Weekly Report
Neutrino Oscillation

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Outlines

- Neutrino oscillation in vacuum
  - Survival probability
  - Transition probability
- Neutrino oscillation in matter
  - Effective potential in matter
  - MSW effect
  - General form of oscillation probability in mattel
  - Survival probability
  - Transition probability
- Next 2 weeks plan

Neutrino oscillation in vacuum

Survival probability $P(\nu_\mu \to \nu_\mu)$

- The general form of survival probability in vacuum

$$
\begin{aligned}
P(\nu_\mu \to \nu_\mu) = 1 \quad & - \quad 4|U_{\mu 1}|^2|U_{\mu 2}|^2 \sin^2 \Delta_{21} \\
& - \quad 4|U_{\mu 1}|^2|U_{\mu 3}|^2 \sin^2 \Delta_{31} \\
& - \quad 4|U_{\mu 2}|^2|U_{\mu 3}|^2 \sin^2 \Delta_{32} \qquad (1)
\end{aligned}
$$

- Where $\Delta_{ij} = \frac{\Delta m_{ij}^2 L}{4E}$.

Neutrino oscillation in vacuum

Survival probability $P(\nu_\mu \rightarrow \nu_\mu)$

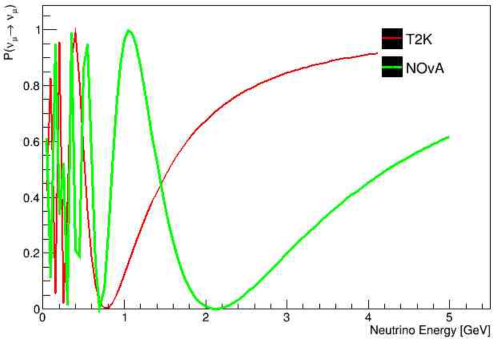
- For LBL neutrino oscillation experiments like T2K with $L = 295km$, $E = 0.6GeV$ and NOvA with $L = 810km$, $E = 2GeV$ we can ignore the Δ_{21} tearm and make an approximation

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta)_{23} \sin^2\left(\frac{1.27\Delta m_{32}^2[eV]^2 L[km]}{E[GeV]}\right) \quad (2)$$

Neutrino oscillation in vacuum

Survival probability $P(\nu_\mu \to \nu_\mu)$

- With the parameters: $\theta_{23} = 0.785, \Delta m^2_{23} = 2.42 \times 10^{-3} eV^2$
  The graph of survival probability with respect to the energy of T2K and NOvA is shown



Neutrino oscillation in vacuum

Survival probability $P(\nu_\mu \to \nu_\mu)$

- Complete formula

$$
\begin{aligned}
P\quad(\nu_\mu \to \nu_\mu) &= 1 - \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} s_{23}^2 c_{13} \cos\delta \\
&\times(\sin^2\Delta_{31} - \sin^2\Delta_{32}) \\
&- \left[\sin^2 2\theta_{12}(c_{23}^4 + s_{23}^4 s_{13}^4) + \sin^2 2\theta_{23}(s_{12}^4 + c_{12}^4) s_{13}^2\right. \\
&+ \sin 4\theta_{12} \sin 2\theta_{23}(c_{23}^2 - s_{23}^2 s_{13}^2) s_{13} \cos\delta \\
&\left.- \sin^2 2\theta_{12} \sin^2 2\theta_{23} s_{13}^2 \cos^2\delta\right] \sin^2\Delta_{21} \\
&- \left[c_{13}^2 \sin^2 2\theta_{23}(s_{12}^2 \sin^2\Delta_{31} + c_{12}^2 \sin^2\Delta_{32})\right. \\
&\left.- s_{23}^4 \sin^2 2\theta_{13}(c_{12}^2 \sin^2\Delta_{31} + s_{12}^2 \sin^2\Delta_{32})\right] \qquad (3)
\end{aligned}
$$

Neutrino oscillation in vacuum

Transition probability $P(\nu_\mu \to \nu_\mu)$

- The general form of transition probability

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= -4Re\left[U^*_{\mu 1} U_{e1} U_{\mu 2} U^*_{e2}\right] \sin^2 \Delta_{21} \\ &\quad -4Re\left[U^*_{\mu 1} U_{e1} U_{\mu 3} U^*_{e3}\right] \sin^2 \Delta_{31} \\ &\quad -4Re\left[U^*_{\mu 2} U_{e2} U_{\mu 3} U^*_{e3}\right] \sin^2 \Delta_{32} \\ &\quad +8Im\left[U^*_{\mu 1} U_{e1} U_{\mu_3} U^*_{e_3}\right] \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32} \end{aligned} \quad (4)$$

In which $\Delta_{ij} = \frac{\Delta m^2_{ij} L}{4E}$.

