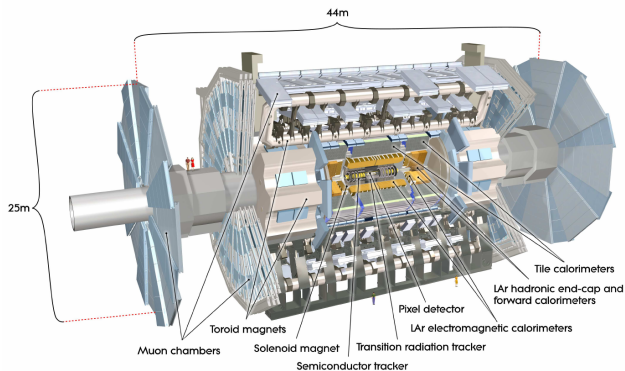
## ATLAS Luminosity Public



# The Large Hadron Collider (LHC) at CERN



# The ATLAS detector

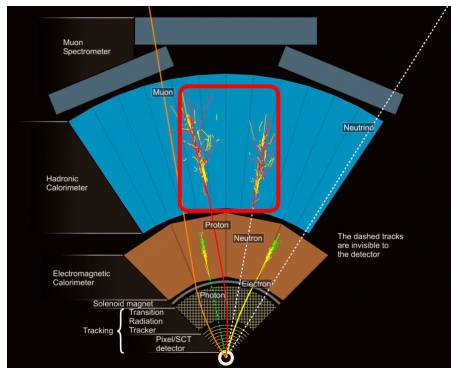


- Multi-purpose detector:
  - Higgs boson physics
  - SM precision measurements
  - New physics searches
- Large  $pp$  collisions dataset in Run 2,  $L \sim 140 \text{ fb}^{-1}$
- 3 sub-components:
  - Inner detector (ID)
  - Calorimeters
  - Muon Spectrometer (MS)

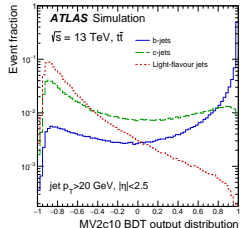
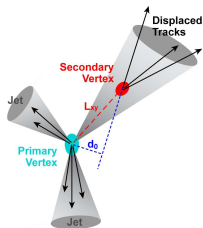
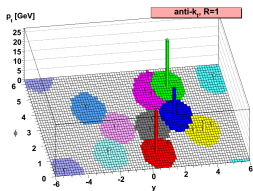
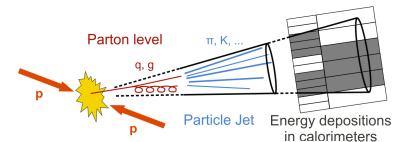


## Overview of particles passage through ATLAS detector

- **Electrons:** Energy deposition in calorimeter and charged track in ID
- **Photons:** Energy deposition in calorimeter, no track in ID
- **Muons:** Combined track in ID and MS
- **Jets:** Energy deposition in calorimeters and charged tracks in ID
- **MET:** negative vectorial sum of selected physics objects and the soft term



# Jet reconstruction and $b$ -jet identification



## ● Jet reconstruction:

- Quarks/ gluons  $\rightarrow$  fragmentation and hadronization  $\rightarrow$  collimated spray of particles, a jet
- Jets reconstructed by **anti- $k_t$**  algorithm

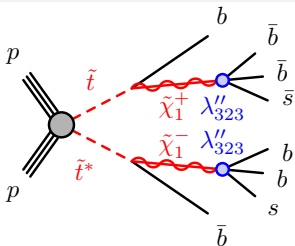
## ● $b$ -jet identification

- $b$ -hadrons travel few hundreds  $\mu\text{m}$  before decay.
- Impact parameter track resolution of  $\sim 10 \mu\text{m}$
- Reconstructs the **Secondary Vertex (SV)** to identify  $b$ -jet
- **Multivariate  $b$ -tagging algorithm** separates  $b$ -jet from light and  $c$ -jet

