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BACHELOR'S DEGREE THESIS

Scattering process $e^+ + e^- \longrightarrow t + \bar{t}$ and fully polarized top quark decays in Standard Model Effective Field Theory

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Contents

Introduction

In the way of searching new physics Beyond the SM (BSM), we assume that new physics effects can be parameterized in Standard Model Effective Field Theory (SMEFT) when we expand the effective Lagrangian by the power of expansion parameter Λ, where Λ is assumed to be the typical energy scale of new physics.

The first two chapterof this thesis are based on Hong Minh's thesis [1] which provide a lot of techniques to calculate several physical quantities of interest for a given scattering process ($e^+ + e^- \to t + \bar{t}$) in both QED and SM.

This bachelor thesis aims to provide an overall perspective for SMEFT as well as studying the effects of several dimension-six (D6) operators in this framework. More specifically, we concentrate on the scattering process $e^+ + e^- \longrightarrow t + \bar{t}$ and fully polarized top-quark decay in SMEFT. From this process, the effects of D6 operators can be obtained via physical observables such as forward-backward asymmetry (for scattering process), W-boson spin observables (for polarized top-quark decay into massive b-quark and polarized W-boson with leptonic decay).

This thesis is presented in the following chapters:

- **Chapter 1: Scattering process $\mathbf{e^+ + e^- \longrightarrow t + \bar{t}}$ in QED**
  We start to calculate this process in the simple case, QED, from this we can perform the calculation for several physical quantities of interest which is usefull for the next chapters.
- **Chapter 2: Scattering process $\mathbf{e^- + e^+ \to t + \bar{t}}$ in SM**
  We improve our calculation in SM case, also we depict the graph of several physical observables.
- **Chapter 3: Scattering process $\mathbf{e^- + e^+ \to t + \bar{t}}$ in SMEFT**
  This is the main chapter of this thesis, we perform an overview of SMEFT, introduce the dimension-six operators, mass eigenstates basic in SMEFT, and of crouse, we re-calculate the Feynman rules which relevant with our process in SMEFT. Then we apply the new Feynman rules to calculate our scattering process.
- **Chapter 4: Fully polarized top-quark decays in SMEFT**
  In this chapter we introduce the spin density matrix of W-boson and the method to calculate spin observables via this matrix. After that we consider the production of polarized W-boson resulting from fully polarized top-quark decay in SMEFT. The results will help us imporve the sensitivity for top-quark new physics.

Chapter 1

Scattering process $\mathbf{e^+ + e^- \longrightarrow t + \bar{t}}$ in QED

1.1 Lagrangian of the process $\mathbf{e^+ + e^- \longrightarrow t + \bar{t}}$ in QED

Quantum Electrodynamics is a quantum field theory that describes the the interaction between charged fermions and photons. Based on this theory, the full QED Lagrangian which relates to our process:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \bar{\psi}[i\gamma^\mu D_\mu - m_f]\psi. \tag{1.1.1}$$

The full Lagrangian in (1.1.1) is a combination of three part which provides the coupling of the electromagnetic current to the photon field, as well as a general propagator. Let's us introduce briefly each part of this Lagrangian:

- The free electromagnetic field with the gauge-fixing term:

$$\mathcal{L}_{gf}^{full} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{1.1.2}$$

where the first term is the well know classical electromagnetic field Lagrangian with the photon field A_μ and the field strengths $F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The next term is the gauge-fixing term with ξ is a real constant called the gauge-fixing parameter. This part will help us derive the general propagator which presented in the following section.

- The fermionic sector

$$\mathcal{L}_{fermion} = \bar{\psi}[i\gamma^\mu D_\mu - m_f]\psi. \tag{1.1.3}$$

According to gauge theory, the interaction of charged fermions with each other could be derived by requiring the invariance of Lagrangian under the local gauge transformation. In order to keep this requirement, we replaced the normal derivative ∂_μ into covariant derivative D_μ which defined as:

$$D_\mu = \partial_\mu - ieQ_f A_\mu. \tag{1.1.4}$$

Substituting equation (1.1.4) into (1.1.3) we obtain the free fermionic component as well as the interaction term between the gauge vector field and the electromagnetic current,

$$\mathcal{L}_{fermion} = \bar{\psi}\left[i\not{\partial} - m\right]\psi + e\bar{\psi}\gamma^\mu\psi A_\mu. \tag{1.1.5}$$

1.1.1 General photon propagator with gauge-fixing parameter

Now we concerntrate on the gauge field with gauge-fixing term, let's first apply the Euler-Lagrange equation for (1.1.2) as below:

$$\begin{cases} \dfrac{\partial \mathcal{L}_{gf}^{full}}{\partial \left[\partial_\sigma A_\lambda\right]} & = -\frac{1}{4}\left[\dfrac{\partial F_{\mu\nu}}{\partial(\partial_\sigma A_\lambda)}F^{\mu\nu} + g^{\alpha\mu}g^{\beta\nu}\dfrac{\partial F_{\alpha\beta}}{\partial(\partial_\sigma A_\lambda)}F_{\mu\nu}\right] + \dfrac{\partial}{\partial\left[\partial_\sigma A_\lambda\right]}\left[-\dfrac{1}{2\xi}(\partial_\mu A^\mu)(\partial_\nu A^\nu)\right] \\ & = F^{\lambda\sigma} - \dfrac{1}{2\xi}g^{\lambda\sigma}\left[\partial_\nu A^\nu + \partial_\mu A^\mu\right]; \\ \dfrac{\partial \mathcal{L}_{gf}^{full}}{\partial A_\lambda} & = 0. \end{cases}$$

With these equations, we obtain the equation of motion of the form:

$$\left[\Box g^{\lambda\rho} - \left(1 - \frac{1}{\xi}\right)\partial^\lambda\partial^\rho\right]A_\rho = 0. \tag{1.1.6}$$

The Feynman propagator of the vector field, $D_{\mu\nu}(x-y)$, is the solution of inhomogeneous equation of motion (1.1.6) with a point-like source [3]:

$$\left[\Box g^{\mu\rho} - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\rho\right]D_{\rho\nu}(x-y) = g^\mu_\nu\delta^4(x-y). \tag{1.1.7}$$

By using Fourier-transformation to express (1.1.7) in momentum-space,

$$\left[\Box g^{\mu\rho} - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\rho\right]\int\frac{d^4p}{(2\pi)^4}D_{\rho\nu}(q)e^{-iq(x-y)} = g^\mu_\nu\int\frac{d^4p}{(2\pi)^4}e^{-iq(x-y)}, \tag{1.1.8}$$

we obtain the algebraic equation for photon propagator $D_{\rho\nu}(q)$ as follow:

$$\left[-q^2 g^{\mu\rho} + \left(1 - \frac{1}{\xi}\right)q^\mu q^\rho\right]D_{\rho\nu}(q) = g^\mu_\nu. \tag{1.1.9}$$

Notice that $D_{\rho\nu}(q)$ has a form of a second rank tensor and depend on two Lorentz indices, thus we can guess that the structure of $D_{\rho\nu}(q)$ should depend on momentum vectors and the metric tensor. The most general form of photon propagator should be:

$$D_{\rho\nu}(q) = A(q^2)q_\rho q_\nu + B(q^2)g_{\rho\nu} \tag{1.1.10}$$

Substituting (1.1.10) into (1.1.9) and equalizing both sides of this equation, we get:

$$\begin{aligned} &\Rightarrow \begin{cases} -B(q^2)q^2 g^\mu_\nu = g^\mu_\nu \\ \left[-A(q^2) + A(q^2)\left(1 - \dfrac{1}{\xi}\right) + B(q^2)\dfrac{1}{q^2}\left(1 - \dfrac{1}{\xi}\right)\right]q^2 q^\mu q_\nu = 0 \end{cases} \\ &\Leftrightarrow \begin{cases} B(q^2) = -\dfrac{1}{q^2} \\ A(q^2) = \dfrac{1-\xi}{q^4} \end{cases} \end{aligned} \tag{1.1.11}$$

Therefore, we have gotten a formula for massless propagator, which is

$$iD_{\rho\nu}(q) = \frac{-i}{q^2 + i\epsilon}\left[g_{\rho\nu} - (1-\xi)\frac{q_\rho q_\nu}{q^2}\right]. \tag{1.1.12}$$

As we can see, without the gauge-fixing term, equation (1.1.9) has no solution, and hence the propagator for massless gauge field can not be defined. One thing to notice that the gauge-fixing term does not satisfy gauge invariant, however, this term can only affect the photon propagator (which is not a physical quantity), therefore breaking the gauge symmetry in this case can be acceptable. Moreover, as a consequence of gauge invariant for all physical quantities (observables), we shall see that the Feynman amplitude must be $\xi - \textit{independent}$. Also, the cancellation of gauge-fixing parameter will be performed in the next section.

1.2 Feynman rules in QED

1.2.1 Vertex factor of QED

As we shall see, the last term of (1.1.5) dictates the coupling of the charged current $j^\mu = e\bar{\psi}\gamma^\mu\psi$ to the gauge vector field A_μ. Nevertherless, this is just a special case for the interaction of elesctrons (positrons), in general, for the Dirac particles with electric charge eQ_f, the Lagragian interaction takes the form

$$\mathcal{L}_{QED}^{int} = -eQ_f\bar{\psi}_f(x)\gamma^\mu\psi_f(x)A_\mu(x). \tag{1.2.1}$$

For instant, an electron has $Q_e = -1$, and an up quark has $Q_t = +2/3$. Notice that, the Lagrangian in (1.2.1) is represented in space coordinate, let's now Fourier transform the fermion and gauge field to momentum space:

$$\mathcal{L}_{QED}^{int} = -eQ_f\tilde{\bar{\psi}}_f(p_1)\gamma^\mu\tilde{\psi}_f(p_2)\tilde{A}_\mu(q)e^{-i(p_1+p_2+q)x} = -eQ_f\tilde{\bar{\psi}}_f(p_1)\gamma^\mu\tilde{\psi}_f(p_2)\tilde{A}_\mu(q). \tag{1.2.2}$$

Since the conservation of momentum must be satisfied at each vertex,

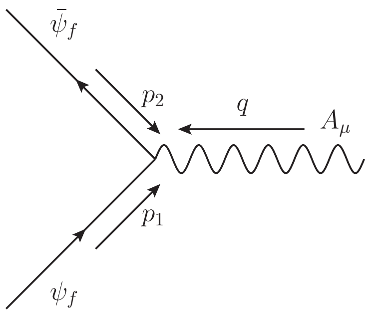
$$p_1 + p_2 + q = 0, \tag{1.2.3}$$

thus, the exponential factor is vanished. Therefore, in the Feynman rules for interaction vertices, all external momenta are considered to be incoming.

1.2.2 Feynman rules in QED

After collecting all informations of the full QED Lagragian, we now introduce a list of Feynman rules in QED as follow:

- Vertex factor in QED:

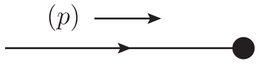


$$= -ieQ_f\gamma^\mu \tag{1.2.4}$$

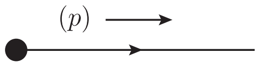
- Photon propagator in R_ξ gauge:

$$= \frac{-i}{q^2}\left[g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right] \tag{1.2.5}$$

- Fermion external lines:
  - Fermion:

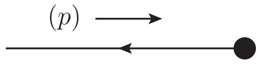
Incoming: 

$= u(p)$ (1.2.6)

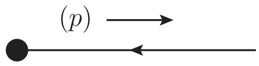
Outgoing: 

$= \bar{u}(p)$ (1.2.7)

  - Anti-fermion:

Incoming: 

$= \bar{v}(p)$ (1.2.8)

Outgoing: 

$= v(p)$ (1.2.9)

1.3 Feynman amplitude

According to the Feynman rules listed in the previous section, we can draw all possible diagrams which contribute to the scattering process $e^+ + e^- \longrightarrow t + \bar{t}$. In case of our process, there is only one Feynman diagram pointed out in Figure (1.1).

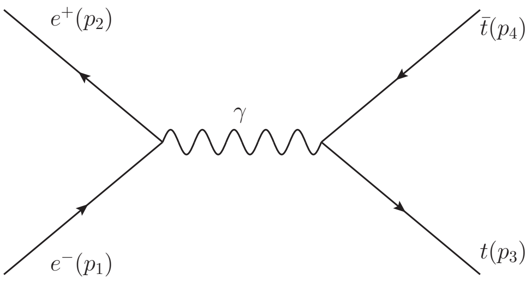


Figure 1.1: The Feynman diagrams of process $e^- + e^+ \longrightarrow t + \bar{t}$ in QED.

It is noticeable that we denote the four-momenta and spin indices of e^-, e^+, t, $\bar{t}$ to be (p_1, s), (p_2, s'), (p_3, r) and (p_4, r'), respectively. Besides, top-quark is described by the triplet colour state, thus the colour indices for top and anti-top quark are sequentially α, β. In order to preserve the charged colour, i.e the summation of all charged colour in final state is zero (since electrons in initial state do not carry charged clour), the delta function $\delta_{\alpha\beta}$ is included. Thus, all colour indices can be contracted. By applying the Feynman rules, we can write down immediately the Feynman amplitude of our process as follow:

$$\begin{aligned}\mathcal{M} &= [\bar{v}_{s'}(p_2)(ie\gamma^\mu)u_s(p_1)] \times \frac{-i}{q^2}\left[g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right] \times \left[\bar{u}^\alpha_r(p_3)\left(-i\frac{2}{3}e\gamma^\nu\right)v^\beta_{r'}(p_4)\right]\delta_{\alpha\beta} \\ &= -i\frac{2e^2}{3q^2}[\bar{v}_{s'}(p_2)\gamma^\mu u_s(p_1)][\bar{u}^\alpha_r(p_3)\gamma_\mu v^\beta_{r'}(p_4)]\delta_{\alpha\beta} + i(1-\xi)\frac{2e^2}{3q^4}[\bar{v}_{s'}(p_2)\not{q}u_s(p_1)][\bar{u}^\alpha_r(p_3)\not{q}v^\beta_{r'}(p_4)]\delta_{\alpha\beta}\end{aligned} \tag{1.3.1}$$

It is clear that the first term of (1.3.1) is ξ-independent while the second term depends on the gauge-fixing parameter. This term can be simplified by using the identity,

$$q_\mu\left[\bar{v}_{s'}(p_2)\gamma^\mu u_s(p_1)\right] = \bar{v}_{s'}(p_2)\left[\not{p}_2 + \not{p}_1\right]u_s(p_1) = \bar{v}_{s'}(p_2)\left[-m_e + m_e\right]u_s(p_1) = 0, \tag{1.3.2}$$

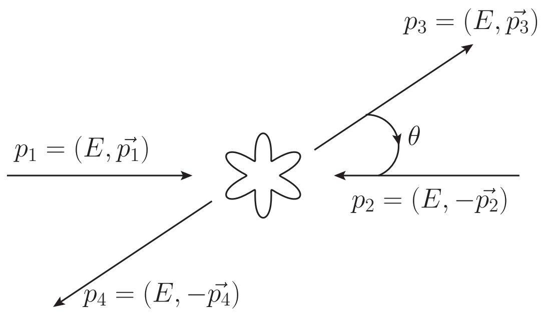


Figure 1.2: The scattering process $e^- + e^+ \longrightarrow t + \bar{t}$ in CMS.

and analogous identity for another fermion line. Notice that we have taken advantage the principle of momentum conservation, i.e $q = p_1 + p_2 = p_3 + p_4$, and the Dirac equations at each vertex. Now, we can see that the gauge-fixing parameter is completely cancelled. From identity (1.3.2), we obtain the ξ-independent Feynman amplitude as follow:

$$\mathcal{M} = -i\frac{2e^2}{3q^2}[\bar{v}_{s'}(p_2)\gamma^\mu u_s(p_1)][\bar{u}^\alpha_r(p_3)\gamma_\mu v^\beta_{r'}(p_4)]\delta_{\alpha\beta}. \tag{1.3.3}$$

In most experiments the initial state of electron and positron beams are unpolarized, moreover, we also ignore the spin states of outgoing beams. Therefore, we have to take average the squared amplitude on initial spin indices s, s' and sum over top quark spin indices r, r'. Taking the hermitian conjugate of (1.3.3), the unpolarized squared amplitude is presented as

$$\begin{aligned}
|\mathcal{M}_0|^2 &= \frac{1}{2}\cdot\frac{1}{2}\sum_{s,s'}\sum_{r,r'}\sum_{\alpha\beta}\mathcal{M}\mathcal{M}^\dagger \\
&= \frac{e^4}{9q^4}\sum_{s,s',r,r'}[\bar{v}_{s'}(p_2)\gamma^\mu u_s(p_1)\bar{u}_s(p_1)\gamma^\nu v_{s'}(p_2)]\,[\bar{v}_{r'}(p_4)\gamma_\nu u_r(p_3)\bar{u}_r(p_3)\gamma_\mu v_{r'}(p_4)] \\
&= \frac{e^4}{9q^4}Tr\left[(\not{p}_2 - m_e)\gamma^\mu(\not{p}_1 + m_e)\gamma^\nu\right]Tr\left[(\not{p}_4 - m_t)\gamma_\nu(\not{p}_3 + m_t)\gamma_\mu\right].
\end{aligned} \tag{1.3.4}$$

Notice that we have used a bit Dirac algebra to derive this equation, all of the techniques for calculating the traces in (1.3.4) have been mentioned in [2], then we get a result:

$$|\mathcal{M}_0|^2 = \frac{32e^4}{3s^2}[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) + m_t^2(p_1\cdot p_2)], \tag{1.3.5}$$

with an approximation $m_e \simeq 0$, since $m_t \gg m_e$. To obtain a more explicit formula, let's choose to work specifically in CM frame and express the vectors p_1, p_2, p_3, p_4 and q in terms of the basic kinematic variables in that frame. All information about energy and momentum of each particles is shown in Figure (1.2). Here is several kinematic relations in CM frame:

$$\Longrightarrow \begin{cases}
|\vec{k}| = \sqrt{E^2 - m_t^2} \\
p_1\cdot p_3 = p_2\cdot p_4 = E^2 - \vec{p}\cdot\vec{k} = E^2 - E|\vec{k}|cos\theta \\
p_1\cdot p_4 = p_2\cdot p_3 = E^2 + \vec{p}\cdot\vec{k} = E^2 + E|\vec{k}|cos\theta \\
q^2 = s = (p_1 + p_2)^2 = p_1^2 + 2p_1\cdot p_2 + p_2^2 = 4E^2 \\
p_1\cdot p_2 = 2E^2 \\
p_3\cdot p_4 = E^2 + |\vec{k}|^2,
\end{cases} \tag{1.3.6}$$

notice that an approximation $m_e \simeq 0$ gave us $|\vec{p}|^2 = E^2$ and $p_1^2 = p_2^2 = m_e^2 = 0$. Substituting these kinematic relations into (1.3.5), we obtain the Feynman squared amplitude, which is

$$|\mathcal{M}_0|^2 = \frac{64e^4E^2}{3s^2}\left[E^2 + |\vec{k}|^2\cos^2\theta + m_t^2\right]. \tag{1.3.7}$$

1.4 Physical results

1.4.1 Total cross-section

For the two body reaction process, the differental cross-section formular is given by

$$\frac{d\sigma}{d\Omega} = \frac{|\mathcal{M}_0|^2}{64\pi^2\left(E_{e^-}+E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|}$$

$$\Rightarrow \frac{d\sigma}{d(cos\theta)} = \int_0^{2\pi} d\phi \frac{|\mathcal{M}_0|^2}{64\pi^2\left(E_{e^-}+E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|} = \frac{e^4|\vec{k}|}{6\pi E s^2}\left[E^2 + |\vec{k}|^2cos^2\theta + m_t^2\right]. \tag{1.4.1}$$

Integrate over θ angle, we obtain the total cross-section as follow:

$$\sigma_T = \int_{-1}^{1}\frac{d\sigma}{d(\cos\theta)}d(\cos\theta) = \frac{e^4|\vec{k}|}{3\pi E s^2}\left[E^2 + \frac{|\vec{k}|^2}{3} + m_t^2\right]. \tag{1.4.2}$$

As we know, *the number of events* in a scattering process is directly proportional to the total cross-section, where L is a proportional factor, which often called *Luminosity*. Hence, the total number of top quarks generated in the annihilated reaction for the pair of electron and positron will be measured by the total cross-section in (1.4.2).

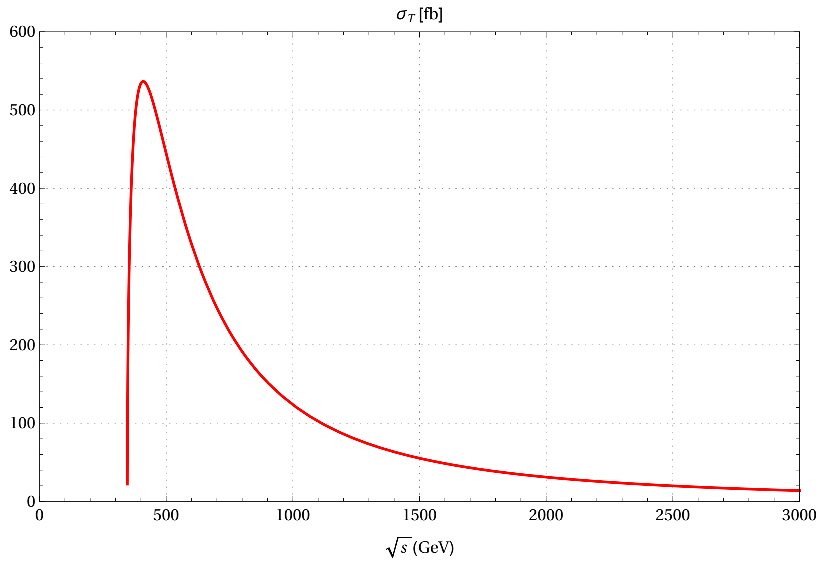


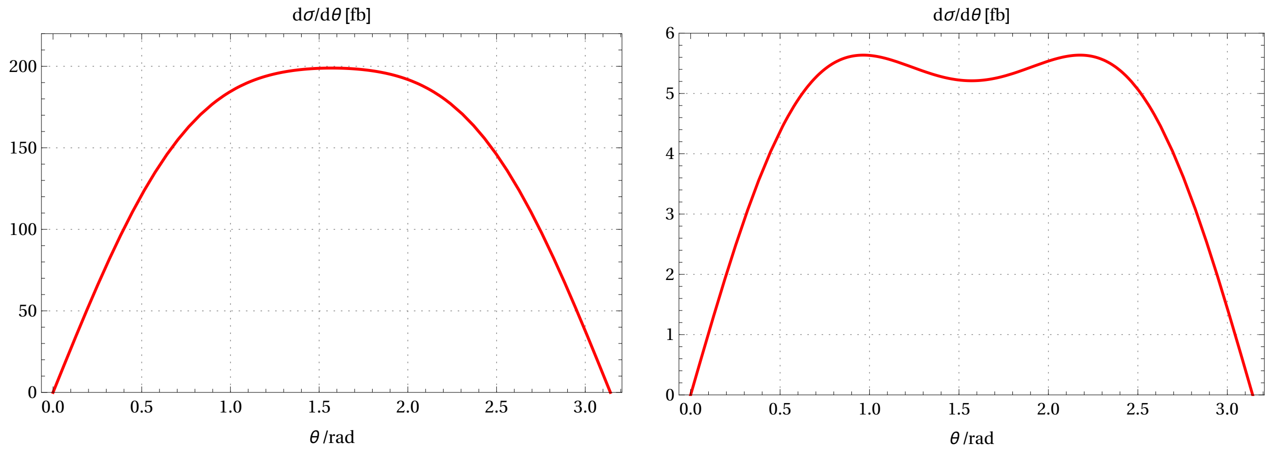
Figure 1.3: The total cross-section of process $e^- + e^+ \longrightarrow t + \bar{t}$ in QED.

The dependence of the total cross-section, σ_T, on the total initial energy-$\sqrt{s}$ is presented in figure (1.3). From this figure, we can see that the threshold energy to produce the pair top, anti-top have to larger than $2m_t \simeq 346.42$ GeV.

1.4.2 Angular distribution of top-quark

In order to obtain more informations about our scattering process at each energy level, let us looking for their distribution. One of a crucial distribution we need to consider is the angular distribution which help us can adjust the detector to obtain most of outgoing particles. We notice that this scattering process is symmetric azimuthal, therefore the angular distribution is nothing but the integration of the differential cross-section with regards to ϕ, we shall have:

$$\frac{d\sigma}{d\theta} = \frac{e^4|\vec{k}|}{6\pi E s^2}\left[E^2 + |\vec{k}|^2 cos^2\theta + m_t^2\right]\sin\theta. \tag{1.4.3}$$



(a) $\sqrt{s} = 500$ GeV (b) $\sqrt{s} = 3$ TeV

Figure 1.4: The angular distribution of process $e^- + e^+ \longrightarrow t + \bar{t}$ in QED.

From figure 1.3, we will choose two values of $\sqrt{s}$ for comparision: $\sqrt{s} = 500$ GeV where the cross-section is maximum, and $\sqrt{s} = 3$ TeV for ultra-relativistic particles. We can see that figure 1.4 illustrates the angular distribution for our process in QED at two difference energy levels, 500 (GeV) and 3 (TeV), respectively. These distributions are perfectly symmertic in non-ultra- and ultra-relativistic cases.

1.4.3 Transverse- and longitudinal-momentum distributions

Transverse momentum distribution:

Another necessary distribution is the transverse and longitudinal momentum distribution of top-quark. We can accquire this quantities by switching from angular distribution to transverse (longitudial) momentum distribution, the transformation formular is given by

$$p(k_t) = \sum_i p(\theta_i)\mathcal{J}_i = \sum_i p(\theta_i)\frac{d\theta}{dk_t(\theta)}\bigg|_{\theta=\theta_i} \tag{1.4.4}$$

Beside, for the transverse momentum, we still have:

$$k_t = |\vec{k}|sin\theta \Rightarrow \begin{cases} \theta_1 = arcsin\dfrac{k_t}{|\vec{k}|} \\ \theta_2 = \pi - arcsin\dfrac{k_t}{|\vec{k}|} \end{cases} \tag{1.4.5}$$

Here we have introduced the factor $\mathcal{J}_i$ which called Jacobian, the explicit form is:

$$\left|\frac{d\theta}{dk_t}\right| = \frac{1}{|\vec{k}||cos\theta|} = \frac{1}{|\vec{k}|\sqrt{1-sin^2\theta}}$$
$$\Rightarrow \left|\frac{d\theta}{dk_t}|_{\theta=\theta_1}\right| = \left|\frac{d\theta}{dk_t}|_{\theta=\theta_2}\right| = \frac{1}{|\vec{k}|\sqrt{1-\frac{k_t^2}{|\vec{k}|^2}}} = \frac{1}{\sqrt{|\vec{k}|^2-k_t^2}} \tag{1.4.6}$$

Substitute equation (1.4.6) into (1.4.4), we shall have:

$$\frac{d\sigma}{dk_t} = \sum_i \frac{d\sigma}{d\theta}|_{\theta=\theta_i}\left|\frac{d\theta}{dk_t}|_{\theta=\theta_i}\right|$$
$$= \sum_{i=1}^{2}\frac{e^4|\vec{k}|}{6\pi Es^2}\left[E^2+|\vec{k}|^2cos^2\theta_i+m_t^2\right]\sin\theta_i\frac{1}{\sqrt{|\vec{k}|^2-k_t^2}} = \frac{e^4k_t\left(2E^2-k_t^2\right)}{3\pi Es^2\sqrt{|\vec{k}|^2-k_t^2}} \tag{1.4.7}$$

Longitudinal momentum distribution:
Similarity calculation, we could find the longitudinal momentum distribution of top-quark, for the longitudinal momentum we have:

$$k_l = |\vec{k}|cos\theta \Rightarrow \begin{cases}\theta_1 = arccos\frac{k_l}{|\vec{k}|}\\ \theta_2 = -arccos\frac{k_l}{|\vec{k}|}\end{cases} \tag{1.4.8}$$

Since $\theta \in [0,\pi]$, the associated Jacobian is:

$$\left|\frac{d\theta}{dk_l}|_{\theta=\theta_1}\right| = \frac{1}{|\vec{k}||cos\theta_1|} = \frac{1}{\sqrt{|\vec{k}|-k_l^2}} \tag{1.4.9}$$

Finally, we obtain the longitudinal momentum distribution of top-quark in CM frame:

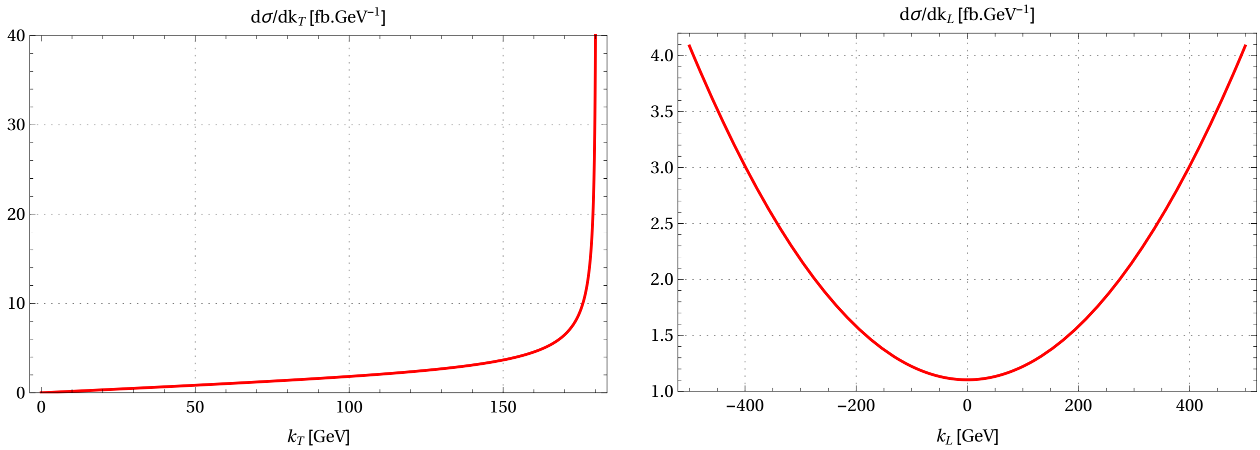
$$\frac{d\sigma}{dk_l} = \frac{d\sigma}{d\theta}|_{\theta=\theta_1}\left|\frac{d\theta}{dk_l}|_{\theta=\theta_1}\right|$$
$$= \frac{e^4|\vec{k}|}{6\pi Es^2}\left[E^2+|\vec{k}|^2\frac{k_l^2}{|\vec{k}|^2}+m_t^2\right]\sqrt{1-\frac{k_l^2}{|\vec{k}|^2}}\frac{1}{\sqrt{|\vec{k}|^2-k_t^2}} = \frac{e^4}{6\pi Es^2}\left[E^2+k_l^2+m_t^2\right] \tag{1.4.10}$$

Combining all ingredients, we now depicts the graph 1.7 of transverse (longgitudinal) momentum distribution of top-quark at threshold $\sqrt{s} = 500$ GeV:

1.4.4 Rapidity and pseudo-rapidity distributions of top-quark

Rapidity distribution:
Basicaly, rapidity is defined as:

$$y = \frac{1}{2}Ln\left[\frac{E+k_l}{E-k_l}\right] \Rightarrow \begin{cases}\vec{k} \parallel Oz \Leftrightarrow y \to \pm\infty\\ \vec{k} \perp Oz \Leftrightarrow y \to 0\end{cases} \tag{1.4.11}$$



(a) Transverse momentum distribution

(b) Longitudinal momentum distribution

Figure 1.5: Transverse and longitudinal momentum distribution at $\sqrt{s} = 500$ GeV.

