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Higgs Boson masses in the Next-to-Minimal Supersymmetric Standard Model with Inverse Seesaw Mechanism

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Abstract

In this thesis, we investigate the masses of the Higgs bosons at one-loop level in the Next-to-Minimal Supersymmetric Standard Model (NMSSM) with the contribution from Inverse Seesaw mechanism (ISS). With the newly discovered particle at 125 GeV in 2012, it is especially meaningful to determine whether this is the particle proposed within the Standard Model (SM) or in theories beyond that. Additionally, neutrino Physics is a hot experimental topic in recent years.

Chapter 2 is devoted to a description of supersymmetric theories in general. While chapter 3 focuses on the Minimal and Next-to-Minimal version of the Standard Model. The mass spectrum and the Higgs sector receives the most attention. The Higgs in NMSSM, while can be a candidate for the experimentally detected Higgs boson, has some differences from the one proposed in SM.

With the discovery of neutrino oscillation, mechanisms to explain neutrinos' masses must be proposed. We discuss the mechanism in chapter 4 with the focus on Inverse Seesaw to obtain the unnatural smallness of neutrino mass so that newly appearing particles can be on TeV scale. The inclusion of such heavy neutrinos surely affects the Higgs masses, and that influence shall be studied.

However, calculation up to one-loop level is essential to investigate the contributions of neutrinos to Higgs masses. Ultraviolet (UV) divergences arise in such computation; thus, a regularization and a renormalization scheme are mandatory to obtain UV-finite result. This will be explored generally in chapter 5 while the method is applied to our problem in chapter 6.

In chapter 7, the new contributions are implemented in the public code `NMSSMCALC` to perform numerical analysis. We investigate the dependence of Higgs boson masses on the neutrino sector with the experimental constrains on the active neutrinos taken into account. We have found that the contribution of inverse seesaw mechanism to the SM-like Higgs boson can be as large as 9%.

Contents

1 The Standard Model of Particle Physics **3**
1.1 Introduction 3
1.2 The SM Lagrangian 4
1.3 Higgs mechanism and SM mass spectrum 6
1.4 The issues of SM 7

2 Supersymmetry **9**
2.1 Introduction 9
2.2 Supersymmetric Algebra 10
2.3 Superfield and superspace 10
2.3.1 Superspace 11
2.3.2 General superfield 11
2.3.3 Chiral superfield 14
2.3.4 Vector superfields 16
2.4 Supersymmetric Lagrangian 17
2.4.1 Chiral sector 18
2.4.2 Super Yang-Mills theory 20
2.4.3 Gauge-matter actions 22
2.4.4 The completed Lagrangian of supersymmetric theories 23

3 Next-to-minimal Supersymmetric Standard Model **25**
3.1 Minimal Supersymmetric Standard Model 25
3.1.1 Field content and the Lagrangian 25
3.1.2 The Higgs sector of MSSM 29
3.2 Next-to-minimal Supersymmetric Standard Model 32
3.2.1 The particle content and Lagrangian 33
3.2.2 Tree level mass spectrum 34

4 Inverse Seesaw Mechanism **42**
4.1 Experimental data and motivation 42
4.2 Type I Seesaw Mechanism 42
4.3 Inverse Seesaw Mechanism 44
4.4 Inverse Seesaw Mechanism in NMSSM 45
4.5 Feynman rules for NMSSM with inverse seesaw mechanism 47
4.5.1 Motivation for Feynman rules for fermion-number-violating interaction . . 47
4.5.2 Feynman rules of NMSSM for new contributions from inverse seesaw mechanism 49

5 Techniques for calculating radiative corrections at one-loop level **54**
5.1 Dimensional regularization . . . 54
5.2 One-loop integral . . . 55
5.2.1 Scalar one-point function . . . 55
5.2.2 Scalar two-point function . . . 57
5.2.3 Reduction of tensor integral to scalar integrals . . . 59
5.3 Renormalization . . . 62

6 One loop corrections to the Higgs boson mass **64**
6.1 Counterterms and renormalization constants . . . 64
6.2 Tadpole and self-energies diagrams . . . 66
6.2.1 Tadpole diagrams . . . 66
6.2.2 Neutrino contribution . . . 66
6.2.3 Neutral Higgs self-energy diagrams . . . 67
6.2.4 Charged Higgs self-energy . . . 69
6.2.5 Z-boson self-energy . . . 70
6.2.6 W-boson self-energy . . . 73
6.3 Calculation of the counterterms and Higgs masses . . . 74
6.3.1 Renormalized self-energy . . . 74
6.3.2 Wave-function renormalization constants . . . 75
6.3.3 Counterterm conditions . . . 75

7 Numerical Analysis **78**
7.1 Numerical parameters . . . 78
7.2 Dependence of Higgs sector parameters in inverse seesaw mechanism . . . 79
7.2.1 Parameter scan for maximal change in Higgs mass . . . 81

8 Conclusion and proposal **84**

A Conventions and Formulas **86**
A.1 Units and metric conventions . . . 86
A.2 Spinor algebra . . . 86
A.3 Calculus of Grassmann variables . . . 87
A.4 Some expansion formulas . . . 88

B Neutral Higgs mass matrix counterterm **89**

Acknowledgement **92**

Bibliography **94**

Chapter 1

The Standard Model of Particle Physics

1.1 Introduction

With the explosion of experimental achievement in particle physics in the last century, demand was high for a theory that can explain those data. Quantum Electrodynamics (QED) was among the earliest theories [1–7], yet it is among the most accurate ones [8, 9]. Further investigations into the beta decay process lead to the proposal of weak force and Electroweak (EW) theory [10–12]. Along the line, Quantum Chromodynamics (QCD) was also developed to describe the strong nuclear force [13–16]. In the end, all of these theories are brought together to the **Standard Model** (SM). It is a very successful theory describing the elementary particles and their interactions. It has so far provided predictions remarkably fit with experimental results. Nearly 50 years after its proposal, the last piece of the model, the Higgs boson, was experimentally confirmed in 2012 [17, 18].

The Standard Model is a renormalizable Yang-Mills theory represented by the gauge group $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$. The group $SU_L(2) \otimes U_Y(1)$ describes a unified theory of electromagnetic and weak interactions, called the electroweak theory. While the group $SU_C(3)$ is the gauge group representing the strong interaction. The particles that undergo these interactions build up to the particle spectrum of the SM. It consists of leptons, quarks divided into three generations ordered by ascending mass (confront table 1.1), together with the gauge bosons mediating the corresponding force and a scalar particle, called the Higgs boson, that is the result of the Higgs mechanism giving mass to other particles [19–25]. Since weak interaction only couples with left-handed fermions, these left-handed particles are arranged in doublet representation of $SU_L(2)$ while their right-handed counterparts are singlet. Neutrino, however, is an exception because only left-handed ones are experimentally confirmed, while right-handed neutrino is yet to be found, thus not included in the SM. The gauge bosons are in the adjoint representations of their gauge group. Unlike gluons, the gauge field of $SU_L(2) \otimes U_Y(1)$ mixes to create the observed $W^{\pm}, Z$ bosons and photon. The particle spectrum is described in table 1.2, where the electric charge is computed through the Gell-Mann-Nishijima formula [26–28]

$$Q = I_3 + Y. \tag{1.1.1}$$

Although some are experimentally confirmed to be massive, gauge bosons have to be massless to conserved gauge invariance. Thus, symmetry must be broken on a low energy scale. The Higgs mechanism explain this through the couplings of every massive field with the Higgs field. Since the W, Z bosons are massive, while the other gauge bosons are not, the Higgs field must be in a representation of the $SU_L(2)$ group. Which means theories with one Higgs singlet is not

Type	1^{st} generation	2^{nd} generation	3^{rd} generation
Quarks	u (up) d (down)	c (charm) s (strange)	t (top) b (bottom)
Leptons	e (electron) ν_e (electron-neutrino)	μ (muon) ν_μ (muon-neutrino)	τ (tau) ν_τ (tau-neutrino)

Table 1.1: 3 generations of quarks and leptons

Particle content	Field	$SU_C(3) \times SU_L(2) \times U_Y(1)$
Quarks	$Q_L = (u_L \ d_L)^T$ $U_R^\dagger = u_R^\dagger$ $D_R^\dagger = d_R^\dagger$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$ $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ $(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
Leptons	$L = (\nu \ e_L)^T$ $E_R^\dagger = e_R^\dagger$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$ $(\mathbf{1}, \mathbf{1}, 1)$
Higgs	$\phi = (\phi^+ \ \phi^0)^T$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
Gluon	g	$(\mathbf{8}, \mathbf{1}, 0)$
W,Z boson, photon (γ)	W^1, W^2, W^3 B	$(\mathbf{1}, \mathbf{3}, 0)$ $(\mathbf{1}, \mathbf{1}, 0)$

Table 1.2: The particle content of the Standard Model

allowed. In SM, a Higgs doublet is introduced.

The SM has a total of 18 free parameters that are fixed by experiment:

- g_s, gauge coupling constant for strong nuclear force $SU_C(3)$,
- g, coupling constant for $SU_L(2)$,
- g', coupling constant for $U_Y(1)$,
- v, vacuum expectation value (VEV) of the Higgs field,
- m_h, the Higgs mass,
- m_f, $f = u, d, c, s, t, b, e, \mu, \tau$, the masses of fermions with the neutrino assumed to be massless,
- The three angles and one phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

1.2 The SM Lagrangian

The entire particle spectrum and interaction can be summarized in a single Lagrangian

$$\mathcal{L}^{SM} = \mathcal{L}_{YM} + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y, \tag{1.2.1}$$

where each term respectively is the kinetic term for gauge boson or Yang-Mills theory, the fermionic term, the Higgs sector and the Yukawa interaction. Each sector will be treated individually below.

Firstly, gauge kinetic sector is given by

$$\mathcal{L}_{YM} = -\frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu} - \frac{1}{4}W^{i\mu\nu}W^{i}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \tag{1.2.2}$$

where $a = 1, \dots, 8, i = 1, \dots, 3$. All repeated indices are implicitly summed over. This convention will be used throughout this thesis whenever repeated indices are encountered, unless otherwise stated. Field strength tensor are determined by

$$G^{a}_{\mu\nu} = \partial_\mu g^a_\nu - \partial_\nu g^a_\mu + g_s f^{abc} g^b_\mu g^c_\nu, \tag{1.2.3}$$

$$W^{i}_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\epsilon^{ijk} W^j_\mu W^k_\nu, \tag{1.2.4}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{1.2.5}$$

Here, gluon field g^a corresponds to $SU_C(3)$ gauge group with structure constant f^{abc}. W^i is the gauge field of $SU_L(2)$ and the structure constant is ϵ^{ijk}. B belongs to the abelian group $U(1)$.

The fermionic sector reads

$$\mathcal{L}_F = \overline{Q_{iL}} i\not{D} Q_{iL} + \overline{L_i} i\not{D} L_i + \overline{U_{iR}} i\not{D} U_{iR} + \overline{D_{iR}} i\not{D} D_{iR} + \overline{E_{iR}} i\not{D} E_{iR}, \tag{1.2.6}$$

where the repeated $i = 1, 2, 3$ is the generation index determined in table 1.1 and table 1.2. The covariant derivative containing the gauge interaction, written for the general case with all interactions is

$$D_\mu = \partial_\mu - ig_s \sum_{a=1}^{8} T^a G^a_\mu - ig \sum_{b=1}^{3} I^b W^b_\mu - ig' Y B_\mu \tag{1.2.7}$$

$$= \partial_\mu - ig_s \sum_{a=1}^{8} T^a G^a_\mu - \frac{ig}{\sqrt{2}}(I^+ W^+_\mu + I^- W^-_\mu) - \frac{ig}{\cos\theta_W}\left(I_3 - Q\sin^2\theta_W\right) Z_\mu - ieQA_\mu, \tag{1.2.8}$$

where T^a and I^b is the generators of their corresponding groups and is respectively half the Gell-Mann matrices and Pauli matrices; the rotated field Z_μ and A_μ are defined as

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \tag{1.2.9}$$

$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \tag{1.2.10}$$

With the convention that $T^a, I^b, I^\pm, I_3$ yields 0 when acts upon the corresponding singlet and Q is the charge operator, this covariant derivative is applicable to all cases of matters listed in table 1.2.

The Lagrangian for the Higgs doublet is given by

$$\mathcal{L}_H = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi) \tag{1.2.11}$$

with the scalar potential

$$V(\phi) = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 \tag{1.2.12}$$

The Yukawa interaction sector reads

$$\mathcal{L}_Y = -Y^L_{ij}\overline{L^i}\phi E^j_R - Y^D_{ij}\overline{Q^i_L}\phi D^j_R - Y^U_{ij}\overline{Q^i_L}\tilde{\phi} U^j_R + h.c. \tag{1.2.13}$$

where Y_L, Y_D, Y_U are the Yukawa couplings whose indices are generation indices, and $\tilde{\phi} = i\sigma_2\phi$ is the charged conjugated Higgs doublet.

1.3 Higgs mechanism and SM mass spectrum

The idea behind Higgs mechanism is that masses of gauge bosons violate gauge symmetry. Thus, to obtain mass, symmetry must somehow be broken. Since the theory itself is gauge invariance, it was proposed that the vacuum must break symmetry so that massive particles may exist. This mechanism is manifest in the SM in its minimal form using only a Higgs doublet.

Since, the W,Z bosons have non-zero mass while gluon and photon are massless. Thus, the electroweak symmetry $SU_L(2) \otimes U_Y(1)$ must be broken to $U_Q(1)$ describing electromagnetic interaction. For the Higgs potential in equation (1.2.12) to break symmetry at its vacuum, its minimum must be degenerate, so that the lowest energy state must pick one among the degenerate options, thus spontaneously breaking symmetry. More specifically, the parameters must obey the conditions:

- The potential must have a minimum; thus, $\lambda > 0$.
- Its global minima must be degenerate; thus, $\mu^2 > 0$.

With these requirements, one can solve for the global minimum of the scalar potential. That is $V_{\min} = -\mu^4/(4\lambda)$ obtained at

$$|\langle\phi\rangle| = \frac{v}{\sqrt{2}} \quad , \quad v = \sqrt{\frac{\mu^2}{\lambda}} \tag{1.3.1}$$

To preserve $U_Q(1)$ symmetry, the upper component of Higgs field must take a global non-degenerate minimum at $|\langle\phi^+\rangle| = 0$. Then, the only solution is $\left|\langle\phi^0\rangle\right| = v/\sqrt{2}$ corresponding to a circle on the complex plane. According to Higgs mechanism, at the lowest energy state, the vacuum is forced to pick one among this collection of states, thus breaking symmetry. At an energy level not much higher than the vacuum, the generation of mass can be explained by first expanding the Higgs field around the chosen minimum

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \begin{pmatrix} G^+(x) \\ (v + H(x) - iG^0(x))/\sqrt{2} \end{pmatrix}. \tag{1.3.2}$$

Substituting the above expansion into the scalar potential and collect the bilinear terms for H, G^+ and G^0, it can immediately be computed that only the neutral CP-even H has a mass of $m_H = \sqrt{2\lambda v^2}$. The charged $G^\pm$ and neutral CP-odd G^0 are massless. They are the unphysical Nambu-Goldstone bosons that are absorbed by the gauge bosons to generate the masses for $W^\pm$ and Z.

To obtain the mass of gauge bosons, equation (1.3.2) is substituted into the kinetic part of Lagrangian (1.2.11) and the mass is obtained from the bilinear term. The mass eigenstates of the gauge bosons read

$$\begin{cases} W^\pm_\mu &= \dfrac{W^1_\mu \mp iW^2_\mu}{\sqrt{2}} \\ Z_\mu &= \cos\theta_W W^3_\mu - \sin\theta_W B_\mu \\ A_\mu &= \sin\theta_W W^3_\mu + \cos\theta_W B_\mu \end{cases} \tag{1.3.3}$$

where θ_W is called the weak mixing angle, defined by

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} \ , \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{1.3.4}$$

In this basis, the masses of the particles are given by

$$M_W = \frac{vg}{2}\ ,\ M_Z = \frac{v\sqrt{g^2+g'^2}}{2}\ ,\ M_A = 0. \tag{1.3.5}$$

The photon remains massless because we demanded $U_Q(1)$ to be not broken by the vacuum. The fermions' masses are obtained through the Yukawa interaction sector of the Lagrangian.

$$\mathcal{L}_{F\ mass} = -\frac{vY^L_{ij}}{\sqrt{2}}\bar{e}^i_L e^j_R - \frac{vY^D_{ij}}{\sqrt{2}}\bar{d}^i_L d^j_R - \frac{vY^U_{ij}}{\sqrt{2}}\bar{u}^i_L u^j_R + h.c. \tag{1.3.6}$$

with $i, j = 1, 2, 3$. Here, no bilinear term for neutrino is found. Thus, the SM predicts that neutrinos are massless. This, however, contradicts with experiment and is discussed in chapter 4. As for the gauge bosons, we need to diagonalize the corresponding Yukawa matrices to obtain their mass

$$m_f = \frac{vy_f}{\sqrt{2}}\ ,\quad f = e, \mu, \tau, u, d, c, s, t, b \tag{1.3.7}$$

where y_f is the respective eigenvalues of the corresponding Yukawa matrix. In reality, without taking neutrino sector into account, no leptonic mixing has been experimentally confirmed, thus their Yukawa matrix is automatically diagonalized. It is, however, not the case for the quark sector. Quarks are observed to mixed among its flavour. The mixing is characterized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [29, 30], defined by

$$U_{CKM} = V^u_L V^{d\dagger}_L \tag{1.3.8}$$

with $i, j = 1, 2, 3$. The matrix has three angle parameters and one CP-violating phase. Such phenomenon is important because it is the only source of CP violations in the SM. Unlike predicted in the SM, neutrinos are also observed to mix through neutrino oscillation experiments [31–33]. This topic will be elaborated further in chapter 4

1.4 The issues of SM

Though being the best fit theory for experiment, the Standard Model is considered to be an effective theory and also fails to explain some questions. These can be categorized into two groups. The first consists of theoretical problems.

- The Quantum theory of Gravity: SM only explain electromagnetic, weak and strong, excluding gravity, and has some unexplained issues. Thus it is expected to be only a low energy approximation for a more general theory.

- The hierarchy problems: The typical scale of SM is about $M_{ew} \sim 100$ GeV, which is multiple of magnitudes smaller than the Planck scale $M_{pl} \sim 10^{19}$ GeV, where the quantum theory of gravity lies. Many questions arises. Why the fundamental scales of the Universe so different? But a more important question is that the quantum correction of Higgs mass is quadratically sensitive to the cut-off mass scale, which is naturally of order M_{pl}. But with the experimentally confirmed mass of Higgs of ~ 125 GeV, the radiative correction to Higgs mass is 34 orders of magnitude larger than the mass itself. This requires extreme fine-tuning to the counterterms of the Higgs mass and is a hierarchy problem that supersymmetric can solve easily.

- Gauge coupling unification: In the SM, the three couplings of tends to meet at the energy scale around GUT scale, but they not actually coincide. However, in supersymmetric Grand Unified Theory (SGUT), the three couplings exactly meet

The second group is suggested from experimental data.

- Neutrino mass: As shown above, the SM neutrinos are massless. However, experimental data gathered from solar, atmospheric, reactor and accelerator neutrinos suggest otherwise. Not only do those neutrinos have non-zero mass, its mass is unnaturally small, of order 10^{-6} that of the lightest lepton, electron. The SM need to be extended to include neutrino masses and to explain the their smallness

- Dark matter: Although observed to be abundant, dark matter has yet to reveal any information about what it is made of. Experiments have shown that it must not have electric or color charge and must be stable. However, SM cannot gives any candidate with such property. Even if we account for massive neutrinos, the amount of them would not be enough to cover all of dark matter. Therefore, a new theory that can give a reliable candidate is attractive.

There are many theoretical ways to try solving the above problems. This thesis focuses on the supersymmetric methods, especially the Minimal and Next-to-Minimal Supersymmetric version of the Standard Model, and the seesaw mechanisms, particularly Inverse Seesaw Mechanism. As shown in the upcoming chapters, these methods provide the solutions to some of the above problems

Chapter 2

Supersymmetry

2.1 Introduction

One possible path to resolves to problems existing in SM is to extend its symmetry group to a more general case. While one popular approach is to extend its internal symmetry such as SO(10) or SU(5) [34], supersymmetry (SUSY) expands the external (or spacetime) symmetry. Contrary to the endless choices available for internal symmetry, Coleman and Mandula [35] together with Haag, Lopuszanski and Sohnius [36] have proven that there is a largest possible spacetime symmetry group. This theorem is the mathematical foundation for the development of SUSY. According to Coleman and Mandula's paper, for a reasonably natural physical theory, spacetime symmetry group contains the following properties:

- Lorentz invariance.
- There is a finite number of types of particle. Or more specifically, each mass multiplet contains a finite number of types of particle.
- Elastic-scattering amplitudes are analytic functions of the center of mass energy $\sqrt{s}$ and momentum transfer in some neighbourhood of physical region.

The most general symmetry of nature is Poincaré $\times$ Internal symmetry. However, in that theorem, only bosonic operators are considered. Later, Haag, Lopuszanski and Sohnius [36] expanded the treatment to include both commuting and anti-commuting operators. In such theories, the largest possible symmetry is

$$\text{SuperPoincaré} \times \text{Internal symmetry}$$

where SuperPoincaré composes of the generators of Poincaré group and fermionic operators $Q^I_\alpha, \bar{Q}^J_{\dot{\alpha}}$ $(I, J = 1, \ldots, N; \alpha = 1, 2; \dot{\alpha} = \dot{1}, \dot{2})$. This operator act as

$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle\ , \quad Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle\,. \tag{2.1.1}$$

That means it changes between boson and fermion or changes the spin of a particles, which is directly related to the generators of Lorentz group $M^{\mu\nu}$; hence, this operators commutes with translations P^μ but not with Lorentz group generators. For more details, see section 2.2.

Although no experimental evidence has yet been found to support supersymmetry, many physicists still find it interesting to investigate deeper into this topic. Moreover, with the recent and upcoming updates of the Large Hadron Collider (LHC) and the plans to build more advanced colliders such as the Future Circular Collider (FCC), physicist may find evidence to prove the validity of supersymmetry if it is the underlying theory of nature.

Next, we shall investigate the algebraic structure of supersymmetry before we move on to construct a supersymmetric Lagrangian using the notion of superfield and superspace.

2.2 Supersymmetric Algebra

According to the paper by Haag, Lopuszanski and Sohnius [36], the generators for the largest symmetry group in nature obey the algebra

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0, && (2.2.1)\\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(g_{\mu\sigma}M_{\nu\rho} + g_{\nu\rho}M_{\mu\sigma} - g_{\mu\rho}M_{\nu\sigma} - g_{\nu\sigma}M_{\mu\rho}), && (2.2.2)\\
[M_{\mu\nu}, P_\rho] &= i(g_{\rho\nu}P_\mu - g_{\rho\mu}P_\nu) && (2.2.3)\\
\left[P_\mu, Q^I_\alpha\right] &= 0, && (2.2.4)\\
\left[P_\mu, \bar{Q}^I_{\dot\alpha}\right] &= 0, && (2.2.5)\\
\left[Q^I_\alpha, M_{\mu\nu}\right] &= (\sigma_{\mu\nu})_\alpha^{\ \beta} Q^I_\beta, && (2.2.6)\\
\left[\bar{Q}^{I\dot\alpha}, M_{\mu\nu}\right] &= (\bar\sigma_{\mu\nu})^{\dot\alpha}_{\ \dot\beta} \bar{Q}^{I\dot\beta}, && (2.2.7)\\
\left\{Q^I_\alpha, \bar{Q}^J_{\dot\beta}\right\} &= 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^{IJ}, && (2.2.8)\\
\left\{Q^I_\alpha, Q^J_\beta\right\} &= \epsilon_{\alpha\beta} Z^{IJ}, && (2.2.9)\\
\left\{\bar{Q}^I_{\dot\alpha}, \bar{Q}^J_{\dot\beta}\right\} &= \epsilon_{\dot\alpha\dot\beta}(Z^{IJ})^*, && (2.2.10)
\end{aligned}
$$

with $I, J = 1, \dots, N$; $\mu, \nu, \rho = 0, \cdots, 3$; $\alpha, \beta = 1, 2$; $\dot\alpha, \dot\beta = \dot1, \dot2$. Z^{IJ}, which has the anti-symmetric property $Z^{IJ} = -Z^{JI}$, are called central charges, are Lorentz scalar, and commute with the whole supersymmetry algebra and the internal symmetry. $Q^I_\alpha, \bar{Q}^I_{\dot\alpha}$ are written in two-component-spinor notation. A brief description of $\sigma_{\mu\nu}, \bar\sigma_{\mu\nu}, \epsilon_{\alpha\beta}, \epsilon_{\dot\alpha\dot\beta}$ and spinor algebra can be found in appendix $A.2$. In some texts, an extra i are added in equation (2.2.6) and (2.2.7). But that is only a matter of convention. From the algebra, we can observe that $P_\mu, M_{\mu\nu}$ are bosonic operators due to their commutation relations, while $Q^I_\alpha, \bar{Q}^I_{\dot\alpha}$ are fermionic operators due to anticommutation relations.

If we expand our consideration to internal symmetry, whose generators are A_l, to get the full symmetry acting on a system, we will have these additional commutators added to our algebra

$$
\begin{aligned}
[A_l, A_m] &= i f^n_{lm} A_n, && (2.2.11)\\
[P_\mu, A_l] &= 0, && (2.2.12)\\
[M_{\mu\nu}, A_l] &= 0, && (2.2.13)\\
\left[Q^I_\alpha, A_l\right] &= (a_l)^I_J Q^J_\alpha, && (2.2.14)\\
\left[\bar{Q}_{I\dot\alpha}, A_l\right] &= -\bar{Q}_{J\dot\alpha}(a_l)^J_I. && (2.2.15)
\end{aligned}
$$

For simplicity and relevant phenomenologies, we only consider the case where $N = 1$.

2.3 Superfield and superspace

Using the generators of supersymmetry algebra, we can, in principle, write the supersymmetric field representations in 4-dimensional spacetime, thus build the Lagrangian as normally do in SM. However, such approach is so complicated because in the SM one has to make sure each term satisfying intenal symmetry separately while in SUSY one has to ensure that the sum of all terms to be invariant. The simpler way to accomplish this task is through the introduction of superspace and superfield. Therefore, this section is devoted to describing these two concepts. In the following one, we will use those definitions to construct a general Lagrangian for supersymmetric Lagrangian

2.3.1 Superspace

To construct the Lagrangian for supersymmetric field theory, we need to be able write down the representation of multiplet of fields. Although this is possible, it poses a problem. To construct a supersymmetric theory, the action should be invariant under a supersymmetric transformation. Checking this using regular Minkowski space is very cumbersome. One way to solve this problem is to introduce a new space called *superspace* which is an extension of Minkowski space. In this formalism, the supersymmetric invariance is realized naturally.

First of all, the Minkowski space can be defined as followed: Minkowski = Poincaré / Lorentz. Since the Poincaré group consists of translation, boost and rotation while the Lorentz group has boost and rotation, the Minkowski space is constructed from only translational transformation. This can be easily understood since the whole Minkowski space is made purely from translating the origin using all arbitrary 4-vectors.

The same idea can be used to extend the regular Minkowski space to superspace. We can extend the set of generators for Poincaré group $\{M^{\mu\nu}, P^{\mu}\}$ to also contain the spinor generators $Q^A_\alpha, \bar{Q}^A_{\dot{\alpha}}$, which becomes the generators for super Poincaré group. The new superspace can now be defined as

$$\text{Superspace} = \text{SuperPoincaré} \ / \ \text{Lorentz}.$$

For convenience, we need to rewrite the whole supersymmetry algebra in term of Lie algebra. To do that we need to introduce the Grassmann parameters $\theta^\alpha, \bar{\theta}_{\dot{\beta}}$ to reduce the anti-commutators in the supersymmetry algebra to commutator. This works as follow

$$\left[\theta^\alpha Q_\alpha, \bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}}\right] = \theta^\alpha\bar{\theta}^{\dot{\beta}}Q_\alpha\bar{Q}_{\dot{\beta}} + \theta^\alpha\bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}}Q_\alpha = \theta^\alpha\bar{\theta}^{\dot{\beta}}\left\{Q_\alpha, \bar{Q}_{\dot{\beta}}\right\} = 2\theta^\alpha(\sigma^\mu)_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}P_\mu, \tag{2.3.1}$$

$$\left[\theta^\alpha Q_\alpha, \theta^\beta Q_\beta\right] = \theta^\alpha\theta^\beta Q_\alpha Q_\beta + \theta^\alpha\theta^\beta Q_\beta Q_\alpha = \theta^\alpha\theta^\beta\{Q_\alpha, Q_\beta\} = 0, \tag{2.3.2}$$

$$\left[\bar{\theta}^{\dot{\alpha}}\bar{Q}_{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}}\right] = \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\alpha}}\bar{Q}_{\dot{\beta}} + \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}}\bar{Q}_{\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}}\left\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\right\} = 0. \tag{2.3.3}$$

Exponentiating this Lie algebra, we get the SuperPoincaré group whose element can be written as

$$G(x, \theta, \bar{\theta}, \omega) = \exp\left(ix^\mu P_\mu + i\theta^\alpha Q_\alpha + i\bar{\theta}^{\dot{\beta}}\bar{Q}_{\dot{\beta}} + \frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}\right). \tag{2.3.4}$$

A "point" in superspace can now be identified through a one-to-one map with the corresponding "super-translation" by omitting the generator of Lorentz group from the above expression

$$(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}) \leftrightarrow e^{ix^\mu P_\mu}e^{i(\theta Q + \bar{\theta}\bar{Q})}. \tag{2.3.5}$$

The 2+2 anti-commuting Grassmann numbers $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ can be thought of as coordinates in superspace. More details on the algebra of Grassmann numbers are presented in appendix $A.2$ while its calculus can be found in appendix $A.3$

2.3.2 General superfield

To define a superfield, which is a field in superspace, we can start from the definition of field. A field, in Minkowski space, is

- a function of spacetime coordinate x^μ, and
- transform under translation as, for example, $\phi(x^\mu) = \exp(-ia_\mu P^\mu)\phi(0)\exp(ia_\mu P^\mu)$.

This very definition can be naturally extended to superfield by replacing ordinary spacetime with superspace and translation with super-translation.

Firstly, a superfield is a function of coordinate in superspace $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$. As consequence of Grassman number properties $A.3$, the Taylor expansion of a general scalar superfield $Y = Y(x,\theta,\bar{\theta})$ is

$$\begin{aligned}Y(x,\theta,\bar{\theta}) = f(x) + \theta\psi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma^\mu\bar{\theta}v_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x)\\ + \bar{\theta}\bar{\theta}\theta\rho(x) + \theta\theta\bar{\theta}\bar{\theta}d(x)\end{aligned} \tag{2.3.6}$$

where $f, \psi, \bar{\chi}, m, n, v_\mu, \bar{\lambda}, \rho, d$ are fields. Since each term in the expansion is automatically Lorentz invariant by definition, the superfield itself is also Lorentz invariance. For the last property, a superfield is a field in superspace that transforms under a super-translation according to the following rule with parameters $\epsilon, \bar{\epsilon}$. Here, we exclude translational parameters coming from Poincaré group because all component fields, thus the general field, have already satisfied such transformation.

$$Y(x+\delta x, \theta+\delta\theta, \bar{\theta}+\delta\bar{\theta}) = \exp\big(-i(\epsilon Q + \bar{\epsilon}\bar{Q})\big)Y(x,\theta,\bar{\theta})\exp\big(i(\epsilon Q + \bar{\epsilon}\bar{Q})\big). \tag{2.3.7}$$

Using this definition, we can now find the appropriate variation property of a superfield so that the suitable transformation relation is realized, which will also be handy in our later discussion about the invariance of the Lagrangian. The variation is defined as

$$\delta_{\epsilon,\bar{\epsilon}}Y(x,\theta,\bar{\theta}) := Y(x+\delta x, \theta+\delta\theta, \bar{\theta}+\delta\bar{\theta}) - Y(x,\theta,\bar{\theta}) \tag{2.3.8}$$

where the variation $\delta x, \delta\theta, \delta\bar{\theta}$ is resulted from the translation (2.3.7) above and depends on the parameter $\epsilon, \bar{\epsilon}$. The explicit expression can be found as followed

$$\begin{aligned}Y(x+\delta x, \theta+\delta\theta, \bar{\theta}+\delta\bar{\theta}) &= \exp\big\{-i(\epsilon Q + \bar{\epsilon}\bar{Q})\big\}Y(x,\theta,\bar{\theta})\exp\big\{i(\epsilon Q + \bar{\epsilon}\bar{Q})\big\}\\ &= e^{-i(\epsilon Q+\bar{\epsilon}\bar{Q})}e^{-i(x\mathcal{P}+\theta Q+\bar{\theta}\bar{Q})}Y(0,0,0)e^{i(x\mathcal{P}+\theta Q+\bar{\theta}\bar{Q})}e^{i(\epsilon Q+\bar{\epsilon}\bar{Q})}.\end{aligned}$$

While the left-hand-side equates

$$Y(x+\delta x, \theta+\delta\theta, \bar{\theta}+\delta\bar{\theta}) = e^{-i(x+\delta x)\mathcal{P}-i(\theta+\delta\theta)Q-i(\bar{\theta}+\delta\bar{\theta})\bar{Q}}Y(0,0,0)e^{i(x+\delta x)\mathcal{P}+i(\theta+\delta\theta)Q+i(\bar{\theta}+\delta\bar{\theta})\bar{Q}},$$

which means

$$\exp\big\{i(x\mathcal{P}+\theta Q+\bar{\theta}\bar{Q})\big\}\exp\big\{i(\epsilon Q+\bar{\epsilon}\bar{Q})\big\} = \exp\big\{i(x+\delta x)\mathcal{P}+i(\theta+\delta\theta)Q+i(\bar{\theta}+\delta\bar{\theta})\bar{Q}\big\}. \tag{2.3.9}$$

Using the Baker-Campbell-Hausdorff formula to the second order (approximating higher terms as zero) for the left-hand-side

$$\begin{aligned}1 &\approx \exp\bigg\{i(x\mathcal{P}+\theta Q+\bar{\theta}\bar{Q}) + i(\epsilon Q+\bar{\epsilon}\bar{Q}) + \frac{1}{2}\big[i(x\mathcal{P}+\theta Q+\bar{\theta}\bar{Q}), i(\epsilon Q+\bar{\epsilon}\bar{Q})\big]\bigg\}\\ &= \exp\bigg\{ix\mathcal{P} + i(\theta+\epsilon)Q + i(\bar{\theta}+\bar{\epsilon})\bar{Q} - \frac{1}{2}\big[\theta Q, \bar{\epsilon}\bar{Q}\big] + \frac{1}{2}\big[\bar{\theta}\bar{Q}, \epsilon Q\big]\bigg\}\\ &= \exp\big\{ix\mathcal{P} + i(\theta+\epsilon)Q + i(\bar{\theta}+\bar{\epsilon})\bar{Q} - \theta\sigma^\mu\bar{\epsilon}\mathcal{P}_\mu + \epsilon\sigma^\mu\bar{\theta}\mathcal{P}_\mu\big\}\\ &= \exp\big\{i(x+i\theta\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu\bar{\theta})\mathcal{P} + i(\theta+\epsilon)Q + i(\bar{\theta}+\bar{\epsilon})\bar{Q}\big\}.\end{aligned}$$

Equating two sides of equation (2.3.9), we obtain

$$\begin{cases}\delta x^\mu &= i\theta\sigma^\mu\bar{\epsilon} - i\epsilon\sigma^\mu\bar{\theta}\\ \delta\theta^\alpha &= \epsilon^\alpha\\ \delta\bar{\theta}^{\dot{\alpha}} &= \bar{\epsilon}^{\dot{\alpha}}\end{cases}. \tag{2.3.10}$$

Although we only consider the generator $Q, \bar{Q}$, due to the supersymmetry algebra $\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} \sim P_\mu$, two subsequent supersymmetric transformation produce spacetime translation. Because there are two ways to express the variation of a superfield. One way is to treat $Y(x,\theta,\bar{\theta})$ as a field operator using the definition (2.3.7)

$$\begin{aligned}\delta_{\epsilon,\bar{\epsilon}} Y(x,\theta,\bar{\theta}) &\approx (1 - i\epsilon Q - i\bar{\epsilon}\bar{Q}) Y(x,\theta,\bar{\theta})(1 + i\epsilon Q + i\bar{\epsilon}\bar{Q}) - Y(x,\theta,\bar{\theta}) \\ &\approx -i\epsilon^\alpha \big[Q_\alpha, Y(x,\theta,\bar{\theta})\big] - i\bar{\epsilon}_{\dot{\alpha}} \big[\bar{Q}^{\dot{\alpha}}, Y(x,\theta,\bar{\theta})\big].\end{aligned}$$

The other is to consider it as a vector in some vector space

$$\begin{aligned}&Y(x+\delta x, \theta + \delta\theta, \bar{\theta} + \delta\bar{\theta}) = \exp\big\{i(\epsilon Q + \bar{\epsilon}\bar{Q})\big\} Y(x,\theta,\bar{\theta}) \approx (1 + i(\epsilon Q + \bar{\epsilon}\bar{Q}))Y(x,\theta,\bar{\theta}) \\ &\qquad \Longrightarrow \delta_{\epsilon,\bar{\epsilon}} Y(x,\theta,\bar{\theta}) \approx i(\epsilon Q + \bar{\epsilon}\bar{Q})Y.\end{aligned}$$

Therefore, we can equate the above two expressions to get $[Y, Q_\alpha] \equiv Q_\alpha Y$, $\big[Y, \bar{Q}_{\dot{\alpha}}\big] \equiv \bar{Q}_{\dot{\alpha}} Y$ and obtain

$$\delta_{\epsilon,\bar{\epsilon}} Y = i(\epsilon Q + \bar{\epsilon}\bar{Q})Y = i\big[Y, (\epsilon Q + \bar{\epsilon}\bar{Q})\big]. \tag{2.3.11}$$

This formula comes directly from the third property of a superfields and completes the definition. It is often used to check whether a field is a superfield. One problem arises, however, is that in order to use that relation, we need to know the differential representations of the operators. To obtain this, we re-express the variation of Y in (2.3.11) using Taylor expansion and equates the two sides

$$\begin{aligned}\delta_{\epsilon,\bar{\epsilon}} Y(x,\theta,\bar{\theta}) &\approx Y(x,\theta,\bar{\theta}) + \delta x^\mu \partial_\mu Y(x,\theta,\bar{\theta}) + \delta\theta^\alpha \partial_\alpha Y(x,\theta,\bar{\theta}) + \delta\bar{\theta}^{\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} Y(x,\theta,\bar{\theta}) - Y(x,\theta,\bar{\theta}) \\ &= \big[\epsilon^\alpha \partial_\alpha + \bar{\epsilon}^{\dot{\alpha}} \bar{\partial}_{\dot{\alpha}} + i(\theta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\theta})\partial_\mu\big] Y(x,\theta,\bar{\theta}) \\ &= \Big[i\epsilon^\alpha(-i\partial_\alpha - \sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu) + i\bar{\epsilon}^{\dot{\alpha}}(-i\bar{\partial}_{\dot{\alpha}} - \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu)\Big] Y(x,\theta,\bar{\theta}).\end{aligned}$$

We obtain the differential representations for the operators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$

$$\begin{cases} Q_\alpha &= -i\partial_\alpha - \sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \\ \bar{Q}_{\dot{\alpha}} &= i\bar{\partial}_{\dot{\alpha}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu \end{cases}. \tag{2.3.12}$$

Using the above results, we can prove the following properties:

- A product of superfields is a superfield.
- Linear combination of superfields is a superfield.
- If S is a superfield, then $\partial_\mu S, D_\alpha S, \bar{D}_{\dot{\alpha}} S$ are also superfields, where D_α and $\bar{D}_{\dot{\alpha}}$ are covariant derivatives

$$D_\alpha \coloneqq \partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu \quad , \quad \bar{D}_{\dot{\alpha}} \coloneqq \bar{\partial}_{\dot{\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu. \tag{2.3.13}$$

Using these properties, we can see that any combination of addition, multiplication or derivations of superfields is a superfield.