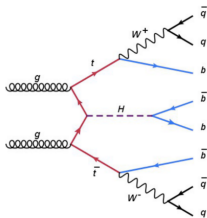
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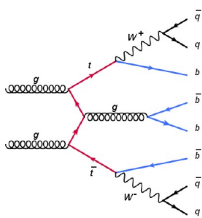
## Signal and background processes



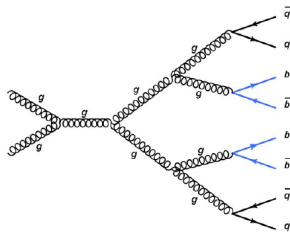
$$\sigma_{pp \rightarrow t\bar{t}} = 0.0017 \sim 2.1 \text{ pb}$$



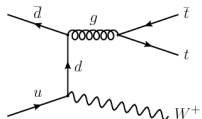
$$\sigma_{t\bar{t}H} = 0.5 \text{ pb}$$



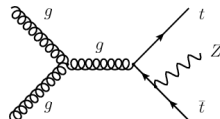
$$\sigma_{t\bar{t}+\text{jets}} = 730 \text{ pb}$$



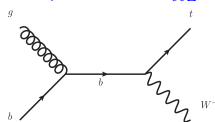
$$\sigma_{\text{multijet}} \sim O(1) \text{ mb}$$



$$\sigma_{t\bar{t}W} = 1.12 \text{ pb}$$

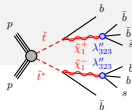


$$\sigma_{t\bar{t}Z} = 0.16 \text{ pb}$$

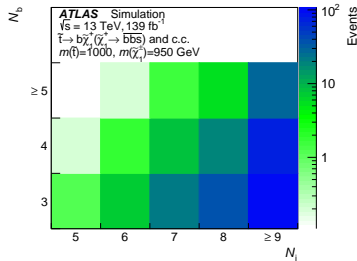
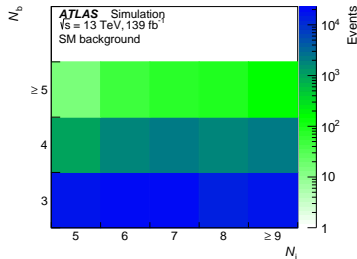


$$\sigma_{Wt} = 76 \text{ pb}$$

## RPV top squark production: Analysis strategy



- $\geq 4$  hadronic jets with  $p_T > 120$  (140) GeV
- additional jets must have  $p_T > 25$  GeV,  $|\eta| < 2.5$
- $\geq 2$   $b$ -tagged jets
- Events containing leptons are vetoed
- **Strategy:** Counting events in different jet and  $b$ -tagged jet multiplicity regions
- Top-quark backgrounds simulated by NLO MC generators
- Most dominant **multijet** background estimated by data-driven ( $\text{TRF}_{\text{MJ}}$ ) method



## Event categorization

Analysis Regions	3	4	$N_b$	$\geq 5$
$N_j$	6	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{SR}_{\text{stop}} \rightarrow$	$\leftarrow \text{SR}_{\text{stop}} \rightarrow$
	7	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{SR}_{\text{stop}} \rightarrow$
	8	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{SR}_{\text{stop,discovery}} \rightarrow$
	$\geq 9$	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{VR-MJ} \rightarrow$	$\leftarrow \text{SR}_{\text{stop,discovery}} \rightarrow$

- $N_j$ : number of jets,  $N_b$ : number of  $b$ -tagged jets

Validation regions for  $\text{TRF}_{\text{MJ}}$  method

Analysis Regions	$N_b$		
	3	4	$\geq 5$
$N_j = 6$	← VR-MJ →	← VR-MJ →	← SR <sub>stop</sub> →
$N_j = 7$	← VR-MJ →	← VR-MJ →	← SR <sub>stop</sub> →
$N_j = 8$	← VR-MJ →	← VR-MJ →	← SR <sub>stop</sub> →
$N_j \geq 9$	← VR-MJ →	← VR-MJ →	← SR <sub>stop</sub> →