On the orther hand, the rapidity can be represented in term of longitudinal momentum:

$$y = arctanh\frac{k_l}{E} \Rightarrow k_l = E\ \tanh(y) \tag{1.4.12}$$

This definition would allow us to switch from longitudinal momentum distribution to rapidity distribution. By using repeatly the procedure of changing variables, we shall have:

$$p(y) = \sum_i p(k_l^i)\frac{dk_l^i}{dy(k_l)} \tag{1.4.13}$$

The associated Jacobian is:

$$\left|\frac{dk_l}{dy}|_{k_l=E\ \tanh\ y}\right| = E\left(1-\tanh^2 y\right) \tag{1.4.14}$$

Finally, the rapidity ditribution of top-quark in CM frame is presented as follow:

$$\begin{aligned}\frac{d\sigma}{dy} &= \frac{d\sigma}{dk_l}|_{k_l=E\ \tanh\ y}\left|\frac{dk_l}{dy}|_{k_l=E\ \tanh\ y}\right| \\ &= \frac{e^4}{6\pi s^2 \cosh^2 y}\left[E^2(1+\tanh^2 y)+m_t^2\right]\end{aligned} \tag{1.4.15}$$

Pesudo-rapidity distribution:
Furthermore, physicstis also use pseudo-rapidity to describe ultra-relativistic particles that is defined as:

$$\eta = \frac{1}{2}Ln\left(\frac{|\vec{k}|+k_l}{|\vec{k}|-k_l}\right) \tag{1.4.16}$$

Notice that we could also express pseudo-rapidity as a function of θ:

$$\eta = -Ln\left(tan\frac{\theta}{2}\right) \Rightarrow \theta = 2\arctan(e^{-\eta}) \tag{1.4.17}$$

Let's switch from angular distribution to pseudo-rapidity distribution as follow:

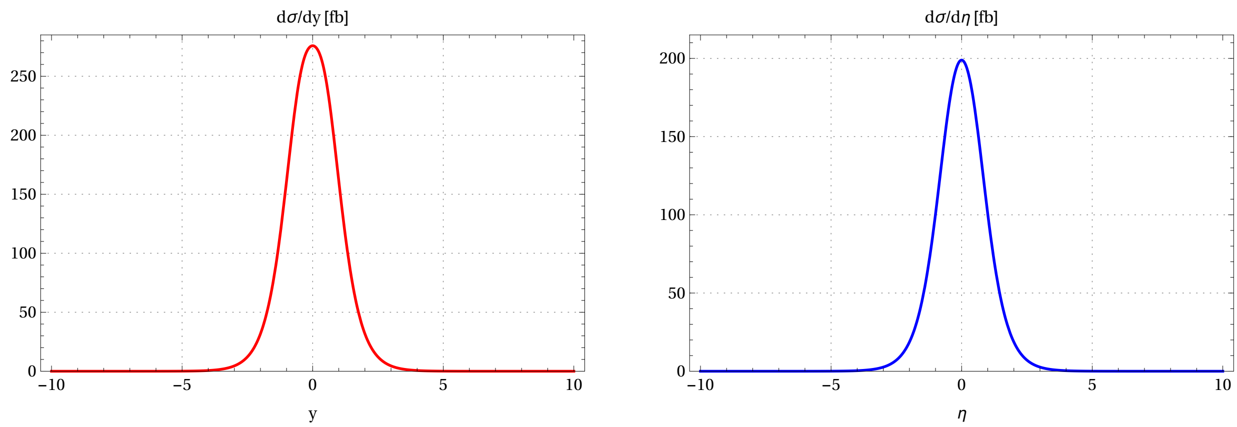
$$p(\eta) = p(\theta)|_{\theta=2\arctan(e^{-\eta})}\left|\frac{d\theta}{d\eta(\theta)}|_{\theta=2\arctan(e^{-\eta})}\right| \tag{1.4.18}$$

The associated Jacobian is:

$$\left|\frac{d\theta}{d\eta(\theta)}|_{\theta=2\arctan(e^{-\eta})}\right| = \left|\frac{-2e^{-\eta}}{1+e^{-2\eta}}\right| = \frac{-2}{e^{\eta}+e^{-\eta}} = \frac{-1}{\cosh\,\eta} \tag{1.4.19}$$

Thus, we obtain the pseudo-rapidity distribution:

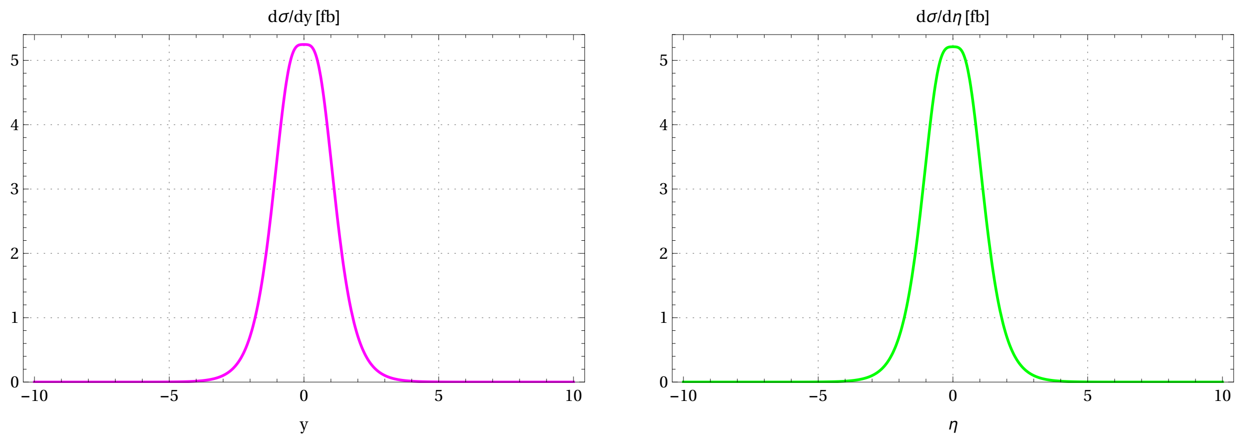
$$\frac{d\sigma}{d\eta} = \frac{e^4|\vec{k}|}{6\pi E s^2 \cosh^2\eta}\left[E^2 + m_t^2 + (E^2 - m_t^2)\tanh^2\eta\right] \tag{1.4.20}$$



(a) Rapidity distribution (b) Pesudo-rapidity distribution

Figure 1.6: Rapidity and pseudo-rapidity distribution for $\sqrt{s} = 500$ GeV.

Although pseudo-rapidity distribution looks like rapidity, however, their numerical values is not similar. Because of the mass of top-quark, the heaviest particle with $m_t \simeq 173.21$ GeV, hence we cannot ignore the mass of top-quark at level $\sqrt{s} = 500$ GeV. Thus, let us consider the high energy case compare with this mass:



(a) Rapidity distribution (b) Pesudo-rapidity distribution

Figure 1.7: Rapidity and pseudo-rapidity distribution for $\sqrt{s} = 3$ TeV.

Now we can see, the two distributions are similar with each other.

Chapter 2

Scattering process $\mathbf{e^- + e^+ \longrightarrow t + \bar{t}}$ in SM

2.1 An overview of Standard Model

The Standard Model is the particular physical model which summarise our present knowledge of the elementary constituents of matter and their interactions. Based on the framework of Quantum Field Theory and group theory, it is invariance under the local gauge transformations with the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with color group $SU(3)_C$ for the strong interaction and with $SU(2)_L \otimes U(1)_Y$ for the electroweak interaction spontaneously broken by the Higgs mechanism. More specifically, the interaction between each elementary particles are described by gauge vector bosons, with gluons for the strong interaction and $W^{\pm}, Z, \gamma$ for the electroweak interaction. Besides, in the Standard Model, the fermion fields are classified into three generations of Leptons and Quarks which are mentioned by a table below:

Standard Model of Elementary Particles					
Fermions				Gauge Bosons	Higgs Boson
Quarks	u	c	t	g	H
	d	s	b	γ	
Leptons	ν_e	ν_μ	ν_τ	$W^{\pm}$	
	e	μ	τ	Z	
	I	**II**	**III**	Three generation of fermions	

Table 2.1: Classification of elementary particles in the SM

According to the SM, the gauge bosons $W^{\pm}, Z$ acquire masses through Higgs mechanism while fermions gain masses with the help of gauge-invarinat Yukawa interactions and Higgs field. Notice that in the genuine SM neutrinos are considered as massless and never exist right-handed neutrino fields. Further information for the formulations of QCD and electroweak interaction can be found in [3]. Now we move on the full Lagrangian of the Standard Model as well as their notations and conventions.

2.2 Standard Model Lagrangian

In the spirit of $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ invariant, that symmetry motivates us to write down the full Lagrangian as follows:

$$\mathcal{L} = \mathcal{L}_{fermionic} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{gauge-fixing} + \mathcal{L}_{FPG}, \tag{2.2.1}$$

where the combination of kinetic fermion term $\mathcal{L}_{fermionic}$, the kinetic of gauge field which also called the pure Yang-Mill field $\mathcal{L}_{gauge}$, the Higgs sector $\mathcal{L}_{Higgs}$ and the Yukawa interaction $\mathcal{L}_{Yukawa}$ gives us the classical SM Lagrangin. In order to quantize this Lagrangian and help us derive the gauge boson propagator, the gauge-fixing term and Faddeev-Popov (Ghost) Lagrangian are introduced. Notice that, the Ghost terms just only appear in the higher order calculations, that means, in the scope of our thesis (at tree-level) these terms are completely absent. Additionally, although the SM's classical Lagrangian is gauge invariant, however, the gauge-fixing terms are not. Fortunately, the full Lagrangian in (2.2.1) preserve another symmetry which called BRST symmerty (furhter reading can be found in [2]). Another thing we should know is that the $\mathcal{L}_{Yukawa}$ which responses for the mysterious massive problem of all matter fields, by expanding this term we are able to collect the mass term of fermions and their couplings with Higgs field (we will get back to this term in next chapter when calculate the vertex factors of fermion-Goldstone boson).

In order to be consistent with the next chapters, we will follow the conventions in [4]. Let us express explicitly the classical Lagrangian of SM as follows:

$$\begin{aligned}\mathcal{L}_{SM} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \left[-\mu^2\phi^\dagger\phi + \frac{\lambda}{2}\left(\phi^\dagger\phi\right)^2\right] \\ &+ i\left(\bar{l}_L\not{D}l_L + \bar{e}_R\not{D}e_R + \bar{q}_L\not{D}q_L + \bar{u}_R\not{D}u_R + \bar{d}_R\not{D}d_R\right) \\ &- \left(\bar{l}_L\Gamma_e e_R\phi + \bar{q}_L\Gamma_u u_R\tilde{\phi} + \bar{q}_L\Gamma_d d_R\phi + h.c\right).\end{aligned} \tag{2.2.2}$$

Notice that the sign convention for covariant derivative is

$$D_\mu = \partial_\mu + ig_s\tau^A G^A_\mu + ig_2\tau^I W^I_\mu + ig_1 Y B_\mu, \tag{2.2.3}$$

where $\tau^A = \frac{1}{2}\lambda^A$ and $\tau^I = \frac{1}{2}\sigma^I$ are the generators of $SU(3)$ and $SU(2)$ group, respectively, besides, λ^A, σ^I are sequentially the Gell-Mann and Pauli matrices. Also, $\Gamma_{e,u,d}$ (Yukawa couplings) are 3×3 matrices in generation space. The notation Y stands for the weak hypercharge, their eigenvalues have been listed in the Table 2.2.

Also, the field strengh tensors are given by

$$\begin{cases} G^A_{\mu\nu} &= \partial_\mu G^A_{\mu\nu} - \partial_\nu G^A_\nu - g_s f^{ABC}G^B_\mu G^C_\nu \\ W^I_{\mu\nu} &= \partial_\mu W^I_{\mu\nu} - \partial_\nu W^I_\nu - g_2\epsilon^{IJK}W^J_\mu W^K_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{cases} \tag{2.2.4}$$

It is crucial to notice that the gauge boson can be collected by expanding the kinetic term of Higgs field. However, to diagonolize the mass terms (identify the physical gauge fields), let us introduce a tranformation from weak eigenstates to mass eigenstates as follow:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right), \tag{2.2.5}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} cos\theta_w & sin\theta_w \\ -sin\theta_w & cos\theta_w \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}, \tag{2.2.6}$$

	Fermions					Scalars
Field	$l^j_{Lp} = \begin{pmatrix} \nu_p \\ e_p \end{pmatrix}_L$	e_{Rp}	$q^{\alpha j}_{Lp} = \begin{pmatrix} u^\alpha_p \\ d^\alpha_p \end{pmatrix}_L$	u^α_{Rp}	d^α_{Rp}	ϕ^j
Hypercharge Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$

Table 2.2: SM hypercharge. Chronologically, $j = 1, 2$, $\alpha = 1 \cdots 3$, $p = 1 \cdots 3$ are isospin, colour and generation indices.

where θ_w is Weinberg angle, and $tan\theta_w = g_1/g_2$. With these transformations, the mass of $W^\pm$, Z boson and photon (A_μ) are presented as

$$\begin{cases} M_W &= \dfrac{g_2 v}{2} \\ M_Z &= \dfrac{1}{2} v \sqrt{g_1^2 + g_2^2} = \dfrac{g_2 v}{2c_w} \\ M_A &= 0 \end{cases}, \tag{2.2.7}$$

for convenience, we denote s_w, c_w abbreviate for $sin\theta_w$ and $cos\theta_w$. And last but not least, the convention for complex conjugate of Higgs field is

$$\tilde{\phi}^j = \epsilon_{jk} \left(\phi^k\right)^*, \quad \text{with} \quad \epsilon_{12} = +1 \quad \text{(totally anti-symmetric tensor).} \tag{2.2.8}$$

2.3 Lagrangian of the process $e^- + e^+ \longrightarrow t + \bar{t}$

Arcorrding to SM Lagrangian as present in the previous section, we could write down Lagrangian sectors which coresspond to our process:

$$\mathcal{L}_{process} = \mathcal{L}_{fermion} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{gauge-fixing}, \tag{2.3.1}$$

in term of this Lgrangian process, equation (2.3.1) is a combination of four essential part which help us derive the interaction of the fermion current with gauge fields, as well as general propagator. In case of our process, the requirement of charge conservation has to include and therefore the weak interaction must be mediated by neutral gauge bosons like Z-boson or photon. For simplicity, we just only consider the combination of W^3_μ and B_μ in term of gauge-field Lagrangian as well as their coressponding gauge-fixing terms. Notice that the matter fields in our process are electron and top-quark, hence we can expand the fermion sector as

$$\begin{aligned} \mathcal{L}_{fermions} &= \sum_f i\bar{L}_f \not{D} L_f + \sum_f i\bar{R} \not{D} R_f \\ &= i\bar{l}_{L1} \not{D} l_{L1} + i\bar{e}_{R1} \not{D} e_{R1} + i\bar{q}_{L3} \not{D} q_{L3} + i\bar{u}_{R3} \not{D} u_{R3}, \end{aligned} \tag{2.3.2}$$

we denote that l_{L1}, q_{L3} are left-handed leptons and quarks which are

$$E_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \qquad Q_L = \begin{pmatrix} t \\ b \end{pmatrix}_L . \tag{2.3.3}$$

2.4 Vertex factors of process $e^- + e^+ \longrightarrow t + \bar{t}$

2.4.1 Neutral-current coupling constant

The interaction of neutral-current mediated by Z-boson and photon could be extracted in the kinetic term of fermionic sector. Since the interaction terms are hidden in the part that involes W^3_μ and B_μ gauge fields, it is reasonable that we can introduce a neutral covariant derivative D^N_μ as follow:

$$\begin{cases} D^N_\mu \psi^L_f &= \left(ig_2 \dfrac{\sigma^3}{2} W^3_\mu + ig_1 Y B_\mu \right) \psi^L_f \\ \\ D^N_\mu \psi^R_f &= \left(ig_1 Y B_\mu \right) \psi^R_f, \end{cases} \tag{2.4.1}$$

substituting (2.4.1) into (2.3.2) the Lagrangian interaction mediated by neutral bosons is:

$$\mathcal{L}^{neutral}_{int} = -\bar{L}_f \gamma^\mu \left(g_2 \frac{\sigma^3}{2} W^3_\mu + g_1 Y B_\mu \right) L_f - \bar{R}_f \gamma^\mu \left(g_1 Y B_\mu \right) R_f. \tag{2.4.2}$$

By using the relation between W^3_μ, B_μ and physical fields like Z_μ, A_μ as we known in (2.2.6), we obtain the expression of Lagrangian interaction:

$$\begin{aligned} \mathcal{L}^{neutral}_{int} &= -\bar{L}_f \gamma^\mu \left[g_2 \frac{\sigma^3}{2} (c_w Z_\mu + s_w A_\mu) + g_1 Y (-s_w Z_\mu + c_w A_\mu) \right] L_f \\ &\quad - \bar{R}_f \gamma^\mu \left[g_1 Y (-s_w Z_\mu + c_w A_\mu) \right] R_f \\ &= -\bar{L}_f \gamma^\mu \left[\frac{g_2}{c_w} \left(c_w^2 I_3 - s_w^2 Y \right) Z_\mu + g_2 s_w Q_f A_\mu \right] L_f \\ &\quad - \bar{R}_f \gamma^\mu \left[-\frac{g_2}{c_W} s_w^2 Y Z_\mu + g_2 s_w Y A_\mu \right] R_f, \end{aligned} \tag{2.4.3}$$

where I_3 is an eigenvalues of $\sigma^3/2$ operator. Substituting all fermion types of our process and collecting all suitable terms we will derive a coupling constants.

Left-handed fermions

1. For $L_f = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$:

$$\begin{aligned} i\bar{L}_e \gamma^\mu D^N_\mu L_e &= -\begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \gamma^\mu \left[\frac{g_2}{c_w} \left(c_w^2 I_3 - s_w^2 Y_e \right) Z_\mu + g_2 s_w \left(I_3 + Y_e \right) A_\mu \right] \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \\ &= -\frac{g_2}{c_w} \left[c_w^2 I^3_e - s_w^2 Y_e \right] Z_\mu \left(\bar{e}_L \gamma^\mu e_L \right) - g s_w Q_e A_\mu \left(\bar{e}_L \gamma^\mu e_L \right) \\ \Rightarrow \mathcal{L}^{int}_{e-L} &= -\frac{g_2}{c_w} \left[-\frac{1}{2} + s_W^2 \right] Z_\mu \left(\bar{e}_L \gamma^\mu e_L \right) + g_2 s_w A_\mu \left(\bar{e}_L \gamma^\mu e_L \right). \end{aligned} \tag{2.4.4}$$

2. For $L_f = \begin{pmatrix} t \\ b \end{pmatrix}_L$:

$$
\begin{aligned}
i\bar{Q}_e\gamma^\mu D_\mu^N Q_e &= -\begin{pmatrix}\bar{t}_e & \bar{b}\end{pmatrix}_L \gamma^\mu \left[\frac{g_2}{c_w}\left(c_w^2 I_3 - s_W^2 Y\right) Z_\mu + g_2 s_w \left(I_3 + Y\right) A_\mu\right] \begin{pmatrix} t \\ b \end{pmatrix}_L \\
&= -\frac{g_2}{c_w}\left[c_w^2 I_t^3 - s_w^2 Y\right] Z_\mu \left(\bar{t}_L\gamma^\mu t_L\right) - g_2 s_w Q_t A_\mu \left(\bar{t}_L\gamma^\mu t_L\right) \\
\Rightarrow \mathcal{L}_{top-L}^{int} &= -\frac{g_2}{c_W}\left[\frac{1}{2} - \frac{2}{3}s_w^2\right] Z_\mu \left(\bar{t}_L\gamma^\mu t_L\right) - \frac{2}{3} g s_w A_\mu \left(\bar{t}_L\gamma^\mu t_L\right). \qquad (2.4.5)
\end{aligned}
$$

Right-handed fermions: In this case we just consider only e_R and t_R as below:

$$
\begin{aligned}
\sum_{e,t} i\bar{R}_f\gamma^\mu D_\mu^N R_f &= -\bar{e}_R\gamma^\mu\left[-\frac{g_2}{c_w}s_w^2 Y Z_\mu + g_2 s_w Y A_\mu\right] e_R - \bar{t}_R\gamma^\mu\left[-\frac{g_2}{c_w}s_w^2 Y Z_\mu + g_2 s_w Y A_\mu\right] t_R \\
&= -\frac{g_2}{c_w}s_w^2 Z_\mu\left(\bar{e}_R\gamma^\mu e_R\right) + g_2 s_w A_\mu\left(\bar{e}_R\gamma^\mu e_R\right) \\
&\quad + \frac{2g_2}{3c_w}s_w^2 Z_\mu\left(\bar{t}_R\gamma^\mu t_R\right) - \frac{2}{3}g_2 s_w A_\mu\left(\bar{t}_R\gamma^\mu t_R\right). \qquad (2.4.6)
\end{aligned}
$$

We also notice that the quantum number, isospin, can be calculated through the weak hypercharge (listed in Table 2.2) with the help of Gell-Mann-Nishijima relation, $Q = I_3 + Y$. The electric charge of electron and top quark are -1 and +2/3, respectively. Now we combinding all ingredients which yields:

$$
\begin{aligned}
\mathcal{L}_{e-int} &= -\frac{g_2}{c_w}\left[-\frac{1}{2} + s_w^2\right] Z_\mu\left(\bar{e}_L\gamma^\mu e_L\right) + \frac{g_2}{c_w}s_w^2 Z_\mu\left(\bar{e}_R\gamma^\mu e_R\right) + g_2 s_w A_\mu\left(\bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R\right) \\
&= g_L^e Z_\mu\left(\bar{e}_L\gamma^\mu e_L\right) + g_R^e Z_\mu\left(\bar{e}_R\gamma^\mu e_R\right) + e A_\mu\left(\bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R\right) \qquad (2.4.7)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{t-int} &= -\frac{g_2}{c_w}\left[\frac{1}{2} - \frac{2}{3}s_w^2\right] Z_\mu\left(\bar{t}_L\gamma^\mu t_L\right) + \frac{2g_2}{3c_w}s_w^2 Z_\mu\left(\bar{t}_R\gamma^\mu t_R\right) - \frac{2}{3}g_2 s_w A_\mu\left(\bar{t}_L\gamma^\mu t_L + \bar{t}_R\gamma^\mu t_R\right) \\
&= g_L^t Z_\mu\left(\bar{t}_L\gamma^\mu t_L\right) + g_R^t Z_\mu\left(\bar{t}_R\gamma^\mu t_R\right) - \frac{2}{3}e A_\mu\left(\bar{t}_L\gamma^\mu t_L + \bar{t}_R\gamma^\mu t_R\right) \qquad (2.4.8)
\end{aligned}
$$

Notice that $g_2 s_w = e$ which also called electric charge.

2.4.2 Vertex factors

To derive the vertex factor of our process, we need to perform the Lagrangian interation form similar with $e\bar{e}A_\mu$, which mentioned in V-A theory, by the way of using Chialrity operators:

$$
\begin{cases}
\psi_L = \mathrm{P}_L\psi = \dfrac{1-\gamma^5}{2}\psi \\[2ex]
\psi_R = \mathrm{P}_R\psi = \dfrac{1+\gamma^5}{2}\psi
\end{cases} \qquad (2.4.9)
$$

Let us transform a bit general Lagrangian interaction to convert like a form vector-axial:

$$
\begin{aligned}
\mathcal{L}_{int} &= g_L A_\mu(\bar{\psi}_L\gamma^\mu\psi_L) + g_R A_\mu(\bar{\psi}_R\gamma^\mu\psi_R) \\
&= g_L A_\mu\psi^\dagger\frac{1-\gamma^5}{2}\gamma^0\gamma^\mu\frac{1-\gamma^5}{2}\psi + g_R A_\mu\psi^\dagger\frac{1+\gamma^5}{2}\gamma^0\gamma^\mu\frac{1+\gamma^5}{2}\psi \\
&= A_\mu\left[\frac{1}{2}(g_L+g_R)(\bar{\psi}\gamma^\mu\psi) - \frac{1}{2}(g_L-g_R)(\bar{\psi}\gamma^\mu\gamma^5\psi)\right] \qquad (2.4.10) \\
&= A_\mu\left[\frac{1}{2}g_V(\bar{\psi}\gamma^\mu\psi) - \frac{1}{2}g_A(\bar{\psi}\gamma^\mu\gamma^5\psi)\right], \qquad (2.4.11)
\end{aligned}
$$

from (2.4.11) we can express the Lagrangian interaction of electron and top quark as follow:

$$\mathcal{L}_{e-int} = Z_\mu \left\{ \frac{1}{2}\left(-\frac{g_2}{2c_w}[-1+4s_w]\right)[\bar{e}\gamma^\mu e] - \frac{1}{2}\left(\frac{g_2}{2c_w}\right)[\bar{e}\gamma^\mu\gamma^5 e]\right\}$$
$$+ A_\mu\left[\frac{1}{2}(2g_2 s_w)\right](\bar{e}\gamma^\mu e), \tag{2.4.12}$$

$$\mathcal{L}_{t-int} = Z_\mu \left\{ \frac{1}{2}\left(-\frac{g_2}{2c_w}[\frac{1}{2}-\frac{4}{3}s_w]\right)[\bar{t}\gamma^\mu t] - \frac{1}{2}\left(-\frac{g_2}{2c_w}\right)[\bar{t}\gamma^\mu\gamma^5 t]\right\}$$
$$+ A_\mu\left[\frac{1}{2}\left(-\frac{4}{3}g_2 s_w\right)\right](\bar{t}\gamma^\mu t). \tag{2.4.13}$$

More convenience, all coupling constants of neutral-current interacting with Z-boson and photon are summerized in the Table 2.3 below:

	fermions	g_L	g_R	g_V	g_A
Z-boson	$e\ \mu\ \tau$	$-\frac{g_2}{c_w}\left(-\frac{1}{2}+s_w^2\right)$	$-\frac{g_2}{c_w}s_w^2$	$-\frac{g_2}{2c_w}(-1+4s_w^2)$	$\frac{g}{2c_w}$
	u c t	$-\frac{g_2}{c_w}\left(\frac{1}{2}-\frac{2}{3}s_w^2\right)$	$\frac{2}{3}s_w^2\frac{g_2}{c_w}$	$-\frac{g_2}{2c_w}\left(\frac{1}{2}-\frac{4}{3}s_w^2\right)$	$-\frac{g_2}{2c_w}$
Photon	$e\ \mu\ \tau$	$g_2 s_w$	$g_2 s_w$	$2g_2 s_w$	0
	u c t	$-\frac{2}{3}g_2 s_w$	$-\frac{2}{3}g_2 s_w$	$-\frac{4}{3}g_2 s_w$	0

Table 2.3: Summerize EW coupling mediated by Z-boson and photon

2.5 Propagator of Z-boson

Analogous with QED case, next step, we will find the propagator of Z-boson. The kinetic and mass temrs of Z-boson are proceeded via expanding the gauge and Higgs kinetic sector of SM Lagragian (2.2.2), then combinding with the gauge-fixing terms we shall have:

$$\mathcal{L}_Z = -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{1}{2\xi_Z}(\partial^\mu Z_\mu)^2 \tag{2.5.1}$$

Applying Euler-Lagrange equation for (2.5.1) provide the equation of motion as

$$\left[(\Box + m_Z^2)g^{\mu\rho} - \left(1-\frac{1}{\xi_Z}\right)\partial^\mu\partial^\rho\right]D_{\rho\nu}(x-y) = g^\mu_\nu\delta^4(x-y), \tag{2.5.2}$$

by Fourier transform, we have

$$\left[(-q^2 + m_Z^2)g^{\mu\rho} + \left(1-\frac{1}{\xi_Z}\right)q^\mu q^\rho\right]D_{\rho\nu}(q) = g^\mu_\nu. \tag{2.5.3}$$

Now we see, (2.5.3) has a similar form with (1.1.9), then applying the same procedure as we have done in QED case, the general propagator of Z-boson is

$$D_{\rho\nu}(q) = \frac{-1}{q^2 - m_Z^2}\left[1 - (1-\xi_Z)\frac{q_\rho q_\nu}{q^2 - \xi_Z m_z^2}\right]. \tag{2.5.4}$$

It is crucial to notice that the propagator (2.5.4) will generate a divergence where $\sqrt{s} = m_Z$. However, this divergence will appear if the mass of mediated gauge bosons, $m_{gauge-boson}$, larger than the threshold energy of their corresponding process. Fortunately, the threshold energy of our process is $2m_t > m_Z$, thus this divergence will not happen.

2.6 Feynman rules

- Photon propagator:

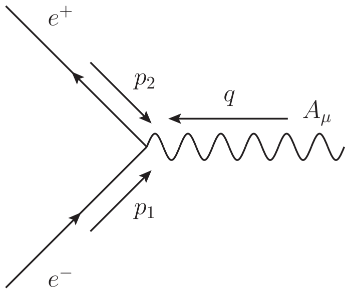
$$= -\frac{i}{q^2}\left[g_{\mu\nu} - (1-\xi_A)\frac{q_\mu q_\nu}{q^2}\right] \tag{2.6.1}$$

- Z-boson propagator:

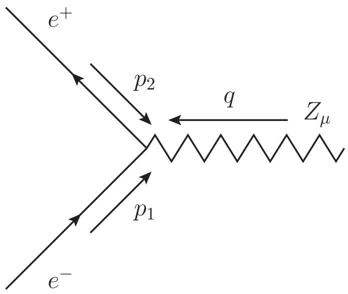
$$= -\frac{i}{q^2 - m_Z^2}\left[g_{\mu\nu} - (1-\xi_Z)\frac{q_\mu q_\nu}{q^2 - \xi m_Z^2}\right] \tag{2.6.2}$$

Notice that, in this chapter we set $\xi \to \infty$ (unitary gauge) when calculate, in the next section we shall work with general propagators in R_ξ-gauge. The Feynman rules for external lines are the same with QED. Collecting all informations from the Lagrangian interaction, the vertex factors are

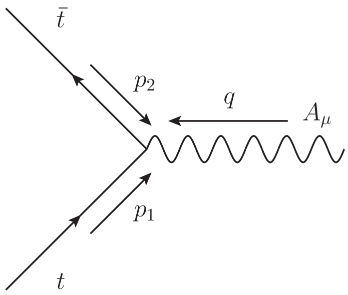
- The vertex factors



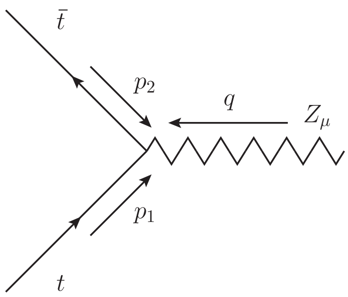
$$= ie\gamma^\mu \tag{2.6.3}$$



$$= i\frac{1}{2}\gamma^\mu\left(g_V^e - g_A^e\gamma^5\right) \tag{2.6.4}$$



$$= -i\frac{2}{3}e\gamma^\mu \tag{2.6.5}$$



$$= i\frac{1}{2}\gamma^\mu\left(g_V^t - g_A^t\gamma^5\right) \tag{2.6.6}$$

2.7 Total cross-section

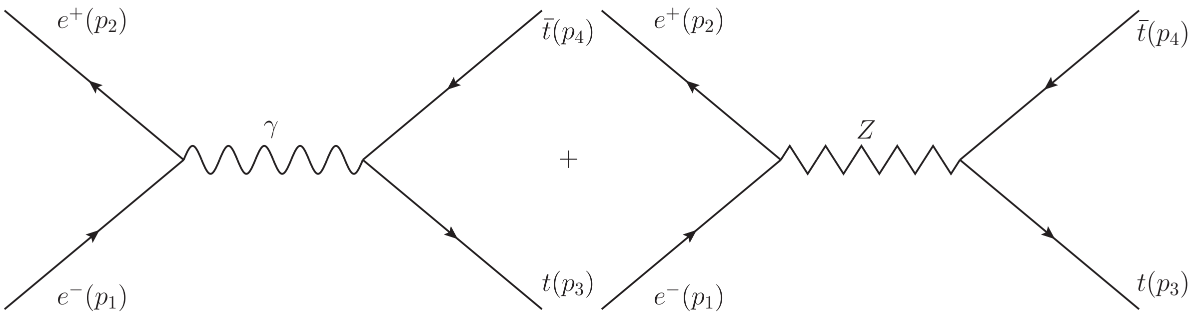


Figure 2.1: The Feynman diagrams of process $e^- + e^+ \longrightarrow t + \bar{t}$ in SM.

