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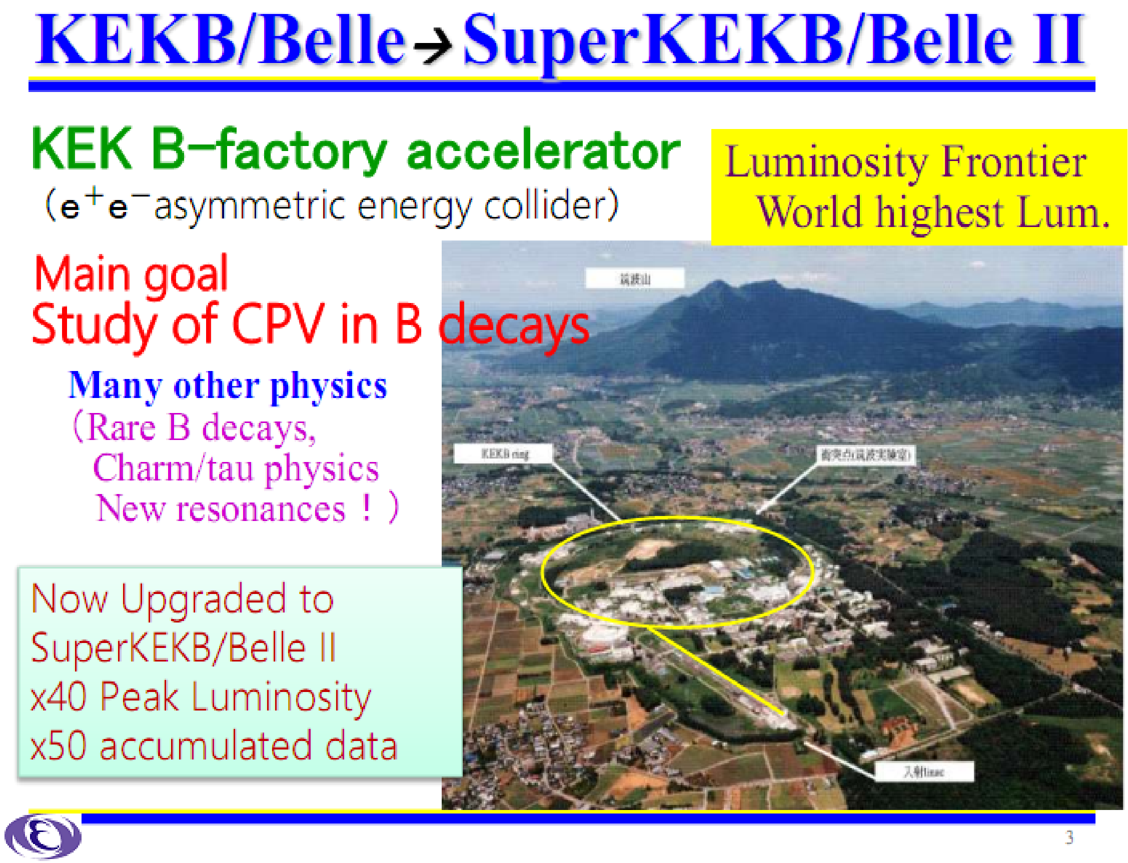