**TRF<sub>MJ</sub> validation regions**

- $N_j$ : number of jets,  $N_b$ : number of  $b$ -tagged jets
- Validation regions (VR-MJ) based on  $C_{\text{mass}}$  ( $H_T/M_{\text{jets}}$ ) cut



## Signal region definition

Analysis Regions	$N_b$		
	3	4	$\geq 5$
6	$\leftarrow$ VR-MJ $\rightarrow$	$\leftarrow$ SR <sub>stop</sub> $\rightarrow$	$\leftarrow$ SR <sub>stop</sub> $\rightarrow$
$N_j$ 7	$\leftarrow$ VR-MJ $\rightarrow$	$\leftarrow$ SR <sub>st</sub> $\rightarrow$	stop $\rightarrow$
8	$\leftarrow$ VR-MJ $\rightarrow$	$\leftarrow$ SR <sub>st</sub> $\rightarrow$	stop,discovery $\rightarrow$
$\geq 9$	$\leftarrow$ VR-MJ $\rightarrow$	$\leftarrow$ SR <sub>st</sub> $\rightarrow$	stop,discovery $\rightarrow$

Signal region at high jet and high b-tagged jet

- $N_j$ : number of jets,  $N_b$ : number of  $b$ -tagged jets
- Signal regions:  $N_j \geq 6$  and  $N_b \geq 4$

## Model-independent test setup

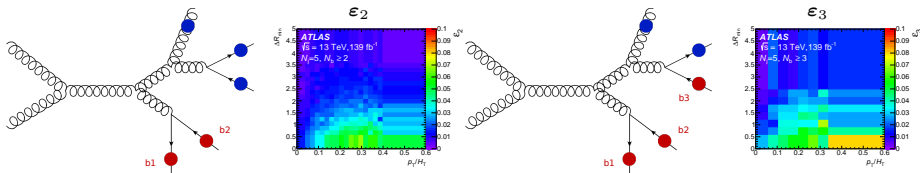
Analysis Regions	3	4	$N_b$	$\geq 5$
6		← $SR_{stop}$ →		← $SR_{stop}$ →
	← VR-MJ →	← VR-MJ →		
$N_j$ 7				← $SR_{stop}$ →
	← VR-MJ →			
8				← $SR_{stop,discovery}$ →
	← VR-MJ →	<b>Discovery regions</b>		
$\geq 9$				← $SR_{stop,discovery}$ →
	← VR-MJ →			

- $N_j$ : number of jets,  $N_b$ : number of  $b$ -tagged jets
- Discovery regions:  $(N_j \geq 8, N_b \geq 5)$  and  $(N_j \geq 9, N_b \geq 5)$

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## Tag-Rate-Function multijet: TRF<sub>MJ</sub> data-driven method

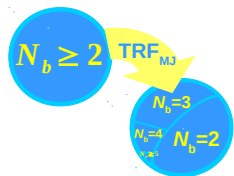
- TRF<sub>MJ</sub> predicts the  $b$ -tagged jet spectrum of multijet events:
  - Based on the probability of  $b$ -tagging a jet in events with 2 ( $\epsilon_2$ ) and 3 ( $\epsilon_3$ )  $b$ -tagged jets estimated in 5 jets events
  - After removing 2 (3) jets with highest  $b$ -tagging weight,  $\epsilon_2$  ( $\epsilon_3$ ) derived as a function of:
    - $p_T/H_T$  and minimum angular distance between the jet and the removed  $b$ -tagged jets



Starting from a sample with  $N(\geq 2b)$ :