Base on the propagators and all vertices which calculated before, we are able to write down all contributing diagrams of process $e^+ + e^- \to t\bar{t}$ up to tree level. In unitary gauge, there are two diagrams performed in the figure 2.1 contribute to our process, by appying Feynman rules, the Feynman amplitude is

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z, \tag{2.7.1}$$

with $\mathcal{M}_\gamma$ and $\mathcal{M}_Z$ are:

$$\mathcal{M}_\gamma = [\bar{v}_{s'}(p_2)(ie\gamma^\mu)u_s(p_1)] \left[\frac{-i}{s}\left(g_{\mu\nu} - (1-\xi_A)\frac{q_\mu q_\nu}{s}\right)\right] \left[\bar{u}_r(p_3)\left(-i\frac{2}{3}e\gamma^\nu\right)v_{r'}(p_4)\right]\delta_{\alpha\beta} \tag{2.7.2}$$

$$\mathcal{M}_Z = \left[\bar{v}_{s'}(p_2)\frac{i}{2}\left(g_V^e\gamma^\mu - g_A^e\gamma^\mu\gamma^5\right)u_s(p_1)\right]\left[\frac{-i}{s-m_Z^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}\right)\right] \times \left[\bar{u}_r(p_3)\frac{i}{2}\left(g_V^t\gamma^\nu - g_A^t\gamma^\nu\gamma^5\right)v_{r'}(p_4)\right]\delta_{\alpha\beta}, \tag{2.7.3}$$

notice that $\delta_{\alpha\beta}$ appeare for colour charge conservation, with this colour indices are contracted. We also have the law of conservation momentum at all vertices, which could be exploited as $q = p_1 + p_2$, thus we can use the Mandelstam variables, $s = q^2$. Let us transform a bit to simplify eqs.(2.7.2) and (2.7.3). First of all, we prove that the Feynman amplitude of $\gamma-$diagram is $\xi-$independent, the terms which proportion with gauge-fixing parameter are

$$\mathcal{M}_\gamma^{gf} = i\frac{2(1-\xi_A)e^2}{3s^2}[\bar{v}_{s'}(p_2)\not{q}u_s(p_1)][\bar{u}_r(p_3)\not{q}v_{r'}(p_4)]. \tag{2.7.4}$$

By using the same identites like 1.3.2, we shall have

$$\bar{v}_{s'}(p_2)\not{q}u_s(p_1) = \bar{v}_{s'}(p_2)(\not{p}_1 + \not{p}_2)u_s(p_1) = m_e\bar{v}_{s'}(p_2)u_s(p_1) - m_e\bar{v}_{s'}(p_2)u_s(p_1) = 0, \tag{2.7.5}$$

from 2.7.5 we can see that $\mathcal{M}_\gamma$ does not depend on ξ_A. At this point, we obtain:

$$\mathcal{M}_\gamma = -i\frac{2e^2}{3s}\left[\bar{v}_{s'}(p_2)\gamma^\mu u_s(p_1)\right]\left[\bar{u}_r(p_3)\gamma_\mu v_{r'}(p_4)\right]\delta_{\alpha\beta}. \tag{2.7.6}$$

With the same procedure have done before, the Feynman amplitude term of Z-boson also simplified as below:

$$\mathcal{M}_Z^{gf} = \frac{-i}{4(s-m_Z^2)m_Z^2}\left[\bar{v}_{s'}(p_2)(g_V^e \not{q} - g_A^e \not{q}\gamma^5)u_s(p_1)\right]\left[\bar{u}_r(p_3)(g_V^t \not{q} - g_A^t \not{q}\gamma^5)v_{r'}(p_4)\right], \quad (2.7.7)$$

here we introduce a new identity

$$\begin{aligned} q_\mu \bar{v}_{s'}(p_2)\gamma^\mu(1-\gamma^5)u_s(p_1) &= 0 - \bar{v}_{s'}(p_2)(\not{p}_1 + \not{p}_2)\gamma^5 u_s(p_1) \\ &= 2m_e \bar{v}_{s'}(p_2)\gamma^5 u_s(p_1), \end{aligned} \quad (2.7.8)$$

and so do with second term, combinding all pices of (2.7.7), we obtain:

$$\mathcal{M}_Z^{gf} = i\frac{m_e m_t}{(s-m_Z^2)m_Z^2}[\bar{v}_{s'}(p_2)g_A^e\gamma^5 u_s(p_1)][\bar{u}_r(p_3)g_A^t\gamma^5 v_{r'}(p_4)] \simeq 0. \quad (2.7.9)$$

Since, the mass of electron is tiny compare with Z-boson and the energy threshold to happen this sacttering, therefore we can set $m_e \simeq 0$, approximately, hence equation (2.7.9) is vanished. The Feynman amplitude for Z-boson is

$$\mathcal{M}_Z = \frac{i}{4(s-m_Z^2)}\left[\bar{v}_{s'}(p_2)\left(g_V^e\gamma^\mu - g_A^e\gamma^\mu\gamma^5\right)u_s(p_1)\right]\left[\bar{u}_r(p_3)\left(g_V^t\gamma_\mu - g_A^t\gamma_\mu\gamma^5\right)v_{r'}(p_4)\right]\delta_{\alpha\beta}. \quad (2.7.10)$$

With all ingredients were calculated explicitly, we shall have the Feynman squared amplitude by taking a sum average of initial spin indices, the sum over spin and colour indices of out-going top-quark as follow:

$$|\mathcal{M}_0|^2 = 3\cdot\frac{1}{2}\cdot\frac{1}{2}\sum_{s,s'r,r'}\left(|\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + \mathcal{M}_\gamma\mathcal{M}_Z^\dagger + \mathcal{M}_Z\mathcal{M}_\gamma^\dagger\right), \quad (2.7.11)$$

with colour indices have been contracted, summing over colour indices yields the factor 3. This Feynman squared amplitude have been calculated by FORM [5], after that we calculate the total cross-section with Mathematica 10.4.

2.8 Physical results

In two-body phase space, the differential cross-section given by

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{|\mathcal{M}_0|^2}{64\pi^2\left(E_{e^-}+E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|} \\ \Rightarrow \sigma_T &= \int_{-1}^{1}\frac{d\sigma}{d(cos\theta)}d(cos\theta) = \int_{-1}^{1}\int_0^{2\pi}\frac{|\mathcal{M}_0|^2}{64\pi^2\left(E_{e^-}+E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|}d\phi\, d(cos\theta). \end{aligned} \quad (2.8.1)$$

Before we present the result of total cross-section, let us briefly introduce the input parameters. In this thesis, we have used data from PDG [6] which are listed as follows:

$$\begin{aligned} M_W &= 80.385 \text{ GeV}; \qquad M_Z = 91.1876 \text{ GeV}; \\ M_{top} &= 173.21 \text{ GeV}; \qquad G_F = 1.166378.10^{-5} \text{ GeV}^{-2}. \end{aligned} \quad (2.8.2)$$

From these input parameters, we can calculate straightforward another physical constant. Remarkable that, we also have some following relations:

$$\cos\theta_w = \frac{m_W}{m_Z}; \qquad \frac{g_2}{2\cos\theta_w} = \left(\sqrt{2}G_F m_Z^2\right)^{1/2}; \qquad \tan\theta_w = \frac{g_1}{g_2}, \tag{2.8.3}$$

moreover, some physical constants can be expressed in term of these parameters as

$$e = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}; \qquad v = \left(\sqrt{2}G_F\right)^{-1/2}. \tag{2.8.4}$$

Now we can depict the dependence of the total cross-section on the center of mass energy as figure 2.2. In this figure, we also plot the QED cross-section to help us comparision. As we can

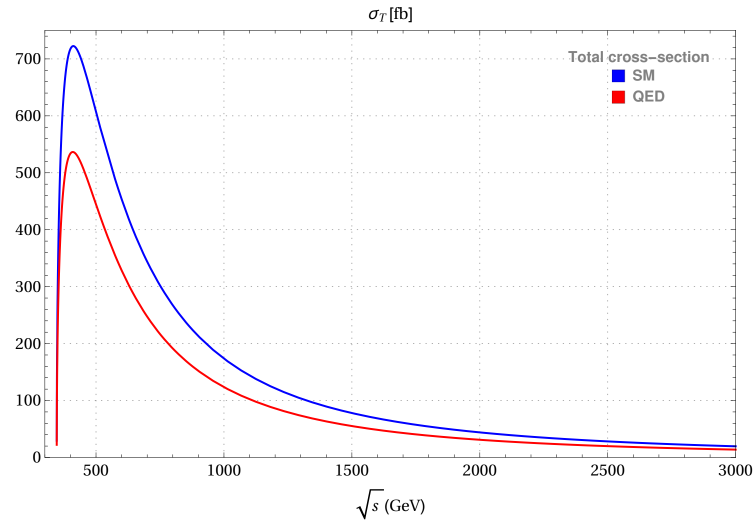
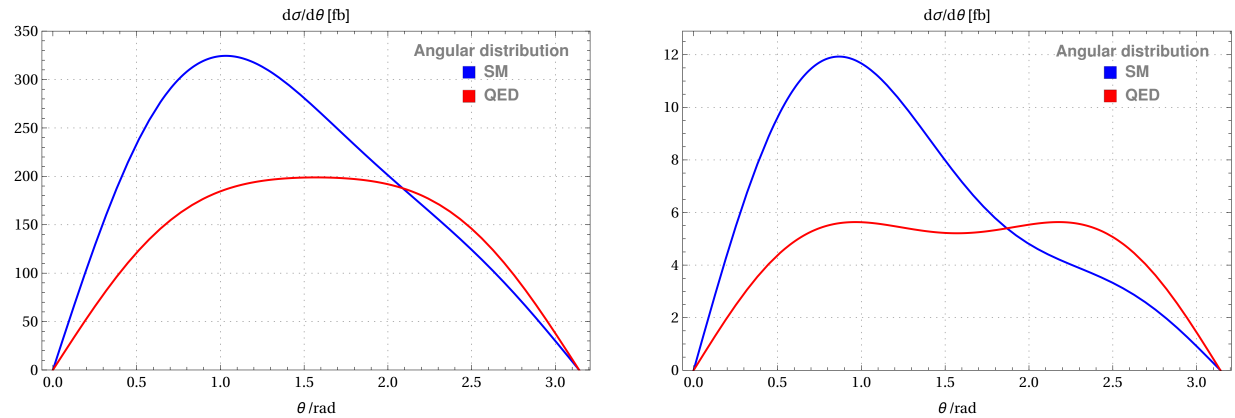


Figure 2.2: The total of process $e^- + e^+ \longrightarrow t + \bar{t}$ in SM and QED.

see, the total cross-section is raising significantly from the threshold energy of our process and reach a peak when $\sqrt{s} = 400 \sim 500$ GeV with the maximum value approximate 717 [fb]. From figure 1.2, we can choose $\sqrt{s} = 500$ GeV to point out some relevant distributions. We also present some distributions as we have done in QED case. Firstly, we present the comparision of angular distribution between QED and SM in figure 2.3:



(a) $\sqrt{s} = 500$ GeV (b) $\sqrt{s} = 3$ TeV

Figure 2.3: The angular distribution in SM and QED.

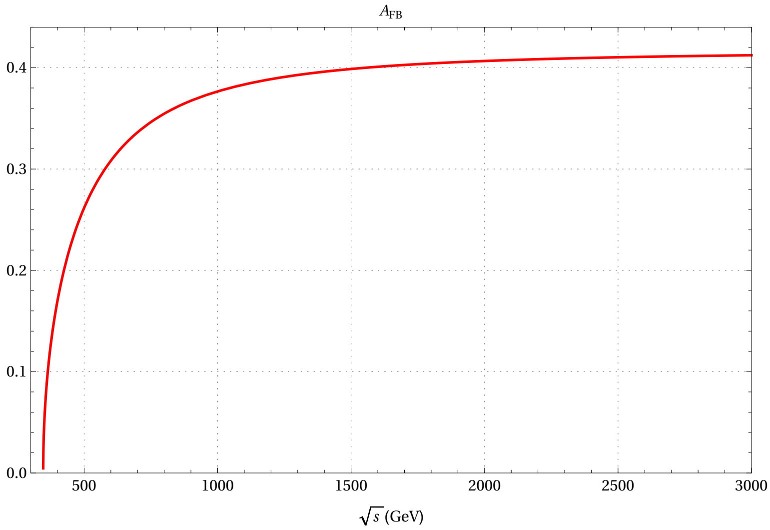


Figure 2.4: The forward-backward asymmetry.

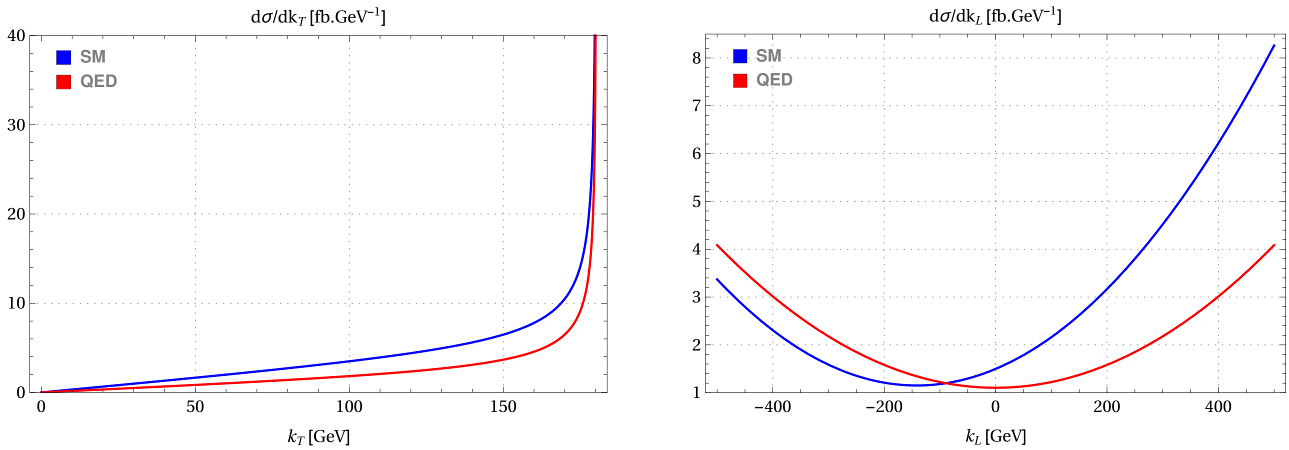
From figure 2.2 we can easily verify that the angular distribution in SM is considerably asymmetric while in QED this asymmetry never appear. In order to understand this asymmertry, physictis introduced a quantity called the forward-backward symmetry, A_{FB}, which defined as:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{\sigma_F - \sigma_B}{\sigma_T}, \tag{2.8.5}$$

where we have

$$\sigma_F = \int_0^{\pi/2} \frac{d\sigma}{d\theta} d\theta, \qquad \text{and} \quad \sigma_B = \int_{\pi/2}^{\pi} \frac{d\sigma}{d\theta} d\theta. \tag{2.8.6}$$

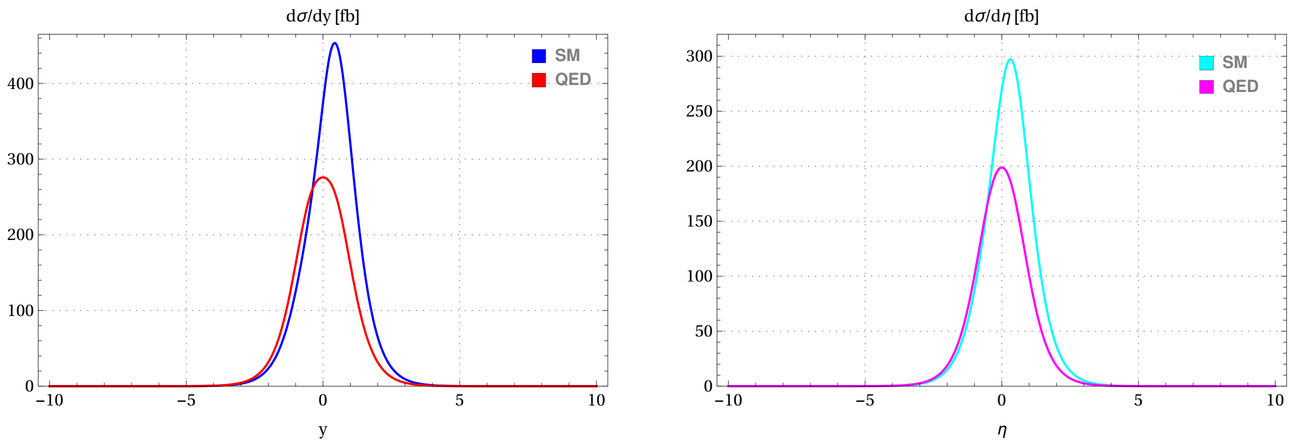
From this formula, we can plot the forward-backward asymmetry as a function with respect to $\sqrt{s}$ as figure 4.1. Now we move on another distribution such as transverse- and longitudinal-momentum distribution, rapidity and pseudo-rapidity distribution as follows:



(a) Transverse momentum distribution

(b) Longitudinal momentum distribution

Figure 2.5: Transverse- and longitudinal-momentum distribution for $\sqrt{s} = 500$ GeV.

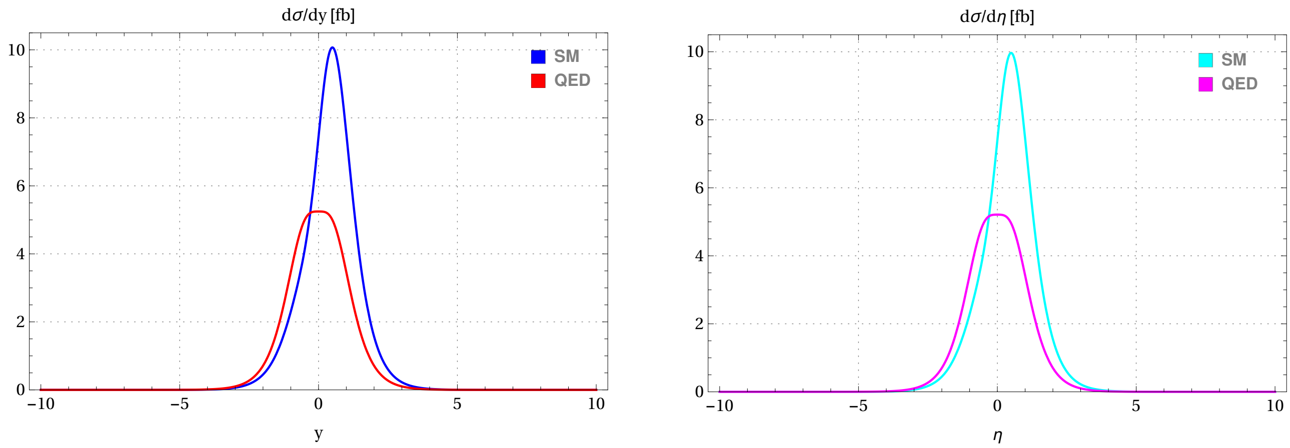


(a) Rapidity distribution

(b) Pseudo-rapidity distribution

Figure 2.6: Rapidity and pseudo-rapidity distribution for $\sqrt{s} = 500$ GeV.

At relativistic high energy, the rapidity and pseudo-rapidity distributions are



(a) Rapidity distribution

(b) Pseudo-rapidity distribution

Figure 2.7: Rapidity and pseudo-rapidity distribution for $\sqrt{s} = 3$ TeV.

Chapter 3

Scattering process $\mathbf{e^- + e^+ \to t + \bar{t}}$ in SMEFT

3.1 An introduction to SMEFT

Until nowadays, the Standard Model for strong and electroweak interactions has been tested with very high accuracy experiments and yielded a great results in phenomenological predictions. Since the Higgs boson has been discovered, the picture of Standard Model has become the theory of spontaneous symmetry breaking at an Electroweeak Scale (EW) with the vaccum expectation value-$v \simeq 246$ GeV. Nevertheless, if we consider the Standard Model just describes well physical phenomenas at EW scale, the signal of new physics beyond the SM may be hidden in the experimental errors of measurements. Thus, we assume that new physics effects can be parameterized in Standard Model Effective Field Theory (SMEFT) when we expand the effective Lagrangian by the power of expansion parameter Λ. Notice that the effective Lagrangian is valid up to a certain energy scale Λ.

Based on effective Lagrangian techniques, the general Lagrangian contains higher-dimensional operators can be written as an expansion in $(1/\Lambda)$:

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda}\sum_i C_i^{(5)} Q_i^{(5)} + \frac{1}{\Lambda^2}\sum_i C_i^{(6)} Q_i^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right), \tag{3.1.1}$$

where $\mathcal{L}_{SM}^{(4)}$ is nothing but the SM Lagrangian which contains only dimension-two and -four operators. In the rest terms of (3.1.1), we denote that $Q_i^{(n)}$ is the dimension-n operators and the corresponding dimensionless coupling constant (Willsion coefficients) of each operator is $C_i^{(n)}$. Remarkable that, the theory vaild above the energy scale Λ should satisfy the requirements below [4]

- The effective Lagrangian in (3.1.1) should be $SU_c(3)\otimes SU_L(2)\otimes U_Y(1)$ invariant, therefore, all dimension-n operators also satisfy this group structure too,
- all the SM degrees of freedom should be incorporated either as fundamental or composite fields,
- up to energy scale Λ, SMEFT does not appear any additional fields. Besides, effective field theory should behave consistently with SM when reduce to low-energy cases.

In the scope of this thesis, we concentrate on the effective Lagrangian constructed in a complete set of gauge invariant operators up to dimension six, which also called *"Warsaw basis"* (further information can be found in [4],[7]). Before we go to further calculation, let us show clearly our conventions of the following chapters.

3.2 Notations and conventions

In order to derive the new effective Lagrangian and cross-check the Feynman rules in [7], we followed the notation and conventions of [4]. For convenience, we also absorb the energy scale Λ by re-defining the Willsion coefficients as

$$\frac{C_i^{(5)}}{\Lambda} \to C_i^{(5)}, \quad \text{and } \frac{C_i^{(6)}}{\Lambda^2} \to C_i^{(6)}. \tag{3.2.1}$$

We have used the SM Lagrangian and the covariant derivative in the previous chapter again (since it is consistent with [4]'s conventions). Other new conventions are the Hermitian derivatives which defined as follow:

$$\phi^\dagger i\overleftrightarrow{D}_\mu \phi \equiv i\phi^\dagger \left(D_\mu - \overleftarrow{D}_\mu\right)\phi, \quad \text{and} \quad \phi^\dagger i\overleftrightarrow{D}^I_\mu \phi \equiv i\phi^\dagger \left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I\right)\phi, \tag{3.2.2}$$

whereas $\phi^\dagger \overleftarrow{D}_\mu \phi \equiv (D_\mu\phi)^\dagger \phi$. From these conventions, we now can expand all dimension-six operators listed in Tables 3.1 and 3.2.

X^3		ϕ^6 and $\phi^4 D^2$		$\psi^2\phi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_ϕ	$\left(\phi^\dagger\phi\right)^3$	$Q_{e\phi}$	$\left(\phi^\dagger\phi\right)(\bar{l}'_p e'_r \phi)$
$Q_{\tilde{G}}$	$f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\phi\Box}$	$\left(\phi^\dagger\phi\right)\Box\left(\phi^\dagger\phi\right)$	$Q_{u\phi}$	$\left(\phi^\dagger\phi\right)(\bar{q}'_p u'_r \tilde{\phi})$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\phi D}$	$\left(\phi^\dagger D^\mu\phi\right)^*\left(\phi^\dagger D_\mu\phi\right)$	$Q_{d\phi}$	$\left(\phi^\dagger\phi\right)(\bar{q}'_p d'_r \phi)$
$Q_{\tilde{W}}$	$\epsilon^{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\phi^2$		$\psi^2 X\phi$		$\psi^2\phi^2 D$	
$Q_{\phi G}$	$\phi^\dagger\phi G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\tau^I\phi W^I_{\mu\nu}$	$Q^{(1)}_{\phi l}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{l}'_p\gamma^\mu l'_r)$
$Q_{\phi\tilde{G}}$	$\phi^\dagger\phi \tilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}'_p\sigma^{\mu\nu}e'_r)\phi B_{\mu\nu}$	$Q^{(3)}_{\phi l}$	$(\phi^\dagger i\overleftrightarrow{D}^I_\mu\phi)(\bar{l}'_p\tau^I\gamma^\mu l'_r)$
$Q_{\phi W}$	$\phi^\dagger\phi W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}'_p\sigma^{\mu\nu}\tau^A u'_r)\tilde{\phi}G^A_{\mu\nu}$	$Q_{\phi e}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{e}'_p\gamma^\mu e'_r)$
$Q_{\phi\tilde{W}}$	$\phi^\dagger\phi \tilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tau^I\tilde{\phi}W^I_{\mu\nu}$	$Q^{(1)}_{\phi q}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{q}'_p\gamma^\mu q'_r)$
$Q_{\phi B}$	$\phi^\dagger\phi B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}'_p\sigma^{\mu\nu}u'_r)\tilde{\phi}B_{\mu\nu}$	$Q^{(3)}_{\phi q}$	$(\phi^\dagger i\overleftrightarrow{D}^I_\mu\phi)(\bar{q}'_p\tau^I\gamma^\mu q'_r)$
$Q_{\phi\tilde{B}}$	$\phi^\dagger\phi \tilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}'_p\sigma^{\mu\nu}\tau^A d'_r)\phi G^A_{\mu\nu}$	$Q^{\phi u}_{\phi q}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{u}'_p\gamma^\mu u'_r)$
$Q_{\phi WB}$	$\phi^\dagger\tau^I\phi W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\tau^I\phi W^I_{\mu\nu}$	$Q_{\phi d}$	$(\phi^\dagger i\overleftrightarrow{D}_\mu\phi)(\bar{d}'_p\gamma^\mu d'_r)$
$Q_{\phi\tilde{W}B}$	$\phi^\dagger\tau^I\phi \tilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}'_p\sigma^{\mu\nu}d'_r)\phi B_{\mu\nu}$	$Q_{\phi ud}$	$i(\tilde{\phi}^\dagger\overleftrightarrow{D}_\mu\phi)(\bar{u}'_p\gamma^\mu d'_r)$

Table 3.1: Dimension-six operators other than the four-fermion ones taken from [4]

3.3 Mass eigenstates in SMEFT

In this section we perform step by step the identification physical (unphysical) degree of freedom process with the presence of spontaneous symmetry breaking (SSB). To achive this goal, we need to represent all fields in mass eigenstates basic. Notice that this procedure will include the field rescaling steps, our further calculations will use the fields which are rescaled.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $\bar{R}L)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(d_p^\alpha)^T C u_r^\beta]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}[(q_p^{\alpha j})^T C q_r^{\beta k}]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}(q_p^{\alpha j})^T C q_r^{\beta k}]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}[(d_p^\alpha)^T C u_r^\beta]\left[(u_s^\gamma)^T C e_t\right]$
Q_{lequ}	$\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$		

Table 3.2: Four-fermion operators taken from [4].

3.3.1 Higgs sector

The full Lagrangian of Higgs field including dimension-six operators is

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger(D^\mu\phi) - \left[-\mu^2\left(\phi^\dagger\phi\right) + \frac{\lambda}{2}\left(\phi^\dagger\phi\right)^2\right] + C^\phi\left(\phi^\dagger\phi\right)^3 + C^{\phi\Box}\left(\phi^\dagger\phi\right)\Box\left(\phi^\dagger\phi\right) + C^{\phi D}\left(\phi^\dagger D_\mu\phi\right)^*\left(\phi^\dagger D^\mu\phi\right). \tag{3.3.1}$$

First of all, we consider the Higgs potential within the correction of dimension-six operators

$$\mathrm{V}(\phi) = -\mu^2\left(\phi^\dagger\phi\right) + \frac{\lambda}{2}\left(\phi^\dagger\phi\right)^2 + C^\phi\left(\phi^\dagger\phi\right)^3, \tag{3.3.2}$$

where $\mu^2, \lambda > 0$. Obviously, $\mathrm{V}(\phi)$ does not minimize with the field configuration $\phi = 0$, instead, this potential reaches an extremum (minimum) at non-zero configuration given by

$$\frac{\partial \mathrm{V}}{\partial\phi} = 0 \quad \Rightarrow \quad \phi^\dagger\phi = \frac{\lambda - \sqrt{\lambda^2 - 12\mu^2 C^\phi}}{6C^\phi}, \tag{3.3.3}$$

notice that we choose this root to obtain the finite results when taking the limit of $C^\phi \to 0$. By expanding (3.3.3) around the small value of C^ϕ, we have the following vaccum state

$$\begin{cases} \langle\phi\rangle & = \dfrac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix} \\ \text{with } v & = \sqrt{\dfrac{2\mu^2}{\lambda}} + \dfrac{3\mu^3}{\sqrt{2}\lambda^{5/2}}C^\phi, \end{cases} \tag{3.3.4}$$

notice that v is called the vaccum expectation value (vev). From now on, we just only use this vev for all expressions and Feynman rules.

