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BACHELOR'S THESIS

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Constraining Wilson coefficients from lepton-flavour non-universal LEP1 data

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Contents

Introduction

A popular method to find new-physics effects is to use the Standard Model Effective Field Theory framework, which has the same symmetries and fields as of the Standard Model, but with higher-dimension operators included in the Lagrangian. In this thesis, from the measurements of Z-pole data at LEP1, we constrain Wilson coefficients of dimension-six (D6) operators involving gauge bosons and fermions.

In the gauge boson sector, using a different set of D6 operators (Warsaw's basis in Ref. [1]), we reproduce and make a comparison with the results from Ref. [2] by O. Nachtmann, F. Nagel, M. Pospischil (NNP). Thenceforth, we expand the constraints by performing fits for Wilson coefficients in fermion sector, first with only leptons, then with quarks included. Both the case of flavor universal leptons and the general case of flavor non-universal leptons are investigated using the relevant set of observables' data. The quarks' coefficients are always considered in flavor universal case.

The thesis is presented in 5 Chapters:

- **Chapter 1: Standard Model Effective Field Theory (SMEFT)**
  We introduce the formalism of SMEFT, with basic notation, conventions, and the set of D6 operators with related Wilson coefficients from Warsaw's basis in Ref. [1].

- **Chapter 2: Wilson coefficients fitting methodology**
  We provide the basic concepts and notation for the statistical fitting method of linear least square (or χ^2). The data set of LEP1 observables for the fitting as well as the usage of χ^2 method in our problem is also discussed.

- **Chapter 3: Constraints on gauge sector Wilson coefficients**
  We start with a simple case, considering only gauge sector and using Warsaw's basis of D6 operators and Wilson coefficients. With a different approach in rescaling the Lagrangian, we compare the results with NNT's, and then improve the constraints using updated observables' data.

- **Chapter 4: Constraints on lepton sector Wilson coeffcients**
  New operators with two leptons are added to the Lagrangian. Flavor universal and various assumptions for flavor non-universal fermions are investigated using the data of flavor non-universal observables of A_{FB}^{0,e_p}, $R_{e_p}^0$.

- **Chapter 5: Constraints on lepton and quark sector Wilson coefficients**
  With the relevant observables for quarks of $A_{FB}^{0,b}$, $A_{FB}^{0,c}$, R_b^0, R_c^0 added, we investigate another perspective of view that all relevant gauge sector Wilson coefficients are nulls. Fits for lepton and quark sectors are performed using several lepton flavor non-universal and quark flavor universal assumptions.

Chapter 1

Standard Model Effective Field Theory (SMEFT)

1.1 Introduction to SMEFT

Standard Model (SM) of Particle Physics has been tremendously successful when describing the electromagnetic, weak and strong interactions as gauge field theories, constructed from an underlying symmetry called local gauge symmetry. The SM interactions and perturbative predictions have been tested and agreed to an amazing precision. Nevertheless, new physics beyond SM could be hidden in the experimental or theoretical errors. To find new physics effects, we use the SMEFT framework. This method can be used to compare theoretical predictions with measurements for energies below a cut-off scale called Λ. In this thesis, we assume that Λ is much larger than the electroweak scale of $v \approx 246\text{GeV}$. A field theory legitimate above that energy scale Λ should satisfy the following requirements (Ref. [1]):

1. Its group must satisfy gauge symmetries and contains $SU(3)_C \times SU(2)_L \times U(1)_Y$ of SM.
2. It includes all SM degrees of freedom as either fundamental or composite fields.
3. It should reduce to SM at low energy scale, provided that no weakly coupled light particle e.g. axions, sterile neutrinos,... exists.

The SMEFT leads to additional terms with higher-dimensional operators to the SM Lagrangian. We have:

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda}\sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) \tag{1.1.1}$$

For the consistence with the notation in "Warsaw" basis in Ref. [1], we absorb the cut-off scale Λ to the Wilson coefficients. We also absorb the Higgs field's vacuum expectation value (vev) of v, for convenience in comparison with the results in Ref. [2]

$$\frac{v}{\Lambda} C_i^{(5)} \to C_i^{(5)}, \qquad \frac{v^2}{\Lambda^2} C_i^{(6)} \to C_i^{(6)}. \tag{1.1.2}$$

These higher-dimensional operators must satisfy Lorentz and gauge invariance. Therefore, our C_i coefficients, which are constrained in this thesis, are dimensionless. They describe new physics

	fermions					scalars
field	$l'^{j}_{Lp} = \begin{pmatrix} \nu'_{Lp} \\ e'_{Lp} \end{pmatrix}$	e'_{Rp}	$q'^{\alpha j}_{Lp} = \begin{pmatrix} u'^{\alpha}_{Lp} \\ d'^{\alpha}_{Lp} \end{pmatrix}$	u'^{α}_{Rp}	d'^{α}_{Rp}	$\varphi^{j} = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$
hypercharge Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 1.1: The SM matter content in the gauge basis, taken from Ref. [1].

effects. We assume throughout this context that the scale Λ is far above the Electro-weak (EW) scale of $v \sim 246$ GeV ($\Lambda \gg v$). Hence, the effects of the new operators are suppressed by the powers of (v/Λ). Here, the operators in dimension five violate the lepton-number conservation, therefore we consider only the leading order terms of dimension six (D6) operators in our analysis.

1.2 Notation and conventions

Our notation and conventions are basically based on the Warsaw's basis in Ref. [1]. We summarize SM matter content in Tab. 1.1 with isospin, colour, and generation indices denoted by $j = 1, 2$, $\alpha = 1, 2, 3$, and $p = 1, 2, 3$, respectively. Here we specify also Chirality indices (L, R) of the fermion fields. Complex conjugate of the Higgs field will always occur either as $\varphi^{\dagger}$ or $\widetilde{\varphi}$, where $\widetilde{\varphi}^{j} = \varepsilon_{jk}(\varphi^{k})^{\star}$, and ε_{jk} stands for totally antisymmetric tensor with $\varepsilon_{12} = +1$.

The SM Lagrangian $\mathcal{L}^{(4)}_{\mathrm{SM}}$ reads

$$\begin{aligned} \mathcal{L}^{(4)}_{\mathrm{SM}} = & -\frac{1}{4} G^{A}_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^{I}_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + (D_{\mu}\varphi)^{\dagger} (D^{\mu}\varphi) + m^{2}\varphi^{\dagger}\varphi - \frac{1}{2}\lambda \left(\varphi^{\dagger}\varphi\right)^{2} \\ & + i\bar{f}\not{D}f - \left(\bar{l}\,\Gamma_{e} e\varphi + \bar{q}\,\Gamma_{u} u\widetilde{\varphi} + \bar{q}\,\Gamma_{d} d\varphi + \text{h.c.}\right), \end{aligned} \tag{1.2.3}$$

where f stands for fermion fields, for which the convention for their corresponding hypercharges are mentioned in Tab. 1.1, with the convention for Gell-Mann–Nishijima formula of $Q = T^{3} + Y$. The Yukawa couplings $\Gamma_{e,u,d}$ are matrices in the generation space. Our convention for covariant derivatives reads,

$$D_{\mu} = \partial_{\mu} + igB_{\mu}Y + igW^{I}_{\mu}T^{I} + ig_{s}G^{A}_{\mu}\mathcal{T}^{A}, \tag{1.2.4}$$

whereas, $\mathcal{T}^{A} = \frac{1}{2}\lambda^{A}$ and $T^{I} = \frac{1}{2}\tau^{I}$ are the $SU(3)$ and $SU(2)$ generators, while λ^{A} and τ^{I} are the Gell-Mann and Pauli matrices, respectively.

We define Hermitian derivative terms as follows:

$$\varphi^{\dagger} i \overleftrightarrow{D}_{\mu} \varphi \equiv i\varphi^{\dagger} \left(D_{\mu} - \overleftarrow{D}_{\mu}\right) \varphi \quad \text{and} \quad \varphi^{\dagger} i \overleftrightarrow{D}^{I}_{\mu} \varphi \equiv i\varphi^{\dagger} \left(\tau^{I} D_{\mu} - \overleftarrow{D}_{\mu}\tau^{I}\right) \varphi, \tag{1.2.5}$$

whereas, $\varphi^{\dagger}\overleftarrow{D}_{\mu}\varphi \equiv (D_{\mu}\varphi)^{\dagger}\varphi$.
The gauge field strength tensors and their covariant derivatives read

$$\begin{aligned} G^{A}_{\mu\nu} &= \partial_{\mu}G^{A}_{\nu} - \partial_{\nu}G^{A}_{\mu} - g_{s}f^{ABC}G^{B}_{\mu}G^{C}_{\nu}, & (D_{\rho}G_{\mu\nu})^{A} &= \partial_{\rho}G^{A}_{\mu\nu} - g_{s}f^{ABC}G^{B}_{\rho}G^{C}_{\mu\nu}, \\ W^{I}_{\mu\nu} &= \partial_{\mu}W^{I}_{\nu} - \partial_{\nu}W^{I}_{\mu} - g\varepsilon^{IJK}W^{J}_{\mu}W^{K}_{\nu}, & (D_{\rho}W_{\mu\nu})^{I} &= \partial_{\rho}W^{I}_{\mu\nu} - g\varepsilon^{IJK}W^{J}_{\rho}W^{K}_{\mu\nu}, \\ B_{\mu\nu} &= \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}, & D_{\rho}B_{\mu\nu} &= \partial_{\rho}B_{\mu\nu}. \end{aligned} \tag{1.2.6}$$

Dual tensors are defined by $\widetilde{X}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$ ($\varepsilon_{0123} = +1$), where X stands for G^A, W^I or B. The Higg doublet is expanded arround vacuum as:

$$\varphi(x) = \langle\varphi\rangle + \Phi = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\Phi^0), \end{pmatrix} \tag{1.2.7}$$

whereas,

$$\langle\varphi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.2.8}$$

with v is the vev of Higgs field. Take into account the contribution from D6 operators in φ^6 and $\varphi^4 D^2$ sector, the effective Higgs potential reads:

$$V(\varphi) = -\mu^2(\varphi^\dagger\varphi) + \frac{\lambda}{2}(\varphi^\dagger\varphi)^2 + C_\varphi(\varphi^\dagger\varphi)^3. \tag{1.2.9}$$

We assume $\mu^2 > 0$ and $\lambda > 0$. The potential reaches its minimum when:

$$\frac{\partial V(\varphi)}{\partial\varphi} = 0, \tag{1.2.10}$$

or

$$\varphi^\dagger\varphi = \frac{\lambda - \sqrt{\lambda^2 - 12\mu^2 C_\varphi}}{6C_\varphi}. \tag{1.2.11}$$

To the first order in D6 Wilson coefficients, the vev reads:

$$v \approx \frac{2\mu^2}{\lambda} + \frac{3\mu^3}{\sqrt{2\lambda^5}}C_\varphi. \tag{1.2.12}$$

Note that when absorbed in to the D6 Wilson coefficients as Eq. (1.1.2), to the linear order of the Wilson coefficients, only the term $2\mu^2/\lambda$ of the vev contributes.

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu} e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q^{(1)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q^{(3)}_{\varphi l}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu} T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu} u_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$	$Q^{(1)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu} u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q^{(3)}_{\varphi q}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu} d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 1.2: D6 operators other than the four-fermion ones, taken from Ref. [1]. Here the indices "p", "r" stand for generation indices, "A", "B", "C" stand for SU(3) indices from 1 to 8, "I", "J", "K" stand for SU(2) indices from 1 to 3.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu \tau^I q_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{q}_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$		

Table 1.3: Four-fermion operators, taken from Ref. [1]. The conventions for indices are the same with Table 1.2 appart from the indices "s" and "t", which also stand for generation indices.

Chapter 2

Wilson coefficients fitting methodology

2.1 Input data and observables

We introduce the P_Z scheme with a set of EW input parameter, including Z boson's mass (m_Z), Fermi's constant (G_F) and the fine structure constant at Z scale [$\alpha(m_Z)$]. Numerical values of these parameters are obtained from Ref. [3]:

Input parameters	Value	Ref.
$\hat{m}_Z$[GeV]	91.1876 ± 0.0021	[4, 5, 6]
$\hat{G}_F$[GeV^{-2}]	$1.1663787(6) \times 10^{-5}$	[5, 6]
$\alpha(m_Z)$	$1/128.886 \pm 0.090$	[4]

Table 2.1: Input parameters values, taken from Ref. [3].

Here, the masses of leptons and light quarks (u,d,c,s,b) are neglected.
The physical positron charge (e) at m_Z is determined as:

$$e = \sqrt{4\pi\alpha(m_Z)}. \tag{2.1.1}$$

The experimental and theoretical values for these observables are also obtained from Ref. [3], and showed in Table 2.2.

Apart from the observables in Table 2.2, we also get the data of $\Gamma_W^{\text{exp.}}$[GeV] $= 2.085 \pm 0.042$ from Ref. [12] and $\Gamma_W^{\text{theo.}}$[GeV] $= 2.0896 \pm 0.0032$ from Ref. [13] for the constraints, with correlation between Γ_W and m_W is -6.7% from Ref. [4].

We got the correlation matrix for these observables from Ref. [4]. To constrain the D6 Wilson coeficients, we define the observables and pseudo-observables obtained from Z-pole data at LEP1 as follows.
The decay width of Z-boson to two fermions:

$$\Gamma_{Zf\bar{f}}^{\text{tree}} = \frac{g_Z^2 m_Z}{48\pi} N_c^f \chi^f, \qquad \chi^f = \left(c_{\text{V}}^f\right)^2 + \left(c_{\text{A}}^f\right)^2, \tag{2.1.2}$$

The hadronic pole cross section, and R_ℓ^0, R_b^0, R_c^0:

$$\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2}\frac{\Gamma_{\text{ee}}\Gamma_{\text{had}}}{\Gamma_Z^2}, \tag{2.1.3}$$

Observable	Experimental Value	Ref.	SM Theoretical Value	Ref.
$\hat{m}_Z$[GeV]	91.1876 ± 0.0021	[4]	-	-
M_W[GeV]	80.385 ± 0.015	[7]	80.365 ± 0.004	[8]
σ_h^0 [nb]	41.540 ± 0.037	[4]	41.488 ± 0.006	[9]
Γ_Z[GeV]	2.4952 ± 0.0023	[4]	2.4943 ± 0.0005	[9]
R_ℓ^0	20.767 ± 0.025	[4]	20.752 ± 0.005	[9]
R_b^0	0.21629 ± 0.00066	[4]	0.21580 ± 0.00015	[9]
R_c^0	0.1721 ± 0.0030	[4]	0.17223 ± 0.00005	[9]
A_{FB}^ℓ	0.0171 ± 0.0010	[4]	0.01626 ± 0.00008	[10]
A_{FB}^c	0.0707 ± 0.0035	[4]	0.0738 ± 0.0002	[10]
A_{FB}^b	0.0992 ± 0.0016	[4]	0.1033 ± 0.0003	[10]

Table 2.2: Experimental and theoretical values of the LEPI observables used in constructing the χ^2 constraint functions. The results are grouped in terms of the precision of the measurements made. The entries above the double line are measured to better than percent accuracy, the entries below the double line are measured to an accuracy of a few percent, taken from Ref. [3]. The observables on the table are indirectly derived from the following input: $\hat{m}_h = 125.09 \pm 0.21 \pm 0.11$ from Ref. [11]; $\hat{m}_t = 173.21 \pm 0.51 \pm 0.71$ and $\hat{\alpha}_s = 0.1185$ from Ref. [5]; $\Delta\hat{\alpha} = 0.0590$ from Ref. [9]

$$R_\ell^0 = \Gamma_{\rm had}/\Gamma_{\ell\ell}, \qquad R_{\rm b}^0 = \Gamma_{\rm b\overline{b}}/\Gamma_{\rm had}, \qquad R_{\rm c}^0 = \Gamma_{\rm c\overline{c}}/\Gamma_{\rm had}, \tag{2.1.4}$$

with the hadronic decay width, implied by quark universal assumption,

$$\Gamma_{\rm had} = 3 \cdot \Gamma_{b\bar{b}} + 2 \cdot \Gamma_{c\bar{c}}. \tag{2.1.5}$$

The forward-backward asymmetries:

$$A_{\rm FB}^{0,{\rm f}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f, \tag{2.1.6}$$

whereas,

$$\mathcal{A}_f = 2c_V^f c_A^f/\chi^f. \tag{2.1.7}$$

The W-boson decay width into two fermions, with the quark flavor-universal assumption, and rejecting the case of decay into top and bottom quarks:

$$\Gamma_W = \frac{g_W^2 m_W}{48\pi}\left(6\chi_W^{d\bar{u}} + \chi_W^{e\bar{\nu}_e} + \chi_W^{\mu\bar{\nu}_\mu} + \chi_W^{\tau\bar{\nu}_\tau}\right), \tag{2.1.8}$$

where as, $\chi_W^{f\bar{f}} = (c_{V,W}^{f\bar{f}})^2 + (c_{A,W}^{f\bar{f}})^2$.
Noticing that the formulas above for those observables and pseudo-observables remain unchanged when we take into account the D6 operators with only gauge boson or with two fermions.

Without lepton universality		Correlations								
$\chi^2/\text{dof} = 32.6/27$		m_Z	Γ_Z	σ^0_{had}	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{FB}$	$A^{0,\mu}_{FB}$	$A^{0,\tau}_{FB}$
m_Z [GeV]	91.1876 ± 0.0021	1.000								
Γ_Z [GeV]	2.4952 ± 0.0023	−0.024	1.000							
σ^0_{had} [nb]	41.541 ± 0.037	−0.044	−0.297	1.000						
R^0_e	20.804 ± 0.050	0.078	−0.011	0.105	1.000					
R^0_μ	20.785 ± 0.033	0.000	0.008	0.131	0.069	1.000				
R^0_τ	20.764 ± 0.045	0.002	0.006	0.092	0.046	0.069	1.000			
$A^{0,e}_{FB}$	0.0145 ± 0.0025	−0.014	0.007	0.001	−0.371	0.001	0.003	1.000		
$A^{0,\mu}_{FB}$	0.0169 ± 0.0013	0.046	0.002	0.003	0.020	0.012	0.001	−0.024	1.000	
$A^{0,\tau}_{FB}$	0.0188 ± 0.0017	0.035	0.001	0.002	0.013	−0.003	0.009	−0.020	0.046	1.000

With lepton universality		Correlations				
$\chi^2/\text{dof} = 36.5/31$		m_Z	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,\ell}_{FB}$
m_Z [GeV]	91.1875 ± 0.0021	1.000				
Γ_Z [GeV]	2.4952 ± 0.0023	−0.023	1.000			
σ^0_{had} [nb]	41.540 ± 0.037	−0.045	−0.297	1.000		
R^0_ℓ	20.767 ± 0.025	0.033	0.004	0.183	1.000	
$A^{0,\ell}_{FB}$	0.0171 ± 0.0010	0.055	0.003	0.006	−0.056	1.000

Table 2.3: Combined results for the Z parameters of nine pseudo-observables. The errors include all common errors except the parametric uncertainty on m_Z due to the choice of m_H, taken from Ref. [4].

2.2 χ^2 fitting method

We introduce a statistical fitting method of linear least-squares fit, or χ^2 fit, which is mentioned in Ref. [14].

Provide that $\lambda(x;\boldsymbol{\theta})$ is a linear function of parameters θ_j

$$\lambda(x;\boldsymbol{\theta}) = \sum_{j=1}^{m} A_{ij}\theta_j, \tag{2.2.9}$$

where $A_{ij} = a_j(x_i)$ are linearly independent functions of x.

With the value of A_{ij} obtained, we now fit for the estimator $\hat{\theta}_j$ from the data set of measured values y_i and predicted values λ_i. The estimators are best fitted when the following quantity of χ^2 is minimized:

$$\chi^2 = (\boldsymbol{y} - \boldsymbol{\lambda})^T V^{-1} (\boldsymbol{y} - \boldsymbol{\lambda}) \tag{2.2.10}$$

$$= (\boldsymbol{y} - A\boldsymbol{\theta})^T V^{-1} (\boldsymbol{y} - A\boldsymbol{\theta}), \tag{2.2.11}$$

whereas, $\boldsymbol{y} = (y_1, ..., y_N)$ is the measured-value vector, while $\boldsymbol{\lambda} = (\lambda_1, ..., \lambda_N)$ is predicted-value vector with the components of $\lambda_i = \lambda(x_i;\boldsymbol{\theta})$.
The χ^2 reaches minimum values when its derivatives with respect to parameters θ_i equals to zero:

$$\nabla\chi^2 = -2(A^T V^{-1}\boldsymbol{y} - A^T V^{-1} A\boldsymbol{\theta}) = 0. \tag{2.2.12}$$

Providing that the $(A^TV^{-1}A)^{-1}$ is not singular, we can solve for the estimators $\hat{\boldsymbol{\theta}}$

$$\hat{\boldsymbol{\theta}} = (A^TV^{-1}A)^{-1}A^TV^{-1}\boldsymbol{y} \equiv B\boldsymbol{y}. \tag{2.2.13}$$

The covariance matrix for the estimators can be derived by using error propagation:

$$U = BVB^T = (A^TV^{-1}A)^{-1}, \tag{2.2.14}$$

whereas $U_{jk} = \mathrm{cov}[\hat{\theta}_j, \hat{\theta}_k]$.

2.3 Usage in our model

For the observable O_i, with $O_{\mathrm{exp},i}$ stands for experimental value and $O_{\mathrm{theo},i}$ for theoretical value

$$O_i = O^{\mathrm{SM}}_{\mathrm{theo},i}\left(1 + \sum_j c_j\hat{C}_j\right) + \Delta\tilde{O}_i, \tag{2.3.15}$$

whereas, $\hat{C}_i$ stand for estimators of Wilson coefficients, c_i are their corresponding constants depending on the input parameters given in Section 2.1. O^{SM}_{theo} stands for complete SM result (with higher order corrections), $\Delta\tilde{O}$ contains the radiative corrections to the dimension six Wilson coefficients. In this context, $\Delta\tilde{O}$ is neglected as we assume it is very small. We have:

$$\boldsymbol{y} = \boldsymbol{O}_{\mathrm{exp}} - \boldsymbol{O}^{\mathrm{SM}}_{\mathrm{theo}} \tag{2.3.16}$$

The matrix element A_{ij} corresponds for $O^{\mathrm{SM}}_{\mathrm{theo},i} \cdot c_j$, where the covariance matrix $V = V_{\mathrm{exp}} + V_{\mathrm{theo}}$, with V_{exp} is the experimental covariance matrix obtained from Table 2.2, Table 2.3, and V_{theo} is the diagonal matrix of theoretical variances obtaining from Table 2.2:

$$V^{ij} = V^{ij}_{\mathrm{exp}} + V^{ij}_{\mathrm{theo}} = \sigma^i_{\mathrm{exp}}\rho^{ij}\sigma^j_{\mathrm{exp}} + \sigma^i_{\mathrm{theo}}\delta^{ij}\sigma^j_{\mathrm{theo}}, \tag{2.3.17}$$

where ρ^{ij} stands for correlation between two observables O_i and O_j; $\delta^{ij} = 1$ when $i = j$ and $\delta^{ij} = 0$ when $i \neq j$.

2.4 P-value of the fit

An alternative way to evaluate the goodness-of-fit test results is to used P-value, or sometime called “observed significance level” or “confidence level” of the test (Ref. [14]).

First, considering the probability distribution function of χ^2 distribution:

$$f(\chi^2;\nu) = \frac{1}{2^{\nu/2}\Gamma\left(\nu/2\right)}(\chi^2)^{\nu/2-1}e^{-\chi^2/2}, \tag{2.4.18}$$

where as ν is the degrees of freedom (d.o.f) and equals the difference between the number of observables and number of Wilson coefficients, $\nu = N_{\mathrm{Obs.}} - N_{\mathrm{Wilson\ coeff.}}$; $\Gamma\left(\nu/2\right)$ is the Euler Gamma function, $\Gamma\left(z\right) = \int_0^\infty x^{z-1}e^{-x}dx$.

P-value is defined as Ref. [14]: The probability, under assumption of the null hypothesis H_0, of obtaining a result as compatible or less with H_0 than the one actually observed and calculated as follow:

$$\text{P-value} = \int_{\chi^2_{\mathrm{min}}}^{\infty} f(\chi^2;\nu)d\chi^2. \tag{2.4.19}$$

2.5 Pull of the observables

Obtained the central values for D6 operators, we calculate the "pull" for the observables, compared to the experimental data,

$$\text{Pull}_{O_i} = \frac{O_{\text{fit},i} - O_{\text{exp.},i}}{\sqrt{(\delta O_{\text{fit},i})^2 + (\delta O_{\text{exp.},i})^2}}, \tag{2.5.20}$$

whereas $O_{\text{fit},i}$ is the fitted observables,

$$O_{\text{fit},i} = O^{\text{SM}}_{\text{theo.},i}\left(1 + \sum_j c^i_j \hat{C}_j\right). \tag{2.5.21}$$

We define:

$$X^i \equiv 1 + \sum_j c^i_j \hat{C}_j. \tag{2.5.22}$$

Now, in order to find $\delta O_{\text{fit},i}$, we use the propagation of uncertainty formula, given in Ref. [14]:

$$\sigma^2(\boldsymbol{y}) \simeq \sum_{i,j=1}^{n}\left[\frac{\partial y}{\partial x_i}\frac{\partial y}{\partial x_j}\right]_{\boldsymbol{x}=\boldsymbol{\mu}} V_{ij}, \tag{2.5.23}$$

where the function $y(\boldsymbol{x})$ is expanded to the first order about the mean value $\boldsymbol{\mu}$ of $\boldsymbol{x}$.

$$y(\boldsymbol{x}) \simeq (\boldsymbol{\mu}) + \sum_{i=1}^{n}\left[\frac{\partial y}{\partial x_i}\right]_{\boldsymbol{x}=\boldsymbol{\mu}}(x_i - \mu_i). \tag{2.5.24}$$

Applied to our problem, assume there is no correlation between the $O^{\text{SM}}_{\text{theo.},i}$ and the Wilson coefficients $\hat{C}_j$. We can obtain the $\delta O_{\text{fit},i}$ by the propagation of uncertainty:

$$(\delta O_{\text{fit},i})^2 = (O^{\text{SM}}_{\text{theo.},i})^2 \cdot (\delta X^i)^2 + (\delta O_{\text{theo.},i})^2 \cdot (X^i)^2, \tag{2.5.25}$$

with the $(\delta X^i)^2$ reads:

$$(\boldsymbol{\delta X})^2 = \text{Diag}\left(M_{\text{Coeff.}} \cdot M_{\text{Cov.}} \cdot M^T_{\text{Coeff.}}\right), \tag{2.5.26}$$

where as, $(\boldsymbol{\delta X})^2$ is a vector with components $(\delta X^i)^2$. The matrices $M_{\text{Coeff.}}$, $M_{\text{Cov.}}$ respectively stands for coefficient matrix ($M^{i,j}_{\text{Coeff.}} = c^i_j$) and covariance matrix (between D6 Wilson coefficients), with "Diag()" stands for taking the diagonal elements of a matrix.