Neutrino oscillation in vacuum

Transition probability $P(\nu_\mu \to \nu_\mu)$

- Insert the PMNS matrix elements

$$
\begin{aligned}
&P(\nu_\mu \to \nu_e) = \qquad (5)\\
&\frac{1}{4}\left[4\sin^2 2\theta_{12}c_{13}^2c_{23}^2 - \sin^2 2\theta_{12}\sin^2 2\theta_{13}s_{23}^2\right.\\
&\left.+\sin 4\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}c_{13}\cos\delta\right]\sin^2\Delta_{21}\\
&+\sin^2 2\theta_{13}s_{23}^2(c_{12}^2\sin^2\Delta_{31}+s_{12}^2\sin^2\Delta_{32})\\
&+\frac{1}{2}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}c_{13}\cos\delta(\sin^2\Delta_{31}-\sin^2\Delta_{32})\\
&-\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}c_{13}\sin\delta\sin\Delta_{21}\sin\Delta_{31}\sin\Delta_{32}
\end{aligned}
$$

Neutrino oscillation in vacuum

Transition probability $P(\nu_\mu \to \nu_\mu)$

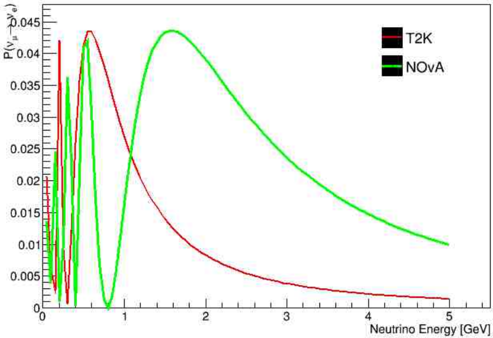
- By ignoring the Δ_{21} term and noting that $\Delta_{31} \approx \Delta_{32}$, we can make an approximation and figure out the dominating term is

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= \sin^2 2\theta_{13} s_{23}^2 \sin^2 \Delta_{31} \\ &= \sin^2 2\theta_{13} s_{23}^2 \sin^2 \frac{1.27 \Delta m_{31}^2 [eV^2] L[km]}{E[GeV]} \end{aligned} \quad (6)$$

Neutrino oscillation in vacuum

Transition probability $P(\nu_\mu \to \nu_\mu)$

- With : $\theta_{13} = 0.15, \theta_{23} = 0.785, \Delta m^2_{31} = 2.42 \times 10^{-3} eV^2$, the transition probability with respect to the energy in vacuum for T2K and NOvA.



Effective potential in matter

- The effective potential of neutrino interactons in matter is mainly from electron, proton and neutron. The one for proton and neutron cancels out each other, so

$$V_\alpha = V_{CC}\delta_{\alpha e} + V_{NC} = \sqrt{2}G_F(N_e\delta_{\alpha e} - \frac{1}{2}N_n) \tag{7}$$

- For antineutrino, V_{NC} is replaced by $-V_{NC}$

Effective potential in matter

- The Schrodinger equation for neutrino in matter

$$i\frac{d}{dt}\nu_\alpha = H\nu_\alpha \tag{8}$$

- Where

$$\nu_\alpha = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}; and$$

$$H = \left[\frac{1}{2E} U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

MSW effect

- Consider now $U = U_{PMNS} = U_{23}U_{13}U_{12}$.
- Subtract the diagonals of the mass matrix bi m_1^2 and take the limit $\Delta m_{21}^2 \to 0$
- Since M_m commute with U_{12}, we can write $M_m = U_{12}MU_{12}^\dagger$
- V_α commute with U_{23}, so we can define new basis $\nu_\alpha = U_{23}\nu_{\alpha m}$
- Then the Schrodinger equation becomes