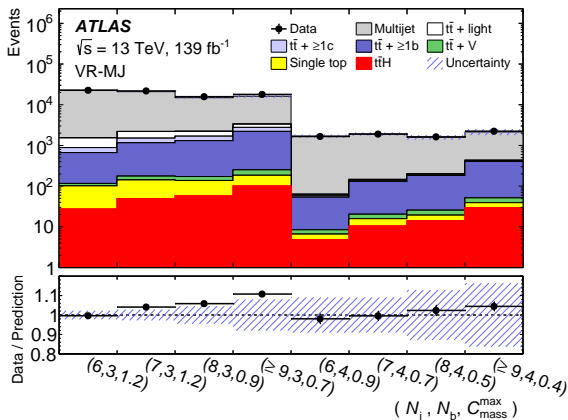
$$N(2b) = \sum_i P^{2b}(i), \text{ where } P^{2b}(i) = \prod_{j \neq b_1, b_2}^{N_i} (1 - \epsilon_2(j))$$

for  $N(3b), N(4b), N(\geq 5b)$   $\epsilon_2$  and  $\epsilon_3$  used in combination



## Validation of TRF<sub>MJ</sub> method in VR-MJ in data

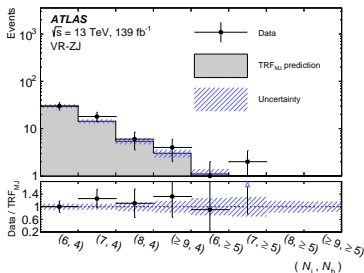
- Number of predicted multijet events with  $N_j \geq 6$  and  $N_b = 3$  or  $N_b = 4$



- $C_{\text{mass}}^{\text{max}}$ : region dependent upper cut on  $C_{\text{mass}}$ 
  - Signal contamination less than 5%
- Systematic uncertainties are represented by the blue hatched area

## Validation of TRF<sub>MJ</sub> method in $Z + \text{jets}$ events

- VR-ZJ event selection:
  - Single isolated lepton trigger ( $e, \mu$ )
  - 2 opposite-sign same flavor leptons  $p_T > 27$  GeV
  - $m_{l\pm l\mp} > 60$  GeV
  - $\geq 5$  jets with  $p_T > 25$  GeV
  - $\geq 2$   $b$ -tagged jets
- TRF<sub>MJ</sub> is derived and applied to estimate the spectrum of number of  $b$ -tagged jets in VR-ZJ
  - Overall agreement within uncertainties
- TRF<sub>MJ</sub> systematic uncertainties derived in MC dijet events
  - Max between residual non-closure and statistical uncertainty in each  $N_j, N_b$



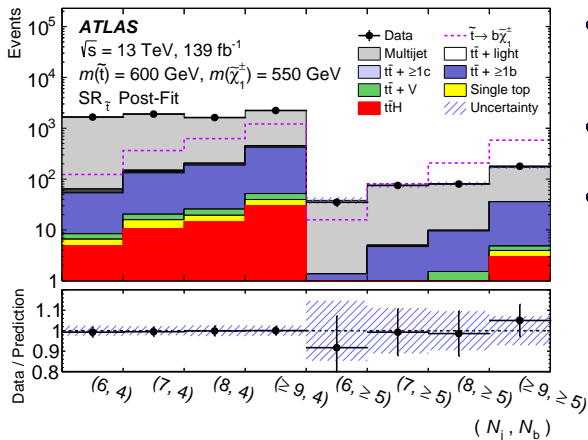
	TRF <sub>MJ</sub> uncertainty	$N_b$ 4	$\geq 5$
$N_j$	6	9%	27%
	7	9%	30%
	8	13%	18%
	$\geq 9$	16%	14%

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## Statistical model

- Profile-likelihood fit is performed on 8 SRs

$$\mathcal{L}(N^{\text{data}} | \mu, \theta) = \prod_{i \in \text{bins}} \mathcal{P}(N_i^{\text{data}} | \mu s_i(\theta) + b_i(\theta)) \Gamma(\theta_i^{\text{stat}}) \prod_{k \in \text{systematics}} \mathcal{N}(\theta_{ki}^{\text{sys}})$$