Chapter 3

Constraints on gauge sector Wilson coefficients

3.1 Gauge sector Lagrangian

In this chapter, we consider only D6 operators related to boson fields, i.e. gauge boson and scalar boson fields. Operators with fermionic fields will be added in the next chapters.

The Lagrangian for the gauge boson propagation:

$$\begin{aligned}\mathcal{L}_{\text{EW}} &= -\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\varphi)^\dagger(D^\mu\varphi) \\ &+ \frac{1}{v^2}\Big[C_{\varphi W}(\varphi^\dagger\varphi)W^I_{\mu\nu}W^{I\mu\nu} + C_{\varphi B}(\varphi^\dagger\varphi)B_{\mu\nu}B^{\mu\nu} + C_{\varphi WB}(\varphi^\dagger\tau^I\varphi)W^I_{\mu\nu}B^{\mu\nu} \\ &+ C_{\varphi D}(\varphi^\dagger D_\mu\varphi)^*(\varphi^\dagger D^\mu\varphi)\Big]\,, \end{aligned} \tag{3.1.1}$$

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} + \frac{1}{v^2}C^{\varphi G}(\varphi^\dagger\varphi)G^A_{\mu\nu}G^{A\mu\nu}\,, \tag{3.1.2}$$

Note that since the operators $\widetilde{B}_{\mu\nu}$, $\widetilde{W}^I_{\mu\nu}$ and $\widetilde{G}^A_{\mu\nu}$ influence only CP-violating vertices and their bilinear terms are total derivatives and do not affect propagators (Ref. [15]). We shall neglect them in our discussion here.

Now consider:

$$(D_\mu\varphi)^\dagger(D^\mu\varphi) \supset g^2\frac{v^2}{8}(W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) \tag{3.1.3}$$

$$+\frac{v^2}{8}\begin{pmatrix}W^3_\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{3\mu} \\ B^\mu\end{pmatrix} \tag{3.1.4}$$

$$(\varphi^\dagger D^\mu\varphi)^*(\varphi^\dagger D^\mu\varphi) \supset \frac{v^2}{16}\begin{pmatrix}W^3_\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{3\mu} \\ B^\mu\end{pmatrix}. \tag{3.1.5}$$

The EW term is rewritten as:

$$\begin{aligned}\mathcal{L}_{EW} =& -\frac{1}{4}(1-2C^{\varphi W})W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}(1-2C^{\varphi B})B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}C^{\varphi WB}W^3_{\mu\nu}B^{\mu\nu} \\ &+ g^2\frac{v^2}{8}(W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) + \frac{v^2}{8}\left(1+\frac{1}{2}C^{\varphi D}\right)\begin{pmatrix}W^3_\mu & B_\mu\end{pmatrix}\begin{pmatrix}g^2 & -gg' \\ -gg' & g'^2\end{pmatrix}\begin{pmatrix}W^{3\mu} \\ B^\mu\end{pmatrix}.\end{aligned} \tag{3.1.6}$$

Now we rescale our gauge fields and couplings as:

$$\bar{W}^I_\mu \equiv \sqrt{1-2C^{\varphi W}} W^I_\mu \approx (1-C^{\varphi W}) W^I_\mu, \qquad \bar{g} \equiv (1+C^{\varphi W}) g \ , \tag{3.1.7}$$

$$\bar{B}_\mu \equiv \sqrt{1-2C^{\varphi B}} B_\mu \approx (1-C^{\varphi B}) B_\mu \ , \qquad \bar{g}' \equiv (1+C^{\varphi B}) g', \tag{3.1.8}$$

$$\bar{G}^A_\mu \equiv \sqrt{1-2C^{\varphi G}} G^A_\mu \approx (1-C^{\varphi G}) G^A_\mu, \qquad \bar{g}_s \equiv (1+C^{\varphi G}) g_s \ , \tag{3.1.9}$$

with the symbol "≈" standing for approximation to the linear order of Wilson coefficients. The gauge invariance is preserved in that such transformation, and also the form of the covariant derivative, which now reads,

$$D_\mu = \bar{D}_\mu = \partial_\mu + i\bar{g}\bar{B}_\mu Y + i\bar{g}\bar{W}^I_\mu T^I + i\bar{g}_s\bar{G}^A_\mu \mathcal{T}^A \ , \tag{3.1.10}$$

while the field strength tensors rescale the same way as their respective fields. Furthermore, we have $\bar{G}^A_\mu \bar{g}_s \approx G^A_\mu g_s$, which makes QCD Lagrangian terms unchanged compared to SM.
The bilinear part of the EW Lagrangian reads:

$$\begin{aligned}
\mathcal{L}^{\text{Bilinear}}_{EW} = & -\frac{1}{4}(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}) - \frac{1}{4}\begin{pmatrix}\bar{W}^3_{\mu\nu} & \bar{B}_{\mu\nu}\end{pmatrix}\begin{pmatrix}1 & \epsilon \\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu}\end{pmatrix} \\
& + \frac{\bar{g}^2 v^2}{8}(\bar{W}^1_\mu \bar{W}^{1\mu} + \bar{W}^2_\mu \bar{W}^{2\mu}) \\
& + \frac{v^2}{8}\left(1 + \frac{C_{\varphi D}}{2}\right)\begin{pmatrix}\bar{W}^3_\mu & \bar{B}_\mu\end{pmatrix}\begin{pmatrix}\bar{g}^2 & -\bar{g}\bar{g}' \\ -\bar{g}\bar{g}' & \bar{g}'^2\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu} \\ \bar{B}^\mu\end{pmatrix}
\end{aligned} \tag{3.1.11}$$

$$\begin{aligned}
= & -\frac{1}{2}W^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4}\begin{pmatrix}\bar{W}^3_{\mu\nu} & \bar{B}_{\mu\nu}\end{pmatrix}\begin{pmatrix}1 & \epsilon \\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu}\end{pmatrix} \\
& + \frac{\bar{g}^2 v^2}{4}W^+_\mu W^{-\mu} \\
& + \frac{v^2}{8}\left(1 + \frac{C_{\varphi D}}{2}\right)\begin{pmatrix}\bar{W}^3_\mu & \bar{B}_\mu\end{pmatrix}\begin{pmatrix}\bar{g}^2 & -\bar{g}\bar{g}' \\ -\bar{g}\bar{g}' & \bar{g}'^2\end{pmatrix}\begin{pmatrix}\bar{W}^{3\mu} \\ \bar{B}^\mu\end{pmatrix}
\end{aligned} \tag{3.1.12}$$

where we have defined,

$$\epsilon \equiv C_{\varphi WB} \ . \tag{3.1.13}$$

In Eq. (3.1.12), we have identified the physical charged gauge bosons $W^\pm_\mu$ as:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(\bar{W}^1_\mu \mp i\bar{W}^2_\mu), \tag{3.1.14}$$

with,

$$\bar{m}^2_W = \frac{v^2\bar{g}^2}{4}. \tag{3.1.15}$$

Now considering the Lagrangian of the form:

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{2}W^+_{\mu\nu}W^-_{\mu\nu} - \frac{1}{4}\bar{W}^3_{\mu\nu}\bar{W}^3_{\mu\nu} - \frac{1}{4}\bar{B}_{\mu\nu}\bar{B}_{\mu\nu} - \frac{1}{2}\mathcal{G}\bar{W}^3_{\mu\nu}\bar{B}_{\mu\nu} \\
& + m^2_W W^+_\mu W^-_\mu + \frac{1}{2}m^2_3\bar{W}^3_\mu\bar{W}^3_\mu + \frac{1}{2}m^2_B\bar{B}_\mu\bar{B}_\mu - m^2_{B3}\bar{W}^3_\mu\bar{B}_\mu.
\end{aligned} \tag{3.1.16}$$

where $\mathcal{G}$, m_W^2, m_3^2 and m_B^2 are free parameters and $m_{B3}^2 = m_B m_3$.
The Lagrangian can be diagonalized and normalized by the transformation:

$$\begin{pmatrix} \bar{W}^{3\mu} \\ \bar{B}^{\mu} \end{pmatrix} = \mathbb{X} \begin{pmatrix} Z^{\mu} \\ A^{\mu} \end{pmatrix}, \tag{3.1.17}$$

where the matrix X expresses the transformation of a rotation, a rescaling, and another rotation as Ref. [16]:

$$\mathbb{X} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{1-\mathcal{G}} & 0 \\ 0 & 1/\sqrt{1+\mathcal{G}} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \tag{3.1.18}$$

with

$$\tan\varphi = \frac{(m_3 + m_B)/\sqrt{1+\mathcal{G}} - (m_3 - m_B)/\sqrt{1-\mathcal{G}}}{(m_3 + m_B)/\sqrt{1+\mathcal{G}} + (m_3 - m_B)/\sqrt{1-\mathcal{G}}}. \tag{3.1.19}$$

We obtain the Lagrangian with canonical kinetic terms and diagonal masses:

$$\mathcal{L} = -\frac{1}{2} W^+_{\mu\nu} W^-_{\mu\nu} + m_W^2 W^+_{\mu} W^-_{\mu} - \frac{1}{4} Z_{\mu\nu} Z_{\mu\nu} + \frac{1}{2} m_Z^2 Z_{\mu} Z_{\mu} - \frac{1}{4} A_{\mu\nu} A_{\mu\nu}, \tag{3.1.20}$$

whereas,

$$m_Z^2 = \frac{m_3^2 + m_B^2 - 2\mathcal{G} m_3 m_B}{1 - \mathcal{G}^2}. \tag{3.1.21}$$

With $\mathcal{G} \equiv -\epsilon$, taking all terms to the linear order of Wilson coefficients, the matrix $\mathbb{X}$ becomes:

$$\mathbb{X} = \begin{pmatrix} 1 & -\frac{\epsilon}{2} \\ -\frac{\epsilon}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos\bar{\theta} & \sin\bar{\theta} \\ -\sin\bar{\theta} & \cos\bar{\theta} \end{pmatrix}. \tag{3.1.22}$$

The mixing angle now reads:

$$\tan\bar{\theta} = \frac{\bar{g}'}{\bar{g}} + \frac{\epsilon}{2}\left(1 - \frac{\bar{g}'^2}{\bar{g}^2}\right), \tag{3.1.23}$$

so that:

$$\bar{s}_W \equiv \sin\bar{\theta}_W = \frac{\bar{g}'}{\sqrt{\bar{g}'^2 + \bar{g}^2}} \left(1 + \frac{\epsilon}{2} \cdot \frac{\bar{g}}{\bar{g}'} \cdot \frac{\bar{g}^2 - \bar{g}'^2}{\bar{g}^2 + \bar{g}'^2}\right), \tag{3.1.24}$$

$$\bar{c}_W \equiv \cos\bar{\theta}_W = \frac{\bar{g}}{\sqrt{\bar{g}'^2 + \bar{g}^2}} \left(1 - \frac{\epsilon}{2} \cdot \frac{\bar{g}'}{\bar{g}} \cdot \frac{\bar{g}^2 - \bar{g}'^2}{\bar{g}^2 + \bar{g}'^2}\right). \tag{3.1.25}$$

For the Z boson mass we obtain

$$\bar{m}_Z^2 = \frac{v^2}{4}\left(\bar{g}^2 + \bar{g}'^2\right)\left(1 + \frac{1}{2} C_{\varphi D} + \frac{2\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2} C_{\varphi WB}\right). \tag{3.1.26}$$

The photon remains massless since the determinant of the mass matrix in Eq. (3.1.12) vanishes. Now for convenience, we introduce some conventions:

$$\tilde{s}_W^+ \equiv \bar{s}_W + \frac{\epsilon}{2}\bar{c}_W \approx \frac{\bar{g}'}{\sqrt{\bar{g}'^2+\bar{g}^2}}\left(1+\frac{\epsilon}{2}\cdot\frac{\bar{g}}{\bar{g}'}\cdot\frac{2\bar{g}^2}{\bar{g}^2+\bar{g}'^2}\right), \tag{3.1.27}$$

$$\tilde{s}_W^- \equiv \bar{s}_W - \frac{\epsilon}{2}\bar{c}_W \approx \frac{\bar{g}'}{\sqrt{\bar{g}'^2+\bar{g}^2}}\left(1-\frac{\epsilon}{2}\cdot\frac{\bar{g}}{\bar{g}'}\cdot\frac{2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\right), \tag{3.1.28}$$

$$\tilde{c}_W^+ \equiv \bar{c}_W + \frac{\epsilon}{2}\bar{s}_W \approx \frac{\bar{g}}{\sqrt{\bar{g}'^2+\bar{g}^2}}\left(1+\frac{\epsilon}{2}\cdot\frac{\bar{g}'}{\bar{g}}\cdot\frac{2\bar{g}'^2}{\bar{g}^2+\bar{g}'^2}\right), \tag{3.1.29}$$

$$\tilde{c}_W^- \equiv \bar{c}_W - \frac{\epsilon}{2}\bar{s}_W \approx \frac{\bar{g}}{\sqrt{\bar{g}'^2+\bar{g}^2}}\left(1-\frac{\epsilon}{2}\cdot\frac{\bar{g}'}{\bar{g}}\cdot\frac{2\bar{g}^2}{\bar{g}^2+\bar{g}'^2}\right), \tag{3.1.30}$$

The matrix $\mathbb{X}$ is rewritten as:

$$\mathbb{X} = \begin{pmatrix} \tilde{c}_W^+ & \tilde{s}_W^- \\ -\tilde{s}_W^+ & \tilde{c}_W^- \end{pmatrix} \tag{3.1.31}$$

We rewrite the EW covariant derivative in term of physical fields:

$$\begin{aligned}\bar{D}_\mu^{\text{EW}} &= \partial_\mu + i\frac{\bar{g}}{\sqrt{2}}(T^+W_\mu^+ + T^-W_\mu^-) + i(\bar{g}\tilde{c}_W^+T^3 - \bar{g}'\tilde{s}_W^+Y)Z_\mu + i(\bar{g}\tilde{s}_W^-T^3 + \bar{g}'\tilde{c}_W^-Y)A_\mu & (3.1.32)\\ &\equiv \partial_\mu + i\bar{g}_W(T^+W_\mu^+ + T^-W_\mu^-) + i\bar{g}_Z(T^3 - \bar{s}_W^2Q)Z_\mu + i\bar{e}QA_\mu, & (3.1.33)\end{aligned}$$

whereas we redefine the effective electric charge and couplings:

$$\begin{aligned}\bar{e} &\equiv \bar{g}\tilde{s}_W^- \approx \bar{g}'\tilde{c}_W^- \approx \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2+\bar{g}'^2}}\left(1-\frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right), & (3.1.34)\\ \bar{g}_Z &\equiv \frac{\bar{g}\cdot\tilde{c}_W^+}{\bar{c}_W^2} \approx \frac{\bar{g}'\cdot\tilde{s}_W^+}{\bar{s}_W^2} \approx \frac{\bar{e}}{\bar{s}_W\bar{c}_W}\left(1+\frac{\bar{g}^2+\bar{g}'^2}{2\bar{g}\bar{g}'}\epsilon\right) \approx \sqrt{\bar{g}^2+\bar{g}'^2}\left(1+\frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2+\bar{g}'^2}\right), & (3.1.35)\\ \bar{g}_W &\equiv \frac{\bar{g}}{\sqrt{2}}\,. & (3.1.36)\end{aligned}$$

We can now derive the neutral gauge boson - fermions interation Lagrangian:

$$\mathcal{L}_{int}^{Zff} = -\bar{g}_Z Z_\mu J_Z^\mu, \tag{3.1.37}$$

with the neutral current,

$$J_Z^\mu = \frac{1}{2}\bar{f}\left[\left(T^3 - 2\bar{s}_W^2Q\right)\gamma^\mu - T^3\gamma^\mu\gamma^5\right]f. \tag{3.1.38}$$

3.2 Comparing with NNP approach

Now we compare the rescaling method above with NNP approach in Ref. [2]. In our approach, our basis of D6 operators in the sectors of EW gauge boson fields and gauge boson fields combined with the SM Higgs fields differs from Ref. [2]. We exclude the operator $Q_\varphi^{(1)}$ of $(\varphi^\dagger\varphi)(D_\mu\varphi)^\dagger(D^\mu\varphi)$ and add two additional terms: Q_φ of $(\varphi^\dagger\varphi)^3$ and $Q_{\varphi\Box}$ of $(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$. The two operators $Q_{\varphi W}$, $Q_{\varphi B}$ are also taken into account with the difference of an additional factor 1/2 in Ref. [2].

With the following notation for D6 operators terms:

$$a = 1 - 2c'_{\rm w}s'_{\rm w}C_{\varphi WB} - c'^2_{\rm w}C_{\varphi W} - s'^2_{\rm w}C_{\varphi B}, \tag{3.2.39}$$

$$b = \left(c'^2_{\rm w} - s'^2_{\rm w}\right) C_{\varphi WB} + c'_{\rm w}s'_{\rm w}\left(C_{\varphi B} - C_{\varphi W}\right), \tag{3.2.40}$$

$$d = 1 + 2c'_{\rm w}s'_{\rm w}C_{\varphi WB} - s'^2_{\rm w}C_{\varphi W} - c'^2_{\rm w}C_{\varphi B}, \tag{3.2.41}$$

$$t = ad - b^2, \tag{3.2.42}$$

the fields and field strengths,

$$\boldsymbol{V}'_\mu = \left(Z'_\mu, A'_\mu\right)^{\rm T}, \tag{3.2.43}$$

$$\boldsymbol{V}'_{\mu\nu} = \partial_\mu \boldsymbol{V}'_\nu - \partial_\nu \boldsymbol{V}'_\mu\ , \tag{3.2.44}$$

$$W'^{\pm}_{\mu\nu} = \partial_\mu W'^{\pm}_\nu - \partial_\nu W'^{\pm}_\mu, \tag{3.2.45}$$

where the prime denotes physical gauge-boson fields if all D6 terms vanish. The physical gauge-boson fields of the full Lagrangian are without the prime, namely A_μ, Z_μ, W_μ. Their approach is to rotate and rescale directly the physical gauge boson fields by the transformation:

$$\boldsymbol{V}'_\mu = C\boldsymbol{V}_\mu, \tag{3.2.46}$$

whereas,

$$C = \begin{pmatrix} \sqrt{d/t} & 0 \\ -b/\sqrt{dt} & 1/\sqrt{d} \end{pmatrix}, \tag{3.2.47}$$

and by the relations of:

$$T = C^{\rm T}T'C = \mathbb{1}, \qquad M = C^{\rm T}M'C = \begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.2.48}$$

whereas,

$$T' = \begin{pmatrix} a & b \\ b & d \end{pmatrix}, \qquad M' = m'^2_Z\left(1 + \frac{1}{2}\left(h^{(1)}_\varphi + h^{(3)}_\varphi\right)\right)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.2.49}$$

with

$$m'^2_Z = (g^2 + g'^2)v^2/4, \qquad m'^2_W = g^2v^2/4, \tag{3.2.50}$$

to simultaneously rescale the kinetic terms and diagonalize the mass terms of the effective Lagrangian:

$$\mathcal{L}^{\rm eff.} = -\frac{1}{4}\boldsymbol{V}'^{\rm T}_{\mu\nu}\ T'\ \boldsymbol{V}'^{\mu\nu} + \frac{1}{2}\boldsymbol{V}'^{\rm T}_\mu\ M'\ \boldsymbol{V}'^\mu \tag{3.2.51}$$

$$-\frac{1}{2}\left(1 - h_{\varphi W}\right)W'^+_{\mu\nu}W'^{-\mu\nu} + m'^2_W\left(1 + h^{(1)}_\varphi/2\right)W'^+_\mu W'^{-\mu}\ , \tag{3.2.52}$$

into the standard form of:

$$\mathcal{L}^{\rm eff.} = -\frac{1}{4}(Z_{\mu\nu}Z^{\mu\nu} + A_{\mu\nu}A^{\mu\nu}) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \tag{3.2.53}$$

$$-\frac{1}{2}W^+_{\mu\nu}W^{\mu\nu-} + m_W^2 W^+_\mu W^{\mu-}, \tag{3.2.54}$$

with:

$$m_W^2 = \frac{1 + h_\varphi^{(1)}/2}{1 - h_{\varphi W}} \cdot m_W'^2, \qquad m_Z^2 = \frac{d}{t}\left(1 + \frac{1}{2}(h_\varphi^{(1)} + h_\varphi^{(3)})\right) \cdot m_Z'^2. \tag{3.2.55}$$

This is also a beautiful approach. Within the approximation to the first order of D6 Wilson coefficients, all the physical results derived by the two approaches, such as effective EW gauge boson masses of $\bar{m}_Z$, $\bar{m}_W$, effective couplings $\bar{g}_Z$, $\bar{g}_W$, effective electric charge $\bar{e}$, and even the effective $\bar{s}_W^2$ (as we shall see in the next section) are exactly the same. It is important to note that the D6 operator basis in Ref. [2] is wrong as we have mentioned earlier, however since the operators of $Q_\varphi^{(1)}$, Q_φ, $Q_{\varphi\Box}^{(1)}$ does not contribute to the physical results above, the final results in the two approach should be the same.

3.3 P_Z scheme

The P_Z scheme is defined in Ref. [2]. Here we use this scheme for our rescaling method with Warsaw basis.

We expand the gauge boson masses to the first order in the Wilson coefficients:

$$\bar{m}_Z^2 = \frac{v^2}{4}\left(\bar{g}^2 + \bar{g}'^2\right)\left(1 + \frac{1}{2}C_{\varphi D} + \frac{2\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2}C_{\varphi WB}\right) \tag{3.3.56}$$

$$\approx m_Z^2\left(1 + 2c_W^2 C_{\varphi W} + 2s_W^2 C_{\varphi B} + \frac{1}{2}C_{\varphi D} + 2s_W c_W C_{\varphi WB}\right), \tag{3.3.57}$$

$$\bar{m}_W^2 = \frac{\bar{g}^2 v^2}{4} \tag{3.3.58}$$

$$\approx m_W^2\left(1 + 2C_{\varphi W}\right). \tag{3.3.59}$$

whereas,

$$c_W = \frac{g}{\sqrt{g^2 + g'^2}}, \qquad s_W = \frac{g'}{\sqrt{g^2 + g'^2}}. \tag{3.3.60}$$

Substitute Eq. (3.3.57) and Eq. (3.3.59) into the equation $m_W^2 = c_W^2 m_Z^2$, we obtain:

$$\bar{m}_W^2 = \frac{(1 + 2C_{\varphi W})}{1 + 2c_W^2 C_{\varphi W} + 2s_W^2 C_{\varphi B} + \frac{1}{2}C_{\varphi D} + 2s_W c_W C_{\varphi WB}} c_W^2 \bar{m}_Z^2. \tag{3.3.61}$$

The charge reads,

$$\bar{e}^2 = \frac{\bar{g}\bar{g}'}{\sqrt{\bar{g}^2 + \bar{g}'^2}}\left(1 - \frac{\epsilon\bar{g}\bar{g}'}{\bar{g}^2 + \bar{g}'^2}\right) \tag{3.3.62}$$

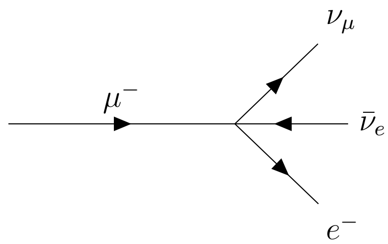
$$\approx e^2\left(1 + 2s_W^2 C_{\varphi W} + 2c_W^2 C_{\varphi B} - 2s_W c_W C_{\varphi WB}\right), \tag{3.3.63}$$

and charged current coupling reads:

$$\bar{g}_W^2 = \frac{\bar{g}^2}{2} \tag{3.3.64}$$

$$= \frac{\bar{e}^2}{2s_W^2}\left(1 - 2s_W^2 C_{\varphi W} - 2c_W^2 C_{\varphi B} + 2s_W c_W C_{\varphi WB}\right)\left(1 + 2C_{\varphi W}\right). \tag{3.3.65}$$

In the low energy limit, where the propagator of W boson become point-like, Fermi's constant is given by interactions of two charged-current, (see Sect. 22.3 of Ref. [17]), including all possible Wilson coefficients, we have:



$$\frac{4}{\sqrt{2}}\bar{G}_F = \frac{\bar{g}_W^2}{\bar{m}_W^2}\left(1 + C_{\varphi l}^{(3)ee} + C_{\varphi l}^{(3)\mu\mu}\right) - \frac{C_{ll}^{\mu ee\mu}}{v^2} - \frac{C_{ll}^{e\mu\mu e}}{v^2}. \tag{3.3.66}$$

For lepton flavor universality:

$$\frac{4}{\sqrt{2}}\bar{G}_F = \frac{\bar{g}_W^2}{\bar{m}_W^2}\left(1 + 2C_{\varphi l}^{(3)}\right) - \frac{2}{v^2}C_{ll}, \tag{3.3.67}$$

$$= \frac{\bar{g}_W^2}{\bar{m}_W^2}\left(1 + 2C_{\varphi l}^{(3)} - C_{ll}\right). \tag{3.3.68}$$

In this analysis, we assume that all the four-fermion contributions are nulls. We have:

$$\bar{G}_F = \frac{\sqrt{2}\bar{g}_W^2}{4\bar{m}_W^2}\left(1 + 2C_{\varphi l}^{(3)}\right). \tag{3.3.69}$$

$$\begin{aligned} &= \frac{\sqrt{2}}{4}\frac{\bar{e}^2}{2s_W^2}\frac{1}{c_W^2\bar{m}_Z^2}\Big[1 + 2(c_W^2 - s_W^2)C_{\varphi W} + 2(s_W^2 - c_W^2)C_{\varphi B} \\ &+ \frac{1}{2}C_{\varphi D} + 4s_W c_W C_{\varphi WB} + 2C_{\varphi l}^{(3)}\Big]. \end{aligned} \tag{3.3.70}$$

From Eq. (3.3.70), we obtain an equation for s_W^2 in quadratic form:

$$\begin{aligned} s_W^4 - s_W^2 + \frac{\bar{e}^2}{4\sqrt{2}\bar{G}_F\bar{m}_Z^2}\Big[1 + 2(c_W^2 - s_W^2)C_{\varphi W} + 2(s_W^2 - c_W^2)C_{\varphi B} \\ + \frac{1}{2}C_{\varphi D} + 4s_W c_W C_{\varphi WB} + 2C_{\varphi l}^{(3)}\Big] = 0. \end{aligned} \tag{3.3.71}$$

Solving for the Eq. (3.3.71):

$$\begin{aligned} s_W^2 =& \frac{1}{2}\Bigg\{1 \pm \Bigg\{1 - \frac{\bar{e}^2}{\sqrt{2}\bar{G}_F\bar{m}_Z^2}\Big[1 + 2(c_W^2 - s_W^2)C_{\varphi W} \\ &+ 2(s_W^2 - c_W^2)C_{\varphi B} + \frac{1}{2}C_{\varphi D} + 4s_W c_W C_{\varphi WB} + 2C_{\varphi l}^{(3)}\Big]\Bigg\}^{\frac{1}{2}}\Bigg\}. \end{aligned} \tag{3.3.72}$$

For the two solutions above, we adopt only the "-" one, since the "+" is far from the physical value. For the case of all Wilson coefficient are set to zero, we denote:

$$s_0^2 = \frac{1}{2}\Bigg\{1 - \left(1 - \frac{\bar{e}^2}{\sqrt{2}\bar{G}_F\bar{m}_Z^2}\right)^{\frac{1}{2}}\Bigg\}, \tag{3.3.73}$$

To the first order in Wilson coefficient, the un-rescaled weak mixing angle reads,

$$s_W^2 \approx s_0^2\left[1 + 2c_0^2(C_{\varphi W} - C_{\varphi B}) + \frac{4s_0c_0^3}{c_0^2 - s_0^2}C_{\varphi WB} + \frac{c_0^2}{2(c_0^2 - s_0^2)}\left(C_{\varphi D} + 4C_{\varphi l}^{(3)}\right)\right], \tag{3.3.74}$$

and the rescaled one:

$$\bar{s}_W^2 \approx s_0^2\left[1 + \frac{c_0}{s_0(c_0^2 - s_0^2)}C_{\varphi WB} + \frac{c_0^2}{2(c_0^2 - s_0^2)}\left(C_{\varphi D} + 4C_{\varphi l}^{(3)}\right)\right]. \tag{3.3.75}$$

This quantity of $\bar{s}_W^2$ is equivalent, and exactly the same with the $\sin^2_{\text{eff}}$ in Ref. [2], expressed in P_Z scheme.