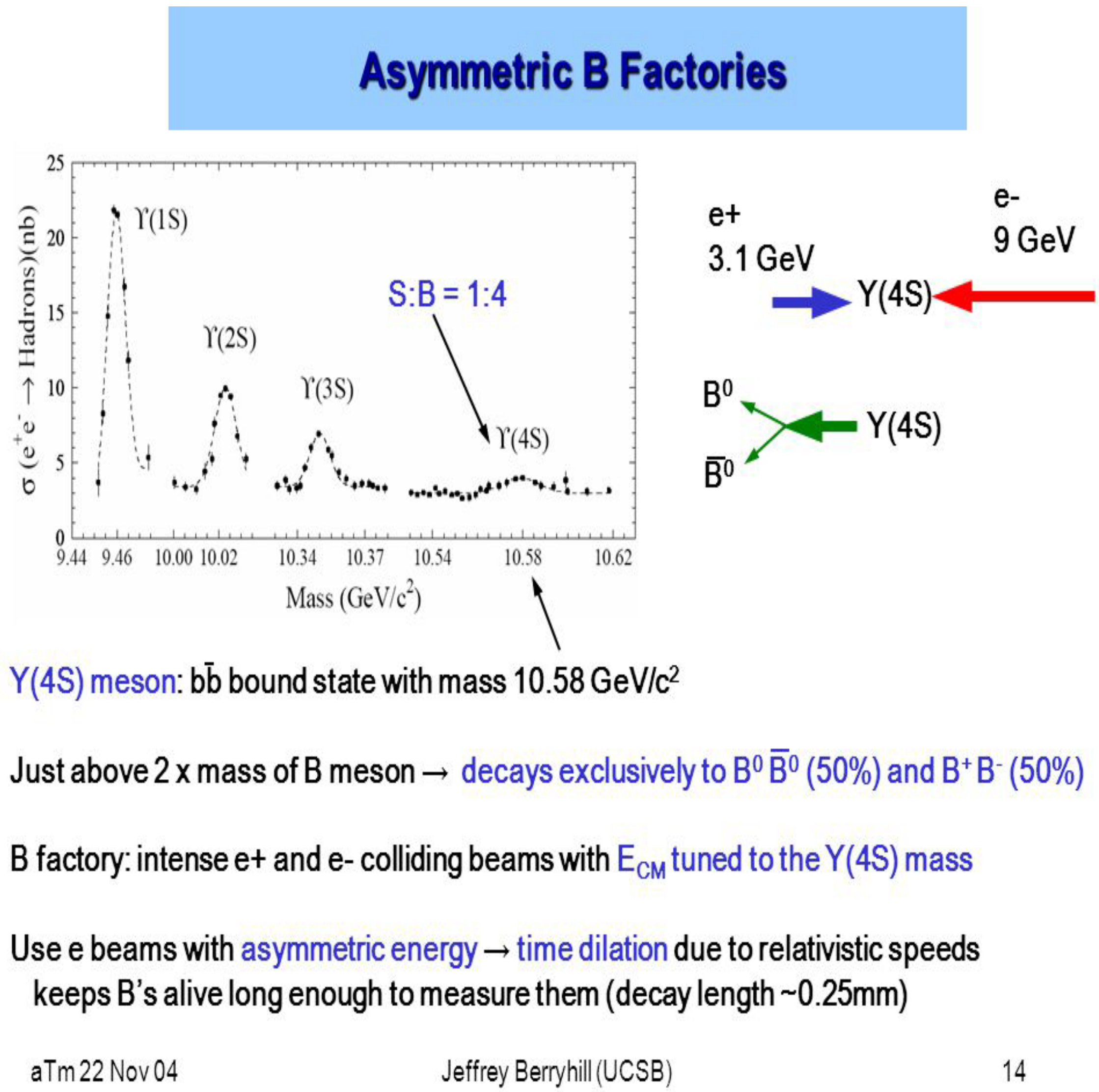
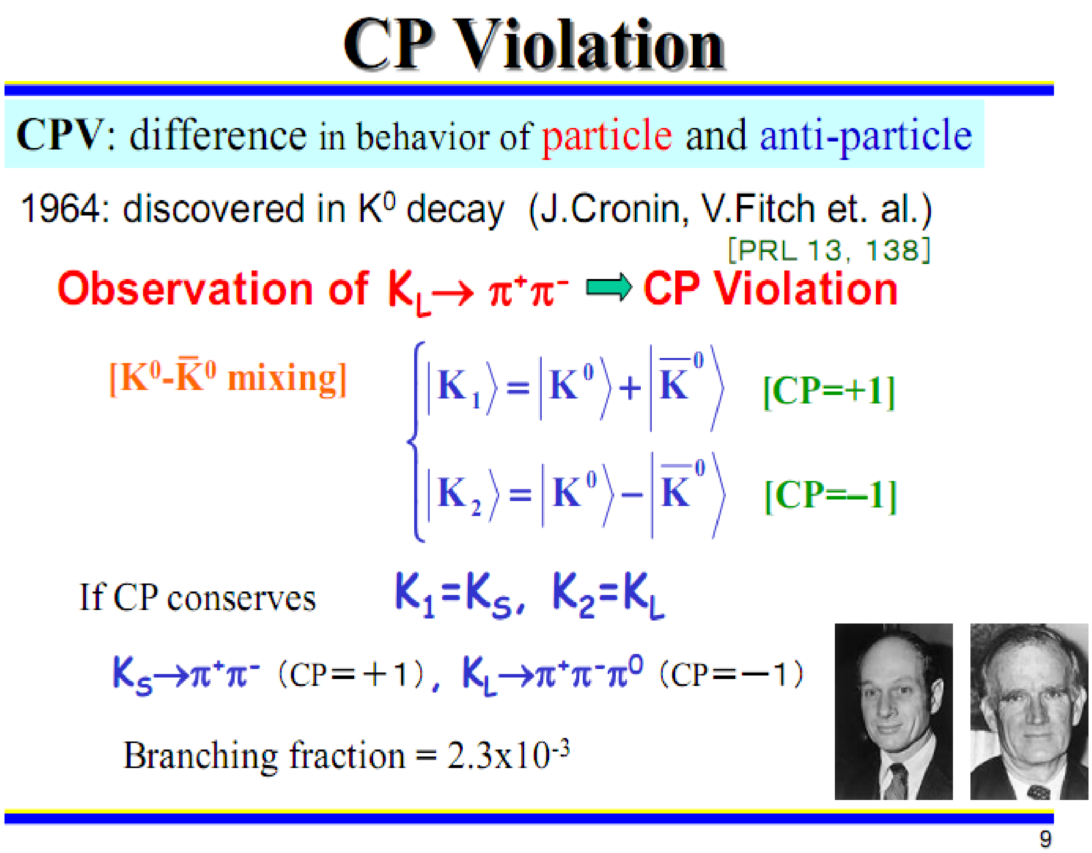
Belle II and B physics