Before we continue, we shall find the variation of the components fields of the general superfield under supersymmetric transformation.

$$\begin{aligned}\delta_{\epsilon,\bar{\epsilon}} Y &= i(\epsilon Q + \bar{\varepsilon}\bar{Q})Y \\ &= \Big(\epsilon^\alpha\partial_\alpha - i\epsilon^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu + \bar{\varepsilon}^{\dot{\alpha}}\bar{\partial}_{\dot{\alpha}} - i\bar{\varepsilon}^{\dot{\alpha}}\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu\Big)Y\end{aligned}$$

$$
\begin{aligned}
&= \epsilon\psi + 2\epsilon\theta m(x) + (\epsilon\sigma^\mu\bar\theta)v_\mu(x) + 2(\epsilon\theta)\bar\theta\bar\lambda(x) + (\bar\theta\bar\theta)\epsilon\rho(x) + 2\epsilon\theta(\bar\theta\bar\theta)d(x) \\
&\quad - i(\epsilon\sigma^\mu\bar\theta)\big(\partial_\mu f + \theta\partial_\mu\psi(x) + \bar\theta\partial_\mu\bar\chi(x) + (\theta\theta)\partial_\mu m(x) + (\theta\sigma^\nu\bar\theta)\partial_\mu v_\nu(x) + (\theta\theta)\bar\theta\partial_\mu\bar\lambda(x)\big) \\
&\quad + \bar\varepsilon\bar\chi + 2\bar\varepsilon\bar\theta n(x) + (\theta\sigma^\mu\bar\varepsilon)v_\mu(x) + (\theta\theta)\bar\varepsilon\bar\lambda(x) + 2(\bar\varepsilon\bar\theta)\theta\rho(x) + 2(\theta\theta)(\bar\varepsilon\bar\theta)d(x) \\
&\quad + i(\theta\sigma^\mu\bar\varepsilon)\big(\partial_\mu f(x) + \theta\partial_\mu\psi(x) + \bar\theta\partial_\mu\bar\chi(x) + (\bar\theta\bar\theta)\partial_\mu n(x) + (\theta\sigma^\nu\bar\theta)\partial_\mu v_\nu(x) + (\bar\theta\bar\theta)\theta\partial_\mu\rho(x)\big) \\
&= (\epsilon\psi + \bar\varepsilon\bar\chi) + \theta[2\epsilon m + \sigma^\mu\bar\varepsilon(i\partial_\mu f + v_\mu)] + \bar\theta[2\bar\varepsilon n - \epsilon\sigma^\mu(i\partial_\mu f - v_\mu)] + (\theta\theta)\left(\bar\varepsilon\bar\lambda - \frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\varepsilon\right) \\
&\quad + (\bar\theta\bar\theta)\left(\epsilon\rho + \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar\chi\right) + (\theta\sigma^\mu\bar\theta)\left[\epsilon\sigma_\mu\bar\lambda + \rho\sigma_\mu\bar\varepsilon + \frac{i}{2}(\partial^\nu\psi\sigma_\mu\bar\sigma_\nu\epsilon - \bar\varepsilon\bar\sigma_\nu\sigma_\mu\partial^\nu\bar\chi)\right] \\
&\quad + (\theta\theta)\bar\theta\left(2\bar\varepsilon d + i\bar\sigma^\mu\epsilon\partial_\mu m + \frac{i}{2}\bar\sigma^\nu\sigma^\mu\bar\varepsilon\partial_\mu v_\nu\right) + (\bar\theta\bar\theta)\theta\left(2\epsilon d + i\sigma^\mu\bar\varepsilon\partial_\mu n - \frac{i}{2}\sigma^\nu\bar\sigma^\mu\epsilon\partial_\mu v_\nu\right) \\
&\quad + (\theta\theta)(\bar\theta\bar\theta)\frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\partial_\mu\bar\lambda - \epsilon\bar\sigma^\mu\partial_\mu\rho) \\
&\equiv \delta f + \theta(\delta\psi) + \bar\theta(\delta\bar\chi) + (\theta\theta)\delta m + (\bar\theta\bar\theta)\delta n + (\theta\sigma^\mu\bar\theta)\delta v_\mu + (\theta\theta)\bar\theta\delta\bar\lambda + (\bar\theta\bar\theta)\theta\delta\rho + (\theta\theta)(\bar\theta\bar\theta)\delta d.
\end{aligned}
$$

Therefore, we obtain the variation of the component fields. These variations will come in handy when we build the Lagrangian for the supersymmetric theory.

$$
\begin{aligned}
\delta f &= \epsilon\psi + \bar\epsilon\bar\chi \\
\delta\psi &= 2\epsilon m + \sigma^\mu\bar\epsilon(i\partial_\mu f + v_\mu) \\
\delta\bar\chi &= 2\bar\epsilon n - \epsilon\sigma^\mu(i\partial_\mu f - v_\mu) \\
\delta m &= \bar\epsilon\bar\lambda - \frac{i}{2}\partial_\mu\psi\sigma^\mu\bar\epsilon \\
\delta n &= \epsilon\rho + \frac{i}{2}\epsilon\sigma^\mu\partial_\mu\bar\chi \\
\delta v_\mu &= \epsilon\sigma_\mu\bar\lambda + \rho\sigma_\mu\bar\epsilon + \frac{i}{2}(\partial^\nu\psi\sigma_\mu\bar\sigma_\nu\epsilon - \bar\epsilon\bar\sigma_\nu\sigma_\mu\partial^\nu\bar\chi) \\
\delta\bar\lambda &= 2\bar\epsilon d + \frac{i}{2}(\bar\sigma^\nu\sigma^\mu\bar\epsilon)\partial_\mu v_\nu + i\bar\sigma^\mu\epsilon\partial_\mu m \\
\delta\rho &= 2\epsilon d - \frac{i}{2}(\sigma^\nu\bar\sigma^\mu\epsilon)\partial_\mu v_\nu + i\sigma^\mu\bar\epsilon\partial_\mu n \\
\delta d &= \frac{i}{2}\partial_\mu(\epsilon\sigma^\mu\bar\lambda - \rho\sigma^\mu\bar\epsilon).
\end{aligned}
$$

Among all the component fields, only the variation of D-term is a total derivative under supersymmetric transformation. Which means, if we construct the 4-dimensional Lagrangian by integrating the superfield

$$
\mathcal{L} = \int d^2\theta d^2\bar\theta Y(x,\theta,\bar\theta) = d(x), \tag{2.3.14}
$$

it is guaranteed to be supersymmetric invariant up to a total derivative. However, the superfield contains so many component fields, this general superfield is reducible representation of supersymmetry. That is, we can eliminate some of its component using some restrictions and it can still be a superfield. Here, we only consider the kinds of irreducible superfields that we will use in our theory

2.3.3 Chiral superfield

The definition of a chiral superfield Φ is

$$
\bar D_{\dot\alpha}\Phi = 0. \tag{2.3.15}
$$

Since the above expression is a superfield, the definition is supersymmetric invariance. A anti-chiral superfield Ψ is defined similarly

$$D_\alpha \Psi = 0. \tag{2.3.16}$$

We shall solve only chiral superfield. The result for the anti-chiral can be obtained similarly. To easily handle chiral superfield, we need to define new coordinates

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \quad , \quad \bar{y}^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta} \tag{2.3.17}$$

which gives

$$\begin{aligned}
\bar{D}_{\dot{\alpha}}\Phi &= \bar{\partial}_{\dot{\alpha}}\Phi + \frac{\partial\Phi}{\partial y^\mu}\frac{\partial y^\mu}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu\Phi \\
&= \bar{\partial}_{\dot{\alpha}}\Phi + \partial_\mu\Phi(-i\theta\sigma^\mu)_{\dot{\alpha}} + i\theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu\Phi \\
&= \bar{\partial}_{\dot{\alpha}}\Phi = 0.
\end{aligned}$$

Therefore the terms depending on $\bar{\theta}$ in $\Phi(y,\theta,\bar{\theta})$ vanishes. Conventionally, it is written as follow

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{2.3.18}$$

To obtain the original $\Phi(x,\theta,\bar{\theta})$, we just need to Taylor-expanding this expression around x

$$\begin{aligned}
\Phi(x,\theta,\bar{\theta}) &= \Phi(y,\theta) = \Phi(x + i\theta\sigma^\mu\bar{\theta}) \\
&= \Phi(x,\theta) + \partial_\mu\Phi(x,\theta)(i\theta\sigma^\mu\bar{\theta}) + \frac{1}{2}\partial_\mu\partial_\nu\Phi(x,\theta)(i\theta\sigma^\mu\bar{\theta})(i\theta\sigma^\nu\bar{\theta}) \\
&= \phi(x) + \sqrt{2}\theta\psi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) + \theta\theta F(x) + \frac{i}{\sqrt{2}}(\theta\theta)(\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi(x).
\end{aligned} \tag{2.3.19}$$

Similarly for anti-chiral superfield

$$\bar{\Phi}(x,\theta,\bar{\theta}) = \bar{\phi}(\bar{y}) + \sqrt{2}\bar{\theta}\bar{\psi}(\bar{y}) + \bar{\theta}\bar{\theta}\bar{F}(\bar{y}) \tag{2.3.20}$$

$$= \bar{\phi}(x) + \sqrt{2}\bar{\theta}\bar{\psi}(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}(x) + \bar{\theta}\bar{\theta}\bar{F}(x) + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}(x) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\bar{\phi}(x). \tag{2.3.21}$$

A note can be made here that the F-term is the same in both coordinate system. Although chiral superfields have 3 component fields, only two of them have physical meaning. The F field is called an auxiliary field and is not physical. This will become clearer in the section 2.4. Next, we need to find the variation of chiral superfield under supersymmetric transformation. Using the new coordinate system $(y,\theta,\bar{\theta})$, the differential representation of $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ is

$$Q^{(y)}_\alpha = -i\partial_\alpha - i\frac{\partial y^\mu}{\partial\theta^\alpha}\partial_\mu - \sigma^\mu_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_\mu = -i\partial_\alpha - i(i\sigma^\mu\bar{\theta})_\alpha\partial_\mu - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu = -i\partial_\alpha, \tag{2.3.22}$$

$$\bar{Q}^{(y)}_{\dot{\alpha}} = i\bar{\partial}_{\dot{\alpha}} + i\frac{\partial y^\mu}{\partial\bar{\theta}^{\dot{\alpha}}}\partial_\mu + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu = -i\partial_\alpha + i(-i\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu = i\bar{\partial}_{\dot{\alpha}} + 2\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\frac{\partial}{\partial y^\mu}. \tag{2.3.23}$$

Plugging these expressions into $\delta_{\epsilon,\bar{\epsilon}}\Phi(y,\theta) = i(\epsilon Q + \bar{\epsilon}\bar{Q})\Phi(y,\theta)$ we obtain

$$\begin{aligned}
\delta_{\epsilon,\bar{\epsilon}}\Phi(y,\theta) &= \left(\epsilon^\alpha\partial_\alpha - \bar{\epsilon}_{\dot{\alpha}}\bar{\partial}^{\dot{\alpha}} + 2i\bar{\epsilon}_{\dot{\alpha}}(\theta\sigma^\mu)^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}\right)\Phi(y,\theta) \\
&= \left(\epsilon^\alpha\partial_\alpha - \bar{\epsilon}_{\dot{\alpha}}\bar{\partial}^{\dot{\alpha}} + 2i(\theta\sigma^\mu)_{\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}\right)\Big(\phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)\Big)
\end{aligned}$$

$$
\begin{aligned}
&= \sqrt{2}\epsilon\psi + 2\epsilon\theta F + 2i\theta\sigma^\mu\bar{\epsilon}\Big(\frac{\partial}{\partial y^\mu}\phi + \sqrt{2}\theta\frac{\partial}{\partial y^\mu}\psi + \theta\theta\frac{\partial}{\partial y^\mu}F\Big) \\
&= \sqrt{2}\epsilon\psi + \sqrt{2}\theta\Big(\sqrt{2}\epsilon F + \sqrt{2}i\sigma^\mu\bar{\epsilon}\frac{\partial}{\partial y^\mu}\phi\Big) + 2\sqrt{2}i(\theta\sigma^\mu\bar{\epsilon})\Big(\theta\frac{\partial\psi}{\partial y^\mu}\Big) \\
&= \sqrt{2}\epsilon\psi + \sqrt{2}\theta\Big(\sqrt{2}\epsilon F + \sqrt{2}i\sigma^\mu\bar{\epsilon}\frac{\partial}{\partial y^\mu}\phi\Big) + 2\sqrt{2}i\Big(-\frac{1}{2}\Big)(\theta\theta)(\partial_\mu\psi\sigma^\mu\bar{\epsilon}) \\
&= \sqrt{2}\epsilon\psi + \sqrt{2}\theta\Big(\sqrt{2}\epsilon F + \sqrt{2}i\sigma^\mu\bar{\epsilon}\frac{\partial}{\partial y^\mu}\phi\Big) - \theta\theta(\sqrt{2}i\partial_\mu\psi\sigma^\mu\bar{\epsilon}) \\
&\equiv \delta\phi + \sqrt{2}\theta\delta\psi + \theta\theta\delta F.
\end{aligned}
$$

Therefore the variation of the different field components of the chiral superfields Φ and $\bar{\Phi}$ are

$$
\begin{cases}
\delta\phi &= \sqrt{2}\epsilon\psi \\
\delta\psi_\alpha &= \sqrt{2}i(\sigma^\mu\bar{\epsilon})_\alpha\partial_\mu\phi + \sqrt{2}\epsilon_\alpha F \\
\delta F &= -i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\epsilon}
\end{cases}
,\quad
\begin{cases}
\delta\bar{\phi} &= \sqrt{2}\bar{\epsilon}\bar{\psi} \\
\delta\bar{\psi}_{\dot{\alpha}} &= \sqrt{2}i(\bar{\sigma}^\mu\epsilon)_{\dot{\alpha}}\partial_\mu\bar{\phi} + \sqrt{2}\bar{\epsilon}_{\dot{\alpha}}F \\
\delta\bar{F} &= -i\sqrt{2}\partial_\mu\bar{\psi}\bar{\sigma}^\mu\epsilon
\end{cases}
. \tag{2.3.24}
$$

It can be observed that upon the three fields, only the F-term is a total derivative.

2.3.4 Vector superfields

The definition for a vector (or real) superfield V is

$$V = \bar{V}. \tag{2.3.25}$$

Plug this definition into the general expression for a superfield, we get

$$f = \bar{f} \quad , \quad \psi = \chi \quad , \quad m = \bar{n} \quad , \quad v_\mu = v_\mu^* \quad , \quad \lambda = \rho \quad , \quad d = d^*. \tag{2.3.26}$$

However, it is conventional and more convenient to express $V(x, \theta, \bar{\theta})$ in the following way, which is equivalent to the above result

$$
\begin{aligned}
V(x,\theta,\bar{\theta}) = C(x) &+ \theta\chi(x) + \bar{\theta}\bar{\chi}(x) + \theta\sigma^\mu\bar{\sigma}v_\mu + \frac{1}{2}\theta\theta(M(x)+iN(x)) + \frac{1}{2}\bar{\theta}\bar{\theta}(M(x)+iN(x)) \\
&+ \theta\theta\bar{\theta}\Big(\bar{\lambda}(x) + \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\chi(x)\Big) + \bar{\theta}\bar{\theta}\theta\Big(\lambda(x) + \frac{i}{2}\sigma^\mu\partial_\mu\bar{\chi}(x)\Big) \\
&+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\Big(D(x) - \frac{1}{2}\partial^2 C(x)\Big).
\end{aligned} \tag{2.3.27}
$$

In this convention, there are 8 bosonic components C, M, N, D, V_μ and $4+4$ fermionic ones, 4 for each of $\chi_\alpha, \lambda_\alpha$. We shall not consider the variation of this superfield because all terms of the general superfield survives; thus, the variation should be similar to that of the general superfield. That is only the D-term transforms like a total derivative under supersymmetric transformation.

In considering the Lagrangian for the Standard Model, we shall also encounter gauge transformation; therefore, it is appropriate here to introduce such formalism in supersymmetric theory. Since $\Phi + \bar{\Phi}$ is a vector superfield if Φ is a chiral superfield, under

$$V \to V + \Phi + \bar{\Phi}, \tag{2.3.28}$$

the vector component in V transforms as

$$v_\mu \to v_\mu - \partial_\mu(2\Im\phi). \tag{2.3.29}$$

This is how an ordinary (abelian) gauge transformation is in non-supersymmetric theory. Therefore, equation (2.3.28) is a natural definition for the supersymmetric gauge transformation. Under such transformation, the component of V transform as

$$\begin{cases} C & \to C + 2\mathrm{Re}\phi \\ \chi & \to \chi - i\sqrt{2}\psi \\ M & \to M - 2\Im F \\ N & \to N + 2\mathrm{Re}F \\ D & \to D \\ \lambda & \to \lambda \\ v^\mu & \to v^\mu - 2\partial_\mu \Im\phi \end{cases} . \tag{2.3.30}$$

Since the theory should be and can be proven to be invariant under gauge transformation, we can choose the component of Φ so that under such gauge, V is suitably simplified. One such choice is the Wess-Zumino gauge, which, although the above argument starts from abelian gauge theory, applies to non-abelian case as well:

$$\mathrm{Re}\phi = -\frac{C}{2} \quad , \quad \psi = -\frac{i}{\sqrt{2}}\chi \quad , \quad \mathrm{Re}F = -\frac{N}{2} \quad , \quad \Im F = \frac{M}{2}, \tag{2.3.31}$$

and the vector superfield reduces to

$$V_{WZ}(x,\theta,\bar\theta) = (\theta\sigma^\mu\bar\theta)v_\mu(x) + (\theta\theta)\big(\bar\theta\bar\lambda(x)\big) + (\bar\theta\bar\theta)(\theta\lambda(x)) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(x). \tag{2.3.32}$$

One important property of this field is that $V_{WZ}^2 = \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)v_\mu v^\mu$ and $V_{WZ}^n = 0, n \geq 3$ because all products involving more than 2 θ or 2 $\bar\theta$ vanish. These identities will be useful later when we construct the Lagrangian.

Before we move on, there are several important properties of chiral, anti-chiral and vector superfields that are needed in our future treatment of the Lagrangian:

- The sums and products of chiral superfields are chiral superfields. Similarly for anti-chiral superfields and vector superfields.
- The sum of a chiral superfield and an anti-chiral superfield is a vector superfield.
- The product of a chiral and an anti-chiral superfield is also a vector superfield.

2.4 Supersymmetric Lagrangian

Now, we have enough tools to construct a Lagrangian for supersymmetric theory composing of chiral, anti-chiral and vector superfields. To start, we can use the observations of the variations of the component fields. Only the D-term of vector superfield and F-term of chiral superfield are invariant under supersymmetric transformation. Thus one illuminating approach would be construct the superfields, and include its invariant component field to the Lagrangian. In this way, we do not need to worry about supersymmetric invariant again. The general Lagrangian one would be

$$\mathcal{L}_{\mathrm{SUSY}} = L_1|_D + (L_2|_F + h.c.) = \int d^2\theta d^2\bar\theta L_1 + \left(\int d^2\theta L_2 + \int d^2\bar\theta \bar L_2\right) \tag{2.4.1}$$

with L_1 being vector superfield, while L_2 being chiral superfield. The notation $|_D$ means taking the D-term of the superfield. Similarly for the F-term.

We shall divides the total Lagrangian into different sectors depending on the fields that participates and investigates them individually before amalgamates them in a complete theory.

2.4.1 Chiral sector

This sector involves only chiral and anti-chiral superfields. Using what we stated above, the only possible combination for Lagrangian in chiral sector is

$$\mathcal{L}_{\text{chiral}} = K(\Phi^i, \bar{\Phi}_i)\big|_D + \big(W(\Phi^i)\big|_F + h.c.\big) \tag{2.4.2}$$

where K is a general superfield known as Kähler potential which contributes to the kinetic part of the Lagrangian. $W(\Phi)$ is known as the superpotential and is a chiral superfield as required since addition and multiplication of chiral superfields are themselves chiral superfields. This is the interaction part. The index i is to be summed over all possible superfields appearing in the theory. Such implicit sum over repeated indices is assumed throughout this paper.

For the theory to be renormalizable, the Kähler potential and superpotential must have some restriction. We know that the general expression for a chiral superfield is

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y). \tag{2.4.3}$$

Since ϕ is a scalar field, while ψ is a spinor, the dimensionality of its component can be found

$$[\Phi] = [\phi] = 1 \quad , \quad [\psi] = \frac{3}{2} \quad \Longrightarrow \quad [\theta] = -\frac{1}{2} \quad , \quad [F] = 2. \tag{2.4.4}$$

Because we want $[\mathcal{L}] = 4$, for the theory to be renormalizable, we must have the dimensionality of the potentials to be at most 4, meaning $[K|_D] \leq 4$ and $[W|_F] \leq 4$. That puts a constraint in the dimensionality of $K = ... + (\theta\theta)(\bar{\theta}\bar{\theta})K|_D$ and $W = ... + (\theta\theta)W|_F$ (since $[\theta] = -\frac{1}{2}$)

$$[K] \leq 2 \quad , \quad [W] \leq 3. \tag{2.4.5}$$

Using this dimensional analysis, the only possible term for K is

$$K(\Phi^i, \bar{\Phi}_i) = \bar{\Phi}_i\Phi^i. \tag{2.4.6}$$

Any other terms either produce a higher dimension than allowed or give a chiral superfield (which we are considering separately) instead of a vector superfield. For W, the highest possible order is 3 and the lowest is 1 because an additive constant does not contribute to the Lagrangian

$$W(\Phi^i) = \lambda_i\Phi^i + \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{3}g_{ijk}\Phi^i\Phi^j\Phi^k. \tag{2.4.7}$$

The general Lagrangian is then

$$\mathcal{L} = \bar{\Phi}_i\Phi^i|_D + \left(\left(\lambda_i\Phi^i + \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{3}g_{ijk}\Phi^i\Phi^j\Phi^k\right)\bigg|_F + h.c.\right). \tag{2.4.8}$$

The next task is to express this in term of component fields. For the superpotential, although direct substitution would yield the same result, the more convenient way is through Taylor expansion use coordinate y^μ to utilize the fact that superpotential is a chiral superfield. Due to being cubic, the superpotential can only be expanded to third order. For conciseness, we will use the notation $\dfrac{\partial W}{\partial \phi^i} \equiv \dfrac{\partial W}{\partial \Phi^i}\bigg|_{\Phi^i=\phi^i}$ and similarly for higher derivatives

$$\begin{aligned}W(\Phi^i) = W(\phi^i) &+ \sum_i(\Phi^i - \phi^i)\frac{\partial W}{\partial\phi^i} + \frac{1}{2}\sum_{i,j}(\Phi^i - \phi^i)(\Phi^j - \phi^j)\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} \\ &+ \frac{1}{6}\sum_{i,j,k}(\Phi^i - \phi^i)(\Phi^j - \phi^j)(\Phi^k - \phi^k)\frac{\partial^3 W}{\partial\phi^i\partial\phi^j\partial\phi^k}\end{aligned}$$

$$
\begin{aligned}
&= W(\phi^i) + \sum_i (\sqrt{2}\theta\psi^i + \theta\theta F^i)\frac{\partial W}{\partial \phi^i} + \frac{1}{2}\sum_{i,j}(\sqrt{2}\theta\psi^i + \theta\theta F^i)(\sqrt{2}\theta\psi^j + \theta\theta F^j)\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} \\
&\quad + \frac{1}{6}\sum_{i,j,k}(\sqrt{2}\theta\psi^i + \theta\theta F^i)(\sqrt{2}\theta\psi^j + \theta\theta F^j)(\sqrt{2}\theta\psi^k + \theta\theta F^k)\frac{\partial^3 W}{\partial\phi^i\partial\phi^j\partial\phi^k} \\
&= W(\phi^i) + \sqrt{2}\theta\sum_i \psi^i\frac{\partial W}{\partial\phi^i} + (\theta\theta)\left(\sum_i F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\sum_{i,j}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}\right).
\end{aligned}
$$

Therefore, the F-term, utilizing the repeated index as summation notation, is

$$
\mathcal{L}_W = W(\Phi^i)\big|_F = F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j}. \tag{2.4.9}
$$

Because the F-term of chiral superfield is the same in both x^μ and y^μ coordinate, although the expression here is derived in y^μ, it is also valid in x^μ. For the Kähler potential, the computation is more complicated. Here, we only focus on the terms that contribute to D-term; other terms are suppressed in "..."

$$
\begin{aligned}
K(\Phi^i, \bar{\Phi}_i) &= \bar{\Phi}_i\Phi^i \\
&= \left[\bar{\phi}_i + \sqrt{2}\bar{\theta}\bar{\psi}_i - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}_i + \bar{\theta}\bar{\theta}\bar{F}_i + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}_i - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\bar{\phi}_i\right] \\
&\quad\times\left[\phi^i + \sqrt{2}\theta\psi^i + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi^i + \theta\theta F^i + \frac{i}{\sqrt{2}}(\theta\theta)(\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi^i) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\phi^i\right] \\
&= \cdots - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\bar{\phi}_i\Box\phi^i + \sqrt{2}\bar{\theta}\bar{\psi}_i\times\frac{i}{\sqrt{2}}(\theta\theta)(\bar{\theta}\bar{\sigma}^\mu\partial_\mu\psi^i) - \left(i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}_i\right)\left(i\theta\sigma^\nu\bar{\theta}\partial_\nu\phi^i\right) \\
&\quad + (\theta\theta)(\bar{\theta}\bar{\theta})\bar{F}_iF^i + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}_i\sqrt{2}\theta\psi^i - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\Box\bar{\phi}_i)\phi^i \\
&= \cdots - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\bar{\phi}_i\Box\phi^i - \frac{i}{2}(\theta\theta)(\bar{\theta}\bar{\theta})(\bar{\psi}_i\bar{\sigma}^\mu\partial_\mu\psi^i) + \frac{1}{2}(\theta\theta)(\bar{\theta}\bar{\theta})g^{\mu\nu}\partial_\mu\bar{\phi}_i\partial_\nu\phi^i \\
&\quad + (\theta\theta)(\bar{\theta}\bar{\theta})\bar{F}_iF^i - \frac{i}{2}(\bar{\theta}\bar{\theta})(\theta\theta)(\psi^i\sigma^\mu\partial_\mu\bar{\psi}_i) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\Box\bar{\phi}_i)\phi^i \\
&= \cdots - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\mu\partial_\mu(\bar{\phi}_i\phi^i) + (\theta\theta)(\bar{\theta}\bar{\theta})\left[\partial_\mu\bar{\phi}_i\partial^\mu\phi^i + \frac{i}{2}\left(\partial_\mu\psi^i\sigma^\mu\bar{\psi}_i - \psi^i\sigma^\mu\partial_\mu\bar{\psi}_i\right) + \bar{F}_iF^i\right].
\end{aligned}
$$

The first term is a total derivative; thus, it can be excluded. The F-term of Kähler potential is then

$$
K(\Phi^i, \bar{\Phi}_i)\big|_D = \partial_\mu\bar{\phi}_i\partial^\mu\phi^i + \frac{i}{2}\left(\partial_\mu\psi^i\sigma^\mu\bar{\psi}_i - \psi^i\sigma^\mu\partial_\mu\bar{\psi}_i\right) + \bar{F}_iF^i. \tag{2.4.10}
$$

Thus

$$
\mathcal{L}_{\text{chiral}} = \partial_\mu\bar{\phi}_i\partial^\mu\phi^i + \frac{i}{2}\left(\partial_\mu\psi^i\sigma^\mu\bar{\psi}_i - \psi^i\sigma^\mu\partial_\mu\bar{\psi}_i\right) + \bar{F}_iF^i + \left(F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} + h.c.\right). \tag{2.4.11}
$$

From the Lagrangian of the chiral sector, we can see that only ϕ, corresponding to a scalar field, and ψ, which is a Dirac field, have kinetic terms. The field F has no such term, thus is not a physical field. Moreover, we can solve the Euler-Lagrange equation for the auxiliary field F

$$
\bar{F}_i = -\frac{\partial W}{\partial\phi^i} \quad , \quad F^i = -\frac{\partial\bar{W}}{\partial\bar{\phi}_i}. \tag{2.4.12}
$$

Substitute this back will eliminate the auxiliary field and give us the on-shell Lagrangian

$$\mathcal{L}_{\text{chiral OS}} = \partial_\mu\bar{\phi}_i\partial^\mu\phi^i + \frac{i}{2}\big(\partial_\mu\psi^i\sigma^\mu\bar{\psi}_i - \psi^i\sigma^\mu\partial_\mu\bar{\psi}_i\big) - \sum_i\left|\frac{\partial W}{\partial\phi^i}\right|^2 - \frac{1}{2}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} - \frac{1}{2}\bar{\psi}_i\bar{\psi}_j\frac{\partial^2\bar{W}}{\partial\bar{\phi}_i\partial\bar{\phi}_j}. \tag{2.4.13}$$

From this Lagrangian, we can read the scalar potential

$$V(\phi,\bar{\phi}) = \sum_i\left|\frac{\partial W}{\partial\phi^i}\right|^2 = \bar{F}_iF^i. \tag{2.4.14}$$

2.4.2 Super Yang-Mills theory

In this section, we will Find the kinetic term for supersymmetric vector field. For a general $SU(N)$ gauge group (this includes the case of abelian gauge group) whose Lie algebra is spanned by hermitian generators T^a

$$\Lambda = \Lambda_aT^a \quad , \quad V = V_aT^a \quad , \quad \Big[T^a,T^b\Big] = if^{abc}T_c \quad , \quad \mathrm{Tr}\Big\{T^aT^b\Big\} = \frac{1}{2}\delta^{ab}. \tag{2.4.15}$$

This definition will be used throughout this section. A general gauge transformation can be defined as

$$e^V \mapsto e^{i\bar{\Lambda}}e^Ve^{-i\Lambda} \tag{2.4.16}$$

with Λ_a being chiral superfields as demonstrated in section 2.3.4 for abelian case. Our theory must be gauge invariant. Therefore, we can work in Wess-Zumino gauge, where exponentials of V higher than 2 will vanish. Therefore, we can have a simple expression

$$e^V = 1 + V + \frac{1}{2}V^2. \tag{2.4.17}$$

The gauge superfield strengths can be defined as

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}\big(e^{-V}D_\alpha e^V\big) \quad , \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}DD\big(e^V\bar{D}_{\dot{\alpha}}e^{-V}\big). \tag{2.4.18}$$

Under gauge transformation, they transform as

$$W_\alpha \to e^{i\Lambda}W_\alpha e^{-i\Lambda} \quad , \quad \bar{W}_{\dot{\alpha}} \to e^{i\bar{\Lambda}}\bar{W}_{\dot{\alpha}}e^{-i\bar{\Lambda}}. \tag{2.4.19}$$

By construction, $D^3 = 0$ and $\bar{D}^3 = 0$ because they involves at least 3 Grassmann variables. Therefore, these superfield strength are chiral and anti-chiral superfields. Thus, we can work in coordinate y^μ without changing the Lagrangian, which is its F-term. To obtain an explicit expression for field strength, we need to rewrite the superfield strength using equation (2.4.17)

$$\begin{aligned}
W_\alpha &= -\frac{1}{4}\bar{D}\bar{D}\bigg[\Big(1 - V + \frac{1}{2}V^2\Big)D_\alpha\Big(1 + V + \frac{1}{2}V^2\Big)\bigg] \\
&= -\frac{1}{4}\bar{D}\bar{D}\bigg[D_\alpha V + \frac{1}{2}D_\alpha V^2 - VD_\alpha V\bigg] \\
&= -\frac{1}{4}\bar{D}\bar{D}\bigg[D_\alpha V + \frac{1}{2}(D_\alpha V)V + \frac{1}{2}V(D_\alpha V) - VD_\alpha V\bigg] \\
&= -\frac{1}{4}\bar{D}\bar{D}D_\alpha V + \frac{1}{8}\bar{D}\bar{D}[V, D_\alpha V].
\end{aligned}$$

Using y^μ, the vector superfield is

$$V(x,\theta,\bar{\theta}) = V(y - i\theta\sigma\bar{\theta},\theta,\bar{\theta}) = V(y,\theta,\bar{\theta}) - i\theta\sigma^\mu\bar{\theta}\partial_\mu V(y,\theta,\bar{\theta})$$

$$= (\theta\sigma^\mu\bar\theta)v_\mu(y) + (\theta\theta)\big(\bar\theta\bar\lambda(y)\big) + (\bar\theta\bar\theta)(\theta\lambda(y)) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D(y) - i\theta\sigma^\mu\bar\theta\partial_\mu\big[(\theta\sigma^\nu\bar\theta)v_\nu(y)\big]$$
$$= (\theta\sigma^\mu\bar\theta)v_\mu(y) + (\theta\theta)\big(\bar\theta\bar\lambda(y)\big) + (\bar\theta\bar\theta)(\theta\lambda(y)) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)[D(y) - i\partial^\mu v_\mu(y)].$$

Although in the y^μ basis, $\bar D_{\dot\beta} = \bar\partial_{\dot\beta}$, the case for D_α is different. Specifically,

$$D_\alpha = \partial_\alpha + \frac{\partial y^\mu}{\partial\theta^\alpha}\frac{\partial}{\partial y^\mu} + i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu = \partial_\alpha + 2i\sigma^\mu_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu. \tag{2.4.20}$$

The superfield strength can be obtained

$$W_\alpha = -\lambda_\alpha - \theta_\alpha D + \theta_\beta(\sigma^{\mu\nu})^\beta_\alpha F_{\mu\nu} - i(\theta\theta)\big(\sigma^\mu D_\mu\bar\lambda\big)_\alpha. \tag{2.4.21}$$

with

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + \frac{i}{2}[v_\mu, v_\nu] \tag{2.4.22}$$
$$D_\mu = \partial_\mu - \frac{i}{2}[v_\mu,]. \tag{2.4.23}$$

If we redefine the vector superfield to take out the coupling constant explicitly $V \to 2gV$, then we need to redefine $W_\alpha \to 2gW_\alpha$ and

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + ig[v_\mu, v_\nu] \tag{2.4.24}$$
$$D_\mu = \partial_\mu - ig[v_\mu, \;] \tag{2.4.25}$$

to keep the analytical expression of W_α. In the case of abelian gauge theory, all component fields commute; thus

$$F_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu \tag{2.4.26}$$
$$D_\mu = \partial_\mu \tag{2.4.27}$$

A reasonable guess for the kinetic term of gauge sector, similar to SM, is

$$W^\alpha W_\alpha = \Big[-\lambda^\alpha - \theta^\alpha D + \theta^\beta(\sigma^{\mu\nu})^\alpha_\beta F_{\mu\nu} - i(\theta\theta)\big(\sigma^\mu D_\mu\bar\lambda\big)^\alpha\Big]$$
$$\times\Big[-\lambda_\alpha - \theta_\alpha D + \theta_\beta(\sigma^{\mu\nu})^\beta_\alpha F_{\mu\nu} - i(\theta\theta)\big(\sigma^\mu D_\mu\bar\lambda\big)_\alpha\Big].$$

Since a product of chiral superfields is itself chiral, we only need to find the F-term while suppressing other terms

$$W^\alpha W_\alpha = \cdots + 2i(\theta\theta)(\lambda\sigma^\mu D_\mu\bar\lambda) + (\theta\theta)D^2 + (\sigma^{\mu\nu})^\alpha_\beta\theta^\beta(\sigma^{\rho\tau})^\gamma_\alpha\theta_\gamma F_{\mu\nu}F_{\rho\tau}.$$

The term $\theta\sigma^{\mu\nu}\theta = 0$ because of the identity $\chi\sigma^\mu\bar\sigma^\nu\psi = \psi\sigma^\nu\bar\sigma^\mu\chi$. Directly obtaining the F-term from this chiral superfield is not trivial because of the last term. Thus, we resort to the integration of the fermionic coordinate

$$W^\alpha W_\alpha|_F = \int d^2\theta W^\alpha W_\alpha = \frac{1}{4}\partial^\rho\partial_\rho W^\alpha W_\alpha$$
$$= -2i(\lambda\sigma^\mu D_\mu\bar\lambda) + D^2 + \frac{1}{4}(\sigma^{\mu\nu})^\alpha_\beta(\sigma^{\rho\tau})^\gamma_\alpha\partial^\rho\partial_\rho\Big(\theta^\beta\theta_\gamma\Big)F_{\mu\nu}F_{\rho\tau}.$$

Since $\partial^\rho\partial_\rho(\theta^\beta\theta_\gamma) = \partial^\rho(\delta^\beta_\rho\theta_\gamma + \theta^\beta\epsilon_{\rho\gamma}) = \delta^\beta_\rho\delta^\rho_\gamma - \epsilon^{\rho\beta}\epsilon_{\rho\gamma} = 2\delta^\beta_\gamma$, and use equation (A.2.5), the above becomes

$$W^\alpha W_\alpha|_F = -2i(\lambda\sigma^\mu D_\mu\bar\lambda) + D^2 + \frac{1}{2}\,\mathrm{Tr}\{(\sigma^{\mu\nu})(\sigma^{\rho\tau})\}F_{\mu\nu}F_{\rho\tau}$$

$$= -2i(\lambda\sigma^\mu D_\mu\bar{\lambda}) + D^2 + \frac{1}{4}(g^{\mu\rho}g^{\nu\tau} - g^{\mu\tau}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\tau})F_{\mu\nu}F_{\rho\tau}$$
$$= -2i(\lambda\sigma^\mu D_\mu\bar{\lambda}) + D^2 + \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{1}{4}i\epsilon^{\mu\nu\rho\tau}F_{\mu\nu}F_{\rho\tau}.$$

The hermitian conjugate of the first term is $2i(\lambda\sigma^\mu D_\mu\bar{\lambda})^* = 2i\big((D_\mu\lambda)\sigma^\mu\bar{\lambda}\big) = 2iD_\mu\big(\lambda\sigma^\mu\bar{\lambda}\big) - 2i(\lambda\sigma^\mu D_\mu\bar{\lambda})$. Eliminating the total derivative, the Lagrangian of this sector is obtained, up to a normalization factor,

$$W^\alpha W_\alpha|_F + h.c. = -4i(\lambda\sigma^\mu D_\mu\bar{\lambda}) + 2D^2 + F^{\mu\nu}F_{\mu\nu}.$$

Unfortunately, the CP-violating is cancelled in the final Lagrangian. However, this term, while proposed by hand in SM to give an extra source of CP-violation, appears naturally in this context. To make the derivation general, that is to keep this term from being cancelled, an appropriate normalization factor τ must be introduced. Let $\tau = -\frac{1}{2} - i\frac{\Theta g^2}{8\pi^2}$. Then, the Lagrangian for this sector can be written down

$$\mathcal{L}_{SYM} = \mathrm{Tr}\Big\{\tau\int d^2\theta W^\alpha W_\alpha + h.c.\Big\} = \mathrm{Tr}\Big\{2i(\lambda\sigma^\mu D_\mu\bar{\lambda}) - D^2 - \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + \frac{\Theta g^2}{16\pi^2}\epsilon^{\mu\nu\rho\tau}F_{\mu\nu}F_{\rho\tau}\Big\}$$
$$= i(\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a) - \frac{1}{2}D^aD^a - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\Theta g^2}{32\pi^2}\epsilon^{\mu\nu\rho\tau}F_{\mu\nu}F_{\rho\tau}.$$

Using the normalization $\mathrm{Tr}\{T^aT^b\} = \frac{1}{2}\delta^{ab}$, we obtain the full expression for Super-Yang-Mills Lagrangian

$$\mathcal{L}_{SYM} = -\frac{1}{4}F^{a\mu\nu}F^a_{\mu\nu} + i(\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a) - \frac{1}{2}D^aD^a + \frac{\Theta g^2}{32\pi^2}\epsilon^{\mu\nu\rho\tau}F^a_{\mu\nu}F^a_{\rho\tau}. \tag{2.4.28}$$

2.4.3 Gauge-matter actions

To incorporate gauge transformation into the matter sector, we first need to redefine the Kähler potential

$$\bar{\Phi}_i\Phi^i \mapsto \bar{\Phi}e^{2g_nV_n}\Phi \tag{2.4.29}$$

where $n = 1, \ldots, N$ with N is the number of gauge field participating in the interaction. Therefore, the complete Lagrangian reads

$$\mathcal{L}_{matter} = \int d^2\theta d^2\bar{\theta}\bar{\Phi}e^{2g_nV_n}\Phi + \int d^2\theta W(\Phi) + \int d^2\bar{\theta}\bar{W}(\bar{\Phi}). \tag{2.4.30}$$

Although the Kähler potential is gauge invariant, the superpotential, built by hand depending on the theory, must be constructed so that it respects gauge symmetry. Thus, it is safe to use the Wess-Zumino gauge to simplify the Kähler potential

$$\bar{\Phi}e^{2g_nV_n}\Phi = \bar{\Phi}\Phi + \bar{\Phi}2g_nV_n\Phi + \frac{1}{2}\bar{\Phi}(2g_nV_n)^2\Phi. \tag{2.4.31}$$

To get the explicit form of the Kähler potential, we need to compute the following (note that all terms not contributing to the D-term is hidden away)

$$\int d^2\theta d^2\bar{\theta}\bar{\Phi}\Phi = \partial_\mu\bar{\phi}_i\partial^\mu\phi^i + \frac{i}{2}\big(\partial_\mu\psi^i\sigma^\mu\bar{\psi}_i - \psi^i\sigma^\mu\partial_\mu\bar{\psi}_i\big) + \bar{F}_iF^i$$
(computed in section 2.4.1)
$$\bar{\Phi}_i2g_nV_n\Phi^i = \Big[\bar{\phi}_i + \sqrt{2}\bar{\theta}\bar{\psi}_i - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}_i + \bar{\theta}\bar{\theta}\bar{F}_i + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\psi}_i - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box\bar{\phi}_i\Big]$$

$$
\begin{aligned}
&\times 2g_n\left[(\theta\sigma^\mu\bar\theta)v_{n\mu} + (\theta\theta)\big(\bar\theta\bar\lambda_n\big) + (\bar\theta\bar\theta)(\theta\lambda_n) + \frac{1}{2}(\theta\theta)(\bar\theta\bar\theta)D_n\right] \\
&\times\left[\phi^i + \sqrt{2}\theta\psi^i + i\theta\sigma^\mu\bar\theta\partial_\mu\phi^i + \theta\theta F^i + \frac{i}{\sqrt{2}}(\theta\theta)(\bar\theta\bar\sigma^\mu\partial_\mu\psi^i) - \frac{1}{4}\theta\theta\bar\theta\bar\theta\Box\phi^i\right] \\
=\ &\cdots + (\theta\theta)(\bar\theta\bar\theta)\bar\phi_i g_n D_n\phi^i + \bar\phi_i 2g_n(\bar\theta\bar\theta)(\theta\lambda_n)\sqrt{2}\theta\psi^i + \bar\phi_i 2g_n(\theta\sigma^\mu\bar\theta)v_{n\mu}i\theta\sigma^\nu\bar\theta\partial_\nu\phi^i \\
&+ 4g_n\bar\theta\bar\psi_i(\theta\sigma^\mu\bar\theta)v_{n\mu}\theta\psi^i + \sqrt{2}\bar\theta\bar\psi_i 2g_n(\theta\theta)\big(\bar\theta\bar\lambda_n\big)\phi^i - i\theta\sigma^\mu\bar\theta\partial_\mu\bar\phi_i 2g_n(\theta\sigma^\nu\bar\theta)v_{n\nu}\phi^i \\
=\ &\cdots + g_n(\theta\theta)(\bar\theta\bar\theta)\bar\phi_i D_n\phi^i - \sqrt{2}g_n\bar\phi_i(\bar\theta\bar\theta)(\theta\theta)\lambda_n\psi^i + i(\theta\theta)(\bar\theta\bar\theta)g_n\bar\phi_i g^{\mu\nu}v_{n\mu}\partial_\nu\phi^i \\
&- 2g_n\bar\theta\bar\psi_i v_{n\mu}(\theta\theta)(\psi^i\sigma^\mu\bar\theta) - \sqrt{2}g_n(\theta\theta)(\bar\theta\bar\theta)(\bar\psi_i\bar\lambda_n)\phi^i - ig_n(\theta\theta)(\bar\theta\bar\theta)g^{\mu\nu}\partial_\mu\bar\phi_i v_{n\nu}\phi^i \\
=\ &\cdots + g_n(\theta\theta)(\bar\theta\bar\theta)\left[\bar\phi_i D_n\phi^i - \sqrt{2}\bar\phi_i\lambda_n\psi^i + i\bar\phi_i v_n^\mu\partial_\mu\phi^i - \bar\psi_i\bar\sigma^\mu v_{n\mu}\psi^i\right. \\
&\left.-\sqrt{2}(\bar\psi_i\bar\lambda_n)\phi^i - i\partial_\mu\bar\phi_i v_n^\mu\phi^i\right] \\
\bar\Phi_i(2g_nV_n)^2\Phi^i =\ &\cdots + \bar\phi_i(2g_n(\theta\sigma^\mu\bar\theta)v_{n\mu})^2\phi^i = \cdots + \bar\phi_i 4g_ng_m(\theta\sigma^\mu\bar\theta)(\theta\sigma^\nu\bar\theta)v_{n\mu}v_{m\nu}\phi^i \\
=\ &\cdots + 2g_ng_m(\theta\theta)(\bar\theta\bar\theta)\bar\phi_i v_n^\mu v_{m\mu}\phi^i.
\end{aligned}
$$

Therefore the Kähler potential Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\text{Kähler}} =\ &\int d^2\theta d^2\bar\theta\bar\Phi\Phi + \int d^2\theta d^2\bar\theta\bar\Phi_i 2g_nV_n\Phi^i + \frac{1}{2}\int d^2\theta d^2\bar\theta\bar\Phi_i(2g_nV_n)^2\Phi^i \\
=\ &\partial_\mu\bar\phi_i\partial^\mu\phi^i + \frac{i}{2}\big(\partial_\mu\psi^i\sigma^\mu\bar\psi_i - \psi^i\sigma^\mu\partial_\mu\bar\psi_i\big) + \bar F_iF^i + g_n\left[\bar\phi_i D_n\phi^i - \sqrt{2}\bar\phi_i\lambda_n\psi^i\right. \\
&\left.+ i\bar\phi_i v_n^\mu\partial_\mu\phi^i - \bar\psi_i\bar\sigma^\mu v_{n\mu}\psi^i - \sqrt{2}(\bar\psi_i\bar\lambda_n)\phi^i - i\partial_\mu\bar\phi_i v_n^\mu\phi^i\right] + g_ng_m\bar\phi_i v_n^\mu v_{m\mu}\phi^i \\
=\ &\frac{i}{2}\big[\partial_\mu(\psi^i\sigma^\mu\bar\psi_i) - 2\psi^i\sigma^\mu\partial_\mu\bar\psi_i\big] + g_n\psi^i\sigma^\mu v_{n\mu}\bar\psi_i + \bar F_iF^i + g_n\bar\phi_i D_n\phi^i - \sqrt{2}g_n\bar\phi_i\lambda_n\psi^i \\
&- \sqrt{2}g_n(\bar\psi_i\bar\lambda_n)\phi^i + \partial_\mu\bar\phi_i\big(\partial^\mu\phi^i - ig_nv_n^\mu\phi^i\big) + ig_mv_{m\mu}\bar\phi_i\big(\partial^\mu\phi^i - ig_nv_n^\mu\phi^i\big) \\
=\ &-i\psi^i\sigma^\mu\big(\partial_\mu\bar\psi_i + iv_{n\mu}\bar\psi_i\big) + \bar F_iF^i + g_n\bar\phi_i D_n\phi^i - \sqrt{2}g_n\bar\phi_i\lambda_n\psi^i \\
&- \sqrt{2}g_n(\bar\psi_i\bar\lambda_n)\phi^i + \big(\partial_\mu\bar\phi_i + ig_mv_{m\mu}\bar\phi_i\big)\big(\partial^\mu\phi^i - ig_nv_n^\mu\phi^i\big).
\end{aligned}
$$

Using the covariant derivative

$$D_\mu = \partial_\mu - ig_nv_{n\mu} = \partial_\mu - ig_nv_{n\mu}^aT^a \tag{2.4.32}$$

Combine with the Lagrangian for superpotential , equation (2.4.9), we have the explicit form of the matter Lagrangian

$$
\begin{aligned}
\mathcal{L}_{matter} =\ &\overline{D_\mu\phi_i}D^\mu\phi^i - i\psi^i\sigma^\mu\overline{D_\mu\psi_i} + \bar F_iF^i + g_n\bar\phi_i D_n\phi^i - \sqrt{2}g_n\bar\phi_i\lambda_n\psi^i - \sqrt{2}g_n(\bar\psi_i\bar\lambda_n)\phi^i \\
&+ \left(F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\frac{\partial^2W}{\partial\phi^i\partial\phi^j} + h.c.\right).
\end{aligned} \tag{2.4.33}
$$

Compare to the Lagrangian without gauge theory (2.4.11), we can see the interaction terms between the matter fields ϕ, ψ and gauge field v_μ, which looks exactly like that of non-supersymmetric theories, and gauginos λ.