Our observables of interest now can be rewritten in terms of $\bar{s}_W^2$:

The Z-boson decay width to two fermions:

$$\Gamma_{f\bar{f}}^{\text{tree}} = \frac{\bar{g}_Z^2\bar{m}_Z}{48\pi}N_c^f\chi^f \tag{3.3.76}$$

with,

$$\chi^\nu = \frac{1}{2}, \qquad \chi^l = 4\bar{s}_W^4 - 2\bar{s}_W^2 + \frac{1}{2}, \tag{3.3.77}$$

$$\chi^u = \frac{16}{9}\bar{s}_W^4 - \frac{4}{3}\bar{s}_W^2 + \frac{1}{2}, \qquad \chi^d = \frac{4}{9}\bar{s}_W^4 - \frac{2}{3}\bar{s}_W^2 + \frac{1}{2}. \tag{3.3.78}$$

The forward-backward asymmetries:

$$A_{\text{FB}}^{0,\text{f}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f, \tag{3.3.79}$$

whereas,

$$\mathcal{A}_f = 2c_V^f c_A^f/\chi^f, \tag{3.3.80}$$

with,

$$\mathcal{A}_\nu = 1, \qquad \mathcal{A}_\ell = \left(\frac{1}{2} - 2\bar{s}_W^2\right)/\chi_\ell, \tag{3.3.81}$$

$$\mathcal{A}_\text{u} = \left(\frac{1}{2} - \frac{4}{3}\bar{s}_W^2\right)/\chi_\text{u}, \quad \mathcal{A}_\text{d} = \left(\frac{1}{2} - \frac{2}{3}\bar{s}_W^2\right)/\chi_\text{d}. \tag{3.3.82}$$

Beside the $\bar{s}_W^2$, we also have these quantities modified:

$$\bar{m}_W = c_0\bar{m}_Z\left[1 + \frac{s_0c_0}{s_0^2 - c_0^2}C_{\varphi WB} + \frac{c_0^2}{4(s_0^2 - c_0^2)}\left(C_{\varphi D} + 4C_{\varphi l}^{(3)}\right)\right]. \tag{3.3.83}$$

$$\bar{g}_W = \frac{\bar{e}}{\sqrt{2}s_0}\left[1 + \frac{s_0c_0}{s_0^2 - c_0^2}C_{\varphi WB} + \frac{c_0^2}{4(s_0^2 - c_0^2)}\left(C_{\varphi D} + 4C_{\varphi l}^{(3)}\right)\right]. \tag{3.3.84}$$

$$\bar{\Gamma}_W = \frac{\bar{e}^2}{48\pi}\frac{\bar{m}_Zc_0}{2s_0^2}\left[1 + \frac{s_0c_0}{s_0^2 - c_0^2}C_{\varphi WB} + \frac{c_0^2}{4(s_0^2 - c_0^2)}\left(C_{\varphi D} + 4C_{\varphi l}^{(3)}\right)\right](6\chi_W^{d\bar{u}} + \chi_W^{e\bar{\nu}_e} + \chi_W^{\mu\bar{\nu}_\mu} + \chi_W^{\tau\bar{\nu}_\tau}). \tag{3.3.85}$$

For the case of Wilson coefficients only from gauge sector, we neglect the contribution from $C_{\varphi l}^{(3)}$.

3.4 Comparing with NNP results

Obtaining the observable data from Ref. [2], the set of theoretical values depending on Higg masses because higher-order corrections to the SM prediction are included. Note that Ref. [2] was done when the Higgs mass was not known.
The set of experimental values:

$$m_Z \text{ [GeV]} = 91.1875 \pm 0.0021, \tag{3.4.86}$$
$$\Gamma_Z \text{ [GeV]} = 2.4952 \pm 0.0023, \tag{3.4.87}$$
$$\sigma^0_{\text{had}} \text{ [nb]} = 41.540 \pm 0.037, \tag{3.4.88}$$
$$R^0_\ell = 20.767 \pm 0.025, \tag{3.4.89}$$
$$A^{0,\ell}_{\text{FB}} = 0.0171 \pm 0.0010, \tag{3.4.90}$$
$$\bar{s}^2_W = 0.23148 \pm 0.00017, \tag{3.4.91}$$
$$m_W = 80.449 \pm 0.034, \tag{3.4.92}$$
$$\Gamma_W = 2.136 \pm 0.069. \tag{3.4.93}$$

Here, NNP use the same correlation matrix with ours in Table 2.3. From that matrix, we keep only the 3×3 correlations between Γ_Z, σ_{had}, R^0_ℓ and drop the rest. By adding an additional row and column for $\bar{s}^2_W$ with no correlation to the other observables, we have for $[\Gamma_Z, \sigma^0_{\text{had}}, R^0_\ell, \bar{s}^2_W]$.

$$\begin{pmatrix} 1 & -0.279 & 0.004 & 0 \\ & 1 & 0.183 & 0 \\ & & 1 & 0 \\ & & & 1 \end{pmatrix}. \tag{3.4.94}$$

In this derivation, we need the SM predictions including higher-order effects as explained in Section 2.3. They are provided in Table 3.1. Adding the theoretical uncertainty to the diagonal components of the covariance matrix coming from (3.4.94), using χ^2 method, we derived the results in Table 3.2:
We now include the data of W mass and width in our analysis. With their error correlation of

m_H	120 GeV	200 GeV	500 GeV	δX
s^2_{eff}	0.23156	0.23180	0.23230	0.00030
Γ_Z [GeV]	2.4952	2.4938	2.4902	0.0026
σ^0_{had} [nb]	41.484	41.485	41.489	0.015
R^0_ℓ	20.737	20.732	20.723	0.018
m_W [GeV]	80.374	80.341	80.269	0.041
Γ_W [GeV]	2.0896	2.0880	2.0832	0.0032

Table 3.1: Values of various observables O predicted by the SM for different Higgs masses. The dependence of their uncertainties δO on m_H is negligibly small, taken from Ref. [2].

$\Gamma_Z,\ \sigma^0_{\text{had}},\ R^0_\ell,\ \bar{s}^2_W$					
m_H		120 GeV	200 GeV	500 GeV	$\delta C \times 10^3$
$C_{\varphi WB}$	$\times 10^3$	-0.0619	-0.2421	-0.5027	0.8138
$C_{\varphi D}$	$\times 10^3$	0.4147	-1.0225	-2.8197	2.8187
χ^2_{min}		2.5225	2.5440	2.4012	
P-value	(d.o.f = 2)	0.2833	0.2803	0.3010	

Table 3.2: Our results computed for Higgs mass of 120 GeV, 200 GeV, 500 Gev repectively, with the correlation between $C_{\varphi WB}$ and $C_{\varphi D}$ is -85.8094%. Note that the dependence of δC and the correlation on m_H is very weak and not visible in this table.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$\bar{s}^2_W$
120 GeV	0.169	-1.410	-0.893	-0.077
200 GeV	0.168	-1.402	-0.921	-0.081
500 GeV	0.159	-1.337	-0.934	-0.085

Table 3.3: The pull for observables following the results in Table 3.2.

m_H		120 GeV	200 GeV	500 GeV	$\delta C \times 10^3$
$C_{\varphi WB}$	$\times 10^3$	-0.26	-0.44	-0.68	0.81
$C_{\varphi D}$	$\times 10^3$	0.38	-0.24	-2.08	2.81

Table 3.4: Same as Table 3.2, but this is the result of Ref. [2]. The correlation is -86%.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$\bar{s}^2_W$
120 GeV	-0.021	-1.412	-0.880	-0.127
200 GeV	-0.019	-1.404	-0.907	-0.135
500 GeV	-0.022	-1.338	-0.925	-0.120

Table 3.5: The pull for observables following the results in Table 3.4.

-6.7% (Ref. [4]), we get the correlation matrix for $[\Gamma_Z, \sigma^0_{\text{had}}, R^0_\ell, \bar{s}^2_W, m_W, \Gamma_W]$:

$$\begin{pmatrix} 1 & -0.279 & 0.004 & 0 & 0 & 0 \\ & 1 & 0.183 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ & & & & 1 & -0.067 \\ & & & & & 1 \end{pmatrix} \tag{3.4.95}$$

We obtain the results:
Comparing to NNP, our results well agree with the error values, however, the central values are totally different. The pulls in both case are acceptable since they are smaller than 2σ. NNP results yield better pull for Γ_Z, while our results yield better pulls for $\bar{s}^2_W$ and m_W.
The reason that we do not agree with Ref. [2] for the central values may be that they have used a non-trivial method to project the 5×5 correlation matrix of Table 2.3 on to the $[\Gamma_Z, \sigma^0_{\text{had}}, R^0_\ell]$

		$\Gamma_Z,\ \sigma^0_{\rm had},\ R^0_\ell,\ \bar{s}^2_W,\ m_W,\ \Gamma_W$			
m_H		120 GeV	200 GeV	500 GeV	$\delta C \times 10^3$
$C_{\varphi WB}$	$\times 10^3$	0.1282	0.2286	0.4478	0.7856
$C_{\varphi D}$	$\times 10^3$	−1.7508	−2.4559	−3.9967	2.3900
$\chi^2_{\rm min}$		3.6637	3.7782	3.7322	
P-value	(d.o.f = 4)	0.4534	0.4369	0.4435	

Table 3.6: The same as Table 3.2, but with the additional observables of m_W and Γ_W included. The correlation is −87.9195%.

Observables	Γ_Z	$\sigma^0_{\rm had}$	R^0_ℓ	$\bar{s}^2_W$	m_W	Γ_W
120 GeV	0.609	−1.415	−0.855	−0.229	−0.517	−0.624
200 GeV	0.635	−1.408	−0.880	−0.243	−0.556	−0.612
500 GeV	0.652	−1.343	−0.891	−0.256	−0.593	−0.603

Table 3.7: The pull for observables following the results in Table 3.6.

m_H		120 GeV	200 GeV	500 GeV	$\delta C \times 10^3$
$C_{\varphi WB}$	$\times 10^3$	−0.04	−0.20	−0.43	0.79
$C_{\varphi D}$	$\times 10^3$	−1.17	−1.88	−3.81	2.39

Table 3.8: Same as Table 3.6, but this is the result of Ref. [2]. The correlation is −88%.

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Observables	Γ_Z	$\sigma^0_{\rm had}$	R^0_ℓ	$\bar{s}^2_W$	m_W	Γ_W
120 GeV	0.466	−1.418	−0.835	−0.303	−0.614	−0.630
200 GeV	0.494	−1.410	−0.862	−0.309	−0.654	−0.619
500 GeV	0.479	−1.340	−0.915	−0.161	−0.776	−0.616

Table 3.9: The pull for observables following the results in Table 3.8.

sub-space. This is not clear from the writing in Ref. [2] and we have contacted the authors without success. It may be that they have used the profiling method (see e.g. Ref. [3]), but we have not yet tried this.[1]

Now we consider the results for our input values:

	$C_{\varphi WB}$	$C_{\varphi D}$
Central Val. $\times 10^3$	0.3996	−1.5490
Errors $\times 10^3$	0.3945	1.8093

Table 3.10: The same approach as Table 3.2 using our input values given in Section 2.1 (without m_W and Γ_W). The correlation is −98%., with $\chi^2_{\rm min} = 2.0630$, d.o.f = 2 and P-value = 0.3565.

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[1]We thank Ian Lewis and Julien Baglio for pointing out this method to us.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$\bar{s}^2_W$
Pull	0.271	-1.383	-0.627	-0.007

Table 3.11: The pull for observables following the results in Table 3.10.

	$C_{\varphi WB}$	$C_{\varphi D}$
Central Val. $\times 10^3$	0.4043	-1.5720
Errors $\times 10^3$	0.2289	0.9166

Table 3.12: The same approach as Table 3.10 but with m_W and Γ_W included in the list of observables. The correlation is -95%, with $\chi^2_{\min} = 2.0831$, d.o.f $= 4$ and P-value $= 0.7205$.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$\bar{s}^2_W$	m_W	Γ_W
Pull	0.350	-1.383	-0.626	-0.008	-0.015	0.141

Table 3.13: The pull for observables following the results in Table 3.12.

Using the new data, our constraints for the errors are 1.5 to 3.6 times better for $C_{\varphi WB}$ and $C_{\varphi D}$ than using the data of Ref. [2], respectively.

Chapter 4

Constraints on lepton sector Wilson coefficients

4.1 Feynman rules for SMEFT

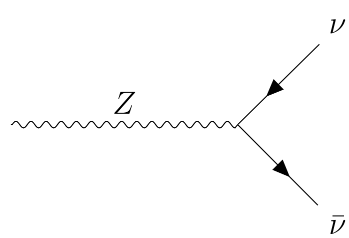
In this chapter, compared to Chapter 3, we have operators with two fermionic fields in addition. Due to our limited number of observables, and also the appearance of tensor basis $\sigma^{\mu\nu}$, which would make the problem much rather complicated, we shall drop all the D6 terms in $\psi^2 X\varphi$ sector. Considering for the weak boson - fermion vertices, we add to the SM Lagrangian the following D6-operator terms for the sector of $\psi^2\varphi^2 D$ and $\psi^2 X\varphi$ in Table 1.2:

$$\begin{aligned}\mathcal{L}_{\text{Weak Boson}-ff} = i\bar{f}\not{D}f + C^{(1)}_{\varphi l}Q^{(1)}_{\varphi l} + C^{(3)}_{\varphi l}Q^{(3)}_{\varphi l} + C_{\varphi e}Q_{\varphi e} \\ + C^{(1)}_{\varphi q}Q^{(1)}_{\varphi q} + C^{(3)}_{\varphi q}Q^{(3)}_{\varphi q} + C_{\varphi u}Q_{\varphi u} + C_{\varphi d}Q_{\varphi d} + C_{\varphi ud}Q_{\varphi ud}.\end{aligned} \tag{4.1.1}$$

Assuming no flavor mixings in the Yukawa sector, the fermionic fields in this Lagrangian are the physical fields, i.e. they are mass eigenstates. The right-handed neutrinos are absent in this work.

We reproduce the Lagrangian for following vertices and confirmed with the Feynman rules in Ref. [15]:

1. *Zνν* vertex

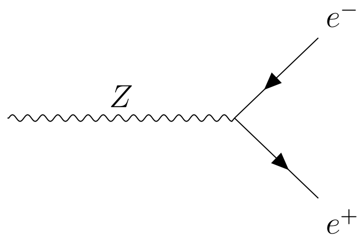


$$\begin{aligned}\mathcal{L}_{Z\nu\nu} = \bar{\nu}\Bigg[-\frac{\sqrt{\bar{g}^2+\bar{g}'^2}}{2}w_- - \frac{\bar{g}\bar{g}'}{2\sqrt{\bar{g}^2+\bar{g}'^2}}C_{\varphi WB}w_- \\ + \frac{\sqrt{\bar{g}^2+\bar{g}'^2}}{2}\gamma^\mu\Big(C^{(1)}_{\varphi l} - C^{(3)}_{\varphi l}\Big)w_-\Bigg]Z_\mu\nu,\end{aligned} \tag{4.1.2}$$

whereas, the left- and right-handed projection operator:

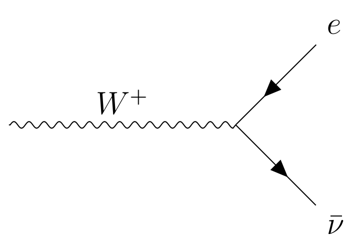
$$w_- = \frac{1-\gamma^5}{2}, \qquad w_+ = \frac{1+\gamma^5}{2}. \tag{4.1.3}$$

2. *Zee* vertex



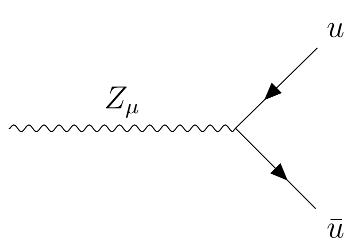
$$\mathcal{L}_{Zee} = \bar{e}\Bigg[-\frac{1}{2\sqrt{\bar{g}^2+\bar{g}'^2}}\gamma^\mu \left(\left(\bar{g}'^2-\bar{g}^2\right) w_- + 2\bar{g}'^2 w_+ \right) + \frac{\bar{g}\bar{g}'}{2(\bar{g}^2+\bar{g}'^2)^{3/2}} C_{\varphi WB}\gamma^\mu \left(\left(\bar{g}'^2-\bar{g}^2\right) w_- - 2\bar{g}^2 w_+ \right) + \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\gamma^\mu \left[\left(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)} \right) w_- + C_{\varphi e} w_+ \right] \Bigg] Z_\mu e. \tag{4.1.4}$$

3. $\boldsymbol{W^+ e\bar{\nu}}$ vertex



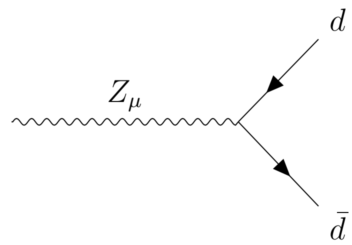
$$\mathcal{L}_{W^+ e\bar{\nu}} = \bar{\nu}\left[-\frac{\bar{g}}{\sqrt{2}} \left(1 + C_{\varphi l}^{(3)}\right) \gamma^\mu w_- \right] W_\mu^+ e. \tag{4.1.5}$$

4. *Zuu* vertex



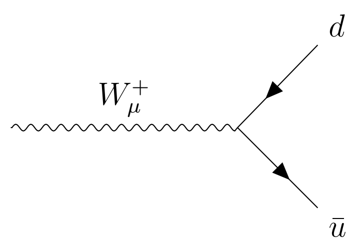
$$\mathcal{L}_{Zuu} = \bar{u}\Bigg[\frac{1}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\gamma^\mu \left(\left(\bar{g}'^2-3\bar{g}^2\right) w_- + 4\bar{g}'^2 w_+ \right) - \frac{\bar{g}\bar{g}'}{6(\bar{g}^2+\bar{g}'^2)^{3/2}} C_{\varphi WB}\gamma^\mu \left(\left(3\bar{g}'^2-\bar{g}^2\right) w_- - 4\bar{g}^2 w_+ \right) + \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\gamma^\mu \left[\left(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)} \right) w_- + C_{\varphi u} w_+ \right] \Bigg] Z_\mu u. \tag{4.1.6}$$

5. *Zdd* vertex



$$\mathcal{L}_{Zdd} = \bar{d}\Bigg[\frac{1}{6\sqrt{\bar{g}^2+\bar{g}'^2}}\gamma^\mu\left(\left(\bar{g}'^2+3\bar{g}^2\right)w_- - 2\bar{g}'^2 w_+\right) + \frac{\bar{g}\bar{g}'}{6(\bar{g}^2+\bar{g}'^2)^{3/2}}C_{\varphi WB}\gamma^\mu\left(\left(3\bar{g}'^2+\bar{g}^2\right)w_- - 2\bar{g}^2 w_+\right) + \frac{1}{2}\sqrt{\bar{g}^2+\bar{g}'^2}\gamma^\mu\left[\left(C_{\varphi q}^{(1)}+C_{\varphi q}^{(3)}\right)w_- + C_{\varphi d}w_+\right]\Bigg]Z_\mu d. \quad (4.1.7)$$

6. $W^+d\bar{u}$ vertex



$$\mathcal{L}_{W^+d\bar{u}} = \bar{u}\left[-\frac{\bar{g}}{\sqrt{2}}\left(1+C_{\varphi q}^{(3)}\right)\gamma^\mu w_- - \frac{\bar{g}}{2\sqrt{2}}\gamma^\mu C_{\varphi ud}\gamma^\mu w_+\right]W_\mu^+ d. \quad (4.1.8)$$

Now for conveniences in constraining, we rewrite the Lagrangian above in the basis of vector and axial vector couplings, and make non-universality assumptions only for D6 operators in $\psi^2\varphi^2 D$ sector. Rewrite it in the vector, axial-vector basis:

$$\mathcal{L}_{Zff} = -\frac{\bar{g}_Z}{2}Z_\mu \bar{f}\left[c_{V,Z}^f\gamma^\mu - c_{A,Z}^f\gamma^\mu\gamma^5\right]f, \qquad \mathcal{L}_{W^+ff} = \frac{\bar{g}_W}{2}W_\mu^+\bar{f}_1\left[c_{V,W}^{f_1f_2}\gamma^\mu - c_{A,W}^{f_1f_2}\gamma^\mu\gamma^5\right]f_2, \quad (4.1.9)$$

where $\bar{g}_Z$ and $\bar{g}_W$ are defined in Eq. (3.1.35) and Eq. (3.1.36) respectively.

1. For the $Z\nu\nu$ vertex:

$$c_{V,Z}^\nu = c_{A,Z}^\nu = \frac{1}{2} + \frac{-C_{\varphi l}^{(1)}+C_{\varphi l}^{(3)}}{2}, \quad (4.1.10)$$

$$\chi^\nu \approx \frac{1}{2} - C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}. \quad (4.1.11)$$

2. For the Zee vertex:

$$c_{V,Z}^l = 2\bar{s}_W^2 - \frac{1}{2} + \frac{-C_{\varphi l}^{(1)}-C_{\varphi l}^{(3)}-C_{\varphi e}}{2}, \qquad c_{A,Z}^l = -\frac{1}{2} + \frac{-C_{\varphi l}^{(1)}-C_{\varphi l}^{(3)}+C_{\varphi e}}{2}, \quad (4.1.12)$$

$$\chi^l \approx 4\bar{s}_W^4 - 2\bar{s}_W^2 + \frac{1}{2} + (-2\bar{s}_W^2 + 1)(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}) - 2\bar{s}_W^2 C_{\varphi e}. \tag{4.1.13}$$

3. For the $W^+ e\bar{\nu}$ vertex:

$$c_{V,W}^{e\bar{\nu}} = c_{A,W}^{e\bar{\nu}} = 1 + C_{\varphi l}^{(3)}, \tag{4.1.14}$$

$$\chi^{e\bar{\nu}} \approx 2\big(1 + 2C_{\varphi l}^{(3)}\big). \tag{4.1.15}$$

4. For the Zuu vertex:

$$c_{V,Z}^u = -\frac{4}{3}\bar{s}_W^2 + \frac{1}{2} + \frac{-C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)} - C_{\varphi u}}{2}, \qquad c_{A,Z}^u = \frac{1}{2} + \frac{-C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)} + C_{\varphi u}}{2}, \tag{4.1.16}$$

$$\chi^u \approx \frac{16}{9}\bar{s}_W^4 - \frac{4}{3}\bar{s}_W^2 + \frac{1}{2} + \left(\frac{4}{3}\bar{s}_W^2 - 1\right)\left(C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)}\right) + \frac{4}{3}\bar{s}_W^2 C_{\varphi u}. \tag{4.1.17}$$

5. For the Zdd vertex:

$$c_{V,Z}^d = \frac{2}{3}\bar{s}_W^2 - \frac{1}{2} + \frac{-C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)} - C_{\varphi d}}{2}, \qquad c_{A,Z}^u = -\frac{1}{2} + \frac{-C_{\varphi q}^{(1)} - C_{\varphi q}^{(3)} + C_{\varphi d}}{2}, \tag{4.1.18}$$

$$\chi^d \approx \frac{4}{9}\bar{s}_W^4 - \frac{2}{3}\bar{s}_W^2 + \frac{1}{2} + \left(-\frac{2}{3}\bar{s}_W^2 + 1\right)\left(C_{\varphi q}^{(1)} + C_{\varphi q}^{(3)}\right) - \frac{2}{3}\bar{s}_W^2 C_{\varphi d}. \tag{4.1.19}$$

6. For the $W^+ d\bar{u}$ vertex:

$$c_{V,W}^{d\bar{u}} = 1 + C_{\varphi q}^{(3)} + \frac{C_{\varphi ud}}{2}, \qquad c_{A,W}^{d\bar{u}} = 1 + C_{\varphi q}^{(3)} - \frac{C_{\varphi ud}}{2}, \tag{4.1.20}$$

$$\chi^{d\bar{u}} \approx 2\left(1 + 2C_{\varphi q}^{(3)}\right). \tag{4.1.21}$$

Here we observe that the $C_{\varphi ud}$ appears only in the $W^+ d\bar{u}$ vertex. However, since it vanishes in $\chi^{d\bar{u}} = (c_{V,W}^{d\bar{u}})^2 + (c_{A,W}^{d\bar{u}})^2$, the $C_{\varphi ud}$ does not contribute hence cannot be constrained by our set of observables.