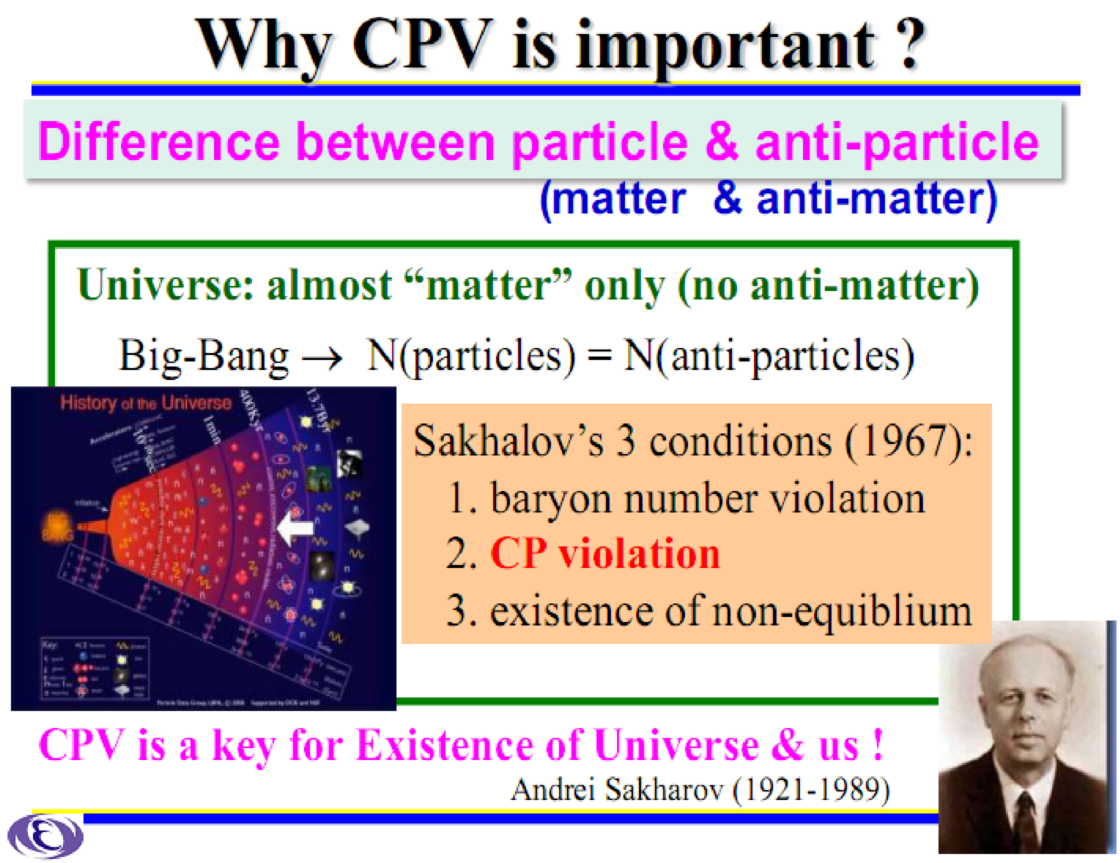


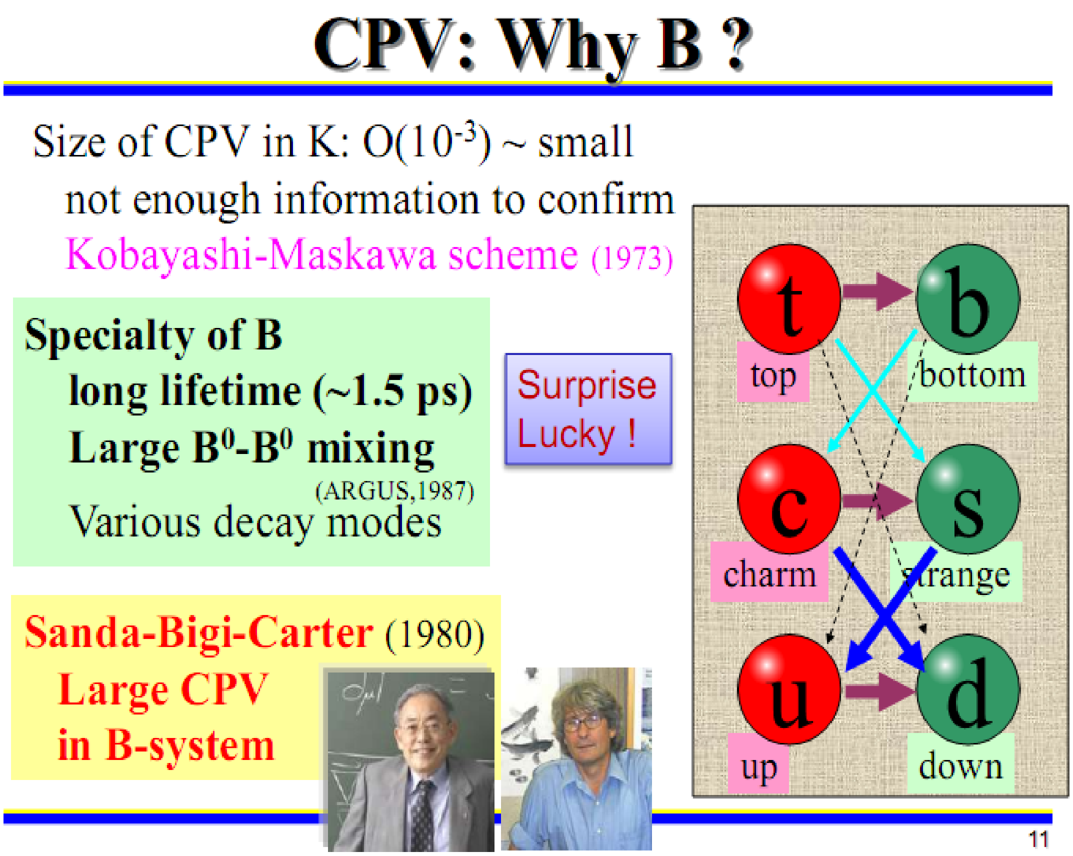
Belle II and B physics




CP Violation
CPV: difference in behavior of particle and anti-particle
1964: discovered in K^0 decay (J.Cronin, V.Fitch et. al.)
[PRL 13, 138]
Observation of $K_L \to \pi^+\pi^-$ ⇒ CP Violation
[K^0-$\bar{K}^0$ mixing]
$|K_1\rangle = |K^0\rangle + |\bar{K}^0\rangle$ [CP=+1]
$|K_2\rangle = |K^0\rangle - |\bar{K}^0\rangle$ [CP=−1]
If CP conserves $K_1 = K_S$, $K_2 = K_L$
$K_S \to \pi^+\pi^-$ (CP=+1), $K_L \to \pi^+\pi^-\pi^0$ (CP=−1)
Branching fraction = 2.3x10^{-3}
9

