# Study Status

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# Non-unitary mixing matrix

# Introduction

The $n \times n$ mixing matrix will indeed to be unitary to preserve probability, the same is not true for any given $m \times m$, with $m < n$

$$U_{\text{PMNS}}^{\text{Extended}} = \begin{pmatrix} \overbrace{\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}}^{U_{\text{PMNS}}^{3\text{x}3}} & \begin{matrix} \cdots & U_{en} \\ \cdots & U_{\mu n} \\ \cdots & U_{\tau n} \end{matrix} \\ \begin{matrix} \vdots & \vdots & \vdots \\ U_{s_n 1} & U_{s_n 2} & U_{s_n 3} \end{matrix} & \begin{matrix} \ddots & \vdots \\ \cdots & U_{s_n n} \end{matrix} \end{pmatrix}$$

# The effective Lagrangian

The effective low-energy Lagrangian in the mass basic

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{1}{2}(\overline{\nu}_i i\partial\nu_i - \overline{\nu^c}_i m_i\nu_i + hc) \\ &- \frac{g}{2\sqrt{2}}(W^+_\mu \bar{l}_\alpha\gamma_\mu(1-\gamma_5)N_{\alpha i}\nu_i + hc) \\ &- \frac{g}{2\cos\theta_W}(Z_\mu\overline{\nu}_i\gamma^\mu(1-\gamma_5)(N^\dagger N)_{ij}\nu_j + hc) + \cdots \quad (1)\end{aligned}$$

The relation between mass and flavour eigenstates

$$\nu_\alpha = N_{\alpha i} \nu_i \tag{2}$$

Mass eigenstates are orthonormal $\langle \nu_i | \nu_j \rangle = \delta_{ij}$
Flavour eigenstates are not orthonormal

$$\langle \nu_\beta | \nu_\alpha \rangle = (\tilde{N} \tilde{N}^\dagger)_{\beta\alpha} \neq \delta_{\alpha\beta}$$

where $\tilde{N}$ is the normalization factor

The effective Lagrangian in the flavour basis

$$\mathcal{L}_{eff} = \frac{1}{2}\Big(i\overline{\nu}_\alpha \partial (NN^\dagger)^{-1}_{\alpha\beta}\nu_\beta - \overline{\nu^c}_\alpha[(N^{-1})^t m N^{-1}]_{\alpha\beta}\nu_\beta + hc\Big)$$
$$- \frac{g}{2\sqrt{2}}[W^+_\mu \bar{l}_\alpha \gamma_\mu (1-\gamma_5)\nu_\alpha + hc]$$
$$- \frac{g}{2\cos\theta_W}[Z_\mu \overline{\nu}_\alpha \gamma^\mu (1-\gamma_5)\nu_\alpha + hc] + \cdots \qquad (3)$$

# Neutrino oscillations in vacuum

We have

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}}} \sum N^*_{\alpha i}|\nu_i\rangle = \sum_i \tilde{N}^*_{\alpha i}|\nu_i\rangle \tag{4}$$

$$\begin{aligned} |\nu_\alpha(L)\rangle &= \sum_i \tilde{N}^*_{\alpha i}|\nu_i(L)\rangle = \sum_i \tilde{N}^*_{\alpha i} e^{-i\phi}\ |\nu_i(0)\rangle \\ &= \sum_i \tilde{N}^*_{\alpha i} e^{-i\phi}\ (\tilde{N}^*_{\beta i})^{-1}|\nu_\beta\rangle \end{aligned} \tag{5}$$

The probability amplitude of flavour transition $\nu_\alpha \to \nu_\beta$ after distance L

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(L) \rangle = \sum_i \tilde{N}^*_{\alpha i} e^{-i\phi} \tilde{N}_{\beta i}$$
$$= \frac{1}{\sqrt{(NN^\dagger)_{\alpha\alpha}(NN^\dagger)_{\beta\beta}}} \sum_i N^*_{\alpha i} e^{-i\phi} N_{\beta i} \qquad (6)$$

The oscillation probability

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta} =& |A_{\nu_\alpha \to \nu_\beta}|^2 = \frac{1}{(NN^\dagger)_{\alpha\alpha}(NN^\dagger)_{\beta\beta}} \Big| \sum_i N^*_{\alpha i} e^{-i\phi} N_{\beta i} \Big|^2 \\ =& \frac{1}{(NN^\dagger)_{\alpha\alpha}(NN^\dagger)_{\beta\beta}} \Big( \sum_i |N^*_{\alpha i}|^2 |N_{\beta i}|^2 \\ &+ 2 \sum_{j>i} Re[N^*_{\alpha i} N_{\beta i} N_{\alpha i} N^*_{\beta i}] \cos \frac{\Delta m^2_{ji} L}{2E} \\ &- 2 \sum_{j>i} Im[N^*_{\alpha i} N_{\beta i} N_{\alpha i} N^*_{\beta i}] \sin \frac{\Delta m^2_{ji} L}{2E} \Big) \end{aligned} \quad (7)$$