4.2 Universal lepton assumption

From now on, we no longer use the pseudo-observables $\bar{s}_W^2$, since, firstly, its experimental value is derived by combining the other pseudo-observables, which contain many assumptions, therefore should not be "clean" data; secondly, more important, these assumptions usually do not take into account the effects of SMEFT. To the best of my knowledge, many groups have performed the fitting without using the data from $\bar{s}_W^2$.

From Table 4.2 we observe that the new D6 two-lepton operators enter into most of the observables Γ_Z, σ_{had}^0, R_l^0, $A_{\text{FB}}^{0,e}$, Γ_W, and dilute the contribution from D6 gauge-sector operators, especially for the σ_{had} and R_ℓ^0.

Our fit results are shown in Tables 4.2, 4.3 and 4.4. Compared to the errors in Table 3.12 the new operators weaken the constraints on D6 gauge-sector operators $C_{\varphi WB}$ and $C_{\varphi D}$ to fifteen times

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$
Γ_Z	−2.055	−1.68	−0.474	−0.157	−0.474
σ^0_{had}	2.309	0.487	104.491	38.256	−60.522
R^0_ℓ	−13.506	−2.847	−44.383	−55.770	38.155
$A^{0,\ell}_{\text{FB}}$	−1.336	−0.282	0.394	−0.733	0.458
m_W	−63.003	−28.727	0.	−114.909	0.
Γ_W	−4.915	−2.241	0.	−7.570	0.

Table 4.1: Coefficient matrix for the case of universal leptons. Analytical results are given in Appendix B.2.1.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$
Central Val. $\times 10^3$	2.82	−5.77	2.72	−0.27	3.72
Errors $\times 10^3$	3.56	6.26	2.33	0.58	3.51

Table 4.2: Results for the case of universal leptons with $\chi^2_{\text{min}} = 0.0187$, d.o.f = 1 and P-value = 0.8911.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$
$C_{\varphi WB}$	1.000				
$C_{\varphi D}$	−0.972	1.000			
$C_{\varphi l}^{(1)}$	0.937	−0.920	1.000		
$C_{\varphi l}^{(3)}$	−0.723	0.549	−0.647	1.000	
$C_{\varphi e}$	0.932	−0.942	0.986	−0.561	1.000

Table 4.3: Correlation matrix for the case of universal leptons.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,l}_{\text{FB}}$	m_W	Γ_W
Pull	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	−0.001	$\mathcal{O}(10^{-4})$	0.137

Table 4.4: Pull values for the case of universal leptons.

(3.56/0.23) and seven times (6.26/0.92) respectively. However, all the D6 operators deviate less than 1.5σ from 0. The P-value (of 0.8911) is very high, which means our null hypothesis is pretty extremer than what was actually observed, the results are strongly statistically insignificant.

Correlations are provided in Table 4.3.

All the recalculated observables deviate much less than 1σ from experimentals, implying that our fitting results are good.

Now we shall consider the non-universal lepton case.

4.3 Non-universal $C_{\varphi e}$ assumption

Now we consider the non-universal lepton case. For the three types of D6 lepton-operators of $C_{\varphi l}^{(1)pp}$, $C_{\varphi l}^{(3)pp}$, $C_{\varphi e}^{pp}$ (where $p = e,\ \mu,\ \tau$), generally, with 3 flavors for each type (assuming no lepton-flavor violation e.g. $\bar{l}_e\gamma^\mu l_\mu$ is forbidden), we can assume 9 operators, adding up the two gauge-sector ones, we have in total 11 operators in our set. However, since there are only 10 components in our set of observables, we must make some additional assumptions to reduce the number of Wilson coefficients. Firstly, for a simple case we consider the non-universality for only operators of the type $C_{\varphi e}^{pp}$.

We rewrite the vector and axial-vector couplings for the involving vertices,
1. For the $Z\nu\nu$ vertex:

$$c_V^\nu = c_A^\nu = \frac{1}{2} + \frac{-C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}}{2}, \tag{4.3.22}$$

$$\chi^\nu \approx \frac{1}{2} - C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}. \tag{4.3.23}$$

2. For the Zee vertex:

$$c_V^{l_p} = 2\bar{s}_W^2 - \frac{1}{2} + \frac{-C_{\varphi l}^{(1)} - C_{(3)\varphi l} - C_{\varphi e}^{pp}}{2}, \qquad c_A^{l_p} = -\frac{1}{2} + \frac{-C_{\varphi l}^{(1)} - C_{\varphi l}^{(3)} + C_{\varphi e}^{pp}}{2}, \tag{4.3.24}$$

$$\chi^{l_p} \approx 4\bar{s}_W^4 - 2\bar{s}_W^2 + \frac{1}{2} + (-2\bar{s}_W^2 + 1)(C_{\varphi l}^{(1)} + C_{\varphi l}^{(3)}) - 2\bar{s}_W^2 C_{\varphi e}^{pp}. \tag{4.3.25}$$

3. For the $W^+e\bar{\nu}$ vertex:

$$c_{V,W}^{e\bar{\nu}} = c_{A,W}^{e\bar{\nu}} = 1 + C_{\varphi l}^{(3)}, \tag{4.3.26}$$

$$\chi^{e\bar{\nu}} \approx 2\big(1 + 2C_{\varphi l}^{(3)}\big). \tag{4.3.27}$$

Our fit results are shown in Tables 4.6, 4.7 and 4.8. As we observed, the constraints on gauge-sector operators are slightly weakened. Considering lepton-sector, the $C_{\varphi l}^{(1)}$, $C_{\varphi l}^{(3)}$ are slightly weakened, the $C_{\varphi e}^{pp}$ are also weaker constrained than the $C_{\varphi e}$ in the universal case.

The P-value (of 0.5289) is high, which means our null hypothesis is much extremer than what was actually observed, the results are strongly statistically insignificant.

For the pull, all the fitted observables greatly agree with the experimentals values.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$
Γ_Z	−2.055	−1.68	−0.474	−0.157	−0.158	−0.158	−0.158
σ^0_{had}	2.309	0.487	104.491	38.256	−71.028	5.253	5.253
R_e^0	−13.506	−2.847	−44.383	−55.770	38.155	0.	0.
R_μ^0	−13.506	−2.847	−44.383	−55.770	0.	38.155	0.
R_τ^0	−13.506	−2.847	−44.383	−55.770	0.	0.	38.155
$A_{\text{FB}}^{0,e}$	−1.336	−0.282	0.394	−0.733	0.458	0.	0.
$A_{\text{FB}}^{0,\mu}$	−1.336	−0.282	0.394	−0.733	0.229	0.229	0.
$A_{\text{FB}}^{0,\tau}$	−1.336	−0.282	0.394	−0.733	0.229	0.	0.229
m_W	−63.003	−28.727	0.	−114.909	0.	0.	0.
Γ_W	−4.915	−2.241	0.	−7.570	0.	0.	0.

Table 4.5: Coefficient matrix A_{ij} for the case of non-universal $C_{\varphi e}$. Analytical results are given in Appendix B.3.1.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$
Central Val. $\times 10^3$	4.08	−7.93	3.58	−0.42	5.12	5.24	4.89
Errors $\times 10^3$	5.28	9.02	3.67	0.77	5.65	4.70	4.74

Table 4.6: Results for the case of non-universal $C_{\varphi e}$ with $\chi^2_{\text{min}} = 2.2154$, d.o.f = 3 and P-value = 0.5289.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$
$C_{\varphi WB}$	1.000						
$C_{\varphi D}$	−0.987	1.000					
$C_{\varphi l}^{(1)}$	0.974	−0.963	1.000				
$C_{\varphi l}^{(3)}$	−0.855	0.764	−0.826	1.000			
$C_{\varphi e}^{ee}$	0.972	−0.972	0.994	−0.787	1.000		
$C_{\varphi e}^{\mu\mu}$	0.958	−0.962	0.977	−0.758	0.980	1.000	
$C_{\varphi e}^{\tau\tau}$	0.945	−0.951	0.962	−0.744	0.966	0.955	1.000

Table 4.7: Correlation matrix for the case of non-universal $C_{\varphi e}$.

Observables	Γ_Z	σ^0_{had}	R_e^0	R_μ^0	R_τ^0	$A_{\text{FB}}^{0,e}$	$A_{\text{FB}}^{0,\mu}$	$A_{\text{FB}}^{0,\tau}$	m_W	Γ_W
Pull	0.003	$\mathcal{O}(10^{-5})$	−0.026	−0.001	0.007	1.046	0.186	−1.022	$\mathcal{O}(10^{-4})$	0.132

Table 4.8: Pull values for the case of non-universal $C_{\varphi e}$.

4.4 Non-universal $C_{\varphi l}^{(1)}$ and $C_{\varphi l}^{(3)}$ assumption

We now make the non-universality assumption for $C_{\varphi l}^{(1)}$, $C_{\varphi l}^{(3)}$ and consider $C_{\varphi e}$ as universal. For the $Z\nu\nu$ vertex:

$$c_V^{\nu_p} = c_A^{\nu_p} = \frac{1}{2} + \frac{-C_{\varphi l}^{(1)pp} + C_{\varphi l}^{(3)pp}}{2} \tag{4.4.28}$$

$$\chi^{\nu_p} = \frac{1}{2} - C_{\varphi l}^{(1)pp} + C_{\varphi l}^{(3)pp}. \tag{4.4.29}$$

For the Zee vertex:

$$c_V^{l_p} = 2\bar{s}_W^2 - \frac{1}{2} + \frac{-C_{\varphi l}^{(1)pp} - C_{\varphi l}^{(3)pp} - C_{\varphi e}}{2}, \quad c_A^{l_p} = -\frac{1}{2} + \frac{-C_{\varphi l}^{(1)pp} - C_{\varphi l}^{(3)pp} + C_{\varphi e}}{2}. \tag{4.4.30}$$

$$\chi^{l_p} = 4\bar{s}_W^4 - 2\bar{s}_W^2 + \frac{1}{2} + (-2\bar{s}_W^2 + 1)(C_{\varphi l}^{(1)pp} + C_{\varphi l}^{(3)pp}) - 2\bar{s}_W^2 C_{\varphi e}. \tag{4.4.31}$$

For the $W^+e\bar{\nu}$ vertex:

$$c_{V,W}^{e\bar{\nu}} = c_{A,W}^{e\bar{\nu}} = 1 + C_{\varphi l}^{(3)pp}, \tag{4.4.32}$$

$$\chi^{e\bar{\nu}} \approx 2\big(1 + 2C_{\varphi l}^{(3)pp}\big). \tag{4.4.33}$$

We obtain the coefficient matrix as in Table 4.9. Due to the numerical issues, the final results

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)ee}$	$C_{\varphi l}^{(3)ee}$	$C_{\varphi l}^{(1)\mu\mu}$	$C_{\varphi l}^{(3)\mu\mu}$	$C_{\varphi l}^{(1)\tau\tau}$	$C_{\varphi l}^{(3)\tau\tau}$	$C_{\varphi e}$
Γ_Z	−2.055	−1.680	−0.158	−0.341	−0.158	−0.341	−0.158	0.525	−0.474
σ_{had}^0	2.309	0.487	93.985	72.231	5.253	−16.501	5.253	−17.474	−60.522
R_e^0	−13.506	−2.847	−44.383	−50.076	0.	−5.693	0.	0.	38.155
R_μ^0	−13.506	−2.847	0.	−5.693	−44.383	−50.076	0.	0.	38.155
R_τ^0	−13.506	−2.847	0.	−5.693	0.	−5.693	−44.383	−44.383	38.155
$A_{\text{FB}}^{0,e}$	−1.336	−0.282	0.394	−0.169	0.	−0.563	0.	0.	0.458
$A_{\text{FB}}^{0,\mu}$	−1.336	−0.282	0.197	−0.366	0.197	−0.366	0.	0.	0.458
$A_{\text{FB}}^{0,\tau}$	−1.336	−0.282	0.197	−0.366	0.	−0.563	0.197	0.197	0.458
m_W	−63.003	−28.727	0.	−57.455	0.	−57.455	0.	0.	0.
Γ_W	−4.915	−2.241	0.	−3.685	0.	−3.685	0.	0.464	0.

Table 4.9: Coefficient matrix A_{ij} for the case of non-universal $C_{\varphi l}^{(1)}$ and $C_{\varphi l}^{(3)}$. Analytical results are given in Appendix B.4.

diverges. We are not be able to constrain the D6 Wilson coefficients using this assumption. However, as we shall see in the Appendix A.1, there is a trick to handle these issues, maintaining the number of 9 variables of the Wilson coefficients set.

Chapter 5

Constraints on lepton and quark sector Wilson coefficients

Now we consider the contributions of quarks. We include four additional observables of R_b^0, R_c^0, A_{FB}^b, A_{FB}^c. Since we could not obtain these type information for up, down, and strange quarks, we use the assumption of quark-flavor universality. We add for each of the assumptions in Chapter 4 the quark contributions to relevant observables using vertices in Eq. (4.1.16) - Eq. (4.1.21). However, as we will see later that gauge, lepton and quarks can not be constrained simultaneously, an alternative assumption that gauge sector's Wilson coefficients are null could be implied, and the fits are performed solely for lepton and quark sector.

5.1 Universal lepton and quark assumption

Simply adding quark observables and coefficients to Table 5.4, we got the Coefficient matrix A_{ij}:

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	-2.055	-1.68	-0.474	-0.157	-0.474	1.183	4.019	0.632	-0.474
σ_{had}^0	2.309	0.487	104.491	38.256	-60.522	-10.886	-36.99	-5.814	4.361
R_ℓ^0	-13.506	-2.847	-44.383	-55.770	38.155	14.234	48.365	7.602	-5.702
$A_{\mathrm{FB}}^{0,\ell}$	-1.336	-0.282	0.394	-0.733	0.458	$0.$	$0.$	$0.$	$0.$
m_W	-63.003	-28.727	$0.$	-114.909	$0.$	$0.$	$0.$	$0.$	$0.$
Γ_W	-4.914	-2.241	$0.$	-7.570	$0.$	$0.$	2.786	$0.$	$0.$
R_b^0	0.031	0.007	$0.$	0.026	$0.$	0.346	-0.009	-0.079	-0.031
R_c^0	-0.048	-0.01	$0.$	-0.040	$0.$	-0.534	0.014	0.122	0.047
$A_{\mathrm{FB}}^{0,b}$	-4.300	-0.906	1.251	-2.374	1.456	0.016	0.016	$0.$	0.089
$A_{\mathrm{FB}}^{0,c}$	-3.331	-0.702	0.894	-1.915	1.04	-0.088	0.088	-0.198	$0.$

Table 5.1: Coefficient matrix A_{ij} for the case of universal leptons and universal quarks. Analytical results are given in Appendix B.2.2.

Here, although the number of observables is more than Wilson coefficients', the results diverge for Table 5.1, and all of the cases including gauge, non-universal lepton and universal quarks

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Central Val. $\times 10^3$	1.02	−0.18	0.59	−1.04	−10.52	18.50	−71.36
Errors $\times 10^3$	0.81	0.13	1.11	3.73	5.44	16.01	32.11

Table 5.2: Results for the case of universal leptons and universal quarks with $\chi^2_{\text{min}} = 0.4331$, d.o.f $= 3$ and P-value 0.9333.

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
$C_{\varphi l}^{(1)}$	1.000						
$C_{\varphi l}^{(3)}$	0.074	1.000					
$C_{\varphi e}$	0.894	0.161	1.000				
$C_{\varphi q}^{(1)}$	−0.243	−0.022	−0.253	1.000			
$C_{\varphi q}^{(3)}$	−0.673	0.024	−0.703	−0.212	1.000		
$C_{\varphi u}$	0.406	−0.021	0.406	0.566	−0.857	1.000	
$C_{\varphi d}$	−0.803	0.001	−0.813	0.363	0.805	−0.410	1.000

Table 5.3: Correlation matrix for the case of universal leptons and universal quarks.

Observables	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,e}_{\text{FB}}$	m_W	Γ_W	R^0_b	R^0_c	$A^{0,b}_{\text{FB}}$	$A^{0,c}_{\text{FB}}$
Pull	0.060	−0.005	−0.009	−0.033	0.001	−0.443	−0.046	−0.192	0.077	0.136

Table 5.4: The "pulls" of the observables for the case of universal leptons and universal quarks.

contemporaneously. Therefore, from now on, we make an assumption that coefficients in gauge sector vanish and exclude their contribution in all of our fits.

The results are presented in Table 5.2, 5.3, 5.4, using universal lepton and quark assumption, with the two first columns in Table 5.1 vanish.
All the central values still lie in the range 2σ from 0, except for the $C_{\varphi d}$, which deviates about 2.2σ from 0. The P-value (of 0.9333) is very high, which mean our null hypothesis is pretty more extreme than what was actually observed, the results are strongly statistically insignificant.

The "Pull" for the observables well agree with the experimental values.

5.2 Non-universal $C_{\varphi e}$ and universal quark assumption

Our results are presented in Table 5.6, 5.7, 5.8, using non-universal $C_{\varphi e}$ and universal quark assumption.

We consider the first non-universal lepton case for $C_{\varphi e}$, with the added quark-sector operators.

Here, the results stay consistent with the universal case above. With the $C^{pp}_{\varphi e}$ are little weaken constrained. The most deviation is less than 2σ from 0, except for $C_{\varphi d}$, differing nearly 2.1σ from 0. The P-value (of 0.7464) is high, which mean our null hypothesis is much more extreme than what was actually observed, the results are strongly statistically insignificant.
The Pull of the observables are still well agreed with the experimental values.

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	−0.474	−0.157	−0.158	−0.158	−0.158	1.183	4.019	0.632	−0.474
σ_{had}^0	104.491	38.256	−71.028	5.253	5.253	−10.886	−36.99	−5.814	4.361
R_e^0	−44.383	−55.770	38.155	0.	0.	14.234	48.365	7.602	−5.702
R_μ^0	−44.383	−55.770	0.	38.155	0.	14.234	48.365	7.602	−5.702
R_τ^0	−44.383	−55.770	0.	0.	38.155	14.234	48.365	7.602	−5.702
$A_{\text{FB}}^{0,e}$	0.394	−0.733	0.458	0.	0.	0.	0.	0.	0.
$A_{\text{FB}}^{0,\mu}$	0.394	−0.733	0.229	0.229	0.	0.	0.	0.	0.
$A_{\text{FB}}^{0,\tau}$	0.394	−0.733	0.229	0.	0.229	0.	0.	0.	0.
m_W	0.	-114.909	0.	0.	0.	0.	0.	0.	0.
Γ_W	0.	-7.570	0.	0.	0.	0.	2.786	0.	0.
R_b^0	0.	0.026	0.	0.	0.	0.346	−0.009	−0.079	−0.031
R_c^0	0.	-0.04	0.	0.	0.	−0.534	0.014	0.122	0.047
$A_{\text{FB}}^{0,b}$	1.251	−2.374	1.456	0.	0.	0.016	0.016	0.	0.089
$A_{\text{FB}}^{0,c}$	0.894	−1.915	1.040	0.	0.	−0.088	0.088	−0.198	0.

Table 5.5: Coefficient matrix A_{ij} for the case of non-universal $C_{\varphi e}$ and universal quarks. Analytical results are given in Appendix B.3.2.

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Central Val. $\times 10^3$	1.00	−0.18	0.58	1.06	0.73	−1.02	−10.45	18.45	−71.03
Errors $\times 10^3$	0.87	0.13	1.22	1.24	1.44	3.76	5.67	16.34	34.45

Table 5.6: Results for the case of non-universal $C_{\varphi e}$ and universal quarks, with $\chi^2_{\min} = 2.6983$, d.o.f = 5 and P-value 0.7464.

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
$C_{\varphi l}^{(1)}$	1.000								
$C_{\varphi l}^{(3)}$	0.050	1.000							
$C_{\varphi e}^{ee}$	0.910	0.125	1.000						
$C_{\varphi e}^{\mu\mu}$	0.567	0.156	0.614	1.000					
$C_{\varphi e}^{\tau\tau}$	0.465	0.135	0.521	0.456	1.000				
$C_{\varphi e}^{\mu\mu}$	−0.264	−0.016	−0.273	−0.173	−0.145	1.000			
$C_{\varphi e}^{\mu\mu}$	−0.705	0.037	−0.731	−0.464	−0.388	−0.171	1.000		
$C_{\varphi e}^{\mu\mu}$	0.444	−0.030	0.444	0.264	0.221	0.530	−0.862	1.000	
$C_{\varphi e}^{\mu\mu}$	−0.830	0.019	−0.840	−0.509	−0.426	0.376	0.882	−0.446	1.000

Table 5.7: Correlation matrix for the case of non-universal $C_{\varphi e}$ and universal quarks.

Observables	Γ_Z	σ_{had}^0	R_e^0	R_μ^0	R_τ^0	$A_{\text{FB}}^{0,e}$	$A_{\text{FB}}^{0,\mu}$
Pull	0.063	−0.005	−0.072	−0.002	0.007	1.022	0.204
Observables	$A_{\text{FB}}^{0,\tau}$	m_W	Γ_W	R_b^0	R_c^0	$A_{\text{FB}}^{0,b}$	$A_{\text{FB}}^{0,c}$
Pull	−1.005	0.001	−0.432	−0.045	−0.192	0.076	0.134

Table 5.8: The "pull" of observables for the case of non-universal $C_{\varphi e}$ and universal quarks.

5.3 Non-universal $C_{\varphi l}^{(1)}$, $C_{\varphi l}^{(3)}$ and universal quark assumption

For the second case of lepton non-universality, as we easily predicted, the added quarks does not solve the numerical-divergence issue of the results.

	$C_{\varphi l}^{(1)ee}$	$C_{\varphi l}^{(3)ee}$	$C_{\varphi l}^{(1)\mu\mu}$	$C_{\varphi l}^{(3)\mu\mu}$	$C_{\varphi l}^{(1)\tau\tau}$	$C_{\varphi l}^{(3)\tau\tau}$	$C_{\varphi e}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	−0.158	−0.341	−0.158	−0.341	−0.158	0.525	−0.474	1.183	4.019	0.632	−0.474
σ^0_{had}	93.985	72.231	5.253	−16.501	5.253	−17.474	−60.522	−10.886	−36.99	−5.814	4.361
R^0_e	−44.383	−50.076	0.	−5.693	0.	0.	38.155	14.234	48.365	7.602	−5.702
R^0_μ	0.	−5.693	−44.383	−50.076	0.	0.	38.155	14.234	48.365	7.602	−5.702
R^0_τ	0.	−5.693	0.	−5.693	−44.383	−44.383	38.155	14.234	48.365	7.602	−5.702
$A^{0,e}_{\text{FB}}$	0.394	−0.169	0.	−0.563	0.	0.	0.458	0.	0.	0.	0.
$A^{0,\mu}_{\text{FB}}$	0.197	−0.366	0.197	−0.366	0.	0.	0.458	0.	0.	0.	0.
$A^{0,\tau}_{\text{FB}}$	0.197	−0.366	0.	−0.563	0.197	0.197	0.458	0.	0.	0.	0.
m_W	0.	−57.455	0.	−57.455	0.	0.	0.	0.	0.	0.	0.
Γ_W	0.	−3.685	0.	−3.685	0.	0.464	0.	0.	2.786	0.	0.
R^0_b	0.	0.013	0.	0.013	0.	0.	0.	0.346	−0.009	−0.079	−0.031
R^0_c	0.	−0.020	0.	−0.020	0.	0.	0.	−0.534	0.014	0.122	0.047
$A^{0,b}_{\text{FB}}$	1.251	−0.561	0.	−1.813	0.	0.	1.456	0.016	0.016	0.	0.089
$A^{0,c}_{\text{FB}}$	0.894	−0.510	0.	−1.404	0.	0.	1.04	−0.088	0.088	−0.198	0.

Table 5.9: Coefficient matrix A_{ij} for the case of non-universal $C_{\varphi l}^{(1)}$, $C_{\varphi l}^{(3)}$ and universal quarks. Analytical results are given in Appendix B.4.2.

Conclusion and outlook

Conclusion

Based on what we have done so far in this thesis, we can draw some important conclusions for the constraints on D6 Wilson coefficients:

- Foremost, in the first two Chapters, we have introduced the formalism of SMEFT, the rescaling of the Lagrangian and the methodology for fitting the D6 Wilson coefficients.
- For the gauge sector, in comparison with NNT's results, our central values for the coefficients differ, however, the errors well agree. The differences are still in 1 σ and may come from their non-trivial method of projecting the 5×5 correlation matrix onto 3 dimensional subspace. The updated set of observables' values improves the errors significantly.
- Regarding the lepton sector, we have performed the fitting for both cases of lepton flavor universal and non-universal, and compared their results. For the non-universal case, we have made several assumptions on the coefficients' universality due to the limited number of observables. The assumption of non-universal $C_{\varphi e}$ yields good fits.
- Adding the quark sector, we can see the fact that fitting can not be done simultaneously for all three sectors. The fits are performed using different perspective of views that gauge sector's Wilson coefficients are nulls. The constrains are good as desired and there is great consistent between results for non-universal $C_{\varphi e}$ and universal case.
- Finally, besides the fitting results, sensitivity of Wilson coefficients towards LEP1 Observables can be evaluated through theirs contributions in the Coefficient matrices A_{ij}.