Search for CPV at Belle II




CPV: Why B ?
Size of CPV in K: O(10^-3) ~ small
not enough information to confirm
Kobayashi-Maskawa scheme (1973)
Specialty of B
long lifetime (~1.5 ps)
Large B0-B0 mixing
(ARGUS,1987)
Various decay modes
Surprise
Lucky !
Sanda-Bigi-Carter (1980)
Large CPV
in B-system
t
top
b
bottom
c
charm
s
strange
u
up
d
down
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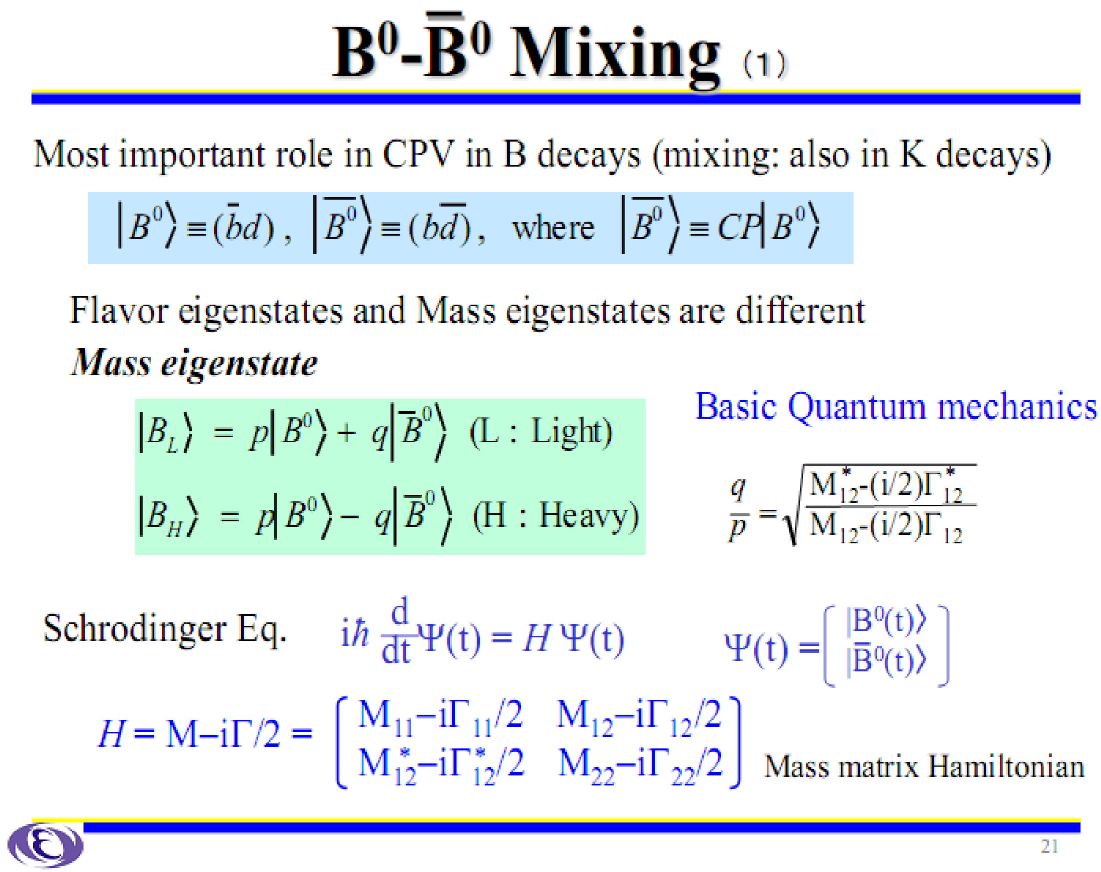
Search for CPV at Belle II

- $B^o - \bar{B}^o$ mixing

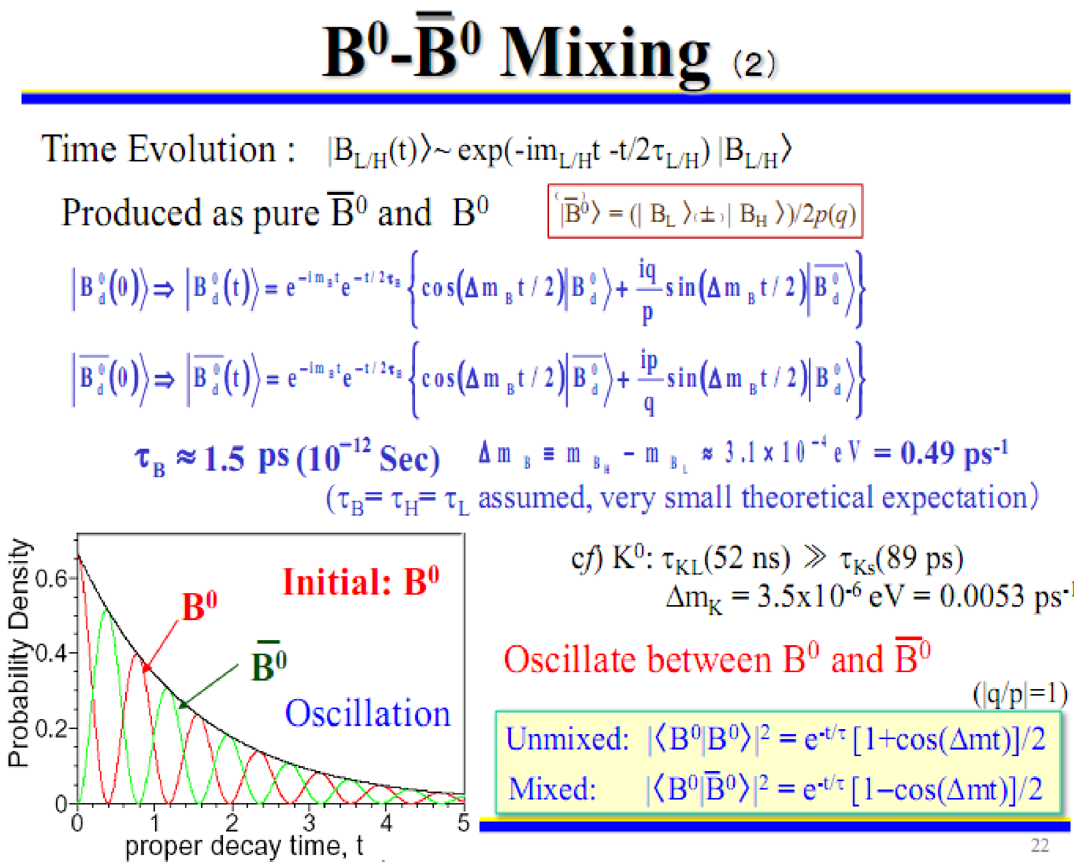


$$p = \frac{1}{\sqrt{2}} \frac{1+\epsilon_B}{\sqrt{1+|\epsilon_B|^2}}, \quad q = \frac{1}{\sqrt{2}} \frac{1-\epsilon_B}{\sqrt{1+|\epsilon_B|^2}}$$