2.4.4 The completed Lagrangian of supersymmetric theories

Combine all of the above terms provides us with the most general $N = 1$ supersymmetric Lagrangian with M chiral superfields and N gauge interactions, among which there are n $U(1)$ factors, is

$$\mathcal{L} = \mathcal{L}_{SYM} + \mathcal{L}_{matter} \tag{2.4.34}$$

$$= \mathrm{Tr}\left\{\tau_n \int d^2\theta W_n^\alpha W_{n\alpha} + h.c.\right\} + \int d^2\theta d^2\bar{\theta}\bar{\Phi}_i e^{2g_n V_n}\Phi^i + \left(\int d^2\theta W(\Phi^i) + h.c.\right) \tag{2.4.35}$$

$$\begin{aligned}
&= -\frac{1}{4}F_n^{a\mu\nu}F^a_{n\mu\nu} + i(\lambda_n^a\sigma^\mu D_\mu\bar{\lambda}_n^a) + \frac{1}{2}D_n^a D_n^a + \frac{\Theta_n g_n^2}{32\pi^2}\epsilon^{\mu\nu\rho\tau}F^a_{n\mu\nu}F^a_{n\rho\tau} \\
&\quad + \overline{D_\mu\phi_i}D^\mu\phi^i - i\psi^i\sigma^\mu\overline{D_\mu\psi_i} + \bar{F}_i F^i + g_n\bar{\phi}_i D_n\phi^i - \sqrt{2}g_n\bar{\phi}_i\lambda_n\psi^i - \sqrt{2}g_n(\bar{\psi}_i\bar{\lambda}_n)\phi^i \\
&\quad + \left(F^i\frac{\partial W}{\partial\phi^i} - \frac{1}{2}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} + h.c.\right).
\end{aligned} \tag{2.4.36}$$

From this Lagrangian, we can see that both F^i and D_n have no kinetic terms. Just like before, we can immediately solve for these auxiliary fields

$$\frac{\partial\mathcal{L}}{\partial\bar{F}_i} = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\bar{F}_i)}\right) = 0 \Longrightarrow F^i = -\frac{\partial\overline{W}}{\partial\bar{\phi}_i} \tag{2.4.37}$$

$$\frac{\partial\mathcal{L}}{\partial D_n^a} = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu D_n^a)}\right) = 0 \Longrightarrow D_n^a = -g_n\bar{\phi}_i T_n^a\phi^i. \tag{2.4.38}$$

Then, we get the on-shell Lagrangian

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F_n^{a\mu\nu}F^a_{n\mu\nu} + i(\lambda_n^a\sigma^\mu D_\mu\bar{\lambda}_n^a) + \frac{\Theta_n g_n^2}{32\pi^2}\epsilon^{\mu\nu\rho\tau}F^a_{n\mu\nu}F^a_{n\rho\tau} \\
&+ \overline{D_\mu\phi_i}D^\mu\phi^i - i\psi^i\sigma^\mu\overline{D_\mu\psi_i} + g_n\bar{\phi}_i D_n\phi^i - \sqrt{2}g_n\bar{\phi}_i\lambda_n\psi^i - \sqrt{2}g_n(\bar{\psi}_i\bar{\lambda}_n)\phi^i \\
&- \frac{1}{2}\psi^i\psi^j\frac{\partial^2 W}{\partial\phi^i\partial\phi^j} - \frac{1}{2}\bar{\psi}_i\bar{\psi}_j\frac{\partial\bar{W}}{\partial\bar{\phi}_i\bar{\phi}_j} - V(\phi^i,\bar{\phi}_i)
\end{aligned} \tag{2.4.39}$$

with

$$V(\phi^i,\bar{\phi}_i) = \frac{\partial W}{\partial\phi^i}\frac{\partial\bar{W}}{\partial\bar{\phi}_i} + \frac{g_n^2}{2}\left|\bar{\phi}_i T_n^a\phi^i\right|^2 \tag{2.4.40}$$

is the potential. It can be seen that $V(\phi^i,\bar{\phi}_i) \geq 0$; thus, the potential is automatically bounded from below, avoiding the existence of infinitely negative energy

Chapter 3

Next-to-minimal Supersymmetric Standard Model

3.1 Minimal Supersymmetric Standard Model

3.1.1 Field content and the Lagrangian

Before we construct such Lagrangian, we must have the content of the Standard Model extended to include their supersymmetric partner. The minimal way to extend the SM is keep as much of its properties as possible. Thus, the Minimal Supersymmetric Standard Model (MSSM) will be based on the gauge group $SU_C(3) \times SU_L(2) \times U_Y(1)$ just like SM. Since the matter fields described in table 1.2 all belongs to different representations of this gauge group, no particles in the SM can be the superpartners of each other, and we must introduce one superfield for each particle. Except for the gauge bosons, which will be described by the vector superfield as in table 3.1, the other particles of the Standard Model belong to their corresponding chiral superfield as in table 3.2. Unlike the Standard Model, there must be at least 2 Higgs supermultiplets in the smallest supersymmetric version of SM because because chiral and its anti-chiral cannot appear simultaneously in the superpotential to conserve its holomorphy. While in the Standard Model, to achieve the interaction for up-type quarks, we used charge conjugated Higgs, the corresponding trick would introduce anti-chiral superfield in superpotential which is not allowed. Since in the Standard Model, matter particles are divided into left-handed and right-handed multiplets, each interact differently with the gauge boson, we have to implement the same idea here. However, the chiral superfield in our theory is purely left-handed. To produce a right-handed superfield, we must use charge conjugation $\psi^c_\alpha \equiv i(\sigma^2)_{\alpha\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}$ to produce a right-handed superfield from left-handed one.

Superfield	spin 1/2	spin 1	$SU_C(3) \times SU_L(2) \times U_Y(1)$	Name (gaugino, gauge boson)	Coupling constant
$\hat{G}$	$\tilde{G}$	G	$(\mathbf{8}, \mathbf{1}, 0)$	gluinos, gluons	g_s
$\hat{W}$	$\tilde{W}$	W	$(\mathbf{1}, \mathbf{3}, 0)$	winos, W-bosons	g
$\hat{B}$	$\tilde{B}$	B	$(\mathbf{1}, \mathbf{1}, 0)$	binos, B-bosons	g'

Table 3.1: The gauge particle content of the supersymmetric Standard Model. The superfields are denoted with a hat, the superpartner of SM particles have a tilde to distinguish. The numbers in the fourth column denote the dimensions of the corresponding representation of the gauge group.

Supermultiplets	spin 0	spin 1/2	Representation	Name
$\hat{Q} = (\hat{u}_L\ \hat{d}_L)^T$	$\tilde{Q} = (\tilde{u}_L\ \tilde{d}_L)^T$	$Q = (u_L\ d_L)^T$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	squarks, quarks
$\hat{U}^c = \hat{u}^c$	$\tilde{U} = \tilde{u}^*_R$	$U = u^\dagger_R$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	
$\hat{D}^d = d^c$	$\tilde{D} = \tilde{d}^*_R$	$D = d^\dagger_R$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	
$\hat{L} = (\hat{\nu}\ \hat{e}_L)^T$	$\tilde{L} = (\tilde{\nu}\ \tilde{e}_L)^T$	$L = (\nu\ e_L)^T$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	sleptons, leptons
$\hat{E}^c = \hat{e}^c$	$\tilde{E} = \tilde{e}^*_R$	$E = e^\dagger_R$	$(\mathbf{1}, \mathbf{1}, 1)$	
$\hat{H}_u = (\hat{H}^+_u\ \hat{H}^0_u)^T$	$H_u = (H^+_u\ H^0_u)^T$	$\tilde{H}_u = (\tilde{H}^+_u\ \tilde{H}^0_u)^T$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	Higgs, Higgsinos
$\hat{H}_d = (\hat{H}^0_d\ \hat{H}^-_d)^T$	$H_d = (H^0_d\ H^-_d)^T$	$\tilde{H}_d = (\tilde{H}^0_d\ \tilde{H}^-_d)^T$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	

Table 3.2: The chiral particle content of the supersymmetric Standard Model. The superfields are denoted with a hat, the superpartner of SM particles have a tilde to distinguish. The generation and color indices are suppressed. The numbers in the fourth column denote the dimensions of the corresponding representation of the gauge group, with the bar denote complex conjugate representation.

The components of the Lagrangian includes the superpotential, Kähler potential, gauge kinetic (or Super Yang-Mill) term and the Fayet-Iliopulos term. In this situation, the super potential for the MSSM, constructed so that the Lagrangian of the non-supersymmetric Standard Model is contained, is

$$\begin{aligned} W_{MSSM} &= \hat{H}_d \cdot \hat{L}\mathbf{Y}_e\hat{E}^c + \hat{H}_d \cdot \hat{Q}\mathbf{Y}_d\hat{D}^c - \hat{H}_u \cdot Q\mathbf{Y}_u\hat{U}^c - \mu\hat{H}_u \cdot \hat{H}_d && (3.1.1)\\ &= \hat{u}^c\mathbf{Y}_u\hat{u}_L\hat{H}^0_u - \hat{u}^c\mathbf{Y}_u\hat{d}_L\hat{H}^+_u - \hat{d}^c\mathbf{Y}_d\hat{u}_L\hat{H}^-_d + \hat{d}^c\mathbf{Y}_d\hat{d}_L\hat{H}^0_d - \hat{e}^c\mathbf{Y}_e\hat{\nu}\hat{H}^-_d + \hat{e}^c\mathbf{Y}_e\hat{e}_L\hat{H}^0_d\\ &\quad + \mu(\hat{H}^+_u\hat{H}^-_d - \hat{H}^0_u\hat{H}^0_d), && (3.1.2)\end{aligned}$$

where $\cdot$ denotes $SU(2)$ product of spinors, that is $H_u \cdot H_d = \epsilon_{\alpha\beta}H^\alpha_u H^\beta_d$. $\mathbf{Y}_f, f = e, d, u$ are 3×3 matrices in flavour space. This superpotential is gauge invariance, with the first three terms produce mass for matter fields since it describes the interaction between them and the Higgs field. The last term yields the mass for the Higgs boson itself. For the vector superfield

$$\hat{G} = T^a\hat{G}^a\ , \quad \hat{W} = I^b\hat{W}^b, \tag{3.1.3}$$

with $a = 1, .., 9, b = 1, 2, 3$, so that we can have the kinetic term for gauge sector, excluding CP-violating term

$$\mathcal{L}_{SYM} = -\frac{1}{2}\,\mathrm{Tr}\Big\{\int d^2\theta W^\alpha(\hat{G})W_\alpha(\hat{G}) + \int d^2\theta W^\alpha(\hat{W})W_\alpha(\hat{W}) + \int d^2\theta W^\alpha(\hat{B})W_\alpha(\hat{B}) + h.c.\Big\} \tag{3.1.4}$$

where

$$W_\alpha(\hat{G}) = -\frac{1}{4}\bar{D}\bar{D}\Big(e^{-\hat{G}}D_\alpha e^{\hat{G}}\Big), \tag{3.1.5}$$

$$W_\alpha(\hat{W}) = -\frac{1}{4}\bar{D}\bar{D}\Big(e^{-\hat{W}}D_\alpha e^{\hat{W}}\Big), \tag{3.1.6}$$

$$W_\alpha(\hat{B}) = -\frac{1}{4}\bar{D}\bar{D}\Big(e^{-\hat{B}}D_\alpha e^{\hat{B}}\Big). \tag{3.1.7}$$

The Kähler potential is

$$K_{MSSM} = \bar{\hat{Q}}\exp\Big(2g_s\hat{G} + 2g\hat{W} + 2g'\hat{B}\times 1/6\Big)\hat{Q}$$

$$
\begin{aligned}
&+ \bar{\hat{U}}^c \exp\Big(-2g_s\hat{G} + 2g'\hat{B} \times (-2/3)\Big)\hat{U}^c \\
&+ \bar{\hat{D}}^c \exp\Big(-2g_s\hat{G} + 2g'\hat{B} \times (1/3)\Big)\hat{D}^c \\
&+ \bar{\hat{L}} \exp\Big(2g_s\hat{W}\Big) \exp\Big(2g'\hat{B} \times (-1/2)\Big)\hat{L} \\
&+ \bar{\hat{E}}^c \exp\Big(2g'\hat{B}\Big)\hat{E}^c \\
&+ \bar{\hat{H}}_u \exp\Big(2g\hat{W}\Big) \exp\Big(2g'\hat{B} \times (1/2)\Big)\hat{H}_u \\
&+ \bar{\hat{H}}_d \exp\Big(2g\hat{W}\Big) \exp\Big(2g'\hat{B} \times (-1/2)\Big)\hat{H}_d
\end{aligned} \tag{3.1.8}
$$

In principle, the combination of the above sectors is enough to construct the Lagrangian. It worth noting that compared to the 18 parameters in SM, the MSSM introduced only one new parameter μ.

Written in terms of component fields, the Super Yang-Mills Lagrangian is

$$
\begin{aligned}
\mathcal{L}^{MSSM}_{\text{OS SYM}} = &-\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} + i\tilde{G}^a\sigma^\mu D^G_\mu\bar{\tilde{G}}^a - \frac{1}{4}W^{b\mu\nu}W^b_{\mu\nu} + i\tilde{W}^a\sigma^\mu D^W_\mu\bar{\tilde{W}}^a \\
&- \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + i\tilde{B}^a\sigma^\mu D^B_\mu\bar{\tilde{B}}^a,
\end{aligned} \tag{3.1.9}
$$

with $G_\mu = T^aG^a_\mu$, $W_\mu = I^bW^b_\mu$, $\tilde{G} = T^a\tilde{G}^a$, $\tilde{W} = I^b\tilde{W}^b$ and

$$
\begin{aligned}
G_{\mu\nu} &= \partial_\mu G_\nu - \partial_\nu G_\mu - ig_s[G_\mu, G_\nu] \ , \qquad & D^G_\mu &= \partial_\mu - ig_s[G_\mu, \ ], \\
W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ig[W_\mu, W_\nu] \ , & D^W_\mu &= \partial_\mu - ig[W_\mu, \ ], \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \ , & D^B_\mu &= \partial_\mu.
\end{aligned}
$$

Here and in the following on-shell parts of the MSSM Lagrangian, the terms involving auxiliary fields will be written separately in the potential. The on-shell Kähler potential part is

$$
\begin{aligned}
\mathcal{L}^{MSSM}_{\text{OS Kähler}} = &\ \overline{D^Q_\mu\tilde{Q}}D^{Q\mu}\tilde{Q} + iQ\sigma^\mu\overline{D^Q_\mu Q} - \sqrt{2}\Big(g_s\bar{\tilde{Q}}\tilde{G}Q + g\bar{\tilde{Q}}\tilde{W}Q + g'\bar{\tilde{Q}}\frac{1}{6}\tilde{B}Q + h.c.\Big) \\
&+ \overline{D^U_\mu\tilde{U}}D^{U\mu}\tilde{U} + iU\sigma^\mu\overline{D^U_\mu U} - \sqrt{2}\Big(-g_s\bar{\tilde{U}}\tilde{G}U + g'\bar{\tilde{U}}\Big(-\frac{2}{3}\tilde{B}\Big)U + h.c.\Big) \\
&+ \overline{D^D_\mu\tilde{D}}D^{D\mu}\tilde{D} + iD\sigma^\mu\overline{D^D_\mu D} - \sqrt{2}\Big(-g_s\bar{\tilde{D}}\tilde{G}D + g'\bar{\tilde{D}}\frac{1}{3}\tilde{B}D + h.c.\Big) \\
&+ \overline{D^L_\mu\tilde{L}}D^{L\mu}\tilde{L} + iL\sigma^\mu\overline{D^L_\mu L} - \sqrt{2}\Big(g\bar{\tilde{L}}\tilde{W}L + g'\bar{\tilde{L}}\Big(-\frac{1}{2}\tilde{B}\Big)L + h.c.\Big) \\
&+ \overline{D^E_\mu\tilde{E}}D^{E\mu}\tilde{E} + iE\sigma^\mu\overline{D^E_\mu E} - \sqrt{2}\Big(g'\bar{\tilde{E}}\tilde{B}E + h.c.\Big) \\
&+ \overline{D^{H_u}_\mu H_u}D^{H_u\mu}H_u + i\tilde{H}_u\sigma^\mu\overline{D^{H_u}_\mu\tilde{H}_u} - \sqrt{2}\Big(g\bar{H}_u\tilde{W}\tilde{H}_u + g'\bar{H}_u\frac{1}{2}\tilde{B}\tilde{H}_u + h.c.\Big) \\
&+ \overline{D^{H_d}_\mu H_d}D^{H_d\mu}H_d + i\tilde{H}_d\sigma^\mu\overline{D^{H_d}_\mu\tilde{H}_d} - \sqrt{2}\Big(g\bar{H}_d\tilde{W}\tilde{H}_d + g'\bar{H}_d\Big(-\frac{1}{2}\tilde{B}\Big)\tilde{H}_d + h.c.\Big),
\end{aligned} \tag{3.1.10}
$$

where the covariant derivatives are

$$
D^Q_\mu = \partial_\mu - ig_sT^aG^a_\mu - igI^bW^b_\mu - ig'\frac{1}{6}B_\mu, \tag{3.1.11}
$$

$$
D^U_\mu = \partial_\mu + ig_sT^aG^a_\mu - ig'\Big(-\frac{2}{3}\Big)B_\mu, \tag{3.1.12}
$$

$$
\begin{aligned}
D_\mu^D &= \partial_\mu + ig_s T^a G_\mu^a - ig'\frac{1}{3}B_\mu, && (3.1.13)\\
D_\mu^L &= \partial_\mu - igI^b W_\mu^b - ig'\left(-\frac{1}{2}\right)B_\mu, && (3.1.14)\\
D_\mu^E &= \partial_\mu - ig' B_\mu, && (3.1.15)\\
D_\mu^{H_u} &= \partial_\mu - igI^b W_\mu^b - ig'\frac{1}{2}B_\mu, && (3.1.16)\\
D_\mu^{H_d} &= \partial_\mu - igI^b W_\mu^b - ig'\left(-\frac{1}{2}\right)B_\mu. && (3.1.17)
\end{aligned}
$$

The on-shell superpotential Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\text{OS }W}^{MSSM} = -\Big[& u_R^\dagger \mathbf{Y}_u u_L H_u^0 + u_R^\dagger \mathbf{Y}_u \tilde{u}_L \tilde{H}_u^0 + \tilde{u}_R^* \mathbf{Y}_u u_L \tilde{H}_u^0 \\
& - u_R^\dagger \boldsymbol{Y}_u d_L H_u^+ - u_R^\dagger \boldsymbol{Y}_u \tilde{d}_L \tilde{H}_u^+ - \tilde{u}_R^* \boldsymbol{Y}_u d_L \tilde{H}_u^+ \\
& - d_R^\dagger \boldsymbol{Y}_d u_L H_d^- - d_R^\dagger \boldsymbol{Y}_d \tilde{u}_L \tilde{H}_d^- - \tilde{d}_R^* \boldsymbol{Y}_d u_L \tilde{H}_d^- \\
& + d_R^\dagger \boldsymbol{Y}_d d_L H_d^0 + d_R^\dagger \boldsymbol{Y}_d \tilde{d}_L \tilde{H}_d^0 + \tilde{d}_R^* \boldsymbol{Y}_d d_L \tilde{H}_d^0 \\
& - e_R^\dagger \boldsymbol{Y}_e \nu H_d^- - e_R^\dagger \boldsymbol{Y}_e \tilde{\nu} \tilde{H}_d^- - \tilde{e}_R^* \boldsymbol{Y}_e \nu \tilde{H}_d^- \\
& + e_R^\dagger \boldsymbol{Y}_e e_L H_d^0 + e_R^\dagger \boldsymbol{Y}_e \tilde{e}_L \tilde{H}_d^0 + \tilde{e}_R^* \boldsymbol{Y}_e e_L \tilde{H}_d^0 \\
& + \mu\left(\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0\right)\Big] + h.c. \qquad (3.1.18)
\end{aligned}
$$

The final part is the potential

$$
\begin{aligned}
\mathcal{L}_V^{MSSM} = -\Big[& \left(\bar{H}_u^0 \bar{\tilde{u}}_L - \bar{H}_u^+ \bar{\tilde{d}}_L\right) \boldsymbol{Y}_u^\dagger \boldsymbol{Y}_u \left(\tilde{u}_L H_u^0 - \tilde{d}_L H_u^+\right) \\
& + \left(-\bar{H}_d^- \bar{\tilde{u}}_L + \bar{H}_d^0 \bar{\tilde{d}}_L\right) \boldsymbol{Y}_d^\dagger \boldsymbol{Y}_d \left(-\tilde{u}_L H_d^- - \tilde{d}_L H_d^0\right) \\
& + \left(-\bar{H}_d^- \bar{\tilde{\nu}} + \bar{H}_d^0 \bar{\tilde{e}}_L\right) \boldsymbol{Y}_e^\dagger \boldsymbol{Y}_e \left(-\tilde{\nu} H_d^- - \tilde{e}_L H_d^0\right) \\
& + \left(H_u^0 \tilde{u}_R^* \boldsymbol{Y}_u - H_d^- \tilde{d}_R^* \boldsymbol{Y}_d\right)\left(\boldsymbol{Y}_u^\dagger \bar{\tilde{u}}_R^* \bar{H}_u^0 - \boldsymbol{Y}_d^\dagger \bar{\tilde{d}}_R^* \bar{H}_d^-\right) \\
& + \left(-H_u^+ \tilde{u}_R^* \boldsymbol{Y}_u + H_d^0 \tilde{d}_R^* \boldsymbol{Y}_d\right)\left(-\boldsymbol{Y}_u^\dagger \bar{\tilde{u}}_R^* \bar{H}_u^+ + \boldsymbol{Y}_d^\dagger \bar{\tilde{d}}_R^* \bar{H}_d^0\right) \\
& + \left(-H_u^+ \tilde{e}_R^* \boldsymbol{Y}_e\right)\left(-\boldsymbol{Y}_e^\dagger \bar{\tilde{e}}_R^* \bar{H}_d^-\right) + \left(H_d^0 \tilde{e}_R^* \boldsymbol{Y}_e\right)\left(\boldsymbol{Y}_e^\dagger \bar{\tilde{e}}_R^* \bar{H}_d^0\right) \\
& + \left(\tilde{u}_R^* \boldsymbol{Y}_u \tilde{u}_L - \mu H_d^0\right)\left(\bar{\tilde{u}}_L \boldsymbol{Y}_u^\dagger \bar{\tilde{u}}_R^* - \mu^* \bar{H}_d^0\right) \\
& + \left(-\tilde{u}_R^* \boldsymbol{Y}_u \tilde{d}_L + \mu H_d^-\right)\left(-\bar{\tilde{d}}_L \boldsymbol{Y}_u^\dagger \bar{\tilde{u}}_R^* + \mu^* \bar{H}_d^-\right) \\
& + \left(\tilde{d}_R^* \boldsymbol{Y}_d \tilde{d}_L + \tilde{e}_R^* - \mu H_u^0\right)\left(\bar{\tilde{d}}_L \boldsymbol{Y}_d^\dagger \bar{\tilde{d}}_R^* + \bar{\tilde{e}}_L \boldsymbol{Y}_e^\dagger \bar{\tilde{e}}_R^* - \mu^* \bar{H}_u^0\right) \\
& + \left(-\tilde{d}_R^* \boldsymbol{Y}_d \tilde{u}_L - \tilde{e}_R^* \boldsymbol{Y}_e \tilde{\nu} + \mu H_u^+\right)\left(-\bar{\tilde{u}}_L \boldsymbol{Y}_d^\dagger \bar{\tilde{d}}_R^* - \bar{\tilde{\nu}} \boldsymbol{Y}_e^\dagger \bar{\tilde{e}}_R^* + \mu^* \bar{H}_u^+\right) \\
& + \frac{g_s^2}{2} \sum_{a=1}^{8} \left| \bar{\tilde{Q}} T^a \tilde{Q} - \bar{\tilde{U}} T^a \tilde{U} - \bar{\tilde{D}} T^a \tilde{D} \right|^2 \\
& + \frac{g^2}{2} \sum_{b=1}^{3} \left| \bar{\tilde{Q}} I^b \tilde{Q} + \bar{\tilde{L}} I^b \tilde{L} + \bar{H}_u I^b H_u + \bar{H}_d I^b H_d \right|^2 \\
& + \frac{g'^2}{2} \left| \frac{1}{6} \bar{\tilde{Q}} \tilde{Q} - \frac{2}{3} \bar{\tilde{U}} \tilde{U} + \frac{1}{3} \bar{\tilde{D}} \tilde{D} - \frac{1}{2} \bar{\tilde{L}} \tilde{L} + \bar{\tilde{E}} \tilde{E} + \frac{1}{2} \bar{H}_u H_u - \frac{1}{2} \bar{H}_d H_d \right|^2 \Big] \qquad (3.1.19)
\end{aligned}
$$

where the last three lines come from the D-terms and the remaining lines come from the F-terms.

However, experiments have yet to found any superpartner of SM particles. Thus, supersymmetry must be broken. Many mechanism of spontaneous SUSY breaking has been proposed, yet no agreement is settled. Fortunately, the terms appearing in the low energy Lagrangian to break SUSY is independent of its mechanism [37], thus, this matter is not discussed in this thesis. To prevent the appearance of quadratic divergence in quantum corrections, which is one of the attractive advantage of SUSY, the newly introduced terms must be soft. Also, though violating supersymmetry, they must obey the other symmetries that are realized in the SM. The possible and suitable terms that violates supersymmetry, followed from [37], is

$$\begin{aligned}
\mathcal{L}_{soft}^{MSSM} = &-\frac{1}{2}\Big(M_3\tilde{G}^a\tilde{G}^a + M_2\tilde{W}^b\tilde{W}^b + M_1\tilde{B}\tilde{B} + h.c.\Big) \\
&- m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - (bH_u \cdot H_d + h.c.) \\
&- \tilde{Q}^\dagger \mathbf{M}_{\tilde{Q}}^2 \tilde{Q} - \tilde{L}^\dagger \mathbf{M}_{\tilde{L}}^2 \tilde{L} - \tilde{U}^\dagger \mathbf{M}_{\tilde{U}}^2 \tilde{U} - \tilde{D}^\dagger \mathbf{M}_{\tilde{D}}^2 \tilde{D} - \tilde{E}^\dagger \mathbf{M}_{\tilde{E}}^2 \tilde{E} \qquad (3.1.20) \\
&- (\tilde{U}^\dagger \mathbf{Y}_u \mathbf{A}_u \tilde{Q} \cdot H_u - \tilde{D}^\dagger \mathbf{Y}_d \mathbf{A}_d \tilde{Q} \cdot H_d - \tilde{E}^\dagger \mathbf{Y}_e \mathbf{A}_e \tilde{L} \cdot H_d + h.c.) \qquad (3.1.21)
\end{aligned}$$

with $M_k, k = 1,2,3$ are the gaugino mass breaking parameter, $\mathbf{M}_{\tilde{F}}^2, F = Q,U,D,L,E$ and $\mathbf{A}_f, f = u,d,e$ are 3×3 matrices in flavour space. Without further constrains, these soft SUSY breaking terms introduces an additional 105 free parameters to MSSM.

The complete Lagrangian for MSSM reads

$$\mathcal{L}^{MSSM} = \mathcal{L}_{\text{OS SYM}}^{MSSM} + \mathcal{L}_{\text{OS Kähler}}^{MSSM} + \mathcal{L}_{\text{OS } W}^{MSSM} + \mathcal{L}_{V}^{MSSM} + \mathcal{L}_{soft}^{MSSM}. \qquad (3.1.22)$$

3.1.2 The Higgs sector of MSSM

To demonstrate the differences between the two supersymmetric version of the SM in this thesis, some analysis on the Higgs sector is done. The Higgs potential can be read off the Lagrangian with some algebra

$$\begin{aligned}
V_H = &(|\mu|^2 + m_{H_u}^2)(|H_u^+|^2 + |H_u^0|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
&+ \frac{g^2}{2}|H_u^+\bar{H}_d^0 + H_u^0\bar{H}_d^-|^2 + \frac{g^2 + g'^2}{8}\Big(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\Big)^2 \\
&+ \big[b(H_u^+H_d^- - H_u^0H_d^0) + h.c.\big] \qquad (3.1.23)
\end{aligned}$$

where, by a redefinition of field, b can be set to real. Following the same procedure as the Higgs mechanism in SM, experimental data confirm that the symmetry group $SU_L(2) \times U_Y(1)$ is broken into $U_Q(1)$ of electromagnetic symmetry. That is:

- The potential must have a stable, global, and non-degenerate minimum for charged Higgs to conserve $U_Q(1)$ symmetry.
- It must have a stable, global but degenerate minimum for neutral Higgs to break the remaining symmetries.

Meaning that the charged Higgs must have vanishing vacuum expectation value (VEV) while the neutral Higgs must have non-vanishing vacuum expectation value

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} = e^{i\xi}\begin{pmatrix} (v_d + \phi_d^0 + i\chi_d^0)/\sqrt{2} \\ \phi_d^- \end{pmatrix}, \qquad (3.1.24)$$

$$H_u = \begin{pmatrix} H_u^+ \\ H_d^0 \end{pmatrix} = \begin{pmatrix} \phi_u^+ \\ (v_u + \phi_u^0 + i\chi_u^0)/\sqrt{2} \end{pmatrix}, \qquad (3.1.25)$$

with ξ is a possible phase difference between the two Higgs doublets. This complex phase is also a source of possible CP-violation. By absorbing the phases into the Higgs field, v_u and v_d can be made real and positive. The condition for charged Higgs then translates to

$$\frac{\partial V_H}{\partial \phi_u^+} = \frac{\partial V_H}{\partial \phi_d^-} = 0 \text{ at } \langle \phi_u^+ \rangle = \langle \phi_d^- \rangle = 0. \tag{3.1.26}$$

The second one is

$$\frac{\partial V_H}{\partial \phi_u^0} = \frac{\partial V_H}{\partial \phi_d^0} = \frac{\partial V_H}{\partial \chi_u^0} = \frac{\partial V_H}{\partial \chi_d^0} = 0 \text{ at } \langle \phi_u^0 \rangle = \langle \phi_d^0 \rangle = \langle \chi_u^0 \rangle = \langle \chi_d^0 \rangle = 0. \tag{3.1.27}$$

While the first condition (3.1.26) is easily checked to automatically satisfied. The second one (3.1.27) is not so obvious. This condition leads to a set of tadpole equations

$$\frac{\partial V_H}{\partial \phi_u^0} = \tilde{m}_{H_u}^2 v_u + \frac{g^2 + g'^2}{8}\left(v_u^2 - v_d^2\right) v_u - b v_d \cos\xi = 0, \tag{3.1.28}$$

$$\frac{\partial V_H}{\partial \phi_d^0} = \tilde{m}_{H_d}^2 v_d + \frac{g^2 + g'^2}{8}\left(v_d^2 - v_u^2\right) v_d - b v_u \cos\xi = 0, \tag{3.1.29}$$

$$\frac{\partial V_H}{\partial \chi_u^0} = b v_d \sin\xi = 0, \tag{3.1.30}$$

$$\frac{\partial V_H}{\partial \chi_d^0} = b v_u \sin\xi = 0, \tag{3.1.31}$$

where

$$\tilde{m}_{H_u}^2 = |\mu|^2 + m_{H_u}^2 \quad , \quad \tilde{m}_{H_d}^2 = |\mu|^2 + m_{H_d}^2. \tag{3.1.32}$$

Equations (3.1.30) and (3.1.31) insists $\xi = 0$, conserving CP symmetry in the Higgs sector at tree-level. This set of equation can have several remarks. Firstly, without the supersymmetry breaking parameters $m_{H_u}^2, m_{H_d}^2, b$, these equations yield $v_u = v_d = 0$, making a non-degenerate minimum. Thus, electroweak symmetry breaking is impossible without supersymmetry breaking. Second, the vacuum expectation values of the Higgs field v_u, v_d are introduced only after supersymmetry breaking; thus, they are not a SUSY-conserving MSSM parameter. Moreover, using these equations, we can compute and replace the mass breaking parameters

$$\tilde{m}_{H_u}^2 = b\frac{v_d}{v_u} - \frac{g^2 + g'^2}{8}\left(v_u^2 - v_d^2\right) \quad , \quad \tilde{m}_{H_d}^2 = b\frac{v_u}{v_d} - \frac{g^2 + g'^2}{8}\left(v_d^2 - v_u^2\right). \tag{3.1.33}$$

It should be noted that the only new SUSY-conserving parameters of MSSM μ is contained within both $m_{H_u}^2$ and $\tilde{m}_{H_d}^2$. Thus, it is reasonable to guess that this parameters is on the SUSY scale. However, it also participates in breaking electroweak symmetry. The right side of this equations only contain parameters on the symmetry breaking scale, which is much lower. This requires very extreme fine-tuning for μ. This hierarchy problem is called the μ-problem [38]. A solution to this is proposed in section 3.2

The gauge boson mass is also considered here. Applying the decomposition of the Higgs doublets (3.1.24) and (3.1.25) to the kinetic term of the Higgs field, the masses of the gauge bosons can be obtained

$$m_W = \frac{g}{2}\sqrt{v_u^2 + v_d^2}, \tag{3.1.34}$$

$$M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\sqrt{v_u^2 + v_d^2}, \tag{3.1.35}$$

$$m_\gamma = m_G = 0. \tag{3.1.36}$$

with $v = \sqrt{v_u^2 + v_d^2}$, these masses are exactly identical to that of the SM. As usual, we also defines the angles θ_W and β as

$$\tan\theta_W = \frac{g}{g'} \quad , \quad \tan\beta = \frac{v_u}{v_d}. \tag{3.1.37}$$

The mass of the Higgs bosons can be obtained from the bilinear terms of the potential

$$V_H = \cdots + \begin{pmatrix} \phi_d^- & \phi_u^- \end{pmatrix} \mathbf{M}_c \begin{pmatrix} \phi_d^+ \\ \phi_u^+ \end{pmatrix} + \begin{pmatrix} \phi_d^0 & \phi_u^0 & \chi_d^0 & \chi_u^0 \end{pmatrix} \mathbf{M}_n \begin{pmatrix} \phi_d^0 \\ \phi_u^0 \\ \chi_d^0 \\ \chi_u^0 \end{pmatrix} \tag{3.1.38}$$

where the mass matrices are

$$\mathbf{M}_c = \begin{pmatrix} M_W^2 \sin^2\beta + b\tan\beta & b + M_W^2 \cos\beta\sin\beta \\ b + M_W^2\cos\beta\sin\beta & M_W^2\cos^2\beta + b\cot\beta \end{pmatrix}, \tag{3.1.39}$$

$$\mathbf{M}_n = \begin{pmatrix} \boldsymbol{M}_{\phi\phi} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\chi\chi} \end{pmatrix}, \tag{3.1.40}$$

$$\mathbf{M}_{\phi\phi} = \begin{pmatrix} M_Z^2\cos^2\beta + b\tan\beta & -b - M_Z^2\cos\beta\sin\beta \\ -b - M_Z^2\cos\beta\sin\beta & b\cot\beta + M_Z^2\sin^2\beta \end{pmatrix}, \tag{3.1.41}$$

$$\mathbf{M}_{\chi\chi} = \begin{pmatrix} b\tan\beta & b \\ b & b\cot\beta \end{pmatrix}. \tag{3.1.42}$$

To find the masses of these bosons, the eigenvalues of their mass matrices must be obtained. For the neutral Higgs, the CP-even and CP-odd Higgs decouples at tree-level, hence CP-conserving. Thus, they can be treated separately. The eigenvalues are derived from diagonalizing the mass matrices by the orthogonal transformation

$$\begin{pmatrix} h \\ H \end{pmatrix} = \boldsymbol{U}_\phi \begin{pmatrix} \phi_d^0 \\ \phi_u^0 \end{pmatrix} \quad , \quad \begin{pmatrix} G \\ A \end{pmatrix} = \boldsymbol{U}_\chi \begin{pmatrix} \chi_d^0 \\ \chi_u^0 \end{pmatrix} \quad , \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \boldsymbol{U}_c \begin{pmatrix} \phi_d^\pm \\ \phi_u^\pm \end{pmatrix} \tag{3.1.43}$$

which satisfy

$$\mathrm{diag}\big(m_G^2, m_A^2\big) = \boldsymbol{U}_\chi \boldsymbol{M}_{\chi\chi} \boldsymbol{U}_\chi^T \ , \ \mathrm{diag}\big(m_h^2, m_H^2\big) = \boldsymbol{U}_\phi \boldsymbol{M}_{\phi\phi} \boldsymbol{U}_\phi^T \ , \ \mathrm{diag}\big(m_{G^\pm}^2, m_{H^\pm}^2\big) = \boldsymbol{U}_c \boldsymbol{M}_c \boldsymbol{U}_c^T, \tag{3.1.44}$$

with the mixing matrices

$$\boldsymbol{U}_c = \boldsymbol{U}_\chi = \begin{pmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \ , \ \boldsymbol{U}_\phi = \begin{pmatrix} -\cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}, \tag{3.1.45}$$

and the masses

$$m_G^2 = m_{G^\pm}^2 = 0, \tag{3.1.46}$$

$$m_A^2 = \frac{2b}{\sin 2\beta}, \tag{3.1.47}$$

$$m_{H^\pm}^2 = m_W^2 + m_A^2, \tag{3.1.48}$$

$$m_h^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{(m_A^2 - M_Z^2)^2 + 4M_Z^2 m_A^2 \sin^2 2\beta}\right], \tag{3.1.49}$$

$$m_H^2 = \frac{1}{2}\left[m_A^2 + m_Z^2 + \sqrt{(m_A^2 - M_Z^2)^2 + 4M_Z^2 m_A^2 \sin^2 2\beta}\right], \tag{3.1.50}$$

and the mixing angle α is

$$\tan\alpha = -\frac{(m_A^2 + M_Z^2)\sin\beta\cos\beta}{M_Z^2\cos^2\beta + m_A^2\sin^2\beta - m_h^2}. \tag{3.1.51}$$

Conventionally, α is chosen so that $\alpha \in (-\pi/2, 0)$. Just like in the SM, there are three Nambu-Goldstone bosons: two charged and one CP-odd neutral boson. If $m_A^2 \geq M_Z^2 \cos 4\beta$ which is a natural requirement, since m_A^2 depends on b, a SUSY-breaking parameter, it can be arbitrarily large, especially when no superpartner has been detected yet, a slight transformation can be made

$$\begin{aligned} m_h^2 &= \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{m_A^4 + M_Z^4 - 2m_A^2 M_Z^2(1 - 2\sin^2 2\beta)}\right] \\ &\leq \frac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{m_A^4 + M_Z^4\cos^2 4\beta - 2m_A^2 M_Z^2 \cos 4\beta}\right] \\ &= \frac{1}{2}\left[m_A^2 + m_Z^2 - |m_A^2 - M_Z^2\cos 4\beta|\right] \\ &= M_Z^2 \cos^2 2\beta. \end{aligned} \tag{3.1.52}$$

Similarly,

$$m_H^2 \geq m_A^2 + M_Z^2 \sin^2 2\beta.$$

This results worth some remarks. Firstly, from these inequality, a neutral Higgs mass hierarchy can be seen

$$m_h^2 < m_A^2 < m_H^2. \tag{3.1.53}$$

If $m_A^2 \gg M_Z^2$, the mass of the three heavy Higgs are similar $m_A^2 \approx m_H^2 \approx m_{H^\pm}^2$ but is much heavier than the light Higgs. Since only one Higgs boson has been experimentally confirmed, if MSSM is to be realized, the lightest Higgs would be a very good candidate, because all the other Higgs are very heavy. Also, in this limit, the couplings of lightest Higgs with fermions and gauge bosons are identical to those of the SM. Thus, h is commonly considered to be the SM-like Higgs boson. Another important point is that, the Higgs mass in the SM is an unbound free parameter, and no prediction can be made regarding at which energy scale a Higgs boson can be found. In contrary, m_h^2 is upper-bounded by $M_Z^2 \cos^2 2\beta$. Although the quantum corrections may raise the mass above this threshold, the bound suggests that the lightest Higgs boson h can be found on the scale of Z-boson mass 91.1876 ± 0.0021 GeV [39]. This prediction was confirmed by experiment with the discovery of a Higgs boson at 125.18 ± 0.16 GeV, not far from the bound. This is a plus to this model since it has correctly predicted something the SM cannot. On the other hand, the experimental value of a Higgs boson mass can be used as a constrains on the supersymmetric parameters. However, for that to be feasible, high order calculation must be made due to the large quantum corrections of the Higgs mass.

3.2 Next-to-minimal Supersymmetric Standard Model

Although having some advantages, the MSSM has one issue mentioned in the previous section. The μ-problem involves extreme fine-tuning of the only SUSY-conserving μ so that a parameter on SUSY scale can participate in electroweak symmetry breaking. This problem is resolved in the Next-to-minimal Supersymmetric Standard Model (NMSSM) by introducing a Higgs singlet, and the parameter μ arises dynamically from electroweak symmetry breaking.

Supermultiplet	spin 0	spin 1/2	$SU_C(3) \times SU_L(2) \times U_Y(1)$
$\hat{S}$	S	$\tilde{S}$	$(\mathbf{1}, \mathbf{1}, 0)$

Table 3.3: The Higgs singlet introduced in NMSSM. The superfield is denoted with a hat, the superpartner has a tilde to distinguish. The numbers in the fourth column denote the dimensions of the corresponding representation of the gauge group.

3.2.1 The particle content and Lagrangian

Compare to MSSM, the particle spectrum of the NMSSM contains one more Higgs singlet described in table 3.3. With the introduction with one more superfield, the most general superpotential and the soft-breaking terms contributing to the Higgs mass are

$$W^{\text{Higgs}}_{NMSSM} = -(\mu + \lambda\hat{S})\hat{H}_d \cdot \hat{H}_u + \xi_F \hat{S} + \frac{1}{2}\mu' \hat{S}^2 + \frac{\kappa}{3}\hat{S}^3 \tag{3.2.1}$$

where the newly introduced terms must stop at third order for a renormalizable theory, since $[W] = 3$, and

$$\mathcal{L}^{NMSSM}_{soft\ \text{Higgs}} = -m_S^2|S|^2 + \Big(A_\lambda \lambda S H_u \cdot H_d - \frac{1}{3}A_\kappa \kappa S^3 + m_3^2 H_u \cdot H_d + \frac{1}{2}{m'_S}^2 S^2 + \xi_S S + h.c.\Big) \tag{3.2.2}$$

while the kinetic term of this superfield gives no modification to the mass spectrum; thus, it is not considered. These superpotential and soft-breaking terms, however, also poses it own hierarchy problem [40]. All newly introduced parameters take part in breaking electroweak symmetry, but at the same times, they should naturally be on SUSY scale. To eliminate this issue, any scale-dependent parameter should vanish. That is, because μ, μ' and ξ_F are dimensionful (μ, μ' being mass while ξ_F being mass^2), they naturally should depend on the energy scale at which they are introduced. Thus, they and their corresponding soft-SUSY-breaking parameters $m_3^2, {m'_S}^2, \xi_S$ should vanish. The remaining parameters are dimensionless, thus, have no dependence on the energy scale at which they appear. Accidentally, this scale independence requirement leaves the complete Lagrangian with a Z_3-symmetry. That is, if each chiral superfield is multiplied with a phase $e^{2\pi i/3}$, called Z_3 charge, the Lagrangian will possesses no phase, or no Z_3 charge. Thus, the name Z_3-invariance NMSSM is often used to refer to the version of NMSSM where the newly introduced terms are scale-independent.

With the restriction of Z_3-symmetry, eliminating hierarchy problem, the superpotential of the NMSSM is

$$W_{NMSSM} = \hat{H}_d \cdot \hat{L}\mathbf{Y}_e\hat{E}^c + \hat{H}_d \cdot \hat{Q}\mathbf{Y}_d\hat{D}^c - \hat{H}_u \cdot Q\mathbf{Y}_u\hat{U}^c - \lambda\hat{S}\hat{H}_d \cdot \hat{H}_u + \frac{\kappa}{3}\hat{S}^3 \tag{3.2.3}$$

$$= \hat{u}^c\mathbf{Y}_u\hat{u}_L\hat{H}_u^0 - \hat{u}^c\mathbf{Y}_u\hat{d}_L\hat{H}_u^+ - \hat{d}^c\mathbf{Y}_d\hat{u}_L\hat{H}_d^- + \hat{d}^c\mathbf{Y}_d\hat{d}_L\hat{H}_d^0 - \hat{e}^c\mathbf{Y}_e\hat{\nu}\hat{H}_d^- + \hat{e}^c\mathbf{Y}_e\hat{e}_L\hat{H}_d^0 + \lambda\hat{S}(\hat{H}_u^+\hat{H}_d^- - \hat{H}_u^0\hat{H}_d^0) + \frac{\kappa}{3}\hat{S}^3. \tag{3.2.4}$$

and the soft-breaking term is

$$\begin{aligned}\mathcal{L}^{NMSSM}_{soft} = &-\frac{1}{2}\Big(M_3\tilde{G}^a\tilde{G}^a + M_2\tilde{W}^b\tilde{W}^b + M_1\tilde{B}\tilde{B} + h.c.\Big)\\ &- m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - m_S^2|S|^2 + \Big(A_\lambda\lambda S H_u \cdot H_d - \frac{1}{3}A_\kappa\kappa S^3 + h.c.\Big)\\ &- \tilde{Q}^\dagger\mathbf{M}^2_{\tilde{Q}}\tilde{Q} - \tilde{L}^\dagger\mathbf{M}^2_{\tilde{L}}\tilde{L} - \tilde{U}^\dagger\mathbf{M}^2_{\tilde{U}}\tilde{U} - \tilde{D}^\dagger\mathbf{M}^2_{\tilde{D}}\tilde{D} - \tilde{E}^\dagger\mathbf{M}^2_{\tilde{E}}\tilde{E} \qquad (3.2.5)\\ &- (\tilde{U}^\dagger\mathbf{Y}_u\mathbf{A}_u\tilde{Q} \cdot H_u - \tilde{D}^\dagger\mathbf{Y}_d\mathbf{A}_d\tilde{Q} \cdot H_d - \tilde{E}^\dagger\mathbf{Y}_e\mathbf{A}_e\tilde{L} \cdot H_d + h.c.). \qquad (3.2.6)\end{aligned}$$

3.2.2 Tree level mass spectrum

With the completed Lagrangian at hand, multiple investigations can be taken. This section focuses on analysing the mass spectrum of NMSSM and compares it with that of the SM and MSSM. Among the SM particles, only the Higgs boson express significantly different behaviour, while the gauge bosons and fermions show quite similar results. This is a rather fortunate finding, since these sectors have been intensively tested by experiment. Any new behaviour could easily falsify the theory. The newly introduced superpartners, however, yield a wide range of interesting behaviours.