$$i\frac{d}{dt}\begin{pmatrix}\nu_{em}\\ \nu_{\mu m}\\ \nu_{\tau m}\end{pmatrix} = \begin{pmatrix}\frac{\Delta m_{31}^2}{2E}s_{13}^2 + V_{CC} & 0 & \frac{\Delta m_{31}^2}{2E}s_{13}c_{13}\\ 0 & 0 & 0\\ \frac{\Delta m_{31}^2}{2E}s_{13}c_{13} & 0 & \frac{\Delta m_{31}^2}{2E}c_{13}^2\end{pmatrix}\begin{pmatrix}\nu_{em}\\ \nu_{\mu m}\\ \nu_{\tau m}\end{pmatrix} \quad (9)$$

The problem now reduces to the 2-flavor oscillation with $\nu_{em} = \nu_e, \nu_{\tau m} = s_{23}\nu_\mu + c_{23}\nu_\tau$.

$$i\frac{d}{dt}\begin{pmatrix}\nu_{em}\\ \nu_{\tau m}\end{pmatrix} = \begin{bmatrix}-\left(\frac{\Delta m_{31}^2}{4E}s_{13}^2 - \frac{V_{CC}}{2}\right) & \frac{\Delta m_{31}^2}{4E}\sin 2\theta_{13}\\ \frac{\Delta m_{31}^2}{4E}\sin 2\theta_{13} & \frac{\Delta m_{31}^2}{4E}\cos 2\theta_{13} - \frac{V_{CC}}{2}\end{bmatrix}\begin{pmatrix}\nu_{em}\\ \nu_{\tau m}\end{pmatrix}$$

The mixing matrix in matter U is of the form

$$U = \begin{pmatrix}\cos\theta_m & \sin\theta_m\\ -\sin\theta_m & \cos\theta_m\end{pmatrix}$$

Then

$$U^\dagger H_m U = \begin{pmatrix} -(A\cos 2\theta_m + C\sin 2\theta_m) & C\cos 2\theta_m - A\sin 2\theta_m \\ C\cos 2\theta_m - A\sin 2\theta_m & A\cos 2\theta_m + C\sin 2\theta_m \end{pmatrix}$$
$$\equiv \begin{pmatrix} -\Delta m_m^2 & 0 \\ 0 & \Delta m_m^2 \end{pmatrix} \qquad (11)$$

Where $A = \frac{\Delta m_{31}^2}{4E} s_{13}^2 - \frac{V_{CC}}{2}$ and $C = \frac{\Delta m_{31}^2}{4E} \sin 2\theta_{13}$
We now can derive

$$\tan 2\theta_m = \frac{C}{A} = \frac{\tan 2\theta_{13}}{1 - \frac{2EV_{CC}}{\Delta m_{31}^2 \cos 2\theta_{13}}} \qquad (12)$$

and

$$\Delta m_m^2 = \sqrt{A^2 + C^2} = \sqrt{\left(\frac{\Delta m^2}{4E}\cos 2\theta_{13} - \frac{V_{CC}}{2}\right)^2 + \left(\frac{\Delta m_{31}^2}{4E}\sin 2\theta_{13}\right)^2} \qquad (13)$$

The resonance occurs when $2EV_{CC} = \Delta m_{31}^2 \cos 2\theta_{13} \Rightarrow$

$$(N_e)_R = \frac{\Delta m_{31}^2 \cos 2\theta_{13}}{2\sqrt{2} G_F E}$$

The mixing now is maximal $\theta_m = \pi/4$ and Δm_m^2 gets minimun $(\Delta m_m^2)_{min} = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{4E}$.
In normal matter $(N_e)_R > 0$, therefore the resonance exists only if

$$\cos 2\theta_{13} > \quad \Leftrightarrow \quad \theta_{13} < \frac{\pi}{4}$$

for matter and

$$\theta_{13} > \frac{\pi}{4}$$

for antimatter.