CP is violated if $\epsilon_B \neq 0 \Leftrightarrow |q/p \neq 1|$

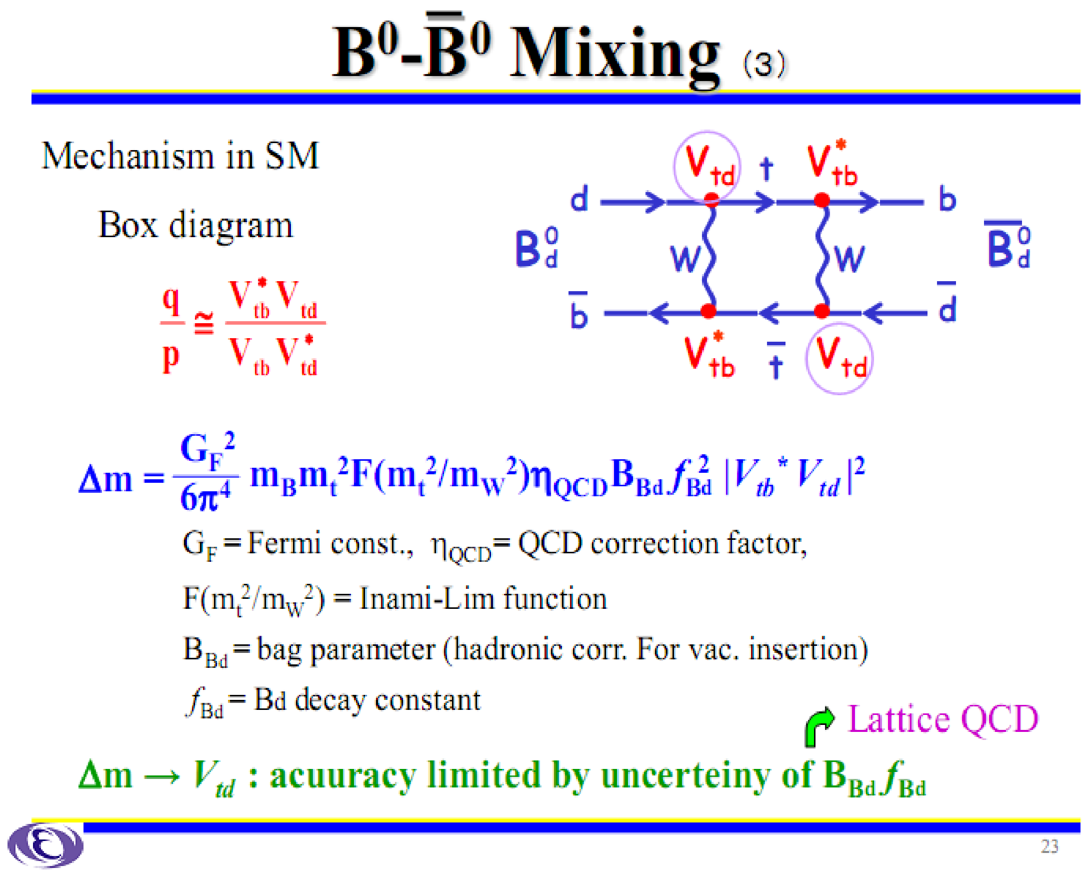
Search for CPV at Belle II

- $B^o - \bar{B}^o$ mixing



Search for CPV at Belle II

- $B^o - \bar{B}^o$ mixing



Statistics and the treatment of experimental data

0.1 Characteristics of probability distribution

Random processes are described by a probability density function (PDF).
PDF gives expected frequency of occurence.
Random variable x can be continuous or discrete.

0.1.1 Cumulative distributions

Probability of finding x with $x_1 \leq x \leq x_2$ for x to be continuous

$$P(x) = \int_{x_1}^{x_2} P(x)dx \tag{1}$$

And for x to be discrete

$$P(x) = \sum_{i=1}^{2} P(x_i) \tag{2}$$

The renormaliation condition for x to be continuous

$$\int P(x)dx = 1 \tag{3}$$

And for x to be discrete

$$\sum_i P(x_i) = 1 \tag{4}$$

0.1.2 Expectation values

For x to be continuous

$$E[x] = \int xP(x)dx \tag{5}$$

And for x to be discrete

$$E[x] = \sum_i x_i P(x_i) \tag{6}$$

If $f(x)$ is a continuous function of x then the expectation value of $f(x)$ is

$$E[f(x)] = \int f(x)P(x)dx \tag{7}$$

0.1.3 Distribution moments. Mean and variance

The n^{th} moment of x about some point x_0 is defined as the expectation value of $(x - x_0)^n$.

Only two first moments are important.

First moment about zero is called ***mean*** or ***average*** of x

$$\mu = E[x] = \int xP(x)dx \tag{8}$$

The second central moment is call ***variance***

$$\sigma^2 = E[(x-\mu)^2] = \int (x-\mu)^2 P(x)dx \tag{9}$$

The square root of variance is call ***standard deviation***

0.1.4 The covariance

The covariance measures linear correlation between two variables

$$cov(x, y) = E[(x - \mu_x)(y - \mu_y)] \tag{10}$$

- μ_x: mean of x.
- μ_y: mean of y.

Correlation coeficient

$$\rho = \frac{cov(x, y)}{\sigma_x . \sigma_y} \tag{11}$$

- $-1 \leq \rho \leq 1$.
- $|\rho| = 1$: perfectly correlated linear.
- $\rho = 0$: x and y are linear independent.

0.2 Some common probability distributions

0.2.1 The binomial distribution

The binomial distribution involves repeated, independent trials, which outcome of a single trial is dichotomous.

The probability of n dichotomous trials, for example *successe* and *failure* is given by

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \tag{12}$$

Where x is number of *successes* (or *failures*), p is probability of success in a single trial.