The Higgs potential

The Higgs potential of the NMSSM reads

$$\begin{aligned} V_H = &\left(|\lambda S|^2 + m_{H_d}^2\right)\left(|H_d^-|^2 + |H_d^0|^2\right) + \left(|\lambda S|^2 + m_{H_u}^2\right)\left(\left|H_u^0\right|^2 + \left|H_u^+\right|^2\right) + m_S^2|S|^2 \\ &+ \frac{1}{2}g^2\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2 + \frac{g^2+g'^2}{8}\left(|H_u^+|^2 + |H_u^0|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\ &+ \left|\lambda(H_u^+ H_d^- - H_u^0 H_d^0) + \kappa S^2\right|^2 + \left[-\lambda A_\lambda S\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \frac{1}{3}\kappa A_\kappa S^3 + h.c.\right]. \end{aligned} \tag{3.2.7}$$

Just as before, experiments confirm that the gauge symmetry $SU_L(2)\times U_Y(1)$ is broken to $U_Q(1)$. For the symmetry to be broken, the Higgs field must obtain a degenerate global minimum so that vacuum can break symmetry spontaneously. However, to preserve electromagnetic $U_Q(1)$ symmetry, the charged Higgs must obtain non-degenerate, thus vanishing, expectation value at the vacuum. That is, the expansion of the Higgs field about its vacuum can be obtained

$$H_d = \begin{pmatrix} (v_d + h_d + ia_d)/\sqrt{2} \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u}\begin{pmatrix} h_u^+ \\ (v_u + h_u + ia_u)/\sqrt{2} \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}}(v_s + h_s + ia_s), \tag{3.2.8}$$

with φ_u, φ_s being the possible phase differences. Since absolute phase can always be absorbed into the field without changing any physical interpretation, it is not considered. By pulling out the phases, the vacuum expectation values (VEV) v_d, v_u and v_s can be chosen to be real and positive.

Here, we can also see how NMSSM circumvent the μ-problem. The term $\lambda\hat{S}\hat{H}_d \cdot \hat{H}_u$ replaces $\mu\hat{H}_d \cdot \hat{H}_u$. Thus, when electroweak symmetry is broken, it becomes

$$\lambda\hat{S}\hat{H}_d \cdot \hat{H}_u \longrightarrow \frac{v_s e^{i\varphi_s}\lambda}{\sqrt{2}}\hat{H}_d \cdot \hat{H}_u. \tag{3.2.9}$$

Instead of directly introducing the parameter μ, NMSSM dynamically generates a similar term, sometimes denoted as

$$\mu_{eff} = \frac{e^{i\varphi_s}v_s\lambda}{\sqrt{2}}. \tag{3.2.10}$$

Since this parameter comes from electroweak symmetry breaking, the conundrum of its participation in the process become natural.

The Higgs potential can now be rewritten in term of expansion around its VEV as

$$V_H = V_H^{const} + t_{h_d}h_d + t_{h_u}h_u + t_{h_s}h_s + t_{a_d}a_d + t_{a_u}a_u + t_{a_s}a_s$$

$$+\frac{1}{2}\phi^{0T}\boldsymbol{M}_{\phi\phi}\phi^0+\phi^{c\dagger}\boldsymbol{M}_{h^+h^-}\phi^c+V_H^{\phi^3,\phi^4}, \tag{3.2.11}$$

with $\phi^0=(h_d,h_u,h_s,a_d,a_u,a_s)^T$ and $\phi^c=\big((h_d^-)^*,h_u^+\big)^T$, $t_\phi(\phi=h_d,h_u,h_s,a_d,a_u,a_s)$ are called the tadpoles coefficients while $\boldsymbol{M}_{\phi\phi}$ and $\boldsymbol{M}_{h^+h^-}$ are the neutral and charged Higgs mass matrix. V_H^{const} is the constant terms while $V_H^{\phi^3,\phi^4}$ is the third and fourth order terms.

The gauge sector

Since the gauge sector is not affected, expanding the kinetic term of the Higgs field still yield the masses of the gauge boson

$$M_W=\frac{g}{2}\sqrt{v_u^2+v_d^2}, \tag{3.2.12}$$

$$M_Z=\frac{1}{2}\sqrt{g^2+g'^2}\sqrt{v_u^2+v_d^2}, \tag{3.2.13}$$

$$M_\gamma=M_G=0, \tag{3.2.14}$$

with $v=\sqrt{v_u^2+v_d^2}$. As usual, we also defines the angles θ_W and β as

$$\tan\theta_W=\frac{g}{g'}\quad,\quad\tan\beta=\frac{v_u}{v_d}. \tag{3.2.15}$$

The Higgs sector

For the desired symmetry breaking mechanism to be realized, when the Higgs acquires its VEV at $\langle h_d\rangle=\langle h_u\rangle=\langle h_s\rangle=\langle a_d\rangle=\langle a_u\rangle=\langle a_s\rangle=0$, the potential must also reach its minimum. That is, its first derivatives must vanish

$$t_\phi=\left\langle\frac{\partial V}{\partial\phi}\right\rangle=0\text{ with }\phi=h_d,h_u,h_s,a_d,a_u,a_s, \tag{3.2.16}$$

where the abbreviation $c_x\equiv\cos x, s_x=\sin x, t_\beta\equiv\tan\beta$ has been used. Although trivially vanishes at tree-level, these terms gain non-trivial quantum corrections at higher levels. Thus, to keep them vanish, that is to keep the minimum of the Higgs potential unshifted, careful consideration of these parameters must be made. For that reason, although zero, they are explicitly kept for the sake of higher order calculation. Using definition (3.2.16), the tadpoles parameters are

$$\begin{aligned}\frac{t_{h_d}}{vc_\beta}&=m_{H_d}^2+\frac{c_{2\beta}M_Z^2}{2}-\frac{|\lambda|t_\beta v_s}{2}\Big(|\kappa|c_{\varphi_y}v_s-\sqrt{2}\Im A_\lambda s_{\varphi_\omega-\varphi_y}+\sqrt{2}\Re A_\lambda c_{\varphi_\omega-\varphi_y}\Big)\\&\quad+\frac{1}{2}|\lambda|^2\big(s_\beta^2v^2+v_s^2\big)\end{aligned} \tag{3.2.17}$$

$$\begin{aligned}\frac{t_{h_u}}{vs_\beta}&=m_{H_u}^2-\frac{c_{2\beta}M_Z^2}{2}-\frac{|\lambda|v_s}{2t_\beta}\Big(|\kappa|c_{\varphi_y}v_s-\sqrt{2}\Im A_\lambda s_{\varphi_\omega-\varphi_y}+\sqrt{2}\Re A_\lambda c_{\varphi_\omega-\varphi_y}\Big)\\&\quad+\frac{1}{2}|\lambda|^2\big(c_\beta^2+v_s^2\big)\end{aligned} \tag{3.2.18}$$

$$\begin{aligned}\frac{t_{h_s}}{v_s}&=m_S^2+|\kappa|^2v_s^2+\frac{|\lambda|^2v^2}{2}+|\lambda|c_\beta s_\beta v^2\left(\frac{\Im A_\kappa s_{\varphi_\omega-\varphi_y}-\Re A_\kappa c_{\varphi_\omega-\varphi_y}}{\sqrt{2}v_2}-|\kappa|c_{\varphi_y}\right)\\&\quad+\frac{|\kappa|v_s(\Re A_\kappa c_{\varphi_\omega}-\Im A_\kappa s_{\varphi_\omega})}{\sqrt{2}}\end{aligned} \tag{3.2.19}$$

$$\frac{t_{a_d}}{vs_\beta}=\frac{1}{2}|\lambda|v_s\Big(-|\kappa|v_s s_{\varphi_y}+\sqrt{2}\Im A_\lambda c_{\varphi_\omega-\varphi_y}+\sqrt{2}\Re A_\lambda s_{\varphi_\omega-\varphi_y}\Big) \tag{3.2.20}$$

$$t_{a_u} = \frac{1}{\tan\beta} t_{a_d} \tag{3.2.21}$$

$$\begin{aligned} t_{a_s} =& \frac{1}{2}|\lambda| c_\beta s_\beta v^2 \Big(2|\kappa| v_s s_{\varphi_y} + \sqrt{2}\Im A_\lambda c_{\varphi_\omega - \varphi_y} + \sqrt{2}\Re A_\lambda s_{\varphi_\omega - \varphi_y} \Big) \\ &- \frac{|\kappa| v_s^2 (\Im A_\kappa c_{\varphi_\omega} + \Re A_\kappa s_{\varphi_\omega})}{\sqrt{2}} \end{aligned} \tag{3.2.22}$$

where the frequently encountered phase combination is abbreviated as

$$\varphi_y = \varphi_\kappa - \varphi_\lambda + 2\varphi_s - \varphi_u, \tag{3.2.23}$$

$$\varphi_\omega = \varphi_\kappa + 3\varphi_s. \tag{3.2.24}$$

It is conventional that the dimensionless parameters λ and κ be expressed by their modulus and phase, while the trilinear couplings A_λ, A_κ are written in terms of their real and imaginary parts. Because the two tadpoles parameters t_{a_d}, t_{a_u} are linearly dependent, only five tadpole equations yield conditions to the Higgs sector. In the case of CP-conserving NMSSM, the pseudoscalar field tadpole conditions are trivially satisfied. On the other hand, if CP is violated, the two minimization conditions $t_{a_d} = t_{a_s} = 0$ can be used to eliminate $\Im A_\lambda$ and $\Im A_\kappa$. The other three tadpole equations $t_{h_d} = t_{h_u} = t_{h_s} = 0$ can be used to replace the soft-breaking mass parameters $m_{H_u}^2, m_{H_d}^2, m_S^2$.

The explicit expression for the mass matrix of the charged Higgs is given by

$$\begin{aligned} \boldsymbol{M}_{h^+h^-} =& \frac{1}{2} \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \left[M_W^2 s_{2\beta} + \frac{|\lambda| v_s \big(|\kappa| v_s c_{\varphi_\omega} + \sqrt{2}\Re A_\lambda \big)}{c_{\varphi_\omega - \varphi_y}} - \frac{1}{2}|\lambda|^2 s_{2\beta} v^2 \right] \\ &+ \begin{pmatrix} \frac{t_{h_d} - t_{a_d}\tan(\varphi_\omega - \varphi_y)}{v c_\beta} & -\frac{t_{a_d}(\tan(\varphi_\omega - \varphi_y) + i)}{v s_\beta} \\ -\frac{t_{a_d}(\tan(\varphi_\omega - \varphi_y) - i)}{v s_\beta} & \frac{s_\beta t_{h_u} - c_\beta t_{a_d}\tan(\varphi_\omega - \varphi_y)}{v s_\beta^2} \end{pmatrix} \end{aligned} \tag{3.2.25}$$

where the tadpoles parameters are explicitly kept. This mass matrix can be diagonalized using the mixing matrix

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \boldsymbol{R}^{G^\pm} \begin{pmatrix} h_d^\pm \\ h_u^\pm \end{pmatrix} \quad , \quad \boldsymbol{R}^{G^\pm} = \begin{pmatrix} -\cos\beta_c & \sin\beta_c \\ \sin\beta_c & \cos\beta_c \end{pmatrix}, \tag{3.2.26}$$

and

$$\mathrm{diag}\big(0, M_{H^\pm}^2\big) = \boldsymbol{R}^{G^\pm} \boldsymbol{M}_{h^+h^-} \Big(\boldsymbol{R}^{G^\pm} \Big)^T. \tag{3.2.27}$$

At tree level, the mixing angle $\beta_c = \beta$. However, the two angles are distinguished, since while β receives quantum correction, β_c is considered renormalized, thus not receiving counterterm. The charged Higgs mass can be obtained

$$\begin{aligned} M_{H^\pm}^2 =& \frac{|\lambda| c_{\beta-\beta_c}^2 v_s \big(|\kappa| v_s c_{\varphi_\omega} + \sqrt{2}\Re A_\lambda \big)}{s_{2\beta} c_{\varphi_y - \varphi_\omega}} - \frac{1}{2}|\lambda|^2 c_{\beta-\beta_c}^2 v^2 + c_{\beta-\beta_c}^2 M_W^2 \\ &+ \frac{s_\beta (c_\beta c_{\beta_c}^2 t_{h_u} + s_\beta s_{\beta_c}^2 t_{h_d} + c_{\beta-\beta_c}^2 t_{a_d} \tan(\varphi_y - \varphi_\omega))}{c_\beta s_\beta^2 v}. \end{aligned} \tag{3.2.28}$$

The tadpoles parameters and the charged mixing angles are explicitly kept for the sake of higher order computation. Using this relation, the parameter $\Re A_\lambda$ can be replaced by the charged Higgs mass. It should also be noted that two massless charged Nambu-Goldstone bosons are also the mass eigenstates

The neutral Higgs mass is described by a 6×6 matrix. However, only 5 of the mass eigenstates are Higgs bosons, while one becomes the Goldstone boson. To separate these two, a rotation is needed. To obtain the final mass eigenstates of the Higgs bosons, one more transformation is required. Thus, two rotations must be performed

$$(h_d, h_u, h_s, a, a_s, G)^T = \boldsymbol{R}^G (h_d, h_u, h_s, a_d, a_u, a_s)^T \tag{3.2.29}$$

$$(G, h_1, h_2, h_3, h_4, h_5)^T = \boldsymbol{R}(h_d, h_u, h_s, a, a_s, G)^T \tag{3.2.30}$$

which consecutively decouples the Goldstone boson and diagonalize the mass matrix

$$\boldsymbol{M}_{hh} = \boldsymbol{R}^G \boldsymbol{M}_{\phi\phi} \big(\boldsymbol{R}^G\big)^T \ , \quad \mathrm{diag}\big(0, m_{h_1}^2, m_{h_2}^2, m_{h_3}^2, m_{h_4}^2, m_{h_5}^2\big) = \boldsymbol{R}\boldsymbol{M}_{hh}\boldsymbol{R}^T \tag{3.2.31}$$

where the rotation matrix capable of decoupling the Goldstone boson is

$$\boldsymbol{R}^G = \begin{pmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \\ \mathbf{0} & \tilde{\boldsymbol{R}}^G \end{pmatrix} \ , \quad \tilde{\boldsymbol{R}}^G = \begin{pmatrix} \sin\beta_n & \cos\beta_n & 0 \\ 0 & 0 & 1 \\ \cos\beta_n & -\sin\beta_n & 0 \end{pmatrix} \tag{3.2.32}$$

which only mixes the CP-odd sector. Thus, the Goldstone boson is a pseudoscalar. The mass eigenstates h_i are sorted from lightest to heaviest $0 \le m_{h_1} \le \cdots \le m_{h_5}$. The mass matrix $\boldsymbol{M}_{hh}$ is

$$\begin{aligned}
(\boldsymbol{M}_{hh})_{h_d h_d} &= \frac{1}{2} v^2 |\lambda|^2 s_\beta^2 - \frac{\left(c_{\beta+\beta_c} - 3c_{\beta-\beta_c}\right) c_{\beta_c} t_{h_d}}{2 v c_{\beta-\beta_c}^2} - \frac{c_{\beta_c}^2 s_\beta t_{h_u}}{v c_{\beta-\beta_c}^2} + c_\beta^2 M_Z^2 + \frac{M_{H^\pm}^2 s_\beta^2}{c_{\beta-\beta_c}^2} - M_W^2 s_\beta^2 \\
(\boldsymbol{M}_{hh})_{h_d h_u} &= \frac{1}{2} v^2 |\lambda|^2 c_\beta s_\beta + \frac{s_\beta s_{\beta_c}^2 t_{h_d}}{v c_{\beta-\beta_c}^2} + \frac{c_\beta c_{\beta_c}^2 t_{h_u}}{v c_{\beta-\beta_c}^2} + c_\beta M_W^2 s_\beta - c_\beta M_Z^2 s_\beta - \frac{M_{H^\pm}^2 s_{2\beta}}{2 c_{\beta-\beta_c}^2} \\
(\boldsymbol{M}_{hh})_{h_d h_s} &= |\lambda|^2 \left(v c_\beta v_s - \frac{v^3 c_\beta s_\beta^2}{2 v_s} \right) - \frac{1}{2} v |\kappa| |\lambda| s_\beta v_s c_{\varphi_y} + \frac{s_\beta^2 s_{\beta_c}^2 t_{h_d}}{c_{\beta-\beta_c}^2 v_s} + \frac{c_{\beta_c}^2 s_{2\beta} t_{h_u}}{2 c_{\beta-\beta_c}^2 v_s} \\
&\quad + \frac{v}{v_s} \left(c_\beta M_W^2 s_\beta^2 - \frac{M_{H^\pm}^2 c_\beta s_\beta^2}{c_{\beta-\beta_c}^2} \right) \\
(\boldsymbol{M}_{hh})_{h_d a} &= \frac{t_{a_d} c_{\beta_n}}{v s_\beta} \\
(\boldsymbol{M}_{hh})_{h_d a_s} &= \frac{t_{a_d}}{v_s} + \frac{3}{2} v |\kappa| |\lambda| s_\beta v_s s_{\varphi_y} \\
(\boldsymbol{M}_{hh})_{h_u h_u} &= \frac{1}{2} v^2 |\lambda|^2 c_\beta^2 - \frac{c_\beta s_{\beta_c}^2 t_{h_d}}{v c_{\beta-\beta_c}^2} + \frac{s_{\beta_c} \left(2 c_\beta c_{\beta_c} + s_\beta s_{\beta_c}\right) t_{h_u}}{v c_{\beta-\beta_c}^2} - c_\beta^2 M_W^2 + \frac{M_{H^\pm}^2 c_\beta^2}{c_{\beta-\beta_c}^2} + M_Z^2 s_\beta^2 \\
(\boldsymbol{M}_{hh})_{h_u h_s} &= |\lambda|^2 \left(v s_\beta v_s - \frac{v^3 c_\beta^2 s_\beta}{2 v_s} \right) - \frac{1}{2} v |\kappa| |\lambda| c_\beta v_s c_{\varphi_y} + \frac{s_{2\beta} s_{\beta_c}^2 t_{h_d}}{2 c_{\beta-\beta_c}^2 v_s} + \frac{c_\beta^2 c_{\beta_c}^2 t_{h_u}}{c_{\beta-\beta_c}^2 v_s} \\
&\quad + \frac{v}{v_2} \left(c_\beta^2 M_W^2 s_\beta - \frac{M_{H^\pm}^2 c_\beta^2 s_\beta}{c_{\beta-\beta_c}^2} \right) \\
(\boldsymbol{M}_{hh})_{h_u a} &= \frac{t_{a_d} s_{\beta_n}}{v s_\beta} \\
(\boldsymbol{M}_{hh})_{h_u a_s} &= \frac{\cot\beta t_{a_d}}{v_s} + \frac{3}{2} v |\kappa| |\lambda| c_\beta v_s s_{\varphi_y} \\
(\boldsymbol{M}_{hh})_{h_s h_s} &= \frac{i v c_\beta \left(-1 + e^{2i\varphi_\omega}\right) t_{a_d}}{v_s^2 \left(1 + e^{2i\varphi_\omega}\right)} - \frac{i \left(-1 + e^{2i\varphi_\omega}\right) t_{a_s}}{v_s \left(1 + e^{2i\varphi_\omega}\right)} + \frac{\sqrt{2} |\kappa| v_s e^{i\varphi_\omega} \Re A_\kappa}{1 + e^{2i\varphi_\omega}} \\
&\quad + \frac{1}{2} v^2 |\kappa| |\lambda| c_\beta s_\beta \left(-c_{\varphi_y} + \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} \right) + \frac{v^4 |\lambda|^2 s_{2\beta}^2}{8 v_s^2} + 2 |\kappa|^2 v_s^2
\end{aligned}$$

$$
\begin{aligned}
& - \frac{v c_\beta s_\beta^2 s_{\beta_c}^2 t_{h_d}}{c_{\beta-\beta_c}^2 v_s^2} - \frac{v c_\beta^2 c_{\beta_c}^2 s_\beta t_{h_u}}{c_{\beta-\beta_c}^2 v_s^2} + \frac{v^2 \left(\frac{M_{H^\pm}^2 s_{2\beta}^2}{4 c_{\beta-\beta_c}^2} - c_\beta^2 M_W^2 s_\beta^2 \right)}{v_s^2} + \frac{t_{h_s}}{v_s} \\
(\boldsymbol{M}_{hh})_{h_s a} &= \frac{t_{a_d} \left(\cot\beta c_{\beta_n} + s_{\beta_n} \right)}{v_s} - \frac{1}{2} v \left|\kappa\right| \left|\lambda\right| v_s c_{\beta-\beta_n} s_{\varphi_y} \\
(\boldsymbol{M}_{hh})_{h_s a_s} &= -\frac{2 v c_\beta t_{a_d}}{v_s^2} + \frac{2 t_{a_s}}{v_s} - 2 v^2 \left|\kappa\right| \left|\lambda\right| c_\beta s_\beta s_{\varphi_y} \\
(\boldsymbol{M}_{hh})_{aa} &= \frac{1}{2} v^2 \left|\lambda\right|^2 c_{\beta-\beta_n}^2 - \frac{t_{h_d} s_{\beta_c-\beta_n} \left(2 s_\beta s_{\beta_c} s_{\beta_n} + c_\beta s_{\beta_c+\beta_n} \right)}{v c_{\beta-\beta_c}^2} \\
& + \frac{t_{h_u} s_{\beta_c-\beta_n} \left(2 c_\beta c_{\beta_c} c_{\beta_n} + s_\beta s_{\beta_c+\beta_n} \right)}{v c_{\beta-\beta_c}^2} - M_W^2 c_{\beta-\beta_n}^2 + \frac{M_{H^\pm}^2 c_{\beta-\beta_n}^2}{c_{\beta-\beta_c}^2} \\
(\boldsymbol{M}_{hh})_{a a_s} &= \frac{v^3 \left|\lambda\right|^2 s_{2\beta} c_{\beta-\beta_n}}{4 v_s} - \frac{3}{2} v \left|\kappa\right| \left|\lambda\right| v_s c_{\beta-\beta_n} c_{\varphi_y} - \frac{s_\beta c_{\beta-\beta_n} s_{\beta_c}^2 t_{h_d}}{c_{\beta-\beta_c}^2 v_s} - \frac{c_\beta c_{\beta_c}^2 c_{\beta-\beta_n} t_{h_u}}{c_{\beta-\beta_c}^2 v_s} \\
& + \frac{v}{v_s} \left(\frac{M_{H^\pm}^2 s_{2\beta} c_{\beta-\beta_n}}{2 c_{\beta-\beta_c}^2} - \frac{1}{2} M_W^2 s_{2\beta} c_{\beta-\beta_n} \right) \\
(\boldsymbol{M}_{hh})_{a_s a_s} &= -\frac{3 i v c_\beta \left(-1 + e^{2 i \varphi_\omega} \right) t_{a_d}}{v_s^2 \left(1 + e^{2 i \varphi_\omega} \right)} + \frac{3 i \left(-1 + e^{2 i \varphi_\omega} \right) t_{a_s}}{v_s \left(1 + e^{2 i \varphi_\omega} \right)} - \frac{3\sqrt{2} \left|\kappa\right| v_s e^{i\varphi_\omega} \Re A_\kappa}{1 + e^{2 i \varphi_\omega}} \\
& + \frac{3}{2} v^2 \left|\kappa\right| \left|\lambda\right| c_\beta s_\beta \left(c_{\varphi_y} - \frac{3 i \left(-1 + e^{2 i \varphi_\omega} \right) s_{\varphi_y}}{1 + e^{2 i \varphi_\omega}} \right) + \frac{v^4 \left|\lambda\right|^2 s_{2\beta}^2}{8 v_s^2} - \frac{v c_\beta s_\beta^2 s_{\beta_c}^2 t_{h_d}}{c_{\beta-\beta_c}^2 v_s^2} \\
& - \frac{v c_\beta^2 c_{\beta_c}^2 s_\beta t_{h_u}}{c_{\beta-\beta_c}^2 v_s^2} + \frac{v^2}{v_s^2} \left(\frac{M_{H^\pm}^2 s_{2\beta}^2}{4 c_{\beta-\beta_c}^2} - c_\beta^2 M_W^2 s_\beta^2 \right) + \frac{t_{h_s}}{v_s}
\end{aligned}
\tag{3.2.33}
$$

In the case of CP-conservation, the CP-odd and CP-even sectors decouple; thus, the mass matrices can be simultaneously diagonalized, and an analytical expression for the mass eigenvalue is possible. If CP-violation is considered, the rotation to obtain the mass eigenvalues cannot be expressed analytically. Just like β_c, at tree level, $\beta_n = \beta_c = \beta$. However, all mixing angles are kept separately because only β receives quantum correction, but not β_c and β_n. However, the relation $\beta_c = \beta_n$ always hold. One remark is that the neutral Goldstone boson derived from NMSSM is also CP-odd, just like SM and MSSM.

The following set of independent parameters shall be used for the Higgs sector

$$
\left\{ t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_{H^\pm}^2, v, \sin\theta_W, v, \tan\beta, |\lambda|, v_s, |\kappa|, \Re A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s \right\}. \tag{3.2.34}
$$

Neutralinos and Charginos

Unlike their SM counterpart, the superpartners of Higgs and gauge bosons mixes to form the mass eigenstates. The two neutral gauginos $(\tilde{B}, \tilde{W}^3)$ mixes with the three neutral Higgsinos $(\tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})$ to form the five mass eigenstates, called neutralinos. Written in the basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S})^T$, their mass matrix is

$$
\boldsymbol{M}_N = \begin{pmatrix}
M_1 & 0 & -M_Z s_W c_\beta & e^{-i\varphi_u} M_Z s_W s_\beta & 0 \\
0 & M_2 & M_Z c_W c_\beta & -e^{-i\varphi_u} M_Z c_W s_\beta & 0 \\
-c_\beta s_W M_Z & c_\beta c_W M_Z & 0 & -e^{i\varphi_s} \lambda v_s/\sqrt{2} & -e^{i\varphi_u} \lambda v_u/\sqrt{2} \\
e^{-i\varphi_u} s_\beta s_W M_Z & -e^{-i\varphi_u} s_\beta c_W M_Z & -e^{i\varphi_s} \lambda v_s/\sqrt{2} & 0 & -\lambda v_d/\sqrt{2} \\
0 & 0 & -e^{i\varphi_s} \lambda v_u/\sqrt{2} & -\lambda v_d/\sqrt{2} & \sqrt{2} e^{i\varphi_s} v_s \kappa
\end{pmatrix}, \tag{3.2.35}
$$

where the short-hand notation $s_x \equiv \sin x$, $c_x \equiv \cos x$ and $s_W \equiv \sin\theta_W, c_W = \cos\theta_W$ have been introduced. Using an appropriate unitary transformation $\mathbf{N}$, the mass matrix can be diagonalized yielding the five neutralinos mass eigenstates $\chi_i^0 (i = 1, \cdots, 5)$

$$\left(\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0\right)^T = \boldsymbol{N}\left(\tilde{B}, \tilde{W}^3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}\right)^T, \tag{3.2.36}$$

and the mass eigenvalues

$$\mathrm{diag}\left(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}, m_{\tilde{\chi}_5^0}\right) = \boldsymbol{N}^* \boldsymbol{M}_N \boldsymbol{N}^\dagger \tag{3.2.37}$$

where they are ordered by ascending mass $m_{\tilde{\chi}_1^0} \leq \cdots \leq m_{\tilde{\chi}_5^0}$. These neutralinos are Majorana fermions and do not interact electromagnetically. Thus, they can be candidates for dark matter, though more analysis should be done for a more definitive answer.

For the charged gauginos $\tilde{W}^\pm$ and charged Higgsinos $\tilde{H}_d^-, \tilde{H}_u^+$, their mass eigenstates are called charginos. Their mass matrix in the basis $(\tilde{W}^+, \tilde{H}_u^+)^T$ on the right and $(\tilde{W}^-, \tilde{H}_d^-)^T$ on the left is

$$\boldsymbol{M}_C = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta M_W e^{-i\varphi_u} \\ \sqrt{2}\cos\beta M_W & \lambda v_s e^{i\varphi_s}/\sqrt{2} \end{pmatrix}. \tag{3.2.38}$$

This matrix is not symmetric; thus, two unitary mixing matrices

$$\begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} \quad , \quad \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix} \tag{3.2.39}$$

are needed to diagonalize it

$$\mathrm{diag}\left(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}\right) = \boldsymbol{U}^* \boldsymbol{M}_C \mathbf{V}^\dagger \tag{3.2.40}$$

where the mass eigenvalues squared are

$$\begin{aligned} m_{\tilde{\chi}_{1,2}^\pm}^2 = \frac{1}{2}\Bigg[&|M_2|^2 + |\mu_{eff}|^2 + 2M_W^2 \\ &\mp\sqrt{\left(|M_2|^2 + |\mu_{eff}|^2 + 2M_W^2\right)^2 - 4\left|M_2\mu_{eff} - e^{-i\varphi_u} M_W^2 \sin 2\beta\right|^2}\,\Bigg], \end{aligned} \tag{3.2.41}$$

with μ_{eff} defined in (3.2.10)

Gluino

Gluino is the superpartner of gluons. They are Majorana fermions that carry color charge. They does not couples with the Higgs field or mix with any other particle in the NMSSM. Thus, at tree-level, the mass of gluino is entirely determined by the soft SUSY breaking parameter M_3

$$m_{\tilde{G}} = |M_3|. \tag{3.2.42}$$

Quarks and leptons

Similar to that of the SM, the masses of quarks and leptons comes from the Yukawa interaction with the Higgs field. The difference is that all fermions in the SM couples with just one Higgs doublet, while the NMSSM H_u couples with up-type quarks and H_d gives mass to down-type quarks and leptons.

Since the mass matrix of leptons is proportional to its Yukawa coupling, let λ_l , $l = e, \mu, \tau$ be the eigenvalues of the Yukawa coupling matrix $\boldsymbol{Y}_e$. The masses of the charged-lepton are

$$m_l = \frac{\lambda_l v_d}{\sqrt{2}} = \frac{\sqrt{2}\lambda_l M_W \sin\theta_W \cos\beta}{e}. \tag{3.2.43}$$

Experimental data suggests that the mass eigenstates of charged leptons can be set identical to its flavour eigenstates

At the same time, let λ_{q_u} , $q_u = u, c, t$ and λ_{q_d}, $q_d = d, s, b$ respectively be the eigenvalues of the Yukawa coupling matrices of up-type and down-type quarks $\boldsymbol{Y}_u, \boldsymbol{Y}_d$. In contrary to leptons, flavour mixing among quarks is non-vanishing. Thus, four unitary matrices $V^{u,d}_{L,R}$ are needed to obtain the eigenvalues, just as in the case of SM. Then, the masses of the quarks are

$$m_{q_u} = \frac{\lambda_{q_u} v_u}{\sqrt{2}} = \frac{\sqrt{2}\lambda_{q_u} M_W \sin\theta_W \sin\beta}{e} \ , \quad m_{q_d} = \frac{\lambda_{q_d} v_d}{\sqrt{2}} = \frac{\sqrt{2}\lambda_{q_d} M_W \sin\theta_W \cos\beta}{e}. \tag{3.2.44}$$

Squarks and sleptons

The mass matrix for the sfermions, excluding sneutrino, can be written in one general way in the basis $(\tilde{f}_L, \tilde{f}_R)^T$ is

$$\boldsymbol{M}_{\tilde{f}} = \begin{pmatrix} M_Z^2 c_{2\beta}\left(I_3^f - Q^f s_W^2\right)\mathbf{I}_{3\times3} + \boldsymbol{M}^2_{\tilde{f}_L} + \mathbf{m}_f^*\mathbf{m}_f^T & \mathbf{m}_f^*\mathbf{X}_f^* \\ \mathbf{X}_f^T\mathbf{m}_f^T & \mathbf{m}_f^T\mathbf{m}_f^* + \boldsymbol{M}^2_{\tilde{f}_R} + M_Z^2 c_{2\beta} Q_f s_W^2 \mathbf{I}_{3\times3} \end{pmatrix}, \tag{3.2.45}$$

with $f = e, u, d$. I_3^f and Q_f are respectively the isospin and electric charge of the fermion. $\boldsymbol{M}^2_{\tilde{f}_L}$ is $\boldsymbol{M}^2_{\tilde{Q}}$ for squarks or $\boldsymbol{M}^2_{\tilde{L}}$ for sleptons. $\boldsymbol{M}^2_{\tilde{f}_R}$ is $\boldsymbol{M}^2_{\tilde{U}}$ for up-type squarks, $\boldsymbol{M}^2_{\tilde{D}}$ for down-type squarks and $\boldsymbol{M}^2_{\tilde{E}}$ for sleptons. $\mathbf{m}_f$ is the corresponding mass matrix for fermions. It is $\mathbf{Y}_e v_d/\sqrt{2}$ for leptons, $\mathbf{Y}_d v_d/\sqrt{2}$ for down-type quarks and $e^{i\varphi_u}\mathbf{Y}_u v_u/\sqrt{2}$ for up-type quarks. And

$$\mathbf{X}_f = \mathbf{A}_f - \mathbf{I}_{3\times3}\, e^{-i\varphi_u}\mu^*_{eff}(\cot\beta)^{2\, I_3^f}, \tag{3.2.46}$$

with $I_3^f = 1/2$ for up-type sfermions and $-1/2$ for down-type sfermions. The mass matrices can be diagonalize analytically if the different generations of sfermions and fermions decouple. Such case yields 2×2 mass matrix

$$\mathbf{M}_{\tilde{f}_i} = \begin{pmatrix} M_Z^2 c_{2\beta}\left(I_3^{f_i} - Q^{f_i} s_W^2\right) + M^2_{\tilde{f}_{iL}} + m^2_{f_i} & m_{f_i} X^*_{f_i} \\ X_{f_i} m_{f_i} & m^2_{f_i} + M^2_{\tilde{f}_{iR}} + M_Z^2 c_{2\beta} Q_{f_i} s_W^2 \mathbf{I}_{3\times3} \end{pmatrix}, \tag{3.2.47}$$

with $i = 1, 2, 3$ being the generation index, the real and positive m_{f_i} is the mass of the corresponding fermion and

$$X_{f_i} = A_{f_i} - e^{-i\varphi_u}\mu^*_{eff}(\cot\beta)^{2\, I_3^f}. \tag{3.2.48}$$

Then, the sfermions $\tilde{f}_{iL}$ and $\tilde{f}_{iR}$ mixes to generate two mass eigenstates. The masses of the sfermions are

$$m^2_{\tilde{f}_i 1,2} = \frac{1}{2}\Big[2m_f^2 + M^2_{\tilde{f}_{iL}} + M^2_{\tilde{f}_{iR}} + I_3^f M_Z^2 \cos 2\beta$$

$$\mp\sqrt{\left[M^2_{\tilde{f}_{iL}} - M^2_{\tilde{f}_{iR}} + M_Z^2 \cos 2\beta (I_3^f - 2Q^f \sin^2\theta_W)\right]^2 + 4m^2_{f_i}|X_{f_i}|^2}\Bigg], \tag{3.2.49}$$

and their eigenstates are obtained by

$$\begin{pmatrix} \tilde{f}_{i1} \\ \tilde{f}_{i2} \end{pmatrix} = \boldsymbol{U}_{\tilde{f}} \begin{pmatrix} \tilde{f}_{iL} \\ \tilde{f}_{iR} \end{pmatrix} \quad , \quad \boldsymbol{U}_{\tilde{f}} = \begin{pmatrix} -\sin\theta_{\tilde{f}_i} & \cos\theta_{\tilde{f}_i} \\ \cos\theta_{\tilde{f}_i} & \sin\theta_{\tilde{f}_i} \end{pmatrix}, \tag{3.2.50}$$

with $\theta_{\tilde{f}_i}$ defined as

$$\begin{aligned}\cot\theta_{\tilde{f}_i} = -\frac{1}{2m_{f_i}X^*_{f_i}} \Bigg[& M^2_{\tilde{f}_{iL}} - M^2_{\tilde{f}_{iR}} + M_Z^2 \cos 2\beta (I_3^f - 2Q^f \sin^2\theta_W) \\ & + \sqrt{\left[M^2_{\tilde{f}_{iL}} - M^2_{\tilde{f}_{iR}} + M_Z^2 \cos 2\beta (I_3^f - 2Q^f \sin^2\theta_W)\right]^2 + 4m^2_{f_i}|X_{f_i}|^2}\Bigg].\end{aligned} \tag{3.2.51}$$

Lastly, the mass for sneutrino is, given that the different flavours decouple and suppressing the generation index,

$$m^2_{\tilde{\nu}} = M^2_{\tilde{L}} + \frac{1}{2}m_Z^2 \cos 2\beta. \tag{3.2.52}$$

Chapter 4

Inverse Seesaw Mechanism

4.1 Experimental data and motivation

Although neutrino is predicted in SM to be massless, experiments with solar, atmospheric, reactor and accelerator neutrino has shown evidence of oscillation [31–33]. Such data implies massive neutrinos and the existence of mixing between different flavours of neutrino. This phenomenon is characterized by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U^{PMNS}

Further investigations not only strengthen the evidence but also report unusually light neutrinos. The upper bound for neutrino mass from cosmological data [41]

$$\sum_{i=1}^{3} m_{n_i} < 0.23 \text{ eV} \tag{4.1.1}$$

is of order 10^6 times smaller than that of electron, the lightest massive particle in SM. However, given that the masses of all particles in the SM is generated through the same way, the Higgs mechanism, the many orders of magnitude difference in their masses is troublesome.

The fact that neutrino has mass proves that the current description of neutrino is incomplete. New ways of explaining the smallness, yet non-zero, of neutrino mass have been proposed. Upon them, the most attractive explanation is probably the seesaw mechanism.

This mechanism has many versions, but they generally introduces right-handed neutrino and Majorana mass term. The smallness of the observed neutrinos is then explained by the arbitrarily large mass of sterile neutrinos due to parameters independent of SM, thus can be on any scale, including GUT or SUSY scale.

4.2 Type I Seesaw Mechanism

The mass terms for a general Dirac fermion $\Psi_D = (f_L, f_R)^T$ is

$$\mathcal{L}^D = -m\bar{\Psi}_D\Psi_D = -m\big(\bar{f}_L f_R + \bar{f}_R f_L\big). \tag{4.2.1}$$

Since right-handed neutrino is not observed in experiment, thus, massless according to SM. However, this argument is not applicable if right-handed neutrino does exist and singlet under gauge group, called sterile right-handed neutrino. Moreover, it is also not the most general mass term and it alone cannot explain the smallness of neutrino mass. Beside Dirac fermion, a different kind of fermion, called Majorana fermion, was proposed. This particle is its own anti-particle, defined by

$$\Psi_M = \Psi_M^c, \tag{4.2.2}$$

with c being the charge-conjugated operator. If Majorana fermion is included, more mass terms are possible

$$\mathcal{L}^M = -\frac{m}{2}\bar{\Psi}_M \Psi_M^c + h.c. = -\frac{m}{2}(\bar{f}_L f_L^c + \bar{f}_R f_R^c) + h.c. \ . \tag{4.2.3}$$

In general, Majorana mass terms do not respect gauge symmetry if it is not a gauge singlet.

Combine both cases, we can write down the general mass term for neutrino

$$\mathcal{L}^{D+M} = -\frac{m_L^M}{2}\bar{f}_L f_L^c - \frac{m_R^M}{2}\bar{f}_R f_R^c - m^D \bar{f}_L f_R + h.c. \ . \tag{4.2.4}$$

For the specific case of neutrino with 3 generations $\nu = (\nu_e, \nu_\mu, \nu_\tau)^T$, that is

$$\begin{aligned}\mathcal{L}^{D+M} &= -\frac{1}{2}\bar{\nu}_L \boldsymbol{M}_L^M \nu_L^c - \frac{1}{2}\bar{\nu}_R \boldsymbol{M}_R^M \nu_R^c - \bar{\nu}_L \boldsymbol{M}^D \nu_R + h.c. &(4.2.5)\\ &= -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L & \bar{\nu}_R^c\end{pmatrix} \boldsymbol{M}_\nu^{D+M} \begin{pmatrix}\nu_L^c \\ \nu_R\end{pmatrix} + h.c., &(4.2.6)\end{aligned}$$

with

$$\boldsymbol{M}_\nu^{D+M} = \begin{pmatrix}\boldsymbol{M}_L^M & \boldsymbol{M}^D \\ (\boldsymbol{M}^D)^T & \boldsymbol{M}_R^M\end{pmatrix}. \tag{4.2.7}$$

Even though the mass term can be constructed using only left-handed neutrino, such Majorana term would violate $SU(2)$ symmetry; thus, we must exclude its Majorana term, meaning $M_L^M = \boldsymbol{0}$, and must introduce sterile right-handed neutrino. Meaning that

$$\boldsymbol{M}_\nu^{D+M} = \begin{pmatrix}\boldsymbol{0} & \boldsymbol{M}^D \\ (\boldsymbol{M}^D)^T & \boldsymbol{M}_R^M\end{pmatrix}. \tag{4.2.8}$$

Diagonalizing this matrix, we will obtain the neutrino mass.

To show how seesaw mechanism work, we shall consider the simplest case of only 1 flavour of neutrino. Then the mass matrix becomes

$$\boldsymbol{M}_\nu^{D+M} = \begin{pmatrix}0 & m^D \\ m^D & m_R\end{pmatrix}. \tag{4.2.9}$$

Diagonalizing this matrix yields the eigenvalues

$$\begin{vmatrix}-\mu & m^D \\ m^D & m_R - \mu\end{vmatrix} = 0 \Longleftrightarrow \mu = \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 + 4(m^D)^2}\right]. \tag{4.2.10}$$

While m^D is generated from electroweak symmetry breaking, m_R is not bound to the SM, which is considered to be an effective theory of a more general theory at higher energy scale; thus, it is natural to expect the value of m^D is of order of electroweak symmetry breaking while m_R can be on the scale of the general theory, which may be SUSY or GUT. That means $m^D \ll m_R$. Using this, the mass eigenvalues can be approximated to first order as

$$\mu = \frac{1}{2}m_R\left[1 \pm \sqrt{1 + \left(\frac{2m^D}{m_R}\right)^2}\right] \approx \frac{1}{2}m_R\left[1 \pm \left(1 + 2\left(\frac{m^D}{m_R}\right)^2\right)\right] \tag{4.2.11}$$

$$\Longrightarrow \mu \approx -\frac{(m^D)^2}{m_R} \quad \text{or} \quad \mu \approx m_R. \tag{4.2.12}$$

Since m_R is large compared to m^D, we have 2 mass eigenvalues, one represents very light neutrino, as demanded by experiment, the other is very heavy, thus the name seesaw mechanism.

In the case of 3 flavours, the mathematical formula is more complicated, but the idea is the same. That is, we can block diagonalize the mass matrix in the case $\boldsymbol{M}^D \ll \boldsymbol{M}_R^M$ [42]

$$\boldsymbol{M}_\nu^{D+M} = \begin{pmatrix} \mathbf{0} & \boldsymbol{M}^D \\ (\boldsymbol{M}^D)^T & \boldsymbol{M}_R^M \end{pmatrix} \approx \boldsymbol{U}_\nu \begin{pmatrix} -(\boldsymbol{M}^D)^T \boldsymbol{M}_R^{-1} \boldsymbol{M}_D & \mathbf{0} \\ \mathbf{0} & \boldsymbol{M}_R \end{pmatrix} \boldsymbol{U}_\nu^T, \tag{4.2.13}$$

with

$$\boldsymbol{U}_\nu = \begin{pmatrix} \mathbf{I}_3 & -\boldsymbol{M}_R^{-1}(\boldsymbol{M}^D)^T \\ (\boldsymbol{M}^D)^*(\boldsymbol{M}_R^{-1})^\dagger & \mathbf{I}_3 \end{pmatrix}. \tag{4.2.14}$$

Since the mass matrix is block diagonalized, we can find the eigenvalues of each block independently. Because $\boldsymbol{M}^D \ll \boldsymbol{M}_R^M$, the upper-left block and its eigenvalues are very small, of order $(m^D)^2/m_R$ like in equation (4.2.12), thus contributing to the unusual smallness of light neutrino, while the lower-left block is on SUSY or GUT scale demanding the existence of very heavy neutrino just as showed in the case of 1 flavour.