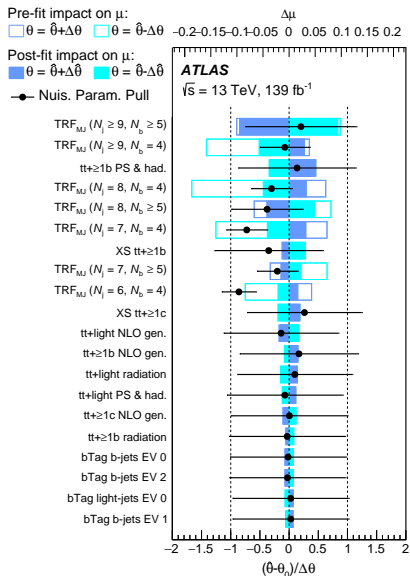
- $\mu = \frac{\sigma_{\text{obs}}}{\sigma_{\text{MSSM}}}$ : signal strength parameter

- $\theta$ : nuisance parameters

- Systematic uncertainties are represented by the blue hatched area

← Background-only fit



Impact of systematics on  $\mu$ 

- $\text{TRF}_{\text{MJ}}$  normalization uncertainty: non-closure as found in MC dijet events
- $tt+\text{jets}$  modeling uncertainties: parton shower (PS), generator (Gen) and initial and final-state radiation (ISR, FSR)
- 50% uncertainties on cross-section of  $tt+\geq 1b$  and  $tt+\geq 1c$  backgrounds
- Jets experimental uncertainties: jet energy scale (JES), jet energy resolution (JER), jet vertex tagging (JVT)
- $b$ -tagging uncertainties

## Model-independent results

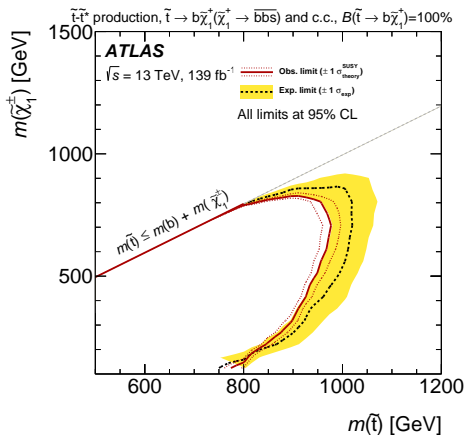
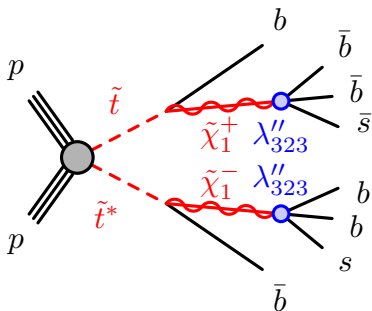
- Fitted background yields in ( $N_j \geq 8, N_b \geq 5$ ) and ( $N_j \geq 9, N_b \geq 5$ ) signal regions

Process	$N_j \geq 8, N_b \geq 5$	$N_j \geq 9, N_b \geq 5$
Multijet	$200 \pm 40$	$123 \pm 20$
$t\bar{t} + \geq 1c$	$0.6 \pm 0.6$	$0.29 \pm 0.33$
$t\bar{t} + \geq 1b$	$26 \pm 20$	$20 \pm 15$
$t\bar{t} + W$	$0.11 \pm 0.05$	$0.09 \pm 0.04$
$t\bar{t} + Z$	$1.4 \pm 0.7$	$0.8 \pm 0.7$
$Wt$	$0.9 \pm 0.8$	$0.9 \pm 1.2$
$t\bar{t}H$	$3.7 \pm 1.6$	$2.9 \pm 1.4$
Total background	$230 \pm 40$	$147 \pm 20$
Data	259	179

- No significant excess observed
- Model-independent limits are set on the contribution of new phenomena to the signal-region yields.