- x events occur with probability p each.
- $n-x$ events occur with probability $1-p$ each.
- Note that $P(x)$ is the x^{th} term of binomial expansion

$$(a+b)^n = \sum_{k=0}^{n} C_k^n a^{n-k} b^k \tag{13}$$

Mean of binomial distribution is defined as the following

$$\mu = \sum_{x=0} xP(x) = \sum_{x=1} \frac{n!}{(n-x)!(x-1)!} p^x (1-p)^{n-x}$$

Let $N = n-1$ and $y = x-1$, we can rewrite the above equation as

$$\begin{aligned}
\mu &= \sum_{x=1}^{n} \frac{n!}{(n-x)!(x-1)!} p^x (1-p)^{n-x} = \sum_{y=0}^{N} \frac{(N+1)!}{(N-y)!(y)!} p^{y+1} (1-p)^{N-y} \\
&= (N+1)p \sum_{y=0}^{N} \frac{(N)!}{(N-y)!(y)!} p^y (1-p)^{N-y} = (N+1)p[p+(1-p)]^N \\
&= (N+1)p
\end{aligned}$$

Or

$$\mu = np \tag{14}$$

Variance of binomial distribution is defined as the following

#1 : By definition

$$\begin{aligned}
\sigma^2 &= \sum_{x=0}^{n} (x-\mu)^2 P(x) = \sum_{x=0}^{n} (x-\mu)^2 \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \\
&= \sum_{x=0}^{n} (x-np)^2 \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \\
&= A+B+C
\end{aligned}$$

Where

$$\begin{aligned}
A &= \sum_{x=0}^{n} x^2 \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} = \sum_{x=1}^{n} \frac{xn!}{(n-x)!(x-1)!} p^x (1-p)^{n-x} \\
&= \sum_{y=0}^{N} \frac{(y+1)(N+1)!}{(N-y)!y!} p^{(y+1)} (1-p)^{N-y} = np + n(n-1)p^2
\end{aligned}$$

$$B = -2np \sum_{x=0}^{n} x \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} = -2(np)^2$$

And

$$C = \sum_{x=0}^{n} (np)^2 \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} = (np)^2$$

Hence

$$\sigma^2 = \sum_x (x-\mu)^2 P(x) = np(1-p) \tag{15}$$

#2 : By using $\sigma^2 = \mu(x^2) - [\mu(x)]^2$

Where $\mu(x^2) = A = np + n(n-1)p^2$ and $[\mu(x)]^2 = (np)^2$. We get the similar result as (15)

$$\sigma^2 = \mu(x^2) - [\mu(x)]^2 = np(1-p)$$

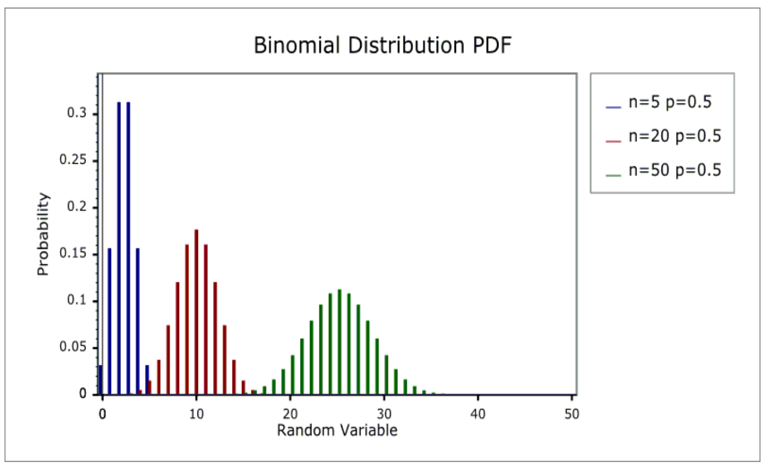


Figure 1: Binomial distribution for several values of n and p

0.2.2 The Poisson distribution

Poisson distribution is the case where we take the limits $p \to 0$ and $n \to \infty$ from binomial distribution such that $np = \mu = const$.

Poisson distribution is an appropriate model if the following asssumptions are hold:

- x can take values: $0, 1, 2, ...$
- Events occur independently.
- The rate at which events occur is constant.
- Two events cannot occur at exactly the same instant.
- Probability of an event in a small sub-interval is proportional to the length of the sub-interval.

Let $p = \frac{\mu}{n}$. When $n \to \infty$, we have following approximations

$$\frac{n!}{(n-x)!} = n(n-1)(n-2)...(n-x-2)(n-x-1) \approx n^x$$

And

$$(1-p)^{n-x} = \left(1 - \frac{\mu}{n}\right)^{-x} \left(1 - \frac{\mu}{n}\right)^{n} \approx e^{-\mu}$$

Note that: $lim_{n\to\infty} \left(1 - \frac{\mu}{n}\right)^n \approx e^{-\mu}$

Then the Poisson distribution is defined as

$$P(x) = lim_{n\to\infty} \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} = \frac{\mu^x}{x!} e^{-\mu} \tag{16}$$

Mean of Poisson distribution

$$\mu_P = \sum_{x=0} x \frac{\mu^x e^{-\mu}}{x!} = \sum_{x=1} \frac{\mu^x e^{-\mu}}{(x-1)!} = \mu e^{-\mu} \sum_{x=0} \frac{\mu^x}{x!} = \mu \tag{17}$$

Variance of Poisson distribution

$$\begin{aligned} \sigma_P^2 &= \mu(x^2) - [\mu(x)]^2 = \sum_{x=0} x^2 \frac{\mu^x e^{-\mu}}{x!} - \mu^2 \\ &= \sum_{x=1} x \frac{\mu^x e^{-\mu}}{(x-1)!} - \mu^2 = \sum_{x=0} \frac{(x+1)\mu^{x+1}}{x!} e^{-\mu} - \mu^2 = \mu^2 + \mu - \mu^2 \\ &= \mu \end{aligned} \tag{18}$$

