Study Status

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PMNS matrix

Neutrino mixing is governed by the Pontecorvo - Maki - Nakagawa - Sakata(PMNS) mixing matrix which relates the mass eigenstates to the flavor eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

We have

$$U^\dagger U = \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Similarly, $UU^\dagger = 1$

$$U^\dagger U = UU^\dagger = 1 \tag{1}$$

$$U_{ij}^* U_{jk} = U_{ij} U_{jk}^* = \delta_{ik} \tag{2}$$

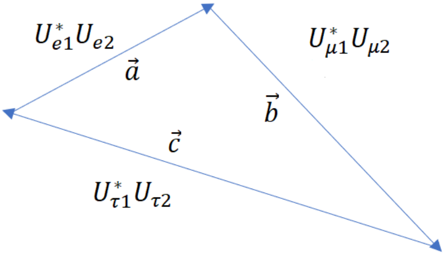
$\rightarrow$ The PMNS mixing matrix is unitary

Unitary triangle

Unitary requires:

$$
\begin{aligned}
& U_{e1}^* U_{e2} + U_{\mu 1}^* U_{\mu 2} + U_{\tau 1}^* U_{\tau 2} = 0 \\
\rightarrow \quad & \vec{a} \quad + \quad \vec{b} \quad + \quad \vec{c} \qquad = 0
\end{aligned} \tag{3}
$$

Such three vectors define triangle in two dimension coordinate



Eq.(2) gives the six unitary triangles, it means these six unitary triangles have same area.
The area of triangle:

$$\begin{aligned} S &= Im\frac{1}{2}[ac^*] = Im\frac{1}{2}[ba^*] = Im\frac{1}{2}[cb^*] \\ &= \frac{1}{2}Im[U_{e1}^* U_{e2} U_{\tau 1} U_{\tau 2}^*] \\ &= \frac{1}{2}Im[U_{\mu 1}^* U_{\mu 2} U_{e1} U_{e2}^*] \\ &= \frac{1}{2}Im[U_{\tau 1}^* U_{\tau 2} U_{\mu 1} U_{\mu 2}^*] \end{aligned} \tag{4}$$

Jarlskog invariant:

$$J = Im\left[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right] \qquad (\alpha \neq \beta, i \neq j) \tag{5}$$

From Eq.(5) and Eq.(4)

$$J = 2S \tag{6}$$

$\rightarrow$ The area of unitary triangles equal to a half of Jarlskog's invariant.