To get a qualitative picture of how massive heavy neutrino should be, we can use some approximation of the values of the parameters. The upper-bound for neutrino mass from cosmological data is 0.23 eV $= 0.23 \times 10^{-9}$ GeV [41]. Since m^D is on electroweak breaking scale. It may ranges from the lightest lepton's mass, electron, to heaviest quark's mass, top quark. That is m^D may varies from 0.5×10^{-3} GeV to 170 GeV; thus, the mass of heavy neutrino ranges from 10^3 GeV to 10^{14} GeV

4.3 Inverse Seesaw Mechanism

Although seesaw mechanism can explain the existence and the smallness of neutrino mass, it is experimentally impractical to detect neutrino as heavy as 10^{14} GeV. Inverse seesaw mechanism is an alternative explanation for the mass of neutrino that does not resort to such high mass. In fact, heavy neutrinos in this mechanism can be as light as TeV scale, which is within grasp of today experimental capability. The trade of is that we need to introduce a fermionic singlet for every light neutrino. That is, we need an additional 3 singlets to describe the 3 generations of light neutrino. The Lagrangian is

$$\mathcal{L}_\nu = -\frac{1}{2}\bar{n}^c \boldsymbol{M}_\nu n + h.c., \tag{4.3.1}$$

with $n = (\nu_L^c, N_R, s)^T$ and

$$\boldsymbol{M}_\nu = \begin{pmatrix} \mathbf{0} & \boldsymbol{M}_D & \mathbf{0} \\ \boldsymbol{M}_D^T & \boldsymbol{\mu}_R & \boldsymbol{\mu}^T \\ \mathbf{0} & \boldsymbol{\mu} & \boldsymbol{\lambda}_X \end{pmatrix}. \tag{4.3.2}$$

To conserve gauge invariant, the terms $\nu_L^c \nu_L^c$ and $\nu_L^c s$ must vanishes. Among the remaining terms, M_D is on the electroweak symmetry breaking scale, μ can be on any arbitrary scale. While $\boldsymbol{\mu}_R$ and $\boldsymbol{\lambda}_X$ can be on any scale, these terms break lepton number, which is highly constrained by experiments. Thus, they have to be very small. Therefore, we have $\boldsymbol{\lambda}_X \ll \boldsymbol{M}_D \ll \boldsymbol{\mu}$. With this, we can approximately, to first order, block diagonalize the mass matrix to obtain the block for light neutrino [42]

$$\boldsymbol{m}_\nu = \boldsymbol{M}_D \boldsymbol{\mu}^{-1} \boldsymbol{\lambda}_X (\boldsymbol{\mu}^{-1})^T \boldsymbol{M}_D^T. \tag{4.3.3}$$

Diagonalizing this matrix gives the mass of the light neutrino. The contribution of $\boldsymbol{\mu}_R$ to $\boldsymbol{m}_\nu$ is to second order. Thus, it is often dropped from the analysis, as will be the case in this thesis.

To have a qualitative picture of the energy scale of each terms, we shall follow the same procedure of that in previous section. That is, the scale for neutrino mass is 0.1 eV $= 0.1 \times 10^{-9}$ GeV. $\boldsymbol{M}_D$ may varies from 0.5×10^{-3} GeV to 170 GeV. With $\boldsymbol{\lambda}_X \ll \boldsymbol{M}_D$, and since lepton number violation should be on the energy scale smaller than the lower bound of $\boldsymbol{M}_D$ because no such violation is observed yet, it is required that $\boldsymbol{\mu} < 2 \times 10^5$ GeV. That is, all parameters in this mechanism is within the TeV scale or lower. This is an interesting result since the newly introduced particles are in the reach of future or even present collider.

4.4 Inverse Seesaw Mechanism in NMSSM

To incorporate the idea of inverse-seesaw into NMSSM, two superfields, each with three generations corresponding to the three flavour of light neutrinos, must be introduced. As mentioned while the Lagrangian for NMSSM was constructed, the requirement of scale-independent, thus producing no hierarchy problem, coincidently forces the Lagrangian to be Z_3-invariant. Thus, in bringing more superfields into the theory, caution should be taken in order to conserve Z_3-symmetry. Following [43], each of the new neutrino superfields is assigned with the appropriate Z_3-charge as described in table 4.1. The superpotential involving only the neutrino sectors is given by

$$W_{ISS} = \hat{N}^c y_\nu \hat{L} \cdot \hat{H}_u + \hat{S} \hat{X} \lambda_X \hat{X} + \hat{N}^c \mu_X \hat{X}, \tag{4.4.1}$$

while the soft SUSY breaking Lagrangian is

$$\mathcal{L}_{soft}^{ISS} = -\tilde{N}^T M_{\tilde{N}}^2 \tilde{N}^* - \tilde{X}^\dagger M_{\tilde{X}}^2 \tilde{X} - \Big(S \tilde{X} \lambda_X A_X \tilde{X} + \tilde{N}^* y_\nu A_\nu \tilde{L} \cdot H_u + \tilde{N}^* \mu_X B_{\mu_X} \tilde{X} + h.c. \Big), \tag{4.4.2}$$

with y_ν, λ_X, μ_X are 3×3 matrices in flavour space. After constructing the Lagrangian, the mass matrix for neutrino can be obtained. In the interaction basis (ν, N^c, X), it is the 9×9 matrix

$$M_\nu = \begin{pmatrix} \mathbf{0} & \dfrac{v_u e^{i\varphi_u} y_\nu}{\sqrt{2}} & \mathbf{0} \\ \dfrac{v_u e^{i\varphi_u}}{\sqrt{2}} y_\nu^T & \mathbf{0} & \mu_X \\ \mathbf{0} & \mu_X^T & \dfrac{v_s e^{i\varphi_s}}{\sqrt{2}} \left(\lambda_X + \lambda_X^T \right) \end{pmatrix}. \tag{4.4.3}$$

In general, we will use four-component spinor in our calculation. In this notation, a Majorana fermion Ψ_M is defined as

$$\Psi_M \equiv \begin{pmatrix} \psi_L \\ \psi_L^c \end{pmatrix} = \Psi_M^c. \tag{4.4.4}$$

The respective chiral components can be obtained using the chiral operator $\mathcal{P}_{L,R} = \frac{1 \pm \gamma_5}{2}$

$$\psi_L = \mathcal{P}_L \Psi_M \quad , \quad \psi_R = \mathcal{P}_R \Psi_M = (\psi_L)^c. \tag{4.4.5}$$

Superfields	spin 0	spin 1/2	$SU_C(3) \times SU_L(2) \times U_Y(1)$	Z_3-charge
$\hat{N}^c$	$\tilde{N}^*$	N^c	$(\mathbf{1}, \mathbf{1}, 0)$	ω^2
$\hat{X}$	$\tilde{X}$	X	$(\mathbf{1}, \mathbf{1}, 0)$	ω

Table 4.1: The newly introduced neutrino superfield. The superfields are denoted with a hat, the superpartner of SM particles have a tilde to distinguish. The generation indices are suppressed. The numbers in the fourth column denote the dimensions of the corresponding representation of the gauge group, with the bar denote complex conjugate representation. The last column is the Z_3-charge with unit charge $\omega = e^{2\pi i/3}$

Since neutrinos are proposed to be Majorana fermions, they also have similar definition

$$\psi_\nu = \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} \quad , \quad \psi_N = \begin{pmatrix} N^c \\ N \end{pmatrix} \quad , \quad \psi_X = \begin{pmatrix} X \\ X^c \end{pmatrix}. \tag{4.4.6}$$

Approximately block diagonalize the matrix (4.4.3), the mass matrix for light neutrino is obtained [42]

$$\begin{aligned} m_\nu &= \left[\left(\frac{v_u e^{i\varphi_u} y_\nu}{\sqrt{2}} \right) (\mu_X^T)^{-1} \right] \frac{v_s e^{i\varphi_s}}{\sqrt{2}} (\lambda_X + \lambda_X^T) \left[\mu_X^{-1} \left(\frac{v_u e^{i\varphi_u}}{\sqrt{2}} y_\nu^T \right) \right] \\ &= \frac{v_u^2 v_s e^{i(2\varphi_u + \varphi_s)}}{2\sqrt{2}} \left[y_v (\mu_X^T)^{-1} \right] (\lambda_X + \lambda_X^T) \left[\mu_X^{-1} y_\nu^T \right]. \end{aligned} \tag{4.4.7}$$

Diagonalizing this 3×3 matrix yields the masses of light neutrinos. An analysis similar to that of the inverse-seesaw mechanism in the SM can be taken. v_u is on the electroweak symmetry breaking scale. The parameter λ_X characterize lepton number violation, which is not detected by experiments. Therefore, it should be sufficiently small. With the help of this lepton number violating parameter, the unusual smallness of neutrino mass matrix only requires μ_X on the TeV scale, within the reach of present or future colliders.

With the introduction of the new superfield, sneutrino sector is also changed. To incorporate CP-violation, each sneutrino field is separate into its CP-even and CP-odd component

$$\tilde{\nu} = \frac{1}{\sqrt{2}} (\tilde{\nu}_+ + i\tilde{\nu}_-), \tag{4.4.8}$$

$$\tilde{N}^* = \frac{1}{\sqrt{2}} \left(\tilde{N}_+ + i\tilde{N}_- \right), \tag{4.4.9}$$

$$\tilde{X} = \frac{1}{\sqrt{2}} \left(\tilde{X}_+ + i\tilde{X}_- \right). \tag{4.4.10}$$

Although the mass matrix for these fields can be computed from the Yukawa couplings and other parameters, in practice, it is the mass matrix that is known beforehand, while the other parameters such as the Yukawa couplings must be computed. Thus, a reverse parametrization is needed. One such way is using the mass and mixing matrices of neutrino as input parameter and the Yukawa coupling matrix have to be computed. Let the mass of neutrino be $m_{\nu_1} \leq m_{\nu_2} \leq m_{\nu_3}$. The unitary mixing matrix U_{PMNS} is defined as

$$m_\nu = U_{PMNS}^T \mathcal{D}_\nu U_{PMNS} \ , \quad \mathcal{D}_\nu = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}). \tag{4.4.11}$$

Note that it is conventional for U_{PMNS} to be defined as transforming the mass eigenstates to the flavour eigenstates, contrary to the convention for mixing matrices used in this thesis. Let

$\sqrt{\lambda_X}$ and $\sqrt{\mathcal{D}_\nu}$ be the matrix such that

$$2\sqrt{\lambda_X}^T\sqrt{\lambda_X} = \lambda_X + \lambda_X^T \ , \quad \sqrt{\mathcal{D}_\nu} = \mathrm{diag}\big(\sqrt{m_{\nu_1}}, \sqrt{m_{\nu_2}}, \sqrt{m_{\nu_3}}\big). \tag{4.4.12}$$

Since no information regarding the new neutrino sector has been confirmed, without lost of generality, the matrices μ_X and λ_X can be made diagonalized by rotating the fields N and X to the appropriate basis. Then, the matrix $\sqrt{\lambda_X}$ can be constructed from taking the square root of the diagonal element of λ_X. Thus, it can always be defined. Then, equation (4.4.7) is equivalent to

$$\mathbf{I}_{3\times 3} = \frac{v_u^2 v_s e^{i(2\varphi_u+\varphi_s)}}{\sqrt{2}}\left[\sqrt{\mathcal{D}_\nu^{-1}}U^*_{PMNS}\ y_v(\mu_X^T)^{-1}\sqrt{\lambda_X}^T\right]\left[\sqrt{\lambda_X}\mu_X^{-1}y_\nu^T U^\dagger_{PMNS}\sqrt{\mathcal{D}_\nu^{-1}}\right]. \tag{4.4.13}$$

Let

$$R = \sqrt{\frac{v_u^2 v_s e^{i(2\varphi_u+\varphi_s)}}{\sqrt{2}}}\sqrt{\lambda_X}\mu_X^{-1}y_\nu^T U^\dagger_{PMNS}\mathcal{D}_\nu^{-1}. \tag{4.4.14}$$

Then, this relation immediately equivalents to $\mathbf{I}_{3\times 3} = R^T R$, which is correct for all complex orthogonal matrix R. Thus, all y_ν satisfying the relation

$$y_\nu = U^T_{PMNS}\sqrt{\mathcal{D}_\nu}R^T\left(\sqrt{\lambda_X}^T\right)^{-1}\mu_X^T \tag{4.4.15}$$

for all complex orthogonal R, generate the desirable mass matrix m_ν

4.5 Feynman rules for NMSSM with inverse seesaw mechanism

In this section, the necessary Feynman rules for the theory is presented. Only those that have new contribution to the Higgs sector of NMSSM is considered. Due to the existence of fermion-number-violating interaction, the usual Feynman rules are not usable. Therefore, the Feynman rules for fermion in such case following the work in [44]. They are then summarized in the last part.

4.5.1 Motivation for Feynman rules for fermion-number-violating interaction

Let us denote ψ as any kind of fermion, ψ_D as Dirac fermion, ψ_M as Majorana fermion and Γ denote the gamma matrix structure, thus is a 4×4 matrix. Without Majorana fermion, it is possible to arrange the contraction among fermions so that it takes the form

$$\langle\cdots|\,(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)\cdots\ \cdots(\bar{\psi}\Gamma\psi)\,|\cdots\rangle \tag{4.5.1}$$

for an open fermion line, or

$$\cdots(\bar{\psi}\Gamma\psi)(\bar{\psi}\Gamma\psi)\cdots\cdots(\bar{\psi}\Gamma\psi)\cdots \tag{4.5.2}$$

for a closed fermion loop. This show that the fermion line in the Feynman diagram is a continuous and oriented line, or that fermion-number is conserved. However, this is not the case if Majorana fermion present. One reason is thatm as opposed to Dirac fermion, where only one contraction is possible, the following contractions are also non-vanishing for Majorana fermion (detailed calculation is presented later)

$$\psi_M\psi_M \quad , \quad \bar{\psi}_M\bar{\psi}_M \tag{4.5.3}$$

Here, we denoted ψ_M for Majorana fermion. Therefore, if the contraction

$$\cdots(\bar{\psi}_M \Gamma \psi_M)\cdots \tag{4.5.4}$$

is possible, so is the contraction

$$\cdots(\bar{\psi}_M \Gamma \psi_M)\cdots \tag{4.5.5}$$

However, while the former conserves fermion-number and show diagrammatically as an oriented continuous line, it is not the case for the later. Firstly, in $\psi\bar{\psi}$, if we denote the fermion-number of ψ as 1 and $\bar{\psi}$ as -1, this contraction yield a fermion-number 0, thus conserved. But, in $\psi\psi$, it is 2 and in $\bar{\psi}\bar{\psi}$, it is -2, violating fermion-number. Secondly, (4.5.5) breaks the continuous flow. One cannot unambiguously determine the direction of particle-number flow with the propagator $\psi\psi$ and $\bar{\psi}\bar{\psi}$.

To resolve these problems and implement Feynman rule in theories with Majorana fermion, we follow the work in [44]. The idea is to keep essence of the concept fermion-number flow by adjusting the interaction Lagrangian. This adjustment is built upon the following observation. Introducing as the charge-conjugating matrix [1], ψ^c and $\bar{\psi}^c$ are the charge conjugated field, transforming according to

$$\psi^c = \mathcal{C}\bar{\psi}^T \quad , \quad \bar{\psi}^c = -\psi^T \mathcal{C}^\dagger, \tag{4.5.6}$$

we can obtain

$$\bar{\psi}_1 \Gamma \psi_2 = (\bar{\psi}_1 \Gamma \psi_2)^T = -\psi_2^T \Gamma^T \bar{\psi}_1^T = (-\psi_2^T \mathcal{C}^\dagger)(\mathcal{C}\Gamma^T \mathcal{C}^\dagger)(\mathcal{C}\bar{\psi}_1^T) = \bar{\psi}_2^c (\mathcal{C}\Gamma^T \mathcal{C}^\dagger)\psi_1^c = \bar{\psi}_2^c \Gamma^c \psi_1^c, \tag{4.5.7}$$

where we introduced $\Gamma^c = \mathcal{C}\Gamma^T \mathcal{C}^\dagger$. In this expression, the fields operator $\bar{\psi}_1$ and ψ_2 switched place and the direction within this interaction Lagrangian is reversed. Note that the coupling constants and non-fermionic field is unaffected by this argument. Using this, one can restore the continuous flow, which is called fermion flow, by replacing

$$\cdots(\bar{\psi}_{M1} \Gamma \psi_{M2})\cdots \longrightarrow \cdots(\bar{\psi}_{M2}^c \Gamma^c \psi_{M1}^c)\cdots \tag{4.5.8}$$

Also, the only contractions are between ψ and $\bar{\psi}$ since we transform the contractions $\psi\psi$ and $\bar{\psi}\bar{\psi}$ to $\psi\bar{\psi}^c$ and $\psi^c\bar{\psi}$ respectively. Additionally, using this trick, we can choose any orientation as the fermion flow. If the chosen flow is in the opposite direction of the fermion-number flow in the ordinary Feynman rules, we just need to use charge conjugation to reverse the flow. The cost is introducing charge conjugated fields in the contraction. Therefore, we must calculate the Feynman rules relating to these fields before we can state it.

We shall denote, for brevity, S_F as the propagator for its corresponding contraction. Then all of the non-vanishing contractions involving charge-conjugated fields are

$$\begin{aligned}
S_F(\psi_a^c \bar{\psi}_b^c) &= -S_F(\mathcal{C}_{ad}\bar{\psi}_d^T \psi_c^T \mathcal{C}_{cb}^\dagger) = -\mathcal{C}_{ad} S_F(\bar{\psi}_d \psi_c)\mathcal{C}_{cb}^\dagger = \mathcal{C}_{ad} S_F(\psi_c \bar{\psi}_d)\mathcal{C}_{cb}^\dagger \\
&= \mathcal{C}_{ad}\frac{i(\not{p}+m)_{cd}}{p^2-m^2+i\varepsilon}\mathcal{C}_{cb}^\dagger = \frac{i(p_\mu \mathcal{C}\gamma^{\mu T}\mathcal{C}^\dagger + m)_{ab}}{p^2-m^2+i\varepsilon} = \frac{i(-\not{p}+m)_{ab}}{p^2-m^2+i\varepsilon} \\
S_F(\psi_a \psi_b^c) &= S_F(\psi_a \mathcal{C}_{bd}\bar{\psi}_d^T) = S_F(\psi_a \bar{\psi}_d)\mathcal{C}_{bd} = \frac{i(\not{p}+m)_{ad}}{p^2-m^2+i\varepsilon}\mathcal{C}_{db}^T = \frac{i\left[(\not{p}+m)\mathcal{C}^T\right]_{ab}}{p^2-m^2+i\varepsilon}
\end{aligned}$$

[1]$\mathcal{C}$ has the properties $\mathcal{C}^\dagger = \mathcal{C}^{-1}$, $\mathcal{C}^{-1}\gamma^\mu \mathcal{C} = -\gamma^{\mu T}$ so that it leaves free Dirac field invariant under charge conjugation

$$S_F(\bar{\psi}^c_a \bar{\psi}_b) = -S_F(\psi^T_c \mathcal{C}^\dagger_{ca} \bar{\psi}_b) = -\mathcal{C}^\dagger_{ca} S_F(\psi_c \bar{\psi}_b) = -\mathcal{C}^\dagger_{ca} \frac{i(\not{p}+m)_{cb}}{p^2 - m^2 + i\varepsilon} = \frac{i\big[-\mathcal{C}^*(\not{p}+m)\big]_{ab}}{p^2 - m^2 + i\varepsilon}$$

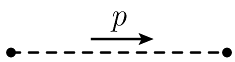
The later 2 propagators are calculated to demonstrate the claim in (4.5.3) that these propagators are not zero for Majorana fermion, where $\psi^c_M = \psi_M$. However, using the rules laid out in the last part, we will only encounter the first of these three propagators. Since we shall not encounter external fermion line in this problem, we shall not include the derivation for external charge conjugated fermion line here.

4.5.2 Feynman rules of NMSSM for new contributions from inverse seesaw mechanism

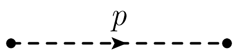
In all of the below Feynman rules, it is understood that momentum is conserved at each vertex, all undetermined momentums are to be integrated over. Loops of fermion receive an extra (-1) factor. There is a potential symmetry factor for each diagram. The momentum of every line is assumed to flow from left to right, unless explicitly shown otherwise.

In this section, only the new contribution to the Higgs sector is considered. The Feynman-'t Hooft gauge $\xi = 1$ is used. For fermion, choose and fix an arbitrary orientation as the fermion flow for every fermion chain. The Feynman rules of fermion will depends on this choice. The thick and short arrow denotes momentum flow. The thin and long arrow denotes fermion flow. Fermion is denoted by a solid line. Dirac fermion has an arrow associated with its line where Majorana fermion does not. Scalar field is denoted by dashed line with complex scalar having an arrow denoting particle flow. Vector field is presented by a wiggly line with an arrow for complex vector field. Since no decay processes is considered, the regulator $i\varepsilon$ in the denominator is set in the limit $\varepsilon \to 0$ without changing the result

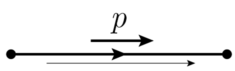
Propagator

- Real scalar: 

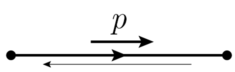
$= \dfrac{i}{p^2 - m^2}$

- Complex scalar: 

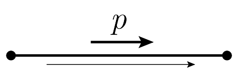
$= \dfrac{i}{p^2 - m^2}$

- Dirac fermion: 

$= \dfrac{i(\not{p}+m)}{p^2 - m^2}$

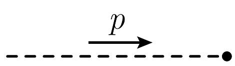


$= \dfrac{i(-\not{p}+m)}{p^2 - m^2}$

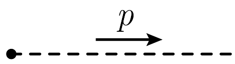
- Majorana fermion: 

$= \dfrac{i(\not{p}+m)}{p^2 - m^2}$

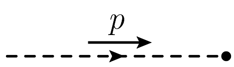
External line

- External real scalar: 

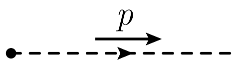
$= 1$ (incoming)



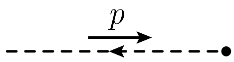
$= 1$ (outgoing)

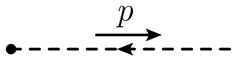
- External complex scalar:
  - Particle: 

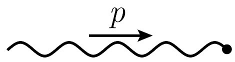
$= 1$ (incoming)

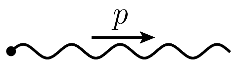


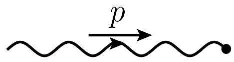
$= 1$ (outgoing)

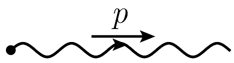
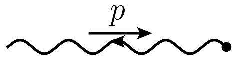
- Antiparticle:  $= 1$ (incoming)

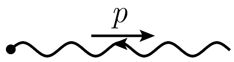
   $= 1$ (outgoing)

- External real vector:  $= \epsilon_\mu(p)$ (incoming)

   $= \epsilon^*_\mu(p)$ (outgoing)

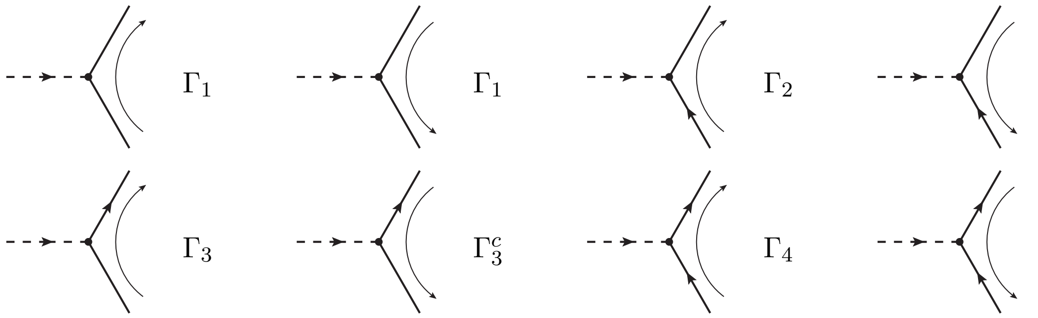
- External complex vector:
  - Particle:  $= \epsilon_\mu(p)$ (incoming)

     $= \epsilon^*_\mu(p)$ (outgoing)
  - Antiparticle:  $= \epsilon_\mu(p)$ (incoming)

     $= \epsilon^*_\mu(p)$ (outgoing)

Vertex

Since we have the freedom to choose the orientation of fermion flow and adjust the vertex accordingly. Let Γ be the gamma matrix structure of the vertex reading off from the Lagrangian in the ordinary sense, the dashed line represent either a scalar field or a vector field. There are 4 possible vertex types, each with 2 possible orientation of fermion flow. All of them and their corresponding Feynman rules are as followed (the subscript is used to distinguish between each vertex, the momentum p is assumed to flow to the right)

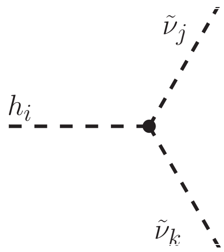


With these rules presented, in the following vertices, we shall present the rules for just one fermion line direction. The reversed one is obtained using the rule above. The vertex is calculated using

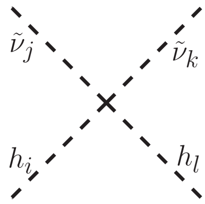
$$-i\lambda = -i \langle 0| \frac{\partial^n \mathcal{L}}{\partial\phi_i \cdots \partial\phi_j} |0\rangle \tag{4.5.9}$$

The momentum p is assumed to flow to the right, the projection operator are $\mathcal{P}_L = \dfrac{1-\gamma_5}{2}, \mathcal{P}_R = \dfrac{1+\gamma_5}{2}$. The vertices relevant to the current problem are

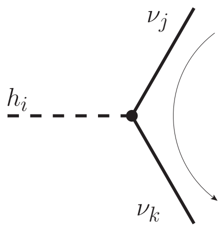
- Neutral Higgs interaction



$$-i\lambda^{h\tilde{\nu}\tilde{\nu}}_{ijk}$$

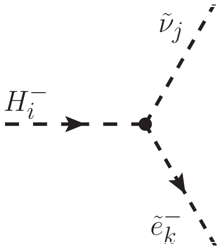


$$-i\lambda^{hh\tilde{\nu}\tilde{\nu}}_{iljk}$$

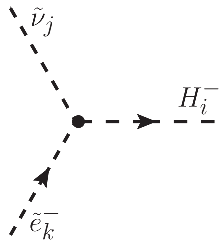


$$-i\Big(\lambda^{h\nu\nu L}_{ijk}\mathcal{P}_L + \lambda^{h\nu\nu R}_{ijk}\mathcal{P}_R\Big)$$

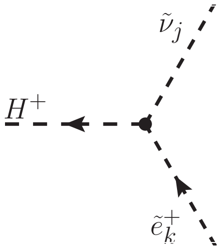
- Charged Higgs interaction



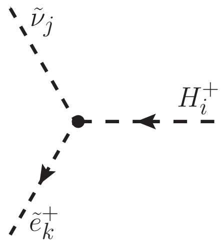
$-i\lambda_{ijk}^{H^-\tilde{\nu}\tilde{e}^+}$



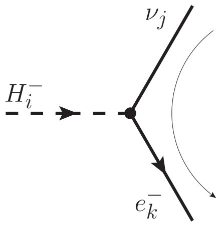
$-i\lambda_{ijk}^{H^+\tilde{\nu}\tilde{e}^-}$



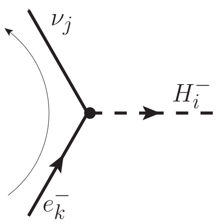
$-i\lambda_{ijk}^{H^+\tilde{\nu}\tilde{e}^-}$



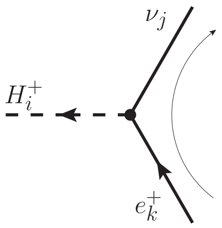
$-i\lambda_{ijk}^{H^-\tilde{\nu}\tilde{e}^+}$



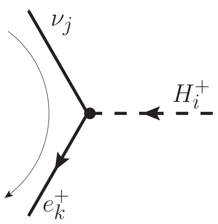
$-i(\lambda_{ijk}^{H^-\nu e^+ L}\mathcal{P}_L+\lambda_{ijk}^{H^-\nu e^+ R}\mathcal{P}_R)$



$-i(\lambda_{ijk}^{H^+\nu e^- L}\mathcal{P}_L+\lambda_{ijk}^{H^+\nu e^- R}\mathcal{P}_R)$

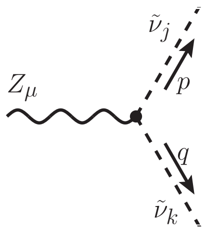


$-i(\lambda_{ijk}^{H^+\nu e^- L}\mathcal{P}_L+\lambda_{ijk}^{H^+\nu e^- R}\mathcal{P}_R)$

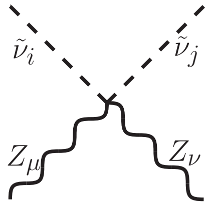


$-i(\lambda_{ijk}^{H^-\nu e^+ L}\mathcal{P}_L+\lambda_{ijk}^{H^-\nu e^+ R}\mathcal{P}_R)$

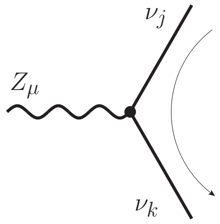
- Z-boson interaction



$-i\lambda^{Z\tilde{\nu}\tilde{\nu}}_{jk}(q_\mu - p_\mu)$

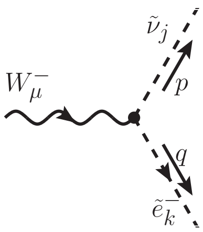


$-i\lambda^{ZZ\tilde{\nu}\tilde{\nu}}_{ij} g_{\mu\nu}$

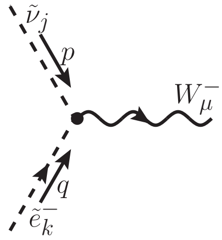


$-i\Big(\lambda^{Z\nu\nu L}_{jk}\gamma_\mu \mathcal{P}_L + \lambda^{Z\nu\nu R}_{jk}\gamma_\mu \mathcal{P}_R\Big)$

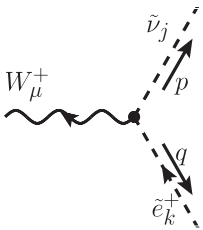
- W-boson interaction



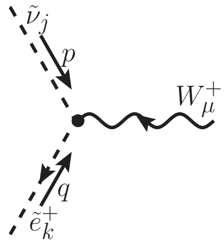
$-i\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}(q_\mu - p_\mu)$



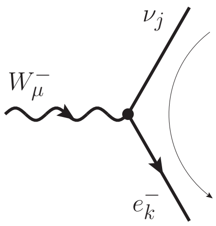
$-i\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{ijk}(-q_\mu + p_\mu)$



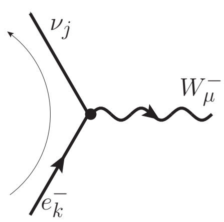
$-i\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{ijk}(q_\mu - p_\mu)$



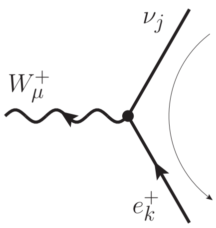
$-i\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{ijk}(-q_\mu + p_\mu)$



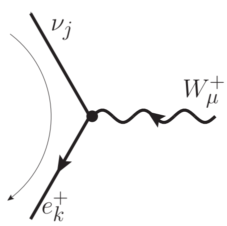
$-i\lambda^{W^-\nu e^+ L}_{ijk}\gamma_\mu \mathcal{P}_L$



$-i\lambda^{W^+\nu e^- L}_{ijk}\gamma_\mu \mathcal{P}_L$



$-i\lambda^{W^+\nu e^- L}_{ijk}\gamma_\mu \mathcal{P}_L$



$-i\lambda^{W^-\nu e^+ L}_{ijk}\gamma_\mu \mathcal{P}_L$

Chapter 5

Techniques for calculating radiative corrections at one-loop level

The calculation of the loop-corrected Higgs boson mass develops ultraviolet (UV) divergences. Thus, approach similar to the calculation at tree-level is not possible. Several treatments must be used to obtain a finite result for the parameters entering the loop calculation. This chapter concentrates on discussing the techniques used in treating the divergences encountered in the calculations of the one-loop Higgs boson mass corrections in chapter 6.

5.1 Dimensional regularization

A general Feynman diagram at one-loop order involves evaluating the integral of the kind

$$T^N_{\mu_1\dots\mu_P}(q_1,\cdots,q_{N-1},m_0,\cdots,m_{N-1}) = \int_{-\infty}^{\infty}\frac{d^4p}{(2\pi)^4}\frac{p_{\mu_1}\cdots p_{\mu_P}}{D_0 D_1\cdots D_{N-1}} \tag{5.1.1}$$

where the denominators is denoted as

$$D_0 = p^2 - m_0^2\ , \quad D_i = (p+q_i) - m_i^2\ , \quad i = 1,\cdots,N-1 \tag{5.1.2}$$

with N being the number of propagator factors in the loop, P is the tensor rank, which for renormalizable theory, $P \leq N$, and q_i are external momenta. Generally, the mass term in the denominator is $m_i^2 - i\varepsilon$, with ε related to the decay width of the particle. For simplicity of writing, we assume that m_i^2 is a complex quantity. Unfortunately, this integral diverges in the limit $p \to \infty$ if $4+P \geq 2N$, hence the name ultraviolet (UV) divergence. To work around this, a regularization scheme must be used.

Regularization is the common name for class of methods that deal with UV-divergent integrals by introducing new parameters to separate its finite and divergent part and compute each individually. For example, Pauli-Villars method [45] uses a momentum cut-off Λ or dimensional regularization [46] utilizes a dimensional parameter ϵ. These parameters are used to separate the UV-finite and UV-divergent part of the integral. After the introduction of counterterms in renormalization process, all of these UV divergences will be precisely cancelled.

The dimensional regularization (DREG) involves moving the divergent integrals from 4 to $D = 4 - 2\epsilon$ dimensional space. In such space, the integral converges, thus can be integrated. The result will be series expanded in term of ϵ. The term proportional $1/\epsilon$ is the divergent part. At one-loop level, we need to take only the $1/\epsilon$ and the term independent of ϵ. Other terms do not contribute. However, changing the dimension of space also changes the dimensionality of the momentum integrals, which usually have physical meaning. To preserves this, an arbitrary parameter μ^{4-D} with fixed dimensionality, called renormalization scale, is introduced. Conventionally, $[\mu] = 1$. Since the dimensionality of mass and momentum is unchanged, only the integral changes its dimensionality, while the integrand does not. Thus, using dimensional regularization, the integral becomes

$$\int_{-\infty}^{\infty} \frac{d^4p}{(2\pi)^4} \to \mu^{4-D} \int_{-\infty}^{\infty} \frac{d^Dp}{(2\pi)^D} \tag{5.1.3}$$

The integral (5.1.1) becomes

$$T^N_{\mu_1 \dots \mu_P}(q_1, \cdots, q_{N-1}, m_0, \cdots, m_{N-1}) = \mu^{4-D} \int_{-\infty}^{\infty} \frac{d^Dp}{(2\pi)^D} \frac{p_{\mu_1} \cdots p_{\mu_P}}{D_0 D_1 \cdots D_{N-1}} \tag{5.1.4}$$

Since the parameter ϵ is unphysical and introduced to treat the UV-divergence, after all divergences are cancelled for observables. The case for μ is more complicated and will be mentioned in section 5.3

This scheme works very well with the SM. The only complication arises from the definition of γ^5 matrix since it is intrinsically a 4-dimensional object. Different definitions for this matrix have been made to resolves the problem. For example 't Hooft and Veltman [46] proposed using the definition $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. This, however, violates the axial-vector Ward identities. In theories free of axial anomalies like the SM, the alternative definition $\{\gamma^5, \gamma^\mu\} = 0$ is preferred.

However, applying DREG the supersymmetric theories has a flaw since changing spacetime dimensions does not preserve supersymmetry [47]. Several ways have been proposed to cure this. One such way is using SUSY restoring counterterms. However, a different technique called dimensional reduction (DRED) [47] is used in this thesis. DRED is a modified version of DREG which changes only the dimensionality of momenta, space-time coordinate vectors and metric tensor, while keeping the vector fields and γ matrices in 4 dimensions. This, however, still possess some mathematical inconsistency related to the analytic continuation [48]. Later, a mathematically consistent formulation of DRED was introduced [49] where the 4-dimensional space is realized as an infinite dimensional space having the properties of 4-dimensional space, called quasi-4-dimensional. In this formulation, γ^5 can be defined in either way presented in DREG, as long as the theory is axial anomaly free. It should be noted that a general proof that DRED preserves in all cases has not existed [50]. While DRED has been checked extensively in one-loop case [51–55], the same cannot be said for higher order corrections. Thus, cautions must be taken if one is to use this regulation for higher order calculation.

5.2 One-loop integral

Since the there is a finite number of integration structures, they shall be considered individually in this section. Specifically, two types of integrations called the scalar one-point and two-point function shall be evaluated, because they are relevant to the calculation of Higgs boson mass at one-loop level. The extension of these results to include integrals with tensor structure is also discussed.

5.2.1 Scalar one-point function

The scalar one-point is usually encountered in computing tadpoles diagrams, that is those with only one external leg. The function is defined as

$$A_0(m) := -(4\pi)^2 i \int \frac{d^Dp}{(2\pi)^D} \frac{1}{p^2 - m^2} \mu^{4-D} \tag{5.2.1}$$

Obviously, evaluating this integral in Minkowski space is more challenging than doing it in Euclidean space. Exploiting this fact, the technique called Wick rotation redefine the integration variable so that the new integral would be in Euclidean space. That is, it defines the vector variable l_E such that

$$p^0 = il_E^0 \ , \quad \vec{p} = \vec{l}_E \tag{5.2.2}$$

While the variable p^0 is rotated counter-clockwise by 90 deg in the complex plane, due to the locations of the poles in the complex plane and the behaviour of the integrand, the new variable l_E^0 is still integrated from $-\infty$ to ∞. The integration now become

$$A_0(m) = -(4\pi)^2 \int \frac{d^D l_E}{(2\pi)^D} \frac{1}{l_E^2 + m^2} \mu^{4-D} \tag{5.2.3}$$

In this Euclidean space, the integration can now be performed in spherical coordinate like usual, but in D-dimension

$$\int d^D l_E = \int d\Omega_D \int_0^\infty l_E^{D-1} dl_E \tag{5.2.4}$$

where the solid angle integration is determined using the trick

$$\begin{aligned}(\sqrt{\pi})^D &= \left(\int_{-\infty}^\infty dx e^{-x^2}\right)^D = \int d^D x \exp\left(-\sum_{i=1}^D x_i^2\right) = \int d\Omega_D \int_0^\infty dr r^{D-1} e^{-r^2} \\ &= \left(\int d\Omega_D\right) \frac{1}{2} \int_0^\infty d(x^2)(x^2)^{D/2-1} e^{-x^2} = \left(\int d\Omega_D\right) \frac{1}{2}\Gamma(D/2) \end{aligned} \tag{5.2.5}$$

$$\Longrightarrow \int d\Omega_D = \frac{2\pi^{D/2}}{\Gamma(D/2)} \tag{5.2.6}$$

Thus,

$$\int d^D l_E = \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty l_E^{D-1} dl_E \tag{5.2.7}$$

And

$$\begin{aligned} A_0(m) &= -(4\pi)^2 \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty \frac{l_E^{D-1} dl_E}{(2\pi)^D} \frac{1}{l_E^2 + m^2} \mu^{4-D} \\ &= -(4\pi)^2 \frac{2}{\Gamma(D/2)(4\pi)^{D/2}} \frac{1}{2} \int_0^\infty \frac{d(l_E^2)}{l_E^2 + m^2} (l_E^2)^{D/2-1} \mu^{4-D} \end{aligned}$$

Let $x = \dfrac{m^2}{l_E^2 + m^2}$ or $l_E^2 = \dfrac{m^2}{x} - m^2$

$$\begin{aligned} A_0(m) &= -8\pi^2 \frac{2}{\Gamma(D/2)(4\pi)^{D/2}} \int_0^1 \frac{dx}{m^2/x} \frac{m^2}{x^2} \left(\frac{m^2}{x} - m^2\right)^{D/2-1} \mu^{4-D} \\ &= -8\pi^2 \frac{2(4\pi)^{-2}}{\Gamma(D/2)(4\pi)^{(D-4)/2}} (m^2)^{D/2-1} \int_0^1 dx \frac{1}{x} \left(\frac{1-x}{x}\right)^{D/2-1} \mu^{4-D} \\ &= -8\pi^2 \frac{2(4\pi)^{-2}}{\Gamma(D/2)(4\pi)^{(D-4)/2}} (m^2)^{D/2-1} \int_0^1 dx\, x^{(1-D/2)-1} (1-x)^{D/2-1} \mu^{4-D} \end{aligned}$$

Using the identity

$$\int_0^1 dx x^{\alpha-1}(1-x)^{\beta-1} = B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \tag{5.2.8}$$

The scalar function can be rewritten

$$A_0(m) = -8\pi^2 \frac{2(4\pi)^{-2}}{\Gamma(D/2)(4\pi)^{(D-4)/2}} (m^2)^{D/2-1} \frac{\Gamma(1-D/2)\Gamma(D/2)}{\Gamma(1)} \mu^{4-D}$$

Therefore, the scalar one-point function is

$$A_0(m) = (-m^2)\left(\frac{m^2}{4\pi\mu^2}\right)^{(D-4)/2} \Gamma\left(1 - \frac{D}{2}\right)$$

To obtain the finite and divergent part, this function need to be expanded about $\epsilon = (4-D)/2$. Using the expression (A.4.4) and (A.4.1), the function reads

$$\begin{aligned} A_0(m) &= (-m^2)\left(1 - \epsilon \ln \frac{m^2}{4\pi\mu^2} + \mathcal{O}(\epsilon^2)\right)\left(-\frac{1}{\epsilon} + \gamma_E - 1 + \mathcal{O}(\epsilon)\right) \\ &= m^2\left(\frac{1}{\epsilon} - \gamma_E + 1 - \ln \frac{m^2}{4\pi\mu^2}\right) + \mathcal{O}(\epsilon) \end{aligned}$$

with the UV-divergence conventionally is contained in

$$\Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi \tag{5.2.9}$$

where γ_E is the Euler-Mascheroni constant. The scalar one-point function yields

$$A_0(m) = m^2\left(\Delta - \log \frac{m^2}{\mu^2} + 1\right) + \mathcal{O}(\epsilon) \tag{5.2.10}$$

Although in principle, only the pole $1/\epsilon$ should vanish when the final, physical, and UV-finite result is obtained, the constants in Δ also disappear along with it since these terms are generated together while doing dimensional regularization. The terms of order $\mathcal{O}(\epsilon)$ is irrelevant for one-loop calculation since they vanish in the limit $\epsilon \to 0$. Though, the same cannot be stated for higher-loop calculations.