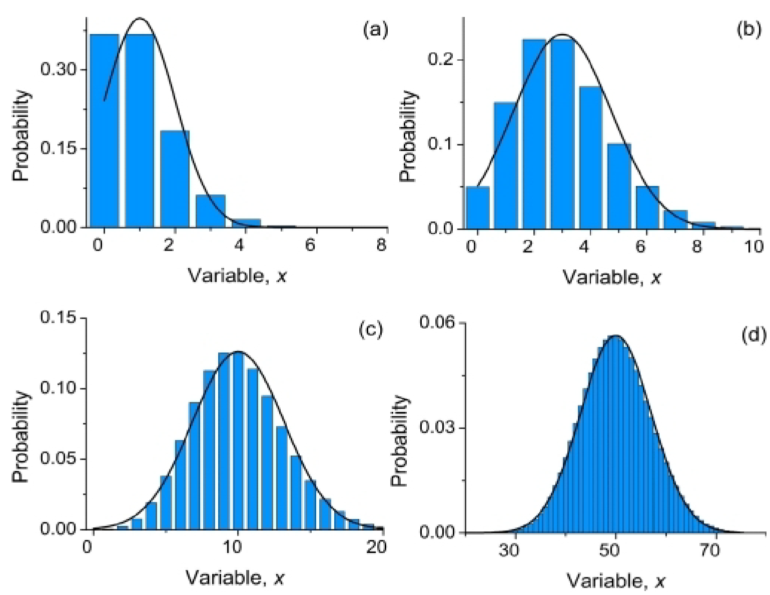


Figure 2: Poisson distribution for several values of μ

0.2.3 The Gaussian distribution

Gaussian is a continuous, symmetric distribution whose density is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{19}$$

Where μ is ***expectation value*** and σ^2 is ***variance***.

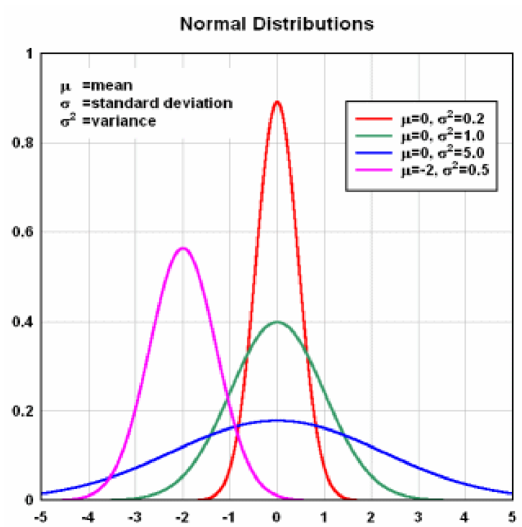


Figure 3: Gaussian distribution for several values of μ and σ^2

• ***Derive Gaussian distribution from binomial distribution***

From the binomial distribution

$$P(x) = \frac{n!}{(n-x)!x!} p^x(1-p)^{n-x}$$

By using Stirling's formula $n! = n^n e^{-n}\sqrt{2\pi n}$, we can rewrite the above equation as

$$\begin{aligned} P(x) &= \frac{n^n e^{-n}\sqrt{2\pi n}}{x^x e^{-x}\sqrt{2\pi x}(n-x)^{n-x}e^{-(n-x)}\sqrt{2\pi(n-x)}} p^x(1-p)^{n-x} \\ &= \left(\frac{np}{x}\right)^x \left(\frac{n(1-p)}{n-x}\right)^{n-x} \sqrt{\frac{n}{2\pi x(n-x)}} \end{aligned}$$

We have

$$\begin{aligned} L &= \ln\left[\left(\frac{np}{x}\right)^x \left(\frac{n(1-p)}{n-x}\right)^{n-x}\right] = -x\ln\left(\frac{x}{np}\right) - (n-x)\ln\left(\frac{n-x}{n(1-p)}\right) \\ &= -x\ln\left(\frac{x}{np}\right) - (n-x)\ln\left(1 + \frac{np-x}{n(1-p)}\right) \end{aligned}$$

Let $\lambda = -(np-x) \quad \Rightarrow \quad x = \lambda + np$.

By using $\ln(1+x) \approx x - \frac{1}{2}x^2 + ...$, we get

$$\begin{aligned}
L &= -(\lambda + np)\ln\left(1 + \frac{\lambda}{np}\right) - (n(1-p) - \lambda)\ln\left(1 - \frac{\lambda}{n(1-p)}\right) \\
&= -(\lambda + np)\left(\frac{\lambda}{np} - \frac{1}{2}\frac{\lambda^2}{(np)^2} + ...\right) - (n(1-p) - \lambda)\left(-\frac{\lambda}{n(1-p)} - \frac{1}{2}\frac{\lambda^2}{n^2(1-p)^2} + ...\right) \\
&= -\left(\frac{\lambda^2}{np} + \lambda - \frac{1}{2}\frac{\lambda^2}{np} + ...\right) - \left(-\lambda + \frac{\lambda^2}{n(1-p)} - \frac{1}{2}\frac{\lambda^2}{n(1-p)} + ...\right) \\
&\approx -\lambda - \frac{\lambda^2}{2np} + \lambda - \frac{\lambda^2}{2n(1-p)} \\
&= -\frac{\lambda}{2np(1-p)}
\end{aligned}$$

Then

$$\left(\frac{np}{x}\right)^x \left(\frac{n(1-p)}{n-x}\right)^{n-x} = e^{-\frac{\lambda^2}{2np(1-p)}}$$

And for $n \to \infty$

$$\begin{aligned}
\sqrt{\frac{n}{2\pi x(n-x)}} &= \sqrt{\frac{n}{2\pi(\lambda + np)(n(1-p) - \lambda)}} \\
&\approx \frac{1}{\sqrt{2\pi np(1-p)}}
\end{aligned}$$

Therefore we finally get

$$P(x) = \frac{1}{\sqrt{2\pi np(1-p)}} e^{-\frac{\lambda^2}{2np(1-p)}}$$

Note that $\mu = np$ and $\sigma^2 = np(1-p)$. The Gaussian distribution is then defined as

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{20}$$

• ***Derive Gaussian distribution from Poisson distribution***

The Poisson distribution is of the form

$$P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

Use the Stirling's formula $x! = x^x e^{-x}\sqrt{2\pi x}$ and let $x = \mu(1+\lambda)$, where $\mu \gg 1$ and $\lambda \ll 1$

$$\begin{aligned}
P(x) &= \frac{\mu^x e^{-\mu}}{x^x e^{-x}\sqrt{2\pi x}} = \left(\frac{\mu}{x}\right)^x \frac{e^{-(\mu - x)}}{\sqrt{2\pi x}} \\
&= \left(\frac{\mu}{\mu(1+\lambda)}\right)^{\mu(1+\lambda)} \frac{e^{-(\mu - \mu(1+\lambda))}}{\sqrt{2\pi\mu(1+\lambda)}} = \frac{1}{\sqrt{2\pi\mu}} \frac{e^{-\mu\lambda}}{(1+\lambda)^{\mu(1+\lambda)} + 1/2}
\end{aligned}$$