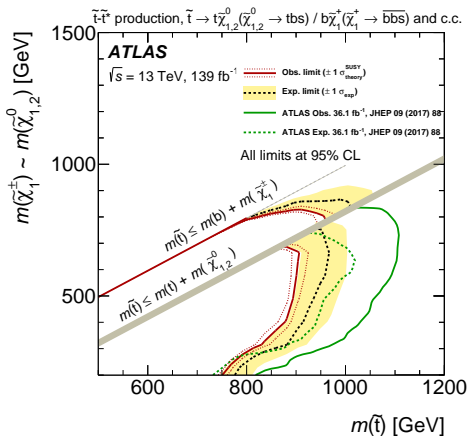
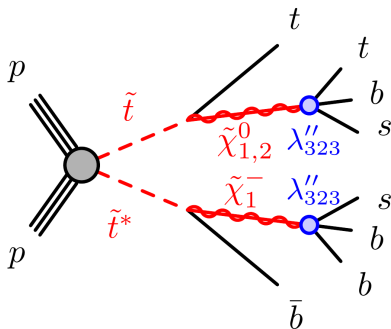
Signal region	$\sigma_{\text{obs}}^{95}$ [fb]	$N_{\text{obs}}^{95}$	$N_{\text{exp}}^{95}$	$p_0 (Z)$
$N_j \geq 8, N_b \geq 5$	0.76	105	$85^{+30}_{-24}$	0.24 (0.7)
$N_j \geq 9, N_b \geq 5$	0.54	75	$52^{+20}_{-15}$	0.11 (1.2)

## Exclusion limits with $\text{BR}(\tilde{t} \rightarrow b\tilde{\chi}^+(\tilde{\chi}^+ \rightarrow bbs)) = 1$



- For the signal model considered, set exclusion limits on  $m_{\tilde{t}}$  up to 950 GeV

## Exclusion limits for $\tilde{H}$ LSP scenario

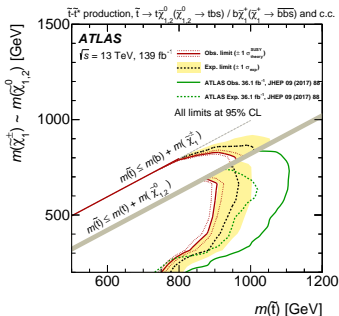


- For the signal model considered, set exclusion limits on  $m_{\tilde{t}}$  up to 950 GeV

- 1 Theoretical framework
- 2 Experimental setup
- 3 Search for new physics in fully hadronic final states
- 4 Conclusions**

## Conclusions

- SUSY is an elegant way to relate fermions and bosons and solving many SM shortcomings
  - Light top squark  $\rightarrow$  solution to the naturalness problem
- Extensive program to explore uncovered phase space for RPV models
  - $pp \rightarrow \tilde{t}\tilde{t}^* \rightarrow b\bar{b}s\bar{b}\bar{b}\bar{s}$  have been a blind spot for natural SUSY
- Search for RPV decay of top squarks pair production in  $139 \text{ fb}^{-1}$  of ATLAS data:
  - Strategy based on a profile likelihood based fit in different  $N_j$  and  $N_b$
  - Multijet background estimated using TRF<sub>MJ</sub> method. Validated in data and MC
  - No significant excess is observed
    - Model-independent observed limit on BSM cross-section is  $0.54 \text{ fb}$  in  $(\geq 9j, \geq 5b)$
    - Observed (Expected) 95% CL exclusion limit is set for top squark mass up to  $0.95 \text{ TeV}$  ( $1 \text{ TeV}$ )
- More information in [2010.01015](https://arxiv.org/abs/2010.01015)