In spite the fact that the Lagrangian in (3.3.1) invariant under the gauge transformation of $SU(2)_L \otimes U(1)_Y$, however, the vaccum configuration $\langle \phi \rangle$ is not. Fortunately, this vaccum state still invariant under the transformation of $U(1)_Q$ group, the electromagnetic subgroup which is generated by the charge Q. These things mean that the $SU(2)_L \otimes U(1)_Y$ symmetry has been spontaneously broken down to $U(1)_Q$ symmetry. Expanding the Higgs doublet field around the vaccum, we shall have:

$$\phi = \langle \phi \rangle + \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} + \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}} \left(H + i\Phi^0 \right) \end{pmatrix} = \begin{pmatrix} \Phi^+ \\ \frac{1}{\sqrt{2}} \left(v + H + i\Phi^0 \right) \end{pmatrix}, \tag{3.3.5}$$

where H is the scalar Higgs field, whereas Φ^+, Φ^0 are unphysical degree of freedom which commonly called *Goldstone-bosons* and can be eliminated by choosing a particular gauge transformation (unitary gauge). Substituting Higgs doublets (3.3.5) into Higss sector of Lagrangian (3.3.1), notice that the contribution of each operators for Higgs sector will be represented in the appendix A.1, thus we have:

$$\begin{aligned} \mathcal{L}_{Higgs} = & \frac{1}{2} \left[1 + \frac{1}{2} C_{\phi D} v^2 - 2 C_{\phi \Box} v^2 \right] \left(\partial_\mu H \right)^2 + \left[\frac{1}{2} \mu^2 - \frac{3}{4} \lambda v^2 + \frac{15}{8} v^2 C_\phi \right] H^2 \\ & + \frac{1}{2} \left[1 + \frac{1}{2} C_{\phi D} v^2 \right] \left(\partial_\mu \Phi^0 \right)^2 + \left(\partial_\mu \Phi^- \right) \left(\partial^\mu \Phi^+ \right). \end{aligned} \tag{3.3.6}$$

In the spirt of presenting the effective Lagrangian (3.3.6) with the standard form

$$\mathcal{L}_{Higgs} = \frac{1}{2} \left(\partial_\mu h \right)^2 - \frac{1}{2} M_H^2 h^2 + \frac{1}{2} \left(\partial_\mu G^0 \right)^2 + \left(\partial_\mu G^- \right) \left(\partial^\mu G^+ \right), \tag{3.3.7}$$

we have to rescale the following fields as below

$$h \equiv \left(1 + \frac{1}{4} C_{\phi D} v^2 - C_{\phi \Box} v^2 \right) H, \quad G^0 \equiv \left(1 + \frac{1}{4} C_{\phi D} v^2 \right) \Phi^0, \quad G^\pm \equiv \Phi^\pm, \tag{3.3.8}$$

and hence we now obtain the kinetic terms which were canonically normalized with $h, G^0, G^\pm$ are sequentially Higgs-field and Goldstone-fields. Furthermore, the squared mass of Higgs boson now is corrected with the constribution of dimension-six operators:

$$M_H^2 = 2\mu^2 \left[1 - \frac{\mu^2}{\lambda^2} \left(3C_\phi - 4\lambda C_{\phi \Box} + \lambda C_{\phi D} \right) \right], \tag{3.3.9}$$

substituting the new vev in (3.3.4) we shall have

$$M_H^2 = \lambda v^2 - \left[3C_\phi - 2\lambda C_{\phi \Box} + \frac{\lambda}{2} C_{\phi D} \right] v^4. \tag{3.3.10}$$

3.3.2 Gauge sector

In order to identify the physical gauge bosons, we now concentrate on the gauge sector within SMEFT Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_{gauge}^{full} = & -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \left(D_\mu \phi \right)^\dagger \left(D^\mu \phi \right) \\ & + C_{\phi G} \left(\phi^\dagger \phi \right) G_{\mu\nu}^A G^{A\mu\nu} \\ & + C_{\phi W} \left(\phi^\dagger \phi \right) W_{\mu\nu}^I W^{I\mu\nu} + C_{\phi B} \left(\phi^\dagger \phi \right) B_{\mu\nu} B^{\mu\nu} + C_{\phi WB} \left(\phi^\dagger \tau^I \phi \right) W_{\mu\nu}^I B^{\mu\nu} \\ & + C_{\phi D} \left(\phi^\dagger D_\mu \phi \right)^* \left(\phi^\dagger D^\mu \phi \right), \end{aligned} \tag{3.3.11}$$

whereas τ^I are the Pauli matrices. Substituting Higgs doulet in (3.3.5) into (3.3.11), let us consider the full correction up to dimension six operators of the kinetic terms of gauge fields:

$$\mathcal{L}_{EW} = -\frac{1}{4}\left(1 - C_{\phi G}v^2\right)^2 G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}\left(1 - C_{\phi W}v^2\right)^2 W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}\left(1 - C_{\phi B}v^2\right)^2 B_{\mu\nu}B^{\mu\nu}, \tag{3.3.12}$$

notice that we have temporarily ignored the mixing terms. One can easily see that, we can re-write this part in canonical normalized form by rescaling the fields as follow

$$\begin{cases} \bar{W}^I_\mu & = \left(1 - C_{\phi W}v^2\right) W^I_\mu \\ \bar{B}_\mu & = \left(1 - C_{\phi B}v^2\right) B_\mu \\ \bar{G}^A_\mu & = \left(1 - C_{\phi G}v^2\right) G^A_\mu \end{cases} \tag{3.3.13}$$

One important things to notice that rescaling these fields does not break the gauge symmetry, besides, they also preserve the form of covariant derivative, which is

$$\bar{D}_\mu = D_\mu = \partial_\mu + i\bar{g}_s\tau^A G^A_\mu + i\bar{g}_2\tau^I\bar{W}^I_\mu + i\bar{g}_1 Y\bar{B}_\mu, \tag{3.3.14}$$

$$\text{with } \bar{g}_1 = \frac{g_1}{1 - C_{\phi B}v^2}, \quad \bar{g}_2 = \frac{g_2}{1 - C_{\phi W}v^2}, \quad \text{and } \bar{g}_s = \frac{g_s}{1 - C_{\phi G}v^2}. \tag{3.3.15}$$

Remarkable that we will use the new rescaled-fields and -coupling constants for all calculations and Feynman rules. Expanding (3.3.11), we now foucus on the electroweak part, which reads

$$\begin{aligned} \mathcal{L}^{full}_{EW} = & -\frac{1}{4}\left(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}\right) + \frac{1}{2}\cdot\frac{\bar{g}_2^2 v^2}{4}\left(\bar{W}^1_{\mu\nu}\bar{W}^{1\mu\nu} + \bar{W}^2_{\mu\nu}\bar{W}^{2\mu\nu}\right) \\ & -\frac{1}{4}\begin{pmatrix}\bar{W}^3_{\mu\nu} & \bar{B}^{\mu\nu}\end{pmatrix}\begin{bmatrix} 1 & C_{\phi WB}v^2 \\ C_{\phi WB}v^2 & 1 \end{bmatrix}\begin{pmatrix}\bar{W}^{3\mu\nu} \\ \bar{B}^{\mu\nu}\end{pmatrix} \\ & -\frac{1}{2}\cdot\frac{v^2}{4}\left[1 + \frac{1}{2}C_{\phi D}v^2\right]\begin{pmatrix}\bar{W}^3_\mu & \bar{B}_\mu\end{pmatrix}\begin{bmatrix} \bar{g}_2^2 & -\bar{g}_1\bar{g}_2 \\ -\bar{g}_1\bar{g}_2 & \bar{g}_1^2 \end{bmatrix}\begin{pmatrix}\bar{W}^{3\mu} \\ \bar{B}^\mu\end{pmatrix} \end{aligned} \tag{3.3.16}$$

In order to diagonal simultaneously the kinetic and the mass terms, let us introduce a following transformations [7]:

$$\begin{cases} W^\pm_\mu = \frac{1}{\sqrt{2}}\left(\bar{W}^1_\mu \mp i\bar{W}^2_\mu\right) \\ \\ \begin{pmatrix}\bar{W}^3_\mu \\ \bar{B}_\mu\end{pmatrix} = \begin{pmatrix} 1 & -\epsilon/2 \\ -\epsilon/2 & 1 \end{pmatrix}\begin{pmatrix} cos\bar{\theta} & sin\bar{\theta} \\ -sin\bar{\theta} & cos\bar{\theta} \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}, \end{cases} \tag{3.3.17}$$

with the identity

$$\epsilon \equiv C_{\phi WB}v^2 \tag{3.3.18}$$

Additionally, the weak mixing angle has been modified as [7]

$$\tan\bar{\theta} = \frac{\bar{g}_1}{\bar{g}_2} + \frac{\epsilon}{2}\left[1 - \frac{\bar{g}_1^2}{\bar{g}_2^2}\right], \tag{3.3.19}$$

thus, $sin\bar{\theta}$ and $cos\bar{\theta}$ are straightforward calculated, their explicit form are

$$\begin{cases} sin\bar{\theta} &= \frac{\bar{g}_1}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1+\frac{\epsilon}{2}\cdot\frac{\bar{g}_2}{\bar{g}_1}\cdot\frac{\bar{g}_2^2-\bar{g}_1^2}{\bar{g}_2^2+\bar{g}_1^2}\right] \\ cos\bar{\theta} &= \frac{\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1-\frac{\epsilon}{2}\cdot\frac{\bar{g}_1}{\bar{g}_2}\cdot\frac{\bar{g}_2^2-\bar{g}_1^2}{\bar{g}_2^2+\bar{g}_1^2}\right]. \end{cases} \tag{3.3.20}$$

Chronologically, the masses of $W^{\pm}$ boson, Z boson, and photon corrected by dimension-six operators are presented as follow:

$$\begin{cases} M_W &= \frac{\bar{g}_2 v}{2} \\ M_Z &= \frac{1}{2}\sqrt{\bar{g}_1^2+\bar{g}_2^2}\; v\left[1+\frac{1}{4}C_{\phi D}v^2\right]\left(1+\epsilon\frac{\bar{g}_2\bar{g}_1}{\bar{g}_2^2+\bar{g}_1^2}\right) \\ M_A &= 0 \end{cases} \tag{3.3.21}$$

3.3.3 Gauge-Goldstone mixing terms

In this chapter, we will work in R_ξ-gauge, thus we have to consider the Goldstone-boson diagrams. Before we calculate the Goldstone-boson propagators, let us consider the "unwanted" gauge-goldstone mixing terms in Lagrangian (3.3.11):

$$\mathcal{L}_{Goldstone} \supset \left(\bar{D}_\mu\phi\right)^\dagger\left(\bar{D}^\mu\phi\right) + C_{\phi D}\left(\phi^\dagger\bar{D}_\mu\phi\right)^*\left(\phi^\dagger\bar{D}^\mu\phi\right). \tag{3.3.22}$$

To convenience, we introduce some notation:

$$\begin{aligned} \bar{D}_\mu\phi &= \left[\partial_\mu + i\bar{g}_2\tau^I\bar{W}_\mu^I + i\bar{g}_1 Y\bar{B}_\mu\right]\left[\langle\phi\rangle+\Phi\right] = \left[\partial_\mu + iP_\mu\right]\left[\langle\phi\rangle+\Phi\right] \\ &= \partial_\mu\Phi + iP_\mu\Phi + iP_\mu\langle\phi\rangle \end{aligned} \tag{3.3.23}$$

From this notation, we expand (3.3.22) and collect the mixing terms of two fileds like-$P_\mu\partial^\mu\Phi$ and their hermitian conjuagates. The Lagrangian corresponding with these mixing terms is

$$\begin{aligned} \mathcal{L}_{Goldstone} = &-i\frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}_\mu^1\left(\partial^\mu\Phi^+ - \partial^\mu\Phi^-\right) + \frac{\bar{g}_2 v}{2\sqrt{2}}\bar{W}_\mu^2\left(\partial^\mu\Phi^+ + \partial^\mu\Phi^-\right) \\ &-\frac{\bar{g}_2 v}{2}\left[1+\frac{1}{4}C_{\phi D}v^2\right]^2\bar{W}_\mu^3\partial^\mu\Phi^0 + \frac{\bar{g}_1 v}{2}\left[1+\frac{1}{4}C_{\phi D}v^2\right]^2\bar{B}_\mu\partial^\mu\Phi^0 \end{aligned} \tag{3.3.24}$$

Using the relatations in (3.3.17) to transform these gauge fields into physical gauge fields (represent these fiels in mass eigenstates basic), yield the result

$$\mathcal{L}_{Goldstone} = iM_W\left(W_\mu^+\partial^\mu G^- - W_\mu^-\partial^\mu G^+\right) - M_Z Z_\mu\partial^\mu G^0. \tag{3.3.25}$$

We will return to eq (3.3.25) when calculate the propagator of Goldstone-bosons in R_ξ-gauge.

3.3.4 Yukawa interactions

For our process, the matter fields are nothing but electron and top-quark, and hence in this section we just only consider the terms which relevant with our matter fields (the full fermion sector and further calculation can be found in [7]). In order to explain the mass term of fermion fields, the Yukawa interaction up to dimension-six for our process can be written as follows:

$$\mathcal{L}_{Yukawa} = -\left(\bar{l}_L^i \Gamma_e^{ij} e_R^j \phi + \phi^\dagger \bar{e}_R^i \Gamma_e^{\dagger ij} l_L^j\right) + \left(\phi^\dagger \phi\right)\left(\bar{l}_L^i C_{e\phi}^{ij} e_R^j \phi\right) + \left(\phi^\dagger \phi\right)\left(\phi^\dagger \bar{e}_R^i C_{e\phi}^{\dagger ij} l_L^j\right)$$
$$- \left(\bar{q}_L^i \Gamma_u^{ij} u_R^j \phi + \phi^\dagger \bar{u}_R^i \Gamma_u^{\dagger ij} q_L^j\right) + \left(\phi^\dagger \phi\right)\left(\bar{q}_L^i C_{u\phi}^{ij} u_R^j \phi\right) + \left(\phi^\dagger \phi\right)\left(\phi^\dagger \bar{u}_R^i C_{u\phi}^{\dagger ij} q_L^j\right), \quad (3.3.26)$$

where $C_{e\phi}, C_{u\phi}$ are general complex 3×3 matrices, $i, j = 1 \cdots 3$ are generation indices. Substituting the Higgs doublet into (3.3.26) and collecting the mass terms, we shall have

$$\mathcal{L}_{mass} = -\bar{e}_L^i \frac{v}{\sqrt{2}} \left(\Gamma_e - C_{e\phi} \frac{v^2}{2}\right)^{ij} e_R^j - \bar{u}_L^i \frac{v}{\sqrt{2}} \left(\Gamma_u - C_{u\phi} \frac{v^2}{2}\right)^{ij} u_R^j + h.c, \quad (3.3.27)$$

notice that we have not obtained the mass of fermion field yet, because the mass matrices are not diagonalized. To diagonalized, we rotate the fermion fields by the unitary matrices:

$$\begin{cases} \psi_X &= U_{\psi X} \psi_X^{mass} \\ \bar{\psi}_X &= \bar{\psi}_X^{mass} U_\psi^\dagger, \end{cases} \quad (3.3.28)$$

where ψ_X^{mass} stand for mass eigenstates fields, $\psi = e, u$ and $X = L, R$. In mass eigenstates, the mass matrices are digonalized as

$$\begin{cases} U_e^\dagger \frac{v}{\sqrt{2}} \left(\Gamma_e - C_{e\phi} \frac{v^2}{2}\right)^{ij} U_e &= M_e^{ij} = diag\left(m_e, m_\mu, m_\tau\right) \\ U_u^\dagger \frac{v}{\sqrt{2}} \left(\Gamma_u - C_{u\phi} \frac{v^2}{2}\right)^{ij} U_u &= M_u^{ij} = diag\left(m_u, m_c, m_t\right). \end{cases} \quad (3.3.29)$$

We will use equation (3.3.29) when calculate the vertex factor of fermion-fermion-Golstone boson in the next section.

3.4 Effective interactions

In this section we focus on pointing out key point steps to derive the effective Lagrangian for each interaction vertices. We can find the contribution of all dimension-six operators in the appendix A. In order to derive the effective interactions, let us perform several intermediate steps as follows:

- For each interaction vertex, we select all operators which can be contribute to our vertex. In our scattering process, the operators contributing to fermion-gauge interaction appear in column $\psi^2 X \phi$ and $\psi^2 \phi^2 D$ of Table 3.1, while the four-fermion interaction operators can be found in Table 3.2.
- For each operators, we need consider all possible case which contribute to the effective Lagrangian. We can do this step by taking hermitian conjugate, or using Fierz transformation. Then, we can expand these operators with all ingredient in the previous sections. Our process just focus on the interaction of fermion-W_μ^3, B_μ and Φ^0 fields, after expanding the operators which coressponds with these fields, we have to express them in mass eigenstates basic that already presented in section 3.3.
- Remarkable that we are still in the coordinates space, let Fourier transform the effective Lagrangian into momentum space. For instant, we will encounter the terms have form $\sigma^{\mu\nu} A_{\mu\nu}$ when expanding dimension-six operators, let's now Fourier transform this term

$$\begin{aligned}\int_{-\infty}^{\infty}\sigma^{\mu\nu}A_{\mu\nu}(x)e^{iqx}d^4x &= \int_{-\infty}^{\infty}\sigma^{\mu\nu}\left[\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x)\right]e^{iqx}d^4x\\ &= -\int_{-\infty}^{\infty}\sigma^{\mu\nu}\left[A_\nu(x)\partial_\mu e^{iqx}-A_\mu(x)\partial_\nu e^{iqx}\right]d^4x\\ &= 2i\int_{-\infty}^{\infty}\sigma^{\mu\nu}q_\nu A_\mu(x)e^{iqx}d^4x = 2i\sigma^{\mu\nu}q_\nu A_\mu(q), \end{aligned} \tag{3.4.1}$$

notice that we have used the integral by part for the first line and exploited the anti-symmetric of $\sigma^{\mu\nu}$ to obtain the final results. In order to help our calculation easier, let us introduce some usefull relations:

$$c_w+\frac{\epsilon}{2}s_w = \frac{\bar{g}_2}{\sqrt{\bar{g}_2^2+\bar{g}_1^2}}\left[1+\frac{\epsilon}{2}\cdot\frac{\bar{g}_1}{\bar{g}_2}\cdot\frac{2\bar{g}_1^2}{\bar{g}_2^2+\bar{g}_1^2}\right], \tag{3.4.2}$$

$$c_w-\frac{\epsilon}{2}s_w = \frac{\bar{g}_2}{\sqrt{\bar{g}_2^2+\bar{g}_1^2}}\left[1-\frac{\epsilon}{2}\cdot\frac{\bar{g}_1}{\bar{g}_2}\cdot\frac{2\bar{g}_2^2}{\bar{g}_2^2+\bar{g}_1^2}\right], \tag{3.4.3}$$

$$s_w+\frac{\epsilon}{2}c_w = \frac{\bar{g}_1}{\sqrt{\bar{g}_2^2+\bar{g}_1^2}}\left[1+\frac{\epsilon}{2}\cdot\frac{\bar{g}_2}{\bar{g}_1}\cdot\frac{2\bar{g}_2^2}{\bar{g}_2^2+\bar{g}_1^2}\right], \tag{3.4.4}$$

$$s_w-\frac{\epsilon}{2}c_w = \frac{\bar{g}_1}{\sqrt{\bar{g}_2^2+\bar{g}_1^2}}\left[1-\frac{\epsilon}{2}\cdot\frac{\bar{g}_2}{\bar{g}_1}\cdot\frac{2\bar{g}_1^2}{\bar{g}_2^2+\bar{g}_1^2}\right]. \tag{3.4.5}$$

and keep in mind that $\epsilon \equiv C_{\phi WB}v^2$.

- Finally, the we collect the vertex factor in Feynman rules by stripping off the fields operators and multiplying with factor i.

3.4.1 $\gamma ee, Zee$ and G^0ee interactions

There are five operators contributing to lepton-gauge interaction of our process. More specifically, Q_{eW}, Q_{eB} contribute for $\gamma ee, Zee$ vertex, $Q_{\phi l}^{(1)}, Q_{\phi l}^{(3)}, Q_{\phi e}$ contribute for Zee and G^0ee vertex. Notice that we not only expand dimension-six operators but also the SM part which relevant to our interaction vertices. In this section we just present the original results, the contribution of each operators are expressed in appendix A.2.

1. **γee vertex**

After expanding all the contributing operators to the interaction vertex γee, we obtain the effective Lagrangian including SM contributions and the correction from dimension-six operators, hence the general γee interaction is

$$\begin{aligned}\mathcal{L}_{\gamma ee} &= \frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\left[1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2+\bar{g}_2{}^2}v^2C_{\phi WB}\right](\bar{e}\gamma^\mu e)A_\mu\\ &+\bar{e}\left[-i\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C_{eW}^*P_L+C_{eW}P_R\right)+i\frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C_{eB}^*P_L+C_{eB}P_R\right)\right]eA_\mu.\end{aligned} \tag{3.4.6}$$

One can easily verify that the first term in (3.4.6) is nothing but the fermion-photon interaction in SM with the new effective coupling which is re-defined in equation (3.4.7). Additionally, the rest term in (3.4.6) will be vanished automatically when we restric our calculation to SM case.

$$\bar{e}_0 = \frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1^2+\bar{g}_2^2}v^2C_{\phi WB}\right] = e_0\left[1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1^2+\bar{g}_2^2}v^2C_{\phi WB}\right] \tag{3.4.7}$$

2. ***Zee*** **vertex**

We express the Zee vertex in terms of SM and dimension-six effective operators as

$$\begin{aligned}
\mathcal{L}_{Zee} = &\bar{e}\left[-\frac{1}{2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L+2\bar{g}_1{}^2P_R\right]\right]eZ_\mu \\
&+\bar{e}\left[\frac{\bar{g}_1\bar{g}_2}{2(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}v^2C_{\phi WB}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L-2\bar{g}_2{}^2P_R\right]\right]eZ_\mu \\
&-i\bar{e}\left[\frac{\sqrt{2}\bar{g}_2v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eW}P_L+C_{eW}P_R\right)+\frac{\sqrt{2}\bar{g}_1v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eB}P_L+C_{eB}P_R\right)\right]eZ_\mu \\
&+\bar{e}\left[\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}\gamma^\mu\left[\left(C^1_{\phi l}+C^3_{\phi l}\right)P_L+C_{\phi e}P_R\right]\right]eZ_\mu. \qquad (3.4.8)
\end{aligned}$$

3. ***G*^{0}*ee*** **vertex**

The effective interaction of Goldstone boson and electron is expressed as follows:

$$\mathcal{L}_{G^0ee} = \bar{e}\left(-\frac{i}{v}m_e\left[1-\frac{1}{4}vC_{\phi D}\right]\gamma^5+iv\not{q}\left[\left(C^1_{\phi l}+C^3_{\phi l}\right)P_L+C_{\phi e}P_R\right]\right)eG^0, \qquad (3.4.9)$$

notice that the first term of (3.4.9) obtained by expanding the Yukawa interaction which is

$$\mathcal{L}_{Yukawa} = -\left[\bar{e}^i_L\frac{i}{\sqrt{2}}\left(\Gamma_e-C_{e\phi}\frac{v^2}{2}\right)^{ij}e^j_R\Phi^0-\bar{e}^i_R\frac{i}{\sqrt{2}}\left(\Gamma_e-C_{e\phi}\frac{v^2}{2}\right)^{ij}e^j_L\Phi^0\right]. \qquad (3.4.10)$$

We present these matter fields in mass eigenstates basic, by using eqs (3.3.28,3.3.29) we have:

$$\mathcal{L}_{Yukawa} = -\left[\bar{e}_L\frac{im_e}{v}e_R\Phi^0-\bar{e}_R\frac{im_e}{v}e_L\Phi^0\right] = -\frac{i}{v}m_e\left[1-\frac{1}{4}C_{\phi D}v^2\right]\left(\bar{e}\gamma^5e\right)G^0, \qquad (3.4.11)$$

notice that Φ^0 has been rescaled as (3.3.8).

3.4.2 $\gamma tt, Ztt$ and G^0tt interactions

Similarity with the lepton-gauge field interaction cases, we also obtain the effective Lagrangian for quark-gauge field interaction including SM and dimension-six operators. There are also five operators contributing to quark-gauge interaction of our process. More specifically, Q_{uW}, Q_{uB} contribute for $\gamma tt, Ztt$ vertex, $Q^{(1)}_{\phi q}, Q^{(3)}_{\phi q}, Q_{\phi u}$ contribute for Ztt and G^0tt vertex. Further contribution of each operators can be found in appendix A.3.

1. ***γtt*** **vertex**

First of all, we consider the interaction of top-photon after being generalized as below:

$$\begin{aligned}
\mathcal{L}_{\gamma tt} = &-\frac{2}{3}\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\left[1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2+\bar{g}_2{}^2}v^2C_{\phi WB}\right]\left(\bar{t}\gamma^\mu t\right)A_\mu \\
&+\bar{t}\left[i\frac{\sqrt{2}\bar{g}_1v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L+C_{uW}P_R\right)+i\frac{\sqrt{2}\bar{g}_2v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L+C_{uB}P_R\right)\right]tA_\mu
\end{aligned}$$
(3.4.12)

2. ***Ztt*** **vertex**

The top-quar electroweak interaction, Ztt vertex, including the SM contributions as well as the correction from dimension-six operators is presented as

$$\begin{aligned}
\mathcal{L}_{Ztt} &= \bar{t}\left[\frac{1}{6\sqrt{\bar{g_1}^2+\bar{g_2}^2}}\gamma^\mu\left[\left(\bar{g_1}^2-3\bar{g_2}^2\right)P_L+4\bar{g_1}^2P_R\right]\right]tZ_\mu \\
&-\bar{t}\left[\frac{\bar{g_1}\bar{g_2}}{6(\bar{g_1}^2+\bar{g_2}^2)^{3/2}}v^2C_{\phi WB}\gamma^\mu\left[\left(3\bar{g_1}^2-\bar{g_2}^2\right)P_L-4\bar{g_2}^2P_R\right]\right]tZ_\mu \\
&+\bar{t}\left[i\frac{\sqrt{2}\bar{g_2}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L+C_{uW}P_R\right)-i\frac{\sqrt{2}\bar{g_1}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L+C_{uB}P_R\right)\right]tZ_\mu \\
&+\bar{t}\left[\frac{1}{2}v^2\sqrt{\bar{g_1}^2+\bar{g_2}^2}\gamma^\mu\left[\left(C^1_{\phi q}-C^3_{\phi q}\right)P_L+C_{\phi u}P_R\right]\right]tZ_\mu. \qquad (3.4.13)
\end{aligned}$$

3. **G^0tt vertex**

The last effective coupling in case of quark-gauge interaction for our process is

$$\mathcal{L}_{G^0tt}=\bar{t}\left(-\frac{i}{v}m_t\left[-1+\frac{1}{4}vC_{\phi D}\right]\gamma^5+iv\not{q}\left[\left(C^1_{\phi q}-C^3_{\phi l}\right)P_L+C_{\phi u}P_R\right]\right)tG^0. \qquad (3.4.14)$$

3.4.3 Four-fermion interactions

First of all, we looking for the operators contribtuting for four-fermion (eett) interaction. There are 7 operators: $Q^{(1)}_{lq}, Q^{(3)}_{lq}, Q_{eu}, Q_{lu}, Q_{qe}, Q^{(1)}_{lequ}$ and $Q^{(3)}_{lequ}$ (their explicit form are expressed in the Table 3.2). Before expanding these operators, for each operators, we have to determine all possible contributions for the effective Lagrangian. This is a crucial step to derive the effective Lagrangian, let us discuss more specifically:

- In the first five operators $Q^{(1)}_{lq}, Q^{(3)}_{lq}, Q_{eu}, Q_{lu}$ and Q_{qe}, Hermitian conjugation is equavalent to transposition of generation indices in each fermionic currents. For our process, the generation indices of lepton are $i=1$ and quark are $i=3$, thus the hermitian conjugate case in the first five operators do not yield any new contributions. Besides, Fierz transformation can not apply to these operators, hene there is only one possible case for each operators.

- For remaining operators, $Q^{(1)}_{lequ}$ and $Q^{(3)}_{lequ}$, we can not obtain the hermition conjugation by running the generation indices. Furthermore, new contribution can be proceeded via taking hermitian conjuagate of this operators. Let us expand the operator $Q^{(1)}_{lequ}$:

$$C^1_{lequ}Q^{(1)}_{lequ}+C^{1*}_{lequ}Q^{(1)\dagger}_{lequ}=C^1_{lequ}\left(\bar{l}^j_pe_r\right)\epsilon_{jk}\left(\bar{q}^k_su_t\right)+C^{1*}_{lequ}\left(\bar{u}_tq^k_s\right)\epsilon_{jk}\left(\bar{e}_rl^j_p\right), \qquad (3.4.15)$$

  for the scattering process $e^++e^-\to t+\bar{t}$, the generation indices are $p=r=1$, $s=t=3$. Additionally, j,k are component indices of left-handed doublet, in case of our process, $j=2$ and $k=1$. Expanding (3.4.15) we shall have:

$$\begin{aligned}
C^1_{lequ}Q^{(1)}_{lequ}+C^{1*}_{lequ}Q^{(1)\dagger}_{lequ} &= C^1_{lequ}\left(\bar{e}^2_Le_R\right)\epsilon_{21}\left(\bar{u}^1_Lu_R\right)+C^{1*}_{lequ}\left(\bar{u}_Ru^1_L\right)\epsilon_{21}\left(\bar{e}_Re^2_L\right) \\
&= -C^1_{lequ}\left(\bar{e}P_Re\right)\left(\bar{t}P_Rt\right)-C^{1*}_{lequ}\left(\bar{t}P_Lt\right)\left(\bar{e}P_Le\right). \qquad (3.4.16)
\end{aligned}$$

  Analogous expansion for $Q^{(3)}_{lequ}$ operator. More information can be found in A.4.

Now we can write down immediately the four-fermion Lagrangian interaction as follows:

$$\begin{aligned}\mathcal{L}_{eett} &= \left[C_{lq}^1 - C_{lq}^3\right] (\bar{e}\gamma^\mu P_L e)_{ss'} (\bar{t}\gamma_\mu P_L t)_{rr'} + C_{eu} (\bar{e}\gamma^\mu P_R e)_{ss'} (\bar{t}\gamma_\mu P_R t)_{rr'} \\ &+ C_{lu} (\bar{e}\gamma^\mu P_L e)_{ss'} (\bar{t}\gamma_\mu P_R t)_{rr'} + C_{qe} (\bar{t}\gamma^\mu P_L t)_{rr'} (\bar{e}\gamma_\mu P_R e)_{ss'} \\ &- \left[C_{lequ}^{1*} (\bar{e}P_L e)_{ss'} (\bar{t}P_L t)_{rr'} + C_{lequ}^1 (\bar{e}P_R e)_{ss'} (\bar{t}P_R t)_{rr'}\right] \\ &- \left[C_{lequ}^{3*} (\bar{e}\sigma^{\mu\nu} P_L e)_{ss'} (\bar{t}\sigma_{\mu\nu} P_L t)_{rr'} + C_{lequ}^3 (\bar{e}\sigma^{\mu\nu} P_R e)_{ss'} (\bar{t}\sigma_{\mu\nu} P_R t)_{rr'}\right], \end{aligned} \tag{3.4.17}$$

notice that we denote s, s', r, r' are sequentially electron, positron, top-quark and anti top-quark spinor indices.

3.5 Propagators in SMEFT

In order to derive the propagator, the gauge fixing and Faddeev-Popov (ghost) Lagrangian have been added into classical Lagrangian. Since our calculation just at leading order (tree-level), we do not consider the ghost terms. In SMEFT, the choice of gauge-fixing terms should be satisfied the following requirements [7]:

- Eliminate the unpleasant Goldstone-gauge mixing terms as well as in SM.
- Althought the full Lagrangian including gauge-fixing and ghost terms is not gauge invariant, instead, preserves the BRST invariant.
- Lead to SM-like propagators in terms of the parameters and fields in mass eigenstates basic.