We see that for $\mu \gg 1$ and $\lambda \ll 1$

$$\begin{aligned}
\ln[(1+\lambda)^{\mu(1+\lambda)} + 1/2] &= (\mu + \mu\lambda + 1/2)\ln(1+\lambda) = (\mu + \mu\lambda + 1/2)(\lambda - \lambda^2/2 + ...) \\
&\approx \mu\lambda + \frac{\mu\lambda^2}{2} \\
\Rightarrow (1+\lambda)^{\mu(1+\lambda)} + 1/2 &\approx e^{\mu\lambda + \frac{\mu\lambda^2}{2}}
\end{aligned}$$

It's then followed that

$$P(x) = \frac{1}{\sqrt{2\pi\mu}} e^{-\frac{\mu\lambda^2}{2}} = \frac{1}{\sqrt{2\pi\mu}} e^{-\frac{(x-\mu)^2}{2\mu}}$$

Note that, for Poisson distribution $\sigma^2 = \mu$. By substituting this into the above equation, we get exactly the same formula as (20).

• ***Derive Gaussian distribution from another way***

Consider we are aiming at the origin of a xy_plane with darts. Assume that:

+1 : Deviation of darts not depend on the origin.

+2 : Deviation in orthogonal directions are independent.

+3 : Large deviation is less likely than small deviation.

Probability that the darts falls in interval $[x, x + \Delta x]$ is

$$P(x)dx$$

Similarly for interval $[y, y + \Delta y]$

Probability of falling in area dA is

$$P(x)P(y)\Delta x \Delta y$$

If we are aiming offset of the origin by a constant μ, then

$$P(x, y) = P(x + \mu)P(y + \mu)\Delta x \Delta y = g(x, y)\Delta x \Delta y$$

Where $g(x, y) = P(x + \mu)P(y + \mu)$.

In term of polar coordiantes where $x = r\cos\theta, y = r\sin\theta$, $g(r, \theta)$ is dependent on r, but not dependent on θ. Then

$$\begin{aligned}
& \frac{dg}{d\theta} = 0 \Rightarrow \\
& P(x+\mu)P'(y+\mu)\frac{dy}{d\theta} + P'(x+\mu)P(y+\mu)\frac{dx}{d\theta} = 0 \\
\Leftrightarrow\ & P(x+\mu)P'(y+\mu)r\cos\theta - P'(x+\mu)P(y+\mu)r\sin\theta = 0 \\
\Leftrightarrow\ & P(x+\mu)P'(y+\mu)x - P'(x+\mu)P(y+\mu)y = 0 \\
\Rightarrow\ & \frac{P'(x+\mu)}{xP(x+\mu)} = \frac{P'(y+\mu)}{yP(y+\mu)} = C; \quad \forall x, y \in R
\end{aligned}$$

We solve for $P(x+\mu)$

$$\frac{P'(x+\mu)}{xP(x+\mu)} = C \quad \Rightarrow \quad P(x+\mu) = Ae^{\frac{Cx^2}{2}}$$
$$\Rightarrow \quad P(x) = P((x-\mu)+\mu) = Ae^{\frac{C(x-\mu)^2}{2}}$$

From assumption +3, we see that C must be negative then

$$P(x) = Ae^{\frac{-C(x-\mu)^2}{2}}; \quad C > 0$$

Normalized condition

$$\int_{-\infty}^{+\infty} Ae^{\frac{-C(x-\mu)^2}{2}} dx = 1 \quad \Rightarrow \int_{-\infty}^{+\infty} e^{\frac{-C(x-\mu)^2}{2}} dx = \frac{1}{A}$$

Use the Gaussian integral $\int_{-\infty}^{+\infty} e^{-a(x+b)^2} dx = \sqrt{\frac{\pi}{a}} \quad \Rightarrow$

$$\int_{-\infty}^{+\infty} e^{\frac{-C(x-\mu)^2}{2}} dx = \frac{\frac{2\pi}{C}}{=}\frac{1}{A}$$
$$\Rightarrow \quad A = \sqrt{\frac{C}{2\pi}}$$

Then

$$P(x) = \sqrt{\frac{C}{2\pi}} e^{\frac{-C}{2}(x-\mu)^2}$$

Let μ and σ^2 are the mean and the variance of the distribution, respectively. The variance is

$$\sigma^2 = \mu(x^2) - [\mu(x)]^2$$

Where

$$\begin{aligned}
\mu(x^2) &= \sqrt{\frac{C}{2\pi}} \int_{-\infty}^{+\infty} x^2 e^{\frac{-C}{2}(x-\mu)^2} dx \\
&= \sqrt{\frac{C}{2\pi}} \int_{-\infty}^{+\infty} (x+\mu)^2 e^{\frac{-C}{2}x^2} dx \\
&= \sqrt{\frac{C}{2\pi}} \int_{-\infty}^{+\infty} x^2 e^{\frac{-C}{2}x^2} dx + \sqrt{\frac{C}{2\pi}} \int_{-\infty}^{+\infty} 2\mu x e^{\frac{-C}{2}x^2} dx + \sqrt{\frac{C}{2\pi}} \int_{-\infty}^{+\infty} \mu^2 e^{\frac{-C}{2}x^2} dx \\
&= \sqrt{\frac{C}{2\pi}} \sqrt{\frac{2\pi}{C}} \frac{1}{C} + \sqrt{\frac{C}{2\pi}}.0 + \sqrt{\frac{C}{2\pi}} \sqrt{\frac{2\pi}{C}} \mu^2 \\
&= \frac{1}{C} + \mu^2
\end{aligned}$$

Then $\sigma^2 = \frac{1}{C}$, and therefore we find exactly the same form as (20)

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$