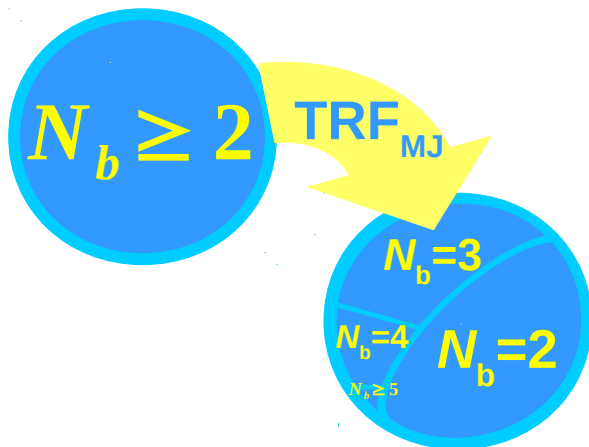
THANK YOU FOR YOUR ATTENTION

BACK UP



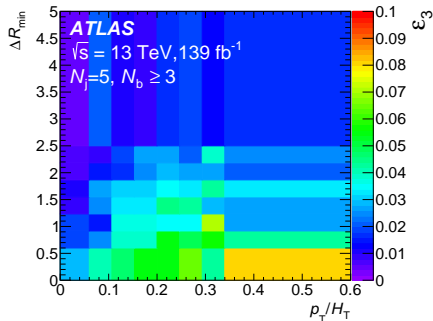
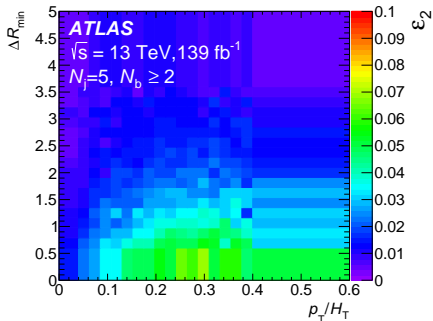
## Multijet background estimation: $\text{TRF}_{\text{MJ}}$ method

- $\text{TRF}_{\text{MJ}}$  based on the **probability of  $b$ -tagging a jet in multijet events**
  - Starting from events with the number of  $b$ -tagged jets  $N_b \geq 2$ , one can predict the  $b$ -tagged jet spectrum of multijet events at high jet multiplicity

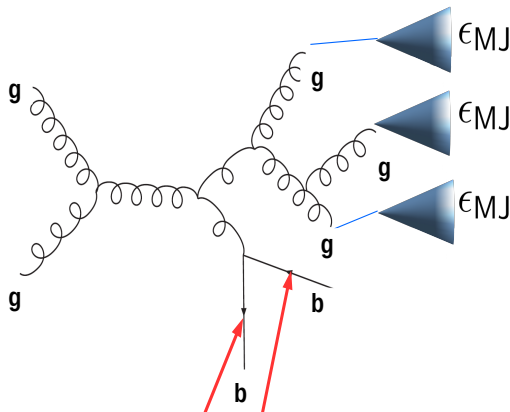


## Deriving $b$ -tagging probability $\varepsilon_2$ and $\varepsilon_3$ in data

- $\varepsilon_2$  and  $\varepsilon_3$  are derived in 5 jets events
  - Low signal contamination and close to search regions
  - MC backgrounds are subtracted from data when estimating  $\varepsilon_2$  and  $\varepsilon_3$
- Parametrization with respect to:
  - $p_T/H_T$
  - $\Delta R_{\min}$ : minimum  $\Delta R$  between jet and highest  $b$ -tagging weight jets



## TRF<sub>MJ</sub>: Predicting $b$ -tagging spectrum



2 jets with highest  $b$ -tag weight removed from estimation

- Take as example events with (5 jets,  $\geq 2$   $b$ -tags)
  - Remove 2 jets with highest  $b$ -tagging weight
- $b$ -tagged probability of a jet  $i$ -th:  $\epsilon_{MJ}(i)$
- Probability to have 3  $b$  tag :  $P_3 = \prod_{i=1}^3 \epsilon_{MJ}(i)$
- Probability to have 0  $b$  tag :  $P_0 = \prod_{i=1}^3 (1 - \epsilon_{MJ}(i))$
- Probability to have 1  $b$  tag :  $P_1 = \sum_{j=1}^3 \left( \epsilon_j \prod_{i \neq j} (1 - \epsilon_i) \right)$
- Permutation is considered for  $P_2$