A review of Electroweak interaction

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The Electromagnetic interaction in QED[1] is

$$-iej_{\mu}^{em}A^{\mu} = -ie(\overline{\psi}\gamma_{\mu}Q\psi)A^{\mu} \quad (1)$$

The Lagrangian of QED[1]:

$$\mathcal{L} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - e\overline{\psi}\gamma^{\mu}Q\psi A_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad (2)$$

Two basic Weak interactions[1]

$$-ig\mathbf{J}_{\mu}\mathbf{W}^{\mu} = -ig\overline{\varphi}_L\gamma_{\mu}\mathbf{T}.\mathbf{W}^{\mu}\varphi_L \quad (3)$$

$$-i\frac{g'}{2}j_{\mu}^{Y}B^{\mu} = -ig'\overline{\psi}\gamma_{\mu}\frac{Y}{2}\psi B^{\mu} \quad (4)$$

We introduce A_{μ} and Z_{μ} are the neutral gauge fields have the form

$$A_{\mu} = B_{\mu}\cos\theta_W + W_{\mu}^3\sin\theta_W \quad (5)$$

$$Z_{\mu} = -B_{\mu}\sin\theta_W + W_{\mu}^3\cos\theta_W \quad (6)$$

We take together two interactions (3) and (4)
The Electroweak interaction is

$$-ig\mathbf{J}_\mu\mathbf{W}^\mu - i\frac{g'}{2}j_\mu^Y B^\mu \tag{7}$$

We have the Electroweak neutral current interaction

$$\begin{aligned}
-igJ_\mu^3(W^3)^\mu - i\frac{g'}{2}j_\mu^Y B^\mu \\
&= -i\left(gJ_\mu^3\sin\theta_W + \frac{g'}{2}j_\mu^Y\cos\theta_W\right)A^\mu \\
&\quad - i\left(gJ_\mu^3\cos\theta_W + \frac{g'}{2}j_\mu^Y\sin\theta_W\right)Z^\mu \\
&= -iej_\mu^{em}A^\mu - \frac{ie}{\sin\theta_W\cos\theta_W}\left(J_\mu^3 - \sin^2\theta_W j_\mu^{em}\right)Z^\mu
\end{aligned} \tag{8}$$

The Glashow - Weinberg - Salam theory

The requirement of the Electroweak Lagrangian is it must have an $SU(2) \times U(1)$ invariant form. For instance, the Lagrangian of the electron-neutrino pair:

$$\mathcal{L} = \overline{\varphi}_L \gamma_\mu \left[i\partial^\mu - g\frac{1}{2}\tau\mathbf{W}^\mu - g'(-\frac{1}{2})B^\mu \right] \varphi_L + \overline{e}_R \gamma_\mu \left[i\partial^\mu - g'(-1)B^\mu \right] e_R - \frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \quad (9)$$

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - g\mathbf{W}_\mu \times \mathbf{W}_\nu \quad (10)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (11)$$

We introduce the Lagrangian for the scalar fields (Higgs fields) ϕ that is $SU(2) \times U(1)$ invariant to add to the Lagrangian (9)

$$\mathcal{L}' = \left| \left(i\partial_\mu - g\mathbf{T}.\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu \right) \phi \right|^2 - V(\phi) \tag{12}$$

where

$$V(\phi) = \alpha^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2 \tag{13}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \text{ with } \phi^+ = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \text{ and } \phi^0 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \tag{14}$$

We get the vacuum expectation value

$$\phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \rho\end{pmatrix} \text{ with } T = \frac{1}{2}, T^3 = -\frac{1}{2}, Y = 1 \tag{15}$$

where $-\alpha^2/\lambda = \rho^2$

Substituting (15) into the Lagrangian (12) we have the term

$$\left|\left(i\partial_\mu - \frac{g}{2}\tau.\mathbf{W}_\mu - \frac{g'}{2}B_\mu\right)\phi\right|^2$$
$$= \frac{1}{4}(g\rho)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}\rho^2\left[g^2\left(W_\mu^3\right)^2 - 2gg'W_\mu^3 B^\mu + g'^2 B_\mu^2\right] \tag{16}$$

We have

$$M_W = \frac{1}{2}g\rho \tag{17}$$

Consider the term

$$N = \frac{1}{8}\rho^2 \left[g^2 \left(W_\mu^3\right)^2 - 2gg' W_\mu^3 B^\mu + g'^2 B_\mu^2 \right]$$
$$= \frac{1}{8}\rho^2 \left(gW_\mu^3 - g' B_\mu\right)^2 + 0 \left(gW_\mu^3 + g' B_\mu\right)^2$$
$$= \frac{1}{8}\rho^2 \left(\frac{g}{\cos\theta_W}\right)^2 Z_\mu^2 + 0 A_\mu^2 \quad (18)$$