5.2.2 Scalar two-point function

Next, the scalar two-point integral shall be evaluated. This integral is usually encountered in diagrams with two external legs, hence the name. Though that is not always the case. The scalar two-point integral was defined as

$$B_0(q, m_j, m_k) := -i(4\pi)^2 \mu^{4-D} \int \frac{d^D p}{(2\pi)^D} \frac{1}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \tag{5.2.11}$$

To evaluate this integral, we first note the identity

$$\frac{1}{AB} = \int_0^1 \frac{dx}{\left[xA + (1-x)B\right]^2} = \int_0^1 dxdy \frac{\delta(x+y-1)}{\left[xA + yB\right]^2} \tag{5.2.12}$$

This identity is capable of fusing the different factors of the denominator into one, with the cost of introducing two auxiliary variables x, y. It will make the integral easier later on. The variables x, y are called Feynman parameters.

$$B_0(q, m_j, m_k) = -i(4\pi)^2 \int_0^1 dxdy \mu^{4-D} \int \frac{d^D p}{(2\pi)^D} \frac{\delta(x+y-1)}{\left\{x\left[(p+q)^2 - m_k^2\right] + y(p^2 - m_j^2)\right\}^2} \tag{5.2.13}$$

The δ function forces us to consider the case where $x+y=1$. The next task is to complete the square in the denominator. Let $\mathcal{D}$ be denoted as as

$$\begin{aligned}\mathcal{D} &= x\big[(p+q)^2 - m_k^2\big] + y(p^2 - m_j^2) = (x+y)p^2 + 2xpq + xq^2 - xm_k^2 - ym_j^2 \\ &= p^2 + 2xpq + x^2q^2 - x^2q^2 + xq^2 - xm_k^2 - ym_j^2 \\ &= (p+xq)^2 + xyq^2 - xm_k^2 - ym_j^2 \end{aligned} \tag{5.2.14}$$

The square in the denominator can now be used as the new variable. That is, let $l = p + xq$. For shortness, let $\Delta_m = -xyq^2 + xm_k^2 + ym_j^2$. The integral is simplified into

$$B_0(q, m_j, m_k) = -i(4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l^2 - \Delta_m)^2} \tag{5.2.15}$$

Next, the coordinates are Wick rotated $l^0 = il_E^0, \vec{l} = \vec{p}_E$

$$B_0(q, m_j, m_k) = -i(4\pi)^2 \int_0^1 dxdy\delta(x+y-1)i \int \frac{d^D l_E}{(2\pi)^D} \frac{1}{\left(l_E^2 + \Delta_m\right)^2} \mu^{4-D} \tag{5.2.16}$$

Similar to section 5.2.1, to evaluate this integral, spherical coordinate is used as in (5.2.7)

$$\begin{aligned} B_0(q, m_j, m_k) &= (4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \frac{2\pi^{D/2}}{\Gamma(D/2)} \int_0^\infty \frac{l_E^{D-1} dl_E}{(2\pi)^D} \frac{1}{\left(l_E^2 + \Delta_m\right)^2} \mu^{4-D} \\ &= (4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \frac{1}{\Gamma(D/2)} \int_0^\infty \frac{d(l_E^2)}{(4\pi)^{D/2}} \frac{(l_E^2)^{D/2-1}}{\left(l_E^2 + \Delta_m\right)^2} \mu^{4-D} \end{aligned} \tag{5.2.17}$$

Then, let $z = \dfrac{\Delta_m}{l_E^2 + \Delta_m}$, or $l_E^2 = \dfrac{\Delta_m}{z} - \Delta_m$,

$$\begin{aligned} B_0(q, m_j, m_k) &= (4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \frac{1}{\Gamma(D/2)} \int_0^1 \frac{dz}{(4\pi)^{D/2}} \frac{\Delta_m}{z^2} \frac{(\Delta_m/z - \Delta_m)^{D/2-1}}{(\Delta_m/z)^2} \mu^{4-D} \\ &= (4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \frac{\Delta_m^{D/2-2}}{\Gamma(D/2)} \frac{\mu^{4-D}}{(4\pi)^{D/2}} \int_0^1 dz\, z^{1-D/2}(1-z)^{D/2-1} \\ &= (4\pi)^2 \int_0^1 dxdy\delta(x+y-1) \frac{\Delta_m^{D/2-2}}{\Gamma(D/2)} \frac{\mu^{4-D}}{(4\pi)^{D/2}} \frac{\Gamma(D/2)\Gamma(2-D/2)}{\Gamma(2)} \\ &= \int_0^1 dxdy\delta(x+y-1) \frac{\mu^{4-D}\Delta_m^{D/2-2}}{(4\pi)^{D/2-2}} \Gamma(2-D/2) \\ &= \int_0^1 dxdy\delta(x+y-1) \left(\frac{\Delta_m}{4\pi\mu^2}\right)^{D/2-2} \Gamma(2-D/2) \end{aligned} \tag{5.2.18}$$

Integrating the δ-function, the function yields

$$B_0(q, m_j, m_k) = \int_0^1 dx \left(\frac{x^2q^2 + x(m_k^2 - m_j^2 - q^2) + m_j^2}{4\pi\mu^2} \right)^{D/2-2} \Gamma(2-D/2) \tag{5.2.19}$$

To obtain the finite and diverged part, this function must be expanded about $\epsilon = (4-D)/2$. Using the expression (A.4.3) and (A.4.1), the function reads

$$B_0(q, m_j, m_k) = \int_0^1 dx \left(1 - \epsilon \ln \frac{x^2q^2 + x(m_k^2 - m_j^2 - q^2) + m_j^2}{4\pi\mu^2} + \mathcal{O}(\epsilon^2) \right) \left(\frac{1}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon) \right)$$

$$= \int_0^1 dx \left(\frac{1}{\epsilon} - \gamma_E - \ln \frac{x^2q^2 + x(m_k^2 - m_j^2 - q^2) + m_j^2}{4\pi\mu^2} \right) + \mathcal{O}(\epsilon)$$

Using the variable Δ in (5.2.9), the function reads

$$B_0(q, m_j, m_k) = \Delta - \int_0^1 dx \ln \frac{x^2q^2 + x(m_k^2 - m_j^2 - q^2) + m_j^2}{\mu^2} + \mathcal{O}(\epsilon) \tag{5.2.20}$$

The integral over x can now be evaluated. If $q^2 = 0$, then there are two possible cases. If $m_j = m_k$

$$B_0(0, m_j, m_j) = \Delta - \ln \frac{m_j^2}{\mu^2} + \mathcal{O}(\epsilon) \tag{5.2.21}$$

if $m_j \neq m_k$

$$B_0(0, m_j, m_k) = \Delta + \ln \mu^2 + 1 - \frac{m_k^2 \ln m_k^2 - m_j^2 \ln m_j^2}{m_k^2 - m_j^2} + \mathcal{O}(\epsilon) \tag{5.2.22}$$

Else, let r_1, r_2 be determined from

$$x^2q^2 + x(m_k^2 - m_j^2 - q^2) + m_j^2 = q^2(x + r_1)(x + r_2) \tag{5.2.23}$$

Then

$$B_0(q, m_j, m_k) = \Delta + 2 - \ln \frac{q^2}{\mu^2} + r_1 \ln \frac{r_1}{1 + r_1} + r_2 \ln \frac{r_2}{1 + r_2} - \ln[(1 + r_1)(1 + r_2)] + \mathcal{O}(\epsilon) \tag{5.2.24}$$

5.2.3 Reduction of tensor integral to scalar integrals

The general structure for one-loop two-point integral is:

$$\mathcal{I}_{\text{1L2P}}(q, m_j, m_k) = \int \frac{d^4p}{(2\pi)^4} \frac{A + Bqp + C^{\mu\nu}p_\mu p_\nu}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \tag{5.2.25}$$

Generally, one may write the numerator as $A + B^\mu p_\mu + C^{\mu\nu}p_\mu p_\nu$. However, only one tensor structure is possible for B^μ, which is Bq^μ with B being a suitable scalar. Therefore, both forms are equivalent. Evaluating each of the three tensor structure separately is tedious. However, it is possible to calculate only the scalar integral and express the other two in term of it. That is, to evaluate this integral, we need two steps. First is to reduce the tensor integral to scalar integral. Then, we evaluate the scalar integral. The method used here is called Pasarino-Veltman reduction method [56]

Firstly, the general tensor structure in the numerator of the one-loop two-point integral can be reduce to purely scalar integrals. Consider the case of 1 propagator factor in the denominator of the following structure

$$T^1(q, m) = \frac{1}{C} \int \frac{d^Dp}{(2\pi)^D} \frac{1}{(p+q)^2 - m^2} \mu^{4-D} \tag{5.2.26}$$

$$T^1_\mu(q, m) = \int \frac{d^Dp}{(2\pi)^D} \frac{p_\mu}{(p+q)^2 - m^2 \mu^{4-D}} \tag{5.2.27}$$

with $C = i/(4\pi)^2$. Both integrals can be evaluated using a change of variable $k = p + q$. The first integral is

$$T^1(q, m) = \frac{1}{C} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m^2} \mu^{4-D} = A_0(m) \tag{5.2.28}$$

For the second integral,

$$T^1_\mu(q, m) = \frac{1}{C} \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu - q_\mu}{k^2 - m^2} \mu^{4-D} = \frac{1}{C} \int \frac{d^D k}{(2\pi)^D} \frac{k_\mu}{k^2 - m^2} \mu^{4-D} - q_\mu \frac{1}{C} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m^2} \mu^{4-D}$$

The first term vanishes, since it is odd symmetric. The integral becomes

$$T^1_\mu(q, m) = -q_\mu A_0(m) \tag{5.2.29}$$

As a special case, $T^1_\mu(0, m) = \frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{p_\mu}{p^2 - m^2} \mu^{4-D} = 0$.

For the case of 2 propagator factor, there are three structures encountered in this thesis

$$B_0(q, m_j, m_k) = \frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{1}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \mu^{4-D} \tag{5.2.30}$$

$$B_\mu(q, m_j, m_k) = \frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{p_\mu}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \mu^{4-D} \tag{5.2.31}$$

$$B_{\mu\nu}(q, m_j, m_k) = \frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{p_\mu p_\nu}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \mu^{4-D} \tag{5.2.32}$$

The first integral is a scalar one. All other tensor integrals will be expressed in term of the scalar ones $A_0(m)$ and $B_0(q, m_j, m_k)$. For the second integral (5.2.31), its result must have the form

$$B_\mu(q, m_j, m_k) = B_1 q_\mu \tag{5.2.33}$$

To find B_1, both sides are contracted with q^μ

$$B_1 q_\mu q^\mu = B_\mu(q, m_j, m_k) q^\mu = \frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{p_\mu q^\mu}{\left[(p+q)^2 - m_k^2\right](p^2 - m_j^2)} \mu^{4-D} \tag{5.2.34}$$

For short, let $D_0 = p^2 - m_j^2$ and $D_1 = (p+q)^2 - m_k^2$. Then, the numerator can be rewritten as

$$p_\mu q^\mu = \frac{1}{2}\left[(p+q)^2 - m_k^2 - \left(p^2 - m_j^2\right) + (m_k^2 - m_j^2 - q^2)\right] = \frac{1}{2}\left[D_1 - D_0 + (m_k^2 - m_j^2 - q^2)\right] \tag{5.2.35}$$

The integral, thus, become

$$\begin{aligned} B_1 q^2 &= \frac{1}{2}\frac{1}{C} \int \frac{d^D p}{(2\pi)^D} \frac{D_1 - D_0 + (m_k^2 - m_j^2 - q^2)}{D_1 D_0} \mu^{4-D} \\ &= \frac{1}{2}\left[T^1(0, m_j) - T^1(q, m_k) + (m_k^2 - m_j^2 - q^2) B_0(q, m_j, m_k)\right] \\ &= \frac{1}{2}\left[A_0(m_j) - A_0(m_k) + (m_k^2 - m_j^2 - q^2) B_0(q, m_j, m_k)\right] \end{aligned} \tag{5.2.36}$$

meaning

$$B_1 = \frac{1}{2q^2}\left[A_0(m_j) - A_0(m_k) + (m_k^2 - m_j^2 - q^2) B_0(q, m_j, m_k)\right] \tag{5.2.37}$$

and

$$B_\mu(q,m_j,m_k) = B_1 q_\mu = \frac{q_\mu}{2q^2}\left[A_0(m_j) - A_0(m_k) + (m_k^2 - m_j^2 - q^2)B_0(q,m_j,m_k)\right] \tag{5.2.38}$$

For the third integral (5.2.32), the tensor structure of the result must be

$$B_{\mu\nu}(q,m_j,m_k) = B_{00}g_{\mu\nu} + B_{11}q_\mu q_\nu \tag{5.2.39}$$

To determine B_{11} and B_{00}, we need two equations. These equations can be obtained by contracting the left-hand-side with appropriate tensor

$$B_{\mu\nu}(q,m_j,m_k)g^{\mu\nu} = DB_{00} + B_{11}q^2 \tag{5.2.40}$$

$$B_{\mu\nu}(q,m_j,m_k)q^\mu q^\nu = q^2 B_{00} + B_{11}\left(q^2\right)^2 \tag{5.2.41}$$

At the same times, the contraction on the left can be evaluated using scalar integral

$$\begin{aligned}
B_{\mu\nu}(q,m_j,m_k)g^{\mu\nu} &= \frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{p_\mu p_\nu}{D_0D_1}\mu^{4-D}g^{\mu\nu} = \frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{D_0+m_j^2}{D_0D_1}\mu^{4-D}\\
&= \frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{D_1}\mu^{4-D} + m_j^2\frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{D_0D_1}\mu^{4-D}\\
&= T^1(q,m_k) + m_j^2 B_0(q,m_j,m_k)\\
&= A_0(m_k) + m_j^2 B_0(q,m_j,m_k)
\end{aligned} \tag{5.2.42}$$

And

$$\begin{aligned}
B_{\mu\nu}(q,m_j,m_k)q^\mu q^\nu &= \frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{pq}{D_0D_1}pq\mu^{4-D}\\
&= \frac{1}{2}\frac{1}{C}\int\frac{d^Dp}{(2\pi)^D}\frac{D_1-D_0+(m_k^2-m_j^2-q^2)}{D_0D_1}pq\mu^{4-D}\\
&= \frac{1}{2}\frac{1}{C}q^\mu\int\frac{d^Dp}{(2\pi)^D}\left[\frac{p_\mu}{D_0}-\frac{p_\mu}{D_1}+(m_k^2-m_j^2-q^2)\frac{p_\mu}{D_0D_1}\right]\mu^{4-D}\\
&= \frac{1}{2}q^\mu\left[T^1_\mu(0,m_j) - T^1_\mu(q,m_k) + (m_k^2-m_j^2-q^2)B_\mu(q,m_j,m_k)\right]\\
&= \frac{1}{2}\Big\{q^2A_0(m_k) + \frac{1}{2}(m_k^2-m_j^2-q^2)\\
&\quad\times\left[A_0(m_j) - A_0(m_k) + (m_k^2-m_j^2-q^2)B_0(q,m_j,m_k)\right]\Big\}\\
&= \frac{1}{4}(m_k^2-m_j^2-q^2)A_0(m_j) + \frac{1}{4}(-m_k^2+m_j^2+3q^2)A_0(m_k)\\
&\quad + \frac{1}{4}(m_k^2-m_j^2-q^2)^2B_0(q,m_j,m_k)
\end{aligned} \tag{5.2.43}$$

Substitute (5.2.42) and (5.2.43) into (5.2.40) and (5.2.41)

$$\begin{aligned}
DB_{00} + B_{11}q^2 &= A_0(m_k) + m_j^2B_0(q,m_j,m_k)\\
q^2B_{00} + B_{11}\left(q^2\right)^2 &= \frac{1}{4}(m_k^2-m_j^2-q^2)A_0(m_j) + \frac{1}{4}(-m_k^2+m_j^2+3q^2)A_0(m_k)\\
&\quad + \frac{1}{4}(m_k^2-m_j^2-q^2)^2B_0(q,m_j,m_k)
\end{aligned}$$

The solution to this system of equations is

$$B_{00} = \frac{1}{4(D-1)q^2}\left\{(m_k^2-m_j^2+q^2)A_0(m_k) + (-m_k^2+m_j^2+q^2)A_0(m_j)\right.$$

$$+\big[4q^2m_j^2-(m_k^2-m_j^2-q^2)^2\big]B_0(q,m_j,m_k)\big\} \tag{5.2.44}$$

$$B_{11}=\frac{1}{4(D-1)(q^2)^2}\big\{D(m_k^2-m_j^2-q^2)A_0(m_j)+(-Dm_k^2+Dm_j^2+(3D-4)q^2)A_0(m_k)$$
$$+\big[D(m_k^2-m_j^2-q^2)^2-4q^2m_j^2\big]B_0(q,m_j,m_k)\big\} \tag{5.2.45}$$

Thus

$$B_{\mu\nu}(q,m_j,m_k)= \tag{5.2.46}$$
$$\frac{g_{\mu\nu}}{4(D-1)q^2}\big\{(m_k^2-m_j^2+q^2)A_0(m_k)+(-m_k^2+m_j^2+q^2)A_0(m_j)$$
$$+\big[4q^2m_j^2-(m_k^2-m_j^2-q^2)^2\big]B_0(q,m_j,m_k)\big\}$$
$$+\frac{q_\mu q_\nu}{4(D-1)(q^2)^2}\big\{D(m_k^2-m_j^2-q^2)A_0(m_j)+(-Dm_k^2+Dm_j^2+(3D-4)q^2)A_0(m_k)$$
$$+\big[D(m_k^2-m_j^2-q^2)^2-4q^2m_j^2\big]B_0(q,m_j,m_k)\big\} \tag{5.2.47}$$

It is important to point out that the derivation here cannot be used in the case $q^2=0$. However, that is suffice for our present purpose. For convenience, the following shall be precomputed.

$$g^{\mu\nu}B_{\mu\nu}(q,m_j,m_k)=A_0(m_k)+m_j^2B_0(q,m_j,m_k) \tag{5.2.48}$$

$$q^\mu q^\nu B_{\mu\nu}(q,m_j,m_k)=\frac{1}{4}\big[(m_k^2-m_j^2-q^2)A_0(m_j)+(-m_k^2+m_j^2+3q^2)A_0(m_k)$$
$$+(m_k^2-m_j^2-q^2)^2B_0(q,m_j,m_k)\big] \tag{5.2.49}$$

5.3 Renormalization

Although regularization schemes can be used as a workaround to compute the loop integrals, they cannot eliminate any divergences. One need to resolve to renormalization theory to complete this task. The theory is based on the idea that the quantities appearing in the Lagrangian are unphysical, called bare parameters, and all the divergences are absorbed into these quantities , leaving only the physical parameters behind. Although there may be other ways to remove divergences, counterterm formalism is most widely used due to its systematic approach.

In this formalism first one chooses a set of independent parameters. Other quantities are calculated based on this set. According to renormalization theory, a bare parameter, denoted as p^0, are split into a UV-finite renormalized part p and a counterterm δp containing the divergence. That is

$$p_0=p+\delta p \tag{5.3.1}$$

For each order of correction, one counterterm is introduced. That is $\delta p=\delta^{(1)}p+\delta^{(2)}(p)+\cdots$, where $\delta^{(n)}$ is the contributions from the n-th order of correction. It is conventional, however, to define the counterterms of the couplings g, and thus electric charge, as

$$g_0=Z_g g=(1+\delta Z_g)g\ ,\quad e_0=Z_e e=(1+\delta Z_e)e \tag{5.3.2}$$

However, not only the parameters, but the fields also need to be renormalized to make the self-energies UV-finite. Commonly, the renormalization of the fields is introduced as

$$\Phi_0=\sqrt{Z_\Phi}\Phi \tag{5.3.3}$$
$$=\sqrt{1+\delta^{(1)}Z_\Phi+\delta^{(2)}Z_\Phi+\cdots}\Phi \tag{5.3.4}$$
$$=\left(1+\frac{1}{2}\delta^{(1)}Z_\Phi-\frac{1}{8}(\delta^{(1)}Z_\Phi)^2+\frac{1}{2}\delta^{(2)}Z_\Phi+\cdots\right)\Phi \tag{5.3.5}$$

Inserting these decompositions into the Lagrangian also splits the bare Lagrangian into finite renormalized part and a counterterm part containing the divergences

$$\mathcal{L}_0 = \mathcal{L} + \delta\mathcal{L} \tag{5.3.6}$$

Note that although any physical quantities has its own counterterm, these counterterms are calculated based only a set of independent parameters and their counterterms.

The problem arises, however, as one still have a freedom in the finite contributions of the counterterm. Thus, one has to make a set of rules to fix how much is absorbed into the counterterm, and what is left for renormalized parameter. This set of rules is called renormalization conditions and is what determine the renormalization scheme. When fixed, a new set of Feynman rules involving the counterterms can be derived. These rules contributes to the counterterm diagrams. If the renormalization procedure is done correctly, the UV-divergences of the counterterm diagrams will precisely cancel those from loop diagrams, yielding finite physical observables.

There are two popular renormalization schemes used. The differences of these schemes lie in the the finite part of the counterterm and the renormalized quantity.

- On-shell (OS) scheme: the on-shell conditions are applied so that the renormalized quantity is equivalent to the physical one for each and every order in the perturbation theory. For examples, tadpole parameters, motivated from their zero at tree-level, are defined so that total contributions from all diagrams, including counterterms diagrams, yield vanishing renormalized tadpole parameters. Renormalization for mass is motivated from the fact that

  $$\text{propagator} \sim \frac{1}{p^2 - m^2 + i\varepsilon}$$

  Thus, the mass counterterm is defined so that the leftover finite renormalized quantity is the real part of the pole of the propagator, or the physical mass. This scheme is useful when a clear physical description of a parameter can be given, such as mass or tadpoles. If this is not the case, other renormalization scheme may be helpful.

- MS/$\overline{\text{MS}}$ or DR/$\overline{\text{DR}}$: the MS renormalization condition dictates that strictly only the divergent parts, terms proportional to $1/\epsilon$, is allowed in the counterterm. All finite parts are absorbed inside the renormalized quantity. Since this is the simplest condition, it is called Minimal Subtraction (MS). On the other hand, the $\overline{\text{MS}}$ absorbs every terms proportional to the variable $\Delta = 1/\epsilon - \gamma_E + \ln 4\pi$ into the counterterms, because, as stated above, these terms are also cancelled along with the pole. The scheme is called Modified Minimal Subtraction scheme ($\overline{\text{MS}}$). The DR, or $\overline{\text{DR}}$, scheme is essentially identical to that of MS, or $\overline{\text{MS}}$, scheme. The only difference is that the MS/$\overline{\text{MS}}$ is used in dimensional regularization, while DR/$\overline{\text{DR}}$ is used with dimensional reduction.

One important point is that if all order of perturbation is included, the final result cannot depend on the renormalization scheme anymore. However, practical calculations always stop at a specific order of correction. Thus, results may vary if one chooses different schemes for computation. The same problem applies to the renormalization scale μ. Since the parameter is only introduced as a reference scale and have no physical meaning, if all order of quantum corrections are taken into account, the physical result should be independent of μ. This is of course impractical. Hence, the dependence of computed predictions on the scale μ is a measure for the missing higher order corrections.

Chapter 6

One loop corrections to the Higgs boson mass

This chapter is devoted for the evaluation of one-loop Higgs boson mass using the techniques presented in chapter 5. The calculations in this chapter is focused to one-loop order and to the new sector in NMSSM appeared because of inverse seesaw mechanism. A mixed renormalization scheme was used that mixes on-shell and $\overline{\mathrm{DR}}$ conditions.

6.1 Counterterms and renormalization constants

The final goal of this chapter is computing the neutral Higgs mass at one-loop level. However, as anticipated, the amalgamation of all diagrams contributing to this quantity UV-diverges. Thus, the counterterm for Higgs mass matrix must be computed to cancel these divergences. The counterterm for Higgs mass matrix in the mass basis

$$\boldsymbol{M}_{hh0} = \boldsymbol{M}_{hh} + \delta \boldsymbol{M}_{hh} \tag{6.1.1}$$

However, the bare mass matrix appearing in the bare Lagrangian is not an input parameter, but rather, it is computed from a set of independent bare parameter. Thus, to obtain the counterterm for mass matrix, the counterterms of independent parameters must be computed first. The following set is chosen as the set of independent input parameter for computation of the Higgs sector

$$\left\{t_{h_d}, t_{h_u}, t_{h_s}, t_{a_d}, t_{a_s}, M_W^2, M_Z^2, M_{H^\pm}^2, v, \tan\beta, v_s, |\lambda|, |\kappa|, \Re A_\kappa, \varphi_\lambda, \varphi_\kappa, \varphi_u, \varphi_s\right\} \tag{6.1.2}$$

The renormalization and counterterms of these parameters are defined as follow

$$\begin{aligned}
t_{\phi 0} &= t_\phi + \delta t_\phi \text{ with } \phi = \{h_d, h_u, h_s, a_d, a_u\} && (6.1.3)\\
M_{W0}^2 &= M_W^2 + \delta M_W^2 && (6.1.4)\\
M_{Z0}^2 &= M_Z^2 + \delta M_Z^2 && (6.1.5)\\
M_{H^\pm 0}^2 &= M_{H^\pm}^2 + \delta M_{H^\pm}^2 && (6.1.6)\\
v_0 &= v + \delta v && (6.1.7)
\end{aligned}$$

These parameters will be renormalized on-shell. Although not part of chosen set of input, the renormalization of electric charge is required for on-shell renormalization of v. Conventionally, that is

$$e_0 = (1 + \delta Z_e)e. \tag{6.1.8}$$

The following will be imposed with $\overline{\mathrm{DR}}$ conditions

$$\tan\beta_0 = \tan\beta + \delta\tan\beta \tag{6.1.9}$$
$$v_{s0} = v_s + \delta v_s \tag{6.1.10}$$
$$|\lambda|_0 = |\lambda| + \delta|\lambda| \tag{6.1.11}$$
$$|\kappa|_0 = |\kappa| + \delta|\kappa| \tag{6.1.12}$$
$$\Re A_{\kappa 0} = \Re A_\kappa + \delta\Re A_\kappa \tag{6.1.13}$$
$$\phi_{p0} = \phi_p + \delta\phi \text{ with } p = \{u, s, \lambda, \kappa\} \tag{6.1.14}$$

Inserting the replacements from equation (6.1.3)-(6.1.14) into the bare Lagrangian and keeping terms linear to the counterterm, meaning quadratic terms such as $\delta|\kappa|\delta|\lambda|$ are not accounted for first order correction of the Higgs mass, the Higgs mass matrix counterterm can be obtained. In other words, expanding the Higgs mass matrix at tree level (3.2.33) around the counterterms to first order, the Higgs mass matrix can be obtained. The analytical expression is presented in B. Thus, to compute the counterterm for Higgs mass matrix to yield UV-finite Higgs mass, the counterterms (6.1.3)-(6.1.14) must be computed first. In order to do that, the Higgs field also need to be renormalized. Let the Higgs field be renormalized as

$$H_{u0} = \left(1 + \frac{1}{2}\delta Z_{H_u}\right)H_u\ , \quad H_{d0} = \left(1 + \frac{1}{2}\delta Z_{H_d}\right)H_d\ , \quad S_0 = \left(1 + \frac{1}{2}\delta S\right)S \tag{6.1.15}$$

Hence the renormalization for the field $\phi = (h_d, h_u, h_s, a_d, a_u, a_s)^T$ is

$$\phi_0 = \mathrm{diag}\left(1 + \frac{1}{2}\delta Z_{H_d}, 1 + \frac{1}{2}\delta Z_{H_u}, 1 + \frac{1}{2}\delta Z_S, 1 + \frac{1}{2}\delta Z_{H_d}, 1 + \frac{1}{2}\delta Z_{H_u}, 1 + \frac{1}{2}\delta Z_S\right)\phi$$

On the other hand, by definition, the renormalization constant is

$$\phi_0 = \left(1 + \frac{1}{2}\delta\boldsymbol{Z}_\phi\right)\phi \tag{6.1.16}$$

Therefore

$$\boldsymbol{Z}_\phi = \mathrm{diag}(\delta Z_{H_d}, \delta Z_{H_u}, \delta Z_S, \delta Z_{H_d}, \delta Z_{H_u}, \delta Z_S) \tag{6.1.17}$$

However, since it will be more convenient working in the basis $\Phi = (h_d, h_u, h_s, a, a_s, G)^T$, these field renormalization will be rotated to the new basis. In this new basis,

$$\Phi_0 = \boldsymbol{R}^G\phi_0\ , \quad \Phi = \boldsymbol{R}^G\phi \quad \text{and} \quad \Phi_0 = \left(1 + \frac{1}{2}\delta\boldsymbol{Z}\right)\Phi \tag{6.1.18}$$

and

$$\boldsymbol{R}^G\left(1 + \frac{1}{2}\delta\boldsymbol{Z}_\phi\right)\phi = \boldsymbol{R}^G\phi_0 = \Phi_0 = \left(1 + \frac{1}{2}\delta\boldsymbol{Z}\right)\Phi = \left(1 + \frac{1}{2}\delta\boldsymbol{Z}\right)\boldsymbol{R}^G\phi \tag{6.1.19}$$

Thus

$$\delta\boldsymbol{Z} = \boldsymbol{R}^G\delta\boldsymbol{Z}_\phi\left(\boldsymbol{R}^G\right)^T = \begin{pmatrix} \delta Z_{H_d} & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta Z_{H_u} & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta Z_S & 0 & 0 & 0 \\ 0 & 0 & 0 & s_\beta^2\delta Z_{H_d} + c_\beta^2\delta Z_{H_u} & 0 & (\delta Z_{H_d} - \delta Z_{H_u})c_\beta s_\beta \\ 0 & 0 & 0 & 0 & \delta Z_S & 0 \\ 0 & 0 & 0 & (\delta Z_{H_d} - \delta Z_{H_u})c_\beta s_\beta & 0 & c_\beta^2\delta Z_{H_d} + s_\beta^2\delta Z_{H_u} \end{pmatrix} \tag{6.1.20}$$

6.2 Tadpole and self-energies diagrams

For the purpose of setting renormalization conditions for counterterms, it is important to compute all the diagrams contributing to a quantity. The diagrams contributing to tadpole parameters are called tadpole diagrams. Those contributing to the mass parameters are called self-energy diagrams. Here, only contributions different than that in the NMSSM without inverse seesaw are computed. Conventionally, all of the diagrams are multiplied with an i

6.2.1 Tadpole diagrams

Figure 6.1: Neutrino and sneutrino contribution to Higgs tadpole diagrams

(a) Sneutrino contribution (b) Neutrino contribution

Sneutrino contribution

For the sneutrino contribution to Higgs tadpole in figure 6.1a, the amplitude is

$$\mathcal{T}_i^{\tilde{\nu}} = (-i)\frac{1}{2}\sum_j \int \frac{d^D p}{(2\pi)^D} \frac{i}{p^2 - m_j^2} (-i)\lambda_{ijj}^{h\tilde{\nu}\tilde{\nu}} \mu^{4-D} \tag{6.2.1}$$

The $\frac{1}{2}$ factor in front is due to the symmetry of the diagram. Using the result in section 5.2.1, the result of this integral is

$$\mathcal{T}_i^{\tilde{\nu}} = \frac{1}{2}\sum_j \frac{\lambda_{ijj}^{h\tilde{\nu}\tilde{\nu}}}{(4\pi)^2} A_0(m_j) \tag{6.2.2}$$

6.2.2 Neutrino contribution

For the neutrino contribution in figure 6.1b, note a minus sign for fermion loop and a $\frac{1}{2}$ factor for the symmetry of the diagram, the amplitude can be written as

$$\begin{aligned}
\mathcal{T}_i^{\nu} &= -(-i)\frac{1}{2}\sum_j \int \frac{d^D p}{(2\pi)^D} \mathrm{Tr}\left[\frac{i(\not{p} + m_j)}{p^2 - m_j^2}(\lambda_{ijj}^{h\nu\nu L}(-i)P_L + \lambda_{ijj}^{h\nu\nu R}(-i)P_R)\right]\mu^{4-D} \\
&= i\frac{1}{2}\sum_j \int \frac{d^D p}{(2\pi)^D} \mathrm{Tr}\left[\frac{(\not{p} + m_j)}{p^2 - m_j^2}\left(\lambda_{ijj}^{h\nu\nu L}\frac{1-\gamma_5}{2} + \lambda_{ijj}^{h\nu\nu R}\frac{1+\gamma_5}{2}\right)\right]\mu^{4-D} \\
&= i\frac{1}{4}\sum_j \int \frac{d^D p}{(2\pi)^D} \left\{\mathrm{Tr}\left[\frac{(\not{p} + m_j)}{p^2 - m_j^2}\left(\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right)\right]\right. \\
&\quad \left. + \mathrm{Tr}\left[\frac{(\not{p} + m_j)\gamma_5}{p^2 - m_j^2}\left(-\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right)\right]\right\}\mu^{4-D}
\end{aligned}$$

Since $\mathrm{Tr}(\gamma_5) = \mathrm{Tr}(\gamma_\mu\gamma_5) = \mathrm{Tr}(\gamma_\mu) = 0$, the second term vanishes and

$$\mathcal{T}_i^{\nu} = i\frac{1}{2}\sum_j \int \frac{d^D p}{(2\pi)^D} \frac{4m_j}{p^2 - m_j^2}\frac{1}{2}\left(\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right)\mu^{4-D}$$

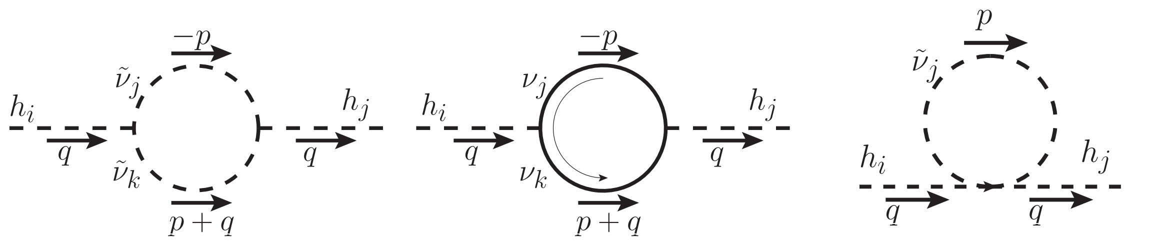
$$= i\frac{1}{2}\sum_j \int \frac{d^D p}{(2\pi)^D}\frac{1}{p^2 - m_j^2}\frac{4m_j}{2}\left(\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right)\mu^{4-D}$$

With the result in section 5.2.1, the outcome is

$$\mathcal{T}_i^{\nu} = i\frac{1}{2}\sum_j \frac{i}{(4\pi)^2} 2m_j \left(\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right) A_0(m_j) = -\sum_j \frac{1}{(4\pi)^2} m_j \left(\lambda_{ijj}^{h\nu\nu L} + \lambda_{ijj}^{h\nu\nu R}\right) A_0(m_j) \tag{6.2.3}$$

6.2.3 Neutral Higgs self-energy diagrams

The self-energy of neutral Higgs is contributed by the 3 processes in figure 6.2



(a) Higgs-sneutrino-sneutrino loop (b) Higgs-neutrino-neutrino loop (c) Higgs-Higgs-sneutrino-sneutrino loop

Figure 6.2: Neutral Higgs self-energy from neutrino and sneutrino sectors

Higgs-sneutrino-sneutrino loop

The diagram of this process is illustrated in figure 6.2a. With the index $j,k = 1,...,18$ represents the different flavour of neutrino and $i = 1,...,5$ represents different flavour of Higgs, the amplitude is

$$\begin{aligned}
\Sigma_{h_i h_j}^{h\tilde{\nu}\tilde{\nu}}(q) &= (-i)\frac{1}{2}\sum_{j,k}\int \frac{d^D p}{(2\pi)^D}(-i\lambda_{ijk}^{h\tilde{\nu}\tilde{\nu}})\frac{i}{p^2 - m_j^2}(-i\lambda_{ijk}^{h\tilde{\nu}\tilde{\nu}})\frac{i}{(p+q)^2 - m_k^2}\mu^{4-D} \\
&= (-i)\frac{1}{2}\sum_{j,k}(\lambda_{ijk}^{h\tilde{\nu}\tilde{\nu}})^2 \int \frac{d^D p}{(2\pi)^D}\frac{1}{(p^2 - m_j^2)\left[(p+q)^2 - m_k^2\right]}\mu^{4-D}
\end{aligned} \tag{6.2.4}$$

With the result from chapter 5, the result reads

$$\Sigma_{h_i h_j}^{h\tilde{\nu}\tilde{\nu}}(q) = \frac{1}{2}\sum_{j,k}(\lambda_{ijk}^{h\tilde{\nu}\tilde{\nu}})^2 \frac{1}{(4\pi)^2} B_0(q, m_j, m_k) \tag{6.2.5}$$

Higgs-neutrino-neutrino loop

With the diagram illustrated in figure 6.2b, $j,k = 1..9$ and $i = 1,...,5$, the amplitude for this contribution is

$$\begin{aligned}
\Sigma_{h_i h_j}^{h\nu\nu}(q) = -(-i)\frac{1}{2}\sum_{j,k}\int \frac{d^D p}{(2\pi)^D}\,\mathrm{Tr}\Bigg\{&(-i)\left(\lambda_{ijk}^{h\nu\nu L}\mathcal{P}_L + \lambda_{ijk}^{h\nu\nu R}\mathcal{P}_R\right)\frac{i(\not{p} + m_j)}{p^2 - m_j^2} \\
&(-i)\left(\lambda_{ijk}^{h\nu\nu L}\mathcal{P}_L + \lambda_{ijk}^{h\nu\nu R}\mathcal{P}_R\right)\frac{i(\not{p} + \not{q} + m_k)}{(p+q)^2 - m_k^2}\Bigg\}\mu^{4-D} \\
= i\frac{1}{2}\sum_{j,k}\int \frac{d^D p}{(2\pi)^D}\frac{1}{4}\,\mathrm{Tr}\Bigg\{&\left[(\lambda_{ijk}^{h\nu\nu L} + \lambda_{ijk}^{h\nu\nu R}) + \gamma_5(\lambda_{ijk}^{h\nu\nu R} - \lambda_{ijk}^{h\nu\nu L})\right]\frac{\not{p} + m_j}{p^2 - m_j^2}
\end{aligned}$$

$$\left[(\lambda_{ijk}^{h\nu\nu L}+\lambda_{ijk}^{h\nu\nu R})+\gamma_5(\lambda_{ijk}^{h\nu\nu R}-\lambda_{ijk}^{h\nu\nu L})\right]\frac{\not p+\not q+m_k}{(p+q)^2-m_k^2}\Bigg\}\mu^{4-D}$$

The trace structure is of the form

$$\begin{aligned}
&\mathrm{Tr}\{(a+\gamma_5 b)(p_\mu\gamma^\mu+m_j)(a+\gamma_5 b)(p_\nu\gamma^\nu+q_\nu\gamma^\nu+m_k)\}\\
=\;&\mathrm{Tr}\big\{a^2(p_\mu\gamma^\mu+m_j)(p_\nu\gamma^\nu+q_\nu\gamma^\nu+m_k)+\gamma_5 b^2(p_\mu\gamma^\mu+m_j)\gamma_5(p_\nu\gamma^\nu+q_\nu\gamma^\nu+m_k)\big\}\\
=\;&\mathrm{Tr}\big\{a^2p_\mu(p_\nu+q_\nu)\gamma^\mu\gamma^\nu+a^2m_jm_kI+b^2p_\mu(p_\nu+q_\nu)\gamma_5\gamma^\mu\gamma_5\gamma^\nu+b^2m_jm_k\gamma_5\gamma_5\big\}\\
=\;&a^2p_\mu(p_\nu+q_\nu)4g^{\mu\nu}+4a^2m_jm_k+4b^2m_jm_k+-\mathrm{Tr}\big\{b^2p_\mu(p_\nu+q_\nu)\gamma^\mu\gamma^\nu\big\}\\
=\;&4(a^2-b^2)p(p+q)+4(a^2+b^2)m_jm_k\\
=\;&4(a^2-b^2)p(p+q)+4(a^2+b^2)m_jm_k
\end{aligned}\tag{6.2.6}$$

Since the trace of one γ_5 with less than 4 gamma matrices vanishes, those terms are eliminated in the second line, trace of an odd number of gamma matrices was eliminated in the third line. Using this, the self-energy yields

$$\Sigma_{h_ih_j}^{h\nu\nu}(q)=i\frac{1}{2}\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\frac{\alpha_{ijk}^{h\nu\nu}g_{\mu\nu}p^\mu p^\nu+\alpha_{ijk}^{h\nu\nu}g_{\mu\nu}q^\nu p^\mu+\beta_{ijk}^{h\nu\nu}m_jm_k}{(p^2-m_j^2)\left[(p+q)^2-m_k^2\right]}\mu^{4-D}$$

where the followings are abbreviated

$$\alpha_{ijk}^{h\nu\nu}=\left[\left(\lambda_{ijk}^{h\nu\nu L}+\lambda_{ijk}^{h\nu\nu R}\right)^2-\left(\lambda_{ijk}^{h\nu\nu R}-\lambda_{ijk}^{h\nu\nu L}\right)^2\right]=4\lambda_{ijk}^{h\nu\nu L}\lambda_{ijk}^{h\nu\nu R}\tag{6.2.7}$$

$$\beta_{ijk}^{h\nu\nu}=\left(\lambda_{ijk}^{h\nu\nu L}+\lambda_{ijk}^{h\nu\nu R}\right)^2+\left(\lambda_{ijk}^{h\nu\nu R}-\lambda_{ijk}^{h\nu\nu L}\right)^2=2\left[(\lambda_{ijk}^{h\nu\nu L})^2+(\lambda_{ijk}^{h\nu\nu R})^2\right]\tag{6.2.8}$$

Evaluating the integrals yields

$$\begin{aligned}\Sigma_{h_ih_j}^{h\nu\nu}(q)=-\frac{1}{2}\sum_{j,k}\frac{1}{(4\pi)^2}&\Big[\alpha_{ijk}^{h\nu\nu}g_{\mu\nu}B^{\mu\nu}(q,m_j,m_k)+\alpha_{ijk}^{h\nu\nu}g_{\mu\nu}q^\nu B^\mu(q,m_j,m_k)\\
&+\beta_{ijk}^{h\nu\nu}m_jm_kB_0(q,m_j,m_k)\Big]\end{aligned}\tag{6.2.9}$$

In term of the scalar functions, we have

$$\begin{aligned}
&g_{\mu\nu}B^{\mu\nu}(q,m_j,m_k)+q^2B_1(q,m_j,m_k)\\
=\;&A_0(m_k)+m_j^2B_0(q,m_j,m_k)+\frac{1}{2}\left[A_0(m_j)-A_0(m_k)+(m_k^2-m_j^2-q^2)B_0(q,m_j,m_k)\right]\\
=\;&\frac{1}{2}\left[A_0(m_j)+A_0(m_k)+(m_k^2+m_j^2-q^2)B_0(q,m_j,m_k)\right]
\end{aligned}$$

The completely reduced amplitude is

$$\begin{aligned}\Sigma_{h_ih_j}^{h\nu\nu}(q)=-\frac{1}{4}\sum_{j,k}\frac{1}{(4\pi)^2}&\Big\{\alpha_{ijk}^{h\nu\nu}\left[A_0(m_j)+A_0(m_k)+(m_k^2+m_j^2-q^2)B_0(q,m_j,m_k)\right]\\
&+2\beta_{ijk}^{h\nu\nu}m_jm_kB_0(q,m_j,m_k)\Big\}\end{aligned}\tag{6.2.10}$$

Higgs-Higgs-sneutrino-sneutrino loop

With the diagram 6.2c, $j=1,...,18$ and $i=1,...,5$, the amplitude is

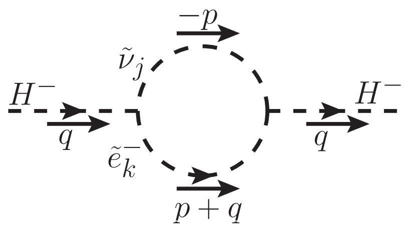
$$\Sigma_{h_ih_j}^{hh\tilde\nu\tilde\nu}(q)=(-i)\frac{1}{2}\sum_j\int\frac{d^Dp}{(2\pi)^D}(-i\lambda_{iijj}^{hh\tilde\nu\tilde\nu})\frac{i}{p^2-m_j^2+}\mu^{4-D}\tag{6.2.11}$$

With the result in section 5.2.1, the outcome is

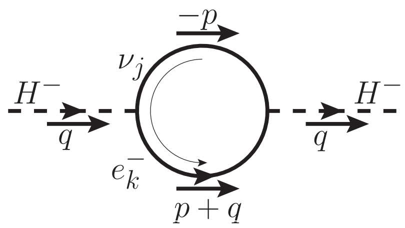
$$\Sigma_{h_ih_j}^{hh\tilde\nu\tilde\nu}(q)=(-i)\frac{1}{2}\sum_j\int\lambda_{iijj}^{hh\tilde\nu\tilde\nu}\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2-m_j^2}\mu^{4-D}=\frac{1}{2}\sum_j\lambda_{iijj}^{hh\tilde\nu\tilde\nu}\frac{1}{(4\pi)^2}A_0(m_j)\tag{6.2.12}$$

6.2.4 Charged Higgs self-energy

The self-energy of charged Higgs is contributed by the 4 diagrams in figure 6.3



(a) Charged Higgs-slepton-sneutrino loop



(b) Charged Higgs-lepton-neutrino loop

Figure 6.3: Charged Higgs self-energy from neutrino and sneutrino sectors

Charged Higgs-sneutrino-slepton loop

The diagram of this process is illustrated in figure 6.3a. The indices are $i = 1, 2$, $j = 1, ..., 18$, $k = 1, 2, 3$. The amplitude is

$$\begin{aligned}\Sigma^{\tilde{\nu}}_{H^\pm H^\mp}(q) &= (-i)\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}(-i\lambda^{H^-\tilde{\nu}\tilde{e}^+}_{ijk})\frac{i}{p^2-m_j^2}(-i\lambda^{H^+\tilde{\nu}\tilde{e}^-}_{ijk})\frac{i}{(p+q)^2-m_k^2}\mu^{4-D}\\ &= (-i)\sum_{j,k}\lambda^{H^-\tilde{\nu}\tilde{e}^+}_{ijk}\lambda^{H^+\tilde{\nu}\tilde{e}^-}_{ijk}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{(p^2-m_j^2)\big[(p+q)^2-m_k^2\big]}\mu^{4-D} \qquad (6.2.13)\end{aligned}$$