I would like to note that all results presented in Chapters 4, 5 are my own results. To the best of my knowledge, they are new results. However, it is not excluded that similar fits have been done elsewhere by other groups. I plan to search in the literature for similar works and compare my results with those of the others, especially for the case including fermionic operators. This has not yet been done due to time constraints.

Outlook

To make further development on the constraints, some augmentations to the observables can be carried out:

- Combining more observables at Z-pole energy.
- Adopting observables at different energy scales.

Appendix A

Non-universal $\Sigma^{+}_{\varphi l}$ and $C_{\varphi e}$ assumption

A.1 Non-universal $\Sigma^{+}_{\varphi l}$, $C_{\varphi e}$ without quark

To get rid of the numerical issues appearing in the non-universal $C^{1}_{\varphi l}$, $C^{3}_{\varphi l}$ assumption in Section 4.4, we now redefine our variables as:

$$\Sigma^{+}_{\varphi l} = \quad C^{(1)}_{\varphi l} + C^{(3)}_{\varphi l}, \tag{A.1.1}$$

$$\Sigma^{-}_{\varphi l} = -C^{(1)}_{\varphi l} + C^{(3)}_{\varphi l}. \tag{A.1.2}$$

We shall apply the non-universal assumptions for $\Sigma^{+}_{\varphi l}$ and $C_{\varphi e}$, but treat $\Sigma^{-}_{\varphi l}$ as flavor universal. Considering the vertices involved,
1. For the $Z\nu\nu$ vertex:

$$c^{\nu}_{V} = c^{\nu}_{A} = \frac{1}{2} + \frac{\Sigma^{-}_{\varphi l}}{2}, \tag{A.1.3}$$

$$\chi^{\nu} = \frac{1}{2} + \Sigma^{-}_{\varphi l}. \tag{A.1.4}$$

2. For the Zee vertex:

$$c^{e_p}_{V} = 2\bar{s}^{2}_{W} - \frac{1}{2} + \frac{-\Sigma^{pp+}_{\varphi l} - C^{pp}_{\varphi e}}{2}, \qquad c^{e_p}_{A} = -\frac{1}{2} + \frac{-\Sigma^{pp+}_{\varphi l} + C^{pp}_{\varphi e}}{2}, \tag{A.1.5}$$

$$\chi^{e_p} = 4\bar{s}^{4}_{W} - 2\bar{s}^{2}_{W} + \frac{1}{2} + (-2\bar{s}^{2}_{W} + 1)\Sigma^{pp+}_{\varphi l} - 2\bar{s}^{2}_{W} C^{pp}_{\varphi e}. \tag{A.1.6}$$

3. For the $W^{+}e\bar{\nu}$ vertex:

$$c^{e_p\bar{\nu}_p}_{V,W} = c^{e_p\bar{\nu}_p}_{A,W} = 1 + \frac{\Sigma^{pp+}_{\varphi l} + \Sigma^{-}_{\varphi l}}{2}, \tag{A.1.7}$$

$$\chi^{e\bar{\nu}} \approx 2\big(1 + \Sigma^{pp+}_{\varphi l} + \Sigma^{-}_{\varphi l}\big). \tag{A.1.8}$$

	$C_{\varphi WB}$	$C_{\varphi D}$	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^{-}_{\varphi l}$
Γ_Z	−2.055	−1.680	−0.249	−0.158	−0.249	−0.158	0.184	−0.158	0.159
σ^0_{had}	2.309	0.487	83.108	−71.028	−5.624	5.253	−6.111	5.253	−33.118
R^0_e	−13.506	−2.847	−47.230	38.155	−2.847	0.	0.	0.	−5.693
R^0_μ	−13.506	−2.847	−2.847	0.	−47.230	38.155	0.	0.	−5.693
R^0_τ	−13.506	−2.847	−2.847	0.	−2.847	0.	−44.383	38.155	−5.693
$A^{0,e}_{\text{FB}}$	−1.336	−0.282	0.112	0.458	−0.282	0.	0.	0.	−0.563
$A^{0,\mu}_{\text{FB}}$	−1.336	−0.282	−0.085	0.229	−0.085	0.229	0.	0.	−0.563
$A^{0,\tau}_{\text{FB}}$	−1.336	−0.282	−0.085	0.229	−0.282	0.	0.197	0.229	−0.563
m_W	−63.003	−28.727	−28.727	0.	−28.727	0.	0.	0.	−57.455
Γ_W	−4.915	−2.241	−1.843	0.	−1.843	0.	0.232	0.	−3.453

Table A.1: Coefficient matrix A_{ij} for the case of non-universal $\Sigma^{+}_{\varphi l}$ and $C_{\varphi e}$. Analytical results are given in Appendix B.5.1.

	$C_{\varphi WB}$	$C_{\varphi D}$	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^{-}_{\varphi l}$
Central Val. $\times 10^3$	6.80	−12.39	1.88	4.79	7.29	10.59	11.66	15.12	−6.19
Errors $\times 10^3$	5.59	9.56	3.56	5.82	5.54	7.30	6.51	8.38	4.65

Table A.2: Results for the case of non-universal $\Sigma^{+}_{\varphi l}$ and $C_{\varphi e}$ with $\chi^2_{\text{min}} = 0.1159$, d.o.f = 1 and P-value = 0.8841.

	$C_{\varphi WB}$	$C_{\varphi D}$	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^{-}_{\varphi l}$
$C_{\varphi WB}$	1.000								
$C_{\varphi D}$	−0.986	1.000							
$\Sigma^{ee+}_{\varphi l}$	0.669	−0.657	1.000						
$C^{ee}_{\varphi e}$	0.862	−0.850	0.947	1.000					
$\Sigma^{\mu\mu+}_{\varphi l}$	0.698	−0.732	0.060	0.352	1.000				
$C^{\mu\mu}_{\varphi e}$	0.776	−0.803	0.164	0.451	0.985	1.000			
$\Sigma^{\tau\tau+}_{\varphi l}$	0.714	−0.707	0.179	0.412	0.724	0.750	1.000		
$C^{\tau\tau}_{\varphi e}$	0.776	−0.770	0.251	0.488	0.745	0.780	0.984	1.000	
$\Sigma^{\text{lept}-}_{\varphi l}$	−0.972	0.955	−0.619	−0.830	−0.782	−0.842	−0.710	−0.767	1.000

Table A.3: Correlation matrix for the case of non-universal $\Sigma^{+}_{\varphi l}$ and $C_{\varphi e}$.

Observables	Γ_Z	σ^0_{had}	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{\text{FB}}$	$A^{0,\mu}_{\text{FB}}$	$A^{0,\tau}_{\text{FB}}$	m_W	Γ_W
Pull	-0.001	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-4})$	$\mathcal{O}(10^{-5})$	$\mathcal{O}(10^{-5})$	0.001	-0.002	-0.003	$\mathcal{O}(10^{-4})$	0.145

Table A.4: Pull values for the case of non-universal $\Sigma^+_{\varphi l}$ and $C_{\varphi e}$.

The great advantage of this redefinition is to eliminate the divergence in Sect. 4.4 and retain also the non-universality of $C_{\varphi e}$ as in Sect. 4.3, so that we can make comparisons.

Our fit results are shown in Tables A.2, A.3 and A.4. Comparing to Sect. 4.3, this assumption yields weaker constraints. For D6 coefficients in gauge-sector, the errors increase very slightly. For the $C_{\varphi e}$ the constraint is about 1.03 to 1.77 times weaken, as an obvious results when we increase our set of Wilson coefficient by the non-universality assumption. Most of the coefficients are still constrained to the power of 10^{-3}, and deviate not greater than 2σ from 0.

The P-value (of 0.8841) is very high, which mean our null hypothesis is pretty more extreme than what was actually observed, the results are strongly statistically insignificant.

From the pull, we see the fitted observables greatly agree with the experimental values.

From this discussion, we ignite another suggestion to switch the non-universality assumption from $\Sigma^{+pp}_{\varphi l}$ or $C^{pp}_{\varphi e}$ to $\Sigma^{-pp}_{\varphi l}$ However, as can be observed in Coefficient matrix A_{ij}, the $\Sigma^-_{\varphi l}$ is universal to the lepton flavors, due to its same contributions to different flavors of each observable.

A.2 Non-universal $\Sigma^+_{\varphi l}$, $C_{\varphi e}$ and universal quark

For this case, our new-defined variables also overcome the numerical problems, the results well converge.

Our fit results are shown in Tables A.5, A.6 and A.7. Comparing to Sect. 5.2, this assumption yields weaker constraints.

For the $C_{\varphi e}$ the constraint is about 1.89 to 3.40 times weaken, as an obvious results when we increase our set of Wilson coefficient by the non-universality assumption. Most of the coefficients are still constrained to the power of 10^{-3}, and deviate not greater than 2σ from 0.

The P-value (of 0.6595) is high, which mean our null hypothesis is much more extreme than what was actually observed, the results are strongly statistically insignificant.

	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^-_{\varphi l}$	$C^{(1)}_{\varphi q}$	$C^{(3)}_{\varphi q}$	$C_{\varphi u}$	$C_{\varphi d}$
Central Val. $\times 10^3$	-1.22	-1.70	3.33	3.83	6.69	7.26	-1.41	1.23	-1.22	2.90	-8.64
Errors $\times 10^3$	2.06	2.30	3.52	3.88	4.27	4.90	1.23	4.24	9.97	21.31	64.8

Table A.5: Results for the case of non-universal $\Sigma^+_{\varphi l}$, $C_{\varphi e}$ and universal quarks with $\chi^2_{\min} = 0.6595$, d.o.f 3 and P-value = 0.8827.

	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^{-}_{\varphi l}$	$C^{(1)}_{\varphi q}$	$C^{(3)}_{\varphi q}$	$C_{\varphi u}$	$C_{\varphi d}$
$\Sigma^{ee+}_{\varphi l}$	1.000										
$C^{ee}_{\varphi e}$	0.971	1.000									
$\Sigma^{\mu\mu+}_{\varphi l}$	−0.715	−0.630	1.000								
$C^{\mu\mu}_{\varphi e}$	−0.730	−0.636	0.963	1.000							
$\Sigma^{\tau\tau+}_{\varphi l}$	−0.475	−0.478	0.384	0.375	1.000						
$C^{\tau\tau}_{\varphi e}$	−0.469	−0.464	0.331	0.344	0.963	1.000					
$\Sigma^{\text{lept}-}_{\varphi l}$	0.196	0.095	−0.809	−0.744	−0.138	−0.068	1.000				
$C^{(1)}_{\varphi q}$	−0.520	−0.523	0.360	0.363	0.258	0.250	−0.079	1.000			
$C^{(3)}_{\varphi q}$	−0.910	−0.917	0.644	0.646	0.452	0.436	−0.148	0.297	1.000		
$C_{\varphi u}$	0.721	0.725	−0.494	−0.504	−0.352	−0.346	0.092	0.061	−0.904	1.000	
$C_{\varphi d}$	−0.950	−0.954	0.653	0.663	0.466	0.456	−0.129	0.571	0.945	−0.726	1.000

Table A.6: Correlation matrix for the case of non-universal $\Sigma^+_{\varphi l}$, $C_{\varphi e}$ and universal quarks.

Observables	Γ_Z	σ^0_{had}	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{\text{FB}}$	$A^{0,\mu}_{\text{FB}}$
Pull	0.099	−0.008	−0.046	−0.004	$\mathcal{O}(10^{-4})$	0.281	0.356
Observables	$A^{0,\tau}_{\text{FB}}$	m_W	Γ_W	R^0_b	R^0_c	$A^{0,b}_{\text{FB}}$	$A^{0,c}_{\text{FB}}$
Pull	0.007	$\mathcal{O}(10^{-4})$	0.052	−0.034	−0.196	−0.014	−0.013

Table A.7: Pull values for the case of non-universal $\Sigma^+_{\varphi l}$, $C_{\varphi e}$ and universal quarks.

	$\Sigma^{ee+}_{\varphi l}$	$C^{ee}_{\varphi e}$	$\Sigma^{\mu\mu+}_{\varphi l}$	$C^{\mu\mu}_{\varphi e}$	$\Sigma^{\tau\tau+}_{\varphi l}$	$C^{\tau\tau}_{\varphi e}$	$\Sigma^{\text{lept}-}_{\varphi l}$	$C^{(1)}_{\varphi q}$	$C^{(3)}_{\varphi q}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	−0.249	−0.158	−0.249	−0.158	0.184	−0.158	0.159	1.183	4.019	0.632	−0.474
σ^0_{had}	83.108	−71.028	−5.624	5.253	−6.111	5.253	−33.118	−10.886	−36.99	−5.814	4.361
R^0_e	−47.230	38.155	−2.847	0.	0.	0.	−5.693	14.234	48.365	7.602	−5.702
R^0_μ	−2.847	0.	−47.230	38.155	0.	0.	−5.693	14.234	48.365	7.602	−5.702
R^0_τ	−2.847	0.	−2.847	0.	−44.383	38.155	−5.693	14.234	48.365	7.602	−5.702
$A^{0,e}_{\text{FB}}$	0.112	0.458	−0.282	0.	0.	0.	−0.563	0.	0.	0.	0.
$A^{0,\mu}_{\text{FB}}$	−0.085	0.229	−0.085	0.229	0.	0.	−0.563	0.	0.	0.	0.
$A^{0,\tau}_{\text{FB}}$	−0.085	0.229	−0.282	0.	0.197	0.229	−0.563	0.	0.	0.	0.
m_W	−28.727	0.	−28.727	0.	0.	0.	−57.455	0.	0.	0.	0.
Γ_W	−1.843	0.	−1.843	0.	0.232	0.	−3.453	0.	2.786	0.	0.
R^0_b	0.007	0.	0.007	0.	0.	0.	0.013	0.346	−0.009	−0.079	−0.031
R^0_c	−0.010	0.	−0.010	0.	0.	0.	−0.020	−0.534	0.014	0.122	0.047
$A^{0,b}_{\text{FB}}$	0.345	1.456	−0.906	0.	0.	0.	−1.813	0.016	0.016	0.	0.089
$A^{0,c}_{\text{FB}}$	0.192	1.040	−0.702	0.	0.	0.	−1.404	−0.088	0.088	−0.198	0.

Table A.8: Coefficient matrix A_{ij} for the case of non-universal $\Sigma^{+}_{\varphi l}$, $C_{\varphi e}$ and universal quarks. Analytical results are given in Appendix B.5.2.

Appendix B

Analytical results for coefficients c_j^i

Here, for the governing equation of

$$O_i = O_{\text{theo},i}^{\text{SM}} \left(1 + \sum_j c_j^i \hat{C}_j\right), \tag{B.0.1}$$

we would like to present the analytical results of c_j^i for each of our assumptions. These results are needed for the calculations in Chapters 4 and 5.

B.1 Quark sector

	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_b	$c_a^{\Gamma_b}$	$c_b^{\Gamma_b}$	0.	$c_d^{\Gamma_b}$
Γ_c	$c_a^{\Gamma_c}$	$c_b^{\Gamma_c}$	$c_c^{\Gamma_c}$	0.
$\Gamma_{\text{had.}}$	$c_a^{\Gamma_{\text{had.}}}$	$c_b^{\Gamma_{\text{had.}}}$	$c_c^{\Gamma_{\text{had.}}}$	$c_d^{\Gamma_{\text{had.}}}$
Γ_{Z}	$c_a^{\Gamma_{\text{Z}}}$	$c_b^{\Gamma_{\text{Z}}}$	$c_c^{\Gamma_{\text{Z}}}$	$c_d^{\Gamma_{\text{Z}}}$
σ_{had}^0	$c_a^{\sigma_{\text{had}}^0}$	$c_b^{\sigma_{\text{had}}^0}$	$c_c^{\sigma_{\text{had}}^0}$	$c_d^{\sigma_{\text{had}}^0}$
R_0^b	$c_a^{R_0^b}$	$c_b^{R_0^b}$	$c_c^{R_0^b}$	$c_d^{R_0^b}$
R_0^c	$c_a^{R_0^c}$	$c_b^{R_0^c}$	$c_c^{R_0^c}$	$c_d^{R_0^c}$
R_0^e	$c_a^{R_0^e}$	$c_b^{R_0^e}$	$c_c^{R_0^e}$	$c_d^{R_0^e}$
R_0^μ	$c_a^{R_0^\mu}$	$c_b^{R_0^\mu}$	$c_c^{R_0^\mu}$	$c_d^{R_0^\mu}$
R_0^τ	$c_a^{R_0^\tau}$	$c_b^{R_0^\tau}$	$c_c^{R_0^\tau}$	$c_d^{R_0^\tau}$

Table B.1: Coefficients for quark sector.

With the following notation:

$$\chi_{\rm SM}^{\nu_{ep}} = \chi_{\rm SM}^{\nu_l} = \frac{1}{2}, \tag{B.1.2}$$

$$\chi_{\rm SM}^{e_p} = \chi_{\rm SM}^{l} = \frac{1}{2} - 2s_0^2 + 4s_0^4, \tag{B.1.3}$$

$$\chi_{\rm SM}^{u} = \chi_{\rm SM}^{c} = \frac{1}{2} - \frac{4}{3}s_0^2 + \frac{16}{9}s_0^4, \tag{B.1.4}$$

$$\chi_{\rm SM}^{d} = \chi_{\rm SM}^{b} = \frac{1}{2} - \frac{2}{3}s_0^2 + \frac{4}{9}s_0^4, \tag{B.1.5}$$

$$\chi_{\rm SM}^{\rm had.} = 2\cdot\chi_{\rm SM}^{c} + 3\cdot\chi_{\rm SM}^{b} = \frac{5}{2} - \frac{14}{3}s_0^2 + \frac{44}{9}s_0^4, \tag{B.1.6}$$

$$\chi_{\rm SM}^{\rm Z} = \chi_{\rm SM}^{\nu_l} + \chi_{\rm SM}^{l} + \chi_{\rm SM}^{\rm had.} = \frac{7}{2} - \frac{20}{3}s_0^2 + \frac{80}{9}s_0^4. \tag{B.1.7}$$

Coefficients for the decay width read:

For d-type quarks:

$$c_a^{\Gamma_b} = \frac{1-\frac{2}{3}s_0^2}{\chi_{\rm SM}^b}, \quad c_b^{\Gamma_b} = \frac{1-\frac{2}{3}s_0^2}{\chi_{\rm SM}^b}, \quad c_d^{\Gamma_b} = \frac{-\frac{2}{3}s_0^2}{\chi_{\rm SM}^b}. \tag{B.1.8}$$

For u-type quarks:

$$c_a^{\Gamma_c} = \frac{-1+\frac{4}{3}s_0^2}{\chi_{\rm SM}^c}, \quad c_b^{\Gamma_c} = \frac{+1-\frac{4}{3}s_0^2}{\chi_{\rm SM}^c}, \quad c_c^{\Gamma_c} = \frac{\frac{4}{3}s_0^2}{\chi_{\rm SM}^c}. \tag{B.1.9}$$

We have:

$$\chi_{\rm SM}^{\rm had.} = 2\cdot\chi_{\rm SM}^{c} + 3\cdot\chi_{\rm SM}^{b}. \tag{B.1.10}$$

The hadrons' coefficients read:

$$c_a^{\Gamma_{\rm had.}} = 3\cdot\frac{\chi_{\rm SM}^b}{\chi_{\rm SM}^{\rm had.}}\cdot c_a^{\Gamma_{\rm b}} + 2\cdot\frac{\chi_{\rm SM}^c}{\chi_{\rm SM}^{\rm had.}}\cdot c_a^{\Gamma_{\rm c}} = \frac{6(3+2s_0^2)}{45-84s_0^2+88s_0^4}\,, \tag{B.1.11}$$

$$c_b^{\Gamma_{\rm had.}} = 3\cdot\frac{\chi_{\rm SM}^b}{\chi_{\rm SM}^{\rm had.}}\cdot c_b^{\Gamma_{\rm b}} + 2\cdot\frac{\chi_{\rm SM}^c}{\chi_{\rm SM}^{\rm had.}}\cdot c_b^{\Gamma_{\rm c}} = \frac{90-84s_0^2}{45-84s_0^2+88s_0^4}\,, \tag{B.1.12}$$

$$c_c^{\Gamma_{\rm had.}} = 2\cdot\frac{\chi_{\rm SM}^c}{\chi_{\rm SM}^{\rm had.}}\cdot c_c^{\Gamma_{\rm c}} = \frac{48s_0^2}{45-84s_0^2+88s_0^4}\,, \tag{B.1.13}$$

$$c_d^{\Gamma_{\rm had.}} = 3\cdot\frac{\chi_{\rm SM}^b}{\chi_{\rm SM}^{\rm had.}}\cdot c_d^{\Gamma_{\rm b}} = \frac{-36s_0^2}{45-84s_0^2+88s_0^4}\,. \tag{B.1.14}$$

For the total Z decay width:

$$c_a^{\Gamma_{\rm Z}} = \frac{\chi^{\rm had.}}{\chi^{\rm Z}}\cdot c_a^{\Gamma_{\rm had.}} = \frac{6(3+2s_0^2)}{63-120s_0^2+160s_0^4}, \tag{B.1.15}$$

$$c_b^{\Gamma_{\rm Z}} = \frac{\chi^{\rm had.}}{\chi^{\rm Z}}\cdot c_b^{\Gamma_{\rm had.}} = \frac{90-84s_0^2}{63-120s_0^2+160s_0^4}, \tag{B.1.16}$$

$$c_c^{\Gamma_{\rm Z}} = \frac{\chi^{\rm had.}}{\chi^{\rm Z}}\cdot c_c^{\Gamma_{\rm had.}} = \frac{48s_0^2}{63-120s_0^2+160s_0^4}, \tag{B.1.17}$$

$$c_d^{\Gamma_{\rm Z}} = \frac{\chi^{\rm had.}}{\chi^{\rm Z}}\cdot c_d^{\Gamma_{\rm had.}} = \frac{-36s_0^2}{63-120s_0^2+160s_0^4}. \tag{B.1.18}$$

We derive the following relations:
For the σ^0_{had}:

$$c^{\sigma^0_{\text{had}}}_{a,b,c,d} = c^{\Gamma_{\text{had.}}}_{a,b,c,d} - 2c^{\Gamma_Z}_{a,b,c,d}. \tag{B.1.19}$$

For the R^c_0:

$$c^{R^c_0}_{a,b,c} = c^{\Gamma_c}_{a,b,c} - c^{\text{had.}}_{a,b,c}, \qquad c^{R^c_0}_{d} = -c^{\text{had.}}_{d}. \tag{B.1.20}$$

For the R^b_0:

$$c^{R^b_0}_{a,b,d} = c^{\Gamma_b}_{a,b,d} - c^{\text{had.}}_{a,b,d}, \qquad c^{R^b_0}_{c} = -c^{\text{had.}}_{c}. \tag{B.1.21}$$

For the $R^{e_p}_0$:

$$c^{R^e_0}_{a,b,c,d} = c^{R^\mu_0}_{a,b,c,d} = c^{R^\tau_0}_{a,b,c,d} = c^{\text{had.}}_{a,b,c,d} \tag{B.1.22}$$

The quarks are considered as flavor-universal and the above set of coefficients is applied to derive observables for all of the cases with quarks.

B.2 Universal lepton and quark

B.2.1 Lepton

	$C_{\varphi WB}$	$C_{\varphi D}$	$C^{(1)}_{\varphi l}$	$C^{(3)}_{\varphi l}$	$C^{\text{uni.}}_{\varphi e}$
Γ_{ν_l}	0.	$c_2^{\Gamma_{\nu_l}}$	$c_3^{\Gamma_{\nu_l}}$	$c_4^{\Gamma_{\nu_l}}$	0.
Γ_l	$c_1^{\Gamma_l}$	$c_2^{\Gamma_l}$	$c_3^{\Gamma_l}$	$c_4^{\Gamma_l}$	$c_5^{\Gamma_l}$
$\Gamma_{\text{had.}}$	$c_1^{\Gamma_{\text{had.}}}$	$c_2^{\Gamma_{\text{had.}}}$	0.	$c_4^{\Gamma_{\text{had.}}}$	0.

Table B.2: Coefficients of decay widths in universal lepton assumption.