Hence

$$M_A = 0, M_Z = \frac{1}{2}\frac{g\rho}{\cos\theta_W}$$

$$\mathcal{L}_1 = -G_e \left[\left(\overline{\nu}_e, \overline{e}\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \overline{e}_R \left(\phi^-, \overline{\phi^0}\right) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right] \tag{19}$$

After spontaneously breaking the symmetry, we choose

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \rho + h(x) \end{pmatrix}$$

$$\mathcal{L}_1 = -\frac{G_e}{\sqrt{2}} \rho \left(\overline{e}_L e_R + \overline{e}_R e_L\right) - \frac{G_e}{\sqrt{2}} \left(\overline{e}_L e_R h + \overline{e}_R e_L h\right) \tag{20}$$

Then

$$M_e = \frac{G_e \rho}{\sqrt{2}}$$

We include the quark Lagrangian

$$\mathcal{L}_2 = -G_d^{ij}\left(\overline{u}_i, \overline{d'}_i\right)_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_{jR} - G_u^{ij}\left(\overline{u}_i, \overline{d'}_i\right)_L \begin{pmatrix} -\overline{\phi^0} \\ \phi^- \end{pmatrix} u_{jR} + \text{h.c} \quad (21)$$

where $i, j = 1, 2, ..., n$ and n is the number of quark doublets

$$d'_i = \sum_j U_{ij} d_j \quad (22)$$

with U is the CKM matrix.

$$\mathcal{L}_2 = -M_d^i \overline{d}_i d_i \left(1 + \frac{h}{\rho}\right) - M_u^i \overline{u}_i u_i \left(1 + \frac{h}{\rho}\right) \quad (23)$$

The masses depend on the couplings G_d and G_u.

The complete Lagrangian in the Glashow - Weinberg - Salam theory:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \overline{L}\gamma_\mu\left[i\partial^\mu - g\frac{1}{2}\tau\mathbf{W}^\mu - g'\frac{Y}{2}B^\mu\right]L \\ &+ \overline{R}\gamma_\mu\left[i\partial^\mu - g'\frac{Y}{2}B^\mu\right]R + \left|\left(i\partial_\mu - g\frac{1}{2}\tau\mathbf{W}_\mu - g'\frac{Y}{2}B_\mu\right)\phi\right|^2 \\ &- V(\phi) - G_1\overline{L}\phi R + G_2\overline{L}\phi_c R + \text{hermitian conjugate} \qquad (24)\end{aligned}$$

The Neutrino

- The masses of Neutrinos are zero
- The left-handed Neutrinos are operative only

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{25}$$

We have

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{26}$$

For example, the oscillation between ν_e and ν_μ[2]. The states ν_1 and ν_2 at the time $t > 0$ is given by

$$\nu_1(t) = e^{-iE_1 t}\nu_1(0), \nu_2(t) = e^{-iE_2 t}\nu_2(0) \tag{27}$$

where $E_i^2 = \overrightarrow{p}^2 + m_i^2, i = 1, 2$ and since $m_i \ll |p_i|$, then $|p_i| \approx E$ and $E_i \approx E + m_i^2/2E$

The Neutrino

$$\begin{aligned} \nu_e(t) &= \left(e^{-iE_1 t}\cos^2\theta + e^{-iE_2 t}\sin^2\theta\right)\nu_e(0) \\ &\quad + \cos\theta\sin\theta\left(e^{-iE_2 t} - e^{-iE_1 t}\right)\nu_\mu(0) \end{aligned} \tag{28}$$

$$\begin{aligned} \nu_\mu(t) &= \left(e^{-iE_1 t}\sin^2\theta + e^{-iE_2 t}\cos^2\theta\right)\nu_\mu(0) \\ &\quad + \sin\theta\cos\theta\left(e^{-iE_2 t} - e^{-iE_1 t}\right)\nu_e(0) \end{aligned} \tag{29}$$

The probability for ν_μ at $t = 0$ remains ν_μ at $t > 0$ is

$$P_1 = |\langle \nu_\mu(t)|\nu_\mu(0)\rangle|^2 = 1 - \frac{1}{2}\sin^2 2\theta + \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{21}^2}{2E}t\right) \tag{30}$$

where $\Delta m_{21}^2 = m_2^2 - m_1^2$.
The probability for ν_μ at $t = 0$ to be converted into ν_e at $t > 0$ is

$$P_2 = |\langle \nu_e(t)|\nu_\mu(0)\rangle|^2 = \frac{1}{2}\sin^2 2\theta - \frac{1}{2}\sin^2 2\theta \cos\left(\frac{\Delta m_{21}^2}{2E}t\right) \tag{31}$$

The Neutrino

The conditions for oscillations are θ and Δm_{21}^2 have nonzero values, the traveling distance L of the Neutrinos must not be too different from the oscillation length L_0

$$L_0 = \frac{4\pi E}{|\Delta m_{21}^2|} \tag{32}$$

References

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