Using the result from chapter 5

$$\begin{aligned}\Sigma^{\tilde{\nu}}_{H^\pm H^\mp}(q) &= (-i)\sum_{j,k}\lambda^{H^-\tilde{\nu}\tilde{e}^+}_{ijk}\lambda^{H^+\tilde{\nu}\tilde{e}^-}_{ijk}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{(p^2-m_j^2)\big[(p+q)^2-m_k^2\big]}\mu^{4-D}\\ &= \sum_{j,k}\lambda^{H^-\tilde{\nu}\tilde{e}^+}_{ijk}\lambda^{H^+\tilde{\nu}\tilde{e}^-}_{ijk}\frac{1}{(4\pi)^2}B_0(q,m_j,m_k) \qquad (6.2.14)\end{aligned}$$

Charged Higgs-neutrino-lepton loop

With the diagram in figure 6.3b, the indices $i = 1, 2, j = 1, ..., 9, k = 1, 2, 3$, the upper sign corresponding to the self energy of minus Higgs while the lower sign being for the plus Higgs, the amplitude yields

$$\begin{aligned}\Sigma^{\nu}_{H^\pm H^\mp}(q) = -(-i)\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\Bigg\{&(-i)(\lambda^{H^-\nu e^+L}_{ijk}\mathcal{P}_L+\lambda^{H^-\nu e^+R}_{ijk}\mathcal{P}_R)\frac{i(\pm\not{p}\pm\not{q}+m_k)}{(p+q)^2-m_k^2}\\ &(-i)(\lambda^{H^+\nu e^-L}_{ijk}\mathcal{P}_L+\lambda^{H^+\nu e^-R}_{ijk}\mathcal{P}_R)\frac{i(\pm\not{p}+m_j)}{p^2-m_j^2}\Bigg\}\mu^{4-D}\\ = i\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\Bigg\{&(\lambda^{H^-\nu e^+L}_{ijk}\mathcal{P}_L+\lambda^{H^-\nu e^+R}_{ijk}\mathcal{P}_R)\frac{\pm\not{p}\pm\not{q}+m_k}{(p+q)^2-m_k^2}\\ &(\lambda^{H^+\nu e^-L}_{ijk}\mathcal{P}_L+\lambda^{H^+\nu e^-R}_{ijk}\mathcal{P}_R)\frac{\pm\not{p}+m_j}{p^2-m_j^2}\Bigg\}\mu^{4-D} \qquad (6.2.15)\\ = i\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{4}\,\mathrm{Tr}\Big\{&\Big[(\lambda^{H^-\nu e^+L}_{ijk}+\lambda^{H^-\nu e^+R}_{ijk})+\gamma_5(\lambda^{H^-\nu e^+R}_{ijk}-\lambda^{H^-\nu e^+L}_{ijk})\Big]\end{aligned}$$

$$\frac{\pm \not{p} \pm \not{q} + m_k}{(p+q)^2 - m_k^2} \left[(\lambda_{ijk}^{H^+\nu e^- L} + \lambda_{ijk}^{H^+\nu e^- R}) + \gamma_5 (\lambda_{ijk}^{H^+\nu e^- R} - \lambda_{ijk}^{H^+\nu e^- L}) \right] \frac{\pm \not{p} + m_j}{p^2 - m_j^2} \right\} \mu^{4-D}$$

The numerator of the above expression has the same matrix structure as that in equation (6.2.6). Using that result, we have

$$\Sigma^{\nu}_{H^{\pm}H^{\mp}\ i}(q) = i \sum_{j,k} \int \frac{d^4p}{(2\pi)^4} \frac{\alpha_{ijk}^{H^{\pm}\nu e^{\mp}} g_{\mu\nu} p^{\mu} p^{\nu} + \alpha_{ijk}^{H^{\pm}\nu e^{\mp}} g_{\mu\nu} p^{\mu} q^{\nu} + \beta_{ijk}^{H^{\pm}\nu e^{\mp}} m_j m_k}{(p^2 - m_j^2)\left[(p+q)^2 - m_k^2\right]} \mu^{4-D} \quad (6.2.16)$$

We abbreviated

$$\alpha_{ijk}^{H^{\pm}\nu e^{\mp}} = 2\lambda_{ijk}^{H^-\nu e^+ L} \lambda_{ijk}^{H^+\nu e^- R} + 2\lambda_{ijk}^{H^-\nu e^+ R} \lambda_{ijk}^{H^+\nu e^- L} \quad (6.2.17)$$

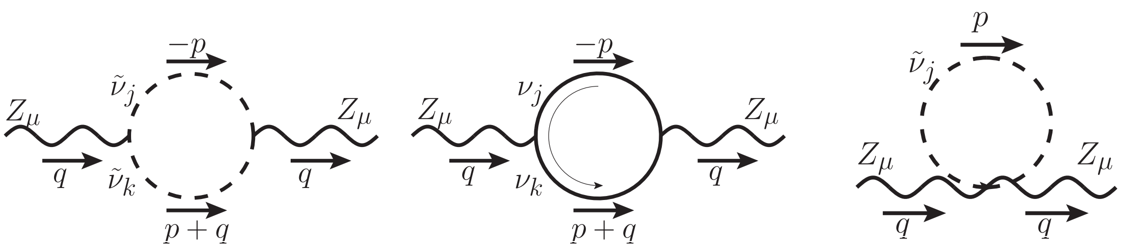
$$\beta_{ijk}^{H^{\pm}\nu e^{\mp}} = 2\lambda_{ijk}^{H^-\nu e^+ L} \lambda_{ijk}^{H^+\nu e^- L} + 2\lambda_{ijk}^{H^-\nu e^+ R} \lambda_{ijk}^{H^+\nu e^- R} \quad (6.2.18)$$

Similar to equation (6.2.10), evaluating the integrals yields

$$\begin{aligned}
\Sigma^{\nu}_{H^{\pm}H^{\mp}\ i}(q) = &-\sum_{j,k} \frac{1}{(4\pi)^2} \Big(\alpha_{ijk}^{H^{\pm}\nu e^{\mp}} g_{\mu\nu} B^{\mu\nu}(q, m_j, m_k) + \alpha_{ijk}^{H^{\pm}\nu e^{\mp}} g_{\mu\nu} q^{\nu} B^{\mu}(q, m_j, m_k) \\
&+ \beta_{ijk}^{H^{\pm}\nu e^{\mp}} m_j m_k B_0(q, m_j, m_k) \Big) \\
= &-\sum_{j,k} \frac{1}{2} \frac{1}{(4\pi)^2} \Big\{ \alpha_{ijk}^{H^{\pm}\nu e^{\mp}} \left[A_0(m_j) + A_0(m_k) + (m_k^2 + m_j^2 - q^2) B_0(q, m_j, m_k) \right] \\
&+ 2\beta_{ijk}^{H^{\pm}\nu e^{\mp}} m_j m_k B_0(q, m_j, m_k) \Big\}
\end{aligned}$$

6.2.5 Z-boson self-energy

The self-energy of Z-boson is contributed by the 3 processes in figure 6.4



(a) Z-sneutrino-sneutrino loop (b) Z-neutrino-neutrino loop (c) Z-Z-sneutrino-sneutrino loop

Figure 6.4: Neutral Higgs self-energy from neutrino and sneutrino sectors

Z-sneutrino-sneutrino loop

The diagram of this process is illustrated in figure 6.4a. With the index $j, k = 1, ..., 18$ representing the different flavour of neutrino. In considering the self-energy, we only care about the transverse amplitude. The reason will be clear in later chapters. The truncated amplitude is

$$\begin{aligned}
\Sigma_{ZZ}^{Z\tilde{\nu}\tilde{\nu}\ \mu\nu}(q) = (-i)\frac{1}{2} \sum_{j,k} \int \frac{d^Dp}{(2\pi)^D} (-i)\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}} (q^{\mu} + 2p^{\mu}) \frac{i}{p^2 - m_j^2} \\
\times (-i)\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}} (-q^{\nu} - 2p^{\nu}) \frac{i}{(p+q)^2 - m_k^2} \mu^{4-D}
\end{aligned}$$

$$= i\frac{1}{2}\sum_{j,k}\left(\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}}\right)^2\int\frac{d^Dp}{(2\pi)^D}\frac{q^\mu q^\nu+2q^\mu p^\nu+2p^\mu q^\nu+4p^\mu p^\nu}{(p^2-m_j^2)\left[(p+q)^2-m_k^2\right]}\mu^{4-D}$$

With the result from chapter 5, we arrive at

$$\begin{aligned}\Sigma_{ZZ}^{Z\tilde{\nu}\tilde{\nu}\ \mu\nu}(q) &= \frac{1}{2}\sum_{j,k}\left(\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}}\right)^2\frac{-1}{(4\pi)^2}\left[q^\mu q^\nu B_0(q,m_j,m_k)\right.\\ &\quad \left.+2q^\mu B^\nu(q,m_j,m_k)+2q^\nu B^\mu(q,m_j,m_k)+4B^{\mu\nu}(q,m_j,m_k)\right]\\ &= -\frac{1}{2}\sum_{j,k}\left(\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}}\right)^2\frac{1}{(4\pi)^2}\left[q^\mu q^\nu B_0(q,m_j,m_k)+4q^\mu q^\nu B_1(q,m_j,m_k)\right.\\ &\quad \left.+4q^\mu q^\nu B_{11}(q,m_j,m_k)+4g^{\mu\nu}B_{00}(q,m_j,m_k)\right]\end{aligned}$$

Because only the transverse amplitude is of our concerned and, since that term is of the form $g^{\mu\nu}\Gamma_T$, we shall extract the coefficient of $g^{\mu\nu}$ in the amplitude expression and omit the rest. Because $B^\mu = q^\mu B_1$ and $B^{\mu\nu} = g^{\mu\nu}B_{00} + q^\mu q^\nu B_{11}$, the transverse amplitude is

$$\Sigma_{ZZ\ T}^{Z\tilde{\nu}\tilde{\nu}}(q) = 2\sum_{j,k}\left(\lambda_{jk}^{Z\tilde{\nu}\tilde{\nu}}\right)^2\frac{1}{(4\pi)^2}B_{00}(q,m_j,m_k) \tag{6.2.19}$$

Z-neutrino-neutrino loop

With the diagram illustrated in figure 6.4b, $j,k = 1..9$, the truncated amplitude for this contribution is

$$\begin{aligned}\Sigma_{ZZ\ \mu\nu}^{Z\nu\nu}(q) &= -(-i)\frac{1}{2}\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left\{(-i)\left(\lambda_{jk}^{Z\nu\nu L}\gamma_\mu\mathcal{P}_L+\lambda_{jk}^{Z\nu\nu R}\gamma_\mu\mathcal{P}_R\right)\frac{i(\not{p}+m_j)}{p^2-m_j^2}\mu^{4-D}\right.\\ &\quad \left.(-i)\left(\lambda_{jk}^{Z\nu\nu L}\gamma_\nu\mathcal{P}_L+\lambda_{jk}^{Z\nu\nu R}\gamma_\nu\mathcal{P}_R\right)\frac{i(\not{p}+\not{q}+m_k)}{(p+q)^2-m_k^2}\right\}\\ &= i\frac{1}{2}\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left\{\gamma_\mu\left(\lambda_{jk}^{Z\nu\nu L}\mathcal{P}_L+\lambda_{jk}^{Z\nu\nu R}\mathcal{P}_R\right)\frac{\not{p}+m_j}{p^2-m_j^2}\gamma_\nu\right.\\ &\quad \left.\left(\lambda_{jk}^{Z\nu\nu L}\mathcal{P}_L+\lambda_{jk}^{Z\nu\nu R}\mathcal{P}_R\right)\frac{\not{p}+\not{q}+m_k}{(p+q)^2-m_k^2}\right\}\mu^{4-D}\\ &= i\frac{1}{2}\sum_{j,k}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{4}\,\mathrm{Tr}\left\{\gamma_\mu\left[(\lambda_{jk}^{Z\nu\nu L}+\lambda_{jk}^{Z\nu\nu R})+\gamma_5(\lambda_{jk}^{Z\nu\nu R}-\lambda_{jk}^{Z\nu\nu L})\right]\frac{\not{p}+m_j}{p^2-m_j^2}\right.\\ &\quad \left.\times\,\gamma_\nu\left[(\lambda_{jk}^{Z\nu\nu L}+\lambda_{jk}^{Z\nu\nu R})+\gamma_5(\lambda_{jk}^{Z\nu\nu R}-\lambda_{jk}^{Z\nu\nu L})\right]\frac{\not{p}+\not{q}+m_k}{(p+q)^2-m_k^2}\right\}\mu^{4-D}\end{aligned}$$

The trace structure of the above expression is of the form

$$\begin{aligned} & \mathrm{Tr}\left\{\gamma_\mu(a+\gamma_5 b)(\not{p}+m_j)\gamma_\nu(a+\gamma_5 b)(\not{p}+\not{q}+m_k)\right\}\\ =\ & \mathrm{Tr}\left\{\gamma_\mu a(\not{p}+m_j)\gamma_\nu(a+\gamma_5 b)(\not{p}+\not{q}+m_k)+\gamma_\mu b\gamma_5(\not{p}+m_j)\gamma_\nu(a+\gamma_5 b)(\not{p}+\not{q}+m_k)\right\}\\ =\ & \mathrm{Tr}\left\{a^2\gamma_\mu(\not{p}+m_j)\gamma_\nu(\not{p}+\not{q}+m_k)+ab\gamma_\mu(\not{p}+m_j)\gamma_\nu\gamma_5(\not{p}+\not{q}+m_k)\right.\\ & \left.+ab\gamma_\mu\gamma_5(\not{p}+m_j)\gamma_\nu(\not{p}+\not{q}+m_k)+b^2\gamma_\mu\gamma_5(\not{p}+m_j)\gamma_\nu\gamma_5(\not{p}+\not{q}+m_k)\right\}\\ =\ & \mathrm{Tr}\left\{a^2p^\rho(p^\sigma+q^\sigma)\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma+a^2m_jm_k\gamma_\mu\gamma_\nu+abp^\rho(p^\sigma+q^\sigma)\gamma_\mu\gamma_\rho\gamma_\nu\gamma_5\gamma_\sigma\right.\\ & \left.+abp^\rho(p^\sigma+q^\sigma)\gamma_\mu\gamma_5\gamma_\rho\gamma_\nu\gamma_\sigma+b^2p^\rho(p^\sigma+q^\sigma)\gamma_\mu\gamma_5\gamma_\rho\gamma_\nu\gamma_5\gamma_\sigma+b^2m_jm_k\gamma_\mu\gamma_5\gamma_\nu\gamma_5\right\}\\ =\ & a^2p^\rho(p^\sigma+q^\sigma)4(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\rho\sigma}+g_{\mu\sigma}g_{\rho\nu})+a^2m_jm_k4g_{\mu\nu}+abp^\rho(p^\sigma+q^\sigma)\times(-4i)\epsilon^{\sigma\mu\rho\nu}\end{aligned}$$

$$+ abp^\rho(p^\sigma + q^\sigma)(-4i)\epsilon^{\rho\nu\sigma\mu} + b^2 p^\rho(p^\sigma + q^\sigma)\,\mathrm{Tr}\{\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\sigma\} - b^2 m_j m_k\,\mathrm{Tr}\{\gamma_\mu\gamma_\nu\}$$
$$= 4(a^2+b^2)p^\rho(p^\sigma+q^\sigma)(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\rho\nu}) + 4(a^2-b^2)m_j m_k g_{\mu\nu} \tag{6.2.20}$$

Due to being contracted with a symmetric tensor, after finish integration, all antisymmetric tensors in this trace vanish. Thus, for simplification, we omit them in this calculation. Let

$$\alpha^{Z\nu\nu}_{jk} = \left(\lambda^{Z\nu\nu L}_{jk} + \lambda^{Z\nu\nu R}_{jk}\right)^2 - \left(\lambda^{Z\nu\nu R}_{jk} - \lambda^{Z\nu\nu L}_{jk}\right)^2 = 4\lambda^{Z\nu\nu L}_{jk}\lambda^{Z\nu\nu R}_{jk} \tag{6.2.21}$$
$$\beta^{Z\nu\nu}_{jk} = \left(\lambda^{Z\nu\nu L}_{jk} + \lambda^{Z\nu\nu R}_{jk}\right)^2 + \left(\lambda^{Z\nu\nu R}_{jk} - \lambda^{Z\nu\nu L}_{jk}\right)^2 = 2\left[(\lambda^{Z\nu\nu L}_{jk})^2 + (\lambda^{Z\nu\nu R}_{jk})^2\right] \tag{6.2.22}$$
$$\beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma} = \beta^{Z\nu\nu}_{jk}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\rho\nu}) \tag{6.2.23}$$

We arrive at

$$\Sigma^{Z\nu\nu}_{ZZ\ \mu\nu}(q) = i\frac{1}{2}\sum_{j,k}\int \frac{d^4p}{(2\pi)^4}\frac{\beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}p^\rho p^\sigma + \beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}p^\rho q^\sigma + \alpha^{Z\nu\nu}_{jk}m_j m_k g_{\mu\nu}}{(p^2-m_j^2)\left[(p+q)^2 - m_k^2\right]} \tag{6.2.24}$$

Using the result from chapter 5, the integral yields

$$\Sigma^{Z\nu\nu}_{ZZ\ \mu\nu}(q) = \frac{1}{2}\sum_{j,k}\frac{-1}{(4\pi)^2}\Big[\beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}B^{\rho\sigma}(q,m_j,m_k) + \beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}q^\sigma B^\rho(q,m_j,m_k)$$
$$+\alpha^{Z\nu\nu}_{jk}m_j m_k g_{\mu\nu}B_0(q,m_j m_k)\Big]$$

We have that

$$\beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}g^{\rho\sigma} = \beta^{Z\nu\nu}_{jk}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\rho\nu})g^{\rho\sigma} = \beta^{Z\nu\nu}_{jk}(2-D)g_{\mu\nu} \tag{6.2.25}$$
$$\beta'^{Z\nu\nu}_{jk\ \mu\nu\rho\sigma}q^\rho q^\sigma = \beta^{Z\nu\nu}_{jk}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\rho\nu})q^\rho q^\sigma = \beta^{Z\nu\nu}_{jk}(2q_\mu q_\nu - q^2 g_{\mu\nu}) \tag{6.2.26}$$

Therefore

$$\Sigma^{Z\nu\nu}_{ZZ\ \mu\nu}(q) = \frac{1}{2}\sum_{j,k}\frac{-1}{(4\pi)^2}\Big[\beta^{Z\nu\nu}_{jk}(2-D)g_{\mu\nu}B_{00}(q,m_j,m_k) +$$
$$\beta^{Z\nu\nu}_{jk}(2q_\mu q_\nu - q^2 g_{\mu\nu})(B_{11}(q,m_j,m_k) + B_1(q,m_j,m_k))$$
$$+\alpha^{Z\nu\nu}_{jk}m_j m_k g_{\mu\nu}B_0(q,m_j m_k)\Big] \tag{6.2.27}$$

Because we only take the transverse amplitude corresponding to the tensor structure $g_{\mu\nu}$, the transverse amplitude is

$$\Sigma^{Z\nu\nu}_{ZZ\ T}(q) = \frac{1}{2}\sum_{j,k}\frac{1}{(4\pi)^2}\Big[\beta^{Z\nu\nu}_{jk}(2-D)B_{00}(q,m_j,m_k) - q^2\beta^{Z\nu\nu}_{jk}(B_{11}(q,m_j,m_k) + B_1(q,m_j,m_k))$$
$$+\alpha^{Z\nu\nu}_{jk}m_j m_k B_0(q,m_j m_k)\Big] \tag{6.2.28}$$

Z-Z-sneutrino-sneutrino loop

With the diagram 6.4c, $j = 1, ..., 18$, the truncated amplitude is

$$\Sigma^{ZZ\tilde{\nu}\tilde{\nu}}_{ZZ}(q) = (-i)\frac{1}{2}\sum_j\int\frac{d^Dp}{(2\pi)^D}(-i\lambda^{ZZ\tilde{\nu}\tilde{\nu}}_j)g_{\mu\nu}\frac{i}{p^2-m_j^2}\mu^{4-D} \tag{6.2.29}$$

From chapter 5,, we obtain

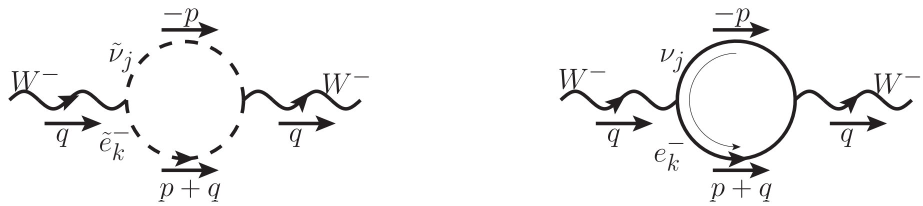
$$\Sigma^{ZZ\tilde{\nu}\tilde{\nu}}_{ZZ}(q) = (-i)\frac{1}{2}\sum_j\lambda^{ZZ\tilde{\nu}\tilde{\nu}}_j g_{\mu\nu}\int\frac{d^Dp}{(2\pi)^D}\frac{1}{p^2-m_j^2}\mu^{4-D} = \frac{1}{2}\sum_j\lambda^{ZZ\tilde{\nu}\tilde{\nu}}_j g_{\mu\nu}\frac{1}{(4\pi)^2}A_0(m_j) \tag{6.2.30}$$

Taking only the transverse amplitude

$$\Sigma^{ZZ\tilde{\nu}\tilde{\nu}}_{ZZ\ T}(q) = -\frac{1}{2}\sum_j\lambda^{ZZ\tilde{\nu}\tilde{\nu}}_j\frac{1}{(4\pi)^2}A_0(m_j) \tag{6.2.31}$$

6.2.6 W-boson self-energy

The self-energy of W-boson is contributed by the 4 diagrams in figure 6.5



(a) W-slepton-sneutrino loop (b) W-lepton-neutrino loop

Figure 6.5: W-boson self-energy from neutrino and sneutrino sectors

W-sneutrino-slepton loop

The diagram of this process is illustrated in figure 6.5a. The indices are $j = 1, ..., 18$, $k = 1, 2, 3$. The truncated amplitude is

$$\begin{aligned}
\Sigma^{\tilde{\nu}}_{W^\pm W^\mp\ \mu\nu}(q) &= (-i)\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}(-i)\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}(q_\mu + 2p_\mu)\frac{i}{p^2 - m_j^2}(-i)\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{jk}(-q_\nu - 2p_\nu) \\
&\quad \times \frac{i}{(p+q)^2 - m_k^2}\mu^{4-D} \\
&= (-i)\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{jk}\frac{-q_\mu q_\nu - 2q_\mu p_\nu - 2p_\mu q_\nu - 4p_\mu p_\nu}{(p^2 - m_j^2)\left[(p+q)^2 - m_k^2\right]}\mu^{4-D}
\end{aligned}$$

Using results from chapter 5, we arrive at

$$\begin{aligned}
\Sigma^{\tilde{\nu}}_{W^\pm W^\mp\ \mu\nu}(q) &= \sum_{j,k}\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{jk}\frac{-1}{(4\pi)^2}\left[q_\mu q_\nu B_0(q, m_j, m_k) + 2q_\nu B_\mu(q, m_j, m_k)\right. \\
&\quad \left. + 2q_\mu B_\nu(q, m_j, m_k) + 4B_{\mu\nu}(q, m_j, m_k)\right] \\
&= \sum_{j,k}\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{jk}\frac{-1}{(4\pi)^2}\left[q_\mu q_\nu B_0(q, m_j, m_k) + 4q_\nu q_\mu B_1(q, m_j, m_k)\right. \\
&\quad \left. + 4q_\mu q_\nu B_{11}(q, m_j, m_k) + 4g_{\mu\nu}B_{00}(q, m_j, m_k)\right]
\end{aligned} \tag{6.2.32}$$

Because we only take the transverse amplitude corresponding to the tensor structure $g_{\mu\nu}$, the transverse amplitude is

$$\Sigma^{\tilde{\nu}}_{W^\pm W^\mp\ T\ i}(q) = \sum_{j,k}\lambda^{W^-\tilde{\nu}\tilde{e}^+}_{jk}\lambda^{W^+\tilde{\nu}\tilde{e}^-}_{jk}\frac{4}{(4\pi)^2}B_{00}(q, m_j, m_k) \tag{6.2.33}$$

W-neutrino-lepton loop

The diagram in figure 6.5b illustrate the neutrino contribution to W-boson self energy, the indices $j = 1, ..., 9, k = 1, 2, 3$, the upper sign corresponding to the self energy of W^- while the lower sign being for the W^+. The truncated amplitude yields

$$\begin{aligned}
&\Sigma^{\nu}_{W^\pm W^\mp\ \mu\nu}(q) \\
&= -(-i)\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left\{(-i\lambda^{W^-\nu e^+ L}_{jk}\gamma^\mu\mathcal{P}_L)\frac{i(\pm\not{p} \pm \not{q} + m_k)}{(p+q)^2 - m_k^2}(-i\lambda^{W^+\nu e^- L}_{jk}\gamma^\nu\mathcal{P}_L)\right.
\end{aligned}$$

$$\times \frac{i(\pm\not{p}+m_j)}{p^2-m_j^2}\Bigg\}\mu^{4-D}$$

$$= i\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}\,\mathrm{Tr}\left\{(\lambda_{jk}^{W^-\nu e^+ L}\gamma^\mu\mathcal{P}_L)\frac{\pm\not{p}\pm\not{q}+m_k}{(p+q)^2-m_k^2}(\lambda_{jk}^{W^+\nu e^- L}\gamma^\nu\mathcal{P}_L)\frac{\pm\not{p}+m_j}{p^2-m_j^2}\right\}\mu^{4-D}$$

$$= i\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}\frac{1}{4}\lambda_{jk}^{W^-\nu e^+ L}\lambda_{jk}^{W^+\nu e^- L}$$

$$\times\,\mathrm{Tr}\left\{\gamma^\nu(1-\gamma_5)\frac{\pm\not{p}+m_j}{p^2-m_j^2}\gamma^\mu(1-\gamma_5)\frac{\pm\not{p}\pm\not{q}+m_k}{(p+q)^2-m_k^2}\right\}\mu^{4-D} \tag{6.2.34}$$

This expression has the similar trace structure as equation (6.2.20) with $a=b=1$. Using that result, we arrive at

$$\Sigma^\nu_{W^\pm W^\mp\ \mu\nu}(q) = i\sum_{j,k}\int \frac{d^Dp}{(2\pi)^D}2\lambda_{jk}^{W^-\nu e^+ L}\lambda_{jk}^{W^+\nu e^- L}\frac{\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}p^\rho p^\sigma+\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}p^\rho q^\sigma}{(p^2-m_j^2)\big[(p+q)^2-m_k^2\big]}\mu^{4-D} \tag{6.2.35}$$

Where we denoted,

$$\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma} = g_{\mu\rho}g_{\nu\sigma}-g_{\mu\nu}g_{\rho\sigma}+g_{\mu\sigma}g_{\rho\nu} \tag{6.2.36}$$

Using the result in chapter 5, we have

$$\Sigma^\nu_{W^\pm W^\mp\ \mu\nu}(q) = \sum_{j,k}2\lambda_{jk}^{W^-\nu e^+ L}\lambda_{jk}^{W^+\nu e^- L}\frac{-1}{(4\pi)^2}$$
$$\times\left[\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}B^{\rho\sigma}(q,m_j,m_k)+\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}q^\sigma B^\rho(q,m_j,m_k)\right] \tag{6.2.37}$$

We have that

$$\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}g^{\rho\sigma} = (2-D)g_{\mu\nu} \tag{6.2.38}$$

$$\beta^{W^\pm\nu e^\mp}_{jk\ \mu\nu\rho\sigma}q^\rho q^\sigma = 2q_\mu q_\nu - g_{\mu\nu}q^2 \tag{6.2.39}$$

The integration becomes

$$\Sigma^\nu_{W^\pm W^\mp\ \mu\nu}(q) = \sum_{j,k}2\lambda_{jk}^{W^-\nu e^+ L}\lambda_{jk}^{W^+\nu e^- L}\frac{-1}{(4\pi)^2}\,[(2-D)g_{\mu\nu}B_{00}(q,m_j,m_k)$$
$$+(2q_\mu q_\nu-g_{\mu\nu}q^2)(B_{11}(q,m_j,m_k)+B_1(q,m_j,m_k))\big] \tag{6.2.40}$$

The transverse amplitude

$$\Sigma^\nu_{W^\pm W^\mp\ T}(q) = \sum_{j,k}2\lambda_{jk}^{W^-\nu e^+ L}\lambda_{jk}^{W^+\nu e^- L}\frac{1}{(4\pi)^2}$$
$$\times\left[(2-D)B_{00}(q,m_j,m_k)-q^2(B_{11}(q,m_j,m_k)+B_1(q,m_j,m_k))\right] \tag{6.2.41}$$

6.3 Calculation of the counterterms and Higgs masses

6.3.1 Renormalized self-energy

With the counterterms of physical quantities and renormalization constants for field renormalization, the renormalized self-energy, derived from the Lagrangian, written in mass basis is

$$\hat{\Sigma}^{(1)}_{h_ih_j}(p^2) = \Sigma^{(1)}_{h_ih_j}(p^2)+\frac{1}{2}p^2\Big[\boldsymbol{R}\Big(\delta\boldsymbol{Z}^\dagger+\delta\boldsymbol{Z}\Big)\boldsymbol{R}^T\Big]_{ij}-\frac{1}{2}\Big[\boldsymbol{R}\Big(\delta\boldsymbol{Z}^\dagger\boldsymbol{M}_{hh}+\boldsymbol{M}_{hh}\delta\boldsymbol{Z}\Big)\boldsymbol{R}^T\Big]_{ij}$$

$$- \left[\boldsymbol{R}\delta\boldsymbol{M}_{hh}\boldsymbol{R}^T\right]_{ij} \tag{6.3.1}$$

where the $\hat{}$ is used to denote renormalized quantities. Here, $\Sigma^{(1)}_{h_ih_j}$ is the unrenormalized Higgs self-energy at one-loop level in the mass basis. Ultimately, the divergent part of the renormalized self-energy has to vanish for all external momentum p.

6.3.2 Wave-function renormalization constants

The $\overline{\text{DR}}$ condition dictates the first derivative divergent part of renormalized self-energy vanish

$$\widetilde{\text{Re}}\, \frac{\partial\hat{\Sigma}^{(1)}_{h_ih_i}}{\partial p^2}\Bigg|^{div}_{p^2=m^2_{h_i}} = 0 \tag{6.3.2}$$

with $\widetilde{\text{Re}}$ only take the real part of loop integrals. This condition leads 6.3.1 to the equation

$$\frac{1}{2}\boldsymbol{R}\Big(\delta\boldsymbol{Z}^\dagger + \delta\boldsymbol{Z}\Big)\boldsymbol{R}^T = -\widetilde{\text{Re}}\, \frac{\partial\Sigma^{(1)}}{\partial p^2}\Bigg|^{div}_{p^2=m^2_{h_i}} \Longleftrightarrow \delta\boldsymbol{Z} = -\boldsymbol{R}^T\widetilde{\text{Re}}\, \frac{\partial\Sigma^{(1)}}{\partial p^2}\Bigg|^{div}_{p^2=m^2_{h_i}}\boldsymbol{R} \tag{6.3.3}$$

where the symmetric matrix $\delta\boldsymbol{Z}$ was assumed to be real. While $\delta\boldsymbol{Z}$ contains only three unknown variables $\delta^{(1)}Z_{H_u}, \delta^{(1)}Z_{H_d}$ and $\delta^{(1)}Z_S$, this matrix equation has much more equations. Such system has no solution except in very special cases. Thus this overdetermined system of equations can be used as a non-trivial double check tool for calculation.

6.3.3 Counterterm conditions

As mentioned in section 6.1, a renormalization scheme mixed between on-shell and $\overline{\text{DR}}$ scheme is used. The on-shell condition is based on physical observables, while $\overline{\text{DR}}$ scheme are used for the parameters whose physical interpretation is not obvious

The following quantities are renormalized on-shell

- Tadpole parameters: to keep the minimum of the Higgs potential from shifting, the tadpole counterterms are defined such that it exactly cancels any contribution from the diagrams at one-loop level

  $$\delta t_\phi + \mathcal{T}_\phi = 0 \text{ with } \phi = h_d, h_u, h_s, a_d, a_s \tag{6.3.4}$$

  where $\mathcal{T}_\phi$ denotes the irreducible one-loop tadpole diagrams. Since the diagrams are computed in mass eigenbasis, to obtain the tadpole diagrams in interaction basis, a rotation must be made

  $$\mathcal{T}_{\phi_i} = (\boldsymbol{R}\boldsymbol{R}^G)^T_{ij}\mathcal{T}_{h_j} \tag{6.3.5}$$

  with T_{h_j} is in the mass basis

- Mass parameters: the condition to renormalized mass parameters is that the pole of the two-point correlation function at one-loop level occurs at exactly the physical mass. For that to occur, the counterterms must cancel all contributions from one-loop diagram computed at the physical mass. That is,

  $$\delta^{(1)}M^2_W = \widetilde{\text{Re}}\Sigma^{(1)T}_{WW}(M^2_W) \tag{6.3.6}$$
  $$\delta^{(1)}M^2_Z = \widetilde{\text{Re}}\Sigma^{(1)T}_{ZZ}(M^2_Z) \tag{6.3.7}$$

$$\delta^{(1)} M_{H^\pm}^2 = \widetilde{\mathrm{Re}}\Sigma^{(1)}_{H^\pm H^\mp}(M_{H^\pm}^2) \tag{6.3.8}$$

$$\tag{6.3.9}$$

with $\Sigma^{(1)T}$ denoting the transverse part of the respective one-loop self-energy

- VEV: the on-shell renormalization condition for VEV counterterm δv is computed from the counterterm for electric charge δe

$$\delta v = \frac{2\big[\delta M_Z^2 M_W^4 + M_Z^2\big(2\delta Z_e M_W^2(M_W^2 - M_Z^2) + \delta M_W^2(-2M_W^2 + M_Z^2)\big)\big]}{e^2 M_Z^4 v} \tag{6.3.10}$$

- Electric charge: the on-shell condition is set so that the electric charge is exactly the electron-position-photon coupling. That is, this coupling receives no quantum correction. In terms of transverse part of photon-photon and photon-Z self-energies, the condition yields [44, 57]

$$\delta Z_e = \frac{1}{2}\left.\frac{\partial \Sigma^T_{\gamma\gamma}(k^2)}{\partial k^2}\right|_{k^2=0} + \frac{s_W}{c_W}\frac{\Sigma^T_{\gamma Z}(0)}{M_Z^2} \tag{6.3.11}$$

While the $\overline{\mathrm{DR}}$ parameters are defined according to the following conditions

- $\tan\beta$: this parameter is computed via [58–60]

$$\delta^{(1)}\tan\beta = \frac{1}{2}\tan\beta\left.\left(\delta^{(1)} Z_{H_u} - \delta^{(1)} Z_{H_d}\right)\right|_{div} \tag{6.3.12}$$

- The remaining $\overline{\mathrm{DR}}$ counterterms are defined so that

$$\left.\hat{\Sigma}^{(1)}_{h_i h_j}\right|_{div} = 0 \tag{6.3.13}$$

  This introduces more equations than counterterms. Thus, the system is overdetermined. However, the suitable solution must render all components of the UV-part of renormalized self-energy UV-finite. It should be noted that these parameters can also be computed from other sectors of the NMSSM just like the case with on-shell parameters. In the end, all of these results must coincide. Thus, these different approaches are a good cross-check for the calculation.

After all counterterms are available, the renormalized self-energy can be calculated. Then, the one-loop masses can be obtained as the poles of the propagators of the Higgs boson. That is, they are from the equation

$$\det\left(\hat{\Gamma}(p^2)\right) = 0 \tag{6.3.14}$$

with $\hat{\Gamma}$ being the renormalized two-point correlation function

$$\hat{\Gamma}(p^2) = i\big(p^2\mathbf{I}_{6\times 6} - \mathcal{M}\big) \tag{6.3.15}$$

where $\mathcal{M}$ is the matrix part defined from the renormalized self-energy $\hat{\Sigma}_{ij}(p^2)$ and the tree-level Higgs mass m_{h_i}

$$\mathcal{M}_{ij} = m_{h_i}^2\delta_{ij} - \hat{\Sigma}_{ij}(p^2) \tag{6.3.16}$$

To solve this equation, iterative method was used [57, 61, 62]. Each time, only one solution to the equation can be found. For the first iteration, the external momentum squared is set equal to the chosen nth tree-level Higgs mass $p^2 = m^2_{h_n}$. In each iteration, an updated guess of the external momentum squared is used to compute the matrix part. That matrix is then diagonalized, turning the determinant equation into a trivial one with the roots being the n-th diagonal element. This value is then used for new iteration. The process is repeated until the change in p^2 between two consecutive iterations is less then 10^{-9}. The algorithm is repeated for all neutral Higgs boson mass.

Chapter 7

Numerical Analysis

This section is devoted for numerical analysis of the one-loop corrected Higgs mass in NMSSM with inverse seesaw mechanism. The couplings and mass matrices are created using the package `SARAH` [63–69] where the inverse seesaw mechanism is newly implemented in the model for NMSSM with CP violation. Then, the generated model file is implemented into `FeynArts` [70–73] and `FeynCalc` [74, 75] to generate amplitudes, manipulate gamma matrices, perform tensor reduction and then factorize the divergent and finite parts. This result is cross-check by hand. The analytical expressions are then implement into the Fortran package `NMSSMCAL` [57, 61, 62, 76–79]. The package `NMSSMCALC` was modified to incorporate the new behaviours and contributions from inverse seesaw mechanism.

7.1 Numerical parameters

When some parameters are being analysed, if no specification is stated, the other parameters are kept unchanged. The current numerical analysis does not consider the case of CP-violation; thus, CP-violating parameters are left out. During calculation, we also take into account the the constraints arise from experimental data. The use of parametrization in this thesis allows simple reproduction of neutrino oscillation data since the masses and the mixing angles can be used as input; thus, constraints on these parameters are controlled beforehand. For the sake of simplicity, the matrix parameters $\mu_X, \lambda_X, A_\nu, A_X, B_{\mu_X}, M^2_{\tilde{N}}, M^2_{\tilde{X}}$ are set to be diagonal without loosing generality.

Follow from experimental data [39], the SM parameters have been chosen as

$$
\begin{aligned}
\alpha_{em}^{-1} &= 127.955 & G_F &= 1.16637 \times 10^{-5}\ \text{GeV}^{-2} \\
\alpha_s &= 0.1181 & M_Z &= 91.1876\ \text{GeV} \\
m_b^{\overline{\text{MS}}}(m_b^{\overline{\text{MS}}}) &= 4.18\ \text{GeV} & m_t &= 172.74\ \text{GeV} \\
m_\tau &= 1.77682\ \text{GeV} & M_W &= 80.379\ \text{GeV} \\
m_e &= 510.99891\ \text{keV} & m_\mu &= 105.658367\ \text{MeV} \\
m_d &= 4.7\ \text{MeV} & m_u &= 2.2\ \text{MeV} \\
m_s &= 95\ \text{MeV} & m_c &= 1.274\ \text{GeV}
\end{aligned}
\tag{7.1.1}
$$

The extra parameters arising from NMSSM are not experimentally measured, though may have constraints on their value. Here, we take the parameter point P1OS in [61]. It satisfies the LHC Run 1 and Run 2 data on the Higgs signal strength.

$$
M_1 = 644.4699\ \text{GeV} \qquad M_2 = 585.2285\ \text{GeV}
$$

$$
\begin{aligned}
&M_3 &&= 1850 \text{ GeV} && A_t &&= -1921.717 \text{ GeV} \\
&A_b &&= -1884.847 \text{ GeV} && A_\tau &&= 1170.264 \text{ GeV} \\
&\tan\beta &&= 4.442242 && M_{H^\pm} &&= 897.8267 \text{ GeV} \\
&|\lambda| &&= 0.301175 && |\kappa| &&= 0.299105 \\
&\mathrm{Re}A_\kappa &&= -791.4436 \text{ GeV} && \mu_{eff} &&= 208.7278 \text{ GeV} \\
&M_{\tilde{L}1} = M_{\tilde{L}2} &&= 3000 \text{ GeV} && M_{\tilde{L}3} &&= 1368.968 \text{ GeV} \\
&M_{\tilde{E}1} = M_{\tilde{E}2} &&= 3000 \text{ GeV} && M_{\tilde{E}3} &&= 2967.018 \text{ GeV} \\
&M_{\tilde{Q}1} = M_{\tilde{Q}2} &&= 3000 \text{ GeV} && M_{\tilde{Q}3} &&= 1226.038 \text{ GeV} \\
&M_{\tilde{U}1} = M_{\tilde{U}2} &&= 3000 \text{ GeV} && M_{\tilde{U}3} &&= 880.8624 \text{ GeV} \\
&M_{\tilde{D}1} = M_{\tilde{D}2} &&= 3000 \text{ GeV} && M_{\tilde{D}3} &&= 2765.234 \text{ GeV}
\end{aligned}
\tag{7.1.2}
$$

While the scale parameter μ arising from regularization is conventionally set at [80, 81]

$$
\mu = M_{SUSY} = \sqrt{M_{\tilde{Q}_3} M_{\tilde{t}_R}} \tag{7.1.3}
$$

Although the exact value is unavailable yet, there are some strict constraints on the mass of neutrinos m_{n_i} and their mixing matrices [31–33]. Cosmological data also put an upper bound to the sum of neutrino masses at 0.23 eV [41]. The unitarity of the neutrino mixing matrix is also tested and restrained [82–84]. Following [85], an upper bound to the Yukawa coupling of neutrino Y_v is set to ensure the perturbativity

$$
\frac{|Y_{ij}|^2}{4\pi} < 1.5 \tag{7.1.4}
$$

Thus, the parameters are carefully picked to satisfy these constrains.

$$
\begin{aligned}
&m_{n_1} = 4.3 \times 10^{-5} \text{ eV} && m_{n_2} = 8.6 \times 10^{-3} \text{ eV} && m_{n_3} = 5.06 \times 10^{-2} \text{ eV} \\
&\theta_{12} = 0.576 && \theta_{13} = 0.147 && \theta_{23} = 0.71 \\
&\mu_{X1} = 10 \text{ TeV} && \mu_{X2} = 20 \text{ TeV} && \mu_{X3} = 30 \text{ TeV} \\
&\lambda_{X1} = 6 \times 10^{-9} && \lambda_{X2} = 7 \times 10^{-9} && \lambda_{X3} = 8 \times 10^{-9} \\
&\theta_1 = 2 && \theta_2 = 3 && \theta_3 = 4
\end{aligned}
$$

The last lines show the values for the mixing angles of Casas-Ibarra parametrization following [86]. For the soft SUSY breaking parameters for the modified neutrino sector, little can be said about their values, except that they should be dependent on their corresponding SUSY-conserving parameters. The solf breaking parameters are of order TeV and chosen as follows

$$
\begin{aligned}
&A_{\nu 1} = 1 \text{ TeV} && A_{\nu 2} = 2 \text{ TeV} && A_{\nu 3} = 3 \text{ TeV} \\
&A_X = 4 \text{ TeV} && A_X = 5 \text{ TeV} && A_X = 6 \text{ TeV} \\
&B_{\mu_X 1} = 1 \text{ TeV} && B_{\mu_X 2} = 2 \text{ TeV} && B_{\mu_X 3} = 3 \text{ TeV} \\
&M_{\tilde{X}1} = 1 \text{ TeV} && M_{\tilde{X}2} = 1.2 \text{ TeV} && M_{\tilde{X}3} = 1.3 \text{ TeV} \\
&M_{\tilde{N}1} = 3 \text{ TeV} && M_{\tilde{N}2} = 4 \text{ TeV} && M_{\tilde{N}3} = 5 \text{ TeV}
\end{aligned}
$$

7.2 Dependence of Higgs sector parameters in inverse seesaw mechanism

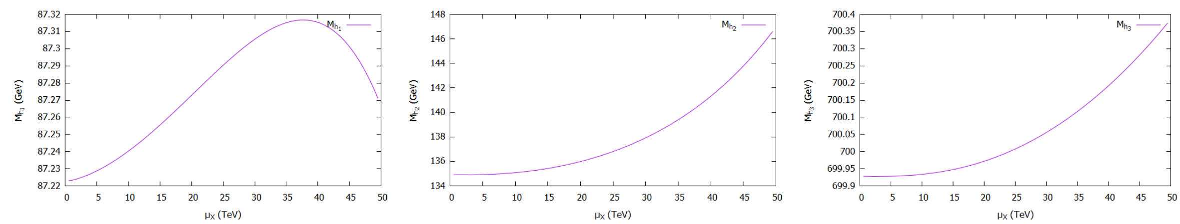
To investigate how the Higgs sector changes with the introduction of the inverse seesaw mechanism, the Higgs masses are plotted against each parameter in the neutrino sector. The idea behind this section is to find out how much a parameter contribute to the masses of the Higgs boson. The parameter range was chosen so that no data point violates experimental constrains. Here, only μ_X and the neutrino mass is considered. Other parameters have negligible effect on the mass of Higgs boson.