For the neutrinos:

$$c_2^{\Gamma_{\nu_l}} = -\frac{1}{2}, \qquad c_3^{\Gamma_{\nu_l}} = -2, \qquad c_4^{\Gamma_{\nu_l}} = 2. \tag{B.2.23}$$

For the charged leptons:

$$c_1^{\Gamma_l} = \frac{4s_0c_0(4s_0^2-1)}{1-6s_0^2+16s_0^4-16s_0^6}, \qquad c_2^{\Gamma_l} = \frac{(-1+2s_0^2+4s_0^4)}{2-4s_0^2(3-8s_0^2+8s_0^4)}, \tag{B.2.24}$$

$$c_3^{\Gamma_l} = \frac{-2s_0^2+1}{\chi^l_{\text{SM}}}, \qquad c_4^{\Gamma_l} = \frac{2-16s_0^2+48s_0^4-32s_0^6}{1-6s_0^2+16s_0^4-16s_0^6}, \tag{B.2.25}$$

$$c_5^{\Gamma_l} = \frac{-2s_0^2}{\chi^l_{\text{SM}}}. \tag{B.2.26}$$

For hadrons:

$$c_1^{\Gamma_{\rm had}} = \frac{4s_0c_0(44s_0^2-21)}{45-174s_0^2+256s_0^4-176s_0^6}, \qquad c_2^{\Gamma_{\rm had}} = \frac{(-45+90s_0^2+4s_0^4)}{90-348s_0^2+512s_0^4-352s_0^6}, \tag{B.2.27}$$

$$c_4^{\Gamma_{\rm had}} = \frac{-8s_0^2(21-65s_0^2+44s_0^4)}{45-174s_0^2+256s_0^4-176s_0^6}. \tag{B.2.28}$$

The coefficients for total Z decay width read:

$$c_1^{\Gamma_Z} = \frac{40s_0c_0(8s_0^2-3)}{63-246s_0^2+400s_0^4-320s_0^6}, \qquad c_2^{\Gamma_Z} = \frac{(-63+126s_0^2+40s_0^4)}{126-492s_0^2+800s_0^4-640s_0^6}, \tag{B.2.29}$$

$$c_3^{\Gamma_Z} = \frac{-2s_0^2}{\chi^Z} = \frac{-36s_0^2}{63-120s_0^2+160s_0^4}, \qquad c_4^{\Gamma_Z} = \frac{4(-9+87s_0^2-238s_0^4+160s_0^6)}{-63+246s_0^2-400s_0^4+320s_0^6}, \tag{B.2.30}$$

$$c_5^{\Gamma_Z} = \frac{-2s_0^2}{\chi^Z} = \frac{-36s_0^2}{63-120s_0^2+160s_0^4}. \tag{B.2.31}$$

For W boson vertices,

$$c_1^{\Gamma_W} = \frac{3s_0c_0}{s_0^2-c_0^2}, \qquad c_2^{\Gamma_W} = \frac{3c_0^2}{4(s_0^2-c_0^2)}, \qquad c_4^{\Gamma_W} = \frac{7c_0^2+2s_0^2}{3(s_0^2-c_0^2)}. \tag{B.2.32}$$

$$c_1^{m_W} = \frac{s_0c_0}{s_0^2-c_0^2}, \qquad c_2^{m_W} = \frac{c_0^2}{4(s_0^2-c_0^2)}, \qquad c_4^{m_W} = \frac{c_0^2}{s_0^2-c_0^2}. \tag{B.2.33}$$

These coefficient remain unchanged for all the cases.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$
Γ_Z	$c_1^{\Gamma_Z}$	$c_2^{\Gamma_Z}$	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$
$\sigma^0_{\rm had}$	$c_1^{\sigma^0_{\rm had}}$	$c_2^{\sigma^0_{\rm had}}$	$c_3^{\sigma^0_{\rm had}}$	$c_4^{\sigma^0_{\rm had}}$	$c_5^{\sigma^0_{\rm had}}$
R^0_ℓ	$c_1^{R^0_\ell}$	$c_2^{R^0_\ell}$	$c_3^{R^0_\ell}$	$c_4^{R^0_\ell}$	$c_5^{R^0_\ell}$
$A^{0,\ell}_{\rm FB}$	$c_1^{A^{0,\ell}_{\rm FB}}$	$c_2^{A^{0,\ell}_{\rm FB}}$	$c_3^{A^{0,\ell}_{\rm FB}}$	$c_4^{A^{0,\ell}_{\rm FB}}$	$c_5^{A^{0,\ell}_{\rm FB}}$
m_W	$c_1^{m_W}$	$c_2^{m_W}$	0.	$c_4^{m_W}$	0.
Γ_W	$c_1^{\Gamma_W}$	$c_2^{\Gamma_W}$	0.	$c_4^{\Gamma_W}$	0.

Table B.3: Coefficients for lepton in universal lepton assumption.

The coefficients for the following observables are derived through relations between coefficients from the above decay widths:
For $\sigma_{\rm had.}$

$$c_{1,2,3,4,5}^{\sigma^0_{\rm had}} = c_{1,2,3,4,5}^{\Gamma_e} - 2\cdot c_{1,2,3,4,5}^{\Gamma_Z}. \tag{B.2.34}$$

For R^0_ℓ:

$$c_{1,2,3,4,5}^{R^0_\ell} = -c_{1,2,3,4,5}^{\Gamma^0_l}. \tag{B.2.35}$$

For $A_{\rm FB}^{0,e_p}$:

$$c_1^{A_{\rm FB}^{0,\ell}} = \frac{32s_0^3c_0}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_2^{A_{\rm FB}^{0,\ell}} = \frac{-16(-s_0^4+s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.2.36}$$

$$c_3^{A_{\rm FB}^{0,\ell}} = \frac{32(-s_0^4+2s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_4^{A_{\rm FB}^{0,\ell}} = \frac{32s_0^4}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.2.37}$$

$$c_5^{A_{\rm FB}^{0,\ell}} = \frac{-16(s_0^2-4s_0^4+4s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.2.38}$$

B.2.2 Lepton and quark

The above coefficients for lepton sector remain unchanged. Now, we add some coefficients for quark sector.
For Γ_W:

$$c_7^{\Gamma_W} = \frac{4}{3} \tag{B.2.39}$$

For $\sigma^0_{\rm had}$:

$$c_{6,7,8,9}^{\sigma^0_{\rm had}} = c_{a,b,c,d}^{\sigma^0_{\rm had}}, \tag{B.2.40}$$

For $c^{R^0_{b,c}}$:

$$c_4^{R_b^0} = \frac{48s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-12s_0^2+8s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.2.41}$$

$$c_4^{R_c^0} = \frac{-72s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-24s_0^2+32s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.2.42}$$

$$c_{6,7,8,9}^{R_b^0} = c_{a,b,c,d}^{R_b^0}, \qquad c_{6,7,8,9}^{R_c^0} = c_{a,b,c,d}^{R_c^0}. \tag{B.2.43}$$

For $c^{R^0_\ell}$:

$$c_{6,7,8,9}^{R_\ell^0} = c_{a,b,c,d}^{R_\ell^0}, \qquad c_{6,7,8,9}^{R_\ell^0} = c_{a,b,c,d}^{R_\ell^0}, \qquad c_{6,7,8,9}^{R_\ell^0} = c_{a,b,c,d}^{R_\ell^0}. \tag{B.2.44}$$

For $A_{\rm FB}^{0,b}$:

$$c_1^{A_{\rm FB}^{0,b}} = \frac{4c_0(120s_0^3-608s_0^5+1216s_0^7-1280s_0^9+512s_0^{11})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.2.45}$$

$$c_2^{A_{\rm FB}^{0,b}} = \frac{-2(-120s_0^4+728s_0^6-1824s_0^8+2496s_0^{10}-1792s_0^{12}+512s_0^{14})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.2.46}$$

$$c_4^{A_{\rm FB}^{0,b}} = \frac{-16s_0^4(-33+184s_0^2-444s_0^4+640s_0^6-480s_0^8+128s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.2.47}$$

$$c_3^{A_{\rm FB}^{0,b}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_5^{A_{\rm FB}^{0,b}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.2.48}$$

$$c_6^{A_{\rm FB}^{0,b}} = \frac{2(-24s_0^4+16s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_7^{A_{\rm FB}^{0,b}} = \frac{16s_0^4(-3+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.2.49}$$

$$c_9^{A_{\rm FB}^{0,b}} = \frac{-8s_0^2(9-12s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.2.50}$$

For $A_{\rm FB}^{0,c}$:

$$c_1^{A_{\rm FB}^{0,c}} = \frac{8c_0(78s_0^3 - 620s_0^5 + 1984s_0^7 - 3200s_0^9 + 2048s_0^{11})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.2.51}$$

$$c_2^{A_{\rm FB}^{0,c}} = \frac{-4(-78s_0^4 + 698s_0^6 - 2604s_0^8 + 5184s_0^{10} - 5248s_0^{12} + 2048s_0^{14})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.2.52}$$

$$c_4^{A_{\rm FB}^{0,c}} = \frac{-16s_0^4(-51 + 446s_0^2 - 1632s_0^4 + 3200s_0^6 - 3072s_0^8 + 1024s_0^{10})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.2.53}$$

$$c_3^{A_{\rm FB}^{0,c}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_5^{A_{\rm FB}^{0,c}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.2.54}$$

$$c_6^{A_{\rm FB}^{0,c}} = \frac{-2(-96s_0^4 + 128s_0^6)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_7^{A_{\rm FB}^{0,c}} = \frac{64s_0^4(-3+4s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.2.55}$$

$$c_8^{A_{\rm FB}^{0,c}} = \frac{16s_0^2(9-24s_0^2+16s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}. \tag{B.2.56}$$

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	$c_1^{\Gamma_Z}$	$c_2^{\Gamma_Z}$	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$	$c_8^{\Gamma_Z}$	$c_9^{\Gamma_Z}$
$\sigma_{\rm had}^0$	$c_1^{\sigma_{\rm had}^0}$	$c_2^{\sigma_{\rm had}^0}$	$c_3^{\sigma_{\rm had}^0}$	$c_4^{\sigma_{\rm had}^0}$	$c_5^{\sigma_{\rm had}^0}$	$c_6^{\sigma_{\rm had}^0}$	$c_7^{\sigma_{\rm had}^0}$	$c_8^{\sigma_{\rm had}^0}$	$c_9^{\sigma_{\rm had}^0}$
R_ℓ^0	$c_1^{R_\ell^0}$	$c_2^{R_\ell^0}$	$c_3^{R_\ell^0}$	$c_4^{R_\ell^0}$	$c_5^{R_\ell^0}$	$c_6^{R_\ell^0}$	$c_7^{R_\ell^0}$	$c_8^{R_\ell^0}$	$c_9^{R_\ell^0}$
$A_{\rm FB}^{0,\ell}$	$c_1^{A_{\rm FB}^{0,\ell}}$	$c_2^{A_{\rm FB}^{0,\ell}}$	$c_3^{A_{\rm FB}^{0,\ell}}$	$c_4^{A_{\rm FB}^{0,\ell}}$	$c_5^{A_{\rm FB}^{0,\ell}}$	0.	0.	0.	0.
m_W	$c_1^{m_W}$	$c_2^{m_W}$	0.	$c_4^{m_W}$	0.	0.	0.	0.	0.
Γ_W	$c_2^{\Gamma_W}$	$c_2^{\Gamma_W}$	0.	$c_4^{\Gamma_W}$	0.	0.	$c_7^{\Gamma_W}$	0.	0.
R_b^0	$c_1^{R_b^0}$	$c_2^{R_b^0}$	0.	$c_4^{R_b^0}$	0.	$c_6^{R_b^0}$	$c_7^{R_b^0}$	$c_8^{R_b^0}$	$c_9^{R_b^0}$
R_c^0	$c_1^{R_c^0}$	$c_2^{R_c^0}$	0.	$c_4^{R_c^0}$	0.	$c_6^{R_c^0}$	$c_7^{R_c^0}$	$c_8^{R_c^0}$	$c_9^{R_c^0}$
$A_{\rm FB}^{0,b}$	$c_1^{A_{\rm FB}^{0,b}}$	$c_2^{A_{\rm FB}^{0,b}}$	$c_3^{A_{\rm FB}^{0,b}}$	$c_4^{A_{\rm FB}^{0,b}}$	$c_5^{A_{\rm FB}^{0,b}}$	$c_6^{A_{\rm FB}^{0,b}}$	$c_7^{A_{\rm FB}^{0,b}}$	0.	$c_9^{A_{\rm FB}^{0,b}}$
$A_{\rm FB}^{0,c}$	$c_1^{A_{\rm FB}^{0,c}}$	$c_2^{A_{\rm FB}^{0,c}}$	$c_3^{A_{\rm FB}^{0,c}}$	$c_4^{A_{\rm FB}^{0,c}}$	$c_5^{A_{\rm FB}^{0,c}}$	$c_6^{A_{\rm FB}^{0,c}}$	$c_7^{A_{\rm FB}^{0,c}}$	$c_8^{A_{\rm FB}^{0,c}}$	0.

Table B.4: Coefficients for lepton and quark in universal lepton assumption.

B.3 Non-universal $C_{\varphi e}$ and quark

B.3.1 Lepton

For the neutrinos:

$$c_2^{\Gamma_{\nu_e}} = c_2^{\Gamma_{\nu_\mu}} = c_2^{\Gamma_{\nu_\tau}} = -\frac{1}{2}, \qquad c_3^{\Gamma_{\nu_e}} = c_3^{\Gamma_{\nu_\mu}} = c_3^{\Gamma_{\nu_\tau}} = -2, \qquad c_4^{\Gamma_{\nu_e}} = c_4^{\Gamma_{\nu_\mu}} = c_4^{\Gamma_{\nu_\tau}} = 2. \tag{B.3.57}$$

For the charged leptons:

$$c_1^{\Gamma_e} = c_1^{\Gamma_\mu} = c_1^{\Gamma_\tau} = \frac{4s_0c_0(4s_0^2-1)}{1-6s_0^2+16s_0^4-16s_0^6}\ , \tag{B.3.58}$$

$$c_2^{\Gamma_e} = c_2^{\Gamma_\mu} = c_2^{\Gamma_\tau} = \frac{-1+2s_0^2+4s_0^4}{2-4s_0^2(3-8s_0^2+8s_0^4)}\ , \tag{B.3.59}$$

$$c_3^{\Gamma_e} = c_3^{\Gamma_\mu} = c_3^{\Gamma_\tau} = \frac{-2s_0^2+1}{\chi_{\text{SM}}^l}\ , \tag{B.3.60}$$

$$c_4^{\Gamma_e} = c_4^{\Gamma_\mu} = c_4^{\Gamma_\tau} = \frac{2(-1+8s_0^2-24s_0^4+16s_0^6)}{-1+6s_0^2-16s_0^4+16s_0^6}, \tag{B.3.61}$$

$$c_5^{\Gamma_e} = c_6^{\Gamma_\mu} = c_7^{\Gamma_\tau} = \frac{-2s_0^2}{\chi_{\text{SM}}^l}\ . \tag{B.3.62}$$

For hadrons:

$$c_1^{\Gamma_{\text{had}}} = \frac{4s_0c_0(44s_0^2-21)}{45-174s_0^2+256s_0^4-176s_0^6}, \qquad c_2^{\Gamma_{\text{had}}} = \frac{(-45+90s_0^2+4s_0^4)}{90-348s_0^2+512s_0^4-352s_0^6}, \tag{B.3.63}$$

$$c_4^{\Gamma_{\text{had}}} = \frac{8s_0^2(21-65s_0^2+44s_0^4)}{-45+174s_0^2-256s_0^4+176s_0^6}. \tag{B.3.64}$$

The coefficients for total Z decay width read:

$$c_1^{\Gamma_Z} = \frac{40s_0c_0(8s_0^2-3)}{63-246s_0^2+400s_0^4-320s_0^6}, \qquad c_2^{\Gamma_Z} = \frac{(-63+126s_0^2+40s_0^4)}{126-492s_0^2+800s_0^4-640s_0^6}, \tag{B.3.65}$$

$$c_3^{\Gamma_Z} = \frac{-2s_0^2}{\chi^Z} = \frac{-36s_0^2}{63-120s_0^2+160s_0^4}, \qquad c_4^{\Gamma_Z} = \frac{4(-9+87s_0^2-238s_0^4+160s_0^6)}{-63+246s_0^2-400s_0^4+320s_0^6}, \tag{B.3.66}$$

$$c_5^{\Gamma_Z} = c_6^{\Gamma_Z} = c_7^{\Gamma_Z} = \frac{1}{3}\cdot\frac{-2s_0^2}{\chi^Z} = \frac{-12s_0^2}{63-120s_0^2+160s_0^4}. \tag{B.3.67}$$

For the total W mass and decay width:

$$c_4^{\Gamma_W} = \frac{7c_0^2+2s_0^2}{3(s_0^2-c_0^2)}, \qquad c_4^{m_W} = \frac{c_0^2}{s_0^2-c_0^2}. \tag{B.3.68}$$

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$
Γ_{ν_e}	0.	$c_2^{\Gamma_{\nu_e}}$	$c_3^{\Gamma_{\nu_e}}$	$c_4^{\Gamma_{\nu_e}}$	0.	0.	0.
Γ_{ν_μ}	0.	$c_2^{\Gamma_{\nu_\mu}}$	$c_3^{\Gamma_{\nu_\mu}}$	$c_4^{\Gamma_{\nu_\mu}}$	0.	0.	0.
Γ_{ν_τ}	0.	$c_2^{\Gamma_{\nu_\tau}}$	$c_3^{\Gamma_{\nu_\tau}}$	$c_4^{\Gamma_{\nu_\tau}}$	0.	0.	0.
Γ_e	$c_1^{\Gamma_e}$	$c_2^{\Gamma_e}$	$c_3^{\Gamma_e}$	$c_4^{\Gamma_e}$	$c_5^{\Gamma_e}$	0.	0.
Γ_μ	$c_1^{\Gamma_\mu}$	$c_2^{\Gamma_\mu}$	$c_3^{\Gamma_\mu}$	$c_4^{\Gamma_\mu}$	0.	$c_6^{\Gamma_\mu}$	0.
Γ_τ	$c_1^{\Gamma_\tau}$	$c_2^{\Gamma_\tau}$	$c_3^{\Gamma_\tau}$	$c_4^{\Gamma_\tau}$	0.	0.	$c_7^{\Gamma_\tau}$
$\Gamma_{\text{had.}}$	$c_1^{\Gamma_{\text{had.}}}$	$c_2^{\Gamma_{\text{had.}}}$	0.	$c_4^{\Gamma_{\text{had.}}}$	0.	0.	0.

Table B.5: Coefficients of decay widths for lepton in non-universal $C_{\varphi e}$ assumption.

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$
Γ_Z	$c_1^{\Gamma_Z}$	$c_2^{\Gamma_Z}$	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$
σ^0_{had}	$c_1^{\sigma^0_{\text{had}}}$	$c_2^{\sigma^0_{\text{had}}}$	$c_3^{\sigma^0_{\text{had}}}$	$c_4^{\sigma^0_{\text{had}}}$	$c_5^{\sigma^0_{\text{had}}}$	$c_6^{\sigma^0_{\text{had}}}$	$c_7^{\sigma^0_{\text{had}}}$
R^0_e	$c_1^{R^0_e}$	$c_2^{R^0_e}$	$c_3^{R^0_e}$	$c_4^{R^0_e}$	$c_5^{R^0_e}$	0.	0.
R^0_μ	$c_1^{R^0_\mu}$	$c_2^{R^0_\mu}$	$c_3^{R^0_\mu}$	$c_4^{R^0_\mu}$	0.	$c_6^{R^0_\mu}$	0.
R^0_τ	$c_1^{R^0_\tau}$	$c_2^{R^0_\tau}$	$c_3^{R^0_\tau}$	$c_4^{R^0_\tau}$	0.	0.	$c_7^{R^0_\tau}$
$A^{0,e}_{\text{FB}}$	$c_1^{A^{0,e}_{\text{FB}}}$	$c_2^{A^{0,e}_{\text{FB}}}$	$c_3^{A^{0,e}_{\text{FB}}}$	$c_4^{A^{0,e}_{\text{FB}}}$	$c_5^{A^{0,e}_{\text{FB}}}$	0.	0.
$A^{0,\mu}_{\text{FB}}$	$c_1^{A^{0,\mu}_{\text{FB}}}$	$c_2^{A^{0,\mu}_{\text{FB}}}$	$c_3^{A^{0,\mu}_{\text{FB}}}$	$c_4^{A^{0,\mu}_{\text{FB}}}$	$c_5^{A^{0,\mu}_{\text{FB}}}$	$c_6^{A^{0,\mu}_{\text{FB}}}$	0.
$A^{0,\tau}_{\text{FB}}$	$c_1^{A^{0,\tau}_{\text{FB}}}$	$c_2^{A^{0,\tau}_{\text{FB}}}$	$c_3^{A^{0,\tau}_{\text{FB}}}$	$c_4^{A^{0,\tau}_{\text{FB}}}$	$c_5^{A^{0,\tau}_{\text{FB}}}$	0.	$c_7^{A^{0,\tau}_{\text{FB}}}$
m_W	$c_1^{m_W}$	$c_2^{m_W}$	0.	$c_4^{m_W}$	0.	0.	0.
Γ_W	$c_1^{\Gamma_W}$	$c_2^{\Gamma_W}$	0.	$c_4^{\Gamma_W}$	0.	0.	0.

Table B.6: Coefficients for lepton in non-universal $C_{\varphi e}$ assumption.

The coefficients for the following observables are derived through relations between coefficients from the above decay widths:
For $\sigma_{\text{had.}}$

$$c_{1,2,3,4,5}^{\sigma^0_{\text{had}}} = c_{1,2,3,4,5}^{\Gamma_e} - 2 \cdot c_{1,2,3,4,5}^{\Gamma_Z}, \qquad c_{5,6,7}^{\sigma^0_{\text{had}}} = -2 \cdot c_{5,6,7}^{\Gamma_Z}. \tag{B.3.69}$$

For $R^0_{e_p}$:

$$c_{1,2,3,4,5}^{R^0_e} = -c_{1,2,3,4,5}^{\Gamma^0_e}, \qquad c_{1,2,3,4,6}^{R^0_\mu} = -c_{1,2,3,4,6}^{\Gamma^0_\mu}, \qquad c_{1,2,3,4,7}^{R^0_\tau} = -c_{1,2,3,4,7}^{\Gamma^0_\tau}. \tag{B.3.70}$$

For A^{0,e_p}_{FB}:

$$c_1^{A^{0,e}_{\text{FB}}} = c_1^{A^{0,\mu}_{\text{FB}}} = c_1^{A^{0,\tau}_{\text{FB}}} = \frac{32 s_0^3 c_0}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.3.71}$$

$$c_2^{A^{0,e}_{\text{FB}}} = c_2^{A^{0,\mu}_{\text{FB}}} = c_2^{A^{0,\tau}_{\text{FB}}} = \frac{-16(-s_0^4 + s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.3.72}$$

$$c_3^{A^{0,e}_{\text{FB}}} = c_3^{A^{0,\mu}_{\text{FB}}} = c_3^{A^{0,\tau}_{\text{FB}}} = \frac{32(-s_0^4 + 2 s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.3.73}$$

$$c_4^{A^{0,e}_{\text{FB}}} = c_4^{A^{0,\mu}_{\text{FB}}} = c_4^{A^{0,\tau}_{\text{FB}}} = \frac{32 s_0^4}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.3.74}$$

$$c_5^{A^{0,e}_{\text{FB}}} = \frac{-16(s_0^2 - 4 s_0^4 + 4 s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1} \tag{B.3.75}$$

$$c_{5,6}^{A^{0,\mu}_{\text{FB}}} = c_{5,7}^{A^{0,\tau}_{\text{FB}}} = \frac{-8(s_0^2 - 4 s_0^4 + 4 s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}. \tag{B.3.76}$$

B.3.2 Lepton and quark

The above coefficients for lepton sector remain unchanged. Now, we add some coefficients for quark sector.

	$C_{\varphi l}^{(1)}$	$C_{\varphi l}^{(3)}$	$C_{\varphi e}^{ee}$	$C_{\varphi e}^{\mu\mu}$	$C_{\varphi e}^{\tau\tau}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$,	$c_8^{\Gamma_Z}$,	$c_9^{\Gamma_Z}$,	$c_{10}^{\Gamma_Z}$,	$c_{11}^{\Gamma_Z}$
$\sigma_{\rm had}^0$	$c_3^{\sigma_{\rm had}^0}$	$c_4^{\sigma_{\rm had}^0}$	$c_5^{\sigma_{\rm had}^0}$	$c_6^{\sigma_{\rm had}^0}$	$c_7^{\sigma_{\rm had}^0}$	$c_8^{\sigma_{\rm had}^0}$	$c_9^{\sigma_{\rm had}^0}$	$c_{10}^{\sigma_{\rm had}^0}$	$c_{11}^{\sigma_{\rm had}^0}$
R_e^0	$c_3^{R_e^0}$	$c_4^{R_e^0}$	$c_5^{R_e^0}$	0.	0.	$c_8^{R_e^0}$	$c_9^{R_e^0}$	$c_{10}^{R_e^0}$	$c_{11}^{R_e^0}$
R_μ^0	$c_3^{R_\mu^0}$	$c_4^{R_\mu^0}$	0.	$c_6^{R_\mu^0}$	0.	$c_8^{R_\mu^0}$	$c_9^{R_\mu^0}$	$c_{10}^{R_\mu^0}$	$c_{11}^{R_\mu^0}$
R_τ^0	$c_3^{R_\tau^0}$	$c_4^{R_\tau^0}$	0.	0.	$c_7^{R_\tau^0}$	$c_8^{R_\tau^0}$	$c_9^{R_\tau^0}$	$c_{10}^{R_\tau^0}$	$c_{11}^{R_\tau^0}$
$A_{\rm FB}^{0,e}$	$c_3^{A_{\rm FB}^{0,e}}$	$c_4^{A_{\rm FB}^{0,e}}$	$c_5^{A_{\rm FB}^{0,e}}$	0.	0.	0.	0.	0.	0.
$A_{\rm FB}^{0,\mu}$	$c_3^{A_{\rm FB}^{0,\mu}}$	$c_4^{A_{\rm FB}^{0,\mu}}$	$c_5^{A_{\rm FB}^{0,\mu}}$	$c_6^{A_{\rm FB}^{0,\mu}}$	0.	0.	0.	0.	0.
$A_{\rm FB}^{0,\tau}$	$c_3^{A_{\rm FB}^{0,\tau}}$	$c_4^{A_{\rm FB}^{0,\tau}}$	$c_5^{A_{\rm FB}^{0,\tau}}$	0.	$c_7^{A_{\rm FB}^{0,\tau}}$	0.	0.	0.	0.
m_W	0.	$c_4^{m_W}$	0.	0.	0.	0.	0.	0.	0.
Γ_W	0.	$c_4^{\Gamma_W}$	0.	0.	0.	0.	$c_9^{\Gamma_W}$	0.	0.
R_b^0	0.	$c_4^{R_b^0}$	0.	0.	0.	$c_8^{R_b^0}$	$c_9^{R_b^0}$	$c_{10}^{R_b^0}$	$c_{11}^{R_b^0}$
R_c^0	0.	$c_4^{R_c^0}$	0.	0.	0.	$c_8^{R_c^0}$	$c_9^{R_c^0}$	$c_{10}^{R_c^0}$	$c_{11}^{R_c^0}$
$A_{\rm FB}^{0,b}$	$c_3^{A_{\rm FB}^{0,b}}$	$c_4^{A_{\rm FB}^{0,b}}$	$c_5^{A_{\rm FB}^{0,b}}$	0.	0.	$c_8^{A_{\rm FB}^{0,b}}$	$c_9^{A_{\rm FB}^{0,b}}$	0.	$c_{11}^{A_{\rm FB}^{0,b}}$
$A_{\rm FB}^{0,c}$	$c_3^{A_{\rm FB}^{0,c}}$	$c_4^{A_{\rm FB}^{0,c}}$	$c_5^{A_{\rm FB}^{0,c}}$	0.	0.	$c_8^{A_{\rm FB}^{0,c}}$	$c_9^{A_{\rm FB}^{0,c}}$	$c_{10}^{A_{\rm FB}^{0,c}}$	0.