General form of oscillation probability in matter

- We have

$$i\frac{d\nu}{dx} = H\nu = (H_0 + H_1)$$

- Where

$$H_0 = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger$$

$$H_1 = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} U^\dagger + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]$$

- and $a = 2EV_{CC} = 2\sqrt{2}G_F E N_e$.

General form of oscillation probability in matter

- Since Δm^2_{21} and $a \ll \Delta m^2_{31}$, we can treat H_1 as a pertubation.
- The Schrodinger equation has a solution of Dyson series form

$$\nu(x) = S(x)\nu(0) \tag{14}$$

- With

$$S(x) \equiv T e^{\int_0^x H(s)ds}$$

- T is the symbol of time ordering. The oscillation probability at distance L then can be calculate through $S(x)$

$$P(\nu_\alpha \to \nu_\beta) = |S_{\beta\alpha}(L)|^2 \tag{15}$$

General form of oscillation probability in matter

- We can calculate the pertubation to the first order in a and Δm^2_{21}.
- We have

$$(S(x)))_{\beta\alpha} = (S_0(x))_{\beta\alpha} + (S_1(x))_{\beta\alpha} \tag{16}$$

- With

$$(S_0(x))_{\beta\alpha} = e^{-iH_0x} = \delta_{\alpha\beta} + U_{\beta 3}U^*_{\alpha 3}\left(e^{-i\frac{\Delta m^2_{31}x}{2E}} - 1\right) \tag{17}$$

General form of oscillation probability in matter

- And

$$(S_1(x))_{\beta\alpha} = e^{-iH_0x}(-i)\int_0^x dsH_1(s) = -i\frac{ax}{2E}e^{-i\frac{\Delta m_{31}^2 x}{2E}}U_{\beta 3}U_{\alpha 3}^*|U_{13}|^2$$
$$-i\frac{x}{2E}\left[\Delta m_{21}^2 U_{\beta 2}U_{\alpha 2}^* + a(\delta_{\alpha 1}\delta_{\beta 1} + U_{\beta 3}U_{\alpha 3}^*(|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}))\right]$$
$$-\frac{a}{\Delta m_{31}^2}\left(e^{-i\frac{\Delta m_{31}^2 x}{2E}} - 1\right)(2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1})U_{\beta 3}U_{\alpha 3}^* \quad (18)$$

General form of oscillation probability in matter

- From (18) and (17) we get

$$(S(x))_{\beta\alpha} = (S_0(x))_{\beta\alpha} + (S_1(x))_{\beta\alpha} \tag{19}$$

$$= \delta_{\alpha\beta} - i2e^{-i\frac{\Delta m_{31}^2 x}{4E}} \sin\frac{\Delta m_{31}^2 x}{4E} U_{\beta 3} U_{\alpha 3}^* \times$$

$$\left[1 - \frac{a}{\Delta m_{31}^2}(2|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}) - \frac{iax}{2E}|U_{13}|^2\right]$$

$$-i\frac{\Delta m_{31}^2 x}{2E} \times$$

$$\left[\frac{\Delta m_{21}^2}{\Delta m_{31}^2} U_{\beta 2} U_{\alpha 2}^* + \frac{a}{\Delta m_{31}^2}(\delta_{\alpha 1}\delta_{\beta 1} + U_{\beta 3} U_{\alpha 3}^*(|U_{13}|^2 - \delta_{\alpha 1} - \delta_{\beta 1}))\right]$$

Next 2 weeks plan

First week

- Complete the general formula of oscillation probability in matter, particularly for $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\mu \to \nu_e)$ (within 2 or 3 days)
- Try to understand and compile Prob++ packet (next 2 days)
- Continue to learn ROOT and Statistics (in extra-time)

Next 2 weeks plan

Second week

- Continue to learn ROOT and Statistics (main activity)
- Try to make similar plots as in Small project and in Prof. Nakaya slides.
- Learn about T2K and NOvA experiments (in extra-time)

References

- E. Kh. Akhmedov (2000), [arXiv: hep-ex/0001.1264v2].
- Fumihiko Suekane (2015), *Neutrino Oscillations*, Springer.
- Jeff Asaf Dror (2014), *Neutrino lecture notes*.
- Mark Thomson (2013), *Modern Particle Physics*, Cambridge University.