The gauge-fixing Lagrangian which relevant to our process is

$$\mathcal{L}_{GF} = -\frac{1}{2\xi_A} (\partial^\mu A_\mu)^2 - \frac{1}{2\xi_Z} \left(\partial^\mu Z_\mu + \xi M_Z G^0\right)^2, \tag{3.5.1}$$

where all Willsion coefficients are absorbed in masses and physical fields, and hence the gauge-fixing terms have a form look like in SM. Notice that the general propagator of photon and Z-boson are the same as before (1.1.2), in this section we just perform a process of calculating the Goldstone-boson propagator. From (3.3.6), (3.3.25) and the gauge-fixing terms in (3.5.1), we shall have:

$$\mathcal{L}_{G^0} = \frac{1}{2} \left[\partial_\mu G^0\right]^2 - m_Z Z^\mu \partial_\mu G^0 - m_Z (\partial_\mu Z^\mu) G^0 - \frac{1}{2}\xi_Z m_Z^2 G^{02} \tag{3.5.2}$$

Applying the Euler-Lagrange equation we obtain

$$\left(\Box + \xi_Z m_Z^2\right) G^0 = 0, \tag{3.5.3}$$

where the propagator $D(x-y)$ is a solution of the inhomogeneous field equation [3]

$$\left(\Box + \xi_Z m_Z^2\right) D(x-y) = -\delta^4(x-y). \tag{3.5.4}$$

A solution of propagator can be obtained by using Fourier transformation which yields a result

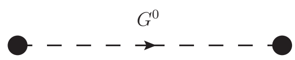
$$D(q) = \frac{1}{q^2 - \xi_Z m_Z^2}. \tag{3.5.5}$$

3.6 Feynman rules for the process in SMEFT

3.6.1 Propagators in the R_ξ-gauges

$$= -\frac{i}{q^2}\left[g_{\mu\nu} - (1-\xi_A)\frac{q_\mu q_\nu}{q^2}\right] \tag{3.6.1}$$

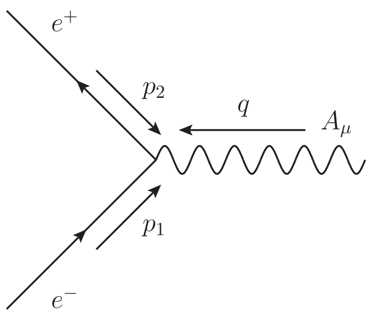
$$= -\frac{i}{q^2 - m_Z^2}\left[g_{\mu\nu} - (1-\xi_Z)\frac{q_\mu q_\nu}{q^2 - \xi m_Z^2}\right] \tag{3.6.2}$$



$$= \frac{i}{q^2 - \xi_Z m_Z^2} \tag{3.6.3}$$

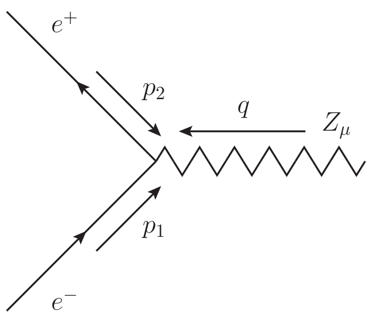
3.6.2 Vertex factors

- Lepton-gauge and -Goldstone boson vertices:



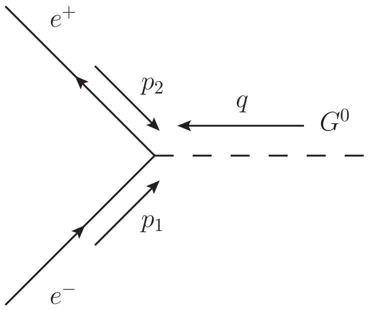
$$= i\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}\left[1 - \frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2 + \bar{g}_2{}^2}v^2 C_{\phi WB}\right]\gamma^\mu$$

$$+ \frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}} q_\rho \sigma^{\mu\rho}\left(C_{eW}^* P_L + C_{eW} P_R\right)$$
$$- \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}} q_\rho \sigma^{\mu\rho}\left(C_{eB}^* P_L + C_{eB} P_R\right) \tag{3.6.4}$$



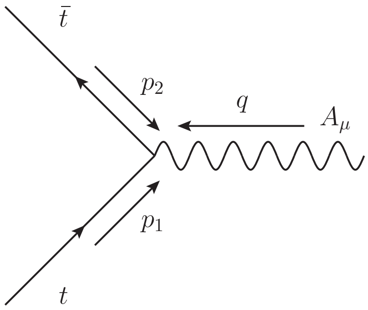
$$= -\frac{i}{2\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2 - \bar{g}_2{}^2\right)P_L + 2\bar{g}_1{}^2 P_R\right]$$

$$+ i\frac{\bar{g}_1\bar{g}_2}{2(\bar{g}_1{}^2 + \bar{g}_2{}^2)^{3/2}}v^2 C_{\phi WB}\gamma^\mu\left[\left(\bar{g}_1{}^2 - \bar{g}_2{}^2\right)P_L - 2\bar{g}_2{}^2 P_R\right]$$
$$+ \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}} q_\rho \sigma^{\mu\rho}\left(C_{eW}^* P_L + C_{eW} P_R\right)$$
$$+ \frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}} q_\rho \sigma^{\mu\rho}\left(C_{eB}^* P_L + C_{eB} P_R\right)$$
$$+ i\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}\gamma^\mu\left[\left(C_{\phi l}^1 + C_{\phi l}^3\right)P_L + C_{\phi e}P_R\right] \tag{3.6.5}$$

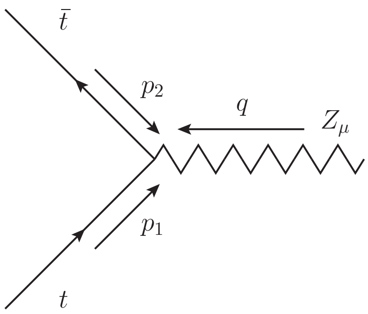


$$= m_e\left[\frac{1}{v} - \frac{1}{4}vC_{\phi D}\right]\gamma^5 - v\not{q}\left[\left(C^1_{\phi l} + C^3_{\phi l}\right)P_L + C_{\phi e}P_R\right] \tag{3.6.6}$$

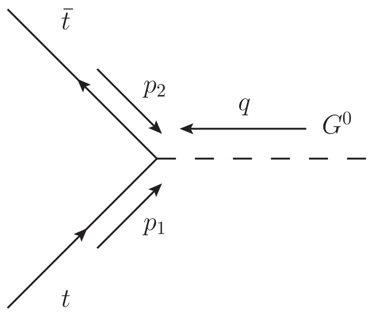
- Quark-gauge and -Goldstone boson vertices:



$$\begin{aligned}
&= -i\frac{2}{3}\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}\left[1 - \frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2 + \bar{g}_2{}^2}v^2 C_{\phi WB}\right]\gamma^\mu \\
&- \frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L + C_{uW}P_R\right) \\
&- \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L + C_{uB}P_R\right)
\end{aligned} \tag{3.6.7}$$

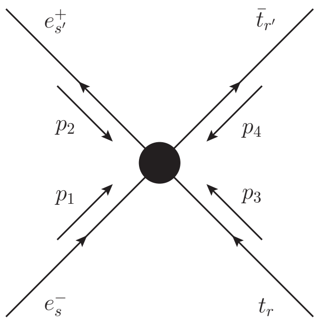


$$\begin{aligned}
&= \frac{i}{6\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2 - 3\bar{g}_2{}^2\right)P_L + 4\bar{g}_1{}^2 P_R\right] \\
&- i\frac{\bar{g}_1\bar{g}_2}{6(\bar{g}_1{}^2 + \bar{g}_2{}^2)^{3/2}}v^2 C_{\phi WB}\gamma^\mu\left[\left(3\bar{g}_1{}^2 - \bar{g}_2{}^2\right)P_L - 4\bar{g}_2{}^2 P_R\right] \\
&- \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L + C_{uW}P_R\right) \\
&+ \frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L + C_{uB}P_R\right) \\
&+ i\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2 + \bar{g}_2{}^2}\gamma^\mu\left[\left(C^1_{\phi q} - C^3_{\phi q}\right)P_L + C_{\phi u}P_R\right]
\end{aligned} \tag{3.6.8}$$



$$= m_t \left[-\frac{1}{v} + \frac{1}{4} v C_{\phi D} \right] \gamma^5 - v \not{q} \left[\left(C^1_{\phi q} - C^3_{\phi l} \right) P_L + C_{\phi u} P_R \right] \tag{3.6.9}$$

- Four-fermion vertex:



$$\begin{aligned}
&= i \left[C^1_{lq} - C^3_{lq} \right] \left(\gamma^\mu P_L \right)_{ss'} \left(\gamma_\mu P_L \right)_{rr'} \\
&+ i C_{eu} \left(\gamma^\mu P_R \right)_{ss'} \left(\gamma_\mu P_R \right)_{rr'} \\
&+ i C_{lu} \left(\gamma^\mu P_L \right)_{ss'} \left(\gamma_\mu P_R \right)_{rr'} \\
&+ i C_{qe} \left(\gamma^\mu P_L \right)_{rr'} \left(\gamma_\mu P_R \right)_{ss'} \\
&- i \left[C^{1*}_{lequ} \left(P_L \right)_{ss'} \left(P_L \right)_{rr'} + C^1_{lequ} \left(P_R \right)_{ss'} \left(P_R \right)_{rr'} \right] \\
&- i \left[C^{3*}_{lequ} \left(\sigma^{\mu\nu} P_L \right)_{ss'} \left(\sigma_{\mu\nu} P_L \right)_{rr'} + C^3_{lequ} \left(\sigma^{\mu\nu} P_R \right)_{ss'} \left(\sigma_{\mu\nu} P_R \right)_{rr'} \right]
\end{aligned} \tag{3.6.10}$$

3.7 Cancellation of gauge-fixing parameter in R_ξ-gauge

As we have done in the two chapter before, in this section we also demonstrate that the Feynman amplitude is $\xi - independence$ again. In R_ξ-gauge, there are 4 diagrams which relevant to our process which is pointed out in figure 3.1

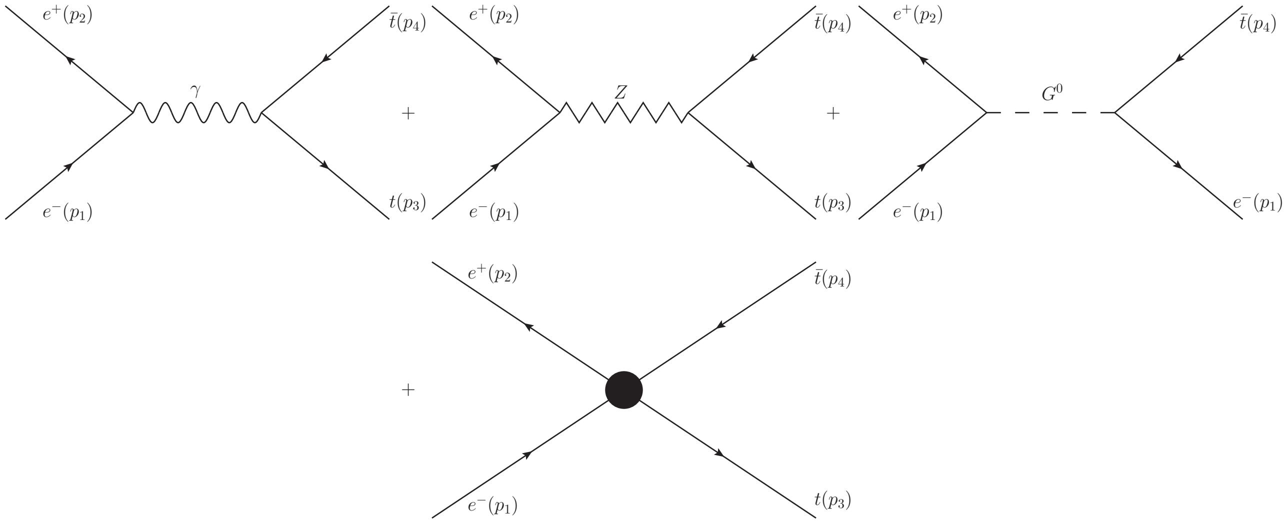


Figure 3.1: The Feynman diagrams of process $e^- + e^+ \longrightarrow t + \bar{t}$ in SMEFT.

Frirstly, from the Feynman rules listed in the previous section before, we can write down the Feynman amplitude for photon diagram in figure (3.1) immediately:

$$\begin{aligned}\mathcal{M}_\gamma = \bar{v}^{s'}(p_2)\Bigg[i\frac{\bar{g_1}\bar{g_2}}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}\left(1-\frac{\bar{g_1}\bar{g_2}}{\bar{g_1}^2+\bar{g_2}^2}v^2C_{\phi WB}\right)\gamma^\mu - \frac{\sqrt{2}\bar{g_1}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eW}P_L + C_{eW}P_R\right) \\ + \frac{\sqrt{2}\bar{g_2}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eB}P_L + C_{eB}P_R\right)\Bigg] u^s(p_1) \times \frac{-i}{q^2}\left[g_{\mu\nu} - (1-\xi_A)\frac{q_\mu q_\nu}{q^2}\right] \\ \times \bar{u}^r(p_3)\Bigg[-i\frac{2}{3}\frac{\bar{g_1}\bar{g_2}}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}\left(1-\frac{\bar{g_1}\bar{g_2}}{\bar{g_1}^2+\bar{g_2}^2}v^2C_{\phi WB}\right)\gamma^\nu - \frac{\sqrt{2}\bar{g_1}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\nu\rho}\left(C^*_{uW}P_L + C_{uW}P_R\right) \\ - \frac{\sqrt{2}\bar{g_2}v}{\sqrt{\bar{g_1}^2+\bar{g_2}^2}}q_\rho\sigma^{\nu\rho}\left(C^*_{uB}P_L + C_{uB}P_R\right)\Bigg] v^{r'}(p_4), \end{aligned} \tag{3.7.1}$$

notice that we need to consider the sign of gauge boson momentum in each vertex carefully, since in Feynman rules all external momenta are considered to be incoming. Similarity with QED, we now looking for the part which multiply with the factor $i(1-\xi_A)q_\mu q_\nu/q^4$, (3.7.3). As we can see, the terms which proportional with γ^μ can be vanished by using the identity 1.3.2,

$$q_\mu\left[\bar{v}^{s'}(p_2)\gamma^\mu u^s(p_1)\right] = \bar{v}^{s'}(p_2)\left[\not{p}_2 + \not{p}_1\right]u^s(p_1) = \bar{v}^{s'}(p_2)\left[-m_e + m_e\right]u^s(p_1) = 0,$$

and so do for analogous terms. Next, the remaning terms of (3.7.3) will multiply with

$$\sigma^{\mu\rho}q_\mu q_\rho = \frac{1}{2}\sigma^{\mu\rho}q_\mu q_\rho + \sigma^{\rho\mu}q_\rho q_\mu = 0, \tag{3.7.2}$$

since $\sigma^{\mu\rho}$ is anti-symmetry under the permutation of Lorentz indices while $q_\mu q_\rho$ is symmetry.

$$\begin{aligned}
\mathcal{M}^{\gamma}_{\xi} &= i\frac{1-\xi}{q^4}\bar{v}^{s'}(p_2)q_\mu \left[i\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\left(1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2+\bar{g}_2{}^2}v^2C_{\phi WB}\right)\gamma^\mu \right.\\
&\quad \left. -\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eW}P_L+C_{eW}P_R\right)+\frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eB}P_L+C_{eB}P_R\right)\right]u^s(p_1)\\
&\quad \times \bar{u}^r(p_3)q_\nu\left[-i\frac{2}{3}\frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\left(1-\frac{\bar{g}_1\bar{g}_2}{\bar{g}_1{}^2+\bar{g}_2{}^2}v^2C_{\phi WB}\right)\gamma^\mu-\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L+C_{uW}P_R\right)\right.\\
&\qquad\qquad \left.-\frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L+C_{uB}P_R\right)\right]v^{r'}(p_4)\\
&= 0
\end{aligned} \tag{3.7.3}$$

Again, the Feynman amplitude in photon diagram is $\xi - \textit{independent}$ as we expect. We now prove the statement (mentioned in section 3.5) that the "unwanted" Goldstone-boson can be eliminated by choosing a particular gauge, $R_\xi - \textit{gauge}$. Let us begin with the Feynman amplitude of Z-boson:

$$\begin{aligned}
\mathcal{M}_Z &= \bar{v}^{s'}(p_2)\left[-\frac{i}{2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L+2\bar{g}_1{}^2P_R\right]\right.\\
&\quad + i\frac{\bar{g}_1\bar{g}_2}{2(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}v^2C_{\phi WB}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L-2\bar{g}_2{}^2P_R\right]\\
&\quad - \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eW}P_L+C_{eW}P_R\right)-\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{eB}P_L+C_{eB}P_R\right)\\
&\quad \left.+i\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}\gamma^\mu\left[\left(C^1_{\phi l}+C^3_{\phi l}\right)P_L+C_{\phi e}P_R\right]\right]u^s(p_1)\times\frac{-i}{q^2-m_Z^2}\left[g_{\mu\nu}-(1-\xi_Z)\frac{q_\mu q_\nu}{q^2-\xi_Z m_Z^2}\right]\\
&\quad \times\bar{u}^r(p_3)\left[\frac{i}{6\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2-3\bar{g}_2{}^2\right)P_L+4\bar{g}_1{}^2P_R\right]\right.\\
&\quad - i\frac{\bar{g}_1\bar{g}_2}{6(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}v^2C_{\phi WB}\gamma^\mu\left[\left(3\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L-4\bar{g}_2{}^2P_R\right]\\
&\quad - \frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uW}P_L+C_{uW}P_R\right)+\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C^*_{uB}P_L+C_{uB}P_R\right)\\
&\quad \left.+i\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}\gamma^\mu\left[\left(C^1_{\phi q}-C^3_{\phi q}\right)P_L+C_{\phi u}P_R\right]\right]v^{r'}(p_4)
\end{aligned} \tag{3.7.4}$$

First step, we simplify the Z-boson propagator as follows:

$$\begin{aligned}
\frac{-i}{q^2-m_Z^2}\left[g_{\mu\nu}-(1-\xi_Z)\frac{q_\mu q_\nu}{q^2-\xi_Z m_Z^2}\right] &= \frac{-i}{q^2-m_Z^2}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_Z^2}+q_\mu q_\nu\left[\frac{1}{m_Z^2}-\frac{1-\xi}{q^2-\xi_Z m_Z^2}\right]\right)\\
&= \frac{-i}{q^2-m_Z^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{m_Z^2}\right]-i\frac{q_\mu q_\nu}{m_Z^2(q^2-\xi_Z m_Z^2)}
\end{aligned} \tag{3.7.5}$$

The first term of (3.7.5) is independent with gauge-fixing parameter, we shall demonstrate that the combination of the rest terms in (3.7.5) and the Goldstone-boson diagram will be vanished. In order to make our calculation more clearly, let us simplify the parts which multiply with the rest terms in (3.7.5) by an indentity:

$$\begin{aligned}
\mathcal{M}_\xi^{Z-e} &= q_\mu \bar{v}^{s'}(p_2)\Bigg[-\frac{i}{2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L+2\bar{g}_1{}^2P_R\right] \\
&+i\frac{\bar{g}_1\bar{g}_2}{2(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}v^2C_{\phi WB}\gamma^\mu\left[\left(\bar{g}_1{}^2-\bar{g}_2{}^2\right)P_L-2\bar{g}_2{}^2P_R\right] \\
&-\frac{\sqrt{2}\bar{g}_2 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C_{eW}^*P_L+C_{eW}P_R\right)-\frac{\sqrt{2}\bar{g}_1 v}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}q_\rho\sigma^{\mu\rho}\left(C_{eB}^*P_L+C_{eB}P_R\right) \\
&+i\frac{1}{2}v^2\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}\gamma^\mu\left[\left(C_{\phi l}^1+C_{\phi l}^3\right)P_L+C_{\phi e}P_R\right]\Bigg]u^s(p_1) \\
&= iM_Z m_e\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi l}^1+C_{\phi l}^3-C_{\phi e}\right)v\right]\left[\bar{v}^{s'}(p_2)\gamma^5u^s(p_1)\right], \qquad (3.7.6)
\end{aligned}$$

and similar calculation for top vertex

$$\mathcal{M}_\xi^{Z-t} = iM_Z m_t\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi u}-C_{\phi q}^1+C_{\phi q}^3\right)v\right]\left[\bar{u}^r(p_3)\gamma^5v^{r'}(p_4)\right]. \qquad (3.7.7)$$

Notice that we have used the same tricks as (1.3.2,2.7.8) to obtain these results. Remember that M_Z here is the effective mass of Z-boson in SMEFT, when calculation we keep only the linear terms of dimension-six operators, any higher order $\mathcal{O}(C_6^2)$ are neglected. Finally, we obtain a result

$$\begin{aligned}
\mathcal{M}_\xi^Z &= \frac{i}{q^2-\xi_Z m_Z^2}\bar{v}^{s'}(p_2)m_e\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi l}^1+C_{\phi l}^3-C_{\phi e}\right)v\right]\gamma^5u^s(p_1) \\
&\times\bar{u}^r(p_3)m_t\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi u}-C_{\phi q}^1+C_{\phi q}^3\right)v\right]\gamma^5v^{r'}(p_4). \qquad (3.7.8)
\end{aligned}$$

For the last step, we write down the Feynman amplitude of Goldstone-boson diagram:

$$\begin{aligned}
\mathcal{M}_{G^0} &= \bar{v}^{s'}(p_2)m_e\left[\frac{m_e}{v}\gamma^5-\frac{1}{4}m_e vC_{\phi D}\gamma^5+v\not{q}P_L(C_{\phi l}^1+C_{\phi l}^3)+v\not{q}P_RC_{\phi e}\right]u^s(p_1)\times\frac{i}{q^2-\xi_Z M_Z^2} \\
&\times\bar{u}^r(p_3)\left[-\frac{m_t}{v}\gamma^5+\frac{1}{4}m_t vC_{\phi D}\gamma^5-v\not{q}P_L(C_{\phi q}^1-C_{\phi q}^3)-v\not{q}P_RC_{\phi u}\right]v^{r'}(p_4) \qquad (3.7.9)
\end{aligned}$$

By using (1.3.2,2.7.8) and analogous indentities for another fermion lines, we shall have:

$$\begin{aligned}
\mathcal{M}_{G^0} &= \frac{-i}{q^2-\xi_Z M_Z^2}\bar{v}^{s'}(p_2)m_e\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi l}^1+C_{\phi l}^3-C_{\phi e}\right)v\right]\gamma^5u^s(p_1) \\
&\times\bar{u}^r(p_3)m_t\left[\frac{1}{v}\left(1-\frac{1}{4}C_{\phi D}v^2\right)+\left(C_{\phi u}-C_{\phi q}^1+C_{\phi q}^3\right)v\right]\gamma^5v^{r'}(p_4) \qquad (3.7.10)
\end{aligned}$$

Now we can easily verify that (3.7.8) perfectly cancels the Goldstone-boson diagram (3.7.10). The remaining terms are independent with gauge-fixing parameters ξ.

3.8 Feynman squared amplitude with FORM

In this section we will manipulate FORM program to calculate the Feynman square amplitude. Notice that the mass of electrons are tiny compare with the threshold energy of our process, that means we can set $m_e \simeq 0$, as a consequence the Goldstone-boson diagram will be eleminated, i.e $\mathcal{M}_{G^0} = 0$. Before we use FORM to evaluate the squared amplitude, let simplify the Feynman amplitude in each diagrams:

Photon diagram:

$$\begin{aligned}\mathcal{M}_\gamma = {} & \bar{v}^{s'}(p_2)\left[i\bar{e}_0\gamma^\mu - G_1 q_\rho \sigma^{\mu\rho}\left(C^*_{eW}P_L + C_{eW}P_R\right) + G_2 q_\rho \sigma^{\mu\rho}\left(C^*_{eB}P_L + C_{eB}P_R\right)\right] u^s(p_1) \\ & \times \frac{-i}{q^2}\left[g_{\mu\nu} - (1-\xi_A)\frac{q_\mu q_\nu}{q^2}\right] \\ & \times \bar{u}^r(p_3)\left[-i\frac{2}{3}\bar{e}_0\gamma^\nu - G_1 q_\rho\sigma^{\nu\rho}\left(C^*_{uW}P_L + C_{uW}P_R\right) - G_2 q_\rho \sigma^{\nu\rho}\left(C^*_{uB}P_L + C_{uB}P_R\right)\right] v^{r'}(p_4)\delta_{\alpha\beta}\end{aligned} \tag{3.8.1}$$

Whereas the compact parameters are

$$\begin{aligned}\bar{e}_0 &= \frac{\bar{g}_1\bar{g}_2}{\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\left[1 - \frac{\bar{g}_1\bar{g}_2}{\bar{g}_1^2+\bar{g}_2^2}v^2 C_{\phi WB}\right] = e_0\left[1 - \frac{\bar{g}_1\bar{g}_2}{\bar{g}_1^2+\bar{g}_2^2}v^2 C_{\phi WB}\right], \\ G_1 &= \frac{v\sqrt{2}g_1}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}, \qquad G_2 = \frac{v\sqrt{2}g_2}{\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}.\end{aligned} \tag{3.8.2}$$

Z-boson diagram:

$$\begin{aligned}\mathcal{M}_Z = {} & \bar{v}^{s'}(p_2)\left[-i\gamma^\mu\left(g^e_V + g^e_A\gamma^5\right) + iv^2 C_{\phi WB}\gamma^\mu\left(z^e_V - z^e_A\gamma^5\right)\right. \\ & \left. - G_2\sigma^{\mu\rho}q_\rho\left[C^*_{eW}P_L + C_{eW}P_R\right] - G_1\sigma^{\mu\rho}q_\rho\left[C^*_{eB}P_L + C_{eB}P_R\right] + iG_3\gamma^\mu\left(C_1^{\phi le} + C_2^{\phi le}\gamma^5\right)\right] u^s(p_1) \\ & \times \frac{-i}{q^2 - m_Z^2}\left[g_{\mu\nu} - (1-\xi_Z)\frac{q_\mu q_\nu}{q^2 - \xi_Z m_Z^2}\right] \times \delta_{\alpha\beta} \\ & \times \bar{u}^r(p_3)\left[i\gamma^\nu\left(g^t_V + g^t_A\gamma^5\right) - iv^2 C_{\phi WB}\gamma^\nu\left(z^t_V - z^t_A\gamma^5\right)\right. \\ & \left. - G_2\sigma^{\nu\theta}q_\theta\left[C^*_{uW}P_L + C_{uW}P_R\right] + G_1\sigma^{\nu\theta}q_\theta\left[C^*_{uB}P_L + C_{uB}P_R\right] + iG_3\gamma^\nu\left(C_1^{\phi qu} + C_2^{\phi qu}\gamma^5\right)\right] v^{r'}(p_4)\end{aligned} \tag{3.8.3}$$

Here we introduce several compact parameters are listed below:

$$g^e_V = \frac{3\bar{g}_1{}^2 - \bar{g}_2{}^2}{4\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}, \quad g^e_A = \frac{\bar{g}_1{}^2 + \bar{g}_2{}^2}{4\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}}; \tag{3.8.4}$$

$$z^e_V = \frac{\bar{g}_1\bar{g}_2}{4(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}(\bar{g}_1{}^2 - 3\bar{g}_2{}^2), \quad z^e_A = \frac{\bar{g}_1\bar{g}_2}{4(\bar{g}_1{}^2+\bar{g}_2{}^2)^{3/2}}(\bar{g}_1{}^2 + \bar{g}_2{}^2); \tag{3.8.5}$$

$$G_3 = \frac{v^2}{2}\sqrt{\bar{g}_1{}^2+\bar{g}_2{}^2}; \quad C_1^{\phi le} = C^1_{\phi l} + C^3_{\phi l} + C_{\phi e}, \quad \text{and} \quad C_2^{\phi le} = C_{\phi e} - C^1_{\phi l} - C^3_{\phi l}. \tag{3.8.6}$$

Four-fermion diagram:
In this diagram we need to carefull about the spinor indices for each fermionic currents.

$$\begin{aligned}\mathcal{M}_{4f} &= i\left[C^1_{lq} - iC^3_{lq}\right]\left[\bar{v}^{s'}(p_2)\gamma^\mu P_L u^s(p_1)\right]\left[\bar{u}^r(p_3)\gamma_\mu P_L v^{r'}(p_4)\right]\delta_{\alpha\beta} \\ &+ iC_{eu}\left[\bar{v}^{s'}(p_2)\gamma^\mu P_R u^s(p_1)\right]\left[\bar{u}^r(p_3)\gamma_\mu P_R v^{r'}(p_4)\right]\delta_{\alpha\beta} \\ &+ iC_{lu}\left[\bar{v}^{s'}(p_2)\gamma^\mu P_L u^s(p_1)\right]\left[\bar{u}^r(p_3)\gamma_\mu P_R v^{r'}(p_4)\right]\delta_{\alpha\beta} \\ &+ iC_{qe}\left[\bar{v}^{s'}(p_2)\gamma^\mu P_R u^s(p_1)\right]\left[\bar{u}^r(p_3)\gamma_\mu P_L v^{r'}(p_4)\right]\delta_{\alpha\beta} \\ &- i\left(C^{1*}_{lequ}\left[\bar{v}^{s'}(p_2)P_L u^s(p_1)\right]\left[\bar{u}^r(p_3)P_L v^{r'}(p_4)\right] + C^1_{lequ}\left[\bar{v}^{s'}(p_2)P_R u^s(p_1)\right]\left[\bar{u}^r(p_3)P_R v^{r'}(p_4)\right]\right)\delta_{\alpha\beta} \\ &- iC^{3*}_{lequ}\left[\bar{v}^{s'}(p_2)\sigma^{\mu\nu}P_L u^s(p_1)\right]\left[\bar{u}^r(p_3)\sigma_{\mu\nu}P_L v^{r'}(p_4)\right]\delta_{\alpha\beta} \\ &- iC^3_{lequ}\left[\bar{v}^{s'}(p_2)\sigma^{\mu\nu}P_R u^s(p_1)\right]\left[\bar{u}^r(p_3)\sigma_{\mu\nu}P_R v^{r'}(p_4)\right]\delta_{\alpha\beta} \qquad (3.8.7)\end{aligned}$$

To compute the unpolarization squared amplitude of our process, we have to take average over initial spins s, s' and sum over final spin r, r' as well as colour indices. More specificly, the squared amplitude is given by

$$|\mathcal{M}|^2 = \frac{3}{4}\sum_{spins}\left(\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{4-f}\right)\left(\mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{4-f}\right)^\dagger, \qquad (3.8.8)$$

notice that the amplitude of four-fermion diagram is proportional with Willsion coefficients, thus we can neglect the $|\mathcal{M}_{4-f}|^2$ term which will be considered in dimension-eight operators. Now, our sum includes 8 terms which are: 2 diagonal terms, i.e $|\mathcal{M}_\gamma|^2, |\mathcal{M}_Z|^2$ and 6 interferences terms ($\mathcal{M}_Z\mathcal{M}^\dagger_\gamma + \mathcal{M}_{4-f}\mathcal{M}^\dagger_\gamma + \mathcal{M}_{4-f}\mathcal{M}^\dagger_Z + h.c$). The analytic results of squared amplitude in R_ξ-gauge calculated by FORM [5].