For illustrative purpose, the neutral Higgs masses at one-loop level are computed without inverse seesaw mechanism to compare with when the mechanism is turned on. The computed one-loop corrected Higgs masses, calculated using `NMSSMCALC`, are

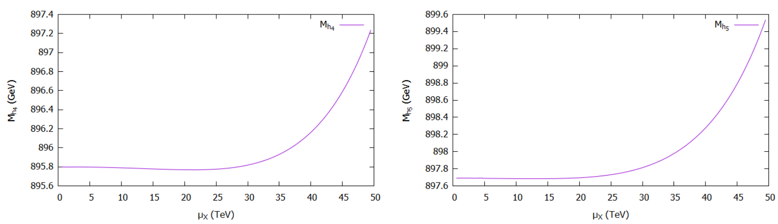
$$m_{h_1} = 86.6934 \text{ GeV} \tag{7.2.1}$$
$$m_{h_2} = 135.065 \text{ GeV} \tag{7.2.2}$$
$$m_{h_3} = 700.112 \text{ GeV} \tag{7.2.3}$$
$$m_{h_4} = 895.828 \text{ GeV} \tag{7.2.4}$$
$$m_{h_5} = 897.833 \text{ GeV} \tag{7.2.5}$$

The Higgs boson h_2 is the candidate for experimentally detected Higgs boson and it receives large negative two-loop correction of order -10 GeV which makes its corrected mass consistent with the Higgs boson mass measured at LHC.

For the sake of simplicity, the MUX matrix is diagonalized with all three diagonal elements equals. That is $\mu_X \equiv \mu_X \mathbf{I}_{3\times3}$ The graph of MUX is plotted from 0.2 TeV to 50 TeV. The lower limit is set according to [85, 87] as this region of energy is very well probed by colliders. The corresponding soft SUSY breaking bilinear coupling is set $B_{\mu_X} \equiv B_{\mu_X} \mathbf{I}_{3\times3}$ with $B_{\mu_X} = 1$ TeV. The plot of the mass of each Higgs boson is plotted in figure 7.1. Although most Higgses gain positive correction from the new sector at high value of μ_X, the lightest Higgs mass experience the opposite after reach its peak. Among them, the second lightest Higgs receive the most correction, as high as 9% compared to that of the NMSSM without inverse seesaw mechanism. Figure 7.2 is graphed to compare between the quantum corrections between each Higgs mass. It can be seen that h_2 receives the most correction from the new sector, while the others deviates very little with h_3 receives the least change.



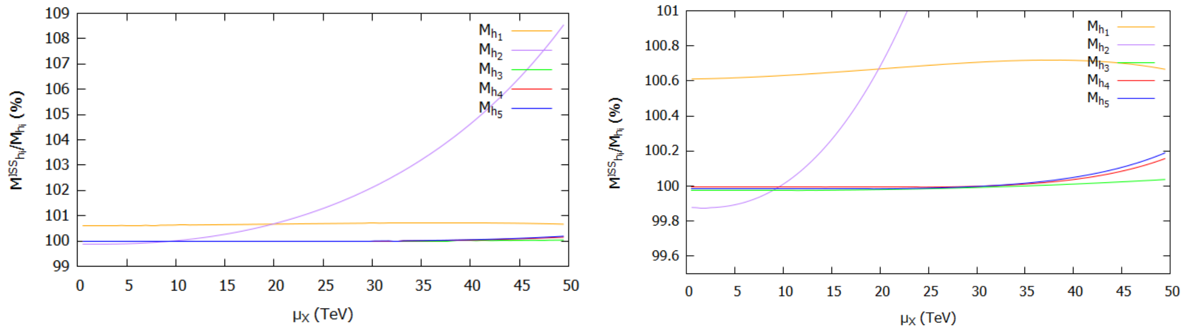
(a) Mass of h_1 against μ_X (b) Mass of h_2 against μ_X (c) Mass of h_3 against μ_X



(d) Mass of h_4 against μ_X (e) Mass of h_5 against μ_X

Figure 7.1: Neutral Higgs mass plotted against μ_X. The Higgs are sorted in ascending mass $m_{h_1} \leq \cdots \leq m_{h_5}$

Just like with μ_X, the same thing is done for m_{ν_1}, the mass of the lightest neutrino. Here, only normal hierarchy $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$ is considered. The other neutrino mass is obtained

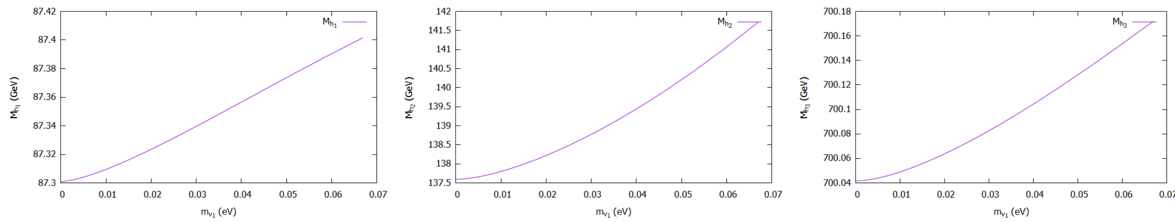


(a) An overview

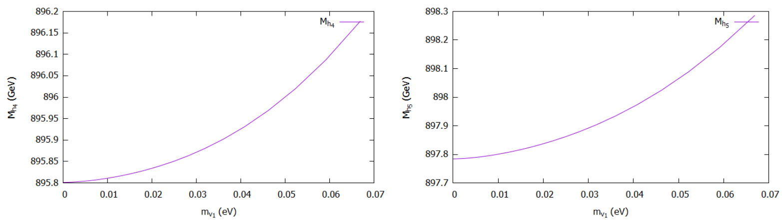
(b) Zoomed in for better structure of graph

Figure 7.2: The deviation (in %) of neutral Higgs masses from their values without inverse seesaw mechanism plotted against μ_X

from the lightest one using the best fitted experimental value [31]. The upper-bound is set to be the value at which cosmological constrain to the sum of neutrino mass is violated. The associated trilinear coupling A_ν also diagonalized with the three diagonal elements the same $A_\nu \equiv A_\nu \mathbf{I}_{3\times3}$ where $A_\nu = 1$ TeV. The result is shown in figure 7.3. The behaviour expressed by this parameter is similar to that of μ_X. However, cosmological data put an upper bound to this parameter, unlike MUX which has yet to be upper-bounded by experiment. Yet, the changes made by this parameter is as much as 5% as shown in figure 7.4



(a) Mass of h_1 against m_{ν_1} (b) Mass of h_2 against m_{ν_1} (c) Mass of h_3 against m_{ν_1}



(d) Mass of h_4 against m_{ν_1} (e) Mass of h_5 against m_{ν_1}

Figure 7.3: Neutral Higgs mass plotted against m_{ν_1}. The Higgs are sorted in ascending mass $m_{h_1} \geq \cdots \geq m_{h_5}$

7.2.1 Parameter scan for maximal change in Higgs mass

To get an idea on the scale of correction contributed by the inverse seesaw mechanism, parameter scan is used, in which all parameters belonging to the inverse seesaw sector take random value in their allowed range. The 6 parameters constrains by oscillation experiments [31–33] $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, \theta_{12}, \theta_{13}, \theta_{23}$ are allowed to take any value within the 3σ uncertainty of experiment, with the exception of m_{ν_1} which takes any value that does not violate the upper bound

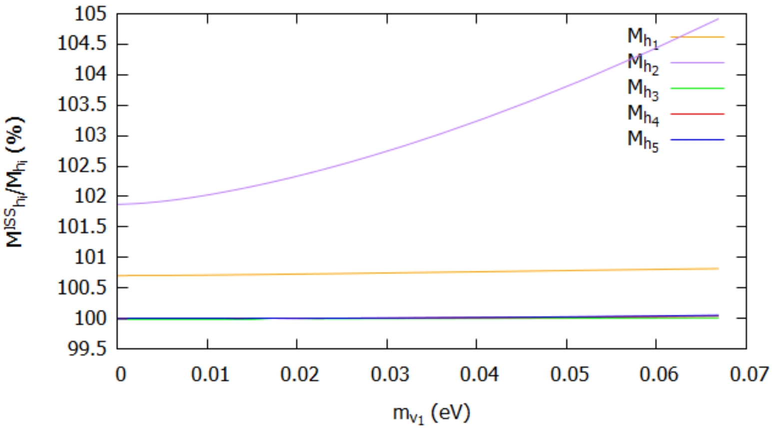


Figure 7.4: The deviation (in %) of neutral Higgs masses from their values without inverse seesaw mechanism plotted against m_{ν_1}

set by cosmological data [41]. These parameters are randomized with a uniform distribution. As before, the other parameters are diagonalized and set so that their diagonal elements are the same. These parameters are distributed in a logarithmically uniform fashion so that any energy scale is equally likely.

$$\begin{aligned}
0.2\text{ TeV} &\leq \mu_X \leq 50\text{ TeV} \\
5\times 10^{-10} &\leq \lambda_X \leq 1\times 10^{-5} \\
1\text{ GeV} &\leq A_\nu \leq 1\text{ TeV} \\
1\text{ GeV} &\leq A_X \leq 1\text{ TeV} \\
1\text{ GeV} &\leq B_{\mu_X} \leq 1\text{ TeV} \\
0.2\text{ TeV} &\leq M_{\tilde{X}} \leq 50\text{ TeV} \\
0.2\text{ TeV} &\leq M_{\tilde{N}} \leq 50\text{ TeV}
\end{aligned}$$

To get a somewhat complete picture, 2000 random points are generated, with those violating any of the mentioned experimental constrains are discarded. The deviation of the result from the one-loop Higgs boson mass without seesaw mechanism is plotted. These values are set at 100%. It can be observed from figure 7.5 that the three heaviest Higgs boson are barely changed by the new sector, although closer inspection reveals that these three masses mostly decreases with the introduction of new parameter. However, the same cannot be said for the two lighter one. The lightest Higgs boson is completely raised to new mass value, more than 0.6% the old one, and deviates about that value. The most significant change belongs to the second lightest Higgs. Although a significant number of its data points lies above the 100% line, its mass can increases as high as 9% its original mass at one-loop level due to the correction from inverse seesaw mechanism. The previous section demonstrated that μ_X is the only parameter in the neutrino sector that can have arbitrary large effect on the corrected mass of Higgs boson. This, however, may require μ_X to be much above the reach of current collider, thus, beating the advantage of inverse seesaw mechanism.

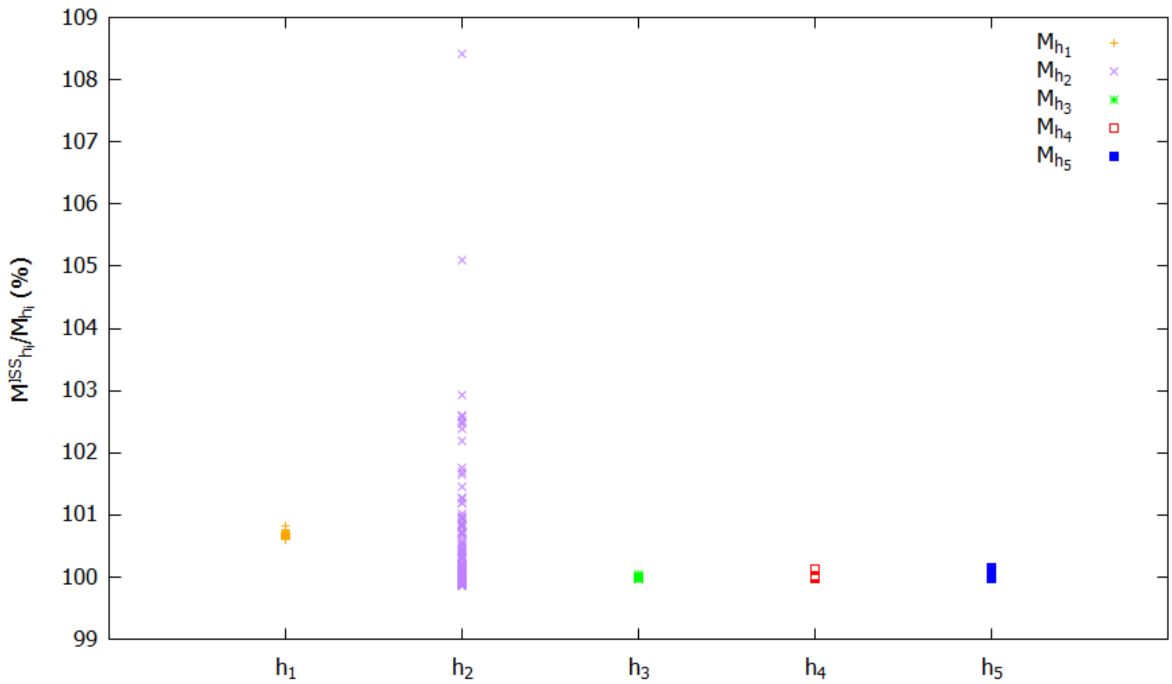


Figure 7.5: The deviation (in %) of neutral Higgs masses from their values without inverse seesaw mechanism with parameter scan

Chapter 8

Conclusion and proposal

This thesis has focused on discussing the Next-to-Minimal Supersymmetric theory (NMSSM) with the concentration on the Higgs sector and the inverse seesaw mechanism and how the two are incorporated with each other. The effect of the inverse seesaw mechanism on the Higgs mass was particularly considered. One-loop calculation was needed for such investigation.

To alleviate some of the shortcomings of the Standard Model (SM), supersymmetric theories was devised. The theory is built based on the idea that the largest spacetime symmetry is Poincaré plus a boson-fermion symmetry. The most minimum theory extended from the SM is called the Minimal supersymmetric theory (MSSM) with only one additional Higgs doublet introduced, along with the superpartners of each of the SM particles. The model has several advantages over the SM; however, it also raises its own hierarchy problem called the μ-problem. To solve this, the NMSSM was proposed where one additional Higgs singlet is introduced to solve the μ-problem by dynamically produce the parameter from the electroweak symmetry breaking (EWSB). The mass spectrum of these theories have been of special interest. While the gauge and fermion sectors behave quite similar to that of the SM, the Higgs sector experience quite dramatic change, such as the lightest Higgs receives an upper bound for its mass. Since experiments have confirmed and measured one Higgs boson, this data can be used to constrain the parameter of supersymmetric theories. For this reason, this sector get more attention throughout the thesis.

Another problem unsolved by the SM that both MSSM and NMSSM fail to explain is the non-zero, yet surprisingly tiny, mass of neutrino constrained by experiments. Seesaw mechanisms is brought in to propose an appealing solution to this discrepancy. The mechanism suggests that if neutrinos are Majorana fermion, their masses can naturally appear without breaking gauge symmetry. The mechanism also proposes that more neutrinos exist in nature than detected by experiment but these sterile neutrinos are so heavy that they render the active ones very light. Inverse seesaw is a variant of this mechanism in which the sterile neutrinos need not be significantly heavy to generate the really low mass of active neutrino by introducing lepton number violating parameters.

The introduction of new and heavy neutrinos surely influence the mass of Higgs boson. However, to investigate such changes, one-loop calculation is necessary. Therefore, the technique for such computation is also studied at length. The problem with calculating one-loop diagrams is that their integration over momentum diverges as $p \to \infty$. Dimensional regularization technique is used to workaround this issue and compute their integrals. The problem still persists, though, as the technique only hide the divergences in new parameters to allow computation, but still fail to yield UV-finite results. An additional method called renormalization theory is used to actually cancel these divergences by introducing counterterms. These counterterms contain its own divergences arriving from bare parameters. Renormalization conditions are required to compute these counterterms and arrive at the result.

Using the technique of one-loop calculation, the corrected Higgs boson mass can be calculated and and its dependence on the inverse seesaw mechanism can be analysed. In the analysis, experimental data is used to constrains the parameters of neutrino sector. Apparently, because of the massive neutrino introduced, the Higgs mass can be corrected by as much as 9% from this sector alone. Although small, the mass of light neutrino can still have noticeable effect on the Higgs mass at one-loop level.

More analysis can still be done in the future. We plan to take into account the constrains from lepton flavour violation data. Moreover, since two-loop quantum corrections is not negligible, to get an accurate bound for the Higgs mass, this perturbation level should also be considered. Also, the observed Higgs boson mass can also be used as a restriction to the parameters of the NMSSM and inverse seesaw mechanism. All of these can be a subject for future studies

Appendix A

Conventions and Formulas

A.1 Units and metric conventions

This thesis uses natural units

$$\hbar = c = 1 \tag{A.1.1}$$

and works with the metric tensor

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A.1.2}$$

Conventionally, in four-component spacetime, the Greek indices runs over $0, 1, 2, 3$, while the Roman indices i, j,etc. denotes spatial components. Repeated indices are implicitly summed over in all cases, unless specifically stated.

A.2 Spinor algebra

Pauli matrices and its extension to include the unit matrix are defined as

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad , \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.2.1}$$

Firstly, we need the definition of generalized Pauli matrix

$$\sigma_\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3) = (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3) \tag{A.2.2}$$

$$\bar{\sigma}_\mu = (\sigma_0, -\sigma_1, -\sigma_2, -\sigma_3) = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \tag{A.2.3}$$

Then, we can define

$$(\sigma^{\mu\nu})_\alpha^{\ \beta} = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu)_\alpha^{\ \beta}, \quad (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\ \dot{\beta}} = \frac{i}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)^{\dot{\alpha}}_{\ \dot{\beta}} \tag{A.2.4}$$

Some properties of the generalized Pauli matrices include

$$\mathrm{Tr}\{\sigma^{\mu\nu}\sigma^{\rho\tau}\} = \frac{1}{2}(g^{\mu\rho}g^{\nu\tau} - g^{\mu\tau}g^{\nu\rho} + i\epsilon^{\mu\nu\rho\tau}) \tag{A.2.5}$$

Since Lorentz group is homomorphic to the group $SL(2,\mathbb{C})$ composed of 2×2 complex matrices of determinant 1, we can use the representation of $SL(2,\mathbb{C})$ for our purpose. The fundamental representation of this group is defined

$$\psi'_{\alpha} = N_{\alpha}^{\beta}\psi_{\beta} \tag{A.2.6}$$

with $\alpha,\beta = 1,2$, while its complex conjugate representation is

$$\bar{\psi}'^{\dot{\alpha}} = N_{\dot{\beta}}^{\dot{\alpha}}\bar{\psi}^{\dot{\beta}} \tag{A.2.7}$$

with $\dot{\alpha},\dot{\beta} = \dot{1},\dot{2}$. It should be noted that these two representations are inequivalent. In $SL(2,\mathbb{C})$, the invariant tensor used for raising and lowering spinor indices is a anti-symmetric tensor conventionally defined as $\epsilon = i\sigma_2$

$$\epsilon^{12} = \epsilon^{\dot{1}\dot{2}} = \epsilon_{12} = \epsilon_{\dot{1}\dot{2}} = 1 \tag{A.2.8}$$

It raises and lowers indices according to

$$\psi^{\alpha} = \epsilon^{\alpha\beta}\psi_{\beta} \quad , \quad \psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \quad , \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}} \quad , \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\psi}^{\dot{\beta}} \tag{A.2.9}$$

Unlike the four-component spinors of Lorentz group $SO^{+}(1,3)$, these representations are called two-component spinors and possess somewhat different rules of algebra and calculus.

A.3 Calculus of Grassmann variables

The two-component spinors have Grassmann numbers, or anti-commuting numbers, as its components. Thus, its rules for calculus are somewhat different. However, an understanding of them is vital in writing down the Lagrangian.

To arrive at the two-component case, the one-dimensional case is considered first. Let κ be a Grassmann number. Due to its anti-commuting property, all quadratic or higher terms vanish since $\kappa\kappa = -\kappa\kappa = 0$. Thus, the most general function for Grassmann number must be linear

$$f(\kappa) = \sum_{k=0}^{\infty} f_k\kappa^k = f_0 + f_1\kappa. \tag{A.3.1}$$

Differentiation is straightforward

$$\frac{df}{d\kappa} = f_1 \tag{A.3.2}$$

For integral, the definition would be

$$\int d\kappa := 0 \quad , \quad \int d\kappa\kappa := 1 \tag{A.3.3}$$

So that Grassmann numbers and ordinary numbers do not equate. From these definitions, one can obtain

$$\int d\kappa f(\kappa) = \int d\kappa(f_0 + f_1\kappa) = f_0\int d\kappa + f_1\int d\kappa\kappa = f_1 = \frac{df}{d\kappa} \tag{A.3.4}$$

$$\int d\kappa\delta(\kappa) = 1 = \int d\kappa\kappa \Longrightarrow \delta(\kappa) = \kappa \tag{A.3.5}$$

In the case of two-component spinors, let the Grassmann numbers $\theta_\alpha, \bar{\theta}_{\dot{\alpha}} (\alpha = 1, 2, \dot{\alpha} = \dot{1}, \dot{2})$ be the components of the two-component spinor $\theta, \bar{\theta}$. One will have that $\theta_\alpha \theta_\beta = 0$ if $\alpha = \beta$, similarly for $\bar{\theta}_{\dot{\alpha}}$. Because that α, β can only take 2 values $1, 2$, any product involving more than two θ's or $\bar{\theta}$'s (whose proof is similar) must vanish because the product such as $\theta_\alpha \theta_\beta \theta_\gamma$ have at least three indices so two of them must equal and vanish. Partial differentiations for two-dimensional cases are defined as

$$\partial_\alpha \equiv \frac{\partial}{\partial \theta^\alpha} \text{ and } \partial^\alpha = -\epsilon^{\alpha\beta} \partial_\beta \quad , \quad \bar{\partial}_{\dot{\alpha}} \equiv \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \text{ and } \bar{\partial}^{\dot{\alpha}} = -\epsilon^{\dot{\alpha}\dot{\beta}} \bar{\partial}_{\dot{\beta}} \tag{A.3.6}$$

From which, the following equalities follow

$$\partial_\alpha \theta^\beta = \delta_\alpha^\beta \quad , \quad \bar{\partial}_{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}} \quad , \quad \partial_\alpha \bar{\theta}_{\dot{\beta}} = 0 \quad , \quad \bar{\partial}^{\dot{\alpha}} \theta^\beta = 0 \tag{A.3.7}$$

Similar to ordinary number, one also need to define

$$d^2\theta := \frac{1}{2} d\theta^1 d\theta^2 \quad , \quad d^2\bar{\theta} := \frac{1}{2} d\bar{\theta}^{\dot{2}} d\bar{\theta}^{\dot{1}} \tag{A.3.8}$$

Together with the relations $\theta^\alpha \theta^\beta = -\frac{1}{2}\epsilon^{\alpha\beta}\theta\theta$ and $\bar{\theta}^{\dot{\alpha}} \bar{\theta}^{\dot{\beta}} = \frac{1}{2}\epsilon^{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}$ one can prove that

$$\int d^2\theta \theta\theta = \frac{1}{2} \int d\theta^1 d\theta^2 \times 2\theta^2\theta^1 = \int d\theta^1 \left(\int d\theta^2 \theta^2 \right) \theta^1 = \int d\theta^1 \theta^1 = 1 \tag{A.3.9}$$

$$\int d^2\bar{\theta} \bar{\theta}\bar{\theta} = \frac{1}{2} \int d\bar{\theta}^{\dot{1}} d\bar{\theta}^{\dot{2}} \times 2\bar{\theta}^{\dot{2}}\bar{\theta}^{\dot{1}} = \int d\bar{\theta}^{\dot{1}} \left(\int d\bar{\theta}^{\dot{2}} \bar{\theta}^{\dot{2}} \right) \bar{\theta}^{\dot{1}} = \int d\bar{\theta}^{\dot{1}} \bar{\theta}^{\dot{1}} = 1 \tag{A.3.10}$$

$$\int d^2\theta \int d^2\bar{\theta} (\theta\theta)(\bar{\theta}\bar{\theta}) = \int d^2\theta \int d^2\bar{\theta} (\bar{\theta}\bar{\theta})(\theta\theta) = \int d^2\theta (\theta\theta) = 1 \tag{A.3.11}$$

From equations (A.3.9), (A.3.10), one may also see the connection between second order integration and differentiation

$$\int d^2\theta = \frac{1}{4} \epsilon^{\alpha\beta} \frac{\partial}{\partial \theta^\alpha} \frac{\partial}{\partial \theta^\beta} \quad , \quad \int d^2\bar{\theta} = -\frac{1}{4} \epsilon^{\dot{\alpha}\dot{\beta}} \frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} \frac{\partial}{\partial \bar{\theta}^{\dot{\beta}}} \tag{A.3.12}$$

A.4 Some expansion formulas

Exponential function

The expansion of infinitesimal exponential is

$$u^\varepsilon = e^{\varepsilon \ln u} \approx 1 + \varepsilon \ln u + \mathcal{O}(\varepsilon^2) \tag{A.4.1}$$

Expansion of gamma function near negative poles

Let $\Gamma(x)$ be the gamma function, $n \in \mathbb{N}$ and ε is infinitesimal, then

$$\Gamma(\varepsilon - n) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi(n+1) + \mathcal{O}(\varepsilon) \right] \tag{A.4.2}$$

where $\psi(1) = -\gamma_E \approx -0.577216$ with γ_E called Euler-Mascheroni number, and $\psi(n+1) = \psi(1) + 1 + \frac{1}{2} + \cdots + \frac{1}{n}$. Some special cases used in the thesis are

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma_E \tag{A.4.3}$$

$$\Gamma(-1+\varepsilon) = -1\left(\frac{1}{\varepsilon} - \gamma_E + 1 \right) = -\frac{1}{\varepsilon} + \gamma_E - 1 \tag{A.4.4}$$

Appendix B

Neutral Higgs mass matrix counterterm

$$\begin{aligned}(\delta \boldsymbol{M}_{hh})_{h_d h_d} = {} & c_\beta^3 s_\beta \delta t_\beta \left(v^2 |\lambda|^2 - 2M_W^2 - 2M_Z^2 + 2M_{H^\pm}^2\right) + v^2 |\lambda| \, s_\beta^2 \delta|\lambda| + v \, |\lambda|^2 \, s_\beta^2 \delta v \\ & - \frac{(c_{2\beta} - 3) \, c_\beta \delta t_{h_d}}{2v} - \frac{c_\beta^2 s_\beta \delta t_{h_u}}{v} + c_\beta^2 \delta M_Z^2 - s_\beta^2 \delta M_W^2 + \delta M_{H^\pm}^2 s_\beta^2\end{aligned} \tag{B.0.1}$$

$$\begin{aligned}(\delta \boldsymbol{M}_{hh})_{h_d h_u} = {} & \frac{1}{2} c_{2\beta} c_\beta^2 \delta t_\beta \left(v^2 |\lambda|^2 + 2M_W^2 - 2M_Z^2 - 2M_{H^\pm}^2\right) + v^2 |\lambda| \, c_\beta s_\beta \delta|\lambda| \\ & + v \, |\lambda|^2 \, c_\beta s_\beta \delta v + \frac{c_\beta^3 \delta t_{h_u}}{v} + c_\beta s_\beta \delta M_W^2 - c_\beta s_\beta \delta M_Z^2 - c_\beta \delta M_{H^\pm}^2 s_\beta + \frac{s_\beta^3 \delta t_{h_d}}{v}\end{aligned} \tag{B.0.2}$$

$$\begin{aligned}(\delta \boldsymbol{M}_{hh})_{h_d h_s} = {} & -\frac{v c_\beta^2 \delta t_\beta}{2 v_s} \Big(2 c_\beta^2 s_\beta \left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right) + |\kappa| \, |\lambda| \, c_\beta v_s^2 c_{\varphi_y} \\ & \qquad + s_\beta^3 \left(-\left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)\right) + 2 \, |\lambda|^2 \, s_\beta v_s^2\Big) \\ & + \frac{v \delta v_s \left(c_\beta \left(s_\beta^2 \left(v^2 |\lambda|^2 - 2M_W^2\right) + 2 \, |\lambda|^2 \, v_s^2\right) + s_\beta \left(M_{H^\pm}^2 s_{2\beta} - |\kappa| \, |\lambda| \, v_s^2 c_{\varphi_y}\right)\right)}{2 v_s^2} \\ & + \frac{\delta v \left(s_\beta \left(M_W^2 s_{2\beta} - |\kappa| \, |\lambda| \, v_s^2 c_{\varphi_y}\right) + c_\beta \left(2 \, |\lambda|^2 \, v_s^2 - s_\beta^2 \left(3 v^2 |\lambda|^2 + 2M_{H^\pm}^2\right)\right)\right)}{2 v_s} \\ & + \delta|\lambda| \left(c_\beta \left(2v \, |\lambda| \, v_s - \frac{v^3 |\lambda| \, s_\beta^2}{v_s}\right) - \frac{1}{2} v \, |\kappa| \, s_\beta v_s c_{\varphi_y}\right) - \frac{1}{2} v \, |\lambda| \, s_\beta v_s \delta|\kappa| c_{\varphi_y} \\ & + \frac{1}{2} v \, |\kappa| \, |\lambda| \, s_\beta v_s s_{\varphi_y} \delta\varphi_y + \frac{c_\beta^3 s_\beta \delta t_{h_u}}{v_s} + \frac{v c_\beta s_\beta^2 \delta M_W^2}{v_s} - \frac{v c_\beta \delta M_{H^\pm}^2 s_\beta^2}{v_s} + \frac{s_\beta^4 \delta t_{h_d}}{v_s}\end{aligned} \tag{B.0.3}$$

$$(\delta \boldsymbol{M}_{hh})_{h_d a} = \frac{\delta t_{a_d} \cot\beta}{v} \tag{B.0.4}$$

$$\begin{aligned}(\delta \boldsymbol{M}_{hh})_{h_d a_s} = {} & \frac{\delta t_{a_d}}{v_s} + \frac{3}{2} v \, |\kappa| \, |\lambda| \, c_\beta^3 v_s s_{\varphi_y} \delta t_\beta + \frac{3}{2} v \, |\kappa| \, |\lambda| \, s_\beta v_s c_{\varphi_y} \delta\varphi_y + \frac{3}{2} v \, |\lambda| \, s_\beta v_s \delta|\kappa| s_{\varphi_y} \\ & + \frac{3}{2} v \, |\kappa| \, s_\beta v_s \delta|\lambda| s_{\varphi_y} + \frac{3}{2} v \, |\kappa| \, |\lambda| \, s_\beta \delta v_s s_{\varphi_y} + \frac{3}{2} \, |\kappa| \, |\lambda| \, s_\beta v_s \delta v s_{\varphi_y}\end{aligned} \tag{B.0.5}$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_u h_u} = {} & c_\beta^3 s_\beta \delta t_\beta \left(-v^2 |\lambda|^2 + 2M_W^2 + 2M_Z^2 - 2M_{H^\pm}^2\right) + v^2 |\lambda| c_\beta^2 \delta|\lambda| + v |\lambda|^2 c_\beta^2 \delta v \\
& - \frac{c_\beta s_\beta^2 \delta t_{h_d}}{v} + \frac{(c_{2\beta}+3) s_\beta \delta t_{h_u}}{2v} - c_\beta^2 \delta M_W^2 + c_\beta^2 \delta M_{H^\pm}^2 + s_\beta^2 \delta M_Z^2
\end{aligned} \tag{B.0.6}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_u h_s} = {} & \frac{v c_\beta^2 \delta t_\beta}{2 v_s} \Big(2 c_\beta \left(|\lambda|^2 v_s^2 + 2 s_\beta^2 \left(M_{H^\pm}^2 - M_W^2\right)\right) + c_\beta^3 \left(-\left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)\right) \\
& + |\lambda| s_\beta \left(|\kappa| v_s^2 c_{\varphi_y} + v^2 |\lambda| s_{2\beta}\right)\Big) \\
& + \frac{1}{2} v \delta v_s \left(\frac{c_\beta^2 s_\beta \left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)}{v_s^2} - |\kappa| |\lambda| c_\beta c_{\varphi_y} + 2 |\lambda|^2 s_\beta\right) \\
& + \delta v \left(-\frac{c_\beta^2 s_\beta \left(3v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)}{2 v_s} - \frac{1}{2} |\kappa| |\lambda| c_\beta v_s c_{\varphi_y} + |\lambda|^2 s_\beta v_s\right) \\
& + \delta|\lambda| \left(-\frac{v^3 |\lambda| c_\beta^2 s_\beta}{v_s} - \frac{1}{2} v |\kappa| c_\beta v_s c_{\varphi_y} + 2v |\lambda| s_\beta v_s\right) - \frac{1}{2} v |\lambda| c_\beta v_s \delta|\kappa| c_{\varphi_y} \\
& + \frac{1}{2} v |\kappa| |\lambda| c_\beta v_s s_{\varphi_y} \delta\varphi_y + \frac{c_\beta s_\beta^3 \delta t_{h_d}}{v_s} + \frac{c_\beta^4 \delta t_{h_u}}{v_s} + \frac{v c_\beta^2 s_\beta \delta M_W^2}{v_s} - \frac{v c_\beta^2 \delta M_{H^\pm}^2 s_\beta}{v_s}
\end{aligned} \tag{B.0.7}
$$

$$
(\delta \boldsymbol{M}_{hh})_{h_u a} = \frac{\delta t_{a_d}}{v} \tag{B.0.8}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_u a_s} = {} & \frac{\cot\beta \delta t_{a_d}}{v_s} - \frac{3}{2} v |\kappa| |\lambda| c_\beta^2 s_\beta v_s s_{\varphi_y} \delta t_\beta + \frac{3}{2} v |\lambda| c_\beta v_s \delta|\kappa| s_{\varphi_y} + \frac{3}{2} v |\kappa| c_\beta v_s \delta|\lambda| s_{\varphi_y} \\
& + \frac{3}{2} v |\kappa| |\lambda| c_\beta \delta v_s s_{\varphi_y} + \frac{3}{2} v |\kappa| |\lambda| c_\beta v_s c_{\varphi_y} \delta\varphi_y + \frac{3}{2} |\kappa| |\lambda| c_\beta v_s \delta v s_{\varphi_y}
\end{aligned} \tag{B.0.9}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_s h_s} = & -\frac{v s_\beta \delta t_{h_u} c_\beta^4}{v_s^2} + \frac{v^2 s_\beta^2 \delta M_{H^\pm}^2 c_\beta^2}{v_s^2} - \frac{v^2 s_\beta^2 \delta M_W^2 c_\beta^2}{v_s^2} \\
& + \frac{1}{2} v^2 c_{2\beta} \left(- |\kappa| \, |\lambda| \, c_{\varphi_y} + \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| \, |\lambda| \, s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} \right. \\
& \left. + \frac{\left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right) s_{2\beta}}{v_s^2} \right) \delta t_\beta c_\beta^2 + \frac{i \left(-1 + e^{2i\varphi_\omega}\right) v \delta t_{a_d} c_\beta}{\left(1 + e^{2i\varphi_\omega}\right) v_s^2} \\
& - \frac{v s_\beta^4 \delta t_{h_d} c_\beta}{v_s^2} + \frac{v^2 |\kappa| \, |\lambda| \, s_\beta \left(3i \left(-1 + e^{2i\varphi_\omega}\right) c_{\varphi_y} + \left(1 + e^{2i\varphi_\omega}\right) s_{\varphi_y}\right) \delta\varphi_y c_\beta}{2 \left(1 + e^{2i\varphi_\omega}\right)} \\
& + \frac{1}{2} v \delta v s_{2\beta} \left(- |\kappa| \, |\lambda| \, c_{\varphi_y} + \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| \, |\lambda| \, s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} \right. \\
& \left. + \frac{\left(v^2 |\lambda|^2 - M_W^2 + M_{H^\pm}^2\right) s_{2\beta}}{v_s^2} \right) \\
& + \left(\frac{1}{2} |\lambda| \, c_\beta s_\beta \left(\frac{3i \left(-1 + e^{2i\varphi_\omega}\right) s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} - c_{\varphi_y} \right) v^2 + v_s \left(\frac{\sqrt{2} e^{i\varphi_\omega} \mathrm{Re} A_\kappa}{1 + e^{2i\varphi_\omega}} + 4 |\kappa| \, v_s \right) \right) \delta|\kappa| \\
& + \frac{1}{4} v^2 s_{2\beta} \left(\frac{|\lambda| \, s_{2\beta} v^2}{v_s^2} - |\kappa| \, c_{\varphi_y} + \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| \, s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} \right) \delta|\lambda| \\
& + \frac{\sqrt{2} e^{i\varphi_\omega} |\kappa| \, v_s \delta \mathrm{Re} A_\kappa}{1 + e^{2i\varphi_\omega}} + \frac{\left(i - i e^{2i\varphi_\omega}\right) \delta t_{a_s}}{e^{2i\varphi_\omega} v_s + v_s} + \frac{\delta t_{h_s}}{v_s} \\
& + \left(- \frac{|\lambda|^2 c_\beta^2 s_\beta^2 v^4}{v_s^3} - \frac{\left(M_{H^\pm}^2 - M_W^2\right) s_{2\beta}^2 v^2}{2 v_s^3} + |\kappa| \left(\frac{\sqrt{2} e^{i\varphi_\omega} \mathrm{Re} A_\kappa}{1 + e^{2i\varphi_\omega}} + 4 |\kappa| \, v_s \right) \right) \delta v_s \\
& + \frac{\left(-6 e^{2i\varphi_\omega} |\kappa| \, |\lambda| \, c_\beta s_\beta s_{\varphi_y} v^2 - i\sqrt{2} e^{i\varphi_\omega} \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| \, \mathrm{Re} A_\kappa v_s\right) \delta\varphi_\omega}{\left(1 + e^{2i\varphi_\omega}\right)^2}
\end{aligned} \tag{B.0.10}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_s a} = & \frac{\delta t_{a_d}}{s_\beta v_s} - \frac{1}{2} v \, |\kappa| \, |\lambda| \, v_s c_{\varphi_y} \delta\varphi_y - \frac{1}{2} v \, |\lambda| \, v_s \delta|\kappa| s_{\varphi_y} \\
& - \frac{1}{2} v \, |\kappa| \, v_s \delta|\lambda| s_{\varphi_y} - \frac{1}{2} v \, |\kappa| \, |\lambda| \, \delta v_s s_{\varphi_y} - \frac{1}{2} |\kappa| \, |\lambda| \, v_s \delta v s_{\varphi_y}
\end{aligned} \tag{B.0.11}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{h_s a_s} = & -\frac{2 v c_\beta \delta t_{a_d}}{v_s^2} + \frac{2 \delta t_{a_s}}{v_s} - 2 v^2 |\kappa| \, |\lambda| \, c_\beta^2 c_{2\beta} s_{\varphi_y} \delta t_\beta - 2 v^2 |\lambda| \, c_\beta s_\beta \delta|\kappa| s_{\varphi_y} \\
& - 2 v^2 |\kappa| \, c_\beta s_\beta \delta|\lambda| s_{\varphi_y} - 2 v^2 |\kappa| \, |\lambda| \, c_\beta s_\beta c_{\varphi_y} \delta\varphi_y - 4 v \, |\kappa| \, |\lambda| \, c_\beta s_\beta \delta v s_{\varphi_y}
\end{aligned} \tag{B.0.12}
$$

$$
(\delta \boldsymbol{M}_{hh})_{aa} = v^2 |\lambda| \, \delta|\lambda| + v \, |\lambda|^2 \delta v - \delta M_W^2 + \delta M_{H^\pm}^2 \tag{B.0.13}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{aa_s} = {}& \frac{v c_{2\beta} c_\beta^2 \delta t_\beta \left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)}{2v_s} \\
& - \frac{v \delta v_s \left(6 |\kappa| |\lambda| v_s^2 c_{\varphi_y} + s_{2\beta} \left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)\right)}{4v_s^2} \\
& + \delta v \left(\frac{s_{2\beta} \left(3v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right)}{4v_s} - \frac{3}{2} |\kappa| |\lambda| v_s c_{\varphi_y}\right) \\
& + \delta|\lambda| \left(\frac{v^3 |\lambda| c_\beta s_\beta}{v_s} - \frac{3}{2} v |\kappa| v_s c_{\varphi_y}\right) - \frac{3}{2} v |\lambda| v_s \delta|\kappa| c_{\varphi_y} \\
& + \frac{3}{2} v |\kappa| |\lambda| v_s s_{\varphi_y} \delta\varphi_y - \frac{c_\beta^3 \delta t_{h_u}}{v_s} - \frac{v c_\beta s_\beta \delta M_W^2}{v_s} + \frac{v c_\beta \delta M_{H^\pm}^2 s_\beta}{v_s} - \frac{s_\beta^3 \delta t_{h_d}}{v_s}
\end{aligned}
\tag{B.0.14}
$$

$$
\begin{aligned}
(\delta \boldsymbol{M}_{hh})_{a_s a_s} = {}& - \frac{v s_\beta \delta t_{h_u} c_\beta^4}{v_s^2} + \frac{v^2 s_\beta^2 \delta M_{H^\pm}^2 c_\beta^2}{v_s^2} - \frac{v^2 s_\beta^2 \delta M_W^2 c_\beta^2}{v_s^2} \\
& + \frac{1}{2} v^2 c_{2\beta} \Bigg(3 |\kappa| |\lambda| c_{\varphi_y} - \frac{9i \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| |\lambda| s_{\varphi_y}}{1 + e^{2i\varphi_\omega}} \\
& + \frac{\left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right) s_{2\beta}}{v_s^2}\Bigg) \delta t_\beta c_\beta^2 - \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) v \delta t_{a_d} c_\beta}{\left(1 + e^{2i\varphi_\omega}\right) v_s^2} - \frac{v s_\beta^4 \delta t_{h_d} c_\beta}{v_s^2} \\
& + \frac{3}{2} v^2 |\kappa| |\lambda| s_\beta \left(-\frac{3i \left(-1 + e^{2i\varphi_\omega}\right) c_{\varphi_y}}{1 + e^{2i\varphi_\omega}} - s_{\varphi_y}\right) \delta\varphi_y c_\beta + \delta v \Bigg(\frac{2 |\lambda|^2 c_\beta^2 s_\beta^2 v^3}{v_s^2} \\
& \qquad + 3 |\kappa| |\lambda| c_\beta s_\beta \left(c_{\varphi_y} - \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) s_{\varphi_y}}{1 + e^{2i\varphi_\omega}}\right) v + \frac{\left(M_{H^\pm}^2 - M_W^2\right) s_{2\beta}^2 v}{2v_s^2}\Bigg) \\
& + \left(\frac{3}{2} v^2 |\lambda| c_\beta s_\beta \left(c_{\varphi_y} - \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) s_{\varphi_y}}{1 + e^{2i\varphi_\omega}}\right) - \frac{3\sqrt{2} e^{i\varphi_\omega} \mathrm{Re} A_\kappa v_s}{1 + e^{2i\varphi_\omega}}\right) \delta|\kappa| \\
& + \frac{1}{4} v^2 s_{2\beta} \left(\frac{|\lambda| s_{2\beta} v^2}{v_s^2} + 3 |\kappa| c_{\varphi_y} - \frac{9i \left(-1 + e^{2i\varphi_\omega}\right) |\kappa| s_{\varphi_y}}{1 + e^{2i\varphi_\omega}}\right) \delta|\lambda| \\
& - \frac{3\sqrt{2} e^{i\varphi_\omega} |\kappa| v_s \delta \mathrm{Re} A_\kappa}{1 + e^{2i\varphi_\omega}} + \frac{3i \left(-1 + e^{2i\varphi_\omega}\right) \delta t_{a_s}}{\left(1 + e^{2i\varphi_\omega}\right) v_s} + \frac{\delta t_{h_s}}{v_s} \\
& + \left(-\frac{v^2 \left(v^2 |\lambda|^2 - 2M_W^2 + 2M_{H^\pm}^2\right) s_{2\beta}^2}{4v_s^3} - \frac{3\sqrt{2} e^{i\varphi_\omega} |\kappa| \mathrm{Re} A_\kappa}{1 + e^{2i\varphi_\omega}}\right) \delta v_s \\
& + \frac{3 e^{i\varphi_\omega} |\kappa| \left(6 e^{i\varphi_\omega} |\lambda| c_\beta s_\beta s_{\varphi_y} v^2 + i\sqrt{2} \left(-1 + e^{2i\varphi_\omega}\right) \mathrm{Re} A_\kappa v_s\right) \delta\varphi_\omega}{\left(1 + e^{2i\varphi_\omega}\right)^2}
\end{aligned}
\tag{B.0.15}
$$

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