Table B.7: Coefficients for lepton and quark in non-universal $C_{\varphi e}$ assumption.

For Γ_W:

$$c_9^{\Gamma_W} = \frac{4}{3} \tag{B.3.77}$$

For $\sigma_{\rm had}^0$:

$$c_{8,9,10,11}^{\sigma_{\rm had}^0} = c_{a,b,c,d}^{\sigma_{\rm had}^0}, \tag{B.3.78}$$

For $c^{R_{b,c}^0}$:

$$c_4^{R_b^0} = \frac{48s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-12s_0^2+8s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.3.79}$$

$$c_4^{R_c^0} = \frac{-72s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-24s_0^2+32s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.3.80}$$

$$c_{8,9,10,11}^{R_b^0} = c_{a,b,c,d}^{R_b^0}\ , \qquad c_{8,9,10,11}^{R_c^0} = c_{a,b,c,d}^{R_c^0}\ . \tag{B.3.81}$$

For $c^{R_{e_p}^0}$:

$$c_{8,9,10,11}^{R_e^0} = c_{a,b,c,d}^{R_e^0}, \qquad c_{8,9,10,11}^{R_e^0} = c_{a,b,c,d}^{R_e^0}, \qquad c_{8,9,10,11}^{R_e^0} = c_{a,b,c,d}^{R_e^0}. \tag{B.3.82}$$

For $A_{\rm FB}^{0,b}$:

$$c_1^{A_{\rm FB}^{0,b}} = \frac{4c_0(120s_0^3 - 608s_0^5 + 1216s_0^7 - 1280s_0^9 + 512s_0^{11})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.3.83}$$

$$c_2^{A_{\rm FB}^{0,b}} = \frac{-2(-120s_0^4 + 728s_0^6 - 1824s_0^8 + 2496s_0^{10} - 1792s_0^{12} + 512s_0^{14})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.3.84}$$

$$c_4^{A_{\rm FB}^{0,b}} = \frac{-16s_0^4(-33 + 184s_0^2 - 444s_0^4 + 640s_0^6 - 480s_0^8 + 128s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.3.85}$$

$$c_3^{A_{\rm FB}^{0,b}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_5^{A_{\rm FB}^{0,b}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.3.86}$$

$$c_{10}^{A_{\rm FB}^{0,b}} = \frac{2(-24s_0^4 + 16s_0^6)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_{11}^{A_{\rm FB}^{0,b}} = \frac{16s_0^4(-3+2s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.3.87}$$

$$c_{13}^{A_{\rm FB}^{0,b}} = \frac{-8s_0^2(9-12s_0^2+4s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}. \tag{B.3.88}$$

For $A_{\rm FB}^{0,c}$:

$$c_1^{A_{\rm FB}^{0,c}} = \frac{8c_0(78s_0^3 - 620s_0^5 + 1984s_0^7 - 3200s_0^9 + 2048s_0^{11})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.3.89}$$

$$c_2^{A_{\rm FB}^{0,c}} = \frac{-4(-78s_0^4 + 698s_0^6 - 2604s_0^8 + 5184s_0^{10} - 5248s_0^{12} + 2048s_0^{14})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.3.90}$$

$$c_4^{A_{\rm FB}^{0,c}} = \frac{-16s_0^4(-51 + 446s_0^2 - 1632s_0^4 + 3200s_0^6 - 3072s_0^8 + 1024s_0^{10})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.3.91}$$

$$c_3^{A_{\rm FB}^{0,c}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_5^{A_{\rm FB}^{0,c}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.3.92}$$

$$c_{10}^{A_{\rm FB}^{0,c}} = \frac{-2(-96s_0^4 + 128s_0^6)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \qquad c_{11}^{A_{\rm FB}^{0,c}} = \frac{64s_0^4(-3+4s_0^2)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}, \tag{B.3.93}$$

$$c_{12}^{A_{\rm FB}^{0,c}} = \frac{16s_0^2(9-24s_0^2+16s_0^4)}{32s_0^6 - 24s_0^4 + 8s_0^2 - 1}. \tag{B.3.94}$$

B.4 Non-universal $C_{\varphi 1}$, $C_{\varphi 3}$ and quark

B.4.1 Lepton

For the neutrinos:

$$c_2^{\Gamma_{\nu_e}} = c_2^{\Gamma_{\nu_\mu}} = c_2^{\Gamma_{\nu_\tau}} = -\frac{1}{2}, \qquad c_3^{\Gamma_{\nu_e}} = c_5^{\Gamma_{\nu_\mu}} = c_7^{\Gamma_{\nu_\tau}} = -2, \qquad c_4^{\Gamma_{\nu_e}} = c_6^{\Gamma_{\nu_\mu}} = c_8^{\Gamma_{\nu_\tau}} = 2. \tag{B.4.95}$$

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)ee}$	$C_{\varphi l}^{(3)ee}$	$C_{\varphi l}^{(1)\mu\mu}$	$C_{\varphi l}^{(3)\mu\mu}$	$C_{\varphi l}^{(1)\tau\tau}$	$C_{\varphi l}^{(3)\tau\tau}$	$C_{\varphi e}^{\text{uni.}}$
Γ_{ν_e}	0.	$c_2^{\Gamma_{\nu_e}}$	$c_3^{\Gamma_{\nu_e}}$	$c_4^{\Gamma_{\nu_e}}$	0.	0.	0.	0.	0.
Γ_{ν_μ}	0.	$c_2^{\Gamma_{\nu_e}}$	0.	0.	$c_5^{\Gamma_{\nu_\mu}}$	$c_6^{\Gamma_{\nu_\mu}}$	0.	0.	0.
Γ_{ν_τ}	0.	$c_2^{\Gamma_{\nu_\tau}}$	0.	0.	0.	0.	$c_7^{\Gamma_{\nu_\tau}}$	$c_8^{\Gamma_{\nu_\tau}}$	0.
Γ_e	$c_1^{\Gamma_e}$	$c_2^{\Gamma_e}$	$c_3^{\Gamma_e}$	$c_4^{\Gamma_e}$	0.	$c_6^{\Gamma_e}$	0.	0.	$c_9^{\Gamma_e}$
Γ_μ	$c_1^{\Gamma_\mu}$	$c_2^{\Gamma_\mu}$	0.	$c_4^{\Gamma_\mu}$	$c_5^{\Gamma_\mu}$	$c_6^{\Gamma_\mu}$	0.	0.	$c_9^{\Gamma_\mu}$
Γ_τ	$c_1^{\Gamma_\tau}$	$c_2^{\Gamma_\tau}$	0.	$c_4^{\Gamma_\tau}$	0.	$c_6^{\Gamma_\tau}$	$c_7^{\Gamma_\tau}$	$c_8^{\Gamma_\tau}$	$c_9^{\Gamma_\tau}$
$\Gamma_{\text{had.}}$	$c_1^{\Gamma_{\text{had.}}}$	$c_2^{\Gamma_{\text{had.}}}$	0.	$c_4^{\Gamma_{\text{had.}}}$	0.	$c_6^{\Gamma_{\text{had.}}}$	0.	0.	$c_9^{\Gamma_{\text{had.}}}$

Table B.8: Coefficients of decay widths for lepton in non-universal $C_{\varphi 1}$, $C_{\varphi 3}$ assumption.

For the charged leptons:

$$c_1^{\Gamma_e} = c_1^{\Gamma_\mu} = c_1^{\Gamma_\tau} = \frac{4s_0c_0(4s_0^2-1)}{1-6s_0^2+16s_0^4-16s_0^6}\ , \tag{B.4.96}$$

$$c_2^{\Gamma_e} = c_2^{\Gamma_\mu} = c_2^{\Gamma_\tau} = \frac{(-1+2s_0^2+4s_0^4)}{2-4s_0^2(3-8s_0^2+8s_0^4)}\ , \tag{B.4.97}$$

$$c_3^{\Gamma_e} = c_5^{\Gamma_\mu} = c_7^{\Gamma_\tau} = c_8^{\Gamma_\tau} = \frac{-2s_0^2+1}{\chi_{\text{SM}}^l}\ , \tag{B.4.98}$$

$$c_4^{\Gamma_e} = c_6^{\Gamma_\mu} = \frac{2(-1+6s_0^2-14s_0^4+8s_0^6)}{-1+6s_0^2-16s_0^4+16s_0^6}\ , \tag{B.4.99}$$

$$c_6^{\Gamma_e} = c_4^{\Gamma_\mu} = c_4^{\Gamma_\tau} = c_6^{\Gamma_\tau} = \frac{4s_0^2(1-5s_0^2+4s_0^4)}{-1+6s_0^2-16s_0^4+16s_0^6}\ , \tag{B.4.100}$$

$$c_9^{\Gamma_e} = c_9^{\Gamma_\mu} = c_9^{\Gamma_\tau} = \frac{-2s_0^2}{\chi_{\text{SM}}^l}\ . \tag{B.4.101}$$

For hadrons:

$$c_1^{\Gamma_{\text{had}}} = \frac{4s_0c_0(44s_0^2-21)}{45-174s_0^2+256s_0^4-176s_0^6}\ , \qquad c_2^{\Gamma_{\text{had}}} = \frac{(-45+90s_0^2+4s_0^4)}{90-348s_0^2+512s_0^4-352s_0^6}\ , \tag{B.4.102}$$

$$c_4^{\Gamma_{\text{had}}} = c_6^{\Gamma_{\text{had}}} = \frac{4s_0^2(21-65s_0^2+44s_0^4)}{-45+174s_0^2-256s_0^4+176s_0^6}\ . \tag{B.4.103}$$

The coefficients for total Z decay width read:

$$c_1^{\Gamma_Z} = \frac{40s_0c_0(8s_0^2-3)}{63-246s_0^2+400s_0^4-320s_0^6}, \qquad c_2^{\Gamma_Z} = \frac{(-63+126s_0^2+40s_0^4)}{126-492s_0^2+800s_0^4-640s_0^6}, \tag{B.4.104}$$

$$c_{3,5,7}^{\Gamma_Z} = \frac{1}{3}\cdot\frac{-2s_0^2}{\chi^Z} = \frac{-12s_0^2}{63-120s_0^2+160s_0^4}, \qquad c_{4,6}^{\Gamma_Z} = \frac{4(-3+39s_0^2-116s_0^4+80s_0^6)}{-63+246s_0^2-400s_0^4+320s_0^6}, \tag{B.4.105}$$

$$c_8^{\Gamma_Z} = \frac{1}{3}\cdot\frac{2c_0^2}{\chi^Z} = \frac{12c_0^2}{63-120s_0^2+160s_0^4}. \qquad c_9^{\Gamma_Z} = \frac{-2s_0^2}{\chi^Z} = \frac{-36s_0^2}{63-120s_0^2+160s_0^4}. \tag{B.4.106}$$

(B.4.107)

For W mass and total decay width:

$$c_{4,6}^{m_W} = \frac{c_0^2}{2(s_0^2-c_0^2)}. \tag{B.4.108}$$

$$c_4^{\Gamma_W} = c_6^{\Gamma_W} = \frac{-4 + 25c_0^2 + 8s_0^2}{18(s_0^2 - c_0^2)}, \qquad c_8^{\Gamma_W} = \frac{2}{9}. \tag{B.4.109}$$

	$C_{\varphi WB}$	$C_{\varphi D}$	$C_{\varphi l}^{(1)ee}$	$C_{\varphi l}^{(3)ee}$	$C_{\varphi l}^{(1)\mu\mu}$	$C_{\varphi l}^{(3)\mu\mu}$	$C_{\varphi l}^{(1)\tau\tau}$	$C_{\varphi l}^{(3)\tau\tau}$	$C_{\varphi e}^{\text{uni.}}$
Γ_Z	$c_1^{\Gamma_Z}$	$c_2^{\Gamma_Z}$	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$	$c_8^{\Gamma_Z}$	$c_9^{\Gamma_Z}$
σ_{had}^0	$c_1^{\sigma_{\text{had}}^0}$	$c_2^{\sigma_{\text{had}}^0}$	$c_3^{\sigma_{\text{had}}^0}$	$c_4^{\sigma_{\text{had}}^0}$	$c_5^{\sigma_{\text{had}}^0}$	$c_6^{\sigma_{\text{had}}^0}$	$c_7^{\sigma_{\text{had}}^0}$	$c_8^{\sigma_{\text{had}}^0}$	$c_9^{\sigma_{\text{had}}^0}$
R_e^0	$c_1^{R_e^0}$	$c_2^{R_e^0}$	$c_3^{R_e^0}$	$c_4^{R_e^0}$	0.	$c_6^{R_e^0}$	0.	0.	$c_9^{R_e^0}$
R_μ^0	$c_1^{R_\mu^0}$	$c_2^{R_\mu^0}$	0.	$c_4^{R_\mu^0}$	$c_5^{R_\mu^0}$	$c_6^{R_\mu^0}$	0.	0.	$c_9^{R_\mu^0}$
R_τ^0	$c_1^{R_\tau^0}$	$c_2^{R_\tau^0}$	0.	$c_4^{R_\tau^0}$	0.	$c_6^{R_\tau^0}$	$c_7^{R_\tau^0}$	$c_8^{R_\tau^0}$	$c_9^{R_\tau^0}$
$A_{\text{FB}}^{0,e}$	$c_1^{A_{\text{FB}}^{0,e}}$	$c_2^{A_{\text{FB}}^{0,e}}$	$c_3^{A_{\text{FB}}^{0,e}}$	$c_4^{A_{\text{FB}}^{0,e}}$	0.	$c_6^{A_{\text{FB}}^{0,e}}$	0.	0.	$c_9^{A_{\text{FB}}^{0,e}}$
$A_{\text{FB}}^{0,\mu}$	$c_1^{A_{\text{FB}}^{0,\mu}}$	$c_2^{A_{\text{FB}}^{0,\mu}}$	$c_3^{A_{\text{FB}}^{0,\mu}}$	$c_4^{A_{\text{FB}}^{0,\mu}}$	$c_5^{A_{\text{FB}}^{0,\mu}}$	$c_6^{A_{\text{FB}}^{0,e}}$	0.	0.	$c_9^{A_{\text{FB}}^{0,\mu}}$
$A_{\text{FB}}^{0,\tau}$	$c_1^{A_{\text{FB}}^{0,\tau}}$	$c_2^{A_{\text{FB}}^{0,\tau}}$	$c_3^{A_{\text{FB}}^{0,\tau}}$	$c_4^{A_{\text{FB}}^{0,\tau}}$	0.	$c_6^{A_{\text{FB}}^{0,\tau}}$	$c_7^{A_{\text{FB}}^{0,\tau}}$	$c_8^{A_{\text{FB}}^{0,\tau}}$	$c_9^{A_{\text{FB}}^{0,\tau}}$
m_W	$c_1^{m_W}$	$c_2^{m_W}$	0.	$c_4^{m_W}$	0.	$c_6^{m_W}$	0.	0.	0.
Γ_W	$c_1^{\Gamma_W}$	$c_2^{\Gamma_W}$	0.	$c_4^{\Gamma_W}$	0.	$c_6^{\Gamma_W}$	0.	$c_8^{\Gamma_W}$	0.

Table B.9: Coefficients for lepton in non-universal $C_{\varphi 1}$, $C_{\varphi 3}$ assumption.

The coefficients for the following observables are derived through relations between coefficients from the above decay widths:
For $\sigma_{\text{had.}}$

$$c_{1,2,3,4,9}^{\sigma_{\text{had}}^0} = c_{1,2,3,4,9}^{\Gamma_e} - 2 \cdot c_{1,2,3,4,9}^{\Gamma_Z}, \qquad c_{5,6,7,8}^{\sigma_{\text{had}}^0} = -2 \cdot c_{5,6,7,8}^{\Gamma_Z}. \tag{B.4.110}$$

For $R_{e_p}^0$:

$$c_{1,2,3,4,6,9}^{R_e^0} = -c_{1,2,3,4,6,9,}^{\Gamma_e^0}, \qquad c_{1,2,4,5,6,9}^{R_\mu^0} = -c_{1,2,4,5,6,9,}^{\Gamma_\mu^0}, \qquad c_{1,2,4,6,7,8,9}^{R_\tau^0} = -c_{1,2,4,6,7,8,9,}^{\Gamma_\tau^0}. \tag{B.4.111}$$

For $A_{\rm FB}^{0,e_p}$:

$$c_1^{A_{\rm FB}^{0,e}} = c_1^{A_{\rm FB}^{0,\mu}} = c_1^{A_{\rm FB}^{0,\tau}} = \frac{32 s_0^3 c_0}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.112}$$

$$c_2^{A_{\rm FB}^{0,e}} = c_2^{A_{\rm FB}^{0,\mu}} = c_2^{A_{\rm FB}^{0,\tau}} = \frac{-16(-s_0^4 + s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.113}$$

$$c_3^{A_{\rm FB}^{0,e}} = \frac{32(-s_0^4 + 2 s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.114}$$

$$c_4^{A_{\rm FB}^{0,e}} = \frac{32 s_0^6}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.115}$$

$$c_6^{A_{\rm FB}^{0,e}} = \frac{32 s_0^4 c_0^2}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.116}$$

$$c_{3,5}^{A_{\rm FB}^{0,\mu}} = c_{3,7,8}^{A_{\rm FB}^{0,\tau}} = \frac{16 s_0^4(-1 + 2 s_0^2)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.117}$$

$$c_{4,6}^{A_{\rm FB}^{0,\mu}} = c_4^{A_{\rm FB}^{0,\tau}} = \frac{16 s_0^4}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}, \tag{B.4.118}$$

$$c_9^{A_{\rm FB}^{0,e}} = c_9^{A_{\rm FB}^{0,\mu}} = c_9^{A_{\rm FB}^{0,\tau}} = \frac{-16(s_0^2 - 4 s_0^4 + 4 s_0^6)}{32 s_0^6 - 24 s_0^4 + 8 s_0^2 - 1}. \tag{B.4.119}$$

B.4.2 Lepton and quark

	$C_{\varphi l}^{(1)ee}$	$C_{\varphi l}^{(3)ee}$	$C_{\varphi l}^{(1)\mu\mu}$	$C_{\varphi l}^{(3)\mu\mu}$	$C_{\varphi l}^{(1)\tau\tau}$	$C_{\varphi l}^{(3)\tau\tau}$	$C_{\varphi e}^{\rm uni.}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$	$c_8^{\Gamma_Z}$	$c_9^{\Gamma_Z}$	$c_{10}^{\Gamma_Z}$	$c_{11}^{\Gamma_Z}$	$c_{12}^{\Gamma_Z}$	$c_{13}^{\Gamma_Z}$
$\sigma_{\rm had}^0$	$c_3^{\sigma_{\rm had}^0}$	$c_4^{\sigma_{\rm had}^0}$	$c_5^{\sigma_{\rm had}^0}$	$c_6^{\sigma_{\rm had}^0}$	$c_7^{\sigma_{\rm had}^0}$	$c_8^{\sigma_{\rm had}^0}$	$c_9^{\sigma_{\rm had}^0}$	$c_{10}^{\sigma_{\rm had}^0}$	$c_{11}^{\sigma_{\rm had}^0}$	$c_{12}^{\sigma_{\rm had}^0}$	$c_{13}^{\sigma_{\rm had}^0}$
R_e^0	$c_3^{R_e^0}$	$c_4^{R_e^0}$	0.	$c_6^{R_e^0}$	0.	0.	$c_9^{R_e^0}$	$c_{10}^{R_e^0}$	$c_{11}^{R_e^0}$	$c_{12}^{R_e^0}$	$c_{13}^{R_e^0}$
R_μ^0	0.	$c_4^{R_\mu^0}$	$c_5^{R_\mu^0}$	$c_6^{R_\mu^0}$	0.	0.	$c_9^{R_\mu^0}$	$c_{10}^{R_\mu^0}$	$c_{11}^{R_\mu^0}$	$c_{12}^{R_\mu^0}$	$c_{13}^{R_\mu^0}$
R_τ^0	0.	$c_4^{R_\tau^0}$	0.	$c_6^{R_\tau^0}$	$c_7^{R_\tau^0}$	$c_8^{R_\tau^0}$	$c_9^{R_\tau^0}$	$c_{10}^{R_\tau^0}$	$c_{11}^{R_\tau^0}$	$c_{12}^{R_\tau^0}$	$c_{13}^{R_\tau^0}$
$A_{\rm FB}^{0,e}$	$c_3^{A_{\rm FB}^{0,e}}$	$c_4^{A_{\rm FB}^{0,e}}$	0.	$c_6^{A_{\rm FB}^{0,e}}$	0.	0.	$c_9^{A_{\rm FB}^{0,e}}$	0.	0.	0.	0.
$A_{\rm FB}^{0,\mu}$	$c_3^{A_{\rm FB}^{0,\mu}}$	$c_4^{A_{\rm FB}^{0,\mu}}$	$c_5^{A_{\rm FB}^{0,\mu}}$	$c_6^{A_{\rm FB}^{0,e}}$	0.	0.	$c_9^{A_{\rm FB}^{0,\mu}}$	0.	0.	0.	0.
$A_{\rm FB}^{0,\tau}$	$c_3^{A_{\rm FB}^{0,\tau}}$	$c_4^{A_{\rm FB}^{0,\tau}}$	0.	$c_6^{A_{\rm FB}^{0,\tau}}$	$c_7^{A_{\rm FB}^{0,\tau}}$	$c_8^{A_{\rm FB}^{0,\tau}}$	$c_9^{A_{\rm FB}^{0,\tau}}$	0.	0.	0.	0.
m_W	0.	$c_4^{m_W}$	0.	$c_6^{m_W}$	0.	0.	0.	0.	0.	0.	0.
Γ_W	0.	$c_4^{\Gamma_W}$	0.	$c_6^{\Gamma_W}$	0.	$c_8^{\Gamma_W}$	0.	0.	$c_{11}^{\Gamma_W}$	0.	0.
R_b^0	0.	$c_4^{R_b^0}$	0.	$c_6^{R_b^0}$	0.	0.	0.	$c_{10}^{R_b^0}$	$c_{11}^{R_b^0}$	$c_{12}^{R_b^0}$	$c_{13}^{R_b^0}$
R_c^0	0.	$c_4^{R_c^0}$	0.	$c_6^{R_c^0}$	0.	0.	0.	$c_{10}^{R_c^0}$	$c_{11}^{R_c^0}$	$c_{12}^{R_c^0}$	$c_{13}^{R_c^0}$
$A_{\rm FB}^{0,b}$	$c_3^{A_{\rm FB}^{0,b}}$	$c_4^{A_{\rm FB}^{0,b}}$	0.	$c_6^{A_{\rm FB}^{0,b}}$	0.	0.	$c_9^{A_{\rm FB}^{0,b}}$	$c_{10}^{A_{\rm FB}^{0,b}}$	$c_{11}^{A_{\rm FB}^{0,b}}$	0.	$c_{13}^{A_{\rm FB}^{0,b}}$
$A_{\rm FB}^{0,c}$	$c_3^{A_{\rm FB}^{0,c}}$	$c_4^{A_{\rm FB}^{0,c}}$	0.	$c_6^{A_{\rm FB}^{0,c}}$	0.	0.	$c_9^{A_{\rm FB}^{0,c}}$.	$c_{10}^{A_{\rm FB}^{0,c}}$	$c_{11}^{A_{\rm FB}^{0,c}}$	$c_{12}^{A_{\rm FB}^{0,c}}$	0.

Table B.10: Coefficients for lepton and quark in non-universal $C_{\varphi 1}$, $C_{\varphi 3}$ assumption.

The above coefficients for lepton sector remain unchanged. Now, we add some coefficients for quark sector.