For the total cross-section, we also used the formula in chapter 2, which is

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \frac{|\mathcal{M}|^2}{64\pi^2\left(E_{e^-} + E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|} \\ \Rightarrow \sigma_T &= \int_{-1}^{1}\frac{d\sigma}{d(cos\theta)}d(cos\theta) = \int_{-1}^{1}\int_0^{2\pi}\frac{|\mathcal{M}|^2}{64\pi^2\left(E_{e^-} + E_{e^+}\right)^2}\frac{|\vec{k}|}{|\vec{p}|}d\phi\ d(cos\theta). \qquad (3.8.9)\end{aligned}$$

From equation (3.8.9) we are able to calculate the total cross-section numerically. Now, we move to the next section to study some physical results in SMEFT.

3.9 Physical results in SMEFT

In first, I have set $C_{\phi B}$, $C_{\phi W}$, $C_{\phi D}$, $C_{\phi WB}$ and C_{ϕ} to zero. With this configuration, we can reuse the coupling constants, mass of fermions and bosons like Standard Model. In order to explore the sensitivity of total cross-section with Willsion coefficients as well as for the convenience of comparision, we have separated these coefficients into three parts:

1. **Top-quark electroweak couplings:** $C_{uW}, C_{uB}, C^1_{\phi q}, C^3_{\phi q}$, and $C_{\phi u}$.
2. **Four-fermion couplings:** $C_{lu}, C_{eu}, C_{qe}, C^1_{lq}$, and C^3_{lq}.
3. **Lepton electroweak couplings:** $C^1_{\phi l}, C^3_{\phi l}$, and $C_{\phi e}$. In this case we do not count C_{eB} and C_{eW} since these coefficients are proportional with m_e in the square amplitude and are neglected.

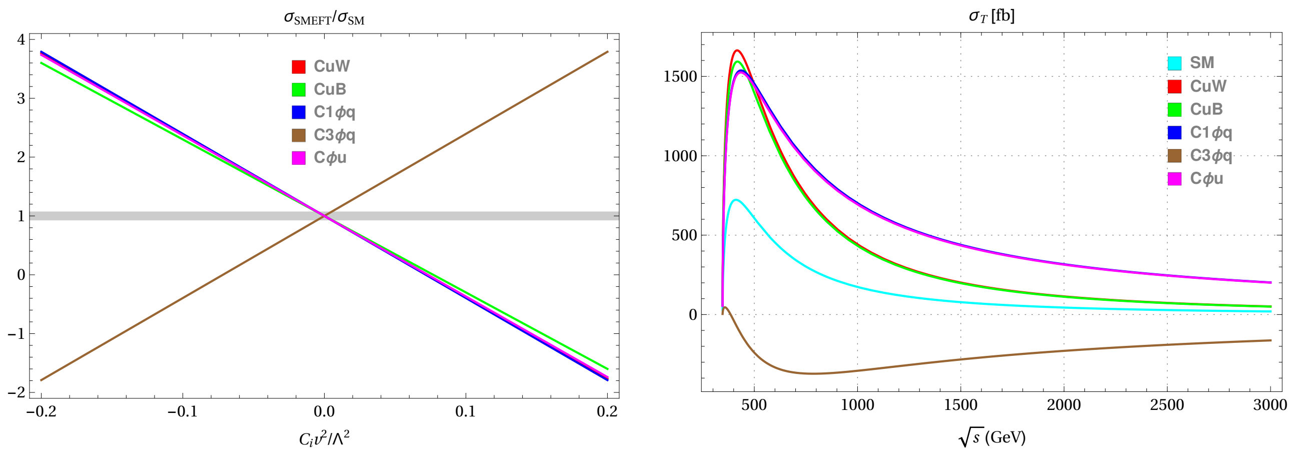
To know how much the sensitivity of each operators is, we exported a results following the ratio $\frac{\sigma_{SMEFT}}{\sigma_{SM}}$. It is important to notice that, we plot a dependence of the physical quantities with respect to a dimensionless parameters, which is

$$\bar{C}_i = C_i \frac{v^2}{\Lambda^2} \tag{3.9.1}$$

where the range of $\bar{C}_i$ can be from -0.2 to $+0.2$. Finally, we depict the effect of each operators into their corresponding physical observables. We also notice that, for each line in all figure below the value of all dimensionless parameters (Willsion coeffcients) are zero except the parameters which denoted in the graph.

3.9.1 Top-quark electroweak couplings

In our process, the measurments are effectively at $\sqrt{s} = 500$ GeV, since the total cross-section reaches a maximum value at this energy level. First of all, let us point out the ratio $\frac{\sigma_{SMEFT}}{\sigma_{SM}}$ for $\sqrt{s} = 500$ GeV as figure 3.2:

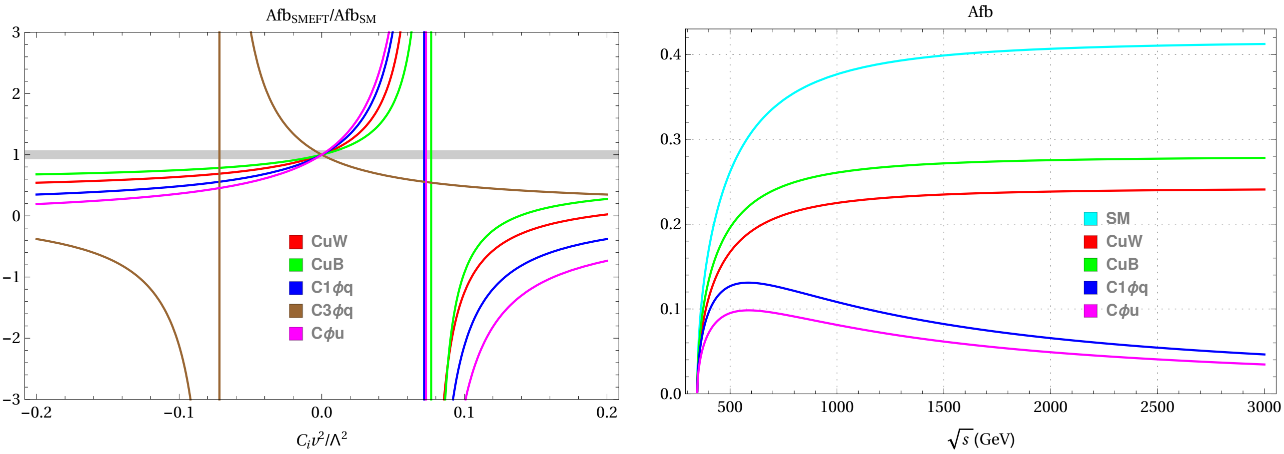


(a) $\sqrt{s} = 500$ (GeV) (b) $\bar{C}_i = -0.1$

Figure 3.2: The dependence of total cross-section on D6 operators for $\sqrt{s} = 500$ GeV (a). The effectof D6 operators on total cross-section at $\bar{C}_i = -0.1$ (b).

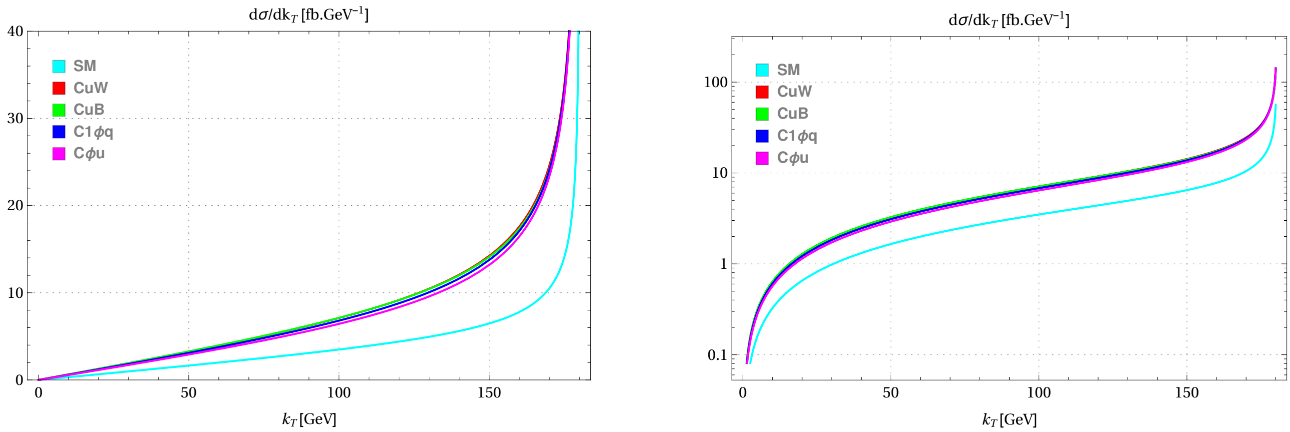
As we can see, the dependence of total cross-section with respect to the dimensionless coefficients $\bar{C}_i$ is linear. According to figure (3.2a) we can choose the values: $\bar{C}_{uW} = \bar{C}_{uB} = \bar{C}^1_{\phi q} = \bar{C}_{\phi u} = \bar{C}_{\phi u} = -0.1$, and notice that we will fix these values to find out which operators are sensitive with physical observables. Therefore, the effect of dimension-six operators are illustrated in figure (3.2b). We also notice that $C^{(3)}_{\phi l}$ has a same contribution with $C^{(1)}_{\phi q}$, but it contain the minus sign thus it is hard to combine all information in one figure, we will try to show more information as much as possible.

Althought the Willsion coefficients affect the total cross-section, however, if $\bar{C}_i$ are small enough, from figure 3.2 we can not distinguish one from the other. Fortunately, each Willsion coefficient may be sensitive with one or more another physical observables. That means, we can find an observable (distribution) which is affected significantly by varying the value of coefficient which we are considering. In this spirit, we now looking for the forward-backward asymmetry, and several distributions are depicted as follows:



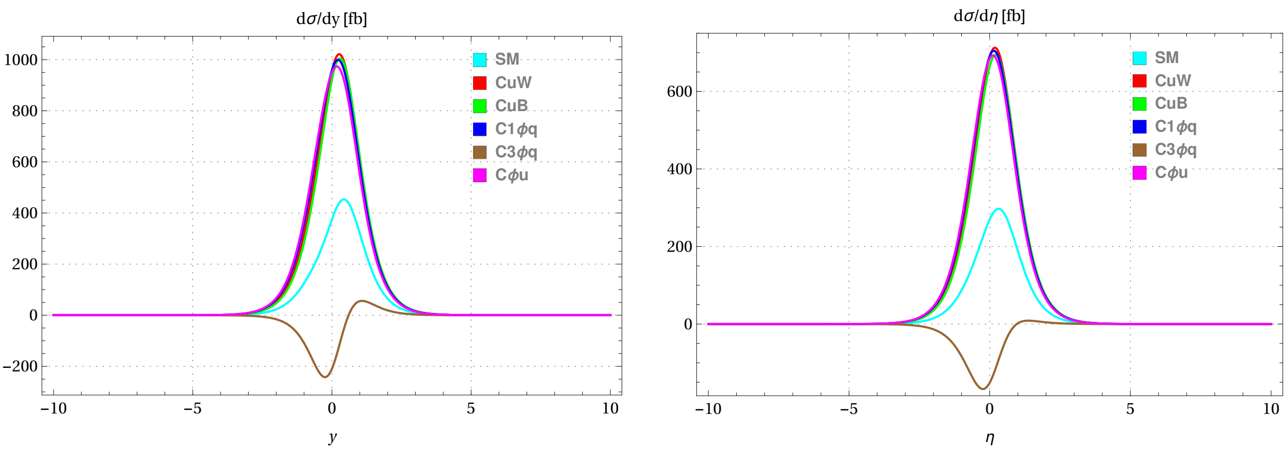
(a) $\sqrt{s} = 500\ (GeV)$ (b) $\bar{C}_i = -0.1$

Figure 3.3: The dependence of forward-backward asymmetry on D6 operators for $\sqrt{s} = 500$ GeV (a). The effects of D6 operators on A_{FB} with $\bar{C}_{uW} = \bar{C}_{uB} = \bar{C}^1_{\phi q} = \bar{C}_{\phi u} = \bar{C}_{\phi u} = -0.1$ each time (b).



(a) $\bar{C}_i = -0.1$ (b) $\bar{C}_i = -0.1$

Figure 3.4: The effect of D6 operators on transverse momentum distribution (a), in Log scale (b).



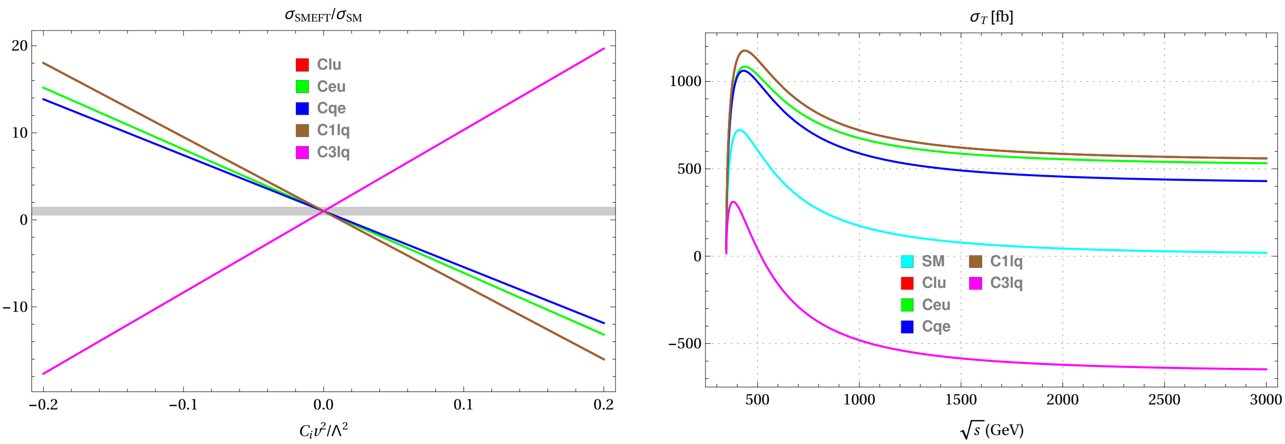
(a) $\bar{C}_i = -0.1$ (b) $\bar{C}_i = -0.1$

Figure 3.5: The effect of D6 operators on rapidity- (a) and pseudo-rapidity distribution (b) for $\sqrt{s} = 500$ GeV with $\bar{C}_{uW} = \bar{C}_{uB} = \bar{C}^1_{\phi q} = \bar{C}_{\phi u} = \bar{C}_{\phi u} = -0.1$.

Base on figure 3.3(a) and 3.3(b), these operators are very sensitive with the forward-backward asymmetry, more specifically, they are sensitive with the asymmetries of cosθ distributions. In case of of F-B asymmetry and transverse momentum distribution, we temporarily do not draw $C^{(3)}_{\phi q}$ since their minus value, however we can see the effect of this operator more clearly in the rapidity and pseudo rapidity distribution at figure 3.5 (a, b).

3.9.2 Four-fermion couplings

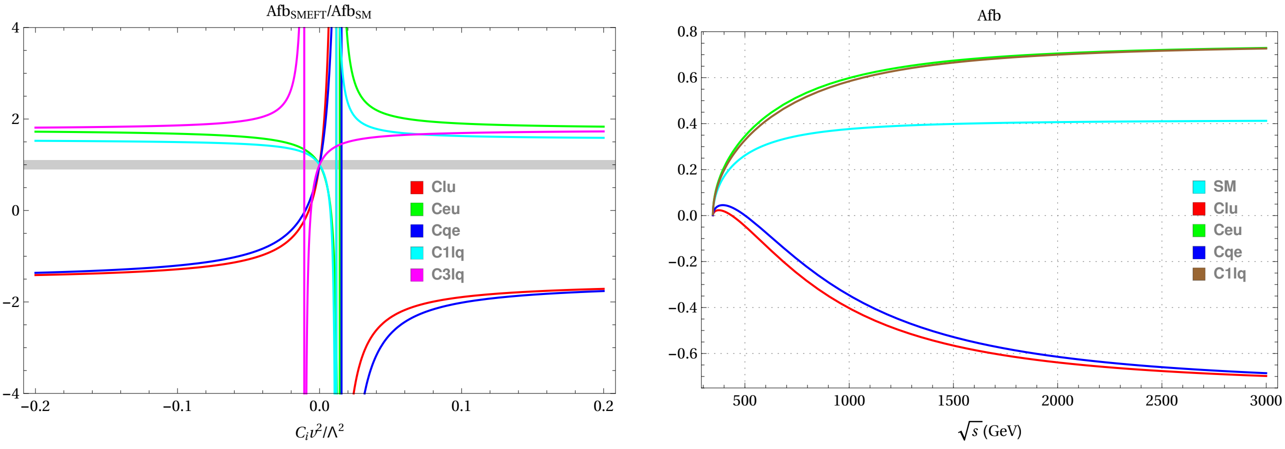
First of all, we also draw the dependence of cross-section on four-fermion operators, let us see figure 3.5. From this figure, we can see that the total cross-section is changed significantly under the varying of four-fermion operators. In that case, we should chose a small value of parameters which is $C_{lu} = C_{eu} = C_{qe} = C^1_{lq} = C^3_{lq} = -0.01$. Here is several results of four-fermion case:



(a) $\sqrt{s} = 500$ (GeV) (b) $\bar{C}_i = -0.01$

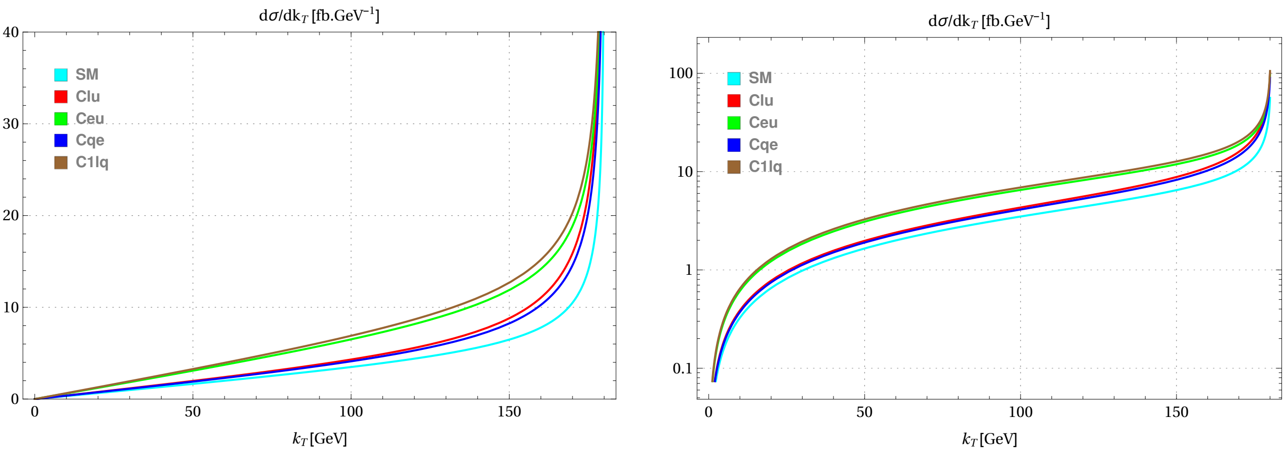
Figure 3.6: The dependence of total cross-section on Four-fermion operators for $\sqrt{s} = 500$ GeV (a), the effects of Four-fermion operators with $C_{lu} = C_{eu} = C_{qe} = C^1_{lq} = C^3_{lq} = -0.01$ (b).

Besides, we also plot some asymmetry and distributions as we have done:

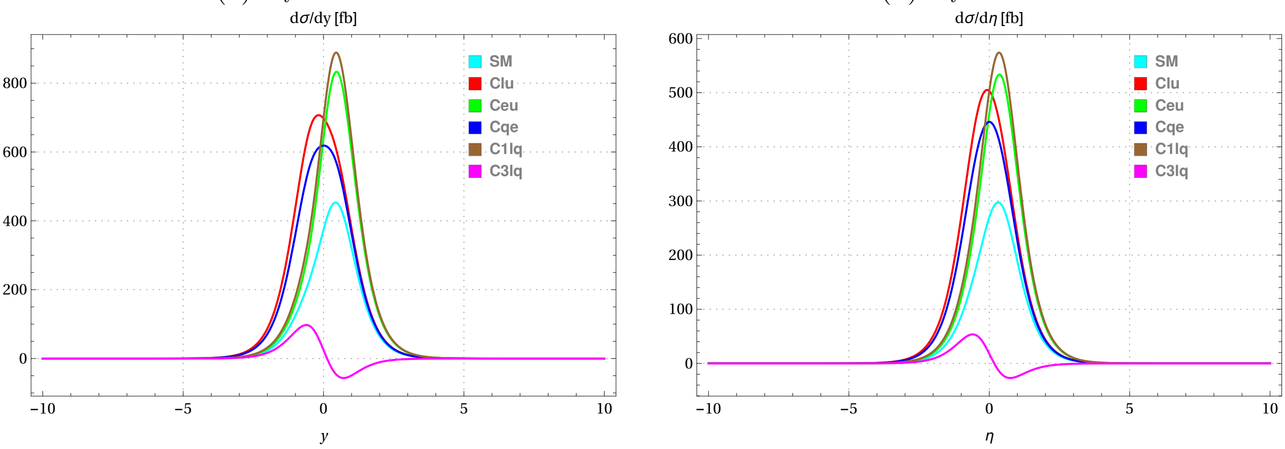


(a) $\sqrt{s} = 500\ (GeV)$ (b) $\bar{C}_i = -0.01$

Figure 3.7: The dependence of F-B asymmetry on Four-fermion operators for $\sqrt{s} = 500$ GeV (a), the effects of Four-fermion operators with $C_{lu} = C_{eu} = C_{qe} = C^1_{lq} = C^3_{lq} = -0.01$ (b).



(a) $\bar{C}_i = -0.01$ (b) $\bar{C}_i = -0.01$

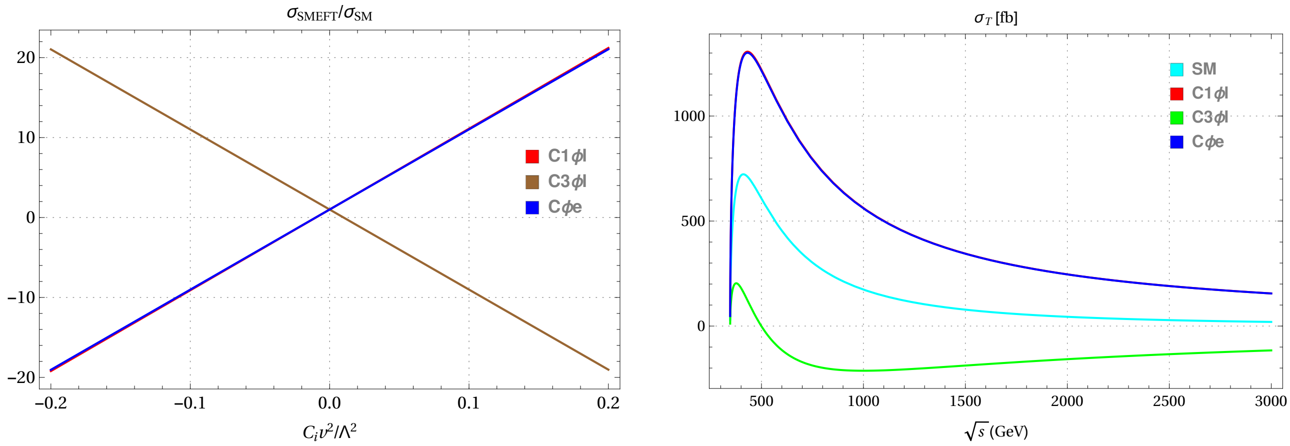


(c) $\bar{C}_i = -0.01$ (d) $\bar{C}_i = -0.01$

Figure 3.8: Transverse momentum distribution (a), transverse momentum in Log scale (b), rapidity and pseudo-rapidity distribution (c,d) for $\sqrt{s} = 500$ with $C_{lu} = C_{eu} = C_{qe} = C^1_{lq} = C^3_{lq} = -0.01$.

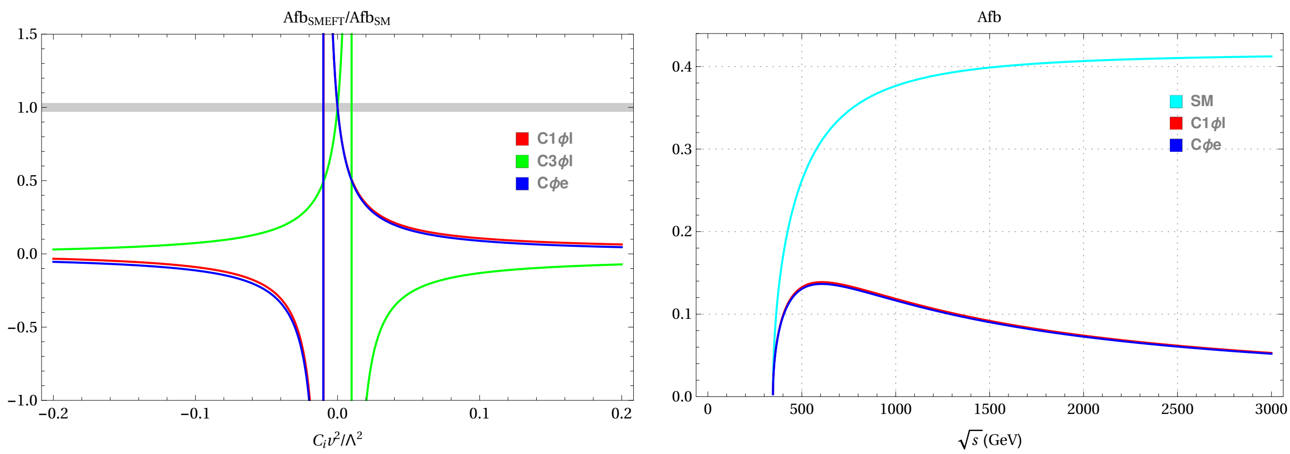
3.9.3 Lepton electroweak couplings

Analogous with the two previous case, we first consider the effect of $C^1_{\phi l}, C^3_{\phi l}$, and $C_{\phi e}$ on the total cross-section in figure (3.6a). In here, the values of parameters are choosen as $C^1_{\phi l}, C_{\phi e} = C^3_{\phi l} = 0.01$. With these values, let us consider the F-B asymmtery and several distributions as follows:



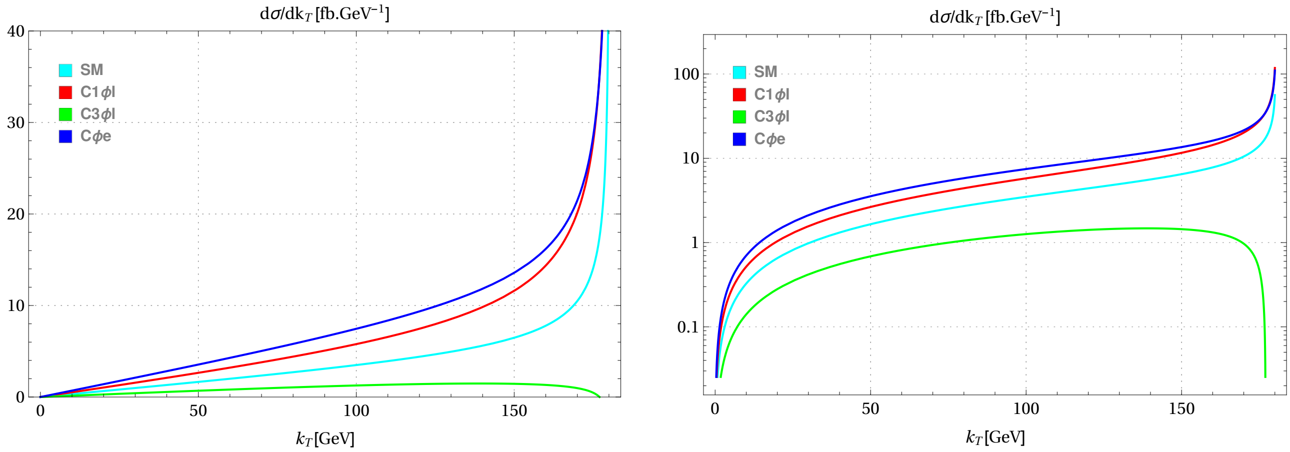
(a) $\sqrt{s} = 500\ (GeV)$ (b) $\bar{C}_i = 0.01$

Figure 3.9: The dependence of total cross-section on D6 operators for $\sqrt{s} = 500$ GeV (a), the effects of D6 operators with $C^1_{\phi l}, C_{\phi e} = C^3_{\phi l} = 0.01$ (b).



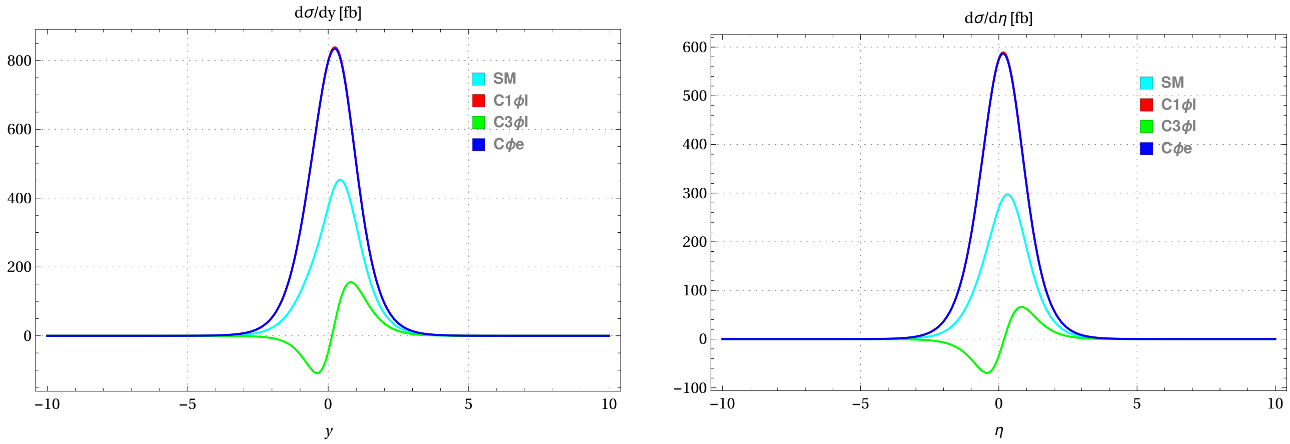
(a) $\bar{C}_i = 0.01$ (b) $\bar{C}_i = 0.01$

Figure 3.10: The dependence of F-B asymmetry on D6 operators for $\sqrt{s} = 500$ GeV (a), the effects of D6 operators with $C^1_{\phi l}, C_{\phi e} = C^3_{\phi l} = 0.01$ (b).



(a) $\bar{C}_i = 0.01$ (b) $\bar{C}_i = 0.01$

Figure 3.11: Transverse momentum distribution (a), transverse momentum in Log scale (b) for $\sqrt{s} = 500$ GeV with $C^1_{\phi l}, C_{\phi e} = C^3_{\phi l} = 0.01$.



(a) $\bar{C}_i = 0.01$ (b) $\bar{C}_i = 0.01$

Figure 3.12: Rapidity and pseudo-rapidity distribution (a, b) for $\sqrt{s} = 500$ with $C^1_{\phi l} = C_{\phi e} = C^3_{\phi l} = 0.01$.