For Γ_W:

$$c_{11}^{\Gamma_W} = \frac{4}{3} \tag{B.4.120}$$

For σ^0_{had}:

$$c_{10,11,12,13}^{\sigma^0_{\text{had}}} = c_{a,b,c,d}^{\sigma^0_{\text{had}}}, \tag{B.4.121}$$

For $c^{R^0_{b,c}}$:

$$c_4^{R_b^0} = c_6^{R_b^0} = \frac{24s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-12s_0^2+8s_0^4)(45-84s_0^2+88s_0^4)}\,, \tag{B.4.122}$$

$$c_4^{R_c^0} = c_6^{R_c^0} = \frac{-36s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-24s_0^2+32s_0^4)(45-84s_0^2+88s_0^4)}\,, \tag{B.4.123}$$

$$c_{10,11,12,13}^{R_b^0} = c_{a,b,c,d}^{R_b^0}\,, \qquad c_{10,11,12,13}^{R_c^0} = c_{a,b,c,d}^{R_c^0}\,. \tag{B.4.124}$$

For $c^{R^0_{e_p}}$:

$$c_{10,11,12,13}^{R_e^0} = c_{a,b,c,d}^{R_e^0}, \qquad c_{10,11,12,13}^{R_e^0} = c_{a,b,c,d}^{R_e^0}, \qquad c_{10,11,12,13}^{R_e^0} = c_{a,b,c,d}^{R_e^0}. \tag{B.4.125}$$

For $A_{\text{FB}}^{0,b}$:

$$c_4^{A_{\text{FB}}^{0,b}} = \frac{16s_0^4(3-2s_0^2-12s_0^4-16s_0^6+32s_0^8)}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.4.126}$$

$$c_6^{A_{\text{FB}}^{0,b}} = \frac{-(32s_0^4(-15+91s_0^2-228s_0^4+312s_0^6-224s_0^8+64s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.4.127}$$

$$c_3^{A_{\text{FB}}^{0,b}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_9^{A_{\text{FB}}^{0,b}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.4.128}$$

$$c_{10}^{A_{\text{FB}}^{0,b}} = \frac{2(-24s_0^4+16s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_{11}^{A_{\text{FB}}^{0,b}} = \frac{16s_0^4(-3+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.4.129}$$

$$c_{13}^{A_{\text{FB}}^{0,b}} = \frac{-8s_0^2(9-12s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.4.130}$$

For $A_{\text{FB}}^{0,c}$:

$$c_3^{A_{\text{FB}}^{0,c}} = \frac{16s_0^4(-1+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.4.131}$$

$$c_4^{A_{\text{FB}}^{0,c}} = \frac{16s_0^4(12-97s_0^2+330s_0^4-608s_0^6+448s_0^8)}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.4.132}$$

$$c_6^{A_{\text{FB}}^{0,c}} = \frac{-(16s_0^4(-39+349s_0^2-1302s_0^4+2592s_0^6-2624s_0^8+1024s_0^{10})}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.4.133}$$

$$c_9^{A_{\text{FB}}^{0,c}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_{10}^{A_{\text{FB}}^{0,c}} = \frac{-2(-96s_0^4+128s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.4.134}$$

$$c_{11}^{A_{\text{FB}}^{0,c}} = \frac{64s_0^4(-3+4s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_{12}^{A_{\text{FB}}^{0,c}} = \frac{16s_0^2(9-24s_0^2+16s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.4.135}$$

(B.4.136)

B.5 Non-universal $\Sigma_{\varphi l}$, $C_{\varphi e}$ and universal quark

B.5.1 Lepton

	$C_{\varphi WB}$	$C_{\varphi D}$	$\Sigma_{\varphi l}^{ee+}$	$C_{\varphi e}^{ee}$	$\Sigma_{\varphi l}^{\mu\mu+}$	$C_{\varphi e}^{\mu\mu}$	$\Sigma_{\varphi l}^{\tau\tau+}$	$C_{\varphi e}^{\tau\tau}$	$\Sigma_{\varphi l}^{\text{uni.}-}$
Γ_{ν_p}	0.	$c_2^{\Gamma_{\nu_p}}$	0.	0.	0.	0.	0.	0.	$c_9^{\Gamma_{\nu_p}}$
Γ_e	$c_1^{\Gamma_e}$	$c_2^{\Gamma_e}$	$c_3^{\Gamma_e}$	$c_4^{\Gamma_e}$	$c_5^{\Gamma_e}$	0.	0.	0.	$c_9^{\Gamma_e}$
Γ_μ	$c_1^{\Gamma_\mu}$	$c_2^{\Gamma_\mu}$	$c_3^{\Gamma_\mu}$	0.	$c_5^{\Gamma_\mu}$	$c_6^{\Gamma_\mu}$	0.	0.	$c_9^{\Gamma_\mu}$
Γ_τ	$c_1^{\Gamma_\tau}$	$c_2^{\Gamma_\tau}$	$c_3^{\Gamma_\tau}$	0	$c_5^{\Gamma_\tau}$	0.	$c_7^{\Gamma_\tau}$	$c_8^{\Gamma_\tau}$	$c_9^{\Gamma_\tau}$
$\Gamma_{\text{had.}}$	$c_1^{\Gamma_{\text{had.}}}$	$c_2^{\Gamma_{\text{had.}}}$	$c_3^{\Gamma_{\text{had.}}}$	0.	$c_5^{\Gamma_{\text{had.}}}$	0.	0.	0.	$c_9^{\Gamma_{\text{had.}}}$

Table B.11: Coefficients of decay widths for lepton in non-universal $\Sigma_{\varphi l}$, $C_{\varphi e}$ assumption.

For the neutrinos:

$$c_2^{\Gamma_{\nu_e}} = c_2^{\Gamma_{\nu_\mu}} = c_2^{\Gamma_{\nu_\tau}} = -\frac{1}{2}, \qquad c_9^{\Gamma_{\nu_e}} = c_9^{\Gamma_{\nu_\mu}} = c_9^{\Gamma_{\nu_\tau}} = 2. \tag{B.5.137}$$

For the charged leptons:

$$c_1^{\Gamma_e} = c_1^{\Gamma_\mu} = c_1^{\Gamma_\tau} = \frac{4s_0c_0(4s_0^2-1)}{1-6s_0^2+16s_0^4-16s_0^6}\ , \tag{B.5.138}$$

$$c_2^{\Gamma_e} = c_2^{\Gamma_\mu} = c_2^{\Gamma_\tau} = \frac{(-1+2s_0^2+4s_0^4)}{2-4s_0^2(3-8s_0^2+8s_0^4)}\ , \tag{B.5.139}$$

$$c_3^{\Gamma_e} = c_5^{\Gamma_\mu} = \frac{2(-1+5s_0^2-9s_0^4+4s_0^6)}{-1+6s_0^2-16s_0^4+16s_0^6}\ , \tag{B.5.140}$$

$$c_5^{\Gamma_e} = c_3^{\Gamma_\mu} = c_3^{\Gamma_\tau} = c_5^{\Gamma_\tau} = \frac{2s_0^2(1-5s_0^2+4s_0^4)}{-1+6s_0^2-16s_0^4+16s_0^6}\ , \tag{B.5.141}$$

$$c_7^{\Gamma_\tau} = \frac{-2s_0^2+1}{\chi_{\text{SM}}^l}\ , \tag{B.5.142}$$

$$c_4^{\Gamma_e} = c_6^{\Gamma_\mu} = c_8^{\Gamma_\tau} = \frac{-2s_0^2}{\chi_{\text{SM}}^l}\ , \tag{B.5.143}$$

$$c_9^{\Gamma_e} = c_9^{\Gamma_\mu} = c_9^{\Gamma_\tau} = \frac{4s_0^2(1-5s_0^2+4s_0^4)}{-1+6s_0^2-16s_0^4+16s_0^6}\ . \tag{B.5.144}$$

For hadrons:

$$c_1^{\Gamma_{\text{had}}} = \frac{4s_0c_0(44s_0^2-21)}{45-174s_0^2+256s_0^4-176s_0^6}, \qquad c_2^{\Gamma_{\text{had}}} = \frac{(-45+90s_0^2+4s_0^4)}{90-348s_0^2+512s_0^4-352s_0^6}, \tag{B.5.145}$$

$$c_3^{\Gamma_{\text{had}}} = c_5^{\Gamma_{\text{had}}} = \frac{2s_0^2(21-65s_0^2+44s_0^4)}{-45+174s_0^2-256s_0^4+176s_0^6}, \quad c_9^{\Gamma_{\text{had}}} = \frac{4s_0^2(21-65s_0^2+44s_0^4)}{-45+174s_0^2-256s_0^4+176s_0^6}\ . \tag{B.5.146}$$

The coefficients for total Z decay width read:

$$c_1^{\Gamma_Z} = \frac{40 s_0 c_0 (8s_0^2 - 3)}{63 - 246 s_0^2 + 400 s_0^4 - 320 s_0^6}, \qquad c_2^{\Gamma_Z} = \frac{(-63 + 126 s_0^2 + 40 s_0^4)}{126 - 492 s_0^2 + 800 s_0^4 - 640 s_0^6}, \tag{B.5.147}$$

$$c_{3,5}^{\Gamma_Z} = \frac{-6 + 84 s_0^2 - 244 s_0^4 + 160 s_0^6}{-63 + 246 s_0^2 - 400 s_0^4 + 320 s_0^6}, \qquad c_7^{\Gamma_Z} = \frac{1}{3} \cdot \frac{-2s_0^2 + 1}{\chi^Z} = \frac{6 - 12 s_0^2}{63 - 120 s_0^2 + 160 s_0^4}, \tag{B.5.148}$$

$$c_{4,6,8}^{\Gamma_Z} = \frac{1}{3} \cdot \frac{-2s_0^2}{\chi^Z} = \frac{-12 s_0^2}{63 - 120 s_0^2 + 160 s_0^4}, \qquad c_9^{\Gamma_Z} = \frac{-18 + 156 s_0^2 - 440 s_0^4 + 320 s_0^6}{63 - 120 s_0^2 + 160 s_0^4}. \tag{B.5.149}$$

(B.5.150)

(B.5.151)

For total W mass and decay width:

$$c_{3,5}^{m_W} = \frac{c_0^2}{4 - 8s_0^2} \qquad c_9^{m_W} = \frac{c_0^2}{4 - 8s_0^2}. \tag{B.5.152}$$

$$c_3^{\Gamma_W} = c_5^{\Gamma_W} = \frac{-4 + 25 c_0^2 + 8 s_0^2}{36(s_0^2 - c_0^2)}, \qquad c_7^{\Gamma_W} = \frac{1}{9}, \qquad c_9^{\Gamma_W} = \frac{-6 + 25 c_0^2 + 12 s_0^2}{-18 + 36 s_0^2}. \tag{B.5.153}$$

The coefficients for the following observables are derived through relations between coefficients

	$C_{\varphi WB}$	$C_{\varphi D}$	$\Sigma_{\varphi l}^{ee+}$	$C_{\varphi e}^{ee}$	$\Sigma_{\varphi l}^{\mu\mu+}$	$C_{\varphi e}^{\mu\mu}$	$\Sigma_{\varphi l}^{\tau\tau+}$	$C_{\varphi e}^{\tau\tau}$	$\Sigma_{\varphi l}^{\text{uni.}-}$
Γ_Z	$c_1^{\Gamma_Z}$	$c_2^{\Gamma_Z}$	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$	$c_8^{\Gamma_Z}$	$c_9^{\Gamma_Z}$
σ_{had}^0	$c_1^{\sigma_{\text{had}}^0}$	$c_2^{\sigma_{\text{had}}^0}$	$c_3^{\sigma_{\text{had}}^0}$	$c_4^{\sigma_{\text{had}}^0}$	$c_5^{\sigma_{\text{had}}^0}$	$c_6^{\sigma_{\text{had}}^0}$	$c_7^{\sigma_{\text{had}}^0}$	$c_8^{\sigma_{\text{had}}^0}$	$c_9^{\sigma_{\text{had}}^0}$
R_e^0	$c_1^{R_e^0}$	$c_2^{R_e^0}$	$c_3^{R_e^0}$	$c_4^{R_e^0}$	$c_5^{R_e^0}$	0.	0.	0.	$c_9^{R_e^0}$
R_μ^0	$c_1^{R_\mu^0}$	$c_2^{R_\mu^0}$	$c_3^{R_\mu^0}$	0.	$c_5^{R_\mu^0}$	$c_6^{R_\mu^0}$	0.	0.	$c_9^{R_\mu^0}$
R_τ^0	$c_1^{R_\tau^0}$	$c_2^{R_\tau^0}$	$c_3^{R_\tau^0}$	0.	$c_5^{R_\tau^0}$	0.	$c_7^{R_\tau^0}$	$c_8^{R_\tau^0}$	$c_9^{R_\tau^0}$
$A_{\text{FB}}^{0,e}$	$c_1^{A_{\text{FB}}^{0,e}}$	$c_2^{A_{\text{FB}}^{0,e}}$	$c_3^{A_{\text{FB}}^{0,e}}$	$c_4^{A_{\text{FB}}^{0,e}}$	$c_5^{A_{\text{FB}}^{0,e}}$	0.	0.	0.	$c_9^{A_{\text{FB}}^{0,e}}$
$A_{\text{FB}}^{0,\mu}$	$c_1^{A_{\text{FB}}^{0,\mu}}$	$c_2^{A_{\text{FB}}^{0,\mu}}$	$c_3^{A_{\text{FB}}^{0,\mu}}$	$c_4^{A_{\text{FB}}^{0,\mu}}$	$c_5^{A_{\text{FB}}^{0,\mu}}$	$c_6^{A_{\text{FB}}^{0,\mu}}$	0.	0.	$c_9^{A_{\text{FB}}^{0,\mu}}$
$A_{\text{FB}}^{0,\tau}$	$c_1^{A_{\text{FB}}^{0,\tau}}$	$c_2^{A_{\text{FB}}^{0,\tau}}$	$c_3^{A_{\text{FB}}^{0,\tau}}$	$c_4^{A_{\text{FB}}^{0,\tau}}$	$c_5^{A_{\text{FB}}^{0,\tau}}$	0.	$c_7^{A_{\text{FB}}^{0,\tau}}$	$c_8^{A_{\text{FB}}^{0,\tau}}$	$c_9^{A_{\text{FB}}^{0,\tau}}$
m_W	$c_1^{m_W}$	$c_2^{m_W}$	$c_3^{m_W}$	0.	$c_5^{m_W}$	0.	0.	0.	$c_9^{m_W}$
Γ_W	$c_1^{\Gamma_W}$	$c_2^{\Gamma_W}$	$c_3^{\Gamma_W}$	0.	$c_5^{\Gamma_W}$	0.	$c_7^{\Gamma_W}$	0.	$c_9^{\Gamma_W}$

Table B.12: Coefficients for lepton in non-universal $\Sigma_{\varphi l}$, $C_{\varphi e}$ assumption.

from the above decay widths:
For $\sigma_{\text{had.}}$

$$c_{1,2,3,4}^{\sigma_{\text{had}}^0} = c_{1,2,3,4}^{\Gamma_e} - 2 \cdot c_{1,2,3,4}^{\Gamma_Z}, \qquad c_{5,6,7,8,9}^{\sigma_{\text{had}}^0} = -2 \cdot c_{5,6,7,8,9}^{\Gamma_Z}. \tag{B.5.154}$$

For $R_{e_p}^0$:

$$c_{1,2,3,4,5,9}^{R_e^0} = -c_{1,2,3,4,5,9}^{\Gamma_e^0}, \qquad c_{1,2,3,5,6,9}^{R_\mu^0} = -c_{1,2,3,5,6,9}^{\Gamma_\mu^0}, \qquad c_{1,2,3,5,7,8,9}^{R_\tau^0} = -c_{1,2,3,5,7,8,9}^{\Gamma_\tau^0}. \tag{B.5.155}$$

For A_{FB}^{0,e_p}:

$$c_1^{A_{\mathrm{FB}}^{0,e}} = c_1^{A_{\mathrm{FB}}^{0,\mu}} = c_1^{A_{\mathrm{FB}}^{0,\tau}} = \frac{32s_0^3c_0}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.156}$$

$$c_2^{A_{\mathrm{FB}}^{0,e}} = c_2^{A_{\mathrm{FB}}^{0,\mu}} = c_2^{A_{\mathrm{FB}}^{0,\tau}} = \frac{-16(-s_0^4+s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.157}$$

$$c_3^{A_{\mathrm{FB}}^{0,e}} = \frac{16s_0^4(-1+3s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \quad c_4^{A_{\mathrm{FB}}^{0,e}} = \frac{-16(s_0^2-4s_0^4+4s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.158}$$

$$c_5^{A_{\mathrm{FB}}^{0,e}} = c_5^{A_{\mathrm{FB}}^{0,\tau}} = \frac{16s_0^4c_0^2}{32s_0^6-24s_0^4+8s_0^2-1}, \quad c_{3,5}^{A_{\mathrm{FB}}^{0,\mu}} = c_3^{A_{\mathrm{FB}}^{0,\tau}} = \frac{16s_0^6}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.159}$$

$$c_7^{A_{\mathrm{FB}}^{0,\tau}} = \frac{16s^4(-1+2s^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \quad c_{4,6}^{A_{\mathrm{FB}}^{0,\mu}} = c_{4,8}^{A_{\mathrm{FB}}^{0,\tau}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.5.160}$$

(B.5.161)

B.5.2 Lepton and quark

	$\Sigma_{\varphi l}^{ee+}$	$C_{\varphi e}^{ee}$	$\Sigma_{\varphi l}^{\mu\mu+}$	$C_{\varphi e}^{\mu\mu}$	$\Sigma_{\varphi l}^{\tau\tau+}$	$C_{\varphi e}^{\tau\tau}$	$\Sigma_{\varphi l}^{\mathrm{uni.}-}$	$C_{\varphi q}^{(1)}$	$C_{\varphi q}^{(3)}$	$C_{\varphi u}$	$C_{\varphi d}$
Γ_Z	$c_3^{\Gamma_Z}$	$c_4^{\Gamma_Z}$	$c_5^{\Gamma_Z}$	$c_6^{\Gamma_Z}$	$c_7^{\Gamma_Z}$	$c_8^{\Gamma_Z}$	$c_9^{\Gamma_Z}$	$c_{10}^{\Gamma_Z}$	$c_{11}^{\Gamma_Z}$	$c_{12}^{\Gamma_Z}$	$c_{13}^{\Gamma_Z}$
σ_{had}^0	$c_3^{\sigma_{\mathrm{had}}^0}$	$c_4^{\sigma_{\mathrm{had}}^0}$	$c_5^{\sigma_{\mathrm{had}}^0}$	$c_6^{\sigma_{\mathrm{had}}^0}$	$c_7^{\sigma_{\mathrm{had}}^0}$	$c_8^{\sigma_{\mathrm{had}}^0}$	$c_9^{\sigma_{\mathrm{had}}^0}$	$c_{10}^{\sigma_{\mathrm{had}}^0}$	$c_{11}^{\sigma_{\mathrm{had}}^0}$	$c_{12}^{\sigma_{\mathrm{had}}^0}$	$c_{13}^{\sigma_{\mathrm{had}}^0}$
R_e^0	$c_3^{R_e^0}$	$c_4^{R_e^0}$	$c_5^{R_e^0}$	0.	0.	0.	$c_9^{R_e^0}$	$c_{10}^{R_e^0}$	$c_{11}^{R_e^0}$	$c_{12}^{R_e^0}$	$c_{13}^{R_e^0}$
R_μ^0	$c_3^{R_\mu^0}$	0.	$c_5^{R_\mu^0}$	$c_6^{R_\mu^0}$	0.	0.	$c_9^{R_\mu^0}$	$c_{10}^{R_\mu^0}$	$c_{11}^{R_\mu^0}$	$c_{12}^{R_\mu^0}$	$c_{13}^{R_\mu^0}$
R_τ^0	$c_3^{R_\tau^0}$	0.	$c_5^{R_\tau^0}$	0.	$c_7^{R_\tau^0}$	$c_8^{R_\tau^0}$	$c_9^{R_\tau^0}$	$c_{10}^{R_\tau^0}$	$c_{11}^{R_\tau^0}$	$c_{12}^{R_\tau^0}$	$c_{13}^{R_\tau^0}$
$A_{\mathrm{FB}}^{0,e}$	$c_3^{A_{\mathrm{FB}}^{0,e}}$	$c_4^{A_{\mathrm{FB}}^{0,e}}$	0.	0.	0.	0.	$c_9^{A_{\mathrm{FB}}^{0,e}}$	0.	0.	0.	0.
$A_{\mathrm{FB}}^{0,\mu}$	$c_3^{A_{\mathrm{FB}}^{0,\mu}}$	$c_4^{A_{\mathrm{FB}}^{0,\mu}}$	$c_5^{A_{\mathrm{FB}}^{0,\mu}}$	$c_6^{A_{\mathrm{FB}}^{0,\mu}}$	0.	0.	$c_9^{A_{\mathrm{FB}}^{0,\mu}}$	0.	0.	0.	0.
$A_{\mathrm{FB}}^{0,\tau}$	$c_3^{A_{\mathrm{FB}}^{0,\tau}}$	$c_4^{A_{\mathrm{FB}}^{0,\tau}}$	0.	0.	$c_7^{A_{\mathrm{FB}}^{0,\tau}}$	$c_8^{A_{\mathrm{FB}}^{0,\tau}}$	$c_9^{A_{\mathrm{FB}}^{0,\tau}}$	0.	0.	0.	0.
m_W	$c_3^{m_W}$	0.	$c_5^{m_W}$	0.	0.	0.	$c_9^{m_W}$	0.	0.	0.	0.
Γ_W	$c_3^{\Gamma_W}$	0.	$c_5^{\Gamma_W}$	0.	$c_7^{\Gamma_W}$	0.	$c_9^{\Gamma_W}$	0.	$c_{11}^{\Gamma_W}$	0.	0.
R_b^0	$c_3^{R_b^0}$	0.	$c_5^{R_b^0}$	0.	0.	0.	$c_9^{R_b^0}$	$c_{10}^{R_b^0}$	$c_{11}^{R_b^0}$	$c_{12}^{R_b^0}$	$c_{13}^{R_b^0}$
R_c^0	$c_3^{R_c^0}$	0.	$c_5^{R_c^0}$	0.	0.	0.	$c_9^{R_c^0}$	$c_{10}^{R_c^0}$	$c_{11}^{R_c^0}$	$c_{12}^{R_c^0}$	$c_{13}^{R_c^0}$
$A_{\mathrm{FB}}^{0,b}$	$c_3^{A_{\mathrm{FB}}^{0,b}}$	$c_4^{A_{\mathrm{FB}}^{0,b}}$	$c_5^{A_{\mathrm{FB}}^{0,b}}$	0.	0.	0.	$c_9^{A_{\mathrm{FB}}^{0,b}}$	$c_{10}^{A_{\mathrm{FB}}^{0,b}}$	$c_{11}^{A_{\mathrm{FB}}^{0,b}}$	0.	$c_{13}^{A_{\mathrm{FB}}^{0,b}}$
$A_{\mathrm{FB}}^{0,c}$	$c_3^{A_{\mathrm{FB}}^{0,c}}$	$c_4^{A_{\mathrm{FB}}^{0,c}}$	$c_5^{A_{\mathrm{FB}}^{0,c}}$	0.	0.	0.	$c_9^{A_{\mathrm{FB}}^{0,c}}$	$c_{10}^{A_{\mathrm{FB}}^{0,c}}$	$c_{11}^{A_{\mathrm{FB}}^{0,c}}$	$c_{12}^{A_{\mathrm{FB}}^{0,c}}$	0.

Table B.13: Coefficients for lepton and quark in non-universal $\Sigma_{\varphi l}$, $C_{\varphi e}$ assumption.

The above coefficients for lepton sector remain unchanged. Now, we add some coefficients for quark sector.
For Γ_W:

$$c_{11}^{\Gamma_W} = \frac{4}{3} \tag{B.5.162}$$

For $\sigma^0_{\rm had}$:

$$c^{\sigma^0_{\rm had}}_{10,11,12,13} = c^{\sigma^0_{\rm had}}_{a,b,c,d}, \tag{B.5.163}$$

For $c^{R^0_{b,c}}$:

$$c^{R^0_b}_3 = c^{R^0_b}_5 = \frac{12s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-12s_0^2+8s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.5.164}$$

$$c^{R^0_b}_9 = \frac{24s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-12s_0^2+8s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.5.165}$$

$$c^{R^0_c}_3 = c^{R^0_c}_5 = \frac{-18s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-24s_0^2+32s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.5.166}$$

$$c^{R^0_c}_9 = \frac{-36s_0^2(-9+45s_0^2-52s_0^4+16s_0^6)}{(-1+2s_0^2)(9-24s_0^2+32s_0^4)(45-84s_0^2+88s_0^4)}\ , \tag{B.5.167}$$

$$c^{R^0_b}_{10,11,12,13} = c^{R^0_b}_{a,b,c,d}, \qquad c^{R^0_c}_{10,11,12,13} = c^{R^0_c}_{a,b,c,d}. \tag{B.5.168}$$

For $c^{R^0_{e_p}}$:

$$c^{R^0_e}_{10,11,12,13} = c^{R^0_e}_{a,b,c,d}, \qquad c^{R^0_e}_{10,11,12,13} = c^{R^0_e}_{a,b,c,d}, \qquad c^{R^0_e}_{10,11,12,13} = c^{R^0_e}_{a,b,c,d}. \tag{B.5.169}$$

For $A^{0,b}_{\rm FB}$:

$$c^{A^{0,b}_{\rm FB}}_3 = \frac{16s_0^4(-12+89s_0^2-240s_0^4+296s_0^6-192s_0^8+64s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.5.170}$$

$$c^{A^{0,b}_{\rm FB}}_4 = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.171}$$

$$c^{A^{0,b}_{\rm FB}}_5 = \frac{-16s_0^4(-15+91s_0^2-228s_0^4+312s_0^6-224s_0^8+64s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.5.172}$$

$$c^{A^{0,b}_{\rm FB}}_9 = \frac{-32s_0^4(-15+91s_0^2-228s_0^4+312s_0^6-224s_0^8+64s_0^{10})}{(-1+2s_0^2)(-3+4s_0^2)(-1+4s_0^2)(9-12s_0^2+8s_0^4)(1-4s_0^2+8s_0^4)}, \tag{B.5.173}$$

$$c^{A^{0,b}_{\rm FB}}_{10} = \frac{2(-24s_0^4+16s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c^{A^{0,b}_{\rm FB}}_{11} = \frac{16s_0^4(-3+2s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.174}$$

$$c^{A^{0,b}_{\rm FB}}_{13} = \frac{-8s_0^2(9-12s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.5.175}$$

For $A_{\rm FB}^{0,c}$:

$$c_3^{A_{\rm FB}^{0,c}} = \frac{8s_0^4(-15+155s_0^2-642s_0^4+1376s_0^6-1728s_0^8+1024s_0^10)}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.5.176}$$

$$c_4^{A_{\rm FB}^{0,c}} = \frac{-8s_0^2(1-4s_0^2+4s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.177}$$

$$c_5^{A_{\rm FB}^{0,c}} = \frac{-8s_0^4(-39+349s_0^2-1302s_0^4+2592s_0^6-2624s_0^8+1024s_0^10)}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.5.178}$$

$$c_9^{A_{\rm FB}^{0,c}} = \frac{-16s_0^4(-39+349s_0^2-1302s_0^4+2592s_0^6-2624s_0^8+1024s_0^10)}{(-1+2s_0^2)(-1+4s_0^2)(-3+8s_0^2)(1-4s_0^2+8s_0^4)(9-24s_0^2+32s_0^4)}, \tag{B.5.179}$$

$$c_{10}^{A_{\rm FB}^{0,c}} = \frac{-2(-96s_0^4+128s_0^6)}{32s_0^6-24s_0^4+8s_0^2-1}, \qquad c_{11}^{A_{\rm FB}^{0,c}} = \frac{64s_0^4(-3+4s_0^2)}{32s_0^6-24s_0^4+8s_0^2-1}, \tag{B.5.180}$$

$$c_{12}^{A_{\rm FB}^{0,c}} = \frac{16s_0^2(9-24s_0^2+16s_0^4)}{32s_0^6-24s_0^4+8s_0^2-1}. \tag{B.5.181}$$

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