Chapter 4

Fully polarized top-quark decays in SMEFT

In this chapter we consider another process to find new physics above the electroweak energy scale. More specifically, we consider the process of fully polarized top-quark decay into a massive b-quark and polarized W-boson with leptonic decay. The prediction of polarization observables for W-boson can help us select which observables are sensitive to new physics in the top quark decay. The calculations here are based on [8], [9].

4.1 Spin density matrix methodology

From the characteristic spin-1, the W-boson provides 8 spin observables including three polarizations and five alignments (tensor polarizations) [9]. To understand more clearly, let us find out a minimal set of parameters to describe the spin state of W-boson as follows:

First of all, W-boson spin state can be described by a 3×3 density matrix with complex elements. Besides, this matrix should be Hermitian with unit trace and positive semidefinite, thus the degree of freedom of this density matrix is

$$9 \times 2 - 3 - 6 - 1 = 8, \tag{4.1.1}$$

which correspond with 8 spin observables of W-boson. By choosing a Cartesian coordinates (x, y, z) in W-boson rest reference frame, the density matrix can be expressed in terms of the three spin operator components S_M and the five tensor operator components T_M as[9]

$$\rho = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{M=-1}^{1} \langle S_M \rangle^* S_M + \sum_{M=-2}^{2} \langle T_M \rangle^* T_M, \tag{4.1.2}$$

where $\langle S_M \rangle$, $\langle T_M \rangle$ are sequentially expectation values of S_M and T_M corresponding operators. We also notice that S_M are spin operator components in coordinate system which can be written in spherical basis

$$S_{\pm 1} = \mp \frac{1}{\sqrt{2}} \left(S_x \pm i S_y\right), \qquad S_0 = S_z, \tag{4.1.3}$$

whereas the explicit form of S_x, S_y, S_z are

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad S_y = \frac{1}{\sqrt{2}i} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \qquad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{4.1.4}$$

Furthermore, T_M are irreducible second rank tensors which are constructed in terms of S_M as

$$T_0 = \frac{1}{\sqrt{6}}\left[S_{+1}S_{-1} + S_{-1}S_{+1} + 2S_0^2\right], \quad T_{\pm 1} \quad = \frac{1}{\sqrt{2}}\left[S_{\pm 1}S_0 + S_0 S_{\pm 1}\right], \quad T_{\pm 2} = S_{\pm 1}^2. \qquad (4.1.5)$$

For convenience, we introduce a following operators in the Cartesian basis:

$$\begin{aligned} A_1 &= \frac{1}{2}\left(T_1 - T_{-1}\right), \qquad A_2 = \frac{1}{2i}\left(T_1 + T_{-1}\right), \\ B_1 &= \frac{1}{2}\left(T_2 + T_{-2}\right), \qquad B_2 = \frac{1}{2i}\left(T_2 - T_{-2}\right). \end{aligned} \qquad (4.1.6)$$

Substituting (4.1.3) and the inverse of (4.1.6), expressing $T_{\pm 1}$, $T_{\pm 2}$ in terms of $A_{1,2}$,$B_{1,2}$ basic, into the density matrix (4.1.2) we shall have:

$$\rho = \frac{1}{3}\mathbb{1} + \frac{1}{2}\sum_{i=1}^{3}\langle S_i\rangle^* S_i + \sum_{j=1}^{2}\langle A_j\rangle^* A_j + \sum_{j=1}^{2}\langle B_j\rangle^* B_j + \langle T_0\rangle^* T_0, \qquad (4.1.7)$$

where $i = 1, \cdots, 3$ stand for x, y, z. Using Cartesian basis in (4.1.4) and the identities (4.1.3, 4.1.5) we can directly calculate the explicit matrix form of A_j, B_j and T_0, which reads

$$\begin{aligned} A_1 &= \frac{1}{2\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad A_2 = \frac{1}{2i\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \\ B_1 &= \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad B_2 = \frac{1}{2i}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \qquad T_0 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \qquad (4.1.8)$$

After that, substituting these results into (4.1.7), we obtain the W-boson density matrix parameterized in terms of expectation values of spin observables, each elements of this matrix are presented as follows:

$$\begin{cases} \rho_{\pm 1 \pm 1} &= \frac{1}{3} \pm \frac{1}{2}\langle S_z\rangle + \frac{1}{\sqrt{6}}\langle T_0\rangle, \qquad \rho_{00} = \frac{1}{3} - \frac{2}{\sqrt{6}}\langle T_0\rangle, \\ \rho_{\pm 10} &= \frac{1}{2\sqrt{2}}\left[\langle S_x\rangle \mp i\langle S_y\rangle\right] \mp \frac{1}{\sqrt{2}}\left[\langle A_1\rangle \mp i\langle A_2\rangle\right], \qquad \rho_{1-1} = \langle B_1\rangle - i\langle B_2\rangle, \end{cases} \qquad (4.1.9)$$

and $\rho_{m'm} = \rho^*_{mm'}$. This matrix will be used to calculate the fully differential decay width of W-boson. For $W^\pm \to l^\pm \nu$, we can manipulate the helicity formalism of Jacob and Wick to write down the amplitude for W-boson leptonic decay process as follows:

$$\mathcal{M}_{m\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} D^{1*}_{m\Lambda}\left(\phi, \theta, 0\right), \qquad (4.1.10)$$

where m is spin components of W-boson ($m = -1, 0, +1$), λ_1 and λ_2 are helicity indices of l and ν, respectively. We also notice that $a_{\lambda_1\lambda_2}$ are constants, $\Lambda = \lambda_1 - \lambda_2$ and $D^{1*}_{m\Lambda}\left(\phi, \theta, 0\right)$ is the so-called Wigner D functions with the explicit form given by

$$D^j_{m'm}\left(\alpha, \beta, \gamma\right) = e^{-i\alpha m'} e^{-i\gamma m} d^j_{m'm}\left(\beta\right), \qquad (4.1.11)$$

where $d^j_{m'm}(\beta)$ is Wigner-d matrix elements (real). In W-boson leptoic decay, the mass of lepton products are approximately considered massless, thus we have $(\lambda_1, \lambda_2) = (\pm 1/2, \mp 1/2)$ for $W^\pm \to l^\pm \nu$. Now, let us write down the Feynman squared amplitude elements which reads

$$|\mathcal{M}_{mm'}|^2 = \mathcal{M}_{m\lambda_1\lambda_2}\mathcal{M}^*_{m'\lambda_1\lambda_2} = |a_{\lambda_1\lambda_2}|^2 \, e^{i(m-m')\phi} d^1_{m\Lambda}(\theta) d^1_{m'\Lambda}(\theta). \tag{4.1.12}$$

As we can see, each elements of density matrix is corresponding to one possibilities contribution of squared amplitude elements. The fully differential decay width of W-boson can presentes as

$$\frac{d\Gamma}{d\text{cos}\theta d\phi} = C \sum_{m,m'} \rho_{mm'} e^{i(m-m')\phi} d^1_{m\Lambda}(\theta) d^1_{m'\Lambda}(\theta), \tag{4.1.13}$$

with (θ, ϕ) are polar and azimuthal angles of the charged lepton momentum in W-boson rest frame, C is a factor including the phase space and the non-zero constant $a_{\lambda_1\lambda_2}$. The index $\Lambda = +1(-1)$ for W^+ or W^- decays, in case of $\Lambda = +1$ the Wigner-d matrix elements for spin-1 particles have form

$$d^1_{11} = \frac{1+\text{cos}\theta}{2}, \qquad d^1_{01} = \frac{\sin\theta}{\sqrt{2}}, \qquad d^1_{-11} = \frac{1-\text{cos}\theta}{2}, \tag{4.1.14}$$

for W^- boson, we can use the identity

$$d^j_{m'm} = (-1)^{m-m'} d^j_{mm'} = d^j_{-m,-m'} \tag{4.1.15}$$

Combinding all ingredients, i.e substituting (4.1.9) and (4.1.14) into (4.1.13) we obtain:

$$\begin{aligned}\frac{d\Gamma}{d\cos\theta d\phi} = C \Bigg[&\frac{1}{2}\left(1+\cos^2\theta\right) + \left[\frac{1}{6} - \frac{1}{\sqrt{6}}\langle T_0\rangle\right]\left(1-3\cos^2\theta\right) + \langle S_z\rangle\cos\theta \\ &+ \langle S_x\rangle \cos\phi\sin\theta + \langle S_y\rangle \sin\phi\sin\theta - \langle A_1\rangle\cos\phi\sin 2\theta - \langle A_2\rangle \sin\phi\sin 2\theta \\ &+ \langle B_1\rangle\cos 2\phi\sin^2\theta + \langle B_2\rangle\sin 2\phi\sin^2\theta \Bigg],\end{aligned} \tag{4.1.16}$$

however, this distribution have not been normalized yet, the factor C can be calculated by

$$\int_{-1}^{1}\int_0^{2\pi} \frac{d\Gamma}{d\text{cos}\theta d\phi} d\phi d(\text{cos}\theta) = 1 \quad \Longrightarrow C = \frac{3}{8\pi} \tag{4.1.17}$$

Finally, we obtain the normalized distribution of W^+ boson, which is

$$\begin{aligned}\frac{1}{\Gamma}\cdot\frac{d\Gamma}{d\text{cos}\theta d\phi} = \frac{3}{8\pi}\Bigg[&\frac{1}{2}\left(1+\text{cos}^2\theta\right) + \left[\frac{1}{6} - \frac{1}{\sqrt{6}}\langle T_0\rangle\right]\left(1-3\text{cos}^2\theta\right) + \langle S_z\rangle\text{cos}\theta \\ &+ \langle S_x\rangle\text{cos}\phi\sin\theta + \langle S_y\rangle\sin\phi\sin\theta - \langle A_1\rangle\text{cos}\phi\sin 2\theta - \langle A_2\rangle\sin\phi\sin 2\theta \\ &+ \langle B_1\rangle\cos 2\phi\sin^2\theta + \langle B_2\rangle\sin 2\phi\sin^2\theta\Bigg].\end{aligned} \tag{4.1.18}$$

Althought the distribution (4.1.18) carry all polarized informations of W-boson with an eight-parameter fit, nevertherless, we can derive another distributions which depend on one or fewer parameters. From this, the connection for decay distributions and spin observables can be constructed directly. In this section we focus on the W^+ boson, the distribution of W^- boson can be derived from W^+ distribution.

From the overall perspective, the distribution (4.1.18) just only depend on the longitudinal polarization $\langle S_z \rangle$ and the alignment $\langle T_0 \rangle$ when we integrate over the azimuthal angle ϕ, so it is reasonable the asymmetry in the $\cos\theta$ distribution may be sensitive with these parameters. Firstly, let us consider the forward-backward asymmetry A_{FB}:

$$\begin{aligned} A_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\cos\theta > 0) - \Gamma(\cos < 0)\right] \\ &= \left[\int_0^{\pi/2}\int_0^{2\pi} - \int_{\pi/2}^{\pi}\int_0^{2\pi}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}d\phi\sin\theta d\theta = \frac{3}{4}\langle S_z\rangle \end{aligned} \tag{4.1.19}$$

Another interesting asymmetry is an "edge-central" asymmetry A_{EC}:

$$\begin{aligned} A_{EC} &= \frac{1}{\Gamma}\left[\Gamma\left(|\cos\theta| > \frac{1}{2}\right) - \Gamma\left(|\cos\theta| < \frac{1}{2}\right)\right] \\ &= \left[\int_0^{\pi/3}\int_0^{2\pi} + \int_{2\pi/3}^{\pi}\int_0^{2\pi} - \int_{\pi/3}^{2\pi/3}\int_0^{2\pi}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}d\phi\sin\theta d\theta = \frac{3}{8}\sqrt{\frac{3}{2}}\langle T_0\rangle \end{aligned} \tag{4.1.20}$$

As we can see, from (4.1.19) and (4.1.20) the spin properties of W-boson, the longitudinal polarization $\langle S_z \rangle$ and the alignment $\langle T_0 \rangle$, can be extracted straightfowrad from the experimential distributions. Furthermore, if $\langle S_z \rangle$ and $\langle T_0 \rangle$ are measured the diagonal elements (4.1.9) of density matrix will be automatically determined.

In order to determine the remaining observables, we need to avoid the cancellation when integrate over ϕ. To do this, we repalce the integration over θ by an integration using a measure $f(\theta)$ to select the desired events. For instant, let us consider the forward-backward asymmetry for $x-$ and $y-$axes, in that case we use

$$f_1(\phi) = \text{sign}[\cos\phi] \Rightarrow \begin{cases} f_1(\phi) = +1, & \text{for events with } \phi \in [-\pi/2, \pi/2] \\ f_1(\phi) = -1, & \text{for events with } \phi \in [\pi/2, 3\pi/2] \end{cases} ; \tag{4.1.21}$$

$$f_2(\phi) = \text{sign}[\sin\phi] \Rightarrow \begin{cases} f_2(\phi) = +1, & \text{for events with } \phi \in [0, \pi] \\ f_2(\phi) = -1, & \text{for events with } \phi \in [\pi, 2\pi] \end{cases} \tag{4.1.22}$$

Integrating with these measures yields the quantities $\delta_k\Gamma$, with $k = 1, 2$, which has an angular distribution as follows:

$$\frac{1}{\Gamma}\frac{d(\delta_k\Gamma)}{d\cos\theta} = \int_0^{2\pi}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}\right]f_k(\phi)d\phi = \frac{3}{2\pi}\langle S_k\rangle\sin\theta - \frac{3}{2\pi}\langle A_k\rangle\sin 2\theta \tag{4.1.23}$$

We also do an analohous calculations like the forward-backward asymmetry, integrating over θ angle, we obtain the following results:

$$\frac{\delta_1\Gamma}{\Gamma} = \frac{1}{\Gamma}\left[\Gamma(\cos\phi > 0) - \Gamma(\cos\phi < 0)\right] = \int_{-1}^{1}\frac{1}{\Gamma}\frac{d(\delta_k\Gamma)}{d\cos\theta}d(\cos\theta) = \frac{3}{4}\langle S_x\rangle, \tag{4.1.24}$$

$$\frac{\delta_2\Gamma}{\Gamma} = \frac{1}{\Gamma}\left[\Gamma(\sin\phi > 0) - \Gamma(\sin\phi < 0)\right] = \int_{-1}^{1}\frac{1}{\Gamma}\frac{d(\delta_2\Gamma)}{d\cos\theta}d(\cos\theta) = -\frac{3}{4}\langle S_y\rangle \tag{4.1.25}$$

As we expected, $\langle S_x\rangle$ and $\langle S_y\rangle$ are relative directly with the forward-backward asymmetry for x- and y-axes. Moreover, we can modify equation (4.1.23) a bit to obtain an observable for tensor polarizations as follows:

$$\begin{aligned}
A^1_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\cos\phi\cos\theta > 0) - \Gamma(\cos\phi\cos\theta < 0)\right] \\
&= \int_0^{\pi}\int_0^{2\pi}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}\right] sign[\cos\phi\cdot\cos\theta] d\phi\sin\theta d\theta \\
&= \left[\int_0^{\pi/2}\int_{-\pi/2}^{\pi/2} + \int_{\pi/2}^{\pi}\int_{\pi/2}^{3\pi/2} - \int_0^{\pi/2}\int_{\pi/2}^{3\pi/2} - \int_{\pi/2}^{\pi}\int_{-\pi/2}^{\pi/2}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi} d\phi\sin\theta d\theta = -\frac{2}{\pi}\langle A_1\rangle,
\end{aligned} \tag{4.1.26}$$

and the same manner to obtain the $\langle A_2\rangle$ quantity:

$$\begin{aligned}
A^2_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\sin\phi\cos\theta > 0) - \Gamma(\sin\phi\cos\theta < 0)\right] \\
&= \int_0^{\pi}\int_0^{2\pi}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}\right] sign[\sin\phi\cdot\cos\theta] d\phi\sin\theta d\theta \\
&= \left[\int_0^{\pi/2}\int_0^{\pi} + \int_{\pi/2}^{\pi}\int_{\pi}^{2\pi} - \int_{\pi/2}^{\pi}\int_0^{\pi} - \int_0^{\pi/2}\int_{\pi}^{2\pi}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi} d\phi\sin\theta d\theta = -\frac{2}{\pi}\langle A_2\rangle.
\end{aligned} \tag{4.1.27}$$

In order to determine $\langle B_j\rangle$ quantities, let us introduce a measure $g_k(\theta)$ as follows:

$$g_1(\phi) = \text{sign}[\cos 2\phi] \Rightarrow \begin{cases} g_1(\phi) = +1, & \text{for } \phi\in[-\pi/2,\pi/2]\cup[3\pi/2,5\pi/2] \\ g_1(\phi) = -1, & \text{for } \phi\in[\pi/2,3\pi/2]\cup[5\pi/2,7\pi/2] \end{cases} \quad ; \tag{4.1.28}$$

$$g_2(\phi) = \text{sign}[\sin 2\phi] \Rightarrow \begin{cases} g_2(\phi) = +1, & \text{for } \phi\in[0,\pi]\cup[2\pi,3\pi] \\ g_2(\phi) = -1, & \text{for } \phi\in[\pi,2\pi]\cup[3\pi,4\pi] \end{cases} \tag{4.1.29}$$

Keep in mind that the range of ϕ is $[0,2\pi]$, hence $2\phi\to[0,4\pi]$. At a glance of (4.1.9), all terms will be eliminated except the terms proportional to $\langle B_j\rangle$. Integrating over cosθ we obtain the azimuthal asymmetries:

$$\begin{aligned}
A^1_{\phi} &= \frac{1}{\Gamma}\left[\Gamma(\cos 2\phi > 0) - \Gamma(\cos 2\phi < 0)\right] \\
&= \int_0^{\pi}\int_0^{2\pi}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi}\right] sign[\cos 2\phi] d\phi d(\cos\theta) \\
&= \int_0^{\pi}\left[\int_{-\pi/2}^{\pi/2} + \int_{3\pi/2}^{5\pi/2} - \int_{\pi/2}^{3\pi/2} - \int_{5\pi/2}^{7\pi/2}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi} d\phi d(\cos\theta) = -\frac{2}{\pi}\langle B_1\rangle,
\end{aligned} \tag{4.1.30}$$

analogous for $g_2(\theta)$ we shall have

$$\begin{aligned}
A^1_{\phi} &= \frac{1}{\Gamma}\left[\Gamma(\sin 2\phi > 0) - \Gamma(\sin 2\phi < 0)\right] \\
&= \int_0^{\pi}\left[\int_0^{\pi} + \int_{2\pi}^{3\pi} - \int_{\pi}^{2\pi} - \int_{3\pi}^{4\pi}\right]\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta d\phi} d\phi d(\cos\theta) = -\frac{2}{\pi}\langle B_2\rangle.
\end{aligned} \tag{4.1.31}$$

For W^- boson, the differential decay distribution can be proceeded via replacing $\sin\theta \to -\sin\theta$ and $\cos\theta \to -\cos\theta$. All relations between asymmetries and spin observables in (4.1.19), (4.1.20), (4.1.24), (4.1.25), (4.1.26), (4.1.27), (4.1.30) and (4.1.31) remain a same form as W^+ boson, instead, with extra minus for A_{FB} and $\delta_k\Gamma$, more specifically we have:

$$\Longrightarrow \begin{cases} A_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\cos\theta > 0) - \Gamma(\cos < 0)\right] = -\frac{3}{4}\langle S_z\rangle \\ A_{EC} &= \frac{1}{\Gamma}\left[\Gamma\left(|\cos\theta| > \frac{1}{2}\right) - \Gamma\left(|\cos\theta| < \frac{1}{2}\right)\right] = \frac{3}{8}\sqrt{\frac{3}{2}}\langle T_0\rangle \\ \frac{\delta_1\Gamma}{\Gamma} &= \frac{1}{\Gamma}\left[\Gamma(\cos\phi > 0) - \Gamma(\cos\phi < 0)\right] = -\frac{3}{4}\langle S_x\rangle \\ \frac{\delta_2\Gamma}{\Gamma} &= \frac{1}{\Gamma}\left[\Gamma(\sin\phi > 0) - \Gamma(\sin\phi < 0)\right] = +\frac{3}{4}\langle S_y\rangle \\ A^1_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\cos\phi\cos\theta > 0) - \Gamma(\cos\phi\cos\theta < 0)\right] = -\frac{2}{\pi}\langle A_1\rangle \\ A^2_{FB} &= \frac{1}{\Gamma}\left[\Gamma(\sin\phi\cos\theta > 0) - \Gamma(\sin\phi\cos\theta < 0)\right] = -\frac{2}{\pi}\langle A_2\rangle \\ A^1_{\phi} &= \frac{1}{\Gamma}\left[\Gamma(\cos 2\phi > 0) - \Gamma(\cos 2\phi < 0)\right] = -\frac{2}{\pi}\langle B_1\rangle \\ A^1_{\phi} &= \frac{1}{\Gamma}\left[\Gamma(\sin 2\phi > 0) - \Gamma(\sin 2\phi < 0)\right] = -\frac{2}{\pi}\langle B_2\rangle \end{cases} \tag{4.1.32}$$

4.2 Fully polarized top-quark decays

The main purpose of this section is to calculate the production of W-boson density matrix resulting from fully polarized top-quark decays in SMEFT.

4.2.1 Effective Wtb interaction

As the way we did in previous chapter before, we also expand the covariant derivative in SM part and the dimension-six operators in SMEFT which are Q_{uW}, Q_{dW} and $Q_{\phi ud}$. The effective Lagrangian include SM contributions and the correction from dimension-six operators are presented as follows:

$$\begin{aligned} \mathcal{L}^{full}_{Wtb} = &-\frac{\bar{g}_2}{\sqrt{2}}\bar{b}\gamma^\mu\left[P_L + \frac{1}{2}C^{3*}_{\phi ud}v^2 P_R\right]tW^-_\mu - \bar{b}i2\sigma^{\mu\nu}q_\nu\left[C^{3*}_{dW}vP_L + C^3_{uW}vP_R\right]tW^-_\mu \\ &-\frac{\bar{g}_2}{\sqrt{2}}\bar{t}\gamma^\mu\left[P_L + \frac{1}{2}C^3_{\phi ud}v^2 P_R\right]bW^+_\mu + \bar{t}i2\sigma^{\mu\nu}q_\nu\left[C^3_{dW}vP_R + C^{3*}_{uW}vP_L\right]bW^+_\mu \end{aligned} \tag{4.2.1}$$

Chronologically, the first line of (4.2.1) describes the top-quark decay process, the last line in nothing but the hermitian conjuagate which desribe the process of anti top-quark decay. In order to consistent with [8] and cross-check with the results of [9] let us parameterized the Lagrangian interaction of top-quark decay as follows:

$$\mathcal{L}_{Wtb} = -\frac{\bar{g}_2}{\sqrt{2}}\bar{b}\gamma^\mu \left[V_L P_L + V_R P_R\right] t W_\mu^- - \frac{\bar{g}_2}{\sqrt{2}}\bar{b}\frac{i\sigma^{\mu\nu}q_\nu}{M_W}\left[g_L P_L + g_R P_R\right] t W_\mu^- \\ - \frac{\bar{g}_2}{\sqrt{2}}\bar{t}\gamma^\mu \left[V_L^* P_L + V_R^* P_R\right] b W_\mu^+ + \frac{\bar{g}_2}{\sqrt{2}}\bar{t}\frac{i\sigma^{\mu\nu}q_\nu}{M_W}\left[g_L^* P_R + g_R^* P_L\right] b W_\mu^+ \tag{4.2.2}$$

whereas the parameters in Lagrangian (4.2.2) are

$$V_L = V_{tb}, \qquad V_R = \frac{1}{2}C^{3*}_{\phi ud}\frac{v^2}{\Lambda^2} \\ g_L = \sqrt{2}C^{3*}_{dW}\frac{v^2}{\Lambda^2}, \qquad g_R = \sqrt{2}C^{3}_{uW}\frac{v^2}{\Lambda^2}. \tag{4.2.3}$$

It is noticeable that V_{tb} is the Cabibbo-Kobayaski-Maskawa matrix element which has a value $V_{tb} \simeq 1$. Besides, we also normalized the $\sigma^{\mu\nu}q_\nu$ term in mass scale $M_W = \bar{g}_2 v/2$. All contribution of dimension six operators for Wtb vertex can be found in appendix A.3. From (4.2.2) we can see that all new physics effects can be parameterized by four parameters which are connected directly with dimension-six operators. Note that, the parameterization (4.2.1) is called EFT framework, while (4.2.2) is called the anomalous coupling framework.

4.2.2 Polarized squared amplitude

The main purpose of this section is calculating the production of polarized W^+ boson via fully polarized top-quark decay. The Feynman diagram of this process is

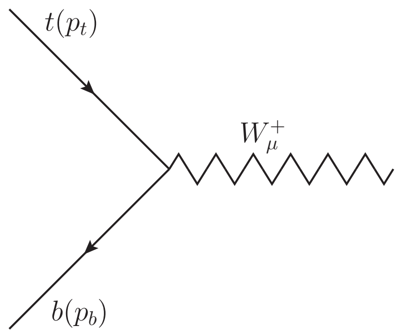


Figure 4.1: The Feynman diagram for top-quark decays.

For our process, we choose top-quark rest frame and setting the positive of z-axis in the direction of W-boson momentum $\vec{q}$. Thus, the W-boson polarization vectors in our top-quark rest frame are

$$\begin{cases} \epsilon_\mu^0 &= \dfrac{1}{M_W}\left(q, 0, 0, E_W\right) \\ \epsilon_\mu^{+1} &= -\dfrac{1}{\sqrt{2}}\left(0, 1, i, 0\right) \\ \epsilon_\mu^{-1} &= \dfrac{1}{\sqrt{2}}\left(0, 1, -i, 0\right) \end{cases} \tag{4.2.4}$$

For polarized top-quark, let us define a spin four-vector which is parameterized as [8]

$$s^t_\mu = (0,\ \sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta) \tag{4.2.5}$$

For the kinematic variables, we define the four energy-momentum vector of top-quark, b-quark and W-boson as follows:

$$p^t_\mu = (m_t, 0, 0, 0)\,, \qquad p^b_\mu = (E_b, 0, 0, -q)\,, \qquad p^W_\mu = (E_W, 0, 0, q)\,, \tag{4.2.6}$$

notice that we can set $m_b \simeq 0$ in two-body decays phase space, approximately, thus the explict form of E_W and E_b are

$$E_W = \frac{1}{2m_t}\left(m_t^2 + m_W^2\right), \qquad E_b = q = \frac{1}{2m_t}\left(m_t^2 - m_W^2\right). \tag{4.2.7}$$

With all ingredients we now can write down the Feynman amplitude of $t \to Wb$ process and their hermitian conjugate as follows:

$$\begin{aligned}\mathcal{M}_i\left(t \to W_i b\right) &= \bar{u}^{\lambda_b}(p_b) i\frac{\bar{g}_2}{\sqrt{2}}\left[-\gamma^\mu\left(V_L^* P_L + V_R^* P_R\right) + i\frac{\sigma^{\mu\nu}q_\nu}{M_W}\left(g_L^* P_R + g_R^* P_L\right)\right] u^{\lambda_t}(p_t)\epsilon^{i*}_\mu(p_W),\\ \mathcal{M}^*_j\left(t \to W_j b\right) &= \epsilon^j_\nu(p_W)\bar{u}^{\lambda_t}(p_t) i\frac{\bar{g}_2}{\sqrt{2}}\left[\gamma^\mu\left(V_L P_L + V_R P_R\right) + i\frac{\sigma^{\mu\nu}q_\nu}{M_W}\left(g_L P_L + g_R P_R\right)\right] u^{\lambda_b}(p_b),\end{aligned} \tag{4.2.8}$$

where $q_\nu = (p_t - p_b)_\nu$ is W-boson momentum. Chronologicaly, $\lambda_t, \lambda_b = [-1/2, +1/2]$ are helicity indices of top quark and bottom quark, $i, j = [-1, 0, +1]$ are W^+ boson helicity indices. Remember that our process is fully polarized top-quark decay into massive b-quark and polarized W-boson, thus we just only sum over final spin state of b-quark. We also notice that quarks have three colour states, in case of our process, we need to take average the initial colour states and sum over final colour states. Combinding all informations, the polarized squared amplitude elements are

$$\begin{aligned}\mathcal{M}_{ij} &= 3\cdot\frac{1}{3}\sum_{\lambda_b}\mathcal{M}_i\left(t \to W_i b\right)\mathcal{M}^*_j\left(t \to W_j b\right)\\ &= \frac{\bar{g}_2^2}{4}\mathrm{Tr}\left\{\left(\not{p}_b + m_b\right)\left[\gamma^\mu\left(V_L^* P_L + V_R^* P_R\right) - i\frac{\sigma^{\mu\nu}q_\nu}{M_W}\left(g_L^* P_R + g_R^* P_L\right)\right]\left(1+\gamma^5\not{s}_t\right)\left(\not{p}_t + m_t\right)\right.\\ &\qquad\left.\times\epsilon^{i*}_\mu(p_W)\epsilon^j_\nu(p_W)\left[\gamma^\mu\left(V_L P_L + V_R P_R\right) + i\frac{\sigma^{\mu\nu}q_\nu}{M_W}\left(g_L P_L + g_R P_R\right)\right]\right\}.\end{aligned} \tag{4.2.9}$$

To derive the polarized squared amplitude (4.2.9) we have used the indentity:

$$u\left(p_t, \lambda_t\right)\bar{u}\left(p_t, \lambda_t\right) = \frac{1}{2}\left(1+\gamma^5\not{s}_t\right)\left(\not{p}_t + m_t\right) \tag{4.2.10}$$

Notice that in (4.2.9) we do not ignore the mass of b-quark, instead, we will performe the final result with respect to m_b order. The analytical result of polarized squared amplitude up to high order of m_b are

$$\mathcal{M}_i\left(t \to W_i b\right)\mathcal{M}^*_j\left(t \to W_j b\right) = \frac{\bar{g}_2^2}{4}m_t^2\rho_{ij}, \tag{4.2.11}$$

where ρ_{ij} is the density matrix elements of W^+ boson. Each matrix elements have a form:

$$\begin{aligned}
\rho_{00} &= A_0 + 2\frac{|\vec{q}|}{m_t}A_1\cos\theta,\\
\rho_{\pm1\pm1} &= B_0(1\pm\cos\theta) \pm 2\frac{|\vec{q}|}{m_t}B_1(1\pm\cos\theta),\\
\rho_{\pm10} = \rho^*_{0\pm1} &= \left[\frac{m_t}{\sqrt{2}M_W}\left(C_0+iD_0\right) \pm \frac{|\vec{q}|}{\sqrt{2}M_W}\left(C_1+iD_1\right)\right]\sin\theta e^{\mp i\phi},\\
\rho_{\pm1\mp1} &= 0.
\end{aligned} \tag{4.2.12}$$

The explicit form of each dimensionless parameters in (4.2.12) are

$$\begin{aligned}
A_0 = {}& \frac{m_t^2}{M_W^2}\left(|V_L|^2+|V_R|^2\right)(1-x_w^2) + \left(|g_L|^2+|g_R|^2\right) - 2\frac{m_t}{M_W}Re\left[V_Lg_R^*+V_Rg_L^*\right](1-x_w^2)\\
&+ 2\frac{m_t}{M_W}x_bRe\left[V_Lg_L^*+V_Rg_R^*\right](1+x_w^2) - 4x_bRe\left[g_Lg_R^*+V_LV_R^*\right]\\
&+ x_b^2\left[-(|g_L|^2+|g_R|^2)\left(2+2\frac{m_t^2}{M_W^2}+\frac{m_t^4}{M_W^4}\right)+4\frac{m_t}{M_W}Re\left[V_Lg_R^*+V_Rg_L^*\right]\right] - 4\frac{m_t^2}{M_W^2}x_b^3Re\left[g_Lg_R^*\right],\\
A_1 = {}& \frac{m_t^2}{M_W^2}\left(|V_L|^2-|V_R|^2\right) - \left(|g_L|^2-|g_R|^2\right) - 2\frac{m_t}{M_W}Re\left[V_Lg_R^*-V_Rg_L^*\right]\\
&+ 2\frac{m_t}{M_W}x_bRe\left[V_Lg_L^*-V_Rg_R^*\right] - x_b^2\frac{m_t^4}{M_W^4}(|g_L|^2-|g_R|^2)(1+2x_w^2),\\
B_0 = {}& \left(|V_L|^2+|V_R|^2\right)(1-x_w^2) + \frac{m_t^2}{M_W^2}\left(|g_L|^2+|g_R|^2\right)(1-x_w^2) - 2\frac{m_t}{M_W}Re\left[V_Rg_L^*+V_Lg_R^*\right](1-x_w^2)\\
&- 4x_bRe\left[V_LV_R^*+g_Lg_R^*\right] + 2\frac{m_t}{M_W}x_bRe\left[V_Lg_L^*+V_Rg_R^*\right](1+x_w^2) + 4\frac{m_t}{M_W}x_b^2Re\left[V_Lg_R^*+V_Rg_L^*\right]\\
&- 2\frac{m_t^2}{M_W^2}x_b^2(|g_L|^2+|g_R|^2)(1+x_w^2) - \frac{m_t^2}{M_W^2}x_b^2(1-x_w^2)(|g_L|^2+|g_R|^2) - 2\frac{m_t^2}{M_W^2}x_b^3Re[g_Lg_R^*],\\
B_1 = {}& -\left(|V_L|^2+|V_R|^2\right) + \frac{m_t^2}{M_W^2}\left(|g_L|^2-|g_R|^2\right) + 2\frac{m_t}{M_W}Re\left[V_Lg_R^*-V_Rg_L^*\right]\\
&+ 2\frac{m_t}{M_W}x_bRe\left[V_Lg_L^*-V_Rg_R^*\right] - \frac{m_t^2}{M_W^2}x_b^2(|g_L|^2-|g_R|^2),\\
C_0 = {}& \left(|V_L|^2+|V_R|^2+|g_L|^2+|g_R|^2\right)(1-x_w^2) - \frac{m_t}{M_W}Re\left[V_Rg_L^*+V_Lg_R^*\right](1-x_w^4)\\
&- 2x_bRe\left[V_LV_R^*+g_Lg_R^*\right](1+x_w^2) + 4x_bx_wRe\left[V_Lg_L^*+V_Rg_R^*\right]\\
&- 2x_b^2\frac{m_t^2}{M_W^2}(|g_L|^2+|g_R|^2)(1-2x_w^2) + 2x_b^2\frac{m_t}{M_W}Re\left[V_Lg_R+g_LV_R\right](1+x_w^2)\\
&- 2x_b^3\frac{m_t^2}{M_W^2}Re\left[g_Lg_R^*\right](1+x_w^2),\\
C_1 = {}& -2\left(-|V_L|^2+|V_R|^2+|g_L|^2-|g_R|^2\right) + 2\frac{m_t}{M_W}Re\left[-V_Rg_L^*+V_Lg_R^*\right](1+x_w^2)\\
&- 4x_b^2\frac{m_t^2}{M_W^2}\left[(|g_R|^2-|g_L|^2)(1-x_w^2)+(|g_L|^2+|g_R|^2)x_w^2\right]\frac{1}{m_t^2-M_W^2},
\end{aligned} \tag{4.2.13}$$

$$
\begin{aligned}
D_0 &= \frac{m_t}{M_W} Im\left[V_R g_L^* + V_L g_R^*\right](1 - x_w^2), \\
D_1 &= -2\frac{m_t}{M_W} Im\left[V_L g_R^* - V_R g_L^*\right](1 - x_w^2) - 4x_b Im\left[V_L V_R^* + g_L g_R^*\right] + 4x_b^2 \frac{m_t}{M_W} Im\left[V_L g_R^* + g_L V_R^*\right] \\
&+ 4x_b^3 \frac{m_t^2}{M_W^2} Im\left[g_L g_R^*\right], \qquad (4.2.14)
\end{aligned}
$$

notice that $x_b = m_b/m_t$ and $x_w = M_W/m_t$. The dimensionless parameters in (4.2.13) are matching with [8] up to $\mathcal{O}(m_b)$ order. By synchronizing the density matrix in (4.2.12) with the density matrix parameterized by expectation values of spin observables, we can obtain the information of polarized quantities. However, the density matrix (4.2.12) have not normalized yet, to do this we exploit the unit trace condition:

$$
C \int_{-1}^{1} \sum_i \rho_{ii} d(\cos\theta) = 1 \qquad (4.2.15)
$$

Substituting the matrix elements in (4.2.12) we obtain the normalized factor as follows:

$$
C = \frac{1}{2\left[A_0 + 2B_0\right]} \qquad (4.2.16)
$$

After that, integrating over $\cos\theta$ and synchronizing with (4.2.12) we shall have

$$
\begin{aligned}
&\langle S_z \rangle = 4\frac{q}{m_t}\frac{B_1}{A_0 + 2B_0}, \qquad \langle S_x \rangle = \frac{\pi}{2}\frac{m_t}{M_W}\frac{C_0}{A_0 + 2B_0}, \qquad \langle S_y \rangle = -\frac{\pi}{2}\frac{q}{M_W}\frac{D_1}{A_0 + 2B_0}, \\
&\langle T_0 \rangle = \sqrt{\frac{2}{3}}\frac{B_0 - A_0}{A_0 + 2B_0}, \qquad \langle A_1 \rangle = -\frac{\pi}{4}\frac{q}{M_W}\frac{C_1}{A_0 + 2B_0}, \qquad \langle A_2 \rangle = \frac{\pi}{4}\frac{m_t}{M_W}\frac{D_0}{A_0 + 2B_0} \\
&\langle B_1 \rangle = \langle B_2 \rangle = 0. \qquad (4.2.17)
\end{aligned}
$$

Now we can see that eight spin-observables are clacuated implying the appearence of dimension-six operators. In order to probe a new physics effect, g_R is the best candidate since its interference terms with V_L are not multiply with the tiny factor x_b.

4.3 Physical results

The goal of probing new physics motivated us to look for the numerical vaule of each spin observables in (4.2.17). In order to cross-checked the results of Ref. [9] we first using their input parameters which is consistency with Ref. [10] as follows:

$$
m_{top} = 172.5 \text{ GeV}, \qquad M_W = 80.385 \text{ GeV}, \qquad m_b = 4.18 \text{ GeV} \qquad (4.3.1)
$$

For the numerical values of g_R, we chose two values to explore the effect of the real and imaginary part of g_R. Here we present the prediction of W-boson spin observables in polarized top-quark decay including dimension-six operators:

		$< S_1 >$	$< S_2 >$	$< S_3 >$	$< T_0 >$	$< A_1 >$	$< A_2 >$
SM	Ref. [9]	0.510	0	-0.302	-0.445	0.255	0
	Results 1	0.510	0	-0.303	-0.445	0.255	0
	δ[%]	0%	0%	0.3%	0%	0%	0%
$g_R = 0.03$	Ref. [9]	0.500	0	**-0.278**	-0.472	0.249	0
	Results 1	0.499	0	**-0.281**	-0.472	0.249	0
	δ[%]	0.2%	0%	**1.08%**	0%	0%	0%
$g_R = 0.10i$	Ref. [9]	0.507	-0.084	**-0.284**	-0.434	0.253	-0.042
	Results1	0.508	-0.084	**-0.312**	-0.434	0.254	- 0.042
	δ[%]	0.2%	0%	**9.85%**	0%	0.3%	0%

Table 4.1: Cross-check with Ref. [9] for W spin observables in polarized top-quark decays with the input parameters in 4.3.1.

Here we denote $\delta[\%] = \left|\frac{Results1 - Ref.[9]}{Ref.[9]}\right| \times 100$ is a relative deviation. As we can see, most of result in Table 4.1 agree with Ref. [9] except $\langle S_3 \rangle$ values in case of $g_R = 0.03$ and $g_R = 0.10i$. We are trying to re-check this values again and contact with the authors of Ref. [9] to let them check their results.

From the Table 4.1, the measurement of these spin observables of W-boson will help us select which physical quatities are sensitive with the anomalous coupling g_R as well as increase the sensitivity for new physics effects in top-quark.

Summary and outlook

Summary

In this part we summarize all what we have done in this thesis. First of all, let us consider the scattering process $e^+ + e^- \longrightarrow t + \bar{t}$ in the first three chapters:

- We first calculated this process in case of QED, SM and also generate several distributions to help us compare. In SM, we could depict the forward-backward asymmetry which does not appear in QED. Notice that, we have worked in unitary-gauge.
- We have used program FORM [5] to calculate the Feynman squared amplitude in both SM and SMEFT case. The total cross-section has been calculated by Mathematica 10.4, we also used this program to plot the graphs in this thesis.
- For SMEFT, we have cross-checked the new Feynman rules and agreed with [7]. Furthermore, in case of SMEFT, we have calculated this process in R_ξ-gauge. After that we study the effect of dimension-six operators on forward-backward asymmetry and some distributions.

For the final chapter we obtain the following results:

- We have re-constructed the density matrix of W-boson and identified eight spin-observables of W-boson by the angular distributions and asymmetries.
- We applied this method to polarized top-quark decays and re-producted the prediction for numerical values of W spin observables. Most of results agreed with [9] except the longitudinal polarization $\langle S_3 \rangle$ with the relative deviation of 9.85% in case of $g_R = 0.1i$.

Outlook

- In this thesis, the total cross-sections are considerd unpolarization, in the future we may re-calculate with polarized case.
- In chapter 3, we have just calculated theoretically. The purpose of this chapter is to show the effect of Willsion coefficients, however we have not compare with experimental measurements. In future we need to fit these parameters using expermental data.

Appendix A

Further calculations in SMEFT

A.1 Operators for Higgs sector

$$(D_\mu\phi)^\dagger (D^\mu\phi) \supset \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \left(\partial_\mu\Phi^-\right)\left(\partial^\mu\Phi^+\right) + \frac{1}{2}\left(\partial_\mu\Phi^0\right)\left(\partial^\mu\Phi^0\right) \tag{A.1}$$

$$C_\phi Q_\phi \supset C_\phi \left[\frac{1}{4}v^4H^2 + \frac{1}{2}v^4H^2 + v^4H^2\right] = C_\phi\frac{15}{8}v^4H^2 \tag{A.2}$$

$$C_{\phi\Box}Q_{\phi\Box} \supset C_{\phi\Box}\left[\partial^\mu\left[(\phi^\dagger\phi)\partial_\mu(\phi^\dagger\phi)\right] - \partial^\mu(\phi^\dagger\phi)\partial_\mu(\phi^\dagger\phi)\right] = -C_{\phi\Box}v^2(\partial_\mu H)(\partial^\mu H) \tag{A.3}$$

$$C_{\phi D}Q_{\phi D} \supset C_{\phi D}\left[\frac{1}{4}v^2(\partial_\mu H)^2 + \frac{1}{4}v^2(\partial_\mu\Phi^0)^2\right] \tag{A.4}$$

A.2 Operators for $\gamma ee, Zee$ and G^0ee vertex

$$\begin{aligned}C_{eW}Q_{eW} + C^*_{eW}Q^\dagger_{eW} \supset & -C_{eW}\frac{v}{\sqrt{2}}\left[Z_{\mu\nu}\left(c_w + \frac{\epsilon}{2}s_w\right) + A_{\mu\nu}\left(s_w - \frac{\epsilon}{2}c_w\right)\right](\bar{e}_L\sigma^{\mu\nu}e_R)\\ & - C^*_{eW}\frac{v}{\sqrt{2}}\left[Z_{\mu\nu}\left(c_w + \frac{\epsilon}{2}s_w\right) + A_{\mu\nu}\left(s_w - \frac{\epsilon}{2}c_w\right)\right](\bar{e}_R\sigma^{\mu\nu}e_L)\end{aligned} \tag{A.1}$$

$$\begin{aligned}C_{eB}Q_{eB} + C^*_{eB}Q^\dagger_{eB} \supset & \, C_{eB}\frac{v}{\sqrt{2}}\left[-Z_{\mu\nu}\left(s_w + \frac{\epsilon}{2}c_w\right) + A_{\mu\nu}\left(c_w - \frac{\epsilon}{2}s_w\right)\right](\bar{e}_L\sigma^{\mu\nu}e_R)\\ & + C^*_{eB}\frac{v}{\sqrt{2}}\left[-Z_{\mu\nu}\left(s_w + \frac{\epsilon}{2}c_w\right) + A_{\mu\nu}\left(c_w - \frac{\epsilon}{2}s_w\right)\right](\bar{e}_R\sigma^{\mu\nu}e_L)\end{aligned} \tag{A.2}$$

$$\begin{aligned}C^{(1)}_{\phi l}Q^{(1)}_{\phi l} \supset C^{(1)}_{\phi l}\Bigg[& -v\partial_\mu\Phi^0 + \frac{v^2}{2}\bar{g}_2\left(Z_\mu\left[c_w + \frac{\epsilon}{2}s_w\right] + A_\mu\left[s_w - \frac{\epsilon}{2}c_w\right]\right)\\ & -\frac{v^2}{2}\bar{g}_1\left(-Z_\mu\left[s_w + \frac{\epsilon}{2}c_w\right] + A_\mu\left[c_w - \frac{\epsilon}{2}s_w\right]\right)\Bigg](\bar{e}_L\gamma^\mu e_L)\end{aligned} \tag{A.3}$$

(A.4)

$$C_{\phi l}^{(3)} Q_{\phi l}^{(3)} \supset C_{\phi l}^{(3)} \left[-v\partial_\mu \Phi^0 + \frac{v^2}{2}\bar{g}_2 \left(Z_\mu \left[c_w + \frac{\epsilon}{2} s_w \right] + A_\mu \left[s_w - \frac{\epsilon}{2} c_w \right] \right) \right.$$
$$\left. - \frac{v^2}{2}\bar{g}_1 \left(-Z_\mu \left[s_w + \frac{\epsilon}{2} c_w \right] + A_\mu \left[c_w - \frac{\epsilon}{2} s_w \right] \right) \right] (\bar{e}_L \gamma^\mu e_L) \tag{A.5}$$

$$C_{\phi e} Q_{\phi e} \supset C_{\phi e} \left[-v\partial_\mu \Phi^0 + \frac{v^2}{2} Z_\mu \left(\bar{g}_2 \left[c_w + \frac{\epsilon}{2} s_w \right] + \bar{g}_1 \left[s_w + \frac{\epsilon}{2} c_w \right] \right) \right.$$
$$\left. + \frac{v^2}{2} A_\mu \left(\bar{g}_2 \left[s_w - \frac{\epsilon}{2} c_w \right] - \bar{g}_1 \left[c_w - \frac{\epsilon}{2} s_w \right] \right) \right] (\bar{e}_R \gamma^\mu e_R) \tag{A.6}$$

A.3 Operators for $\gamma tt, Ztt$ and $G^0 tt$ vertex

$$C_{uW} Q_{uW} + C_{uW}^* Q_{uW}^\dagger \supset C_{uW} \frac{v}{\sqrt{2}} \left[Z_{\mu\nu} \left(c_w + \frac{\epsilon}{2} s_w \right) + A_{\mu\nu} \left(s_w - \frac{\epsilon}{2} c_w \right) \right] (\bar{t}_L \sigma^{\mu\nu} t_R)$$
$$+ C_{uW}^* \frac{v}{\sqrt{2}} \left[Z_{\mu\nu} \left(c_w + \frac{\epsilon}{2} s_w \right) + A_{\mu\nu} \left(s_w - \frac{\epsilon}{2} c_w \right) \right] (\bar{t}_R \sigma^{\mu\nu} t_L) \tag{A.1}$$

$$C_{uB} Q_{uB} + C_{uB}^* Q_{uB}^\dagger \supset C_{uB} \frac{v}{\sqrt{2}} \left[-Z_{\mu\nu} \left(s_w + \frac{\epsilon}{2} c_w \right) + A_{\mu\nu} \left(c_w - \frac{\epsilon}{2} s_w \right) \right] (\bar{t}_L \sigma^{\mu\nu} t_R)$$
$$+ C_{uB}^* \frac{v}{\sqrt{2}} \left[-Z_{\mu\nu} \left(s_w + \frac{\epsilon}{2} c_w \right) + A_{\mu\nu} \left(c_w - \frac{\epsilon}{2} s_w \right) \right] (\bar{t}_R \sigma^{\mu\nu} t_L) \tag{A.2}$$

$$C_{\phi q}^{(1)} Q_{\phi q}^{(1)} \supset C_{\phi q}^{(1)} \left[-v\partial_\mu \Phi^0 + \frac{v^2}{2}\bar{g}_2 \left(Z_\mu \left[c_w + \frac{\epsilon}{2} s_w \right] + A_\mu \left[s_w - \frac{\epsilon}{2} c_w \right] \right) \right.$$
$$\left. + \frac{v^2}{2}\bar{g}_1 \left(Z_\mu \left[s_w + \frac{\epsilon}{2} c_w \right] + A_\mu \left[c_w - \frac{\epsilon}{2} s_w \right] \right) \right] (\bar{t}_L \gamma^\mu t_L) \tag{A.3}$$

$$C_{\phi q}^{(3)} Q_{\phi q}^{(3)} \supset C_{\phi q}^{(3)} \left[v\partial_\mu \Phi^0 - \frac{v^2}{2}\bar{g}_2 \left(Z_\mu \left[c_w + \frac{\epsilon}{2} s_w \right] + A_\mu \left[s_w - \frac{\epsilon}{2} c_w \right] \right) \right.$$
$$\left. + \frac{v^2}{2}\bar{g}_1 \left(-Z_\mu \left[s_w + \frac{\epsilon}{2} c_w \right] + A_\mu \left[c_w - \frac{\epsilon}{2} s_w \right] \right) \right] (\bar{t}_L \gamma^\mu t_L) \tag{A.4}$$

$$C_{\phi u} Q_{\phi u} \supset C_{\Phi u} \left[-v\partial_\mu \Phi^0 + \frac{v^2}{2} Z_\mu \left(\bar{g}_2 \left[c_w + \frac{\epsilon}{2} s_w \right] + \bar{g}_1 \left[s_w + \frac{\epsilon}{2} c_w \right] \right) \right.$$
$$\left. + \frac{v^2}{2} A_\mu \left(\bar{g}_2 \left[s_w - \frac{\epsilon}{2} c_w \right] + \bar{g}_1 \left[c_w - \frac{\epsilon}{2} s_w \right] \right) \right] (\bar{t}_R \gamma^\mu t_R) \tag{A.5}$$

A.4 Operators for four-fermion vertex

$$C_{lq}^{(1)} Q_{lq}^1 \supset C_{lq}^{(1)} \left(\bar{e} \gamma^\mu P_L e \right)_{ss'} \left(\bar{t} \gamma_\mu P_L t \right)_{rr'} \tag{A.1}$$
$$C_{lq}^{(3)} Q_{lq}^3 \supset -C_{lq}^{(3)} \left(\bar{e} \gamma^\mu P_L e \right)_{ss'} \left(\bar{t} \gamma_\mu P_L t \right)_{rr'} \tag{A.2}$$
$$C_{eu} Q_{eu} \supset C_{eu} \left(\bar{e} \gamma^\mu P_R e \right)_{ss'} \left(\bar{t} \gamma_\mu P_R t \right)_{rr'} \tag{A.3}$$

$$C_{lu}Q_{lu} \supset C_{lu}\left(\bar{e}\gamma^{\mu}P_L e\right)_{ss'}\left(\bar{t}\gamma_{\mu}P_R t\right)_{rr'} \tag{A.4}$$

$$C_{qe}Q_{qe} \supset C_{qe}\left(\bar{t}\gamma^{\mu}P_L t\right)_{rr'}\left(\bar{e}\gamma_{\mu}P_R e\right)_{ss'} \tag{A.5}$$

$$C^{(1)}_{lequ}Q^{(1)}_{lequ} + C^{(1)*}_{lequ}Q^{(1)\dagger}_{lequ} \supset -\left[C^{1*}_{lequ}\left(\bar{e}P_L e\right)_{ss'}\left(\bar{t}P_L t\right)_{rr'} + C^{1}_{lequ}\left(\bar{e}P_R e\right)_{ss'}\left(\bar{t}P_R t\right)_{rr'}\right] \tag{A.6}$$

$$C^{(3)}_{lequ}Q^{(3)}_{lequ} + C^{(3)*}_{lequ}Q^{(3)\dagger}_{lequ} \supset -\left[C^{3*}_{lequ}\left(\bar{e}\sigma^{\mu\nu}P_L e\right)_{ss'}\left(\bar{t}\sigma_{\mu\nu}P_L t\right)_{rr'} + C^{3}_{lequ}\left(\bar{e}\sigma^{\mu\nu}P_R e\right)_{ss'}\left(\bar{t}\sigma_{\mu\nu}P_R t\right)_{rr'}\right] \tag{A.7}$$

A.5 Feynman squared amplitude

Diagonal terms:

$$\begin{aligned}\left|\bar{\mathcal{M}}_{\gamma}\right|^2 = &\frac{64E^2e_0^4}{3s^2}\left(1-2G_1G_2Re\left[C_{\phi WB}\right]\right)\left[E^2+k^2cos^2\theta+m_t^2\right]\\ &-\frac{64E^2e_0^3}{s^2}m_t\left(G_1Re[C_{uW}]+G_2Re[C_{uB}]\right)\left[k^2+3E^2+m_t^2\right]\end{aligned} \tag{A.1}$$

$$\begin{aligned}\left|\bar{\mathcal{M}}_{Z}\right|^2 = &\frac{48E^2}{(s-M_Z^2)^2}\Big[(g_V^{e\,2}+g_A^{e\,2})(g_V^{t\,2}+g_A^{t\,2})(E^2+k^2cos^2\theta)\\
&\quad + 8Ek\cos\theta g_V^e g_A^e g_V^t g_A^t + m_t^2(g_V^{e\,2}+g_A^{e\,2})(g_V^{t\,2}-g_A^{t\,2})\Big]\\
&+\frac{48E^2G_3}{(s-M_Z^2)^2}Re\left[C_{\phi u}-C^1_{\phi q}+C^3_{\phi q}\right]\left[g_A^t(g_V^{e\,2}+g_A^{e\,2})(E^2+k^2cos^2\theta-m_t^2)+4Ek\cos\theta g_V^e g_A^e g_V^t\right]\\
&+\frac{48E^2G_3}{(s-M_Z^2)^2}Re\left[C^1_{\phi q}-C^3_{\phi q}+C_{\phi u}\right]\left[g_V^t(g_V^{e\,2}+g_A^{e\,2})(E^2+k^2cos^2\theta+m_t^2)+4Ek\cos\theta g_V^e g_A^e g_A^t\right]\\
&-\frac{48E^2G_3}{(s-M_Z^2)^2}Re\left[C_{\phi e}-C^1_{\phi l}-C^3_{\phi l}\right]\Big[g_A^e(g_V^{t\,2}+g_A^{t\,2})(E^2+k^2cos^2\theta)\\
&\quad +4Ek\cos\theta g_V^e g_V^t g_A^t - m_t^2 g_A^e(g_A^{t\,2}-g_V^{t\,2})\Big]\\
&-\frac{48E^2G_3}{(s-M_Z^2)^2}Re\left[C^1_{\phi l}+C^3_{\phi l}+C_{\phi e}\right]\Big[g_V^e(g_V^{t\,2}+g_A^{t\,2})(E^2+k^2cos^2\theta)\\
&\quad +4Ek\cos\theta g_A^e g_V^t g_A^t - m_t^2 g_V^e(g_A^{t\,2}-g_V^{t\,2})\Big]\\
&+\frac{96E^2m_t}{(s-M_Z^2)^2}\left(G_2Re[C_{uW}]-G_1Re[C_{uB}]\right)\\
&\quad\times\left[g_V^t(g_A^{e\,2}+g_V^{e\,2})(3E^2+k^2+m_t^2)+8Ek\cos\theta g_V^e g_A^e g_A^t\right]\\
&+\frac{96E^2}{(s-M_Z^2)^2}v^2Re\left[C_{\phi WB}\right]\\
&\quad\times\Big\{(E^2+k^2cos^2\theta)\left[(g_A^e z_A^e-g_V^e z_V^e)(g_A^{t\,2}+g_V^{t\,2})+(g_A^{e\,2}+g_V^{e\,2})(g_A^t z_A^t-g_V^t z_V^t)\right]\\
&\quad +4g_V^t g_A^t(-g_A^e z_V^e+g_V^e z_A^e)+4g_V^e g_A^e(-g_A^t z_V^t+g_V^t z_A^t)\Big\}\\
&+\frac{96E^2m_t^2}{(s-M_Z^2)^2}v^2Re\left[C_{\phi WB}\right]\left[(g_A^e z_A^e-g_V^e z_V^e)(-g_A^{t\,2}+g_V^{t\,2})-(g_A^{e\,2}+g_V^{e\,2})(g_A^t z_A^t+g_V^t z_V^t)\right]\end{aligned} \tag{A.2}$$

Interference terms:

$$\mathcal{M}_{4-f}\mathcal{M}_\gamma^\dagger + \mathcal{M}_\gamma^\dagger\mathcal{M}_{4-f} = \frac{16E^2e_0}{s}Re\left[C_{qe}+C_{lu}\right](-k^2cos^2\theta+2Ek\cos\theta-E^2-m_t^2)$$
$$+\frac{16E^2e_0}{s}Re\left[-C_{eu}+C_{lq}^3-C_{lq}^1\right](k^2cos^2\theta+2Ek\cos\theta+E^2+m_t^2) \quad (A.3)$$

$$\mathcal{M}_Z\mathcal{M}_\gamma^\dagger + M_\gamma^\dagger\mathcal{M}_Z = \frac{64E^2e_0^2}{s(s-M_Z^2)}\left\{g_V^eg_V^t(E^2+k^2cos^2\theta+m_t^2)+2g_A^eg_A^tEk\cos\theta\right\}$$
$$+\frac{64E^2e_0^2}{s(s-M_Z^2)}G_1G_2Re[C_{\phi WB}]\left\{g_V^eg_V^t(E^2+k^2cos^2\theta+m_t^2)+2g_A^eg_A^tEk\cos\theta\right\}$$
$$+\frac{64e_0^2}{s(s-M_Z^2)}G_3g_A^eRe\left[C_{\phi u}-C_{\phi q}^1+C_{\phi q}^3\right]E^3k\cos\theta$$
$$-\frac{64e_0^2}{s(s-M_Z^2)}G_3g_A^tRe\left[C_{\phi e}-C_{\phi l}^1-C_{\phi l}^3\right]E^3k\cos\theta$$
$$+\frac{32E^2e_0^2}{s(s-M_Z^2)}G_3g_V^eRe\left[C_{\phi q}^1-C_{\phi q}^3+C_{\phi u}\right](E^2+k^2cos^2\theta)$$
$$-\frac{32E^2e_0^2}{s(s-M_Z^2)}G_3g_V^tRe\left[C_{\phi l}^1+C_{\phi l}^3+C_{\phi e}\right](E^2+k^2cos^2\theta)$$
$$-\frac{128E^2e_0^2}{s(s-M_Z^2)}G_1G_2Re\left[C_{\phi WB}\right]\left\{g_V^eg_V^t[E^2+k^2cos^2\theta+m_t^2]+2g_A^eg_A^tEk\cos\theta\right\}$$
$$-\frac{64E^2e_0^2}{s(s-M_Z^2)}v^2Re\left[C_{\phi WB}\right]\left\{(g_V^tz_V^e+g_V^ez_V^t)(E^2+k^2cos^2\theta+m_t^2)\right.$$
$$\left.-2Ek\cos\theta(g_A^tz_A^t+g_A^ez_A^t)\right\}$$
$$-\frac{96E^2e_0}{s(s-M_Z^2)}m_t\left(G_1Re[C_{uW}]+G_2Re[C_{uB}]\right)\left[g_V^eg_V^t(k^2+3E^2)+4g_A^eg_A^tEk\cos\theta\right]$$
$$+\frac{64E^2e_0^2}{s(s-M_Z^2)}m_t\left(G_2Re[C_{uW}]-G_1Re[C_{uB}]\right)g_V^e(k^2+3E^2)$$
$$+\frac{32E^2e_0^2}{s(s-M_Z^2)}G_3\left(g_V^eRe\left[C_{\phi q}^1-C_{\phi q}^3+C_{\phi u}\right]-g_V^tRe\left[C_{\phi l}^1+C_{\phi l}^3+C_{\phi e}\right]\right)$$
$$-\frac{96E^2e_0}{s(s-M_Z^2)}m_t^3g_V^eg_V^t\left(G_1Re[C_{uW}]+G_2Re[C_{uB}]\right)$$
$$+\frac{64E^2e_0^2}{s(s-M_Z^2)}m_t^3g_V^e\left(-G_1Re[C_{uB}]+G_2Re[C_{uW}]\right) \quad (A.4)$$

$$
\begin{aligned}
\mathcal{M}_{4-f}\mathcal{M}_Z^\dagger + \mathcal{M}_Z^\dagger\mathcal{M}_{4-f} = & \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta - 2Ek\cos\theta + E^2 - m_t^2\right] Re[C_{qe}]\left(g_A^e + g_V^e\right)g_A^t \\
& + \frac{24E^2}{s-M_Z^2}\left[-k^2cos^2\theta + 2Ek\cos\theta - E^2 - m_t^2\right] Re[C_{qe}]\left(g_A^e + g_V^e\right)g_V^t \\
& + \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta - 2Ek\cos\theta + E^2 - m_t^2\right] Re[C_{lu}]\left(g_A^e - g_V^e\right)g_A^t \\
& + \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta - 2Ek\cos\theta + E^2 + m_t^2\right] Re[C_{lu}]\left(g_A^e - g_V^e\right)g_V^t \\
& + \frac{24E^2}{s-M_Z^2}\left[-k^2cos^2\theta - 2Ek\cos\theta - E^2 + m_t^2\right] Re[C_{eu}]\left(g_A^e + g_V^e\right)g_A^t \\
& - \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta + 2Ek\cos\theta + E^2 + m_t^2\right] Re[C_{eu}]\left(g_A^e + g_V^e\right)g_V^t \\
& + \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta + 2Ek\cos\theta + E^2 - m_t^2\right]\left(Re[C_{lq}^3] + Re[C_{lq}^1]\right)\left(g_A^e - g_V^e\right)g_A^t \\
& - \frac{24E^2}{s-M_Z^2}\left[k^2cos^2\theta + 2Ek\cos\theta + E^2 + m_t^2\right]\left(Re[C_{lq}^3] - Re[C_{lq}^1]\right)\left(g_A^e - g_V^e\right)g_V^t
\end{aligned}
\tag{A.5}
$$

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